

# Tutorial 4

## Solving navigation equations: Least squares and Kalman filter

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# Navigation equations system and Least Squares solution

## Exercise 1:

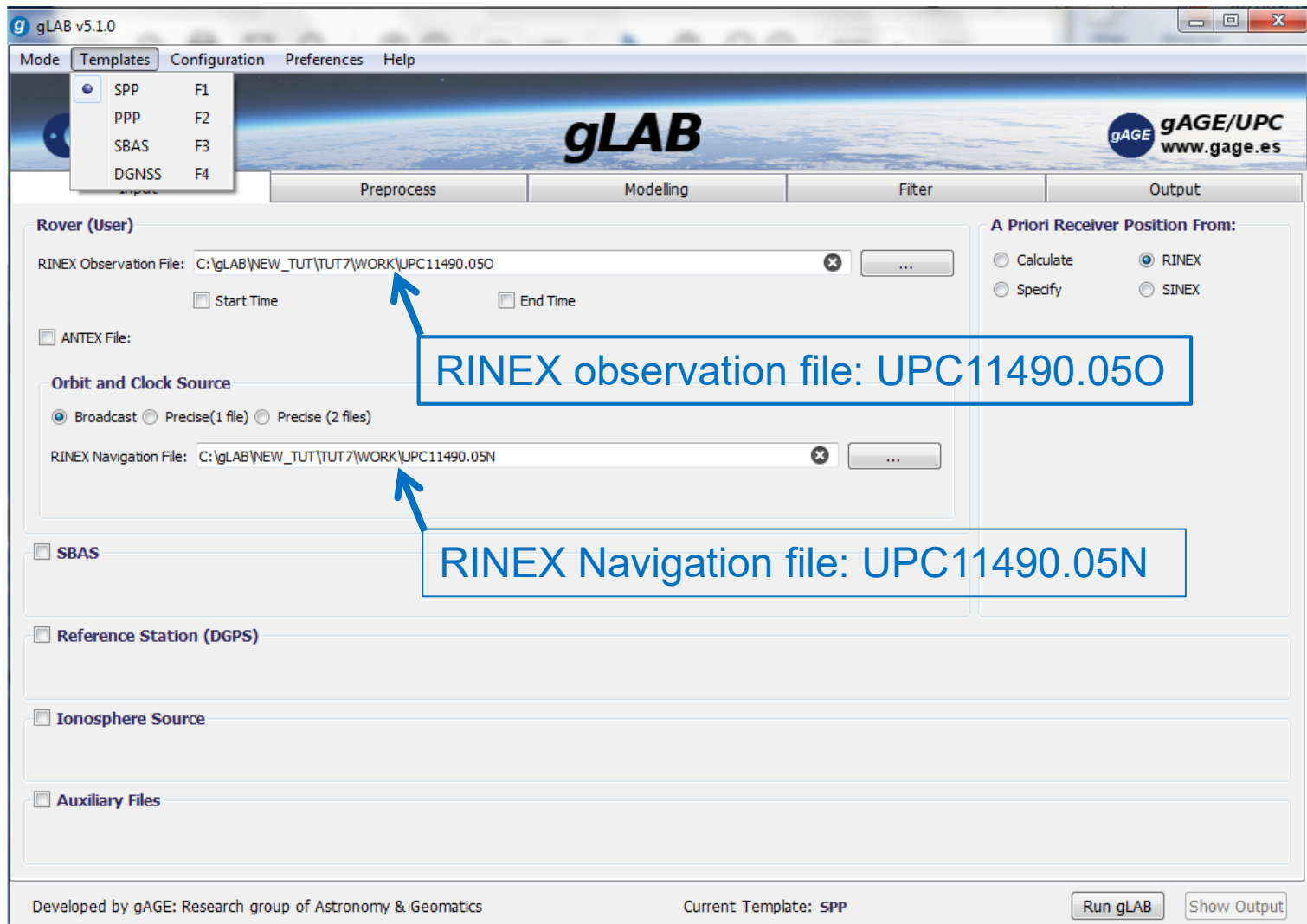
The measurement file **UPC11490.05O** has been collected by a receiver with fixed coordinates. Using navigation file **UPC11490.05N**, compute the **SPP solution** in static mode and check *by hand* the computation of the **Least Square solution** at time **t = 300** seconds.

Complete the next steps:

- 1.- Use gLAB to compute the prefit-residuals, elevation and azimuth of all satellites at time t=300 sec.
- 2.- Build up the navigation equations system to compute the solution in (ENU) coordinates. Solve the system with MATLAB (octave).

$$\begin{bmatrix} Prefit^1 \\ Prefit^2 \\ ..... \\ Prefit^n \end{bmatrix} = \begin{bmatrix} -\cos el^1 \sin az^1 & -\cos el^1 \cos az^1 & -\sin el^1 & 1 \\ -\cos el^2 \sin az^2 & -\cos el^2 \cos az^2 & -\sin el^2 & 1 \\ ..... \\ -\cos el^n \sin az^n & -\cos el^n \cos az^n & -\sin el^n & 1 \end{bmatrix} \begin{bmatrix} \Delta e_{rec} \\ \Delta n_{rec} \\ \Delta u_{rec} \\ c dt_{rec} \end{bmatrix}$$

# 1.- Process the data files in de default SPP mode:



In the **Output** section, unselect all messages except “print PREFIT”

The screenshot shows the gLAB v5.1.0 software interface. The top menu bar includes Mode, Templates, Configuration, Preferences, and Help. The main window has a header with the ESA logo, gLAB logo, and gAGE/UPC logo with the website www.gage.es. Below the header is a tabbed interface with tabs for Input, Preprocess, Modelling, Filter, and Output. The Output tab is selected and highlighted with a blue box. In the Output section, the 'Output Destination' area shows 'Output File: C:\gLAB\win\gLAB.out' and checkboxes for KML File, KML0 File, and SP3 File. The 'Common Navigation Messages' section has two buttons, 'All' and 'None', and a list of messages with checkboxes. The 'Print PREFIT' checkbox is checked and highlighted with a blue box. A blue box also highlights the 'PREFIT messages' section, which contains a list of 16 fields and a sample output. The 'Run gLAB' button is highlighted with a red box. The 'Show Output' button is also visible.

**Output Destination**

Output File: C:\gLAB\win\gLAB.out

☐ KML File:

☐ KML0 File:

☐ SP3 File:

**Common Navigation Messages**

All None

☐ Print INFO ☒ Print PREFIT

☐ Print CS (Cycle-slip) ☐ Print POSTFIT

☐ Print INPUT ☐ Print SATSEL

☐ Print MEAS ☐ Print FILTER

☐ Print MODEL ☐ Print OUTPUT

☐ Print EPOCHSAT ☐ Print USERADDEDERROR

**PREFIT messages**

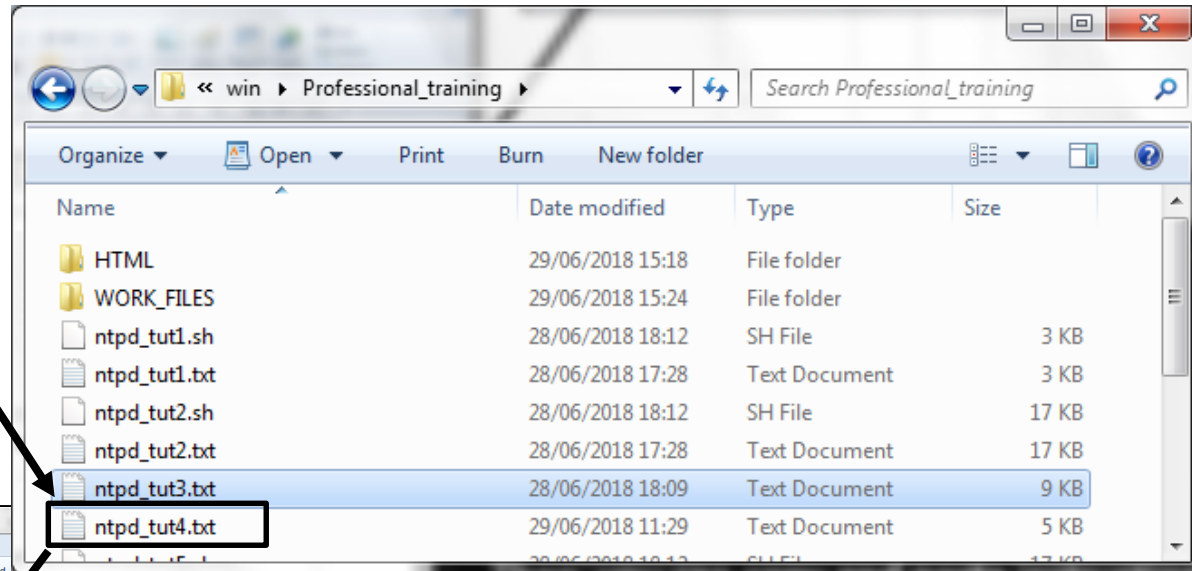
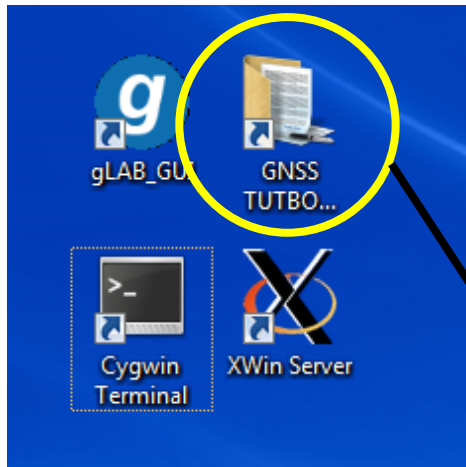
Prefilter values message. It provides the measurement-model values. It is shown in each filter execution.

- **Field 1:** 'PREFIT' (if the satellite is used in the computation) or 'PREFIT\*' (if it is not)
- **Field 2:** Year
- **Field 3:** Day
- **Field 4:** Seconds of day
- **Field 5:** GNSS System (GPS, GAL, GLO or GEO)
- **Field 6:** PRN satellite identifier
- **Field 7:** Measurement identifier (as string)
- **Field 8:** Measurement-model value (prefit) [m]
- **Field 9:** Measurement value [m]
- **Field 10:** Model value [m]
- **Field 11:** X-partial derivative (-X component of the satellite line-of-sight vector)
- **Field 12:** Y-partial derivative (-Y component of the satellite line-of-sight vector)
- **Field 13:** Z-partial derivative (-Z component of the satellite line-of-sight vector)
- **Field 14:** T-partial derivative
- **Field 15:** Elevation of the satellite [degrees]
- **Field 16:** Azimuth of the satellite [degrees]
- **Field 17:** Standard deviation of the measurement (for the filter) [m]
- **Field 18:** Troposphere wet mapping
- **Field 19:** Arc number
- **Sample:** PREFIT 2006 200 300.00 GPS 19 LC -7.3029 22982271.7155 22982279.0184 28.28 0.3931 -0.4834 0.7822 1.0000 77.912 130.010 17.883 2

Developed by gAGE: Research group of Astronomy & Geomatics

Run gLAB Show Output

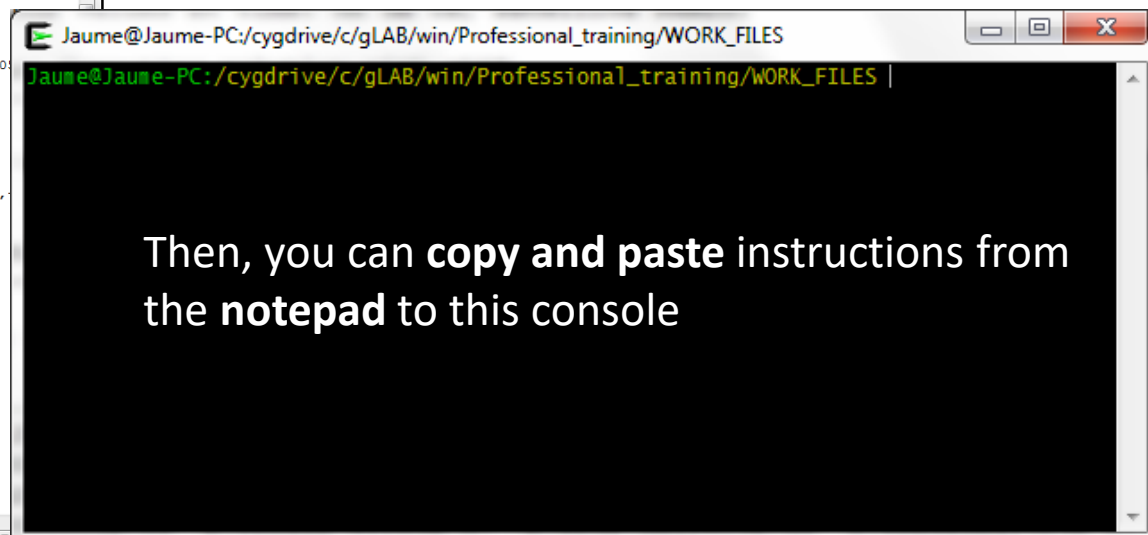
# How to use the notepad ?



```
# Ex 1: Navigation Equations System and LS solution (ENU) [Ex. 6 session 5.2, GNSS BOOK]
#
#Previous:
./gLAB_linux -input:cfg gLAB_ex1.cfg -input:obs UPC11490.050 -input:nav UPC11490.050

# a) Prefit residual vector (Y) and Geometry matrix (G) generation
#
awk '{if ($1=="PREFIT" && $4==300) print $6,$8,$15,$16}' gLAB.out > dat
cat dat | gawk 'BEGIN{g2r=atan2(1,1)/45}{e=$3*g2r;a=$4*g2r;print $2,-cos(e)*sin(a),sin(e)*sin(a)}'

# b) LS solution with Octave or MATLAB
#
#
octave
load M.dat
Y=M(:,1)
G=M(:,2:5)
x=inv(G'*G)*G'*Y
exit
#
#
#
```



## 2.- Pre-fit residual vector (y) and Geometry matrix (G) generation

2. 1. From gLAB.out file, print the pre-fit residual, the elevation and azimuth of all satellites at epoch  $t=300$  sec:

Next “awk” sentence performs this selection:

```
awk '{if ($1=="PREFIT" && $4==300) print $6,$8,$15,$16}' gLAB.out > dat
```

dat (t=300)			
PRN	Pre-fit	Elev (Deg)	Azim (deg)
25	-7,3516	32,97	-49,15
9	-5,3107	15,05	141,52
6	-7,4675	71,83	-144,91
1	-6,8946	30,40	-68,99
2	-6,5154	27,58	62,57
5	-5,2957	38,89	81,01
30	-10,313	72,25	25,35
14	-5,6687	32,79	-117,19

### PREFIT messages

Prefilter values message. It provides the measurement-model values. It is shown in each filter execution.

- Field 1: 'PREFIT' (if the satellite is used in the computation) or 'PREFIT\*' (if it is not)
- Field 2: Year
- Field 3: Day
- Field 4: Seconds of day
- Field 5: Measurement value (m)
- Field 6: PRN satellite identifier
- Field 7: Measurement identifier (as string)
- Field 8: Measurement-model value (prefit) [m]
- Field 9: Measurement value (m)
- Field 10: Model value [m]
- Field 11: X-partial derivative (-X component of the satellite line-of-sight vector)
- Field 12: Y-partial derivative (-Y component of the satellite line-of-sight vector)
- Field 13: Z-partial derivative (-Z component of the satellite line-of-sight vector)
- Field 14: T-partial derivative
- Field 15: Elevation of the satellite [degrees]
- Field 16: Azimuth of the satellite [degrees]
- Field 17: Standard deviation of the measurement (for the filter) [m]
- Field 18: Troposphere wet mapping
- Field 19: Arc number
- Sample: PREFIT 2006 200 300.00 GPS 19 LC -7.3029 22982271.7155 22982279.0184 28.28 0.3931 -0.4834 0.7822 1.0000 77.912 130.010 17.883 2

In general Field 8 = Field 9 - Field 10, but this is no longer true when using smoothing, as the Field 9 is the raw measurement without smoothing, but the Field 8 computation takes smoothing into account.



## 2.- Pre-fit residual vector (y) and Geometry matrix (G) generation

2. 1. From gLAB.out file, print the pre-fit residual, the elevation and azimuth of all satellites at epoch  $t=300$  sec:

Next “awk” sentence performs this selection:

Selected values are sent to file “dat”

```
awk '{if ($1=="PREFIT" && $4==300) print $6,$8,$15,$16}' gLAB.out > dat
```

Selects rows with "PREFIT" in column 1

In column 4 Selects 300

For the selected rows prints columns: 6, 8, 15 and 16

MODEL	2005	149	300.00	GPS	5	C1C	22038212.7089	22038213.2315	12778295.7294	17448678.1282	15370073.0526	-80.2710	1993.9191	-2156.3800	22068872.6572				
MODEL	2005	149	300.00	GPS	6	C1C	22038210.6753	22038213.7948	12778295.7294	17448678.1282	15370073.0526	-80.2710	1993.9191	-2156.3800	22068872.6572				
MODEL	2005	149	300.00	GPS	1	C1C	20171035.5300	20171045.8437	15457014.5128	3212767.4871	21120060.1514	58.4512	2745.4928	-443.5206	20235646.5964				
MODEL	2005	149	300.00	GPS	14	C1C	0.06750	0.06750	0.06750	0.06750	0.06750	0.06750	0.06750	0.06750	0.06750	0.06750			
MODEL	2005	149	300.00	GPS	14	C1C	0.07525	22567003.4670	22567010.3236	20295740.0580	-16130611.9693	5772245.1697	731.1558	-183.3408	-3112.7660	22558161.1977			
MODEL	2005	149	300.00	GPS	14	L1P	0.07525	22567004.5274	22567008.3542	20295740.0580	-16130611.9693	5772245.1697	731.1558	-183.3408	-3112.7660	22558161.1977			
MODEL	2005	149	300.00	GPS	14	L1P	0.07525	22567002.6161	22567006.7490	20295740.0580	-16130611.9693	5772245.1697	731.1558	-183.3408	-3112.7660	22558161.1977			
PREFIT	2005	149	300.00	GPS	25	C1C	-7.3516	22857303.9960	22857311.3476	-0.0689	0.6325	-0.7715	1.0000	32.972	-49.150	5.000	1.833	0	
PREFIT	2005	149	300.00	GPS	9	C1C	-5.3107	24466601.3370	24466606.6477	-0.6720	-0.6261	0.3955	1.0000	15.048	141.520	5.000	3.800	0	
PREFIT	2005	149	300.00	GPS	6	C1C	-7.4675	20405995.0110	20406002.4785	-0.8876	0.1466	-0.4367	1.0000	71.830	-144.914	5.000	1.052	0	
PREFIT	2005	149	300.00	GPS	1	C1C	-6.8946	22758443.9140	22758450.8086	-0.2048	0.7982	-0.5665	1.0000	30.401	-68.993	5.000	1.970	0	
PREFIT	2005	149	300.00	GPS	2	C1C	-6.5154	22847797.9790	22847804.4944	-0.0484	-0.7890	-0.6124	1.0000	27.580	62.571	5.000	2.152	0	
PREFIT	2005	149	300.00	GPS	5	C1C	-5.2957	22038213.1210	22038218.4167	-0.3620	-0.7826	-0.5064	1.0000	38.891	81.008	5.000	1.590	0	
PREFIT	2005	149	300.00	GPS	30	C1C	-10.3137	20171035.5300	20171045.8437	-0.5272	-0.1500	-0.8364	1.0000	72.250	25.346	5.000	1.050	0	
PREFIT	2005	149	300.00	GPS	14	C1C	-5.6687	22567004.8560	22567010.5247	-0.6874	0.7229	-0.0699	1.0000	32.794	-117.190	5.000	1.842	0	
EPOCHSAT	2005	149	300.00	C1C	8	25	9	6	1	2	5	30	14						
POSTFIT	2005	149	300.00	GPS	25	C1C	-0.1118	22857303.9960	22857304.1078	32.972	-49.150								
POSTFIT	2005	149	300.00	GPS	9	C1C	-0.9728	24466601.3370	24466602.3098	15.048	141.520								
POSTFIT	2005	149	300.00	GPS	6	C1C	0.6484	20405995.0110	20405994.3626	71.830	-144.914								
POSTFIT	2005	149	300.00	GPS	1	C1C	-0.0580	22758443.9140	22758443.9720	30.401	-68.993								
POSTFIT	2005	149	300.00	GPS	2	C1C	-0.0304	22847797.9790	22847798.0094	27.580	62.571								
POSTFIT	2005	149	300.00	GPS	5	C1C	1.6222	22038213.1210	22038211.4988	38.891	81.008								
POSTFIT	2005	149	300.00	GPS	30	C1C	-1.6633	20171035.5300	20171037.1933	72.250	25.346								
POSTFIT	2005	149	300.00	GPS	14	C1C	0.5656	22567004.8560	22567004.2904	32.794	-117.190								
FILTER	2005	149	300.00				4789035.2870	176594.9809	4195017.0369	-4.0917									
OUTPUT	2005	149	300.00				9.6891	4789035.2870	176594.9809	4195017.0369	2.6593	-0.0689	3.7866	7.3157	2.8466	5.6796	41.388672834	2.111816713	170.

## 2.- Pre-fit residual vector (y) and Geometry matrix (G) generation

2.2. Using the previous “**dat**” file, build-up the navigation equations system:

dat			
PRN	Pre-fit	Elev (Deg)	Azim (deg)
25	-7,3516	32,97	-49,15
9	-5,3107	15,05	141,52
6	-7,4675	71,83	-144,91
1	-6,8946	30,40	-68,99
2	-6,5154	27,58	62,57
5	-5,2957	38,89	81,01
30	-10,313	72,25	25,35
14	-5,6687	32,79	-117,19

$$\begin{bmatrix} Prefit^1 \\ Prefit^2 \\ \dots\dots \\ Prefit^n \end{bmatrix} = \begin{bmatrix} -\cos el^1 \sin az^1 & -\cos el^1 \cos az^1 & -\sin el^1 & 1 \\ -\cos el^2 \sin az^2 & -\cos el^2 \cos az^2 & -\sin el^2 & 1 \\ \dots\dots\dots & \dots\dots\dots & \dots\dots\dots & \dots\dots\dots \\ -\cos el^n \sin az^n & -\cos el^n \cos az^n & -\sin el^n & 1 \end{bmatrix} \begin{bmatrix} \Delta e_{rec} \\ \Delta n_{rec} \\ \Delta u_{rec} \\ c dt_{rec} \end{bmatrix}$$

Execute next sentence in a single line

```
cat dat| gawk 'BEGIN{g2r=atan2(1,1)/45}{e=$3*g2r;a=$4*g2r;
print $2,-cos(e)*sin(a),-cos(e)*cos(a),-sin(e),1}' > M.dat
```

y	G			
Pre-fit	-cos(e)*sin(a)	-cos(e)*cos(a)	-sin(e)	1
-7,3516	0,634592	-0,548732	-0,544229	1
-5,3107	-0,600904	0,755981	-0,259628	1
-7,4675	0,179246	0,255174	-0,950135	1
-6,8946	0,805180	-0,309192	-0,506049	1
-6,5154	-0,786722	-0,408303	-0,462987	1
-5,2957	-0,768776	-0,121652	-0,627841	1
-10,3137	-0,130507	-0,275518	-0,952396	1
-5,6687	0,747731	0,384117	-0,541620	1

Next, we will use MATLAB (octave) to solve this equations system by Least Squares

## 2.3. Computing the LS solution with octave (or MATLAB)

```
octave

load M.dat

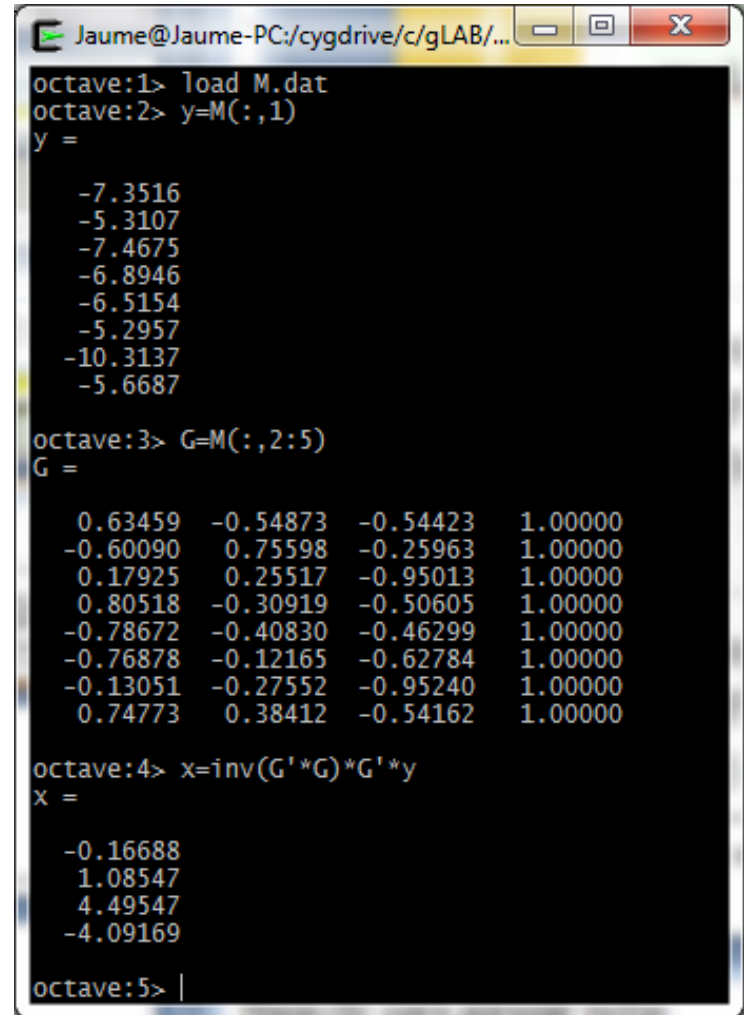
y=M(:,1)

G=M(:,2:5)

x=inv(G'*G)*G'*y

exit
```

**Positioning error**, regarding to the reference coordinates given in the header of RINEX file



```
Jaume@Jaume-PC:/cygdrive/c/gLAB/...
octave:1> load M.dat
octave:2> y=M(:,1)
y =

-7.3516
-5.3107
-7.4675
-6.8946
-6.5154
-5.2957
-10.3137
-5.6687

octave:3> G=M(:,2:5)
G =

0.63459 -0.54873 -0.54423 1.00000
-0.60090 0.75598 -0.25963 1.00000
0.17925 0.25517 -0.95013 1.00000
0.80518 -0.30919 -0.50605 1.00000
-0.78672 -0.40830 -0.46299 1.00000
-0.76878 -0.12165 -0.62784 1.00000
-0.13051 -0.27552 -0.95240 1.00000
0.74773 0.38412 -0.54162 1.00000

octave:4> x=inv(G'*G)*G'*y
x =

-0.16688
1.08547
4.49547
-4.09169

octave:5> |
```

→  $\begin{bmatrix} \Delta\text{East} & \Delta\text{North} & \Delta\text{Up} & dT \\ -0.16688 & 1.08547 & 4.49547 & -4.09169 \end{bmatrix}$

**Receiver clock  
offset**

## 2.4. Check results with gLAB

The screenshot shows the gLAB v5.1.0 interface. The 'Output Destination' section has 'Output File' set to 'C:\gLAB\NEW\_TUT\TUT7\WORK'. Under 'Common Navigation Messages', 'Print FILTER' and 'Print OUTPUT' are checked. A yellow box highlights 'Positioning error, with respect to the a priori coordinates.' pointing to fields 18-20 in the list. Another yellow box highlights 'Receiver clock offset' pointing to 'Field 8: Receiver clock [m]' in the 'FILTER messages' list. A red box highlights the 'Output' button in the top right. Another red box highlights the 'Run gLAB' button at the bottom.

Receiver solution message. This message provides direct information on the filter estimates. It is shown in each filter execution.

- Field 1: 'OUTPUT'
- Field 2: Year
- Field 3: Day
- Field 4: Seconds of day
- Field 5: 3D Formal Error:  $(\sigma_x^2 + \sigma_y^2 + \sigma_z^2)^{1/2}$
- Field 6: Receiver X position [m]
- Field 7: Receiver Y position [m]
- Field 8: Receiver Z position [m]
- Field 9: Receiver X position - Nominal a priori X position [m]
- Field 10: Receiver Y position - Nominal a priori Y position [m]
- Field 11: Receiver Z position - Nominal a priori Z position [m]
- Field 12: Receiver X formal error [m]
- Field 13: Receiver Y formal error [m]
- Field 14: Receiver Z formal error [m]
- Field 15: Receiver latitude [degrees]
- Field 16: Receiver longitude [degrees]
- Field 17: Receiver height [m]
- Field 18: Receiver North difference in relation to nominal a priori position [m]
- Field 19: Receiver East difference in relation to nominal a priori position [m]
- Field 20: Receiver Up difference in relation to nominal a priori position [m]
- Field 21: Receiver formal error in North direction [m]
- Field 22: Receiver formal error in East direction [m]
- Field 23: Receiver formal error in Up direction [m]
- Field 24: Geometric Dilution of Precision (GDOP)
- Field 25: Position Dilution of Precision (PDOP)
- Field 26: Time Dilution of Precision (TDOP)
- Field 27: Horizontal Dilution of Precision (HDOP)
- Field 28: Vertical Dilution of Precision (VDOP)
- Field 29: Zenith Tropospheric Delay (including nominal value) [m]
- Field 30: Zenith Tropospheric Delay (excluding nominal value) [m]
- Field 31: Zenith Tropospheric Delay formal error [m]
- Field 32: Number of satellites used in the navigation solution
- Field 33: Processing mode indicator
- Sample: OUTPUT 2006 200 300.00 2.6219 4849203.1236 -360328.5229 4114913.9535 0.7693 0.4145 0.7580 1.9353 0.6998 1.6246 40.429162956 -4.249653155 830.480629026 0.0993 0.4704 1.0522 1.1365 0.6772 2.2637 5.0394 5.5472 6.3482 2.4261 2.2142 2.1982 0.0097 0.4995 6 0

Providing a nominal a priori position is optional of the processing, but if it is given, fields 9, 10, 11, 18, 19 and 20 will be given in relation to this a priori position. See the option 'A priori Receiver Position' in the Input section for more details.

Filter solution message. This message provides direct information on the filter estimates. It is shown in each filter execution.

- Field 1: 'FILTER'
- Field 2: Year
- Field 3: Day
- Field 4: Seconds of day
- Field 5: Filter estimates. The order is: 3D estimated position, clock, troposphere and ambiguities. The number of fields is variable in this message. With a full filter (troposphere and ambiguities estimation), the Fields are as follows:
  - Field 5: Receiver X position [m]
  - Field 6: Receiver Y position [m]
  - Field 7: Receiver Z position [m]
  - Field 8: Receiver clock [m]
  - Field 9: Zenith Tropospheric Delay [m]
  - Field 10: Carrier
- Sample: FILTER 2006 20 4114913.9184 -7.4867 2.00897 0.0001 0.3845 0.2

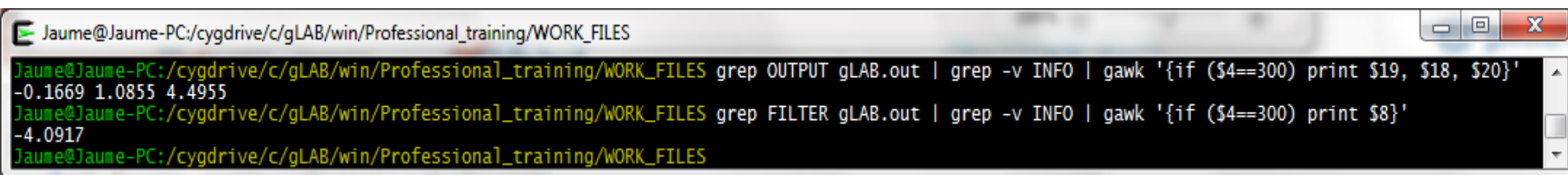
## Checking gLAB values:

```
grep OUTPUT gLAB.out | grep -v INFO |  
    gawk '{if ($4==300) print $19, $18, $20}'
```

➔ -0.1669 1.0855 4.4955

```
grep FILTER gLAB.out | grep -v INFO |  
    gawk '{if ($4==300) print $8}'
```

➔ -4.0917



```
Jaume@Jaume-PC:/cygdrive/c/gLAB/win/Professional_training/WORK_FILES  
Jaume@Jaume-PC:/cygdrive/c/gLAB/win/Professional_training/WORK_FILES$ grep OUTPUT gLAB.out | grep -v INFO | gawk '{if ($4==300) print $19, $18, $20}'  
-0.1669 1.0855 4.4955  
Jaume@Jaume-PC:/cygdrive/c/gLAB/win/Professional_training/WORK_FILES$ grep FILTER gLAB.out | grep -v INFO | gawk '{if ($4==300) print $8}'  
-4.0917  
Jaume@Jaume-PC:/cygdrive/c/gLAB/win/Professional_training/WORK_FILES$
```

## Checking gLAB values:

This is a “pipe”: the output of the first “grep” is sent to the second one.

```
grep OUTPUT gLAB.out
```

```
| grep -v INFO > results
```

Select rows containing “**OUTPUT**”  
(in any of its columns)

Skip rows containing “**INFO**”  
(in any of its columns)

The left screenshot shows the output of the command `grep OUTPUT gLAB.out`. The output is a list of rows from the gLAB.out file, with the row containing 'OUTPUT' highlighted. The right screenshot shows the output of the command `grep -v INFO gLAB.out`, which filters out all rows containing 'INFO'. The row 'INFO OUTPUT Satellite Velocity: ITRF' is highlighted, showing that it was not filtered out because it contains 'INFO'.

Line	Content
POSTFIT	2005 149 300.00 GPS 6 C1C 0.6484 20405995.0110 20405994.3626
POSTFIT	2005 149 300.00 GPS 1 C1C -0.0580 22758443.9140 22758443.9720
POSTFIT	2005 149 300.00 GPS 2 C1C -0.0304 22847797.9790 22847798.0094
POSTFIT	2005 149 300.00 GPS 5 C1C 1.6222 22038213.1210 22038211.4988
POSTFIT	2005 149 300.00 GPS 30 C1C -1.6633 20171035.5300 20171037.1933
POSTFIT	2005 149 300.00 GPS 14 C1C -0.5656 22567004.8560 22567004.2904
OUTPUT	2005 149 300.00 4789035.2870 176594.9809 4195017.0369
INPUT	2005 149 600.00 GPS 25 20 22704167.0660 22704167.0660 22704164.80
INPUT	2005 149 600.00 GPS 9 20 24680382.4850 24680382.4850 24680383.88
INPUT	2005 149 600.00 GPS 6 20 20349857.7450 20349857.7450 20349856.38
INPUT	2005 149 600.00 GPS 1 20 22751281.7810 22751281.7810 22751279.45
INPUT	2005 149 600.00 GPS 2 20 22887681.2390 22887681.2390 22887677.78
INPUT	2005 149 600.00 GPS 5 20 22171761.1020 22171761.1020 22171759.81
INPUT	2005 149 600.00 GPS 30 20 20195547.7030 20195547.7030 20195545.64
INPUT	2005 149 600.00 GPS 14 20 22696142.8270 22696142.8270 22696141.43
MODEL	2005 149 600.00 GPS 25 C1C 0.07583 22704167.0660 22704174.3737 7
MODEL	2005 149 600.00 GPS 25 C2P 0.07583 22704164.8040 22704174.4706 7
MODEL	2005 149 600.00 GPS 25 L1P 0.07583 22704167.2058 22704171.8407 7
MODEL	2005 149 600.00 GPS 25 L2P 0.07583 22704166.6614 22704170.2988 7
MODEL	2005 149 600.00 GPS 9 C1C 0.08228 24680382.4850 24680387.9379 21
MODEL	2005 149 600.00 GPS 9 C2P 0.08228 24680383.8880 24680389.3799 21
MODEL	2005 149 600.00 GPS 9 L1P 0.08228 24680382.6080 24680381.8048 21
MODEL	2005 149 600.00 GPS 9 L2P 0.08228 24680370.6066 24680379.2790 21
MODEL	2005 149 600.00 GPS 6 C1C 0.06844 20349857.7450 20349865.1650 22
MODEL	2005 149 600.00 GPS 6 C2P 0.06844 20349856.3810 20349865.2654 22
MODEL	2005 149 600.00 GPS 6 L1P 0.06844 20349858.2258 20349863.4586 22665682.8664 -2500244.0198 13904392.7113 -1309.7863 1168.8346 2368.4366 20518580.3768 -1
MODEL	2005 149 600.00 GPS 6 L2P 0.06844 20349856.9027 20349862.4551 22665682.8664 -2500244.0198 13904392.7113 -1309.7863 1168.8346 2368.4366 20518580.3768 -1
MODEL	2005 149 600.00 GPS 1 C1C 0.07630 22751281.7810 22751288.1415 10044390.2875 -18356826.5210 16526319.9209 1856.6088 -901.6986 -2156.8573 22872857.7895 -1



# Navigation equations system and least Squares solution

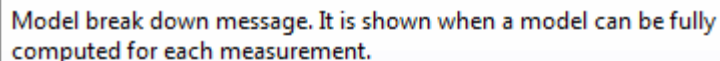
## Exercise 2:

Repeat the previous exercise, but writing the system and computing the solution in (XYZ) coordinates.

Complete the next steps:

- 1.- Use gLAB to compute the prefit-residuals and satellite **(x,y,z)** coordinates at time **t=300** seconds.
- 2.- Build up the navigation equations system to compute the solution in (XYZ) coordinates. Solve the system with MATLAB (octave).

$$\begin{bmatrix} Prefit^1 \\ Prefit^2 \\ \dots\dots \\ Prefit^n \end{bmatrix} = \begin{bmatrix} \frac{x_{0,rec} - x^{sat1}}{\rho_{0,rec}^{sat1}} & \frac{y_{0,rec} - y^{sat1}}{\rho_{0,rec}^{sat1}} & \frac{z_{0,rec} - z^{sat1}}{\rho_{0,rec}^{sat1}} & 1 \\ \frac{x_{0,rec} - x^{sat2}}{\rho_{0,rec}^{sat2}} & \frac{y_{0,rec} - y^{sat2}}{\rho_{0,rec}^{sat2}} & \frac{z_{0,rec} - z^{sat2}}{\rho_{0,rec}^{sat2}} & 1 \\ \dots\dots\dots & \dots\dots\dots & \dots\dots\dots & \dots\dots\dots \\ \frac{x_{0,rec} - x^{satn}}{\rho_{0,rec}^{satn}} & \frac{y_{0,rec} - y^{satn}}{\rho_{0,rec}^{satn}} & \frac{z_{0,rec} - z^{satn}}{\rho_{0,rec}^{satn}} & 1 \end{bmatrix} \begin{bmatrix} \Delta x_{rec} \\ \Delta y_{rec} \\ \Delta z_{rec} \\ c dt_{rec} \end{bmatrix}$$



- ## Satellite coordinates

Field 9 is the direct measurement (as in the RINEX file), but scaled to metres for carrier phase measurements. Field 10 is the model computed for this measurement. Field 10 is the direct sum of fields 17 to 28.





```

Jaume@Jaume-PC:/cygdrive/c/gLAB/NEW_TUT/TUT7/WORK
2.10 OBSERVATION DATA GPS/GEO RINEX VERSION / TYPE
B2AConv V2.0 gAGE/UPC 21-Dec-09 19:17 PGM / RUN BY / DATE
BIT 2 OF LLI (+4) FLAGS DATA COLLECTED UNDER "AS" CONDITION COMMENT
UPC1 MARKER NAME
gAGE / UPC gAGE / UPC OBSERVER / AGENCY
1 NOVATEL MILLENIUM OEM-3 REC # / TYPE / VERS
1 NOVATEL PTNWELL ANT # / TYPE
4789032.6277 176595.0498 4195013.2503 APPROX POSITION XYZ
0.0000 0.0000 0.0000 ANTENNA: DELTA H/E/N
1 1 WAVELENGTH FACT L1/2
6 C1 P2 L1 L2 D1 D2 # / TYPES OF OBSERV
SNR is mapped to signal strength [0-9]
L1 SNR: >44 >35 >26 >17 >8 >0 COMMENT
sig: 9 8 7 6 5 4 COMMENT
L2 SNR: >50 >42 >34 >26 >18 >8 >0 n/a COMMENT
sig: 9 8 7 6 5 4 3 0 COMMENT
1 INTERVAL
2005 5 29 0 0 1.000000 TIME OF FIRST OBS
2005 5 29 23 59 58.000000 TIME OF LAST OBS
END OF HEADER
5 5 29 0 0 1.000000 0 9G25G 9G 6G 1G21G 2G 5G30G14
23014409.454 23014407.0624 120941560.43748 94240180.12946 2797.89748
2180.13646
24255343.500 24255342.1054 127462772.33948 99321651.17545 -3695.18948
-2879.38745
20470437.022 20470435.1684 107572939.39549 83823051.74446 1206.77349
940.33646
22776509.627 22776510.7274 119691395.32948 93266004.45346 413.23948
1,6 Top

```

Receiver coordinates to linearize the equations (*a priori*)

$(x_{0,rec}, y_{0,rec}, z_{0,rec})$

$$\begin{bmatrix} Prefit^1 \\ Prefit^2 \\ \dots \\ Prefit^n \end{bmatrix} = \begin{bmatrix} \frac{x_{0,rec} - x^{sat1}}{\rho_{0,rec}^{sat1}} & \frac{y_{0,rec} - y^{sat1}}{\rho_{0,rec}^{sat1}} & \frac{z_{0,rec} - z^{sat1}}{\rho_{0,rec}^{sat1}} & 1 \\ \frac{x_{0,rec} - x^{sat2}}{\rho_{0,rec}^{sat2}} & \frac{y_{0,rec} - y^{sat2}}{\rho_{0,rec}^{sat2}} & \frac{z_{0,rec} - z^{sat2}}{\rho_{0,rec}^{sat2}} & 1 \\ \dots & \dots & \dots & \dots \\ \frac{x_{0,rec} - x^{satn}}{\rho_{0,rec}^{satn}} & \frac{y_{0,rec} - y^{satn}}{\rho_{0,rec}^{satn}} & \frac{z_{0,rec} - z^{satn}}{\rho_{0,rec}^{satn}} & 1 \end{bmatrix} \begin{bmatrix} \Delta x_{rec} \\ \Delta y_{rec} \\ \Delta z_{rec} \\ c dt_{rec} \end{bmatrix}$$

$x_{0,rec}=4789032.6277$   
 $y_{0,rec}=176595.0498$   
 $z_{0,rec}=4195013.2503$

## 2.1 Pre-fit residual vector (y) and Geometry matrix (G) generation

```

grep "MODEL " gLAB.out | grep -v INFO | grep C1C |
awk 'BEGIN {x=4789032.6277;y=176595.0498;z=4195013.2503
{if ($4==300)
{r1=x-$11;r2=y-$12;r3=z-$13;r=sqrt(r1*r1+r2*r2+r3*r3);
print $9-$10,r1/r,r2/r,r3/r,1}}}'>M.dat

```

$$\begin{bmatrix} Prefit^1 \\ Prefit^2 \\ \dots \\ Prefit^n \end{bmatrix} = \begin{bmatrix} \frac{x_{0,rec} - x^{sat1}}{\rho_{0,rec}^{sat1}} & \frac{y_{0,rec} - y^{sat1}}{\rho_{0,rec}^{sat1}} & \frac{z_{0,rec} - z^{sat1}}{\rho_{0,rec}^{sat1}} & 1 \\ \frac{x_{0,rec} - x^{sat2}}{\rho_{0,rec}^{sat2}} & \frac{y_{0,rec} - y^{sat2}}{\rho_{0,rec}^{sat2}} & \frac{z_{0,rec} - z^{sat2}}{\rho_{0,rec}^{sat2}} & 1 \\ \dots & \dots & \dots & \dots \\ \frac{x_{0,rec} - x^{satn}}{\rho_{0,rec}^{satn}} & \frac{y_{0,rec} - y^{satn}}{\rho_{0,rec}^{satn}} & \frac{z_{0,rec} - z^{satn}}{\rho_{0,rec}^{satn}} & 1 \end{bmatrix} \begin{bmatrix} \Delta x_{rec} \\ \Delta y_{rec} \\ \Delta z_{rec} \\ c dt_{rec} \end{bmatrix}$$

y	M			
Pre-fit	$\frac{x_{0,rec} - x^{sat1}}{\rho_{0,rec}^{sat1}}$	$\frac{y_{0,rec} - y^{sat1}}{\rho_{0,rec}^{sat1}}$	$\frac{z_{0,rec} - z^{sat1}}{\rho_{0,rec}^{sat1}}$	1
-7,3516	-0,068854	0,632491	-0,771501	1
-5,3107	-0,671994	-0,626096	0,395511	1
-7,4675	-0,887550	0,146645	-0,436749	1
-6,8946	-0,204785	0,79818	-0,566544	1
-6,5154	-0,048353	-0,789036	-0,612441	1
-5,2957	-0,362015	-0,782645	-0,506372	1
-10,3137	-0,527188	-0,150041	-0,836398	1
-5,6687	-0,687410	0,722896	-0,069918	1

$$\begin{aligned}x_{0,rec} &= 4789032.6277 \\ y_{0,rec} &= 176595.0498 \\ z_{0,rec} &= 4195013.2503\end{aligned}$$

## Checking gLAB equations values with gLAB:

```
grep "PREFIT " gLAB.out | grep -v INFO |  
  gawk '{if ($4==300) print $8,$11,$12,$13, $14}'
```

y	M			
Pre-fit	Previous computations			
-7,3516	-0,068854	0,632491	-0,771501	1
-5,3107	-0,671994	-0,626096	0,395511	1
-7,4675	-0,887550	0,146645	-0,436749	1
-6,8946	-0,204785	0,79818	-0,566544	1
-6,5154	-0,048353	-0,789036	-0,612441	1
-5,2957	-0,362015	-0,782645	-0,506372	1
-10,3137	-0,527188	-0,150041	-0,836398	1
-5,6687	-0,687410	0,722896	-0,069918	1

y	M			
Pre-fit	From gLAB			
-7,3516	-0,0689	0,6325	-0,7715	1
-5,3107	-0,6720	-0,6261	0,3955	1
-7,4675	-0,8876	0,1466	-0,4367	1
-6,8946	-0,2048	0,7982	-0,5665	1
-6,5154	-0,0484	-0,7890	-0,6124	1
-5,2957	-0,3620	-0,7826	-0,5064	1
-10,3137	-0,5272	-0,1500	-0,8364	1
-5,6687	-0,6874	0,7229	-0,0699	1

## 2.3. Computing the LS solution with octave (or MATLAB)

```
octave

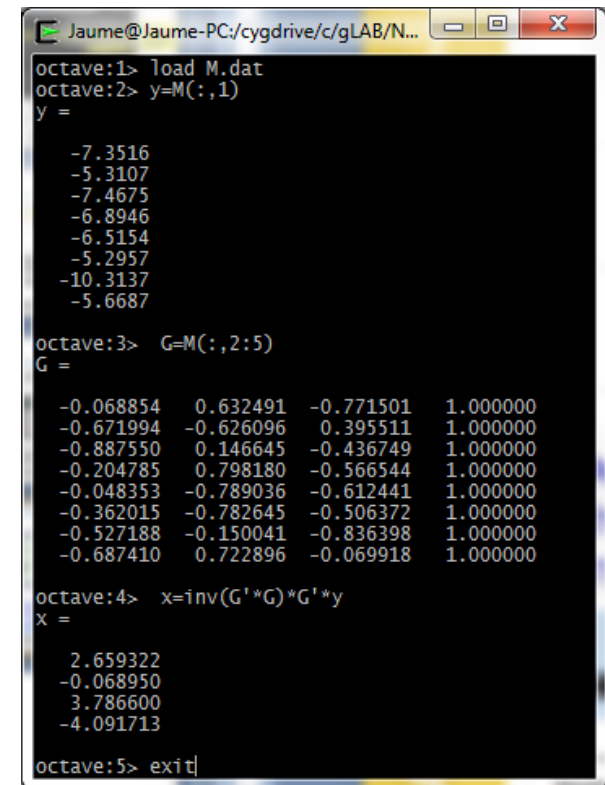
load M.dat

y=M(:,1)

G=M(:,2:5)

x=inv(G'*G)*G'*y

exit
```



```
Jaume@Jaume-PC:/cygdrive/c/gLAB/N...
octave:1> load M.dat
octave:2> y=M(:,1)
y =
-7.3516
-5.3107
-7.4675
-6.8946
-6.5154
-5.2957
-10.3137
-5.6687

octave:3> G=M(:,2:5)
G =
-0.068854    0.632491   -0.771501    1.000000
-0.671994   -0.626096    0.395511    1.000000
-0.887550    0.146645   -0.436749    1.000000
-0.204785    0.798180   -0.566544    1.000000
-0.048353   -0.789036   -0.612441    1.000000
-0.362015   -0.782645   -0.506372    1.000000
-0.527188   -0.150041   -0.836398    1.000000
-0.687410    0.722896   -0.069918    1.000000

octave:4> x=inv(G'*G)*G'*y
x =
2.659322
-0.068950
3.786600
-4.091713

octave:5> exit
```

**Positioning error**, with respect to the a priori coordinates given in the header of RINEX file:

	$\Delta x$	$\Delta y$	$\Delta z$	$dT$
→	[2.659322	-0.068950	3.786600	-4.091713]

**Receiver clock offset**

Finally, the receiver coordinates are:

$$\begin{aligned}(\mathbf{x}, \mathbf{y}, \mathbf{z}) &= (\mathbf{x}_{0,\text{rec}}, \mathbf{x}_{0,\text{rec}}, \mathbf{x}_{0,\text{rec}}) + (\Delta x, \Delta y, \Delta z) \\ &= (4789032.6277, 176595.0498, 4195013.2503) + (2.659322, -0.068950, 3.786600) \\ &= (4789035.2870, 176594.9808, 4195017.0369)\end{aligned}$$

# Navigation equations system and Kalman Filter

## Exercise 3:

The measurement file **UPC11490.05O** has been collected by a receiver with fixed coordinates. Using navigation file **UPC11490.05N**, compute the **SPP solution** in static mode and check *by hand* the computation of the Kalman filter solution for the first three epochs (i.e.  $t = 300$ ,  $t = 600$  and  $t = 900$  seconds).

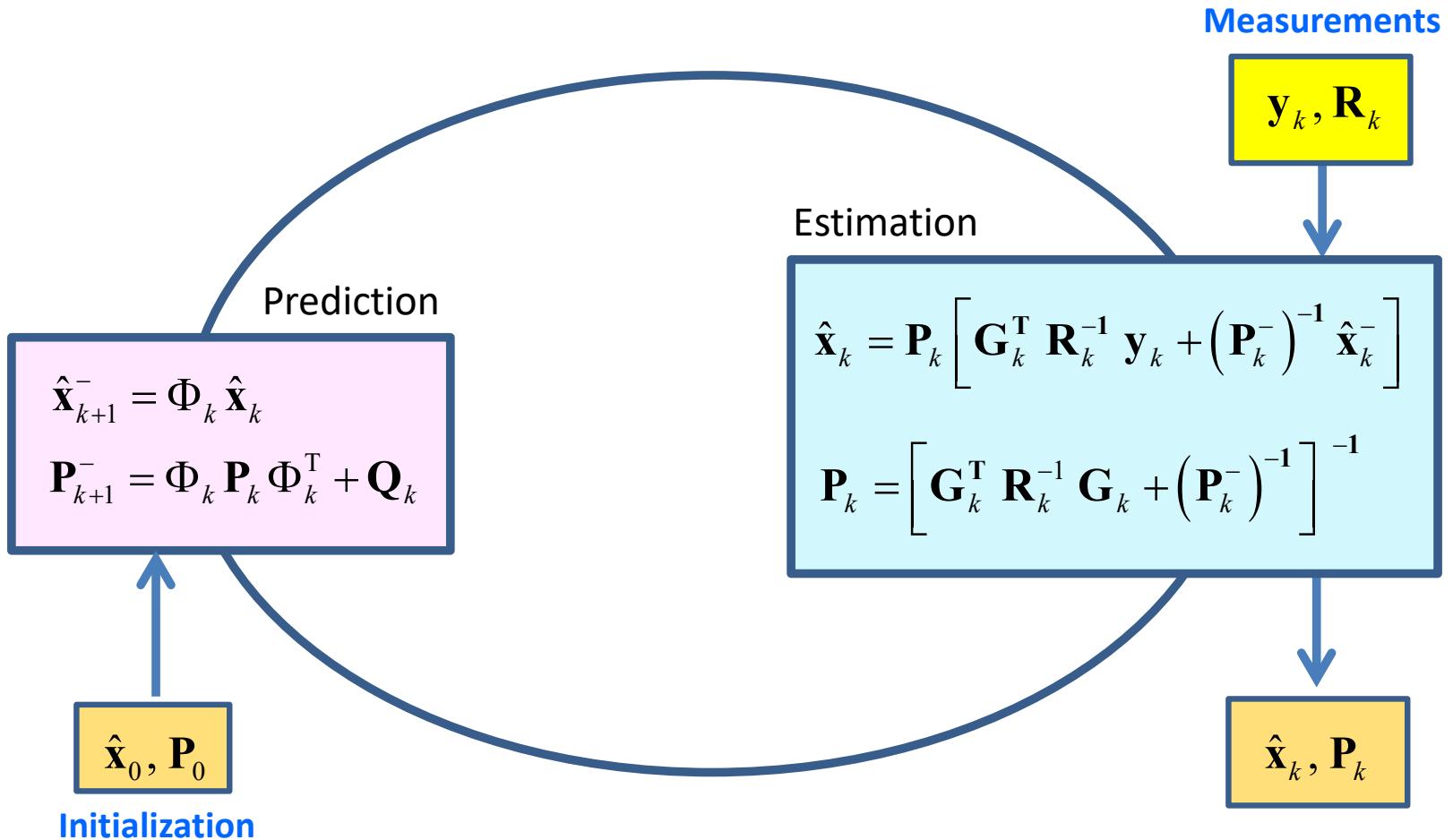
Complete the next steps:

1. Set the default configuration of gLAB for the SPP mode. Then, in section [Filter], **select [ Static] in the Receiver Kinematics option**. To process the data, click Run button.
2. Write the Kalman filter equations. Check the configuration parameters applied by gLAB and compute by hand the solution for the first three epochs in file (i.e.  $t = 300$ ,  $t = 600$  and  $t = 900$  s).

*Note: Use prefit-residuals vector  $\mathbf{y}(k)$  and design matrix  $\mathbf{G}(k)$  from gLAB*

# Kalman filter

(see kalman.f)



gLAB v5.1.0

Mode Templates Configuration Preferences Help

eesa gLAB gAGE/UPC www.gage.es

Input Preprocess Modelling **Filter** Output

**Measurements**

**Selection**

☒ Pseudorange  
☐ Pseudorange + Carrier phase

**Smoothing**

☐ Pseudorange Smoothing

**Measurement Configuration and Noise**

C1C ☒ Fixed StdDev 1 (m) ☐ Elevation StdDev

**Parameters**

	Phi	Q	Po
Coordinates	1 (m <sup>2</sup> )	0 (m <sup>2</sup> )	1e8 (m <sup>2</sup> )
Receiver Clock	0 (m <sup>2</sup> )	9e10 (m <sup>2</sup> )	9e10 (m <sup>2</sup> )

**Available Frequencies**

☒ Single-frequency  
☐ Dual-frequency

**Troposphere**

☐ Estimate wet troposphere residual

**Ionosphere**

☒ Use Sigma Ionosphere

**Receiver Kinematics**

☒ Static  
☐ Kinematic

**Other Options**

☐ Backward Filtering  
☐ Max. GDOP (m)  
☐ Prefit Outlier Detector Threshold (m)

Developed by gAGE: Research group of Astronomy & Geomatics Current Template: SPP **Run gLAB** Show Output

## Filter configuration: From previous gLAB panel, it follows:

i. Filter configuration (according to gLAB):

- Initialisation:

$$\hat{\mathbf{x}}_0 \equiv \hat{\mathbf{x}}(0) = (0, 0, 0, 0),$$

$$\mathbf{P}_0 \equiv \mathbf{P}(0) = \sigma_0^2 \mathbf{I}, \text{ with } \sigma_0 = 3 \cdot 10^5 \text{ m. (see comment [*)]}$$

- Process noise  $\mathbf{Q}$  and transition matrices  $\Phi$ :

$$\mathbf{Q} \equiv \mathbf{Q}(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{dt}^2 \end{bmatrix}, \quad \Phi \equiv \Phi(k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

with  $\sigma_{dt} = 3 \cdot 10^5 \text{ m}$ .

- Measurement covariance matrix:

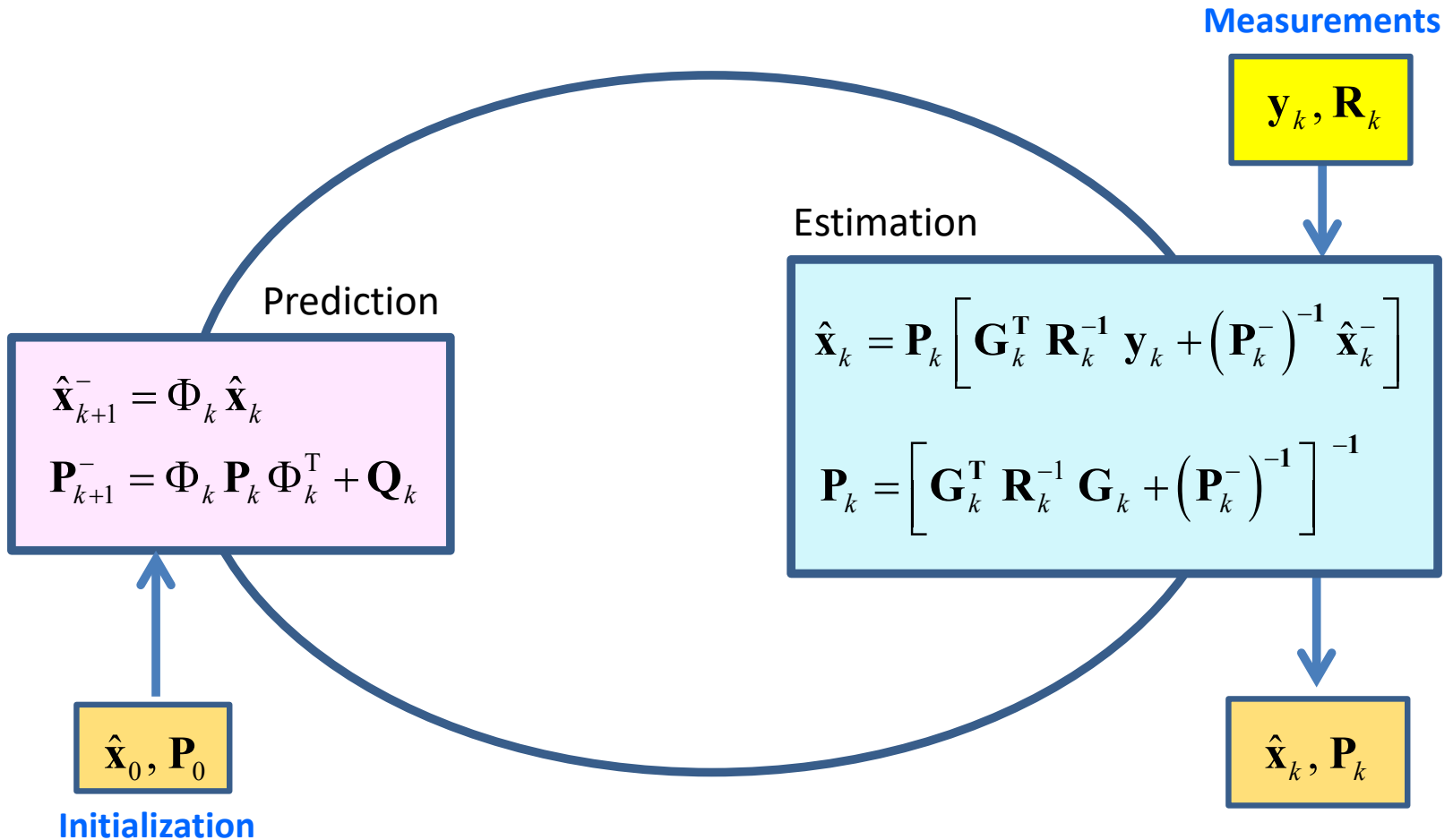
$$\mathbf{R}_k \equiv \mathbf{R}(k) = \sigma_y^2 \mathbf{I}, \text{ with } \sigma_y = 1 \text{ m}.$$

[\*]: By default, gLAB takes  $\mathbf{P}_0$  as:  $\sigma_0 = 1 \cdot 10^4 \text{ m}$  (coord.) and  $\sigma_0 = 3 \cdot 10^5 \text{ m}$  (clock),  
But the results will be basically the same.  
(we use here  $\sigma_0 = 3 \cdot 10^5 \text{ m}$  for coordinates and clock to easier computations)



# Kalman filter

(see kalman.f)



ii. Kalman filter iterations:

k=1:

Prediction

$$\mathbf{x}1^- = \Phi \cdot \hat{\mathbf{x}}0$$

$$\mathbf{P}1^- = \Phi \cdot \mathbf{P}0 \cdot \Phi^T + \mathbf{Q}$$

Estimation

$$\mathbf{P}1 = [\mathbf{G}1^T \cdot \mathbf{R}1^{-1} \cdot \mathbf{G}1 + (\mathbf{P}1^-)^{-1}]^{-1}$$

$$\hat{\mathbf{x}}1 = \mathbf{P}1 \cdot [\mathbf{G}1^T \cdot \mathbf{R}1^{-1} \cdot \mathbf{y}1 + (\mathbf{P}1^-)^{-1} \cdot \mathbf{x}1^-]$$

k=2:

Prediction

$$\mathbf{x}2^- = \Phi \cdot \hat{\mathbf{x}}1$$

$$\mathbf{P}2^- = \Phi \cdot \mathbf{P}1 \cdot \Phi^T + \mathbf{Q}$$

Estimation

$$\mathbf{P}2 = [\mathbf{G}2^T \cdot \mathbf{R}2^{-1} \cdot \mathbf{G}2 + (\mathbf{P}2^-)^{-1}]^{-1}$$

$$\hat{\mathbf{x}}2 = \mathbf{P}2 \cdot [\mathbf{G}2^T \cdot \mathbf{R}2^{-1} \cdot \mathbf{y}2 + (\mathbf{P}2^-)^{-1} \cdot \mathbf{x}2^-]$$

k=3:

...

## Computation of pre-fit residuals and geometry matrix using gLAB (see previous exercise)

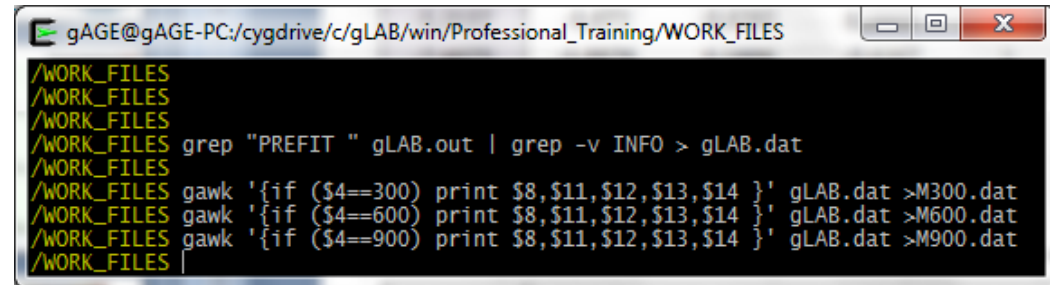
```
grep "PREFIT " gLAB.out | grep -v INFO > gLAB.dat
```

```
gawk '{if ($4==300) print $8,$11,$12,$13,$14 }' gLAB.dat >M300.dat
```

```
gawk '{if ($4==600) print $8,$11,$12,$13,$14 }' gLAB.dat >M600.dat
```

```
gawk '{if ($4==900) print $8,$11,$12,$13,$14 }' gLAB.dat >M900.dat
```

y300	M300			
Pre-fit	1			
-7,3516	-0,0689	0,6325	-0,7715	1
-5,3107	-0,672	-0,6261	0,3955	1
-7,4675	-0,8876	0,1466	-0,4367	1
-6,8946	-0,2048	0,7982	-0,5665	1
-6,5154	-0,0484	-0,7890	-0,6124	1
-5,2957	-0,362	-0,7826	-0,5064	1
-10,3137	-0,5272	-0,1500	-0,8364	1
-5,6687	-0,6874	0,7229	-0,0699	1



y600	M600			
Pre-fit	1			
-7,3077	-0,1017	0,6223	-0,7761	1
-5,4529	-0,6574	-0,6196	0,4289	1
-7,4200	-0,8712	0,1305	-0,4732	1
-6,3605	-0,2298	0,8103	-0,5391	1
-7,1679	-0,0198	-0,7741	-0,6327	1
-5,4569	-0,3589	-0,8043	-0,4735	1
-10,1605	-0,5277	-0,1904	-0,8278	1
-5,6403	-0,6925	0,7209	-0,0281	1

y900	M900			
Pre-fit	1			
-6,8505	-0,1351	0,6124	-0,7789	1
-3,6943	-0,6418	-0,6128	0,4612	1
-7,0965	-0,8537	0,1131	-0,5084	1
-6,6825	-0,2535	0,8219	-0,5101	1
-7,3007	0,0096	-0,7590	-0,6510	1
-5,6064	-0,3561	-0,8245	-0,4398	1
-9,7147	-0,5288	-0,2302	-0,8170	1
-5,0505	-0,6959	0,7180	0,0136	1

## i. Loading data files in octave and filter configuration

octave

load M300.dat

load M600.dat

load M900.dat

$x_0 = [0 \ 0 \ 0 \ 0]'$

$\sigma_0 = 3e5$

$P_0 = (\sigma_0)^2 * \text{eye}(4, 4)$

$\sigma_{dt} = 3e5$

$Q = \text{zeros}(4, 4)$

$Q(4, 4) = (\sigma_{dt})^2$

$f_i = \text{eye}(4, 4)$

$f_i(4, 4) = 0$

```
Jaume@Portatil_Jaume:/cygdrive/c/gLAB/wi... - □ ×
ining/WORK_FILES octave
octave:1> load M300.dat
octave:2> load M600.dat
octave:3> load M900.dat
octave:4> x0=[0 0 0 0]
x0 =

    0
    0
    0
    0

octave:5> sigma0=3e5
sigma0 = 300000
octave:6> P0=(sigma0)^2*eye(4,4)
P0 =

Diagonal Matrix

    9.0000e+10    0    0    0
    0    9.0000e+10    0    0
    0    0    9.0000e+10    0
    0    0    0    9.0000e+10

octave:7> sig_dt=3e5
sig_dt = 300000
octave:8> Q=zeros(4,4)
Q =

    0    0    0    0
    0    0    0    0
    0    0    0    0
    0    0    0    0

octave:9> Q(4,4)=(sig_dt)^2
Q =

    0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    0.0000e+00    0.0000e+00    0.0000e+00    9.0000e+10

octave:10> fi=eye(4,4)
fi =

Diagonal Matrix

    1    0    0    0
    0    1    0    0
    0    0    1    0
    0    0    0    1

octave:11> fi(4,4)=0
fi =

Diagonal Matrix

    1    0    0    0
    0    1    0    0
    0    0    1    0
    0    0    0    0

octave:12>
```

```

y1=M300(:,1);
G1=M300(:,2:5);

sigma_y=1
R1=(sigma_y)^2*eye(size(y1),size(y1))

y2=M600(:,1);
G2=M600(:,2:5);

sigma_y=1
R2=(sigma_y)^2*eye(size(y2),size(y2))

y3=M900(:,1);
G3=M900(:,2:5);

sigma_y=1
R3=(sigma_y)^2*eye(size(y3),size(y3))

```

```

Jaume@Portatil_Jaume:/cygdrive/c/gLAB/wi...
octave:12> Y1=M300(:,1);
octave:13> G1=M300(:,2:5);
octave:14> sigma_y=1
sigma_y = 1
octave:15> R1=(sigma_y)^2*eye(size(Y1),size(Y1))
R1 =

Diagonal Matrix

   1   0   0   0   0   0   0   0
   0   1   0   0   0   0   0   0
   0   0   1   0   0   0   0   0
   0   0   0   1   0   0   0   0
   0   0   0   0   1   0   0   0
   0   0   0   0   0   1   0   0
   0   0   0   0   0   0   1   0
   0   0   0   0   0   0   0   1

octave:16> Y2=M600(:,1);
octave:17> G2=M600(:,2:5);
octave:18> sigma_y=1
sigma_y = 1
octave:19> R2=(sigma_y)^2*eye(size(Y2),size(Y2))
R2 =

Diagonal Matrix

   1   0   0   0   0   0   0   0
   0   1   0   0   0   0   0   0
   0   0   1   0   0   0   0   0
   0   0   0   1   0   0   0   0
   0   0   0   0   1   0   0   0
   0   0   0   0   0   1   0   0
   0   0   0   0   0   0   1   0
   0   0   0   0   0   0   0   1

octave:20> Y3=M900(:,1);
octave:21> G3=M900(:,2:5);
octave:22> sigma_y=1
sigma_y = 1
octave:23> R3=(sigma_y)^2*eye(size(Y3),size(Y3))
R3 =

Diagonal Matrix

   1   0   0   0   0   0   0   0
   0   1   0   0   0   0   0   0
   0   0   1   0   0   0   0   0
   0   0   0   1   0   0   0   0
   0   0   0   0   1   0   0   0
   0   0   0   0   0   1   0   0
   0   0   0   0   0   0   1   0
   0   0   0   0   0   0   0   1

```

## ii. Computations:

### • First iteration (t=300)

$x1\_ = fi * x0$

$P1\_ = fi * P0 * fi' + Q$

$P1 = inv(G1' * inv(R1) * G1 + inv(P1\_))$

$x1 = P1 * (G1' * inv(R1) * y1 + inv(P1\_)*x1\_)$

### • Second iteration (t=600)

$x2\_ = fi * x1$

$P2\_ = fi * P1 * fi' + Q$

$P2 = inv(G2' * inv(R2) * G2 + inv(P2\_))$

$x2 = P2 * (G2' * inv(R2) * y2 + inv(P2\_)*x2\_)$

### • Third iteration (t=900)

$x3\_ = fi * x2$

$P3\_ = fi * P2 * fi' + Q$

$P3 = inv(G3' * inv(R3) * G3 + inv(P3\_))$

$x3 = P3 * (G3' * inv(R3) * y3 + inv(P3\_)*x3\_)$

exit

```
Jaume@Portatil_Jaume:/cygdrive/c/gLAB... - □ ×
octave:24> x1_=fi*x0
x1_ =
    0
    0
    0
    0

octave:25> P1_=fi*P0*fi'+Q
P1_ =
    9.0000e+10    0.0000e+00    0.0000e+00    0.0000e+00
    0.0000e+00    9.0000e+10    0.0000e+00    0.0000e+00
    0.0000e+00    0.0000e+00    9.0000e+10    0.0000e+00
    0.0000e+00    0.0000e+00    0.0000e+00    9.0000e+10

octave:26> P1=inv(G1'*inv(R1)*G1+inv(P1_))
P1 =
    2.14078    0.11270    0.92176    1.31834
    0.11270    0.32412    0.13636    0.10867
    0.92176    0.13636    1.29029    0.94834
    1.31834    0.10867    0.94834    1.09910

octave:27> x1=P1*(G1'*inv(R1)*Y1+inv(P1_)*x1_)
x1 =
    2.659434
   -0.069009
    3.786612
   -4.091662

octave:28> x2_=fi*x1
x2_ =
    2.65943
   -0.06901
    3.78661
   -0.00000

octave:29> P2_=fi*P1*fi'+Q
P2_ =
    2.1408e+00    1.1270e-01    9.2176e-01    0.0000e+00
    1.1270e-01    3.2412e-01    1.3636e-01    0.0000e+00
    9.2176e-01    1.3636e-01    1.2903e+00    0.0000e+00
    0.0000e+00    0.0000e+00    0.0000e+00    9.0000e+10

octave:30> P2=inv(G2'*inv(R2)*G2+inv(P2_))
P2 =
    1.079337    0.062567    0.442905    0.651389
    0.062567    0.161736    0.065505    0.056361
    0.442905    0.065505    0.611456    0.446232
    0.651389    0.056361    0.446232    0.592655

octave:31> x2=P2*(G2'*inv(R2)*Y2+inv(P2_)*x2_)
x2 =
    2.48617
    0.12145
    3.63518
   -4.28497
```

```

Jaume@Portatil_Jaume:/cygdrive/c/gLAB... - □ ×
octave:24> x1_=fi*x0
x1_ =
    0
    0
    0
    0

octave:25> P1_=fi*P0*fi'+Q
P1_ =
    9.0000e+10    0.0000e+00    0.0000e+00    0.0000e+00
    0.0000e+00    9.0000e+10    0.0000e+00    0.0000e+00
    0.0000e+00    0.0000e+00    9.0000e+10    0.0000e+00
    0.0000e+00    0.0000e+00    0.0000e+00    9.0000e+10

octave:26> P1=inv(G1'*inv(R1)*G1+inv(P1_))
P1 =
    2.14078    0.11270    0.92176    1.31834
    0.11270    0.32412    0.13636    0.10867
    0.92176    0.13636    1.29029    0.94834
    1.31834    0.10867    0.94834    1.09910

octave:27> x1=P1*(G1'*inv(R1)*Y1+inv(P1_)*x1_)
x1 =
    2.659434
   -0.069009
    3.786612
   -4.091662

octave:28> x2_=fi*x1
x2_ =
    2.65943
   -0.06901
    3.78661
   -0.00000

octave:29> P2_=fi*P1*fi'+Q
P2_ =
    2.1408e+00    1.1270e-01    9.2176e-01    0.0000e+00
    1.1270e-01    3.2412e-01    1.3636e-01    0.0000e+00
    9.2176e-01    1.3636e-01    1.2903e+00    0.0000e+00
    0.0000e+00    0.0000e+00    0.0000e+00    9.0000e+10

octave:30> P2=inv(G2'*inv(R2)*G2+inv(P2_))
P2 =
    1.079337    0.062567    0.442905    0.651389
    0.062567    0.161736    0.065505    0.056361
    0.442905    0.065505    0.611456    0.446232
    0.651389    0.056361    0.446232    0.592655

octave:31> x2=P2*(G2'*inv(R2)*Y2+inv(P2_)*x2_)
x2 =
    2.48617
    0.12145
    3.63518
   -4.28497

```

```

Jaume@Portatil_Jaume:/cygdrive/c/gLAB... - □ ×
octave:32> x3_=fi*x2
x3_ =
    2.48617
    0.12145
    3.63518
   -0.00000

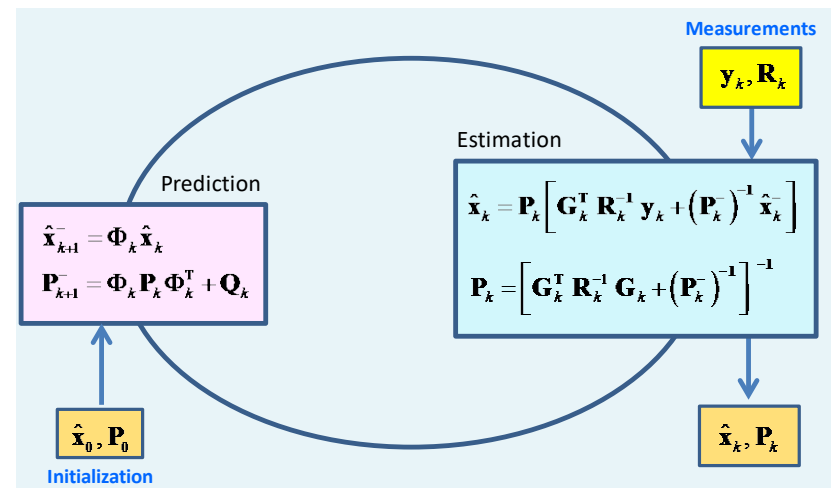
octave:33> P3_=fi*P2*fi'+Q
P3_ =
    1.0793e+00    6.2567e-02    4.4290e-01    0.0000e+00
    6.2567e-02    1.6174e-01    6.5505e-02    0.0000e+00
    4.4290e-01    6.5505e-02    6.1146e-01    0.0000e+00
    0.0000e+00    0.0000e+00    0.0000e+00    9.0000e+10

octave:34> P3=inv(G3'*inv(R3)*G3+inv(P3_))
P3 =
    0.724451    0.045809    0.283082    0.428130
    0.045809    0.107696    0.041886    0.038868
    0.283082    0.041886    0.386279    0.279090
    0.428130    0.038868    0.279090    0.423394

octave:35> x3=P3*(G3'*inv(R3)*Y3+inv(P3_)*x3_)
x3 =
    2.29035
    0.17603
    3.86966
   -3.94417

octave:36> exit

```



# Cross-checking results with gLAB:

## Coordinates (x,y,z)

```
grep OUTPUT gLAB.out | grep -v INFO > gLAB.tmp1
```

```
gawk '{if ($4==300) print $9,$10,$11}' gLAB.tmp1  
➔ 2.6593 -0.0689 3.7866
```

```
gawk '{if ($4==600) print $9,$10,$11}' gLAB.tmp1  
➔ 2.4861 0.1214 3.6351
```

```
gawk '{if ($4==900) print $9,$10,$11}' gLAB.tmp1  
➔ 2.2904 0.1760 3.8697
```

## Clock (dt)

```
grep FILTER gLAB.out | grep -v INFO > gLAB.tmp2
```

```
gawk '{if ($4==300) print $8}' gLAB.tmp2 ➔ -4.0917
```

```
gawk '{if ($4==600) print $8}' gLAB.tmp2 ➔ -4.2850
```

```
gawk '{if ($4==900) print $8}' gLAB.tmp2 ➔ -3.9442
```



# Homework. Predicted Accuracy: Dilution Of Precision

## Exercise 4:

Using navigation file **UPC11490.05N**, compute the **Dilution Of Precision** at time **t = 300** seconds: GDOP, PDOP, TDOP, HDOP, VDOP.

Complete the next steps:

1.- Using previous results of exercises 1 and 2, compute the Geometry matrix **G** in (x,y,z,t) and (e,n,u,t) coordinates at **t = 300** seconds :

$$\mathbf{G}_{xyzt} = \begin{bmatrix} \frac{x_{0,rec} - x^{sat1}}{\rho_{0,rec}^{sat1}} & \frac{y_{0,rec} - y^{sat1}}{\rho_{0,rec}^{sat1}} & \frac{z_{0,rec} - z^{sat1}}{\rho_{0,rec}^{sat1}} & 1 \\ \frac{x_{0,rec} - x^{sat2}}{\rho_{0,rec}^{sat2}} & \frac{y_{0,rec} - y^{sat2}}{\rho_{0,rec}^{sat2}} & \frac{z_{0,rec} - z^{sat2}}{\rho_{0,rec}^{sat2}} & 1 \\ \dots\dots\dots & \dots\dots\dots & \dots\dots\dots & \dots\dots\dots \\ \frac{x_{0,rec} - x^{satn}}{\rho_{0,rec}^{satn}} & \frac{y_{0,rec} - y^{satn}}{\rho_{0,rec}^{satn}} & \frac{z_{0,rec} - z^{satn}}{\rho_{0,rec}^{satn}} & 1 \end{bmatrix}$$

$$\mathbf{G}_{enut} = \begin{bmatrix} -\cos el^1 \sin az^1 & -\cos el^1 \cos az^1 & -\sin el^1 & 1 \\ -\cos el^2 \sin az^2 & -\cos el^2 \cos az^2 & -\sin el^2 & 1 \\ \dots\dots\dots & \dots\dots\dots & \dots\dots\dots & \dots\dots\dots \\ -\cos el^n \sin az^n & -\cos el^n \cos az^n & -\sin el^n & 1 \end{bmatrix}$$

## 2.- Compute GDOP, PDOP, TDOP, HDOP, VDOP at time **t = 300**

- *Geometric Dilution Of Precision:*

$$\text{GDOP} = \sqrt{q_{xx} + q_{yy} + q_{zz} + q_{tt}}$$

- *Position Dilution Of Precision:*

$$\text{PDOP} = \sqrt{q_{xx} + q_{yy} + q_{zz}}$$

- *Time Dilution Of Precision:*

$$\text{TDOP} = \sqrt{q_{tt}}$$

- *Horizontal Dilution Of Precision:*

$$\text{HDOP} = \sqrt{q_{ee} + q_{nn}}$$

- *Vertical Dilution Of Precision:*

$$\text{VDOP} = \sqrt{q_{uu}}$$

$$\mathbf{Q}_{xyzt} = (\mathbf{G}_{xyzt}^T \mathbf{G}_{xyzt})^{-1} = \begin{pmatrix} q_{xx} & q_{xy} & q_{xz} & q_{xt} \\ q_{xy} & q_{yy} & q_{yz} & q_{yt} \\ q_{xz} & q_{yz} & q_{zz} & q_{zt} \\ q_{xt} & q_{yt} & q_{zt} & q_{tt} \end{pmatrix}$$

$$\mathbf{Q}_{enut} = (\mathbf{G}_{enut}^T \mathbf{G}_{enut})^{-1} = \begin{pmatrix} q_{ee} & q_{en} & q_{eu} & q_{et} \\ q_{en} & q_{nn} & q_{na} & q_{nt} \\ q_{eu} & q_{nu} & q_{uu} & q_{ut} \\ q_{et} & q_{nt} & q_{ut} & q_{tt} \end{pmatrix}$$

### 3.- Check the coordinate transformation

Let  $\mathbf{R}$  be the transformation matrix of (e,n,u,t) coordinates to (x,y,z,t)

$$\mathbf{R} = \begin{pmatrix} -\sin \lambda & -\sin \varphi \cos \lambda & \cos \varphi \cos \lambda & 0 \\ \cos \lambda & -\sin \varphi \sin \lambda & \cos \varphi \sin \lambda & 0 \\ 0 & \cos \varphi & \sin \varphi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

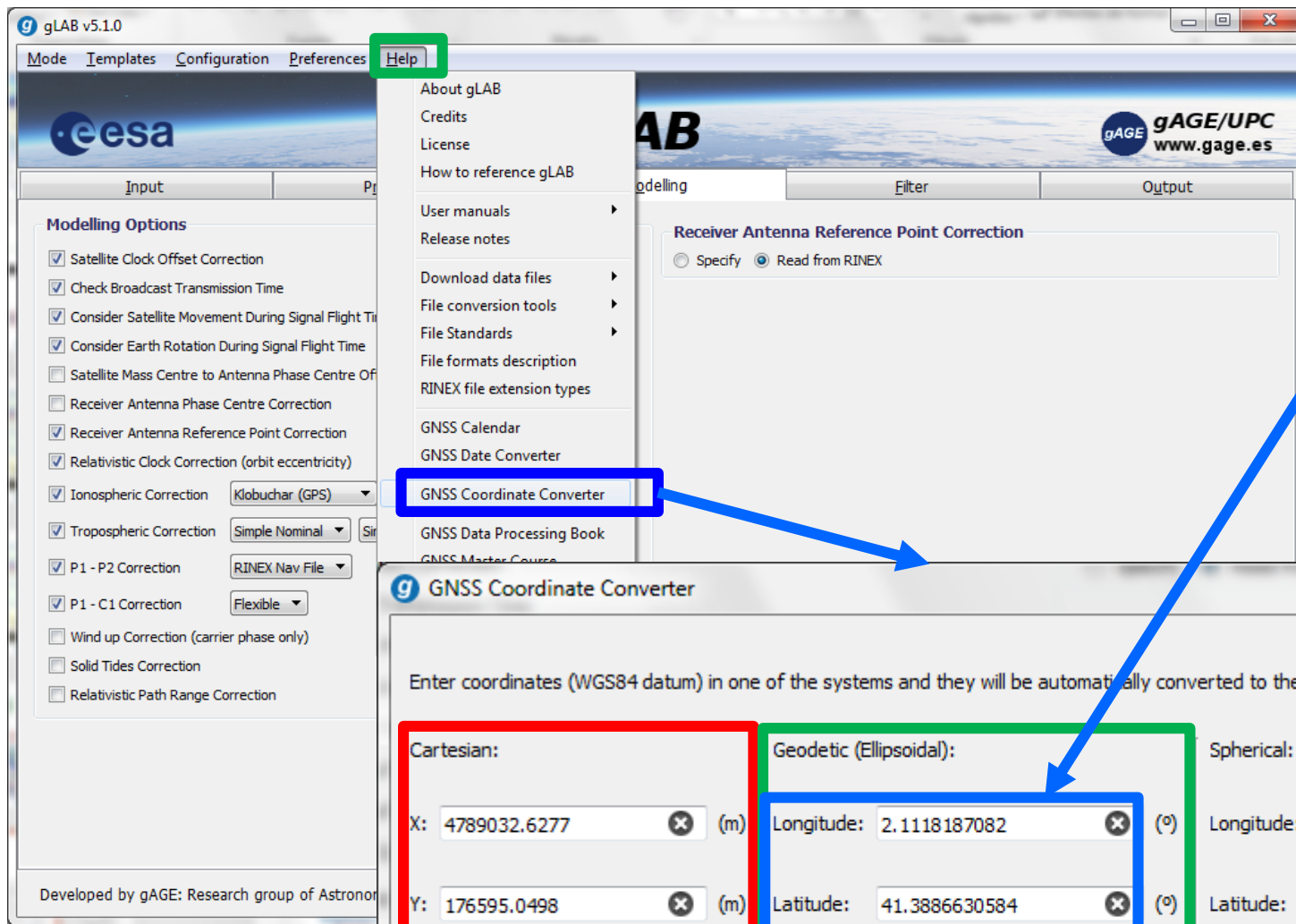
Verify that the estimated coordinates in exercises 1 and 2 fulfill next relationship:

$$\mathbf{R} \begin{pmatrix} e \\ n \\ u \\ t \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$$

Verify the transformation:

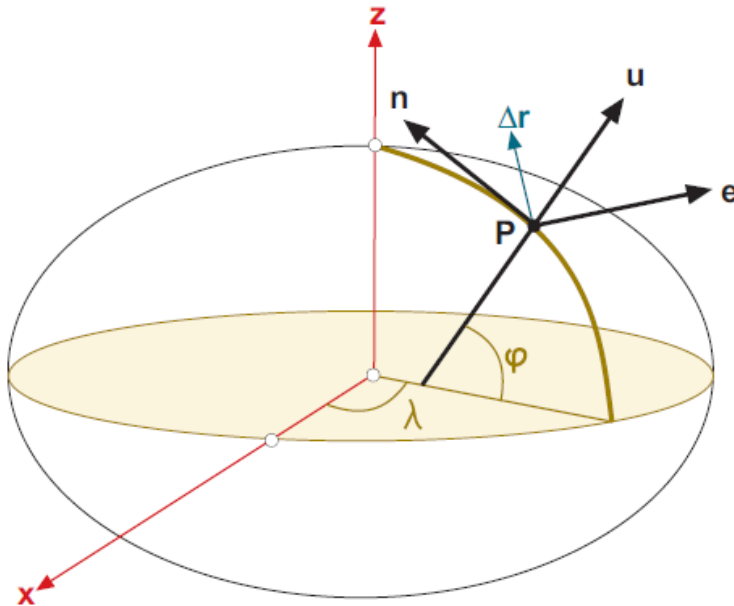
$$\mathbf{Q}_{enut} = \mathbf{R}^T \mathbf{Q}_{xyzt} \mathbf{R}$$

Justify the matrix  $\mathbf{R}$  and previous expressions. Crosscheck results with gLAB.



Hint: use gLAB converter to compute the latitude and longitude from (x,y,z) coordinates

# From ECEF (x,y,z) to Local (e,n,u) coordinates



$$\begin{bmatrix} \Delta e \\ \Delta n \\ \Delta u \end{bmatrix} = \mathbf{R}_1[\pi/2 - \varphi] \mathbf{R}_3[\pi/2 + \lambda] \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}$$

$$\hat{\mathbf{e}} = (-\sin \lambda, \cos \lambda, 0)$$

$$\hat{\mathbf{n}} = (-\cos \lambda \sin \varphi, -\sin \lambda \sin \varphi, \cos \varphi)$$

$$\hat{\mathbf{u}} = (\cos \lambda \cos \varphi, \sin \lambda \cos \varphi, \sin \varphi)$$

$$\begin{bmatrix} \Delta e \\ \Delta n \\ \Delta u \end{bmatrix} = \begin{bmatrix} -\sin \lambda & \cos \lambda & 0 \\ -\cos \lambda \sin \varphi & -\sin \lambda \sin \varphi & \cos \varphi \\ \cos \lambda \cos \varphi & \sin \lambda \cos \varphi & \sin \varphi \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}$$

# Thank you

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Bing

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  - GNSS Course and associated Tutorials
  - GNSS Format Descriptions
  - ▶ GNSS Webinars
- ▶ Software Tools

### Projects

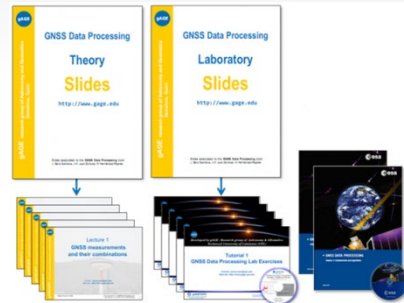
- ▶ gAGE/UPC
- ▶ gAGE-NAV, S.L.

### Patents

- WARTK
- Fast-PPP
- Iono. Corrections
- Iono. Disturb. Mitig.
- Receiver orientation

## GNSS Tutorials

- **GNSS Course** (associated to the **GNSS Data Processing Book**)
  - About the course
  - **GNSS Data Processing: Theory Slides (Full compendium)**
    - Lecture 0: Introduction
    - Lecture 1: GNSS measurements and their combinations
    - Lecture 2: Satellite orbits and clocks computation accuracy
    - Lecture 3: Position estimation with pseudoranges
    - Lecture 4: Introduction to DGNSS
    - Lecture 5: Precise positioning with carrier phase (PPP)
    - Lecture 6: Differential positioning with code pseudoranges
    - Lecture 7: Carrier based differential positioning. Ambiguity resolution techniques
  - **GNSS Data Processing: Laboratory Exercises (Full compendium)**
    - Tutorial 0: UNIX enviroment, tools and skills. GNSS standard file formats [Format files decription]
    - Tutorial 1: GNSS data processing laboratory exercises
    - Tutorial 2: Measurement analysis and error budget
    - Tutorial 3: Differential positioning with code measurements
    - Tutorial 4: Carrier ambiguity fixing
    - Tutorial 5: Analysis of propagation effects from GNSS observables based on laboratory exercises
    - Tutorial 6: Differential positioning and carrier ambiguity fixing
- Associated **Software and Data Files (Linux)**
  - CDROM zipped tar file. How to install the CDROM [Linux]
  - CDROM ISO. How to install the CDROM [Linux]
- Associated **Software and Data Files (Windows)**
  - **Instalable Toolkit (gLAB + Cygwin)**
  - **Data Files**
  - How to install the Software
- **Bootable USB stick (Linux live)**
  - **gAGE-GLUE** (to build-up a botable USB stick). How to burn the gAGE-GLUE. **How to use the bootable USB stick.**
  - **How to start-up the laboratory session.**
- **Useful tools for Windows:** Windows users can install the next ports of Linux tools (instead of Cygwin) at [gnuwin32.sourceforge.net/packages.html](http://gnuwin32.sourceforge.net/packages.html):



### About us

*gAGE is a research group of the Technical University of Catalonia (UPC). UPC is a public university located in Barcelona, Spain.*

### gAGE Brochure

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- GNSS Data Processing Book
- **GNSS Course and associated Tutorials**
- GNSS Webinars
- gLAB Tool Suite
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- Master MAST (UPC)
- Master Of Science (ENAC)
- gAGE upload file facility

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- Log in using OpenID
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# Acknowledgements

- The ESA/UPC GNSS-Lab Tool suit (gLAB) has been developed under the ESA Education Office contract N. P1081434.
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