FORMULAE FOR THE RESOLUTION OF FLUID DYNAMICS AND HEAT AND MASS TRANSFER PROBLEMS

CONTENTS:

A. BASIC MATHEMATICAL RELATIONS

A1. Mass, momentum and energy transport equations in integral form. Moving control volumes:

$$\frac{d}{dt}\int_{V_a(t)} \rho dV + \int_{S_a(t)} \rho(\vec{v}-\vec{v}_b)\cdot\vec{n}dS = 0 \tag{1.1}$$

$$\frac{d}{dt}\int_{V_a(t)} \vec{v}\rho dV + \int_{S_a(t)} \vec{v}\rho(\vec{v}-\vec{v}_b)\cdot\vec{n}dS = \int_{S_a(t)} \vec{f}_{(\vec{n})}dS + \int_{V_a(t)} \vec{g}\rho dV \tag{1.2}$$

$$\frac{d}{dt}\int_{V_a(t)} (u+e_k)\rho dV + \int_{S_a(t)} (u+e_k)\rho(\vec{v}-\vec{v}_b)\cdot\vec{n}dS = -\int_{S_a(t)} \vec{q}\cdot\vec{n}dS + \int_{S_a(t)} \vec{v}\cdot\vec{f}_{(\vec{n})}dS + \int_{V_a(t)} \vec{v}\cdot\vec{g}\rho dV \tag{1.3}$$

Assuming static control volumes ($\vec{v}_b = 0$):

$$\frac{\partial}{\partial t}\int_{V_a} \rho dV + \int_{S_a} \rho\vec{v}\cdot\vec{n}dS = 0 \tag{1.4}$$

$$\frac{\partial}{\partial t}\int_{V_a} \vec{v}\rho dV + \int_{S_a} \vec{v}\rho\vec{v}\cdot\vec{n}dS = \int_{S_a} \vec{f}_{(\vec{n})}dS + \int_{V_a} \vec{g}\rho dV \tag{1.5}$$

$$\frac{\partial}{\partial t}\int_{V_a} (u+e_k)\rho dV + \int_{S_a} (u+e_k)\rho\vec{v}\cdot\vec{n}dS = -\int_{S_a} \vec{q}^{C+R}\cdot\vec{n}dS + \int_{S_a} \vec{v}\cdot\vec{f}_{(\vec{n})}dS + \int_{V_a} \vec{v}\cdot\vec{g}\rho dV \tag{1.6}$$

The energy equation (1.6) can also be rewritten as:

$$\frac{\partial}{\partial t}\int_{V_a} \left(h-\frac{p}{\rho}+e_k+e_p\right)\rho dV + \int_{S_a} (h+e_k+e_p)\rho\vec{v}\cdot\vec{n}dS = -\int_{S_a} \vec{q}^{C+R}\cdot\vec{n}dS + \int_{S_a} \vec{v}\cdot\vec{f}^{\tau}_{(\vec{n})}dS \tag{1.7}$$

where $\vec{f}^{\tau}_{(\vec{n})}$ accounts only for the viscous force vector (per unit surface).

Other important transport equations:

Entropy transport equation: $\frac{\partial}{\partial t}\int_{V_a} s\rho dV + \int_{S_a} s\rho\vec{v}\cdot\vec{n}dS = -\int_{S_a}\frac{\vec{q}}{T}\cdot\vec{n}dS + \int_{V_a}\dot{s}_{gen}dV \quad (\dot{s}_{gen}\geq 0)$ (1.8)

Transport equation of species k: $\frac{\partial}{\partial t}\int_{V_a} Y_k\rho dV + \int_{S_a} Y_k\rho\vec{v}\cdot\vec{n}dS = -\int_{S_a}\vec{j}_k\cdot\vec{n}dS + \int_{V_a}\dot{\omega}_k dV$ (1.9)

A2. Basic transport equations in differential form (Navier-Stokes equations)

In case of **forced convection**, assuming Newtonian fluid, constant density and viscosity, non-participating radiative medium and negligible viscous dissipation:

$$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}=0 \tag{2.1}$$

$$\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}=-\frac{1}{\rho_o}\frac{\partial p}{\partial x}+\frac{\mu_o}{\rho_o}\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)+g_x \tag{2.2}$$

$$\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}=-\frac{1}{\rho_o}\frac{\partial p}{\partial y}+\frac{\mu_o}{\rho_o}\left(\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_y}{\partial z^2}\right)+g_y \tag{2.3}$$

$$\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}=-\frac{1}{\rho_o}\frac{\partial p}{\partial z}+\frac{\mu_o}{\rho_o}\left(\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+g_z \tag{2.4}$$

$$\rho_o c_p\left(\frac{\partial T}{\partial t}+v_x\frac{\partial T}{\partial x}+v_y\frac{\partial T}{\partial y}+v_z\frac{\partial T}{\partial z}\right)=\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right) \tag{2.5}$$

For **natural or mixed convection**, assuming the aforementioned hypothesis, except for the influence of the temperature on density variations in the buoyancy terms of the momentum equations (Boussinesq approach):

$$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}=0 \tag{2.6}$$

$$\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}=-\frac{1}{\rho_o}\frac{\partial p}{\partial x}+\frac{\mu_o}{\rho_o}\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)+g_x-\beta_o(T-T_o)g_x \tag{2.7}$$

$$\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}=-\frac{1}{\rho_o}\frac{\partial p}{\partial y}+\frac{\mu_o}{\rho_o}\left(\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_y}{\partial z^2}\right)+g_y-\beta_o(T-T_o)g_y \tag{2.8}$$

$$\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}=-\frac{1}{\rho_o}\frac{\partial p}{\partial z}+\frac{\mu_o}{\rho_o}\left(\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+g_z-\beta_o(T-T_o)g_z \tag{2.9}$$

$$\rho_o c_p\left(\frac{\partial T}{\partial t}+v_x\frac{\partial T}{\partial x}+v_y\frac{\partial T}{\partial y}+v_z\frac{\partial T}{\partial z}\right)=\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right) \tag{2.10}$$

In this case, it is usual to merge the gravity term $\vec{g}$ with the pressure gradient term. Consequently, the dynamic pressure p_d appears instead of the thermodynamic pressure p. Hence, equation (2.7) now reads as:

$$\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}=-\frac{1}{\rho_o}\frac{\partial p_d}{\partial x}+\frac{\mu_o}{\rho_o}\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)-\beta_o(T-T_o)g_x \tag{2.11}$$

In the same way, equations (2.8) and (2.9) can be rewritten.

For **gases at high velocity**, the Navier-Stokes equations, assuming semi-perfect gas behaviour, can be expressed in vectorial notation as:

$$\frac{\partial \rho}{\partial t}+\nabla\cdot(\rho\vec{v})=0 \tag{2.12}$$

$$\rho\left(\frac{\partial \vec{v}}{\partial t}+\vec{v}\cdot\nabla\vec{v}\right)=-\nabla p+\nabla\cdot\vec{\tau}+\rho\vec{g} \tag{2.13}$$

$$\rho c_v\left(\frac{\partial T}{\partial t}+\vec{v}\cdot\nabla T\right)=\nabla\cdot(\lambda\nabla T)-\nabla\cdot\vec{q}^R-p\nabla\cdot\vec{v}+\vec{\tau}:\nabla\vec{v} \tag{2.14}$$

$$p=\rho RT \tag{2.15}$$

where, $\vec{\tau}=\mu(\nabla\vec{v}+\nabla\vec{v}^T)-\frac{2}{3}\mu(\nabla\cdot\vec{v})\vec{\delta}$.

The kinetic energy equation can be written as: $\rho\left(\frac{\partial e_k}{\partial t}+\vec{v}\cdot\nabla e_k\right)=-\vec{v}\cdot\nabla p+\vec{v}\cdot\nabla\cdot\vec{\tau}+\rho\vec{v}\cdot\vec{g}$ (2.16)

For solids, the energy equation is simply: $\rho c_p\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right)+\dot{q}_v$ (2.17)

A3. Analytical solutions for conduction heat transfer in walls and extended surfaces (fins)

In this section, one-dimensional steady-state temperature profiles, and constant thermophysical properties and $\dot{q}_v$ are considered. For the case of extended surfaces, it is assumed constant external temperature and convective heat transfer coefficient (T_g and α_g) in the fin. The temperature profiles and heat fluxes are presented as follows:

3.1 **Plane walls**: $T = -\frac{\dot{q}_v}{2\lambda}x^2 + C_1 x + C_2; \quad \dot{q}_x = \dot{q}_v x - \lambda C_1$ (3.1)

3.2 **Cylindrical walls**: $T = -\frac{\dot{q}_v}{4\lambda}r^2 + C_1 ln(r) + C_2; \quad \dot{q}_r = \frac{1}{2}\dot{q}_v r - \lambda\frac{C_1}{r}$ (3.2)

3.3 **Spherical walls**: $T = -\frac{\dot{q}_v}{6\lambda}r^2 + \frac{C_1}{r} + C_2; \quad \dot{q}_r = \frac{1}{3}\dot{q}_v r + \lambda\frac{C_1}{r^2}$ (3.3)

3.4 **Fins with constant cross-section**: $T - T_{ext} = C_1 e^{mx} + C_2 e^{-mx}; \quad \dot{q}_x = -\lambda_f m(C_1 e^{mx} - C_2 e^{-mx})$ (3.4)

where $m = \sqrt{\alpha_{ext} P_f/(\lambda_f S_f)}$; P_f is the perimeter of the fin; S_f is the cross-section of the fin, . α_{ext} is the external heat transfer coefficient.

Heat flux delivered by the fin: $\dot{Q}_f = \eta_f \alpha_{ext}(T_w - T_{ext})A_f$ (3.5)

where the efficiency of the fin is evaluated from: $\eta_f = \frac{th[mL_f]}{mL_f}$ (assuming adiabatic fin end), L_f is the length of the fin, A_f is the heat transfer surface ($A_f = P_f L_f$), T_w is the temperature of the fin at its base, and T_{ext} is the external temperature.

3.5 **Circular fins**: $T - T_{ext} = C_1 I_o(mr) + C_2 K_o(mr); \quad \dot{q}_r = -\lambda_f\frac{dT}{dr} = -\lambda_f m[C_1 I_1(mr) - C_2 K_1(mr)]$ (3.6)

where I_o and K_o are the modified zero order Bessel functions of first and second class, respectively. See Table B on page 6.

Heat flux delivered by the fin: $\dot{Q}_f = \eta_f \alpha_{ext}(T_w - T_{ext})A_f$ (3.7)

where $\eta_f \approx \frac{th[mR_i\phi]}{mR_i\phi}$ (assuming adiabatic fin end), $\phi = \left(\frac{R_e}{R_i} - 1\right)\left[1 + 0.35 ln\left(\frac{R_e}{R_i}\right)\right]$, $A_f = 2\pi(R_e^2 - R_i^2)$, $m = \sqrt{2\alpha_{ext}/(\lambda_f e_f)}$, being R_i, R_e and e_f the inner fin radius, outer fin radius and fin thickness, respectively.

A4. Analytical solutions for conduction heat transfer in transient problems

4.1 **Plate of thickness $2e$**. Unsteady-state (heating/cooling) of a flat plate with initial uniform temperature T_o. The plate is suddenly submerged into an atmosphere at temperature T_{ext} (which is considered constant throughout the process). The convective heat transfer coefficient α_{ext} is constant. Thermophysical properties of the material are considered constant as well ($\rho, c_p, \lambda, a = \lambda/\rho c_p$). The temperature follows:

$$\Phi = \sum_{k=1}^{\infty}\frac{2sin(u_k)}{u_k + sin(u_k)cos(u_k)}cos(u_k X)e^{-u_k^2 Fo} \quad (4.1)$$

where, $\Phi = \frac{\mathrm{T}-\mathrm{T_{ext}}}{\mathrm{T_o}-\mathrm{T_{ext}}}$, $X = \frac{x}{e}$, $Fo = \frac{at}{e^2}$. The variables u_k are the solutions to equation $cot(u) = u/Bi$, with $Bi = \alpha_{ext}e/\lambda$.

4.2 **Cylinder of radius r_o**. The analysis of the problem is the same as the previous case. The temperature profile is obtained from:

$$\Phi = \sum_{k=1}^{\infty}\frac{2I_1(u_k)}{u_k[I_o^2(u_k) + I_1^2(u_k)]}I_o^2(u_k R)e^{-u_k^2 Fo} \quad (4.2)$$

where, I_o is a first-class zero-order Bessel function, and I_1 is a first-class first-order; $\Phi = \frac{\mathrm{T}-\mathrm{T_{ext}}}{\mathrm{T_o}-\mathrm{T_{ext}}}$, $R = \frac{r}{r_o}$, $Fo = \frac{at}{r_o^2}$, and u_k are the solutions to the equation $I_o(u)/I_1(u) = u/Bi$, with $Bi = \alpha_{ext}r_o/\lambda$.

4.3 **Sphere of radius r_o**. For this case, the following temperature profile is obtained:

$$\Phi = \sum_{k=1}^{\infty} \frac{2sin(u_k R)[sen(u_k) - u_k cos(u_k)]}{u_k R[u_k - sen(u_k)cos(u_k)]} e^{-u_k^2 Fo} \quad (4.3)$$

where, $\Phi = \frac{\mathrm{T-T_{ext}}}{\mathrm{T_o - T_{ext}}}$, $R = \frac{r}{r_o}$, $Fo = \frac{at}{r_o^2}$, and u_k are solutions of $tan(u) = -u/(Bi-1)$; $Bi = \alpha_{ext} r_o/\lambda$.

A5. Radiation and atmosphere conditions

5.1 **Snell's law**: $n_{A,\nu} sin\beta_A = n_{B,\nu} sin\beta_B$ (5.1)

where β_A and β_B are the angles of incidence and refraction, respectively, and $n_{A,\nu}$ and $n_{B,\nu}$ are the refracted indices of medium A and B, at frequency ν.

5.2 **Stefan-Boltzmann's constant**: $\sigma = \frac{2\pi^5 k^4}{15h^3 c_0^2} = 5.67 \cdot 10^{-8} \frac{W}{m^2 K^4}$ (5.2)

5.3 **Wien's law** for black body radiation: $(\lambda T)_{max.power} = 2897.8\ \mu mK$ (5.3)

5.4 **Solid angle differential**: $d\omega = sin\theta d\theta d\varphi$ (5.4)

where θ and φ are the polar and azimuthal angles, respectively.

5.5 Generalized expression of the **view factor** between surface A_i and surface A_k:

$$F_{ik} = \frac{1}{\pi A_i} \int_{A_i} \int_{A_k} \frac{cos\theta_i cos\theta_k}{d_{ik}^2} dA_i dA_k \quad (5.5)$$

Hottel's crossed string rule. View factor F_{ij} in 2D cases: $F_{ij} = \frac{SUM\ crossed\ strings - SUM\ uncrossed\ strings}{twice\ surface\ A_i\ per\ unit\ depth}$ (5.6)

5.6 **Sky temperature**: $T_{sky} \approx 0.0552 T_{air}^{1.5}$ (simplified correlation) (5.7)

where T_{sky} and T_{air} correspond to the sky and ambient temperatures (both in K).

More accurate correlation: $T_{sky} \approx T_{air} \left[0.711 + 0.0056 T_{dp} + 7.3 \cdot 10^{-5} T_{dp}^2 + 0.013 cos\left(\frac{2\pi t}{24}\right)\right]^{\frac{1}{4}}$ (5.8)

where T_{dp} is the dew-point temperature, and t the time counted from midnight ($t = 0, 1, 2, \ldots, 24$). In this second expression, proposed by Verdal and Martin, all the temperatures are expressed in °C.

5.7 **Black body radiation. Planck's law**: $I_{b,\lambda\omega}^{(e)} = \frac{2hc^2}{\lambda^5} \cdot \frac{1}{exp\left(\frac{hc}{\lambda kT}\right) - 1}$ (5.9)

where, $c = c_o/n$, $c = \lambda\nu$; $h = 6.6261 \cdot 10^{-34}\ Js$; $k = 1.3807 \cdot 10^{-23}\ J/K$; $c_o = 2.9979 \cdot 10^8\ m/s$.

Integrating Planck's law in all directions at a given wave length λ of a body at temperature T, the spectral emissive power is obtained:

$$\dot{q}_{b,\lambda}^{(e)} = \int_{2\pi} I_{b,\lambda\omega}^{(e)} cos\theta d\omega = \pi I_{b,\lambda\omega}^{(e)} \quad (5.10)$$

Integrating Planck's law in all directions and wave lengths ($\lambda = 0\ a\ \infty$) of a body at temperature T:

$$\dot{q}_{b}^{(e)} = \int_{\lambda=0}^{\infty} \pi I_{b,\lambda\omega}^{(e)} d\lambda = \sigma T^4 \quad (5.11)$$

With the attached Table A (see page 6), the following integral can be evaluated from $\lambda = 0$ to λ:

$$\dot{q}_{b,(\lambda=0\to\lambda)}^{(e)} = \int_{\lambda=0}^{\lambda} \pi I_{b,\lambda\omega}^{(e)} d\lambda = f_{\lambda T} \sigma T^4 \quad (5.12)$$

where the fraction $f_{\lambda T}$ only depends on λT (μmK).

5.9 **Radiation in an absorbing medium. The Lambert-Beer law**

From the radiative transport equation (RTE), assuming a purely absorbing medium characterized by a spectral extinction coefficient $\beta_\nu = \kappa_\nu + \sigma_{s,\nu}$ (being κ_ν and $\sigma_{s,\nu}$ the absorption and scattering coefficients, respectively), the specific radiant intensity in a given direction x is given by: $dI_{\nu\omega}/dx = -\beta_\nu I_{\nu\omega}$. Integrating from $x = 0$ to x (assuming constant β_ν): $I_{\nu\omega} = I_{o\nu} e^{-\beta_\nu x}$ (5.13)

where $I_{o\nu}$ is the value of the specific intensity at $x = 0$.

5.10 **Reference values for temperature and pressure distribution in the atmosphere according to ISA** (*International Standard Atmosphere*). The main parameters are included in the next table. The rest of values can be computed with the attached expressions (which allows us to evaluate T and p at any altitude z).

Zone	***Zone***	***Layer k***	$\boldsymbol{z_k\ (m)}$	$\boldsymbol{\beta_k\ (K/m)}$	$\boldsymbol{T_k\ (K)\ en\ z_k}$	$\boldsymbol{p_k\ (Pa)}$	$\boldsymbol{\rho_k\ (kg/m^3)}$
Troposphere		**0**	0	$-6{,}50 \times 10^{-3}$	288.15	101325	1.2252
	Stratosphere	**1** (tropopause)	11000	0,00	216.65	22649	0.3643
		2	20000	$+1{,}00 \times 10^{-3}$	216.65	5482.8	0.0882
		3	32000	$+2{,}80 \times 10^{-3}$	228.65	870.06	0.0133
Mesosphere		**4** (stratopause)	47000	0,00	270.65	111.28	0.0014
		5	51000	$-2{,}80 \times 10^{-3}$	270.65	67.181	0.0009
		6	71000	$-2{,}00 \times 10^{-3}$	214.65	3.9763	0.0001
		7 (mesopause)	84852	-	186.95	0.3757	0.0000

In this table: z_k is the geopotential height; β_{kj} is the variation of temperature per unit of height between layer k and $k+1$; T_k is the temperature at height z_k; p_k is the pressure at position z_k.

Assuming linear temperature changes between layers: $T(z) = T_k + \beta_k (z - z_k)$ (5.14)

From a momentum balance ($dp/dz = -\rho g$), pressure is obtained: $p(z) = p_k [T_k/T(z)]^{\frac{g}{R\beta_{kj}}}$ (5.15)

where $\rho_k = p_k/RT_k$, $g = 9.8\ m/s^2$ and $R = 287\ J/kgK$ are used. In these equations, k is the reference value on the base of the layer. Calculated $p(z)$ and $T(z)$ are in the layer k and $k+1$.

A6. Numerical methods

6.1 **Thermal conductivity at a CV face (harmonic mean)**: $\lambda_f = d_{PF} / \left[\frac{d_{Pf}}{\lambda_P} + \frac{d_{fF}}{\lambda_F}\right]$ (6.1)

6.2 **Over-relaxation and under-relaxation factors**: $\phi^{*(new)} = \phi^{*(old)} + f_r\left[\phi^{equation} - \phi^{*(old)}\right]$ (6.2)

6.3 **TDMA** (Tri-Diagonal Matrix Algorithm) for solving linear discretized equations of this type:

$$a_P[i]T[i] = a_E[i]T[i+1] + a_W[i]T[i-1] + b_P[i] \qquad (6.3)$$

Two steps:

1) Evaluation from $i = 1\ to\ N$ of: $P[i] = \frac{a_E[i]}{a_P[i] - a_W[i]P[i-1]}$, and $R[i] = \frac{b_P[i] + a_W[i]R[i-1]}{a_P[i] - a_W[i]P[i-1]}$ (6.4)

2) Temperatures are obtained from $i = N\ to\ 1$: $T[i] = P[i]T[i+1] + R[i]$ (6.5)

A7. Momentum, mass and heat transfer analogy

7.1 Colburn-Chilton analogy is based on similarities between momentum, mass and heat transport mechanisms ($Re > 10^4$, $0.7 < Pr < 160$, in case of tubes $L/D > 60$):

$$f_{smooth}/2 = j_M = j_H \qquad (7.1)$$

where j_M and j_H are the Colburn-Chilton j-factor for mass and heat (see Nomenclature) defined as:

$$j_M = St_M Sc^{2/3}; \qquad j_H = St_H \mathrm{Pr}^{2/3} \qquad (7.2)$$

Table A. Black body radiation to evaluate energy emitted from $\lambda = 0$ to λ

λT (μmK)	$f_{\lambda T}$	λT (μmK)	$f_{\lambda T}$
200	0.000000	6200	0.754140
400	0.000000	6400	0.769234
600	0.000000	6600	0.783199
800	0.000016	6800	0.796129
1000	0.000321	7000	0.808109
1200	0.002134	7200	0.819217
1400	0.007790	7400	0.829527
1600	0.019718	7600	0.839102
1800	0.039341	7800	0.848005
2000	0.066728	8000	0.856288
2200	0.100888	8500	0.874608
2400	0.140256	9000	0.890029
2600	0.183120	9500	0.903085
2800	0.227897	10,000	0.914199
3000	0.273232	10,500	0.923710
3200	0.318102	11,000	0.931890
3400	0.361735	11,500	0.939959
3600	0.403607	12,000	0.945098
3800	0.443382	13,000	0.955139
4000	0.480877	14,000	0.962898
4200	0.516014	15,000	0.969981
4400	0.548796	16,000	0.973814
4600	0.579280	18,000	0.980860
4800	0.607559	20,000	0.985602
5000	0.633747	25,000	0.992215
5200	0.658970	30,000	0.995340
5400	0.680360	40,000	0.997967
5600	0.701046	50,000	0.998953
5800	0.720158	75,000	0.999713
6000	0.737818	100,000	0.999905

(Table A: Y.A.Cengel, Heat Transfer, A Practical Approach, McGraw-Hill, Boston, 1998)

(Table B: Incropera et al, Fundamental of Heat and Mass Transfer, John Whiley&Sons, 2007)

Table B. Modified Bessel Functions of the first and second kinds

x	$e^{-x}I_o(x)$	$e^{-x}I_1(x)$	$e^{x}K_0(x)$	$e^{x}K_1(x)$
0.0	1.0000	0.0000	∞	∞
0.2	0.8269	0.0823	2.1407	5.8334
0.4	0.6974	0.1368	1.6627	3.2587
0.6	0.5993	0.1722	1.4167	2.3739
0.8	0.5241	0.1945	1.2582	1.9179
1.0	0.4657	0.2079	1.1445	1.6361
1.2	0.4198	0.2152	1.0575	1.4429
1.4	0.3831	0.2185	0.9881	1.3010
1.6	0.3533	0.2190	0.9309	1.1919
1.8	0.3289	0.2177	0.8828	1.1048
2.0	0.3085	0.2153	0.8416	1.0335
2.2	0.2913	0.2121	0.8056	0.9738
2.4	0.2766	0.2085	0.7740	0.9229
2.6	0.2639	0.2046	0.7459	0.8790
2.8	0.2528	0.2007	0.7206	0.8405
3.0	0.2430	0.1968	0.6978	0.8066
3.2	0.2343	0.1930	0.6770	0.7763
3.4	0.2264	0.1892	0.6579	0.7491
3.6	0.2193	0.1856	0.6404	0.7245
3.8	0.2129	0.1821	0.6243	0.7021
4.0	0.2070	0.1787	0.6093	0.6816
4.2	0.2016	0.1755	0.5953	0.6627
4.4	0.1966	0.1724	0.5823	0.6453
4.6	0.1919	0.1695	0.5701	0.6292
4.8	0.1876	0.1667	0.5586	0.6142
5.0	0.1835	0.1640	0.5478	0.6003
5.2	0.1797	0.1614	0.5376	0.5872
5.4	0.1762	0.1589	0.5279	0.5749
5.6	0.1728	0.1565	0.5188	0.5633
5.8	0.1696	0.1542	0.5101	0.5525
6.0	0.1666	0.1520	0.5019	0.5422
6.4	0.1611	0.1479	0.4865	0.5232
6.8	0.1561	0.1441	0.4724	0.5060
7.2	0.1515	0.1405	0.4595	0.4905
7.6	0.1473	0.1372	0.4476	0.4762
8.0	0.1434	0.1341	0.4366	0.4631
8.4	0.1398	0.1312	0.4264	0.4511
8.8	0.1365	0.1285	0.4168	0.4399
9.2	0.1334	0.1260	0.4079	0.4295
9.6	0.1305	0.1235	0.3995	0.4198
10.0	0.1278	0.1213	0.3916	0.4108

$I_{n+1}(x) = I_{n-1}(x) - (2n/x)I_n(x)$

B. CONVECTION HEAT TRANSFER COEFFICIENTS

This section presents more correlations of both natural and forced convection in different situations, considering single phase (subsections B1 to B7; from H.Y.Wong, Handbook of Essential Formulae and Data on Heat Transfer for Engineers, Longman, London, 1977) and two-phase flows (subsection B8).

The attached correlations are:

- Section B1: Heat transfer coefficients in natural (or free) convection
- Section B2: Heat transfer coefficients in forced convection in ducts
- Section B3: Heat transfer coefficients in forced convection in flat plates (liquids and gases at low Mach)
- Section B4: Heat transfer coefficients in forced convection in flat plates (gases at high Mach number)
- Section B5: Heat transfer coefficients in forced convection around tubes and pipe bundles
- Section B6: Heat transfer coefficients in forced convection on rotating surfaces
- Section B7: Friction factors for flows inside ducts (single-phase)
- Section B8: Pressure drop and heat transfer coefficients in two-phase flow.

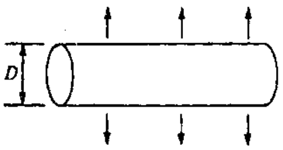
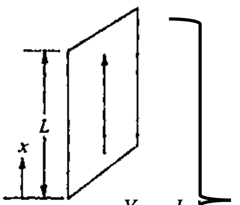
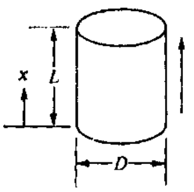
B1. Natural/free convection (1/3)

Formulae: $\overline{Nu} = CRa^n K$ (laminar: $10^3 < Ra < 10^9$; turbulent: $Ra \geq 10^9$) (property values at T_m)

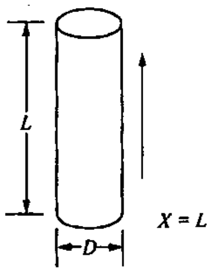
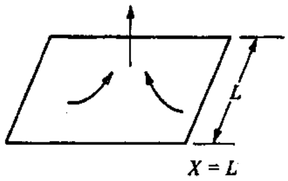
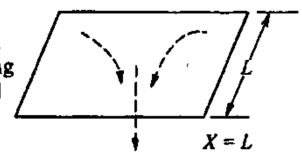
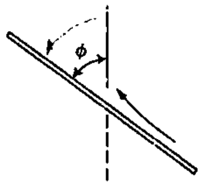
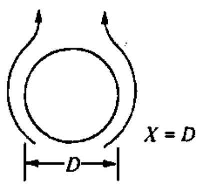
Heat flux: $\dot{q}_w = \bar{\alpha}(T_w - T_f)$

Notation:

C Constant in Nusselt equation
c_p Specific heat at constant pressure (J/kgK)
g Gravitational acceleration, $g = 9.81\ m/s^2$
Gr Grashof number, $Gr = g\beta\rho^2|T_w - T_f|X^3/\mu^2$
K Dimensionless correction function in Nusselt eq.
n Constant in Nusselt equation
$\overline{Nu}$ Mean Nusselt number, $\overline{Nu} = \bar{\alpha}X/\lambda$
$\dot{q}_w$ Heat transfer rate (W/m^2)
Pr Prandtl number, $Pr = \mu c_p/\lambda$
Ra Rayleigh number, $Ra = GrPr$
T_f Fluid bulk temperature (°C or K)
T_m Film temperature (°C or K), $T_m = (T_w + T_f)/2$
T_w Temperature of the wall (°C or K)
X Characteristic length (m)
$\bar{\alpha}$ Overall heat transfer coefficient (W/m^2K)
β Volumetric thermal expansion coefficient (K^{-1})
λ Thermal conductivity (W/mK)
μ Dynamic viscosity (kg/ms)
ρ Density (kg/m^3)

No	System	Schematic presentation	C	n	K	Operating conditions	References
	Exposed surfaces						
1	Horizontal cylinder	 $X = D$	0.47 0.1	1/4 1/3	1 1	Laminar flow Turbulent flow	3.9 3.9
2	Vertical plate or vertical cylinder of large diameter	 $X = L$  $X = L$	0.8	1/4	$\left[1+\left(1+\frac{1}{\sqrt{Pr}}\right)^2\right]^{-1/4}$	Laminar flow; to obtain local Nu, use $C = 0.6$, $X = x$: formula applicable to vertical cylinder when $\frac{D}{L} \geqslant 38\ (Gr)^{-1/4}$	3.8
			0.0246	2/5	$\left[\frac{Pr^{1/6}}{1+0.494Pr^{2/3}}\right]^{2/5}$	Turbulent flow; to obtain local Nu, use $C = 0.0296$ and $X = x$	3.10

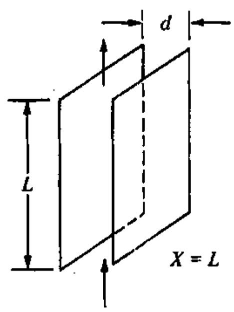
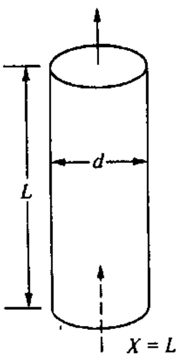
B1. Natural/free convection (2/3)

No.	System	Schematic presentation	C	n	K	Operating conditions	References
3	Vertical cylinder with small diameter		0.686	1/4	$[Pr/(1+1.05\,Pr)]^{1/4}$	Laminar flow; $\bar{Nu}_{total} = \bar{Nu} + 0.52\frac{L}{D}$ $\frac{D}{L} < 38Gr^{-1/4}$	3.13
4	Heated horizontal plate facing upward		0.54	1/4	1	Laminar flow; for circular disc of diameter D, use $X = 0.9D$	3.9
			0.14	1/3	1	Turbulent flow	3.9
5	Heated horizontal plate facing downward		0.27	1/4	1	Laminar flow only	3.20
6	Moderately inclined plate		0.8	1/4	$\left[\frac{\cos\phi}{1+\left(1+\frac{1}{\sqrt{Pr}}\right)^2}\right]^{1/4}$	Laminar flow (multiply Gr. by $\cos\phi$ in the formula for vertical plate)	
7	Sphere		0.49	1/4	1	Laminar flow (air)	3.13

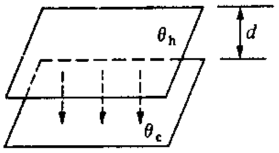
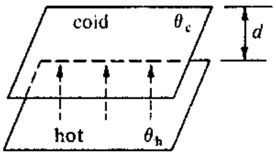
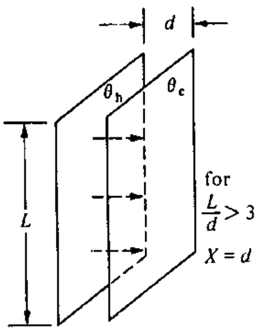
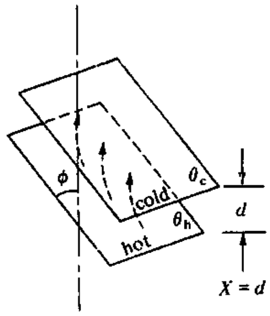
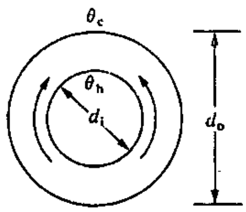
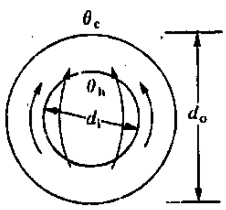
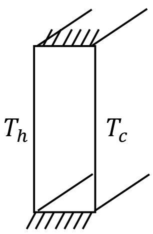
Correlation by Churchill (2002) for $Gr_D Pr < 10^{11}$ and $Pr > 0.7$:

$$Nu_D = 2 + \frac{0.589(Gr_D Pr)^{1/4}}{[1+(0.469/Pr)^{9/16}]^{4/9}}$$

Semi-closed and closed systems:

No.	System	Schematic presentation	C	n	K	Operating conditions	References
8	Two vertical parallel plates at the same temperature		0.04	1	$(d/L)^3$	Air layer	3.7
9	Hollow vertical cylinder with open ends		0.01	1	$(d/L)^3$	Air column	3.7

B1. Natural/free convection (3/3)

No.	System	Schematic presentation	C	n	K	Operating conditions	References
10	Two horizontal parallel plates hot plate uppermost	 $\Delta\theta = \theta_h - \theta_c$ $X = d$	*Note:* $\theta_h = T_h$; $\theta_c = T_c$			Pure conduction $\dot{q} = \bar{\alpha}(T_h - T_c)$	
			0.27	1/4	1	Laminar (air) $3 \times 10^5 < Gr \cdot Pr < 3 \times 10^{10}$	3.20
11	Two horizontal parallel plates cold plate uppermost	 $\Delta\theta = \theta_h - \theta_c$ $X = d$	0.195	1/4	$Pr^{-1/4}$	Laminar (air $10^4 < Gr < 4 \times 10^5$	3.19
			0.068	1/3	$Pr^{-1/3}$	Turbulent (air) $Gr > 4 \times 10^5$ $\dot{q} = \bar{\alpha}(T_h - T_c)$	3.19
12	Two vertical parallel plates at different temperatures (h for both surfaces)	 $\Delta\theta = \theta_h - \theta_c$	0.18	1/4	$(L/d)^{-1/9}(Pr)^{-1/4}$	Laminar (air) $2 \times 10^4 < Gr < 2 \times 10^5$	3.19
			0.065	1/4	$(L/d)^{-1/9}(Pr)^{-1/3}$	Turbulent (air) $2 \times 10^5 < Gr < 10^7$ $\dot{q} = \bar{\alpha}(T_h - T_c)$	3.19
			See Section C4 in case of natural convection in closed cavities with isothermal differentially heated vertical walls.				
13	Two inclined parallel plates		$\overline{Nu} = \frac{1}{2}[\overline{Nu}_{vert}\cos\phi + \overline{Nu}_{horit}\sin\phi]$				
14	Two concentric cylinders	 $X = \frac{1}{2}(d_o - d_i)$ $A = 2\pi X L$	0.317	1/4	$\left[X^3\left(\frac{1}{d_i^{3/5}} + \frac{1}{d_o^{3/5}}\right)^5\right]^{-1/4}$	Laminar flow $\dot{q} = \bar{\alpha}(T_h - T_c)$	3.48
15	Two concentric spheres	 $X = \frac{1}{2}(d_o - d_i)$ $A = 2\pi X(d_o + d_i)$	0.61	1/4	$\frac{1}{2(d_o + d_i)} \times \left[X^3\left(\frac{1}{d_i^{7/5}} + \frac{1}{d_o^{7/5}}\right)^5\right]^{-1/4}$	Laminar flow $\dot{q} = \bar{\alpha}(T_h - T_c)$	3.48
16	**Closed cavity with vertical isothermal walls:** see section C4						

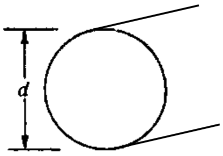
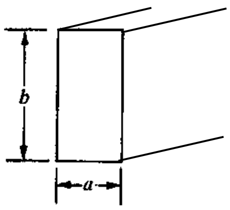
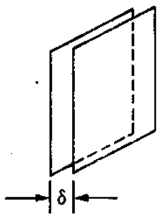
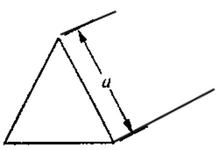
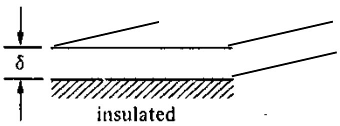
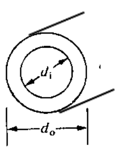
B2. Forced convection inside ducts (liquids and gases at low Mach number)

Formulae: $\overline{Nu} = CRe^m Pr^n K$ (laminar: $Re < 2000$; turbulent: $Re \geq 2000$) (fluid properties at T_f, except μ_w which is calculated at T_w)

Heat flux: $\dot{q}_w = \bar{\alpha}(T_w - T_f)$

Notation:

C	Constant in Nusselt equation
c_p	Specific heat at constant pressure (J/kgK)
D	Hydraulic diameter (m), $D = 4S/P$ (see also Section C3)
Gz	Graetz number, $Gz = RePrD/\ell$
K	Dimensionless correction function in Nusselt equation
ℓ	Length (m)
n	Constant in Nusselt equation
$\overline{Nu}$	Mean Nusselt number, $\overline{Nu} = \bar{\alpha}D/\lambda$
m	Constant in Nusselt equation
$\dot{m}$	Mass flow rate (kg/s)
P	Wet perimeter (m)
Pr	Prandtl number, $Pr = \mu c_p/\lambda$
$\dot{q}_w$	Heat transfer rate at the wall (W/m^2)
Re	Reynolds number, $Re = \rho\bar{v}D/\mu$
S	Cross sectional area (m^2)
T_f	Fluid bulk temperature (°C or K)
T_w	Temperature of the wall (°C or K)
$\bar{v}$	Mean fluid velocity, $\bar{v} = \dot{m}/(\rho S)$
$\bar{\alpha}$	Overall heat transfer coefficient (W/m^2K)
λ	Thermal conductivity (W/mK)
ρ	Density (kg/m^3)
μ	Dynamic viscosity (kg/ms)

No.	Cross-section	D	C	m	n	K	Operating conditions
1	Circular tube	d	1.86	$\frac{1}{3}$	$\frac{1}{3}$	$\left(\frac{d}{l}\right)^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14}$	Laminar flow short tube, Re < 2 000, Gz > 10
		d	3.66	0	0	1	Laminar flow long tube, Re < 2 000, Gz < 10
		d	0.023	0.8	0.4	1	Turbulent flow of gases, Re > 2 000
		d	0.027	0.8	0.33	$\left(\frac{\mu}{\mu_w}\right)^{0.14}$	Turbulent flow of highly viscous liquids, Re > 2000, 0.6 < Pr < 100
2	Rectangular tube  $\frac{b}{a} = 1$	a	2.98	0	0	1	Laminar flow, Re < 2 000
	1.4	1.17 a	3.08	0	0	1	Laminar flow, Re < 2 000
	2	1.33 a	3.39	0	0	1	Laminar flow, Re < 2 000
	3	1.5 a	3.96	0	0	1	Laminar flow, Re < 2 000
	4	1.6 a	4.44	0	0	1	Laminar flow, Re < 2 000
	8	1.78 a	5.95	0	0	1	Laminar flow, Re < 2 000
	∞	2.0 a	7.54	0	0	1	Laminar flow, Re < 2 000
3	Slit or parallel plates 	2δ	1.85	$\frac{1}{3}$	$\frac{1}{3}$	$\left(\frac{2\delta}{l}\right)^{1/3}$	Laminar flow, Re < 2 000, $\left(\mathrm{Re}\cdot\mathrm{Pr}\,\frac{2\delta}{l}\right) > 70$
		2δ	7.54	0	0	1	$\left(\mathrm{Re}\cdot\mathrm{Pr}\,\frac{2\delta}{l}\right) < 70$
4	Equilateral triangle 	0.58 a	1.3	$\frac{1}{3}$	$\frac{1}{3}$	$\left(\frac{0.58\,a}{l}\right)^{1/3}$	Laminar flow, Re < 2 000, $\left(\mathrm{Re}\cdot\mathrm{Pr}\,\frac{0.58\,a}{l}\right) > 7$
		0.58 a	2.47	0	0	1	$\left(\mathrm{Re}\cdot\mathrm{Pr}\,\frac{0.58\,a}{l}\right) < 7$
5	Two parallel plates 	4δ	4.86	0	0	1	Laminar flow
6	**Concentric tube annulus:** see section C3 						

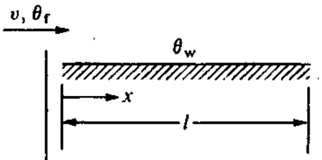
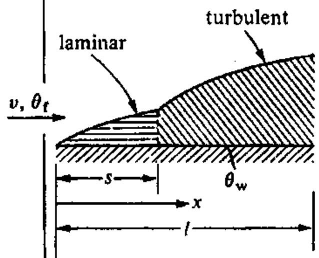
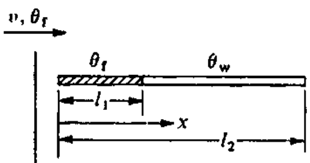
B3. Forced convection in isothermal flat plates (liquids and gases at low Mach number)

Formulae: $Nu_x = C Re_x^m \, Pr^n \, K$ (laminar: $Re_x < Re_{cr}$; turbulent: $Re_x \geq Re_{cr}$) (fluid properties at T_m)

Heat flux: $\dot{q}_w = \alpha_x (T_w - T_f)$;

Notation:

C	Constant in Nusselt equation
c_p	Specific heat at constant pressure (J/kgK)
K	Dimensionless correction function in Nusselt equation
l	Length of the plate (m)
n	Constant in Nusselt equation
Nu_x	Local Nusselt number at location x, $Nu_x = \alpha_x x/\lambda$
$\overline{Nu}_x$	Mean Nusselt number from x=0 to x $(\overline{Nu}_x = \bar{\alpha} x/\lambda)$
m	Constant in Nusselt equation
Pr	Prandtl number $(Pr = \mu c_p/\lambda)$
$\dot{q}_w$	Heat transfer rate at the wall (W/m^2)
Re_x	Local Reynolds number $(Re = \rho v_\infty x/\mu)$
T_f	Free stream temperature (°C or K)
T_m	Film temperature (°C or K) $(T_m = (T_w + T_f)/2)$
T_w	Temperature of the wall (°C or K)
v_∞	Free stream velocity (m/s)
W	Width of the plate
x	Distance measured from the leading edge (m)
$\bar{\alpha}$	Overall heat transfer coefficient from $x = 0$ to x (W/m^2K)
α_x	Local heat transfer coefficient at x (W/m^2K)
λ	Thermal conductivity (W/mK)
μ	Dynamic viscosity (kg/ms)
ρ	Density (kg/m^3)

No.	Flow along a plane surface	Formulae	Operating conditions
1	Pure flow regime 	$Nu_x = 0.332\, Re_x^{1/2} Pr^{1/3}$ (local) $\overline{Nu}_x = 0.664\, Re_x^{1/2} Pr^{1/3}$ (mean)	Laminar flow, $Re < 5 \times 10^5$, $0 < x < l$
		$Nu_x = 0.029\, Re_x^{4/5} Pr^{1/3}$ (local) $\overline{Nu}_x = 0.037\, Re_x^{4/5} Pr^{1/3}$ (mean)	Turbulent flow, $Re > 5 \times 10^5$, $0 < x < l$
2	Mixed flow regime 	Mean $\overline{Nu}_x$ over the distance $0 < x < l$ $\overline{Nu}_x = 0.037\, Pr^{1/3} (Re_x^{4/5} - C)$ where $C = 23\,500$ for $(Re)_{cr} = 5 \times 10^5$ $C = 14\,200$ for $(Re)_{cr} = 3 \times 10^5$ $C = 4\,300$ for $(Re)_{cr} = 10^5$	Laminar flow up to distance s where the critical $(Re)_{cr}$ occurs, and thereafter turbulent flow to $x > s$.
3	Partial wall heating 	$Nu_x = 0.332\, Re_x^{1/2} Pr^{1/3} [1 - (l_1/x)^{3/4}]^{-1/3}$ (local) $\overline{Nu}_x = 0.664\, Re_x^{1/2} Pr^{1/3} \left[1 - \left(\frac{l_1}{x}\right)^{3/4}\right]^{2/3} \Big/ \left(1 - \frac{l_1}{x}\right)$ (mean)	Laminar flow, $Re < 5 \times 10^5$, $l_1 < x < l_2$
	Note: in all these figures $\theta = T$	$Nu_x = 0.029\, Re_x^{4/5} Pr^{1/3} \left[1 - \left(\frac{l_1}{x}\right)^{9/10}\right]^{-1/9}$ (local) $\overline{Nu}_x = 0.037\, Re_x^{4/5} Pr^{1/3} \left[1 - \left(\frac{l_1}{x}\right)^{9/10}\right]^{8/9} \Big/ \left(1 - \frac{l_1}{x}\right)$ (mean)	Turbulent flow, $Re > 5 \times 10^5$, $l_1 < x < l_2$

B4. Forced convection in flat plates (compressible gas flow at high Mach number)

Formulae: $Nu_x = CRe_x^m\, Pr^n\, K$ (laminar: $Re_x < Re_{cr}$; turbulent: $Re_x \geq Re_{cr}$) (fluid properties at T_{ref})

Heat flux: $\dot{q}_w = \alpha_x(T_w - T_r)$ or $\dot{q}_w = \hat{\alpha}_x(h_w - h_r)$; note: $\alpha_x = c_{p,wr}\hat{\alpha}_x$, where $c_{p,wr} = \frac{1}{T_w - T_r}\int_{T_r}^{T_w} c_p dT$

Notation:

c	Sound speed (m/s), $c = \sqrt{\gamma RT}$, T in K
C_f	Skin friction coefficient, $C_f = \tau_w/0.5\rho v_\infty^2$
c_p	Specific heat at constant pressure (J/kgK)
c_{pr}	Mean specific heat (see below) (J/kgK)
c_v	Specific heat at constant volume (J/kgK)
h_x	Specific enthalpy at x conditions ($x = \infty, o, r, w, ref$ means free stream, stagnation, recovery, wall and reference conditions respectively)
L	Length of the plate (m)
M	Mach number, $M = v/c$
Nu_x	Local Nusselt number at location x, $Nu_x = \alpha_x x/\lambda$
Pr	Prandtl number, $Pr = \mu c_p/\lambda$
$\dot{q}_w$	Heat transfer rate at the wall (W/m^2)
r	Recovery factor (see below)
R	Specific gas constant (for air, $R = 289\ J/kgK$)
Re_x	Local Reynolds number, $Re = \rho v x/\mu$
Re_{cr}	Critical Reynolds number (starting turbulent, $Re_{cr} = 0$) (starting laminar, $Re_{cr} \approx 5 \times 10^5$)
T_f	Free stream temperature (°C or K)
T_{ref}	Reference temperature (°C or K) (*see below*)
T_w	Temperature of the wall (°C or K)
v	Free stream velocity (m/s)
x	Distance measured from the leading edge (m)
x_{cr}	Critical distance for transition from laminar to turbulent flow, $Re_{cr} = \rho v_\infty x_{cr}/\mu$
α_x	Local heat transfer coefficient based on T (W/m^2K)
$\hat{\alpha}_x$	Local heat transfer coefficient based on h (kg/m^2s)
γ	Specific heat ratio, $\gamma = c_p/c_v$
λ	Thermal conductivity (W/mK)
μ	Dynamic viscosity (kg/ms)
ρ	Density (kg/m^3)

Reference temperature: $T_{ref} = T(h_{ref})$, where $h_{ref} = \frac{h_w + h_f}{2} + 0.22(h_r - h_f)$. Note: for small variation of c_p, $T_{ref} \approx \frac{T_w + T_f}{2} + 0.22(T_r - T_f)$.

Recovery factor definition: $r = \frac{h_r - h_f}{h_o - h_f} = \frac{h_r - h_f}{v^2/2}$.

Recovery factor empirical expressions: $r = \sqrt{Pr}$ if laminar flow; $r = \sqrt[3]{Pr}$ if turbulent flow.

Recovery temperature (from the above recovery factor definition): $T_r = T_f + rv^2/2c_{p,rf}$, where $c_{p,rf} = \frac{1}{T_r - T_f}\int_{T_f}^{T_r} c_p dT$.

Local Nusselt number (same correlations that the ones used for low Mach number but using the new definition of α and evaluating the thermophysical properties at T_{ref}):

- $Nu_x = 0.332 Re_x^{1/2} Pr^{1/3}$ (Pohlhausen; laminar flow, $Re_x < Re_{cr}$)
- $Nu_x = 0.029 Re_x^{4/5} Pr^{1/3}$ (Blasius; turbulent flow, $Re_{cr} < Re_x < 10^7$)
- $Nu_x = 0.144 Re_x Pr^{1/3}/(\log_{10} Re_x)^{2.45}$ (Prandtl-Schlichting; turbulent flow, $Re_{cr} < Re_x < 10^9$)

Local friction factor. From the Reynolds analogy: $\frac{C_f}{2} = \frac{Nu}{RePr^{1/3}}$.

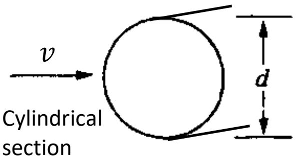
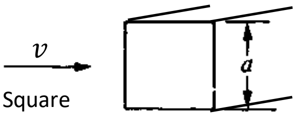
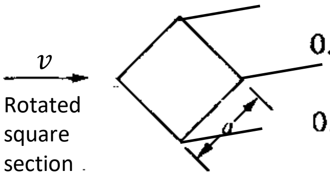
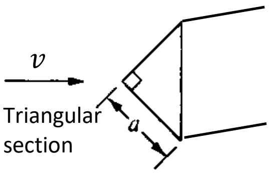
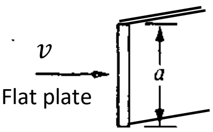
B5. Forced convection around tubes and pipe bundles (liquids and gases at low Mach number) (1/2)

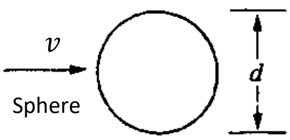
Formulae: $\overline{Nu} = CRe^m$ for air and circular cylinder; $\overline{Nu} = 0.43 + CRe^m$ for air and cylinders of other cross-sections; $\overline{Nu} = 0.43 + CRe^m \mathrm{Pr}^{0.31}$ for liquids (fluid properties at T_m)

Heat flux: $\dot{q}_w = \bar{\alpha}(T_w - T_f)$

Notation:

- C Constant in Nusselt equation
- c_p Specific heat at constant pressure (J/kgK)
- $\overline{Nu}$ Mean Nusselt number, $\overline{Nu} = \bar{\alpha}X/\lambda$
- m Constant in Nusselt equation
- Pr Prandtl number, $Pr = \mu c_p/\lambda$
- $\dot{q}_w$ Heat transfer rate at the wall (W/m^2)
- Re Reynolds number, $Re = \rho v X/\mu$
- T_f Fluid bulk temperature (°C or K)
- T_m Film temperature (°C or K), $T_m = (T_w + T_f)/2)$
- T_w Temperature of the wall (°C or K)
- v Free flow velocity (m/s)
- X Characteristic length (see below)
- $\bar{\alpha}$ Overall heat transfer coefficient (W/m^2K)
- λ Thermal conductivity (W/mK)
- μ Dynamic viscosity (kg/ms)
- ρ Density (kg/m^3)

Cross-section	C	m	Range of Re	Characteristic length X
 Cylindrical section	0.437	0.0895	$10^{-4} - 4 \times 10^{-3}$	d
	0.565	0.136	$4 \times 10^{-3} - 9 \times 10^{-2}$	d
	0.800	0.280	$9 \times 10^{-2} - 1$	d
	0.795	0.384	$1 - 35$	d
	0.583	0.471	$35 - 5 \times 10^3$	d
	0.148	0.633	$5 \times 10^3 - 5 \times 10^4$	d
	0.0208	0.814	$5 \times 10^4 - 5 \times 10^5$	d
 Square section	0.178	0.699	$2.5 \times 10^3 - 8 \times 10^3$	$\frac{4a}{\pi}$
	0.102	0.675	$5 \times 10^3 - 10^5$	$\frac{4a}{\pi}$
 Rotated square section	0.290	0.624	$2.5 \times 10^3 - 7.5 \times 10^3$	$\frac{4a}{\pi}$
	0.246	0.588	$5 \times 10^3 - 10^5$	$\frac{4a}{\pi}$
 Triangular section	0.276	0.61	$3 \times 10^3 - 2 \times 10^4$	$1.09\,a$
 Flat plate	0.227	0.731	$4 \times 10^3 - 1.5 \times 10^4$	$\frac{2a}{\pi}$


Sphere

For flows around a **SPHERE**, Whitaker suggests ($3.5 < Re_d < 7.6 \cdot 10^4$, $0.71 < Pr < 380$, $1.0 < \mu/\mu_w < 3.20$):

$$\overline{Nu}_d = 2 + \left(0.4Re_d^{1/2} + 0.06Re_d^{2/3}\right) \mathrm{Pr}^{0.4} (\mu/\mu_w)^{1/4}$$

where the thermophysical properties are evaluated at the external temperature (T_f), except the dynamic viscosity μ_w , which is evaluated at the temperature of the surface of the sphere.

B5. Forced convection around tubes and pipe bundles (liquids and gases at low Mach number) (2/2)

Formulae: $\overline{Nu} = CRe^{0.6}\mathrm{Pr}^{0.3}(\mu/\mu_w)^{0.14}$ (correlation valid for $2000 < Re < 40000$) (fluid properties at T_f, except μ_w which is calculated at T_w)

Heat flux: $\dot{q}_w = \bar{\alpha}(T_w - T_f)$

Notation:

C Constant in Nusselt equation

c_p Specific heat at constant pressure (J/kgK)

d Tube diameter (m)

e_1, e_2 Horizontal and vertical distances between tubes (m)

$\overline{Nu}$ Mean Nusselt number, $\overline{Nu} = \bar{\alpha}d/\lambda$

Pr Prandtl number, $Pr = \mu c_p/\lambda$

$\dot{q}_w$ Heat transfer rate at the wall (W/m^2)

Re Reynolds number, $Re = \rho v d/\mu$

T_f Fluid bulk temperature (°C or K)

T_w Temperature of the wall (°C or K)

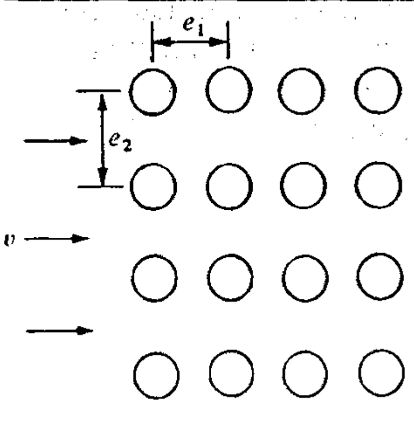
v Free flow velocity (m/s)

$\bar{\alpha}$ Overall heat transfer coefficient (W/m^2K)

λ Thermal conductivity (W/mK)

μ Dynamic viscosity (kg/ms)

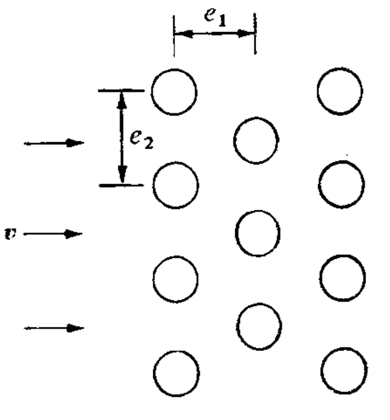
ρ Density (kg/m^3)



in-line pipe bank

Values of C

e_2/d \ e_1/d	1.25	2	3	4
1.25	0.888	0.890	0.880	0.835
2	0.613	0.613	0.638	0.632
3	0.427	0.427	0.500	0.504
4	0.356	0.356	0.421	0.421



staggered pipe bank

Values of C

e_2/d \ e_1/d	1.25	2	3	4
1.25	0.953	0.937	0.875	0.812
2	0.686	0.669	0.638	0.611
3	0.559	0.544	0.506	0.500
4	0.489	0.488	0.466	0.442

B6. Mixed convection on rotating surfaces (liquids and gases at low Mach number)

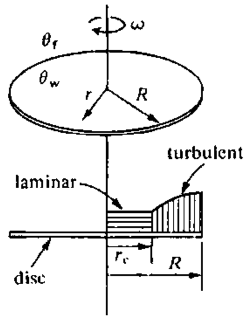
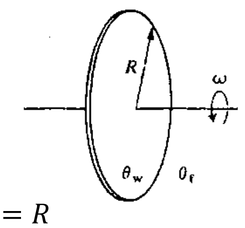
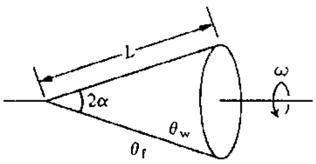
Formulae: $\overline{Nu} = f(Re, Pr, K)$ (see below) (fluid properties at T_m)

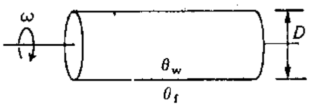
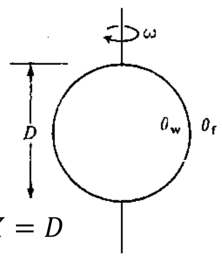
Heat flux: $\dot{q}_w = \bar{\alpha}(T_w - T_f)$

Notation:

C_D Surface drag coefficient
c_p Specific heat at constant pressure (J/kgK)
D Diameter (m)
g gravitational acceleration, $g = 9.81\ m/s^2$
Gr Grashof number, $Gr = \beta g \rho^2 |T_w - T_f| X^3/\mu^2$
$\overline{Nu}$ Mean Nusselt number, $\overline{Nu} = \bar{\alpha}X/\lambda$
Pr Prandtl number, $Pr = \mu c_p/\lambda$
$\dot{q}_w$ Heat transfer rate at the wall (W/m^2)
R Radius (m)
Re Reynolds number, $Re = \rho\omega X^2/\mu$
T_f Fluid bulk temperature (°C or K)
T_m Film temperature (°C or K), $T_m = (T_w + T_f)/2$
T_w Temperature of the wall (°C or K)
X Characteristic length (m)
v_∞ Fluid crossflow velocity (m/s)

$\bar{\alpha}$ Overall heat transfer coefficient (W/m^2K)
α Half vertex angle cone
β Volumetric thermal expansion coefficient (K^{-1})
λ Thermal conductivity (W/mK)
μ Dynamic viscosity (kg/ms)
ν Kinematic viscosity (m^2/s), $\nu = \mu/\rho$
ρ Density (kg/m^3)
ω Angular velocity of rotation (rad/s)

No.	System	Schematic presentation	Formulae	Conditions	Reference
1	Rotating disc		$\bar{Nu} = (0.277 + 0.105\ Pr)\ Re^{0.5}$	Laminar flow, $Re < 2.5 \times 10^5$, $0.7 < Pr < 5.0$	3.33, 3.31
			$\bar{Nu} = 1.1\ Re^{0.5}$	Laminar flow, $Re < 2.5 \times 10^5$, $Pr = 10$	3.33
			$\bar{Nu} = 0.015\ Re^{0.8}$	Turbulent flow, $Re > 2.5 \times 10^5$, $Pr = 0.72$	3.33
		$X = R$	$\bar{Nu} = 0.015\ Re^{0.8} - 100\left(\frac{r_c}{R}\right)^2$	Laminar flow between $r = 0$ and $r = r_c$, turbulent flow between $r = r_c$ and $r = R$ where $r_c = (2.5 \times 10^5\ \nu/\omega)^{1/2}$, $Pr = 0.72$	3.31
		 $X = R$	$\bar{Nu} = 0.4\ (Re^2 + Gr)^{0.25}$ where $\bar{Nu} = \frac{hR}{k}$, $Re = \frac{\omega R^2}{\nu}$, $Gr = \frac{\beta g R^3 \pi^{3/2} \Delta\theta}{\nu^2}$	Combined effects of free convection and rotation in laminar flow (axis horizontal)	3.32
				Note: in items 2, 3 and4, h refers to the heat transfer coefficient, i.e. $h \equiv \alpha$, and $\Delta\theta = \lvert T_w - T_f \rvert$	
2	Rotating cone	 $X = L$	$\bar{Nu} = 0.515\ (Gr)^{0.25}$	Laminar free convection, $Pr = 0.72$, $Gr/Re^2 > 2.0$	3.38
			$\bar{Nu} = 0.33\ Re^{0.5}$	Forced convection, $Pr = 0.72$, $Gr/Re^2 < 0.05$	3.32, 3.40
			$\bar{Nu} = Re^{0.5}\ [0.331 + 0.412(Gr/Re^2) + \cdots]$	Combined free and forced convection, $Pr = 0.72$, $0.2 < Gr/Re^2 < 1.0$	3.32
		Note: in all these figures $\theta = T$	where $\bar{Nu} = \frac{hL}{k}$, $Re = \frac{\omega L^2 \sin\alpha}{\nu}$, $Gr = \frac{\beta g L^3 \cos\alpha \Delta\theta}{\nu^2}$		

No.	System	Schematic presentation	Formulae	Conditions	Reference
3	Rotating cylinder	 $X = D$	$\bar{Nu} = 0.456 (Gr \cdot Pr)^{0.25}$	Free convection, $Re < (Gr/Pr)^{0.5}$	3.34
			$\bar{Nu} = 0.18 [(0.5\, Re^2 + Gr)Pr]^{0.315}$	Combined free and forced convection, $Re \leqslant 5 \times 10^4$	3.32
			$\bar{Nu} = \dfrac{Re \cdot Pr \sqrt{C_D/2}}{5\, Pr + 5 \ln (3\, Pr + 1) + \sqrt{2/C_D} - 12}$ C_D from: $\dfrac{Re}{B} = -1.828 + 1.77 \ln B$ for $B > 950$ $\dfrac{Re}{B} = -3.68 + 2.04 \ln B$ for $B < 950$ where $B = Re\sqrt{C_D}$	Forced convection, $Re > 10^5$	3.39
			$\bar{Nu} = 0.135[(0.5\, Re^2 + Re_f^2 + Gr)\, Pr]^{0.33}$ where $\bar{Nu} = \dfrac{hD}{k}$, $Re = \dfrac{\omega D^2}{\nu}$, $Re_f = \dfrac{v_\infty D}{\nu}$, $Gr = \dfrac{\beta g D^3 \Delta\theta}{\nu^2}$	Combined effects of rotation, free convection and crossflow, $Re_f < 1.5 \times 10^4$, $0.6 < Pr < 15$, $10^3 < Re < 5 \times 10^4$, value in square bracket [ ] $< 10^9$	3.39
4	Rotating sphere	 $X = D$	$\bar{Nu} = 0.43\, Re^{0.5}\, Pr^{0.4}$	Laminar flow, $Gr/Re^2 < 0.1$, $Re < 5 \times 10^4$, $0.7 < Pr < 217$	3.36
			$\bar{Nu} = 0.066\, Re^{0.67}\, Pr^{0.4}$	Turbulent flow, $Gr/Re^2 < 0.1$, $5 \times 10^4 < Re < 7 \times 10^5$, $0.7 < Pr < 7$	3.36
			$\bar{Nu} = 2\, (Re^2 + Gr)^{0.164}$ where $\bar{Nu} = \dfrac{hD}{k}$, $Re = \dfrac{\omega D^2}{\nu}$, $Gr = \dfrac{\beta g D^3 \Delta\theta}{\nu^2}$	Combined free and forced convection, $Gr/Re^2 > 0.1$, $10^3 < Re < 2 \times 10^4$, $4 \times 10^6 < Gr < 2 \times 10^7$	3.32

Note: in all these figures $\theta = T$

B7. Friction factors for flows inside ducts

In this section, the skin friction coefficient is defined as: $f = \frac{\tau_w}{\rho \bar{v}^2/2}$.

Notation: $\bar{v}$ is the average velocity of the fluid (*m/s*); ρ the density (*kg/m*³); D the hydraulic diameter (see Section B2 or C3) (*m*); τ_w the viscous shear stresses at the wall (*N/m*²); Re the Reynolds number ($Re = \rho \bar{v} D/\mu$); μ the dynamic viscosity (*kg/ms*); ε the absolute roughness (*m*); and $\varepsilon_r = \varepsilon/D$ the relative roughness.

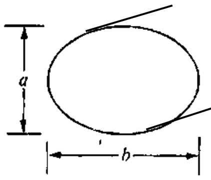
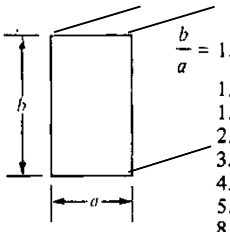
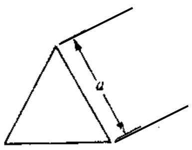
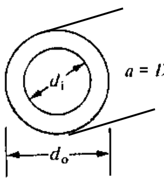
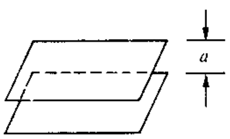
No.	Duct	Cross-sectional shape	Hydraulic Diameter D	Friction factor f	Operating conditions
1	Circular tube		$D = a$	$16\ Re^{-1}$	$Re \leq 2\,000$
				$0.079\ Re^{-0.25}$	$5 \times 10^3 < Re < 3 \times 10^4$ for $\frac{\epsilon}{D} < 0.0001$
				$0.096\ Re^{-0.25}$	$5 \times 10^3 < Re < 3 \times 10^4$ for $\frac{\epsilon}{D} \simeq 0.004$
				$0.046\ Re^{-0.2}$	$3 \times 10^4 < Re < 3 \times 10^6$ for $\frac{\epsilon}{D} < 0.0001$
				$0.078\ Re^{-0.2}$	$3 \times 10^4 < Re < 3 \times 10^6$ for $\frac{\epsilon}{D} \simeq 0.004$

Instead of the previous four correlations for turbulent flow, the more general **Churchill expression** (1997) can be employed for a wide range of Re and relative roughness, $\varepsilon_r = \varepsilon/D$:

$$f = 2\left[\left(\frac{8}{Re}\right)^{12} + \frac{1}{(A+B)^{3/2}}\right]^{1/12}$$

where $A = \left\{2.457 ln\left[\frac{1}{(7/Re)^{0.9} + 0.27\varepsilon_r}\right]\right\}^{16}$ y $B = (37530/Re)^{16}$.

Alternatively, a simpler correlation by **Swamee-Jain** (1976) can be used: $f = \frac{0.0625}{\left[\log_{10}\left(\frac{\varepsilon_r}{3.7} + \frac{5.74}{Re^{0.9}}\right)\right]^2}$ (Re > 4000)

No.	Duct	Cross-sectional shape	Hydraulic Diameter D	Friction factor f	Operating conditions
2	Ellipse	 $\frac{a}{b} = 0.7$	$1.17\,a$	$16.25\ Re^{-1}$	Laminar flow
		0.5	$1.3\,a$	$17\ Re^{-1}$	Laminar flow
		0.3	$1.44\,a$	$18.25\ Re^{-1}$	Laminar flow
		0.2	$1.5\,a$	$19\ Re^{-1}$	Laminar flow
		0.1	$1.55\,a$	$19.5\ Re^{-1}$	Laminar flow
3	Rectangle	 $\frac{b}{a} = 1.0$	a	$14.25\ Re^{-1}$	Laminar flow
		1.25	$1.11\,a$	$14.4\ Re^{-1}$	Laminar flow
		1.67	$1.25\,a$	$15.0\ Re^{-1}$	Laminar flow
		2.0	$1.33\,a$	$15.5\ Re^{-1}$	Laminar flow
		3.0	$1.5\,a$	$17.25\ Re^{-1}$	Laminar flow
		4.0	$1.6\,a$	$18.25\ Re^{-1}$	Laminar flow
		5.0	$1.67\,a$	$19.0\ Re^{-1}$	Laminar flow
		8.0	$1.78\,a$	$20.75\ Re^{-1}$	Laminar flow
		10.0	$1.82\,a$	$21.25\ Re^{-1}$	Laminar flow
4	Equilateral triangle		$0.58\,a$	$13.25\ Re^{-1}$	Laminar flow
5	Circular annulus		$(d_o - d_i)$	$22.3\ Re^{-1}$	Laminar flow, $d_i/d_o = 0.1$
			$(d_o - d_i)$	$24\ Re^{-1}$	Laminar flow, $d_i/d_o > 0.5$
			$(d_o - d_i)$	$0.085\ Re^{-0.25}$	$6 \times 10^3 < Re < 5 \times 10^5$, $d_i/d_o < 0.56$
6	Two parallel plates		$2a$	$24\ Re^{-1}$	Laminar flow

B8. Two-phase flow. Condensation and evaporation

Notation:

Bo Boiling number, $Bo = \frac{\dot{q}_w}{\dot{G}\Delta h_{fg}}$

Co Convective number, $Co = \left(\frac{1-x_g}{x_g}\right)^{0.8}\left(\frac{\rho_g}{\rho_l}\right)^{0.5}$

c_p Specific heat at constant pressure $[J/kgK]$

D Inside diameter $[m]$

Fr Froude number, $Fr = \dot{G}^2/(gD\rho_H^2)$, $Fr_{lo} = \dot{G}^2/(gD\rho_l^2)$

f Friction factor (single-phase flow)

g Gravity acceleration [m/s²]

$\dot{G}$ Mass flow rate per unit area, $\dot{G} = \rho v\ \ [kg/m^s s]$

Ga Galileo number, $Ga = \frac{\rho_l(\rho_l-\rho_g)D^3 g}{\mu_l^2}$

Ja_l Jacob number for liquid, $Ja_l = \frac{c_{pl}(T_{sat}-T_w)}{\Delta h_{fg}}$

Nu Nusselt number, $Nu = \alpha D/\lambda$

Pr Prandtl number, $Pr = \frac{\mu c_p}{\lambda}$; $Pr_l = \frac{\mu_l c_{pl}}{\lambda_l}$, $Pr_g = \frac{\mu_g c_{pg}}{\lambda_g}$

$\dot{q}_w$ Heat flux at the wall $[W/m^2]$

Re Reynolds number, $Re = \frac{\dot{G}D}{\mu}$, $Re_{lo} = \frac{\dot{G}D}{\mu_l}$, $Re_l = \frac{(1-x_g)\dot{G}D}{\mu_l}$, $Re_{go} = \frac{\dot{G}D}{\mu_g}$, $Re_g = \frac{x_g\dot{G}D}{\mu_l}$

T Temperature $[K]$

v Velocity $[m/s]$

We Weber number, $We = \dot{G}^2 D/(\rho_H\sigma)$

x_g Vapour mass fraction (vapour quality)

X_{tt} Martinelli parameter, $X_{tt} = \left(\frac{1-x_g}{x_g}\right)^{0.9}\left(\frac{\rho_g}{\rho_l}\right)^{0.5}\left(\frac{\mu_l}{\mu_g}\right)^{0.1}$

α Local heat transfer coefficient $[\mathrm{W/m^2K}]$

Δh_{fg} Latent heat of vaporization $[J/kg]$

ϵ_g Vapour volumetric fraction (void fraction)

λ Thermal conductivity $[\mathrm{W/mK}]$

μ Dynamic viscosity [Pa s]

ρ Density $[kg/m^3]$

ρ_H Homogeneous density, $\rho_H = \left[\frac{x_g}{\rho_g} + \frac{1-x_g}{\rho_l}\right]^{-1}$

τ_w Two-phase flow viscous shear stresses at the wall, $\tau_w = \phi_{lo}^2\tau_{w,lo}$ $[\mathrm{N/m^2}]$

$\tau_{w,lo}$ Viscous shear stresses at the wall (all liquid), $\tau_{w,lo} = f_l\frac{\dot{G}^2}{2\rho_l}$ $[N/m^2]$

ϕ Two-phase pressure drop factor

σ Surface tension [N/m]

Subscripts:

g Gas phase (vapour)

go It is considered all flow as gas

l Liquid phase

lo It is considered all flow as liquid

sat Saturation conditions

TF Two-phase

w Wall

B8.1 Friction pressure drop correlation for horizontal and vertical two-phase pipe flow[1]

$$\phi_{lo}^2 = \frac{\tau_{w,TF}}{\tau_{w,lo}} = E + \frac{3.23FH}{Fr^{0.045}We^{0.035}}$$

where:

$$E = \left(1-x_g\right)^2 + \frac{\rho_l f_{go}}{\rho_g f_{lo}}x_g^2;\quad F = x_g^{0.78}\left(1-x_g\right)^{0.224};\quad H = \left(\frac{\rho_l}{\rho_g}\right)^{0.91}\left(\frac{\mu_g}{\mu_l}\right)^{0.19}\left(1-\frac{\mu_g}{\mu_l}\right)^{0.7}$$

The term $\tau_{w,lo}$ is calculated considering that all the flow (liquid and vapour) circulates as liquid (table B-7 can be used with $Re = Re_{lo} = \dot{G}D/\mu_l$).

B8.2 Condensation inside smooth horizontal tubes[2]

Two distinct regimes are identified: annular and wavy condensation. Nusselt numbers are obtained from:

$$Nu_{annular} = 0.023Re_l^{0.8}Pr_l^{0.4}\left[1 + \frac{2.22}{X_{tt}^{0.889}}\right]$$

$$Nu_{wavy} = \frac{0.023Re_{go}^{0.12}}{1+1.11X_{tt}^{0.58}}\left[\frac{GaPr_l}{Ja_l}\right] + \left(1-\frac{\phi_1}{\pi}\right)Nu_{forced}$$

[1] Correlation by F. Friedel, Improved Friction Pressure Drop Correlation for Horizontal and vertical Two-phase Pipe Flow, European Two-phase Flow Group Meeting, Ispra, Italy, Paper E2, 1979. Correlation recommended when $\boldsymbol{\mu_l/\mu_g < 1000}$.

[2] Correlation by M.K.Dobson and J.C.Chato, Condensation in Smooth Horizontal Tubes, Journal of Heat Transfer, vol.120, pp.193-213, 1998.

where: $1-\frac{\phi_1}{\pi} \approx \frac{cos^{-1}(2\varepsilon_g-1)}{\pi}$ (from Jaster and Kosky, 1976)

$\varepsilon_g = \left[1+\frac{1-x_g}{x_g}\left(\frac{\rho_g}{\rho_l}\right)^{2/3}\right]^{-1}$ (from Zivi, 1964)

$$Nu_{forced} = 0.0195 Re_l^{0.8} Pr_l^{0.4}\Phi(X_{tt}), \text{ where: } \begin{cases} \Phi(X_{tt}), = \sqrt{1.376+\frac{c_1}{X_{tt}^{c_2}}} \\ 0 < Fr_l < 0.7: c_1 = 4.172 + 5.48 Fr_l - 1.564 Fr_l^2, \\ c_2 = 1.773 - 0.169 Fr_l \\ Fr_l \geq 0.7: c_1 = 1.7242, c_2 = 1.655 \end{cases}$$

Selection criteria:

- If $\dot{\boldsymbol{G}} \geq \mathbf{500}\ \boldsymbol{kg/m^2s} \rightarrow Nu_{TF} = Nu_{annular}$, where $Nu_{TF} = \alpha_{TH} D/\lambda_l$
- If $\dot{\boldsymbol{G}} < \mathbf{500}\boldsymbol{kg/m^2s} \rightarrow Nu_{TF} = Nu_{annular}\ (if\ Fr_{so} < 20)\ or\ Nu_{TF} = Nu_{wavy}\ (if Fr_{so} \geq 20)$

where: $Fr_{so} = C\frac{Re_l^m}{\sqrt{Ga}}\left(\frac{1+1.09X_{tt}^{0.039}}{X_{tt}}\right)^{1.5}$ (if $Re_l \leq 1250 \rightarrow C = 0.025,\ m = 1.59$) (if $Re_l \leq 1250 \rightarrow C = 1.26,\ m = 1.04$).

In case of zeotropic mixtures: $Nu_{annular}^{zeotr} = 0.7\left(\frac{\dot{G}}{300}\right)^{0.3} Nu_{annular}$, and $Nu_{wavy}^{zeotr} = \left(\frac{\dot{G}}{300}\right)^{0.3} Nu_{wavy}$.

B8.3 Evaporation inside horizontal and vertical tubes[3]

Two distinct regimes are identified, the nucleated boiling (α_{nb}) and the convective boiling (α_{cb}). Their evaluation is indicated below. The two-phase heat transfer coefficient is the maximum among them.

$$\alpha_{nb} = \left[0.6683 Co^{-0.2}(25Fr_{lo})^m + 1058 Bo^{0.7} F_{fl}\right]\left(1-x_g\right)^{0.8}\alpha_{lo}$$

$$\alpha_{cb} = \left[1.136 Co^{-0.9}(25Fr_{lo})^m + 667.2 Bo^{0.7} F_{fl}\right]\left(1-x_g\right)^{0.8}\alpha_{lo}$$

$$\alpha_{TF} = max(\alpha_{nb}, \alpha_{cb})$$

where:

- Liquid-only heat transfer coefficient: i) $5\times10^6 \geq Re_{lo} \geq 10^4$: $\alpha_{lo} = \frac{\lambda_l}{D}\cdot\frac{2f Re_{lo}\mathrm{Pr}_l}{1+12.7\left(Pr_l^{2/3}-1\right)(2f)^{0.5}}$; ii) $10^4 > Re_{lo} \geq 3000$: $\alpha_{lo} = \frac{\lambda_l}{D}\cdot\frac{2f(Re_{lo}-1000)\mathrm{Pr}_l}{1+12.7\left(Pr_l^{2/3}-1\right)(2f)^{0.5}}$; iii) $3000 > Re_{lo} > 1600$: α_{lo} from linear interpolation between ii and iv sections; iv) $1600 \geq Re_{lo} \geq 410$: $\alpha_{lo} = \lambda_l Nu_{lo}/D$, where $Nu_{lo} = 4.36$ ($\dot{q}_w = constant$) or $Nu_{lo} = 3.66$ ($T_w = constant$); v) $Re_{lo} < 410$: see paper by Peters & Kandlikar). Note: friction factor in i and ii from correlations in Table B7 using Re_{lo}.
- Froude number dependence: i) horizontal tube **and** $Fr_l \leq 0.04$: $m = 0.3$; ii) vertical tube **or** $Fr_l > 0.04$: $m = 0$.
- Fluid/surface interaction parameter: i) Copper and brass surfaces: $F_{fl} = 1.0$ (water); 1.30 (R11); 1.50 (R12); 1.31 (R13B1); 2.20 (R22); 1.30 (R113); 1.24 (R-114); 1.63 (R134a); 1.10 (R152a); 3.30 (R32/R132); 1.80 (R141b); 1.00 (R124); 0.616 (R-123); 4.70 (N_2), 3.50 (Ne); 0.488 (kerosene); ii) stainless steel surfaces: $F_{fl} = 1.0$ (all fluids).

As a first approximation, subcooled region can be solved as pure liquid and the post dryout region as pure gas. For a more precise analysis of the subcooled region see: S.G.Kandlikar, Heat Transfer Characteristics in Partial Boiling, Fully Developed Boiling, and Significant Void Flow Regions of Subcooled Flow Boiling, J.Heat Transfer, Vol. 120, pp. 395-401. 1998.. Similarly, for the post-dryout region see: D.C. Groeneveld, J.Q.Shan, A.Z.Vasić, L.K.H.Leung, A.Durmayaz, J.Yang, S.C.Cheng, A.Tanase, The 2006 CHF look-up table, Nuclear Engineering and Design 237, 1909–1922, 2007.

[3] Correlation by S.G.Kandlikar, A General Correlation for Predicting the Two-Phase Flow Boiling Heat Transfer Coefficient Inside Horizontal and Vertical Tubes, ASME, Journal of Heat Transfer, vol.112, pp 219-228, 1990. See also Peters & Kandlikar, ICNMM2007-30027, 2007.

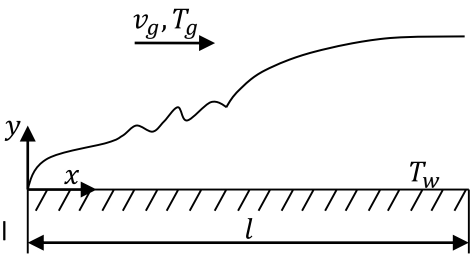
C. ALTERNATIVE CONVECTION HEAT TRANSFER CORRELATIONS

The correlations from Sections C1 to C3 have been extracted from the book by V.Isachenko, V.Osipova and A.Sukomel, “Heat transfer”, Ed. Marcombo, 1979. In Section C4, the correlations have been extracted from N.Seki, S.Fukusako, S. and H.Inaba, “Heat Transfer of Natural Convection in a Rectangular Cavity”, Bulletin of the JSME, Vol. 21, No. 152, 1978.

Index for the attached correlations for liquids and gases at low Mach number: Section C1: Convective heat transfer coefficient in flat plates; Section C2: Convective heat transfer coefficient in circular-section ducts; Section C3: Convective heat transfer coefficient in arbitrary cross-sections; Section C4: Convective heat transfer coefficient in cavities with differentially heated vertical walls.

C1. Convective heat transfer coefficient for flat plates

Alternative correlations can also be seen in section B3. Critical Reynolds number: $Re_{cr,x} \approx 3.3 \cdot 10^5$ (it is assumed that the boundary layer starts as laminar flow).



Subindices:

- x: distance from the origin along the x-direction
- l: overall flat plate length
- f: properties at temperature T_g of the fluid far from the wall
- w: properties at temperature T_w of the fluid in contact with the wall

C1a. Laminar regime

Assumed hypothesis: 1) Laminar regimen with constant thermophysical properties; 2) steady-state; 3) two-dimensional flow; 4) negligible body forces; 5) constant wall temperature and external fluid velocity, T_w and v_g; 6) negligible viscous dissipation.

$$Nu_x = 0.500\sqrt{Re_x Pr} \qquad \text{if } Pr < 0.1 \tag{1}$$

$$Nu_x = 0.332\sqrt{Re_x}\,\sqrt[3]{Pr} \qquad \text{if } Pr > 0.1 \tag{2}$$

In case of **variable thermophysical properties** (but accepting the rest of hypothesis, from 2 to 6):

$$Nu_{fx} = 0.33 Re_{fx}^{0.5} Pr_f^{0.33} (Pr_f/Pr_w)^{0.25} \tag{3}$$

In case of **non-constant wall temperature** (the following temperature distribution is assumed, $T_w - T_g = Kx^m$):

$$Nu_{fx} = 0.33 Re_{fx}^{0.5} Pr_f^{0.33} (Pr_f/Pr_w)^{0.25} \varepsilon \tag{4}$$

where ε is obtained from this table:

m	-0.25	0*	0.1	0.2	0.3	0.4	0.5**	0.8	1.0	2.0
ε	0.655	1.00	1.09	1.17	1.25	1.30	1.36	1.52	1.60	1.98

(*) $T_w = constant$; (**) $\dot{q}_w = constant$.

In case of an **initial adiabatic wall segment** of length from $x = 0$ to $x = x_o$ (being $x_1 = x - x_o$):

$$Nu_{fx_1} = 0.33 Re_{fx_1}^{0.5} Pr_f^{0.33} (x_1/x)^{0.2} (Pr_f/Pr_w)^{0.25} \varepsilon \qquad x > x_o \tag{5}$$

C1b. Turbulent regime

$$Nu_{fx} = 0.0296 Re_{fx}^{0.8} Pr_f^{0.43} (Pr_f/Pr_w)^{0.25} \tag{6}$$

$$\overline{Nu}_{fl} = 0.037 Re_{fl}^{0.8} Pr_f^{0.43} (Pr_f/Pr_w)^{0.25} \tag{7}$$

Expression (6) gives the local Nusselt number, while equation (7) gives an averaged value assuming full turbulent boundary layer along the flat plate. Both expressions can be applied for isothermal walls ($T_w = constant$), or in cases where $T_w - T_g = Kx^m$. In case of $x_o \neq 0$, the variable x starts at x_o.

C2. Convection heat transfer correlations in circular-section ducts

Alternative correlations can also be seen in section B2. Critical Reynolds: $Re_{D,cr} \approx 2500$.

Subscripts:

- D: internal pipe diameter
- x: distance in the x-direction (from the beginning of the tube)
- f: refers to the average temperature of the flow that circulates inside the pipe
- w: refers to the wall temperature
- $f(x)$: refers to the average temperature of the flow that circulates inside the pipe at section x
- $w(x)$: refers to the temperature of the wall at section x

C2a. Laminar regime

Assumed hypothesis: 1) Laminar regime and constant fluid thermophysical properties; 2) negligible body forces; 3) steady-state; 4) axial-symmetric flow; 5) negligible viscous dissipation; 6) **constant heat flux** at the wall ($\dot{q}_w = constant$):

$$Nu_D = 4.36 \tag{8}$$

Under the above mentioned hypothesis, a parabolic velocity profile is obtained.

In case of **isothermal ducts** ($T_w = constant$):

$$Nu_D = 3.66 \tag{9}$$

In case of **short tubes** ($l/D < 216$) with **non-constant thermophysical properties**:

$$Nu_{f(x)x} = 0.33 Re_{f(x)x}^{0.5} Pr_{f(x)}^{0.43} \left(Pr_{f(x)}/Pr_{w(x)}\right)^{0.25} (x/D)^{0.1} \tag{10}$$

If $\boldsymbol{l/D > 216}$:

$$\overline{Nu}_{fD} = 0.15 Re_{fD}^{0.33} Pr_f^{0.43} \left(Pr_{f(x)}/Pr_{w(x)}\right)^{0.25} \tag{11}$$

In case of **non-negligible body forces** (mixed convection):

$$\overline{Nu}_{fD} = 0.15 Re_{fD}^{0.33} Pr_f^{0.33} (Gr_{fD} Pr_f)^{0.1} \left(Pr_{f(x)}/Pr_{w(x)}\right)^{0.25} \bar{\varepsilon}_l \tag{12}$$

where the expression for the non-dimensional Grashof number Gr is defined in Section B1, and $\bar{\varepsilon}_l$ is obtained in the following table:

l/d	1	2	5	10	15	20	30	40	≥50
$\bar{\varepsilon}_l$	1.90	1.70	1.44	1.28	1.18	1.13	1.05	1.02	1

C2b. Turbulent regime

$$Nu_{f(x)D} = 0.022 Re_{f(x)D}^{0.8} Pr_{f(x)}^{0.43} \varepsilon_l \tag{13}$$

where $\varepsilon_l = 1$ if $l/D > 15$. Otherwise, the following expression is applied: $\varepsilon_l = 1.38(x/D)^{-0.12}$.

Average convective heat transfer coefficient can be calculated from:

$$\overline{Nu}_{fD} = 0.021 Re_{fD}^{0.8} Pr_f^{0.43} (Pr_f/Pr_w)^{0.25}\, \bar{\varepsilon}_l \tag{14}$$

where $\bar{\varepsilon}_l$ is obtained from the following table:

Re	***l/D***								
	1	2	5	10	15	20	30	40	≥50
1·10⁴	1.65	1.50	1.34	1.23	1.17	1.13	1.07	1.03	1
2·10⁴	1.51	1.40	1.27	1.18	1.13	1.10	1.05	1.02	1
5·10⁴	1.34	1.27	1.18	1.13	1.10	1.08	1.04	1.02	1
1·10⁵	1.28	1.22	1.15	1.10	1.08	1.06	1.03	1.02	1
1·10⁶	1.14	1.11	1.08	1.05	1.04	1.03	1.02	1.01	1

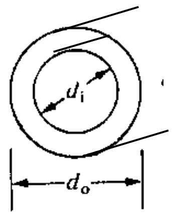
C3. Convective heat transfer coefficients inside ducts of arbitrary cross-sections

For turbulent flows in non-circular tubes, and in absence of specific correlations, it is advisable to use the correlations obtained for circular pipes (see Section B2 or Section C2), replacing in both the Nusselt number and the Reynolds number the diameter D by the hydraulic diameter D_h, which is defined as: $D_h = 4S/P$ (S is the flow cross-section and P the "wet" perimeter).

Some authors recommend the use of the hydraulic diameter D_h for the Reynolds number, and a thermal diameter, D_{th}, for the Nusselt number. The thermal diameter is defined as: $D_{th} = 4S/P_t$, where P_t refers to the perimeter in which the heat transfer takes place.

In any case, the best option is to employ specific correlations. For example, for the case of flow in annular cross-sections, of inner diameter d_i and outer diameter d_o, Monrad and Pelton (1942) suggest the following correlation:

$$Nu_{D_h} = 0.020 Re_{D_h}^{0.8} Pr^{1/3} \left(\frac{D_2}{D_1}\right)^{0.53} \qquad (15)$$



This correlation was obtained in experiments with water and oil, for $d_o/d_i = 1.65$, $2.45,\ and\ 17$, and within the range $Re_{D_h} = 12.000 \div 220.000$. For annular sections, $D_h = d_o - d_i$. More correlations can be found in the technical literature.

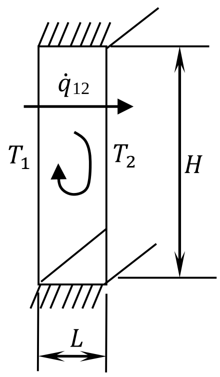
A different approach is proposed by Petukov and Roizen introducing a correction factor Φ on Gnielinski's formula (turbulent flows in tubes, $3000 < Re < 10^6$) and using the hydraulic diameter:

$$Nu_{d_h} = \frac{(f/8)\left(Re_{D_h} - 1000\right)Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}\Phi \qquad (16)$$

where $\Phi = 0.86(d_i/d_o)^{-0.16}$ for heat transfer through the inner wall with the outer wall insulated, and $\Phi = 1 - 0.14(d_i/d_o)^{0.6}$ when the heat transfer is only through the outer wall.

C4. Convection heat transfer coefficients in cavities with isothermal vertical walls

The cavity has a height H and width L. The vertical walls are isothermal (left wall at T_1 and right wall at T_2). Horizontal top and bottom walls are adiabatic. The heat flux per unit surface that goes from wall 1 to wall 2 is expressed by: $\dot{q}_{12} = \bar{\alpha}\,(T_1 - T_2)$.:



$$\overline{Nu}_L = 0.18\left(\frac{Pr Ra_L}{0.2 + Pr}\right)^{0.29} \quad wih\ 1 \lesssim \frac{H}{L} \lesssim 2;\ \ 10^{-3} \lesssim Pr \lesssim 10^5;\ \ 10^3 \lesssim \frac{Pr Ra_L}{0.2 + Pr}$$

$$\overline{Nu}_L = 0.22\left(\frac{Pr Ra_L}{0.2+Pr}\right)^{0.28}\left(\frac{H}{L}\right)^{-1/4} \quad wih\ 2 \lesssim \frac{H}{L} \lesssim 10;\ \ Pr \lesssim 10^5;\ \ 10^3 \lesssim Ra_L \lesssim 10^{10}$$

$$\overline{Nu}_L = 0.42 Ra_L^{1/4} Pr^{0.012}\left(\frac{H}{L}\right)^{-0.3} \quad wih\ 10 \lesssim \frac{H}{L} \lesssim 40;\ \ 1 \lesssim Pr \lesssim 2\times 10^4;\ \ 10^4 \lesssim Ra_L \lesssim 10^7$$

$$\overline{Nu}_L = 0.046 Ra_L^{1/3} \quad wih\ 1 \lesssim \frac{H}{L} \lesssim 40;\ \ 1 \lesssim Pr \lesssim 20;\ \ 10^6 \lesssim Ra_L \lesssim 10^9$$

Rayleigh number, $Ra_L = Pr(g\beta\rho^2|T_1 - T_2|L^3/\mu^2)$. Thermophysical properties are evaluated at $T_m = (T_1 + T_2)/2$. Note: all these correlations are mentioned in Incropera and DeWitt book.

D. THERMOPHYSICAL PROPERTIES

In this Section the following information is given: i) **Table D0**: Algebraic correlations for the evaluation of thermophysical properties of dry air, humid air, mass diffusivities, water and two different thermal oils (Therminol 66 and Mobiltherm 605); ii) **Table D1**: Metallic materials ($\rho, c_p, k, a = \lambda/\rho c_p$); iii) **Table D2**: Liquids (water at saturated conditions, oil, glycerine and mercury) ($\rho, c_p, \nu = \mu/\rho, \lambda, a, Pr, \beta$); iv) **Table D3**: Gases at atmosphere pressure conditions (air, steam, hydrogen, oxygen and nitrogen) ($\rho, c_p, \mu, \nu, k, a, Pr$); v) **Table D4**: Non-metal materials (ρ, c_p, λ, a); vi) **Table D5**: Insulating materials (λ); vii) **Table D6**: Radiative properties of different materials ($\varepsilon_n, \varepsilon$).

Most of the tables have been extracted from the book by Eckert and Drake (Analysis of Heat and Mass Transfer, McGraw-Hill, 1972). Be careful, in some tables the values must be multiplied by the number 10^n showed at the top of the corresponding column.

D0. Thermophysical properties for dry air, humid air, water and thermal oils

Basic thermodynamic relations

Semiperfect liquid ($\rho = constant; c_v = c_p;\ \beta = \kappa = 0$)

$$du = c_p dT$$
$$dh = c_p dT + dp/\rho$$
$$ds = c_p dT/T$$

Semiperfect gas ($\rho = \frac{p}{RT};\ c_p - c_v = R;\ \ \beta = \frac{1}{T};\ \ \kappa = \frac{1}{p}$)

$$du = c_v dT$$
$$dh = c_p dT$$
$$ds = c_p dT/T - R dp/p$$
$$R = \mathcal{R}/W;\quad \mathcal{R} = 8.31447\ kJ/kmol$$

Dry air (range: $T = 100 \div 2500\ K$, except λ) (T in K, p in Pa) (μ_1 for $T < 1500\ K$; μ_2 for $T \geq 1500\ K$):

$$\boldsymbol{\rho} = \frac{p}{287T};\qquad \boldsymbol{\lambda}\left(\frac{\boldsymbol{W}}{\boldsymbol{mK}}\right) = \frac{2.648 \cdot 10^{-3}\sqrt{T}}{1 + (245.4/T)\cdot 10^{-12/T}}\qquad for\ T \leq 1300\ K$$

$$\boldsymbol{c_p}\left(\frac{\boldsymbol{J}}{\boldsymbol{kgK}}\right) = 1034.09 - 2.849 \cdot 10^{-1}T + 7.817 \cdot 10^{-4}T^2 - 4.971 \cdot 10^{-7}T^3 + 1.077 \cdot 10^{-10}T^4$$

$$\boldsymbol{\mu_1}\left(\frac{\boldsymbol{kg}}{\boldsymbol{ms}}\right) = \frac{1.458 \cdot 10^{-6}T^{1.5}}{T + 110.40};\qquad \boldsymbol{\mu_2}\left(\frac{\boldsymbol{kg}}{\boldsymbol{ms}}\right) = \frac{2.5393 \cdot 10^{-5}\sqrt{T/273.15}}{1 + (122/T)}$$

$$\boldsymbol{Pr} = \frac{\mu c_p}{\lambda}\ \ if\ \ T < 1100\ K;\qquad \boldsymbol{Pr} = 0.71\ \ if\ \ T \geq 1100\ K;\qquad \boldsymbol{\beta}(\boldsymbol{K^{-1}}) = 1/T$$

Simplified expressions for dry air (range: $T = 200 \div 400\ K$) (T in K, p in Pa):

$$\boldsymbol{\rho} = p/(287T);\qquad \boldsymbol{c_p}(\boldsymbol{J/kgK}) = 1031.5 - 0.210T + 4.143 \cdot 10^{-4}T^2$$

$$\boldsymbol{\lambda}(\boldsymbol{W/mK}) = 2.728 \cdot 10^{-3} + 7.776 \cdot 10^{-5}T;\qquad \boldsymbol{\mu}(\boldsymbol{kg/ms}) = \frac{2.5393\cdot 10^{-5}\sqrt{T/273.15}}{1+(122/T)};\qquad \boldsymbol{\beta}(\boldsymbol{K^{-1}}) = 1/T$$

Humid air (from ASHRAE Fundamentals):

- **Saturation vapour pressure** (T in K and p_{vs} in Pa) : $\ln p_{vs} = -5.8002206 \times 10^3/T + 1.3914993 - 4.8640239 \times 10^{-2}T + 4.1764768 \times 10^{-5}T^2 - 1.4452093 \times 10^{-8}T^3 + 6.5459673 \ln T$ $\quad \boldsymbol{if\ 273.15 \leq \mathrm{T}(\mathrm{K}) \leq 473.15}$; $\ln p_{vs} = -5.6745359 \times \frac{10^3}{T} + 6.3925247 - 9.677843 \times 10^{-3}T + 6.2215701 \times 10^{-7}T^2 + 2.0747825 \times 10^{-9}T^3 - 9.484024 \times 10^{-13}T^4 + 4.1635019 \ln T$ $\quad \boldsymbol{if\ 173.15 \leq \mathrm{T}(\mathrm{K}) < 273.15.}$

- **Relative humidity:** $\varphi = \left(\frac{n_v}{n_{vs}}\right)_{p,T} \approx \frac{p_v}{p_{vs}}$, where n represents the moles of water vapour contained in the air and p_v is the water partial pressure in air (v: vapour; vs: saturated vapour).

- **Moist air density**: $\rho = \rho_{da} + \rho_v = \frac{p-p_v}{R_{da}T} + \frac{p_v}{R_vT}$ $\quad$ ($R_{da} = 287.042\,J/kgK;\ R_v = 461.524\,J/kgK$) ($da$: dry air)

- **Humidity ratio**: $\phi = \frac{m_v}{m_{da}} = \frac{W_v n_v}{W_{da} n_{da}} \approx \frac{R_{da}}{R_v}\frac{p_v}{p-p_v}$

- **Vapour mass fraction** (or specific humidity): $Y_v = \frac{m_v}{m_{da}+m_v} = \frac{\phi}{1+\phi}$

- **Dew-point temperature**: $T_{dp} = 6.54 + 14.526\alpha + 0.7389\alpha^2 + 0.09486\alpha^3 + 0.4569(p_v)^{0.1984}$ $\boldsymbol{if\ 0 \le T_{dp} \le 93}$, and $T_{dp} = 6.09 + 12.608\alpha + 0.4959\alpha^2$ $\boldsymbol{if\ T_{dp} < 0}$, where $T_{dp}(°C)$, $\alpha = \ln(p_v)$ and $p_v(kPa)$

- **Absolute specific humid air enthalpy**: $h_{ha}(T,p,Y_v) = (1-Y_v)h_{da} + Y_v h_v$, where,

$$h_v(T,p) = h^o_{fv} + \int_{T^o}^{T} c_{pv}dT\,; \quad h^o_{fv} = -13423959\frac{J}{kg}; \quad c_{pv} = 1860\frac{J}{kgK} \qquad (1.3.8)$$

$$h_{da}(T,p) = h^o_{da} + \int_{T^o}^{T} c_{pda}dT\,; \quad h^o_{da} = 0; \quad c_{pda} \approx 1006\frac{J}{kgK} \qquad (1.3.9)$$

Absolute enthalpy of liquid water: $h_l(T,p) = h^o_{fl} + \int_{T^o}^{T} c_{pl}dT + \frac{p-p^o}{\rho_l}$; $\quad h^o_{fl} = -15865987\frac{J}{kg}$; $\quad c_{pl} \approx 4186\frac{J}{kgK}$. Note: $h^o_{fv} - h^o_{fl} = 2.4420 \times 10^6\ kJ/kg$.

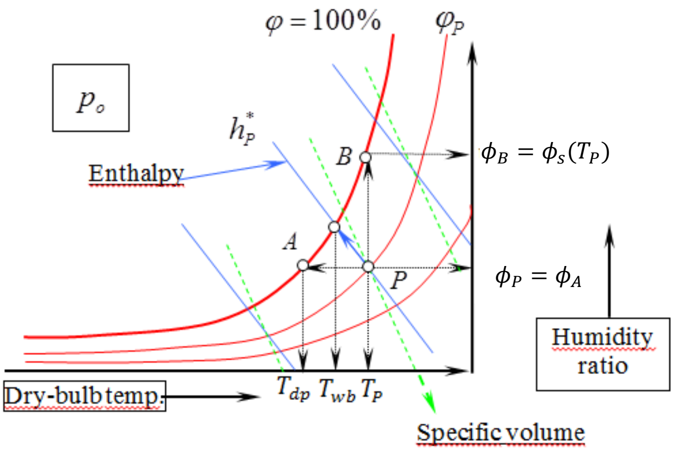
Note. $T^o = 298\ K$, $p^o = 1\ atm$.

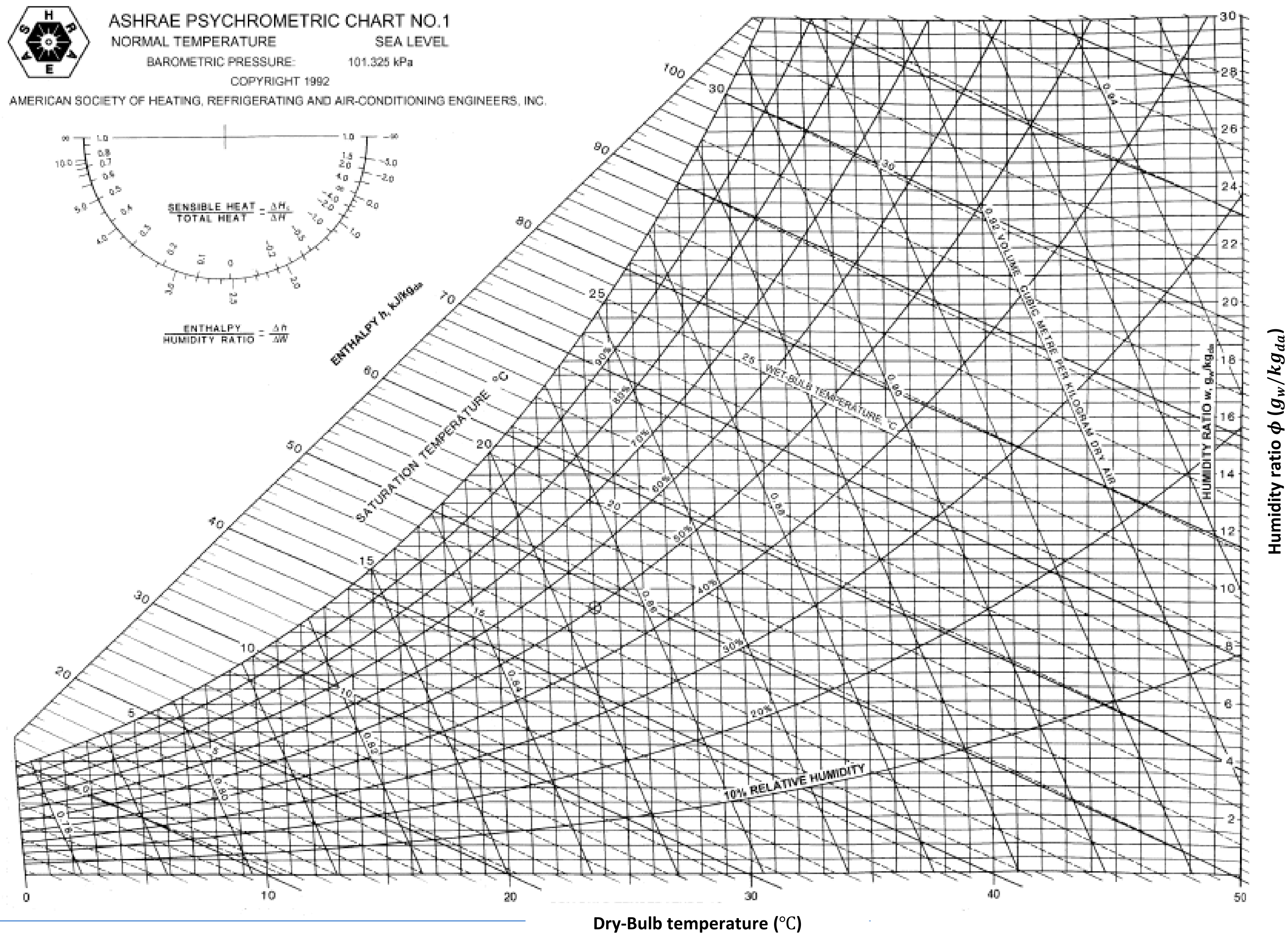
- **Wet-bub temperature** (or adiabatic saturation temperature). Temperature when air is brought to saturation adiabatically (this is an isenthalpic process). Form an energy balance of humid air at given T, p and ϕ, the wet-bulb temperature T_{wb} can be obtained: $T_{wb} = T - \frac{(\phi_{wb}-\phi)[h_v(T_{wb})-h_l(T_{wb})]}{1006+1860\phi}$. This equation must be iteratively solved.

- **Mass diffusivity of water vapour in humid air** (up to 1100°C; empirical expression by Sherwood and Pigford): $D_v = \frac{0.926}{p}\left(\frac{T^{2.5}}{T+245}\right)$ (D_v in mm^2/s, T in K, p in kPa).

- **Thermal conductivity of humid air**: $\lambda = \left[1 + \frac{\chi_{da}(1-\chi_{da})}{2.75}\right](\chi_{da}\lambda_{da} + \chi_v\lambda_v)$, where $\lambda_v(W/mK) = -7.145 \cdot 10^{-3} + 8.4 \cdot 10^{-5}T$ (T in K), $\chi_{da} = n_{da}/n$, $\chi_v = 1 - \chi_{da}$.

- **Psychrometric chart** (see next page):




ASHRAE PSYCHROMETRIC CHART NO.1
NORMAL TEMPERATURE
SEA LEVEL
BAROMETRIC PRESSURE: 101.325 kPa
COPYRIGHT 1992
AMERICAN SOCIETY OF HEATING, REFRIGERATING AND AIR-CONDITIONING ENGINEERS, INC.
SENSIBLE HEAT / TOTAL HEAT = ΔHs / ΔH
ENTHALPY / HUMIDITY RATIO = Δh / ΔW
ENTHALPY h, kJ/kg da
SATURATION TEMPERATURE °C
WET BULB TEMPERATURE, °C
0.92 VOLUME CUBIC METRE PER KILOGRAM DRY AIR
10% RELATIVE HUMIDITY
HUMIDITY RATIO w, gw/kgda
Humidity ratio ϕ (g_w/kg_{da})
Dry-Bulb temperature (°C)

Diffusion coefficients of gases and vapours in air at 25℃ and 1 bar

Substance	D, cm²/s	$Sc = \frac{\nu}{D}$	Substance	D, cm²/s	$Sc = \frac{\nu}{D}$
Ammonia	0.28	0.78	Formic acid	0.159	0.97
Carbon dioxide	0.164	0.94	Acetic acid	0.133	1.16
Hydrogen	0.410	0.22	Aniline	0.073	2.14
Oxygen	0.206	0.75	Benzene	0.088	1.76
Water	0.256	0.60	Toluene	0.084	1.84
Ethyl ether	0.093	1.66	Ethyl benzene	0.077	2.01
Methanol	0.159	0.97	Propyl benzene	0.059	2.62
Ethyl alcohol	0.119	1.30			

(J.H.Perry, Chemical Engineers Handbookm Mcgraw-Hill, 1963)

Water at saturation conditions (range: $T = 273 \div 573\ K$) (μ_1 for $T < 353\ K$; μ_2 for $T \geq 353\ K$) (***T* in *K***):

$$\boldsymbol{\rho}\left(\frac{kg}{m^3}\right) = 847.2 + 1.298T - 2.657 \cdot 10^{-3}T^2; \quad \boldsymbol{\beta} = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_p; \quad \boldsymbol{c_p}\left(\frac{J}{kgK}\right) = 5648.8 - 9.140T + 14.21 \cdot 10^{-3}T^2$$

$$\boldsymbol{\lambda}\left(\frac{W}{mK}\right) = -1.176 + 7.915 \cdot 10^{-3}\,T + 1.486 \cdot 10^{-5}\,T^2 - 1.317 \cdot 10^{-7}\,T^3 + 2.476 \cdot 10^{-10}\,T^4 - 1.556 \cdot 10^{-13}\,T^5$$

$$\boldsymbol{\mu_1}\left(\frac{kg}{ms}\right) = 0.9149 - 1.2563 \cdot 10^{-2}\,T + 6.9182 \cdot 10^{-5}\,T^2 - 1.9067 \cdot 10^{-7}\,T^3 + 2.6275 \cdot 10^{-10}\,T^4 - 1.4474 \cdot 10^{-13}\,T^5$$

$$\boldsymbol{\mu_2}\left(\frac{kg}{ms}\right) = 3.7471 \cdot 10^{-2} - 3.5636 \cdot 10^{-4}\,T + 1.3725 \cdot 10^{-6}\,T^2 - 2.6566 \cdot 10^{-9}\,T^3 + 2.5766 \cdot 10^{-12}\,T^4 - 1 \cdot 10^{-15}\,T^5$$

*Simplified expressions for water at saturation conditions (range: $T = 273 \div 400\ K$) (**T in K**):*

$$\boldsymbol{\rho}\left(\frac{kg}{m^3}\right) = 847.2 + 1.298T - 2.657 \cdot 10^{-3}T^2; \boldsymbol{c_p}\left(\frac{J}{kgK}\right) = 5648.79 - 9.140T + 14.21 \cdot 10^{-3}T^2;$$

$$\boldsymbol{\lambda}\left(\frac{W}{mK}\right) = -0.722 + 7.168 \cdot 10^{-3}T - 9.137 \cdot 10^{-6}T^2; \quad \boldsymbol{\mu}\left(\frac{kg}{ms}\right) = e^{7.867 - 0.077T + 9.04 \cdot 10^{-5}T^2}; \quad \boldsymbol{\beta} = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_p$$

Therminol 66 thermal oil (range: $T = 273 \div 653\ K$) (***T in K***):

$$\boldsymbol{\rho}\left(\frac{kg}{m^3}\right) = 1164.45 - 0.4389T - 3.21 \cdot 10^{-4}T^2; \qquad \boldsymbol{c_p}\left(\frac{J}{kgK}\right) = 658 + 2.82T + 8.97 \cdot 10^{-4}T^2$$

$$\boldsymbol{\lambda}\left(\frac{W}{mK}\right) = 0.116 + 4.9 \times 10^{-5}T - 1.5 \cdot 10^{-7}T^2; \qquad \boldsymbol{\nu}\left(\frac{m^2}{s}\right) = \frac{\mu}{\rho} = e^{-16.096 + \frac{586.38}{T - 210.65}}; \quad \boldsymbol{\beta} = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_p$$

Mobiltherm 605 thermal oil (range: $T = 300 \div 600\ K$) (***T in K***):

$$\boldsymbol{\rho}\left(\frac{kg}{m^3}\right) = 1059.6 - 0.65T; \qquad \mathbf{c_p}\left(\frac{\mathbf{J}}{\mathbf{kgK}}\right) = 829.2 + 3.61T$$

$$\boldsymbol{\lambda}\left(\frac{\mathbf{W}}{\mathbf{mK}}\right) = 0.154 - 7.063 \cdot 10^{-5}T; \qquad \boldsymbol{\nu}\left(\frac{\mathbf{m^2}}{\mathbf{s}}\right) = \frac{\mu}{\rho} = e^{5.47 - 0.069T + 6.09 \cdot 10^{-5}T^2}; \quad \boldsymbol{\beta} = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_p$$

Solar molten salt ($NaNO_3 - KNO_3$, $60 - 40\%wt$) (range: $T = 260 \div 620$ ℃) (***T in*** ℃):

$$\boldsymbol{\rho}\left(\frac{kg}{m^3}\right) = 2090 - 0.636T; \qquad \mathbf{c_p}\left(\frac{\mathbf{J}}{\mathbf{kgK}}\right) = 1443 + 0.172T$$

$$\boldsymbol{\lambda}\left(\frac{\mathbf{W}}{\mathbf{mK}}\right) = 0.443 + 1.93 \cdot 10^{-4}T; \qquad \boldsymbol{\beta} = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_p;$$

$$\boldsymbol{\mu}\left(\frac{\mathbf{kg}}{\mathbf{ms}}\right) = 22.714 \cdot 10^{-3} - 0.120 \cdot 10^{-3}T + 2.281 \cdot 10^{-7}T^2 - 1.474 \cdot 10^{-10}T^3$$

Table D1. Thermophysical properties for metals

Metal	Properties at 20 C				Thermal conductivity λ (W/mK)									
	ρ, kg/m³	c_p Ws/kg K	λ W/m K	a m²/s	−100 C −148 F	0 C 32 F	100 C 212 F	200 C 392 F	300 C 572 F	400 C 752 F	600 C 1112 F	800 C 1472 F	1000 C 1832 F	1200 C 2192 F
Aluminum:														
Pure	2,707	0.896 ×	204	8.418 $\times 10^{-5}$	215	202	206	215	228	249				
Al-Cu (Duralumin) 94–96 Al, 3–5 Cu, trace Mg	2,787	0.883	164	6.676	126	159	182	194						
Al-Mg (Hydronalium) 91–95 Al, 5–9 Mg	2,611	0.904	112	4.764	93	109	125	142						
Al-Si (Silumin) 87 Al, 13 Si	2,659	0.871	164	7.099	149	163	175	185						
Al-Si (Silumin, copper bearing) 86.5 Al, 1 Cu	2,659	0.867	137	5.933	119	137	144	152	161					
Al-Si (Alusil) 78–80 Al, 20–22 Si	2,627	0.854	161	7.172	144	157	168	175	178					
Al-Mg-Si 97 Al, 1 Mg, 1 Si, 1 Mn	2,707	0.892	177	7.311		175	189	204						
Lead	11,373	0.130	35	2.343	36.9	35.1	33.4	31.5	29.8					
Iron:														
Pure	7,897	0.452	73	2.034	87	73	67	62	55	48	40	36	35	36
Wrought iron (C H 0.5 %)	7,849	0.46	59	1,626		59	57	52	48	45	36	33	33	33
Cast iron (C ≈ 4 %)	7,272	0.42	52	1.703										
Steel (C max ≈ 1.5 %)														
Carbon steel C ≈ 0.5 %	7,833	0.465	54	1.474		55	52	48	45	42	35	31	29	31
1.0 %	7,801	0.473	43	1.172		43	43	42	40	36	33	29	28	29
1.5 %	7,753	0.486	36	0.970		36	36	36	35	33	31	28	28	29
Nickel steel Ni ≈ 0 %	7,897	0.452	73	2.026										
10 %	7,945	0.46	26	0.720										
20 %	7,993	0.46	19	0.526										
30 %	8,073	0.46	12	0.325										
40 %	8,169	0.46	10	0.279										
50 %	8,266	0.46	14	0.361										
60 %	8,378	0.46	19	0.493										
70 %	8,506	0.46	26	0.666										
80 %	8,618	0.46	35	0.872										
90 %	8,762	0.46	47	1.156										
100 %	8,906	0.448	90	2.276										
Copper:														
Pure	8,954	0.3831 $\times 10^3$	386	11.234 $\times 10^{-5}$	407	386	379	374	369	363	353			
Aluminum bronze 95 cu, 5 Al	8,666	0.410	83	2.330										
Bronze 75 Cu, 25 Sn	8,666	0.343	26	0.859										
Red Brass 85 Cu, 9 Sn, 6 Zn	8,714	0.385	61	1.804		59	71							
Brass 70 Cu, 30 Zn	8,522	0.385	111	3.412	88		128	144	147	147				
German silver 62 Cu, 15 Ni, 22 Zn	8,618	0.394	24.9	0.733	19.2		31	40	45	48				
Constantan 60 Cu, 40 Ni	8,922	0.410	22.7	0.612	21		22.2	26						
Magnesium:														
Pure	1,746	1.013	171	9.708	178	171	168	163	157					
Mg-Al (electrolytic) 6–8 % Al, 1–2 % Zn	1,810	1.00	66	3.605		52	62	74	83					
Mg-Mn 2 % Mn	1,778	1.00	114	6.382	93	111	125	130						
Mg-Mn 2 % Mn	1,778	1.00	114	6.382	93	111	125	130						
Molybdenum	10,220	0.251	123	4.790	138	125	118	114	111	109	106	102	99	92
Nickel:														
Pure (99.9 %)	8,906	0.4459	90	2.266	104	93	83	73	64	59				
Impure (99.2 %)	8,906	0.444	69	1.747		69	64	59	55	52	55	62	67	69
Ni-Cr 90 Ni, 10 Cr	8,666	0.444	17	0.444		17.1	18.9	20.9	22.8	24.6				
80 Ni, 20 Cr	8,314	0.444	12.6	0.343		12.3	13.8	15.6	17.1	18.9	22.5			
Silver:														
Purest	10,524	0.2340	419	17.004	419	417	415	412						
Pure (99.9 %)	10,524	0.2340	407	16.563	419	410	415	374	362	360				
Tungsten	19,350	0.1344	163	6.271		166	151	142	133	126	112	76		
Zinc, pure	7.144	0.3843	112.2	4.106	114	112	109	106	100	93				
Tin, pure	7,304	0.2265	64	3.884	74	65.9	59	57						

Table D2. Thermophysical properties of fluids in a saturated state

T, C	ρ, kg/m³	c_p, Ws/kg K	ν, m²/s	λ W/m K	a, m²/s	Pr	β, K⁻¹
Water, H_2O							
0	1,002.28	4.2178 × 10³	1.788 × 10⁻⁶	0.552	1.308 × 10⁻⁷	13.6	0.18 × 10⁻³
20	1,000.52	4.1818	1.006	0.597	1.430	7.02	
40	994.59	4.1784	0.658	0.628	1.512	4.34	
60	985.46	4.1843	0.478	0.651	1.554	3.02	
80	974.08	4.1964	0.364	0.668	1.636	2.22	
100	960.63	4.2161	0.294	0.680	1.680	1.74	
120	945.25	4.250	0.247	0.685	1.708	1.446	
140	928.27	4.283	0.214	0.684	1.724	1.241	
160	909.69	4.342	0.190	0.680	1.729	1.099	
180	889.03	4.417	0.173	0.675	1.724	1.004	
200	866.76	4.505	0.160	0.665	1.706	0.937	
220	842.41	4.610	0.150	0.652	1.680	0.891	
240	815.66	4.756	0.143	0.635	1.639	0.871	
260	785.87	4.949	0.137	0.611	1.577	0.874	
280.6	752.55	5.208	0.135	0.580	1.481	0.910	
300	714.26	5.728	0.135	0.540	1.324	1.019	
Engine oil (unused)							
0	899.12	1.796 × 10³	0.00428	0.147	0.911 × 10⁻⁷	47,100	0.70 × 10⁻³
20	888.23	1.880	0.00090	0.145	0.872	10,400	
40	876.05	1.964	0.00024	0.144	0.834	2,870	
60	864.04	2.047	0.839 × 10⁻⁴	0.140	0.800	1,050	
80	852.02	2.131	0.375	0.138	0.769	490	
100	840.01	2.219	0.203	0.137	0.738	276	
120	828.96	2.307	0.124	0.135	0.710	175	
140	816.94	2.395	0.080	0.133	0.686	116	
160	805.89	2.483	0.056	0.132	0.663	84	
Glycerin, $C_3H_5(OH)_3$							
0	1,276.03	2.261 × 10³	0.00831	0.282	0.983 × 10⁻⁷	84.7 × 10³	0.50 × 10⁻³
10	1,270.11	2.319	0.00300	0.284	0.965	31.0	
20	1,264.02	2.386	0.00118	0.286	0.947	12.5	
30	1,258.09	2.445	0.00050	0.286	0.929	5.38	
40	1,252.01	2.512	0.00022	0.286	0.914	2.45	
50	1,244.96	2.583	0.00015	0.287	0.893	1.63	
Mercury, Hg							
0	13,628.22	0.1403 × 10³	0.124 × 10⁻⁶	8.20	42.99 × 10⁻⁷	0.0288	1.82 × 10⁻⁴
20	13,579.04	0.1394	0.114	8.69	46.06	0.0249	
50	13,505.84	0.1386	0.104	9.40	50.22	0.0207	
100	13,384.58	0.1373	0.0928	10.51	57.16	0.0162	
150	13,264.28	0.1365	0.0853	11.49	63.54	0.0134	
200	13,144.94	0.1570	0.0802	12.34	69.08	0.0116	
250	13,025.60	0.1357	0.0765	13.07	74.06	0.0103	
315.5	12,847	0.134	0.0673	14.02	81.5	0.0083	

Table D3 (1/2). Thermophysical properties of gases at atmospheric

Air

T, K	ρ kg/m³	c_p, Ws/kg K	μ, kg/ms	ν, m²/s	λ W/m K	a , m²/s	Pr
100	3.6010	1.0266 ×1	0.6924 $\times 10^{-5}$	1.923 $\times 10^{-6}$	0.009246	0.02501 $\times 10^{-4}$	0.770
150	2.3675	1.0099	1.0283	4.343	0.013735	0.05745	0.753
200	1.7684	1.0061	1.3289	7.490	0.01809	0.10165	0.739
250	1.4128	1.0053	1.600	9.49	0.02227	0.13161	0.722
300	1.1774	1.0057	1.847	15.68	0.02624	0.22160	0.708
350	0.9980	1.0090	2.075	20.76	0.03003	0.2983	0.697
400	0.8826	1.0140	2.286	25.90	0.03365	0.3760	0.689
450	0.7833	1.0207	2.484	28.86	0.03707	0.4222	0.683
500	0.7048	1.0295	2.671	37.90	0.04038	0.5564	0.680
550	0.6423	1.0392	2.848	44.34	0.04360	0.6532	0.680
600	0.5879	1.0551	3.018	51.34	0.04659	0.7512	0.680
650	0.5430	1.0635	3.177	58.51	0.04953	0.8578	0.682
700	0.5030	1.0752	3.332	66.25	0.05230	0.9672	0.684
750	0.4709	1.0856	3.481	73.91	0.05509	1.0774	0.686
800	0.4405	1.0978	3.625	82.29	0.05779	1.1951	0.689
850	0.4149	1.1095	3.765	90.75	0.06028	1.3097	0.692
900	0.3925	1.1212	3.899	99.3	0.06279	1.4271	0.696
950	0.3716	1.1321	4.023	108.2	0.06525	1.5510	0.699
1000	0.3524	1.1417	4.152	117.8	0.06752	1.6779	0.702
1100	0.3204	1.160	4.44	138.6	0.0732	1.969	0.704
1200	0.2947	1.179	4.69	159.1	0.0782	2.251	0.707
1300	0.2707	1.197	4.93	182.1	0.0837	2.583	0.705
1400	0.2515	1.214	5.17	205.5	0.0891	2.920	0.705
1500	0.2355	1.230	5.40	229.1	0.0946	3.262	0.705
1600	0.2211	1.248	5.63	254.5	0.100	3.609	0.705
1700	0.2082	1.267	5.85	280.5	0.105	3.977	0.705
1800	0.1970	1.287	6.07	308.1	0.111	4.379	0.704
1900	0.1858	1.309	6.29	338.5	0.117	4.811	0.704
2000	0.1762	1.338	6.50	369.0	0.124	5.260	0.702
2100	0.1682	1.372	6.72	399.6	0.131	5.715	0.700
2200	0.1602	1.419	6.93	432.6	0.139	6.120	0.707
2300	0.1538	1.482	7.14	464.0	0.149	6.540	0.710
2400	0.1458	1.574	7.35	504.0	0.161	7.020	0.718
2500	0.1394	1.688	7.57	543.5	0.175	7.441	0.730

Steam (H_2O vapor)

T, K	ρ kg/m³	c_p, Ws/kg K	μ, kg/ms	ν, m²/s	λ W/m K	a , m²/s	Pr
380	0.5863	2.060 $\times 10^{3}$	12.71 $\times 10^{-6}$	2.16 $\times 10^{-5}$	0.0246	0.2036 $\times 10^{-4}$	1.060
400	0.5542	2.014	13.44	2.42	0.0261	0.2338	1.040
450	0.4902	1.980	15.25	3.11	0.0299	0.307	1.010
500	0.4405	1.985	17.04	3.86	0.0339	0.387	0.996
550	0.4005	1.997	18.84	4.70	0.0379	0.475	0.991
600	0.3652	2.026	20.67	5.66	0.0422	0.573	0.986
650	0.3380	2.056	22.47	6.64	0.0464	0.666	0.995
700	0.3140	2.085	24.26	7.72	0.0505	0.772	1.000
750	0.2931	2.119	26.04	8.88	0.0549	0.883	1.005
800	0.2739	2.152	27.86	10.20	0.0592	1.001	1.010
850	0.2579	2.186	29.69	11.52	0.0637	1.130	1.019

Table D3 (2/2). Thermophysical properties of gases at atmospheric pressure

T, K	ρ, kg/m³	c_p, Ws/kg K	μ, kg/ms	ν, m²/s	λ W/m K	a, m²/s	Pr
Hydrogen							
30	0.84722	10.840 × 10³	1.606 × 10⁻⁶	1.895 × 10⁻⁶	0.0228	0.02493 × 10⁻⁴	0.759
50	0.50955	10.501	2.516	4.880	0.0362	0.0676	0.721
100	0.24572	11.229	4.212	17.14	0.0665	0.2408	0.712
150	0.16371	12.602	5.595	34.18	0.0981	0.475	0.718
200	0.12270	13.540	6.813	55.53	0.1282	0.772	0.719
250	0.09819	14.059	7.919	80.64	0.1561	1.130	0.713
300	0.08185	14.314	8.963	109.5	0.182	1.554	0.706
350	0.07016	14.436	9.954	141.9	0.206	2.031	0.697
400	0.06135	14.491	10.864	177.1	0.228	2.568	0.690
450	0.05462	14.499	11.779	215.6	0.251	3.164	0.682
500	0.04918	14.507	12.636	257.0	0.272	3.817	0.675
550	0.04469	14.532	13.475	301.6	0.292	4.516	0.668
600	0.04085	14.537	14.285	349.7	0.315	5.306	0.664
700	0.03492	14.574	15.89	455.1	0.351	6.903	0.659
800	0.03060	14.675	17.40	569	0.384	8.563	0.664
900	0.02723	14.821	18.78	690	0.412	10.217	0.676
1000	0.02451	14.968	20.16	822	0.440	11.997	0.686
1100	0.02227	15.165	21.46	965	0.464	13.726	0.703
1200	0.02050	15.366	22.75	1107	0.488	15.484	0.715
1300	0.01890	15.575	24.08	1273	0.512	17.394	0.733
1333	0.01842	15.638	24.44	1328	0.519	18.013	0.736
Oxygen							
100	3.9918	0.9479 × 10³	7.768 × 10⁻⁶	1.946 × 10⁻⁶	0.00903	0.023876 × 10⁻⁴	0.815
150	2.6190	0.9178	11.490	4.387	0.01367	0.05688	0.773
200	1.9559	0.9131	14.850	7.593	0.01824	0.10214	0.745
250	1.5618	0.9157	17.87	11.45	0.02259	0.15794	0.725
300	1.3007	0.9203	20.63	15.86	0.02676	0.22353	0.709
350	1.1133	0.9291	23.16	20.80	0.03070	0.2968	0.702
400	0.9755	0.9420	25.54	26.18	0.03461	0.3768	0.695
450	0.8682	0.9567	27.77	31.99	0.03828	0.4609	0.694
500	0.7801	0.9722	29.91	38.34	0.04173	0.5502	0.697
550	0.7096	0.9881	31.97	45.05	0.04517	0.6441	0.700
600	0.6504	1.0044	33.92	52.15	0.04832	0.7399	0.704
Nitrogen							
100	3.4808	1.0722 × 10³	6.862 × 10⁻⁶	1.971 × 10⁻⁶	0.009450	0.025319 × 10⁻⁴	0.786
200	1.7108	1.0429	12.947	7.568	0.01824	0.10224	0.747
300	1.1421	1.0408	17.84	15.63	0.02620	0.22044	0.713
400	0.8538	1.0459	21.98	25.74	0.03335	0.3734	0.691
500	0.6824	1.0555	25.70	37.66	0.03984	0.5530	0.684
600	0.5687	1.0756	29.11	51.19	0.04580	0.7486	0.686
700	0.4934	1.0969	32.13	65.13	0.05123	0.9466	0.691
800	0.4277	1.1225	34.84	81.46	0.05609	1.1685	0.700
900	0.3796	1.1464	37.49	91.06	0.06070	1.3946	0.711
1000	0.3412	1.1677	40.00	117.2	0.06475	1.6250	0.724
1100	0.3108	1.1857	42.28	136.0	0.06850	1.8591	0.736
1200	0.2851	1.2037	44.50	156.1	0.07184	2.0932	0.748

Table D4. Thermophysical properties for nonmetals

Material	T, C	ρ, kg/m³	c_p, Ws/kg K	λ W/m K	a, m²/s
Aerogel, silica	120	136.2		0.022	
Asbestos	−200	469.3		0.074	
	0	469.3		0.156	
	0	576.7	0.816×10^3	0.151	
	100	576.7	0.816	0.192	
	200	576.7		0.208	
	400	576.7		0.223	
	−200	696.8		0.156	
	0	696.8		0.234	
Brick, dry	20	1,762–1,810	0.84	0.38–0.52	$0.028–0.034\times10^{-5}$
Bakelite	20	1,273.5	1.59	0.232	0.0114
Cardboard, corrugated				0.064	
Clay	20	1,457.7	0.88	1.279	0.101
Concrete	20	1,906–2,307	0.88	0.81–1.40	0.049–0.070
Coal, anthracite	20	1,201–1,506	1.26	0.26	0.013–0.015
Powdered	30	737	1.30	0.116	0.013
Cotton	20	80	1.30	0.059	0.194
Cork, board	30	160		0.043	
Expanded scrap	20	44.9–118.5	1.88	0.036	0.015–0.044
Ground	30	150.6		0.043	
Diatomaceous earth	38	320.4		0.062	
	871	320.4		0.142	
Earth, coarse gravelly	20	2,050	1.84	0.52	0.0139
Felt, wood	30	330.0		0.05	
Fiber, insulating board	21	237.1		0.048	
Red	20	1,289.5		0.47	
Glass plate	20	2,707	0.8	0.76	0.034
Glass, borosilicate	30	2,227		1.09	
Wool	20	200.2	0.67	0.040	0.028
Granite				1.7–4.0	
Ice	0	913	1.93	2.22	0.124
Marble	20	2,499–2,707	0.808	2.8	0.139
Rubber, hard	0	1,198.2		0.151	
Sandstone	20	2,162–2,307	0.71	1.63–2.1	0.106–0.126
Silk	20	57.7	1.38	0.036	0.044
Wood, oak radial	20	609–801	2.39	0.17–0.21	0.0111–0.0121
Fir (20% moisture) radial	20	416.5–421.3	2.72	0.14	0.0124

Table D5. Thermophysical properties (termal conductivity) for insulating materials at high temperatures†

Material	*Mean temperature*										*Limiting-use temperature*	
	37.8 C 100 F	93.3 C 200 F	148.9 C 300 F	204.4 C 400 F	260 C 500 F	315.6 C 600 F	426.7 C 800 F	537.8 C 1000 F	815.6 C 1500 F	1093.3 C 2000 F	C	F
Asbestos (577 kg/m³) laminated asbestos felt	0.168	0.190	0.202	0.209	0.213	0.216	0.225					
Approx. 146 laminations/m	0.057	0.064	0.069	0.076	0.083						371.1	700
Approx. 73 laminations/m	0.078	0.087	0.095	0.104	0.112						260	500
Corrugated asbestos (14.6 plies/m)	0.087	0.100	0.119								148.9	300
85% magnesia (208 kg/m³)	0.059	0.062	0.066	0.069							315.6	600
Diatomaceous earth, asbestos and bonder	0.078	0.081	0.085	0.087	0.092	0.095	0.104	0.112			871.1	1600
Diatomaceous earth, brick	0.093	0.097	0.100	0.104	0.109	0.112	0.119	0.126			871.1	1600
Diatomaceous earth, brick	0.220	0.225	0.230	0.237	0.242	0.247	0.260	0.273	0.305		1093.3	2000
Diatomaceous earth, brick	0.222	0.227	0.234	0.241	0.247	0.256	0.268	0.282	0.317	0.351	1371.1	2500
Diatomaceous earth, powder (density, 288 kg/m³)	0.067	0.073	0.076	0.083	0.088	0.093	0.106	0.118				
Rock wool	0.052	0.059	0.067	0.076	0.087	0.099						

† L. S. Marks, "Mechanical Engineers' Handbook," 5th ed. Copyright 1951. McGraw-Hill Book Company. Used by permission.

Table D6a. Solar absorptivity (α_s) and thermal emissivity (ε) for different materials at room temperatures

Surface	α_s	ε
Aluminum		
Polished	0.09	0.03
Anodized	0.14	0.84
Foil	0.15	0.05
Copper		
Polished	0.18	0.03
Tarnished	0.65	0.75
Stainless steel		
Polished	0.37	0.60
Dull	0.50	0.21
Plated metals		
Black nickel oxide	0.92	0.08
Black chrome	0.87	0.09
Concrete	0.60	0.88
White marble	0.46	0.95
Red brick	0.63	0.93
Asphalt	0.90	0.90
Black paint	0.97	0.97
White paint	0.14	0.93
Snow	0.28	0.97
Human skin (caucasian)	0.62	0.97

Table D6a: from Y.A.Cengel, “Heat Transfer. A Practical Approach”, McGraw-Hill, 1998.

Table D6b: Ecker and Drake (see beginning Section D).

Table D6b. Emissivities ϵ_n of the radiation in the direction of the normal to the surface and ϵ of the total hemispherical radiation for various materials for the temperature t†‡

Surface	T (°C)	ϵ_n	ϵ
Gold, polished	130	0.018	
	400	0.022	
Silver	20	0.020	
Copper, polished	20	0.030	
Lightly oxidized	20	0.037	
Scraped	20	0.070	
Black oxidized	20	0.78	
Oxidized	131	0.76	0.725
Aluminum, bright rolled	170	0.039	0.049
	500	0.050	
Aluminum paint	100	0.20–0.40	
Silumin, cast polished	150	0.186	
Nickel, bright matte	100	0.041	0.046
Polished	100	0.045	0.053
Manganin, bright rolled	118	0.048	0.057
Chrome, polished	150	0.058	0.071
Iron, bright etched	150	0.128	0.158
Bright abrased	20	0.24	
Red rusted	20	0.61	
Hot rolled	20	0.77	
	130	0.60	
Hot cast	100	0.80	
Heavily rusted	20	0.85	
Heat-resistant oxidized	80	0.613	
	200	0.639	
Zinc, gray oxidized	20	0.23–0.28	
Lead, gray oxidized	20	0.28	
Bismuth, bright	80	0.340	0.366
Corundum, emery rough	80	0.855	0.84
Clay, fired	70	0.91	0.86
Lacquer, white	100	0.925	
Red lead	100	0.93	
Enamel, lacquer	20	0.85–0.95	
Lacquer, black matte	80	0.970	
Bakelite lacquer	80	0.935	
Brick, mortar, plaster	20	0.93	
Porcelain	20	0.92–0.94	
Glass	90	0.940	0.876
Ice, smooth, water	0	0.966	0.918
Rough crystals	0	0.985	
Waterglass	20	0.96	
Paper	95	0.92	0.89
Wood, beech	70	0.935	0.91
Tarpaper	20	0.93	

† From measurements by E. Schmidt and E. Eckert.

‡ For metals, the emissivities rise with rising temperature, but for nonmetallic substances (metal oxides, organic substances) this rule is sometimes not correct. Where the exact measurements are not given, take for bright metal surfaces an average ratio $\epsilon/\epsilon_n = 1.2$ and for other substances with smooth surfaces $\epsilon/\epsilon_n = 0.95$; for rough surfaces use $\epsilon/\epsilon_n = 0.98$.

E. NOMENCLATURE[4]

a	Thermal diffusivity (m^2/s), $a=\lambda/\rho c_p$
c	Speed of light; at vacuum, $c=c_o=3\times 10^8\ m/s$
c_p	Specific heat at constant pressure (J/kgK)
d_{ik}	Distance between surface dA_i and d_A_k (m^2)
D, d	Diameter (m)
D_v	Mass diffusion coefficient (m^2/s)
e_k	Specific kinetic energy (J/kg)
e_p	Specific potential energy (J/kg)
f	Friction factor
$\vec{f}_{(\vec{n})}$	Stress vector acting on surface of unit normal vector $\vec{n}$ (N/m^2)
$\vec{g}$	Gravity vector (m/s^2)
h	Specific enthalpy (J/kg)
h^o_{da}	Specific enthalpy of formation of dry air ($p^o=1$ atm, $T^o=25$°C) (J/kg)
h^o_{fv}	Specific enthalpy of formation of water vapour ($p^o=1$ atm, $T^o=25$°C) (J/kg)
$\vec{j}_k$	Mass diffusion of species k (kg/m^2s)
j_H, j_M	Colburn coefficients for heat and mass respectively
$\vec{n}$	Unit vector normal to CV surface and pointing outwards
n_{da}, n_v	Moles of dry air and vapour
Nu	Nusselt number, $Nu=\alpha X/\lambda$
Pr	Prandtl number
p	Pressure (Pa)
p_d	Dynamic pressure (Pa) (Section A2)
p_v	Water vapour pressure (Pa)
p_{vs}	Saturation vapour pressure (Pa)
$\vec{q}$	Heat transfer rate per unit surface (W/m^2)
$\vec{q}^{C+R}$	Conduction+radiation heat transfer rate per unit surface (W/m^2)
$\vec{q}^R$	Radiative heat transfer rate per unit surface (W/m^2)
$\dot{q}_v$	Internal heat generation per unit volume (W/m^3)
r	Radius (m)
R	a) Radius (m); b) Gas constant (J/kgK), $\mathcal{R}/W$
$\mathcal{R}/W$	Universal gas constant, $\mathcal{R}=8.31447\ kJ/kmol$
Re	Reynolds number, $Re=\rho vX/\mu$
s	Specific entropy (J/kgK)
$\dot{s}_{gen}$	Entropy generated per unit volume (J/m^3K), $\dot{s}_{gen}\geq 0$
S_a	Surface associated to V_a (m^2)
Sc	Schmidt number, $Sc=\mu/\rho D_v$
Sh	Sherwood number, $Sh=\alpha_M X/D_v$
St_H	Stanton number for heat, $St_H=\frac{Nu}{RePr}=\frac{\alpha}{c_p\rho v}$
St_M	Stanton number for mass, $St_M=\frac{Sh}{ReSc}=\frac{\alpha_M}{v}$
t	Time (s)
T	Temperature (K or °C)
u	Specific internal energy (J/kg)
$\vec{v}$	Fluid velocity (m/s), $\vec{v}=v_x\vec{i}+v_y\vec{j}+v_z\vec{k}$
$\vec{v}_b$	Velocity of surface S_a (m/s)
V_a	Arbitrary control volume (m^3)
W	Molecular mass $(kg/kmol)$
X	Characteristic length (m)
Y_k	Mass fraction of species k, $Y_k=m_k/m$

α	Heat transfer coefficient (W/m^2K)
α_M	Mass transfer coefficient (m/s)
α_s	Solar absorptivity
β	Volumetric thermal expansion coefficient (K^{-1}), $\beta=-\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_p$
$\vec{\vec{\delta}}$	Unit tensor
ε	Emissivity
κ	Isothermal compressibility coefficient (Pa^{-1}), $\kappa=\frac{1}{\rho}\left(\frac{\partial\rho}{\partial p}\right)_T$
λ	a) Thermal conductivity (W/mK); b) Wavelength of radiation (m) (section A5)
μ	Dynamic viscosity (kg/ms), $\mu=\rho\nu$
ν	Kinematic viscosity (m^2/s), $\nu=\mu/\rho$
ρ	Density (kg/m^3)
σ	Stefan-Boltzmann's constant (W/m^2K^4) (Section A5)
$\vec{\vec{\tau}}$	Viscous-stress tensor (N/m^2)
φ	a) Angle; b) Relative humidity
$\dot{\omega}_k$	Generation (or destruction) of species k per unit volume and time (kg/m^3s)

[4] In general, nomenclature is explicitly indicated in the different sections.