

Developed by gAGE: Research group of Astronomy & GEomatics Technical University of Catalonia (UPC)

Tutorial 4 Carrier ambiguity fixing

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Slides associated to *gLAB* version 2.0.0



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24 April 2014



Aim of this tutorial

- ★ This tutorial is devoted to analyse and assess the ambiguity fixing and the differential positioning with carrier phase measurements (L1, L2). Two receivers UPC1 and UPC2 with a baseline of about 40 metres are considered.
- ★ This study includes ambiguity fixing using the "cascade method" (i.e., fixing one at a time) and with the LAMBDA method.
 - The effect of synchronization errors between the reference station and the user is and its effect on the navigation and ambiguity fixing is also analysed
- ▲ All software tools (including *gLAB*) and associated files for the laboratory session are included in the CD-ROM or USB stick associated with this tutorial.

OVERVIEW

- ▲ Introduction: gLAB processing in command line.
- ▲ Preliminary computations: Data files.
- ▲ Session A: Fixing DD ambiguities one at a time: UPC1-UPC2.
- ▲ **Session B:** Assessing the fixed ambiguities in navigation: Differential positioning of UPC1-UPC2 receivers.
- ▲ Session C: Fixing DD ambiguities with LAMBDA method.

Note: UPC1-UPC2 receivers baseline: 37.95 metres.

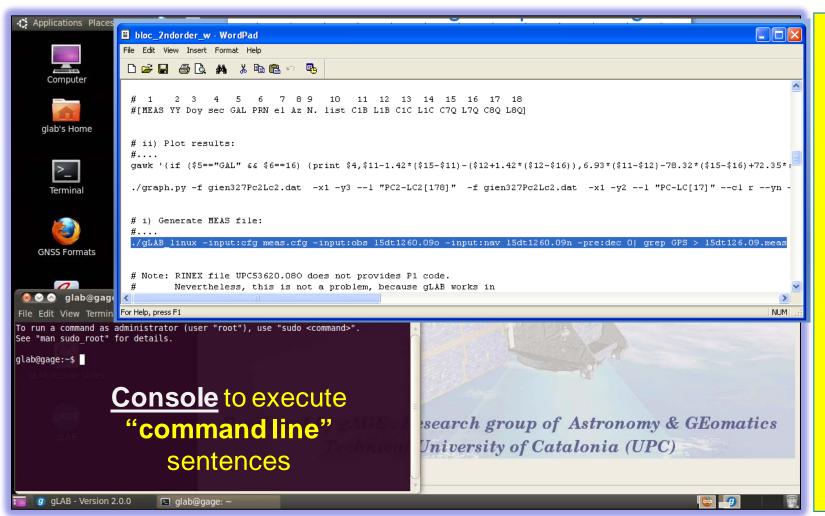


OVERVIEW

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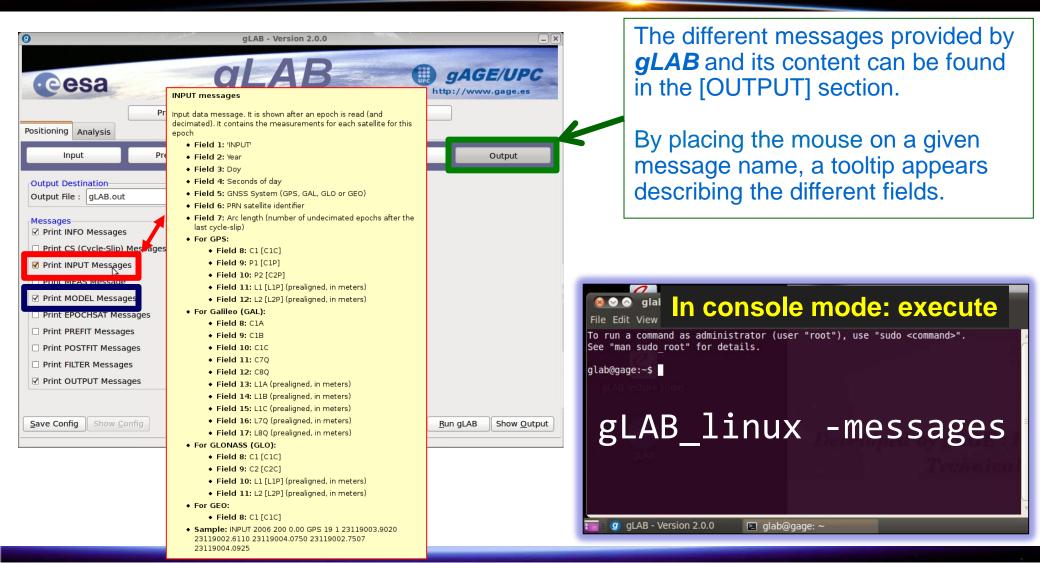
Note: UPC1-UPC2 receivers baseline: 37.95 metres.

gLAB processing in command line



A "notepad" with the command line sentence is provided to facilitate the sentence writing: just paste" from notepad to the working terminal.

gLAB processing in command line



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Note: UPC1-UPC2 receivers baseline: 37.95 metres.

Previous

Preliminary Computations

- ★ This section is devoted to prepare the data files to be used in the exercises.
- ★ These data files will include the code and carrier measurements and the model components: geometric range, nominal troposphere and ionosphere corrections, satellite elevation and azimuth from each receiver...
- ★ This data processing will be done with gLAB for each individual receiver.
- ★ This preliminary processing will provide the baseline data files to perform computations easily using basic tools (such as awk for data files handling, to compute Double Differences of measurements) or using octave (MATLAB) scripts for the LAMBDA method implementation.
- ▲ Detailed guidelines for self learning students are provided in this tutorial and in its associated notepad text file.

P1. Model Components computation

• The script "ObsFile.scr" generates a data file with the following content

```
1 2 3 4 5 6 7 8 9 10 11 12 13 [sta sat DoY sec P1 L1 P2 L2 Rho Trop Ion elev azim]
```

• Run this script for all receivers:

```
ObsFile.scr UPC10770.11o brdc0770.11n ObsFile.scr UPC20770.11o brdc0770.11n
```

Merge all files into a single file:

cat ????.obs > ObsFile.dat

Selecting measurements: Time interval [18000:19900]

- To simplify computations, a time interval with always the same set of satellites in view and without cycle-slips is selected.
- Moreover an elevation mask of 10 degrees will be applied.

If the satellites change or cycle-slips appear during the data processing interval, care with the associated parameters handling must be taken in the navigation filter. Set up new parameters when new satellites appear and treat the ambiguities as constant between cycle-slips and white noise when a cycle-slip happens.

Selecting measurements: Time interval [18000:19900]

Select the satellites in the time interval [18000:19900] with elevation over 10°

```
cat ObsFile.dat|gawk '{if ($4>=18000 && $4<=19900 && $12>10) print $0}' > obs.dat
```

Reference satellite (over the time interval [18000:19900])

Confirm that the satellite PRN06 is the satellite with the highest elevation (this satellite will be used as the reference satellite)

obs.dat →

1 2 3 4 5 6 7 8 9 10 11 12 13 [sta sat DoY sec P1 L1 P2 L2 Rho Trop Ion elev azim]



P2. Double differences between receivers and satellites computation

The script "DDobs.scr" computes the double differences between receivers and satellites from file obs.dat.

1 2 3 4 5 6 7 8 9 10 11 12 13

[sta sat DoY sec P1 L1 P2 L2 rho Trop Ion elev azim]

For instance, the following sentence:

DDobs.scr obs.dat UPC1 UPC2 06 03

generates the file

Where the elevation (EL) and azimuth (AZ) are taken from station #2. and where (EL1, AZ1) are for satellite #1 and (EL1, AZ1) are for satellite #2.



Compute the double differences between receivers UPC1 (reference) and UPC2 and satellites PRN06 (reference) and [PRN 03, 07,16, 18, 19, 21,

22, 24]

```
DDobs.scr obs.dat UPC1 UPC2 06 03
DDobs.scr obs.dat UPC1 UPC2 06 07
DDobs.scr obs.dat UPC1 UPC2 06 16
DDobs.scr obs.dat UPC1 UPC2 06 18
DDobs.scr obs.dat UPC1 UPC2 06 19
DDobs.scr obs.dat UPC1 UPC2 06 21
DDobs.scr obs.dat UPC1 UPC2 06 22
DDobs.scr obs.dat UPC1 UPC2 06 24
```

Merge the files in a single file and sort by time:

cat DD_UPC1_UPC2_06_??.dat sort -n -k +6 > DD_UPC1_UPC2_06_ALL.dat



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- ▲ **Session C:** Fixing DD ambiguities with <u>LAMBDA method</u>.

Note: UPC1-UPC2 receivers baseline: 37.95 metres.

A. Fixing DD ambiguities one at a time: UPC1-UPC2

- ▲ This exercise is devoted to study the ambiguity fixing using the cascade method, that is, fixing the ambiguities one at a time.
- ▲ In the first part, we are going to assess this approach for a single frequency receiver, trying to fix DDN1 and DDN2 independently.
- ▲ In the second part, we are going to assess this approach for dual frequency receivers, fixing first the wide-lane ambiguity DDNw and afterwards DDN1 and DDN2.
- ▲ The results (i.e. the DDN1 and DDN2 ambiguities) will be assessed in the next "Session B" by computing the navigation solution using the carrier phases repaired with the fixed DDN1 and DDN2 ambig.
- ▲ Finally, in Session C, the LAMBDA method will be applied for comparison.





Resolving ambiguities one at a Time: single Freq.

A simple trial would be (for instance using L1 and P1):

N-1

$$P_{1}^{jk} = \rho^{jk} + \nu_{P_{1}}^{jk}$$

$$L_{1}^{jk} = \rho^{jk} + \lambda_{1} N_{1}^{jk} + \nu_{L_{1}}^{jk}$$

 $ightharpoonup L_1^{jk} - P_1^{jk} = \lambda_1 N_1^{jk} + \nu_{P_1}^{jk}
ightharpoonup$

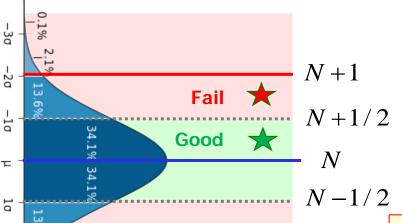
 $\hat{N}_{1}^{jk} = \left[rac{L_{1}^{jk} - P_{1}^{jk}}{\lambda_{1}}
ight]_{roundoff}$

$$\lambda_{l} \approx 20 \, \text{cm}$$

$$\sigma_{P_{l}^{jk}} \approx 1 \, \text{m}$$

$$\sigma_{L_{l}^{jk}} \approx 1 \, \text{cm}$$

 $\sigma_{\hat{N}_1^{jk}} \simeq \frac{1}{\lambda_1} \sigma_{P_1^{jk}} \approx 5$



To much error (5 wavelengths)!

Note that, assuming a Gaussian distribution of errors, $\sigma_{\hat{N}^{jk}} \simeq 1/2$ guarantee only the 68% of success

Similar results with *L2*, *P2* measurements

As the ambiguity is constant (between cycleslips), we would try to reduce uncertainty by averaging the estimate on time, but we will need 100 epochs to reduce noise up to ½ (but measurement errors are highly correlated on time!)

A1.1 Fixing N1 and N2 independently:

Estimate graphically values of DDN1 and DDN2 (i.e. try to identify the true ambiguity from the plot).

Hint:

From file DD_UPC1_UPC2_06_ALL.dat, generate the file DDN1N2.dat with the following

content:

where:

DDN1=[DDL1 -DDP1]/
$$\lambda_1$$
; DDN2=[DDL2 -DDP2]/ λ_2

Note:

"nint" means near-integer

Be careful: "nint" in awk: nint(x)must be generated as: int(x+0.5*sign(x))

```
a) From file DD_UPC1_UPC2_06_ALL.dat, generate the file DDN1N2.dat with the following content:

1 2 3 4 5 6

[PRN sec DDN1 DDN2 nint(DDN1) nint(DDN2)]
```

Execute, for instance:



b) Plot DDN1 and DDN2 for the different satellites and discuss if the ambiguity

DDN1 and DDN2 can be fixed:

```
1 2 3 4 5 6
[PRN sec DDN1 DDN2 nint(DDN1) nint(DDN2)]
```

b1) DDN1 plot:

```
graph.py -f DDN1N2.dat -x2 -y3 -c '($1==16)' -s. -f DDN1N2.dat -x2 -y4 -c '($1==16)' -sx --cl r --yn -4 --yx 10 --xl "time (s)" --yl "cycles L1" -t "DDN1 ambiguity: PRN16"
```

b2) DDN2 plot:

```
graph.py -f DDN1N2.dat -x2 -y5 -c '($1==16)' -s. -f DDN1N2.dat -x2 -y6 -c '($1==16)' -sx --cl r --yn -8 --yx 6 --xl "time (s)" --yl "cycles L2" -t "DDN2 ambiguity: PRN16"
```

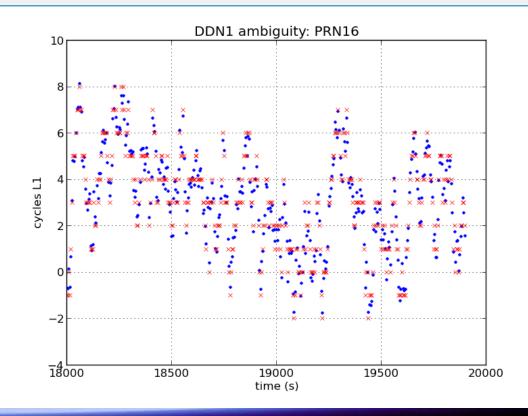


b1) DDN1 plot:

```
graph.py -f DDN1N2.dat -x2 -y3 -c '($1==16)' -s. -f DDN1N2.dat -x2 -y4 -c '($1==16)' -sx --cl r --yn -4 --yx 10 --xl "time (s)" --yl "cycles L1" -t "DDN1 ambiguity: PRN16"
```

Questions:

- 1.- Explain what is the meaning of this plot.
- 2.- Is it possible to identify the integer ambiguity?
- 3.- How reliability can be improved?

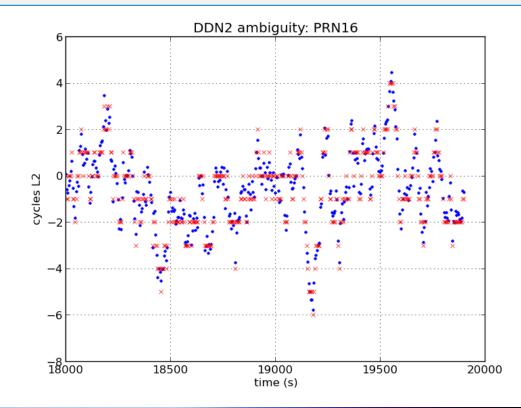


b2) DDN2 plot:

```
graph.py -f DDN1N2.dat -x2 -y5 -c '($1==16)' -s. -f DDN1N2.dat -x2 -y6 -c '($1==16)'
  -sx --cl r --yn -8 --yx 6 --xl "time (s)" --yl "cycles L2" -t "DDN2 ambiguity: PRN16"
```

Questions:

- 1.- Explain what is the meaning of this plot.
- 2.- Is it possible to identify the integer ambiguity?
- 3.- How reliability can be improved?



c) Make plots to analyze the DDP1 and DDP2 code noise.

Hint:

From file DD_UPC1_UPC2_06_ALL.dat, generate the file P1P2noise.dat with

the following content:

Execute, for instance:

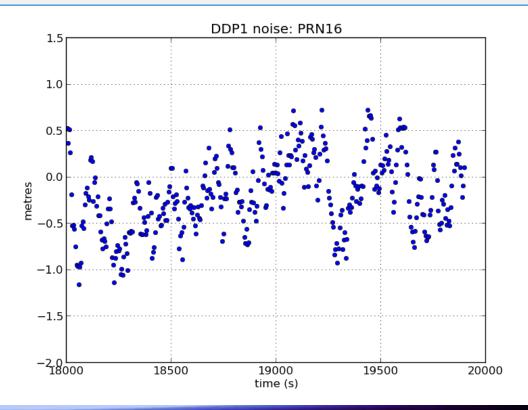
```
gawk '{print $4,$6,$7-$11,$9-$11}' DD_UPC1_UPC2_06_ALL.dat > P1P2noise.dat
```



c1) Depict the DDP1 code noise:

Questions:

Discuss why the ambiguities cannot be fixed by rounding-off the expression DDN1=[DDL1 -DDP1]/ λ_1

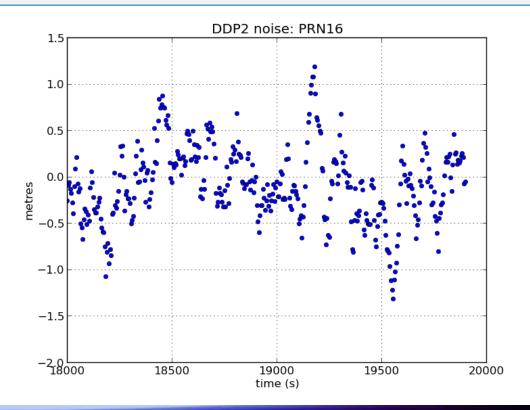




c2) Depict the DDP2 code noise:

Questions:

Discuss why the ambiguities cannot be fixed by rounding-off the expression DDN2=[DDL2 -DDP2]/ λ_2





```
DD UPC1 UPC2 06 ALL.dat
                                          10
[UPC1 UPC2 06 PRN DoY sec DDP1 DDL1 DDP2 DDL2 DDRho DDTrop DDIon El1 Az1 El2 Az2
```

Make plots to analyze the DDL1 and DDL2 carrier noise.

Hint:

From file DD_UPC1_UPC2_06_ALL.dat, generate the file L1L2noise.dat with

the following content:

Execute, for instance:

```
gawk '{print $4,$6,$8-$11,$10-$11}' DD_UPC1_UPC2_06_ALL.dat > L1L2noise.dat
```



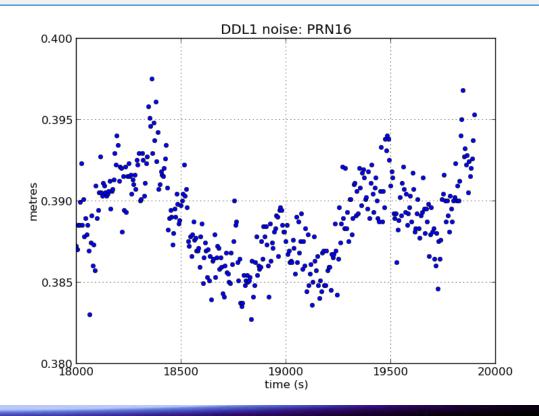
d1) Depict the DDL1 code noise:

graph.py -f L1L2noise.dat -x2 -y3 -c '(\$1==16)' -so --yn 0.38 --yx 0.40 --xl "time (s)" --yl "metres" -t "DDL1 noise: PRN16"

Questions:

Discuss the plot.
What is the level of noise?

Compare the noise with the wavelength $\lambda_1 = 19.0cm$



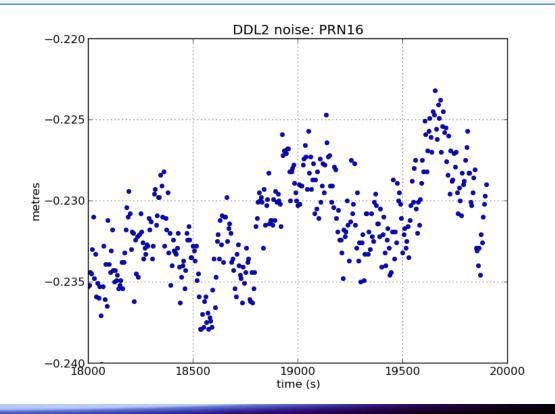
d2) Depict the DDL2 code noise:

graph.py -f L1L2noise.dat -x2 -y4 -c'(\$1==16)' -so --yn -0.24 --yx -0.22 --xl "time (s)" --yl "metres" -t "DDL2 noise: PRN16"

Questions:

Discuss the plot.
What is the level of noise?

Compare the noise with the wavelength $\lambda_2=24.4$ cm



Resolving ambiguities one at a Time: Dual Freq.

Dual frequency measurements: wide-laning with the <u>Melbourne-Wübbena</u> combination

$$P_{1}^{jk} = \rho^{jk} + v_{P_{1}}^{jk}$$

$$P_{2}^{jk} = \rho^{jk} + v_{P_{2}}^{jk}$$

$$L_{1}^{jk} = \rho^{jk} + \lambda_{1} N_{1}^{jk} + v_{L_{1}}^{jk}$$

$$L_{2}^{jk} = \rho^{jk} + \lambda_{2} N_{2}^{jk} + v_{L_{2}}^{jk}$$

$$L_{W}^{jk} = \frac{f_{1}P_{1}^{jk} + f_{2}P_{2}^{jk}}{f_{1} + f_{2}} = \rho^{jk} + \lambda_{W} N_{W}^{jk} + v_{L_{W}}^{jk}$$

$$\downarrow \qquad \qquad \downarrow \qquad \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \qquad \downarrow \qquad \qquad$$

$$P_N^{jk} = \frac{f_1 P_1^{jk} + f_2 P_2^{jk}}{f_1 + f_2} = \rho^{jk} + \nu_{P_N}^{jk}$$

$$L_W^{jk} = \frac{f_1 L_1^{jk} - f_2 L_2^{jk}}{f_1 - f_2} = \rho^{jk} + \lambda_W N_W^{jk} + \nu_{L_W}^{jk}$$

$$L_W^{jk} - P_N^{jk} = \lambda_W N_W^{jk} + v_{P_N}^{jk}
ightarrow \hat{N}_W^{jk} = \left[rac{L_W^{jk} - P_N^{jk}}{\lambda_W}
ight]$$

$$N_W = N_1 - N_2$$

$$\lambda_W = \frac{c}{f_1 - f_2} \approx 86.2 \, cm$$

$$\sigma_{P_N^{jk}} \approx \sigma_{P_1^{jk}} / \sqrt{2} \approx 71 cm$$

$$\sigma_{L_W^{jk}} \approx 6 \sigma_{L_1^{jk}} \approx 6 cm$$

$$\hat{N}_{W}^{jk} = \left[rac{L_{W}^{jk} - P_{N}^{jk}}{\lambda_{W}}
ight]_{roundoff}$$

$$\sigma_{\hat{N}_{W}^{jk}} \approx \frac{1}{\lambda_{W}} \sigma_{P_{N}^{jk}} \simeq \frac{71cm}{86.2cm} \simeq 0.8$$

Now, with uncorrelated measurements from 10 epochs will reduce noise up to about 1/4.

Fixing N_I (after fixing N_W)

$$L_{1}^{jk} - L_{2}^{jk} = \lambda_{1} N_{1}^{jk} - \lambda_{2} N_{2}^{jk} + \nu_{L_{1}-L_{2}}^{jk}$$
$$= (\lambda_{1} - \lambda_{2}) N_{1}^{jk} + \lambda_{2} N_{W}^{jk} + \nu_{L_{1}-L_{2}}^{jk}$$

$$\lambda_{1} = 19.0 \text{ cm}$$

$$\lambda_{2} = 24.4 \text{ cm}$$

$$\lambda_{2} - \lambda_{1} = 5.4 \text{ cm}$$

$$\sigma_{L_{1}^{jk}} \approx 1 \text{ cm}$$

$$\hat{N}_{2}^{\,jk} = \hat{N}_{1}^{\,jk} - \hat{N}_{W}^{\,jk}$$

$$\hat{N}_{1}^{jk} = \left[\frac{L_{1}^{jk} - L_{2}^{jk} - \lambda_{2} \hat{N}_{W}^{jk}}{\lambda_{1} - \lambda_{2}}\right]_{roundoff}$$

$$\hat{N}_{2}^{jk} = \hat{N}_{1}^{jk} - \hat{N}_{W}^{jk} \qquad \sigma_{\hat{N}_{1}^{jk}} \approx \frac{1}{\lambda_{1} - \lambda_{2}} \sqrt{2} \sigma_{\frac{jk}{\lambda_{1}}} \approx \frac{1.4cm}{5.4cm} \approx 1/4$$

A2.1 Fixing Wide-lane ambiguity (Nw):

Estimate graphically values of DDNw (i.e. try to identify the true ambiguity from the plot).

Hint:

From file **DD_UPC1_UPC2_06_ALL.dat**, generate the file **DDNw.dat** with the following content:

1 2 3 4
[PRN sec DDNw nint(DDNw)]

where: $DDN1=[DDL_{W} - DDP_{N}]/\lambda_{W}$

Note:

$$L_{W} = \frac{\beta L_{1} - L_{2}}{\beta - 1} \quad ; \quad P_{N} = \frac{\beta P_{1} + P_{2}}{\beta + 1} \quad ; \quad \beta = \frac{f_{1}}{f_{2}} = \frac{154}{120} \quad ; \quad \lambda_{W} = \frac{c}{f_{1} - f_{2}} = 86.2cm$$

```
a) From file DD_UPC1_UPC2_06_ALL.dat, generate the file DDNw.dat with the following content: 1 2 3 4 where: DDN1=[DDL<sub>W</sub> -DDP<sub>N</sub>]/\lambda_W [PRN sec DDNw nint(DDNw)]
```

Execute, for instance:

```
cat DD_UPC1_UPC2_06_ALL.dat| gawk 'BEGIN{s12=154/120}
{mw=(s12*$8-$10)/(s12-1)-(s12*$7+$9)/(s12+1);if(mw!=0){sign=mw/sqrt(mw*mw)}
else{sign=0};printf "%02i %i %14.4f %i \n", $4,$6, mw/0.862,int(mw/0.862+0.5*sign)}'>
DDNw.dat
```

b) Plot DDNw for the different satellites and discuss if the ambiguity DDNw

can be fixed:

- Example PRN03 plot:

```
graph.py -f DDNw.dat -x2 -y3 -c '($1==03)' -s. -f DDNw.dat -x2 -y4 -c '($1==03)' -sx --cl r --yn -1 --yx 7 --xl "time (s)" --yl "cycles Lw" -t "DDNw ambiguity: PRN03"
```

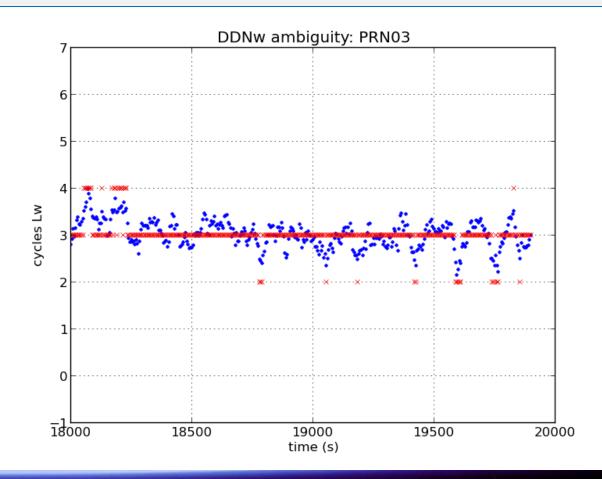
- Example PRN07 plot:

```
graph.py -f DDNw.dat -x2 -y3 -c '($1==07)' -s. -f DDNw.dat -x2 -y4 -c '($1==07)' -sx --cl r --yn -4 --yx 4 --xl "time (s)" --yl "cycles Lw" -t "DDNw ambiguity: PRN07"
```



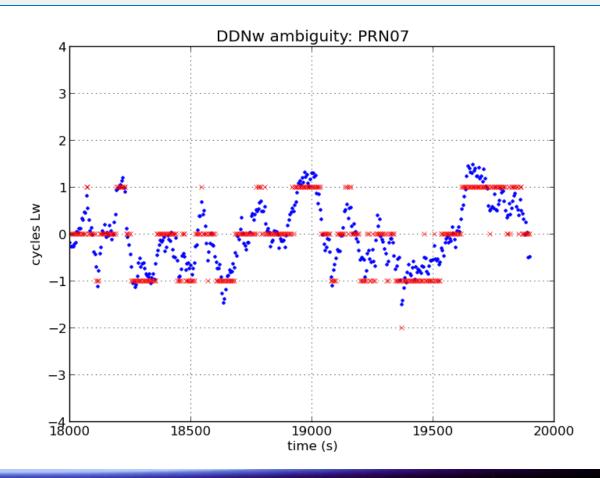
PRN03 plot:

→ DDNw= 3 ?



PRN07 plot:

→ DDNw= 0 ?



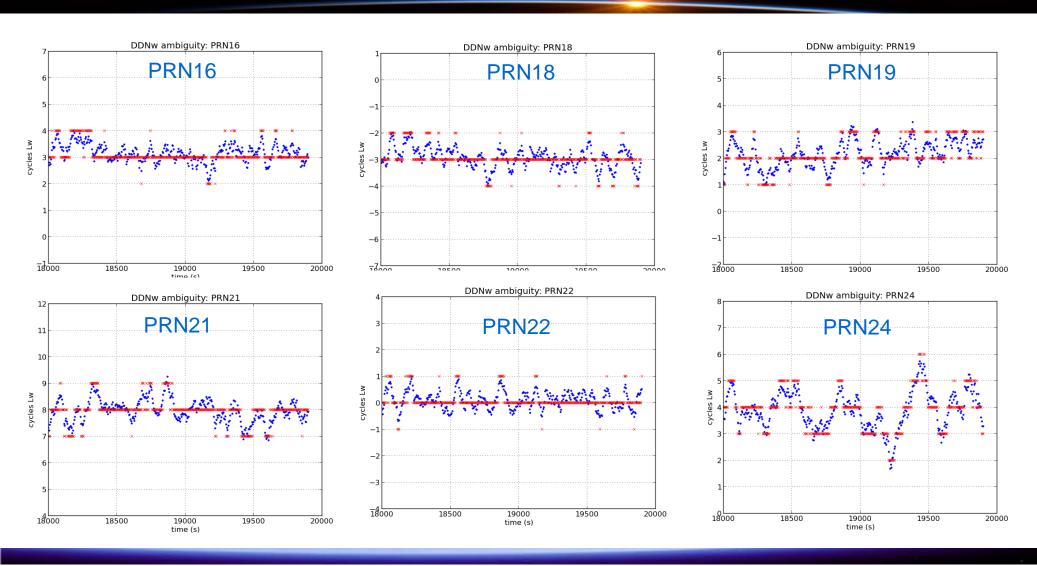
The remaining plots:

```
graph.py -f DDNw.dat -x2 -y3 -c '($1==16)' -s. -f DDNw.dat -x2 -y4 -c '($1==16)'
 -sx --cl r --yn -1 --yx 7 --xl "time (s)" --yl "cycles Lw" -t "DDNw ambiguity: PRN16"
graph.py -f DDNw.dat -x2 -y3 -c ($1==18)' -s. -f DDNw.dat -x2 -y4 -c ($1==18)'
 -sx --cl r --yn -7 --yx 1 --xl "time (s)" --yl "cycles Lw" -t "DDNw ambiguity: PRN18"
graph.py -f DDNw.dat -x2 -y3 -c ($1==19)' -s. -f DDNw.dat -x2 -y4 -c ($1==19)'
 -sx --cl r --yn -2 --yx 6 --xl "time (s)" --yl "cycles Lw" -t "DDNw ambiguity: PRN19"
graph.py -f DDNw.dat -x2 -y3 -c '($1==21)' -s. -f DDNw.dat -x2 -y4 -c '($1==21)'
 -sx --cl r --yn 4 --yx 12 --xl "time (s)" --yl "cycles Lw" -t "DDNw ambiguity: PRN21"
graph.py -f DDNw.dat -x2 -y3 -c '($1==22)' -s. -f DDNw.dat -x2 -y4 -c '($1==22)'
 -sx --cl r --yn -4 --yx 4 --xl "time (s)" --yl "cycles Lw" -t "DDNw ambiguity: PRN22"
```



-sx --cl r --yn 0 --yx 8 --xl "time (s)" --yl "cycles Lw" -t "DDNw ambiguity: PRN24"

graph.py -f DDNw.dat -x2 -y3 -c '(\$1==24)' -s. -f DDNw.dat -x2 -y4 -c '(\$1==24)'



A.2.2 Smoothing the DDNw:

Smooth the DDNw with a 300 seconds sliding window, in order to improve the ambiguity fixing.

Hint: From file DD_UPC1_UPC2_06_ALL.dat, generate the file DDNws.dat with the content:

```
1 2 3 4 5 6
[PRN sec DDNw DDNws nint(DDNw) nint(DDNws)]
```

Execute, for instance (to smooth the DDNw):

Estimate again the ambiguity from the raw DDNw and smoothed DDNws:

```
cat DDNws.tmp | gawk '{sign3=$3/sqrt($3*$3);sign4=$4/sqrt($4*$4);
print $1,$2,$3,$4,int($3+0.5*sign3),int($4+0.5*sign4)}' > DDNws.dat
```



b) Plot DDNw and DDNws for the different satellites and discuss if the

ambiguity DDNw can be fixed:

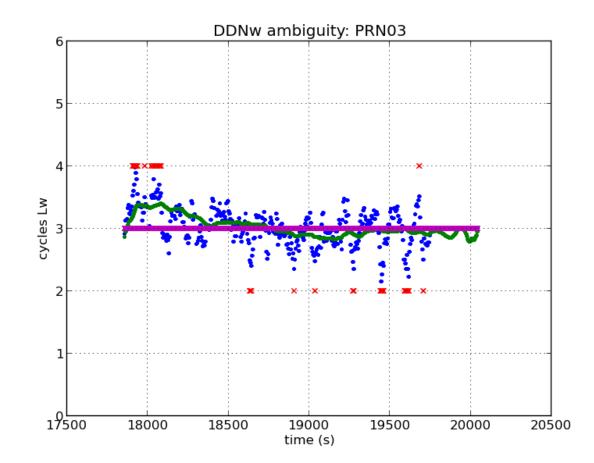
```
1 2 3 4 5 6
[PRN sec DDNw DDNws nint(DDNw) nint(DDNws)]
```

Example: PRN03 plot:

```
graph.py -f DDNws.dat -x2 -y3 -c '($1==03)' -s.
-f DDNws.dat -x2 -y5 -c '($1==03)' -sx --cl r
-f DDNws.dat -x2 -y4 -c '($1==03)' -s. --cl g
-f DDNws.dat -x2 -y6 -c '($1==03)' -sx --cl m
--yn 0 --yx 6 --xl "time (s)" --yl "cycles Lw" -t "DDNw ambig. PRN03"
```

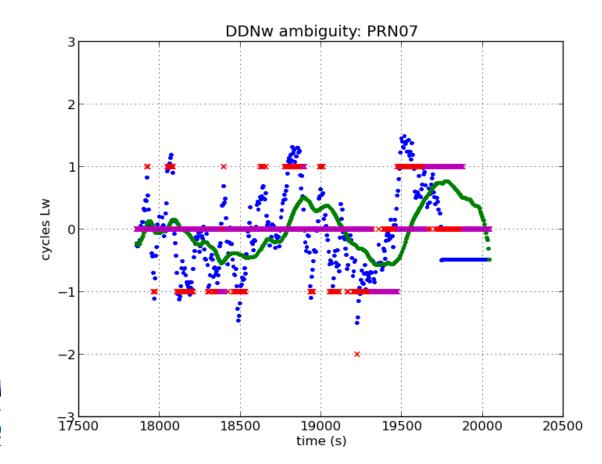
```
graph.py -f DDNws.dat -x2 -y3 -c '($1==03)' -s.
    -f DDNws.dat -x2 -y5 -c '($1==03)' -sx --cl r
    -f DDNws.dat -x2 -y4 -c '($1==03)' -s. --cl g
    -f DDNws.dat -x2 -y6 -c '($1==03)' -sx --cl m
    --yn 0 --yx 6 --xl "time (s)" --yl "cycles Lw" -t "DDNw ambig. PRN03"
```

PRN03 plot:



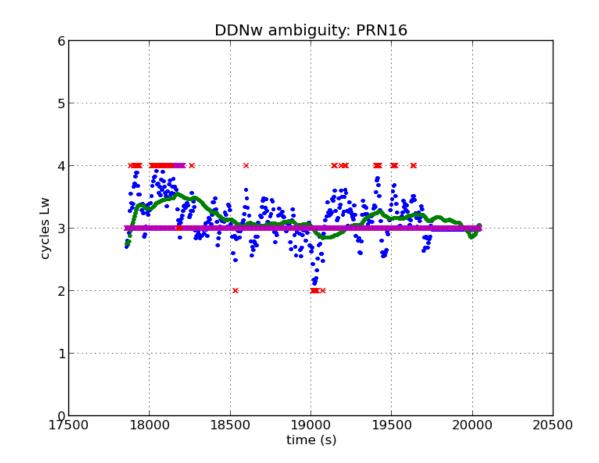
```
graph.py -f DDNws.dat -x2 -y3 -c '($1==07)' -s.
    -f DDNws.dat -x2 -y5 -c '($1==07)' -sx --cl r
    -f DDNws.dat -x2 -y4 -c '($1==07)' -s. --cl g
    -f DDNws.dat -x2 -y6 -c '($1==07)' -sx --cl m
    --yn -3 --yx 3 --xl "time (s)" --yl "cycles Lw" -t "DDNw ambig. PRN07"
```

PRN07 plot:



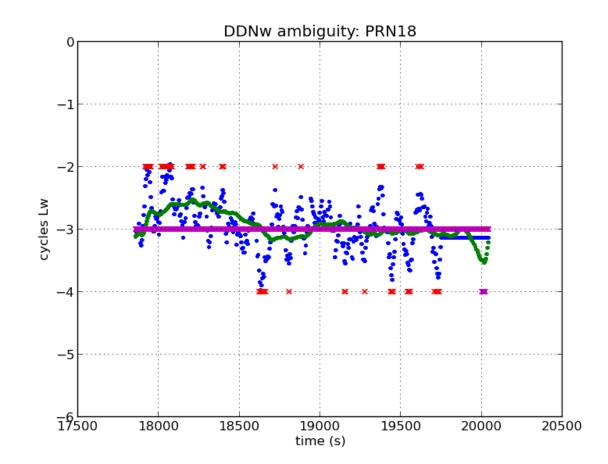
```
graph.py -f DDNws.dat -x2 -y3 -c '($1==16)' -s.
-f DDNws.dat -x2 -y5 -c '($1==16)' -sx --cl r
-f DDNws.dat -x2 -y4 -c '($1==16)' -s. --cl g
-f DDNws.dat -x2 -y6 -c '($1==16)' -sx --cl m
--yn 0 --yx 6 --xl "time (s)" --yl "cycles Lw" -t "DDNw ambig. PRN16"
```

PRN16 plot:



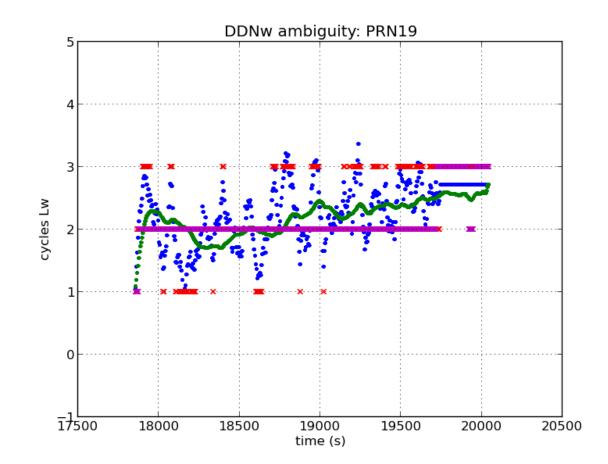
```
graph.py -f DDNws.dat -x2 -y3 -c '($1==18)' -s.
-f DDNws.dat -x2 -y5 -c '($1==18)' -sx --cl r
-f DDNws.dat -x2 -y4 -c '($1==18)' -s. --cl g
-f DDNws.dat -x2 -y6 -c '($1==18)' -sx --cl m
--yn -6 --yx 0 --xl "time (s)" --yl "cycles Lw" -t "DDNw ambig. PRN18"
```

PRN18 plot:



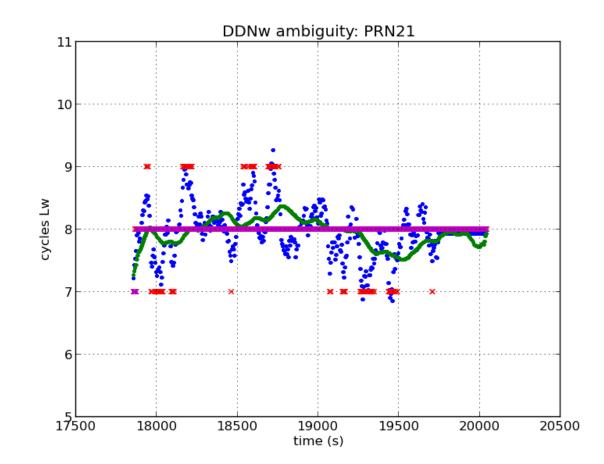
```
graph.py -f DDNws.dat -x2 -y3 -c '($1==19)' -s.
    -f DDNws.dat -x2 -y5 -c '($1==19)' -sx --cl r
    -f DDNws.dat -x2 -y4 -c '($1==19)' -s. --cl g
    -f DDNws.dat -x2 -y6 -c '($1==19)' -sx --cl m
    --yn -1 --yx 5 --xl "time (s)" --yl "cycles Lw" -t "DDNw ambig. PRN19"
```

PRN19 plot:



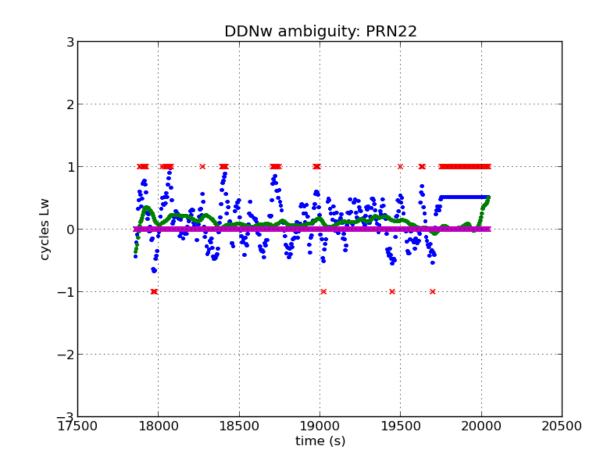
```
graph.py -f DDNws.dat -x2 -y3 -c '($1==21)' -s.
    -f DDNws.dat -x2 -y5 -c '($1==21)' -sx --cl r
    -f DDNws.dat -x2 -y4 -c '($1==21)' -s. --cl g
    -f DDNws.dat -x2 -y6 -c '($1==21)' -sx --cl m
    --yn 5 --yx 11 --xl "time (s)" --yl "cycles Lw" -t "DDNw ambig. PRN21"
```

PRN21 plot:



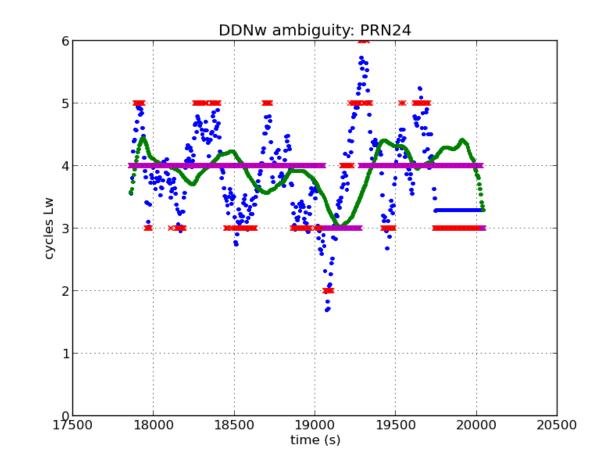
```
graph.py -f DDNws.dat -x2 -y3 -c '($1==22)' -s.
-f DDNws.dat -x2 -y5 -c '($1==22)' -sx --cl r
-f DDNws.dat -x2 -y4 -c '($1==22)' -s. --cl g
-f DDNws.dat -x2 -y6 -c '($1==22)' -sx --cl m
--yn -3 --yx 3 --xl "time (s)" --yl "cycles Lw" -t "DDNw ambig. PRN22"
```

PRN22 plot:



```
graph.py -f DDNws.dat -x2 -y3 -c '($1==24)' -s.
-f DDNws.dat -x2 -y5 -c '($1==24)' -sx --cl r
-f DDNws.dat -x2 -y4 -c '($1==24)' -s. --cl g
-f DDNws.dat -x2 -y6 -c '($1==24)' -sx --cl m
--yn 0 --yx 6 --xl "time (s)" --yl "cycles Lw" -t "DDNw ambig. PRN24"
```

PRN24 plot:



A2.2 Fixing DDN1 from DDNw and DDL1, DDL2

Using the previous DDNw fixed values, estimate graphically the DDN1 ambiguity (i.e. try to identify the true ambiguity from the plot).

Hint:

From file DD_UPC1_UPC2_06_ALL.dat, and using the fixed DDNw, generate the file

DDN1.dat with the following content:

where:

DDN1=(DDL1 - DDL2 -
$$\lambda_2$$
 DDNw)/(λ_1 - λ_2)



```
DD_UPC1_UPC2_06_ALL.dat

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17

[UPC1 UPC2 06 PRN DoY sec DDP1 DDL1 DDP2 DDL2 DDRho DDTrop DDIon El1 Az1 El2 Az2 ]
```

Execute, for instance for PRN24 (with DDNw=4):

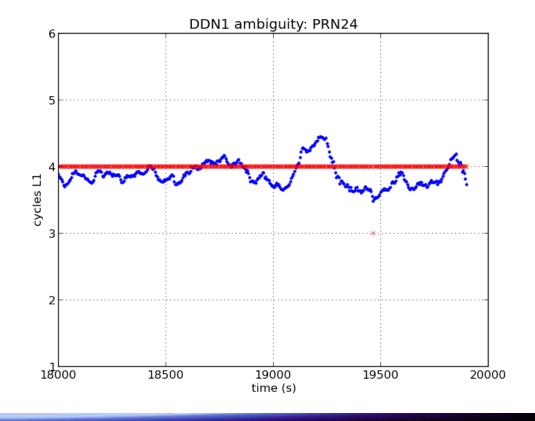
```
gawk 'BEGIN{c=299792458;f0=10.23e+6;f1=154*f0;f2=120*f0;l1=c/f1;l2=c/f2}
{\text{Nw=4};if ($4==24) {amb=($8-$10-l2*\text{Nw})/(l1-l2);}
print $4,$6,amb,int(amb+0.5*amb/sqrt(amb*amb))}}' \text{DD_UPC1_UPC2_06_ALL.dat > DD\n1_PR\n24}
```



```
graph.py -f DDN1_PRN24 -x2 -y3 -c '($1==24)' -s.
-f DDN1_PRN24 -x2 -y4 -c '($1==24)' -sx
--cl r --yn 1 --yx 6 --xl "time (s)"
--yl "cycles L1" -t "DDN1 ambiguity: PRN24"
```

PRN24 plot:

→ DDN1=4



```
From file DD_UPC1_UPC2_06_ALL.dat, and using the fixed DDNw, generate the file DDN1.dat with the following content:

1 2 3 4

[PRN sec DDN1 nint(DDN1)]
```

Other possibility is to execute the following sentence to generate the file for all satellites:

Where **sat.ambNw** is a file containing the DDNw ambiguities:



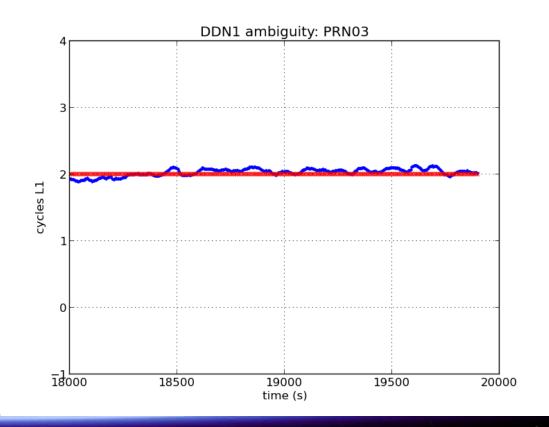
03 3 07 0

24 4

```
graph.py -f DDN1_PRN24 -x2 -y3 -c '($1==03)' -s.
    -f DDN1_PRN24 -x2 -y4 -c '($1==03)' -sx
    --cl r --yn -1 --yx 4 --xl "time (s)"
    --yl "cycles L1" -t "DDN1 ambiguity: PRN03"
```

PRN03 plot:

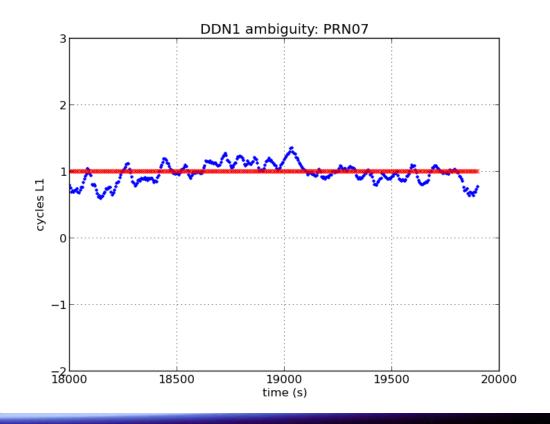
→ DDN1=2

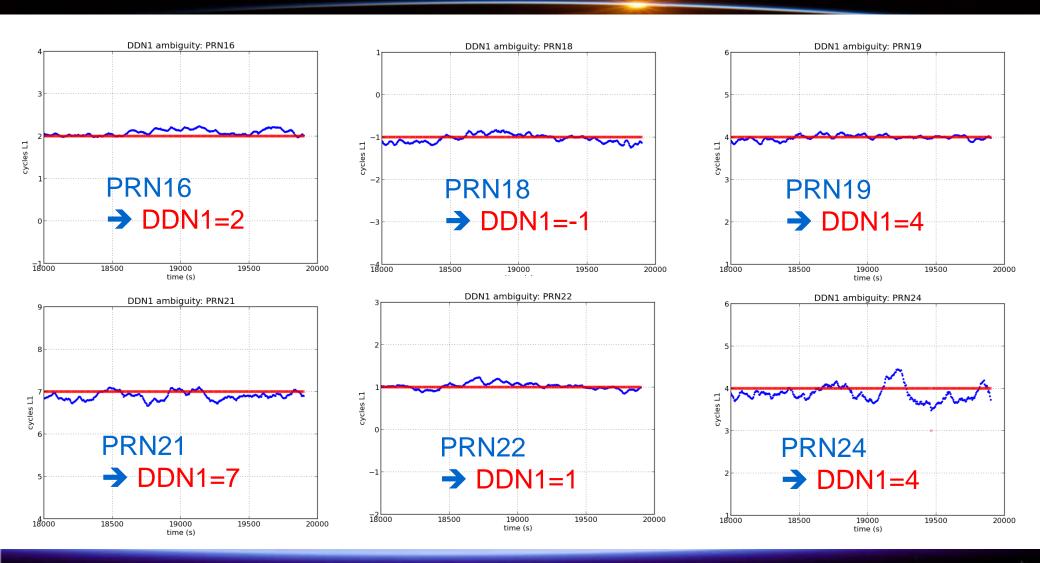


```
graph.py -f DDN1_PRN24 -x2 -y3 -c '($1==07)' -s.
-f DDN1_PRN24 -x2 -y4 -c '($1==07)' -sx
--cl r --yn -2 --yx 3 --xl "time (s)"
--yl "cycles L1" -t "DDN1 ambiguity: PRN07"
```

PRN07 plot:

→ DDN1=1





A2.3 Fixing DDN2

Using the previous DDNw and DDN1 ambiguities fixed, fix the DDN2 ambiguity:

Hint: The DDN2 can be easily computed by: DDN2=DDN1-DDNw

Then:

OVERVIEW

- ▲ Introduction: gLAB processing in command line.
- → Preliminary computations: Data files.
- ▲ Session A: Fixing DD ambiguities one at a time: UPC1-UPC2.
- > Session B: Assessing the fixed ambiguities in navigation: Differential positioning of UPC1-UPC2 receivers.
- ▲ **Session C:** Fixing DD ambiguities with <u>LAMBDA method</u>.

Note: UPC1-UPC2 receivers baseline: 37.95 metres.

Session B

Assessing the fixed ambiguities in navigation: Differential positioning of UPC1-UPC2 receivers

(baseline: 37.95 metres)

B. Assessing the fixed ambiguities in navigation

- ▲ The DDN1 and DDN2 ambiguities have been fixed in the previous Session A using the "cascade method".
- ▲ The obtained results (i.e. the DDN1 and DDN2 ambiguities) will be assessed in this Session B by computing the navigation solution using the carrier phases repaired with the fixed DDN1 and DDN2 ambiguities
- ▲ Finally, in next Session C, the ambiguities will be fixed using the LAMBDA method and the performance with the cascade method will be compared.



B. Assessing the fixed ambiguities in navigation

- ▲ After repairing the carrier ambiguities, these measurements will be used to navigate.
- ▲ Indeed, once the integer ambiguities are known, the carrier phase measurements become like "unambiguous pseudoranges", accurate at the centimetre level or better.
- ▲ Thence, the positioning is straightforward following the same procedure as with pseudoranges.
- ▲ Nevertheless, a wrong ambiguity fix can degrade the position solution significantly.

B1. Repair the DDL1 and DDL2 carrier measurements with the DDN1 and DDN2 ambiguities FIXED and plot results to analyze the data.

```
Write in it the DDN1 and DDN2 ambiguities fixed in previous exercise in file N1N2.dat →
```

```
03 2-1
07 1 1
16 2-1
18-1 2
19 4 2
21 7-1
22 1 1
24 4 0
```

Using the previous file N1N2.dat and "DD_UPC1_UPC2_06_ALL.dat", generate a file with the following content:

Note: This file is identical to file "DD_UPC1_UPC2_06_ALL.dat", but with the ambiguities added in the last fields #18 and #19.



a) From previous file, generate a the file "sat.ambL1L2" with the following content:
 1 2 3 4 5
 [PRN DDN1 DDN2 λ₁DDN1 λ₂DDN2]

b) Generate the "DD_UPC1_UPC2_06_30.fixL1L2" file:



c) Make and discuss the following plots for DDL1

```
graph.py -f DD_UPC1_UPC2_06_ALL.fixL1 -x6 -y'($8-$18-$11)'
-so --yn -0.06 --yx 0.06 -l "(DDL1-lambda1*DDN1)-DDrho" --xl "time (s)" --yl "m"
```

```
graph.py -f DD_UPC1_UPC2_06_ALL.fixL1 -x6 -y'($8-$11)'
-so --yn -5 --yx 5 -l "(DDL1)-DDrho" --xl "time (s)" --yl "metres"
```

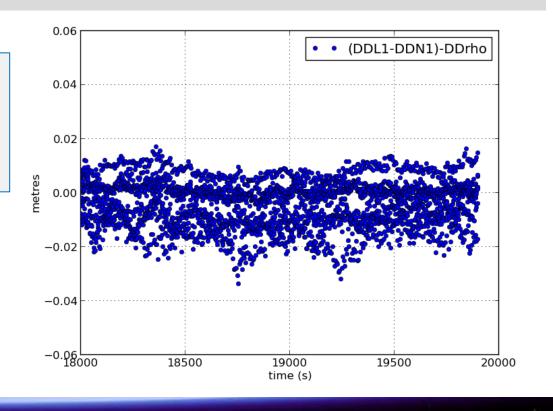
```
graph.py -f DD_UPC1_UPC2_06_ALL.fixL1 -x6 -y'($8-$18)'
-so --yn -20 --yx 20 -l "(DDL1-lambda1*DDN1)" --xl "time (s)" --yl "metres"
```



```
graph.py -f DD_UPC1_UPC2_06_ALL.fixL1
-x6 -y'($8-$18-$11)'
-so --yn -0.06 --yx 0.06
-l "(DDL1-λ<sub>1</sub> DDN1)-DDrho"
--xl "time (s)" --yl "m"
```

Questions:

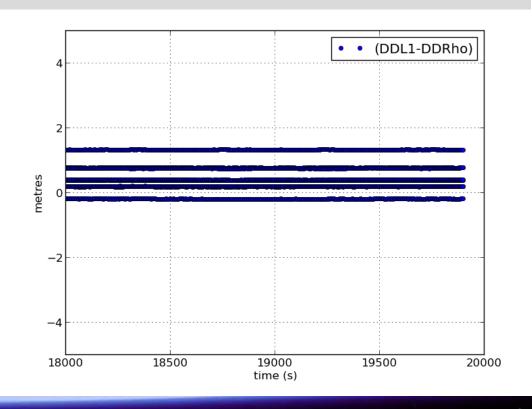
Explain what is the meaning of this plot.



```
graph.py -f DD_UPC1_UPC2_06_ALL.fixL1
    -x6 -y'($8-$11)'
    -so --yn -5 --yx 5
    -1 "DDL1-DDrho"
    --xl "time (s)" --yl "m"
```

Questions:

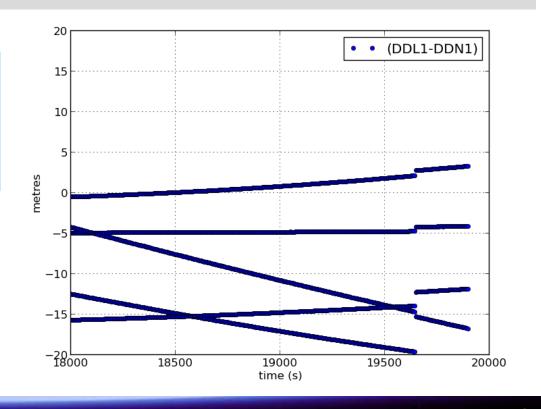
Explain what is the meaning of this plot.



```
graph.py -f DD_UPC1_UPC2_06_ALL.fixL1
-x6 -y'($8-$18)'
-so --yn -20 --yx 20
-l "(DDL1-λ<sub>1</sub> DDN1)"
--xl "time (s)" --yl "m"
```

Questions:

- 1.- Explain what is the meaning of this plot.
- 2.- Why a trend and a discontinuity appears?



d) Make and discuss the following plots for DDL2

```
graph.py -f DD_UPC1_UPC2_06_ALL.fixL1 -x6 -y'($10-$19-$11)'
-so --yn -0.06 --yx 0.06 -l "(DDL2-lambda2*DDN2)-DDrho" --xl "time (s)" --yl "m"
```

```
graph.py -f DD_UPC1_UPC2_06_ALL.fixL1 -x6 -y'($10-$11)'
-so --yn -5 --yx 5 -l "(DDL2)-DDrho" --xl "time (s)" --yl "metres"
```

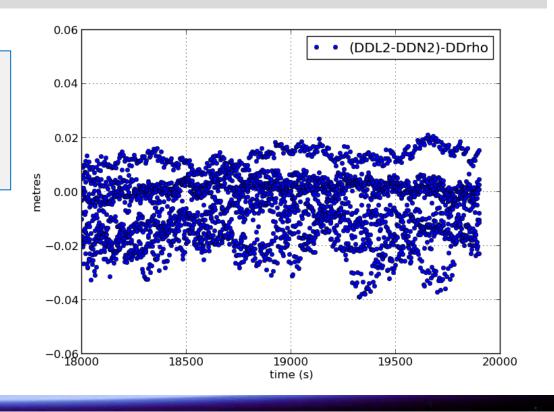
```
graph.py -f DD_UPC1_UPC2_06_ALL.fixL1 -x6 -y'($10-$19)'
-so --yn -20 --yx 20 -l "(DDL2-lambda2*DDN2)" --xl "time (s)" --yl "metres"
```



```
graph.py -f DD_UPC1_UPC2_06_ALL.fixL1
-x6 -y'($9-$19-$11)'
-so --yn -0.06 --yx 0.06
-l "(DDL2-λ<sub>2</sub> DDN2)-DDrho"
--xl "time (s)" --yl "m"
```

Questions:

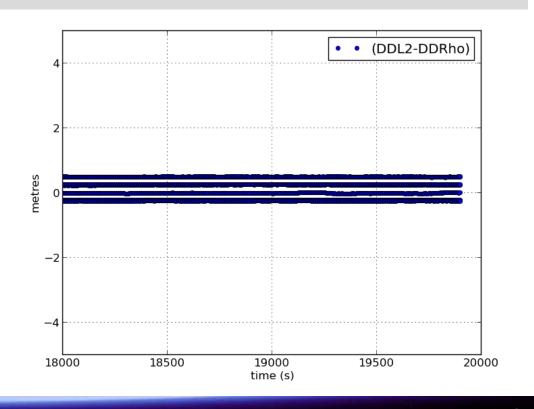
Explain what is the meaning of this plot.



```
graph.py -f DD_UPC1_UPC2_06_ALL.fixL1
    -x6 -y'($9-$11)'
    -so --yn -5 --yx 5
    -1 "DDL2-DDrho"
    --xl "time (s)" --yl "m"
```

Questions:

Explain what is the meaning of this plot.

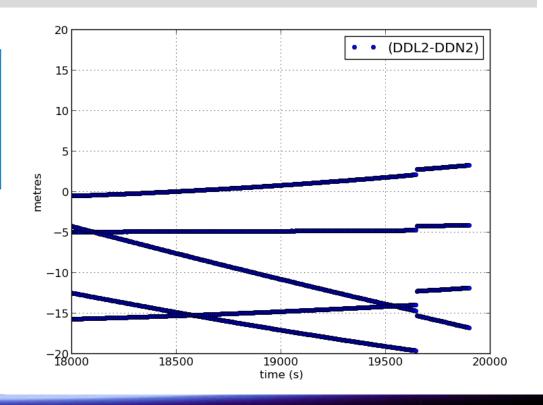


```
DD UPC1 UPC2 06 ALL.fixL1L2
                                                                 13
                                                                       14 15 16 17
                                                                                                  19
[UPC1 UPC2 06 PRN DoY sec DDP1 DDL1 DDP2 DDL2 DDRho DDTrop DDIon El1 Az1 El2 Az2 \lambda_1 DDN1
                                                                                                 \lambda_2 DDN2
                                                                      <---- UPC2 ---->
```

```
graph.py -f DD UPC1 UPC2 06 ALL.fixL1
         -x6 -y'($9-$198)'
          -so --yn -20 --yx 20
         -1 "(DDL2-\lambda_2 DDN2)"
          --xl "time (s)" --yl "m"
```

Questions:

- 1.- Explain what is the meaning of this plot.
- 2.- Why a trend and a discontinuity appears?



B2 Assessing the fixed ambiguities in navigation

- ▲ After repairing the carrier ambiguities, these measurements will be used to navigate.
- ▲ Indeed, once the integer ambiguities are known, the carrier phase measurements become like "unambiguous pseudoranges", accurate at the centimetre level or better.
- ▲ Thence, the positioning is straightforward following the same procedure as with pseudoranges.
- ▲ Nevertheless, a wrong ambiguity fix can degrade the position solution significantly.

B2.1 UPC1-UPC2 Baseline vector estimation with DDL1 carrier (using the time-tagged reference station measurements)

- ▲ In this exercise we will considerer an implementation of differential positioning where the user estimates the baseline vector using the time-tagged measurements of the reference station.
- ↑ This approach is **usually referred to as relative positioning** and can be applied in some applications where the coordinates of the reference station are not accurately known and where the relative position vector between the reference station and the user is the main interest. Examples are formation flying, automatic landing on ships...
- ▲ Of course, the knowledge of the reference receiver location would allow the user to compute its absolute coordinates.
- ▲ This is a simple approach, where synchronism delays between the time tag measurements of the reference station and the user must be taken into account for real-time positioning.



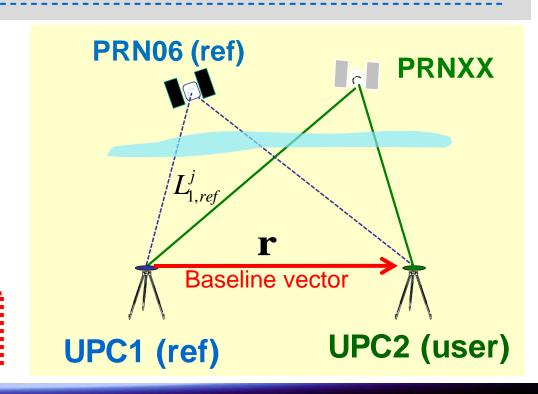
B2.1 UPC1-UPC2 Baseline vector estimation with L1 carrier (using the time-tagged reference station measurements)

Data file DD_UPC1_UPC2_06_ALL.fixL1L2 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 [UPC1 UPC2 06 PRN DoY sec DDP1 DDL1 DDP2 DDL2 DDRho DDTrop DDIon El1 Az1 El2 Az2 λ₁ DDN1 λ₂ DDN2] <---- UPC2 ---->

Where the elevation (EL) and azimuth (AZ) are taken from station **UPC2** (the user)

and where, (EL1, AZ1) are for satellite PNR06 (reference) and (EL1, AZ1) are for satellite PRNXX

 $L_{1,ref}^{j}$ Measurements broadcast by the reference station.





Estimate the baseline vector between UPC1 and UPC2 receivers using the code measurements of file (DD_UPC1_UPC2_06_ALL.dat).

Note: Use the entire file (i.e. time interval [18000:19900]).

[DDL1-
$$\lambda_1$$
 DDN1]=[Los_k - Los_06]*[baseline]

Notation

$$\begin{bmatrix} DDL_{1}^{6,03} - \lambda_{1}DDN_{1}^{6,03} \\ DDL_{1}^{6,07} - \lambda_{1}DDN_{1}^{6,07} \\ \vdots \\ DDL_{1}^{6,24} - \lambda_{1}DDN_{1}^{6,24} \end{bmatrix} = \begin{bmatrix} -(\hat{\boldsymbol{\rho}}^{3} - \hat{\boldsymbol{\rho}}^{6})^{T} \\ -(\hat{\boldsymbol{\rho}}^{7} - \hat{\boldsymbol{\rho}}^{6})^{T} \\ \vdots \\ -(\hat{\boldsymbol{\rho}}^{24} - \hat{\boldsymbol{\rho}}^{6})^{T} \end{bmatrix} \mathbf{r}$$

 $DDL_1^{k j} \equiv DDL1(\text{involving satellites } j \text{ and } k)$

$$DDL_{1}^{k \ j} = DL_{1,usr}^{k \ j} - DL_{1,ref}^{k \ j}$$

$$= \left(L_{1,usr}^{j} - L_{1,usr}^{k}\right) - \left(L_{1,ref}^{j} - L_{1,ref}^{k}\right)$$

 $L_{\mathrm{l},\mathit{ref}}^{\mathit{j}}$ Measurements broadcast by the reference station.

Estimate the baseline vector between UPC1 and UPC2 receivers using the code measurements of file (DD_UPC1_UPC2_06_ALL.dat).

Note: Use the entire file (i.e. time interval [18000:19900]).

[DDL1-
$$\lambda_1$$
 DDN1]=[Los_k - Los_06]*[baseline]

Notation

$$\begin{bmatrix} DDL_{1}^{6,03} - \lambda_{1}DDN_{1}^{6,03} \\ DDL_{1}^{6,07} - \lambda_{1}DDN_{1}^{6,07} \\ \vdots \\ DDL_{1}^{6,24} - \lambda_{1}DDN_{1}^{6,24} \end{bmatrix} = \begin{bmatrix} -(\hat{\boldsymbol{\rho}}^{3} - \hat{\boldsymbol{\rho}}^{6})^{T} \\ -(\hat{\boldsymbol{\rho}}^{7} - \hat{\boldsymbol{\rho}}^{6})^{T} \\ \vdots \\ -(\hat{\boldsymbol{\rho}}^{24} - \hat{\boldsymbol{\rho}}^{6})^{T} \end{bmatrix} \mathbf{r}$$

$$r \equiv$$
 Baseline vector

$$DDL_1^{k j} \equiv DDL1(\text{involving satellites } j \text{ and } k)$$

$$\hat{\rho}^k \equiv \text{Line-Of-Sight unit vector to satelite } k$$

$$\hat{\mathbf{\rho}}^k \equiv \left[\cos(El_k)\sin(Az_k), \cos(El_k)\cos(Az_k), \sin(El_k)\right]$$

Using the DDL1 carrier with the ambiguities FIXED, compute the LS single epoch solution for the whole interval 180000< t <199000 with the program LS.f

Note: The program LS.f computes the Least Square solution for each measurement epoch of the input file (see the FORTRAN code LS.f)

The following procedure can be applied:

a) generate a file with the following content;

```
[Time], [DDL1-\lambda_1DDN1], [Los_k - Los_06]
```

where:

Time= seconds of day

DDL1 - λ_1 **DDN1** = Prefit residulas (i.e., "y" values in program LS.f)

Los_k-Los_06 = The three components of the geometry matrix

(i.e., matrix "a" in program LS.f)



Justify that the next sentence builds the navigation equations system

```
See file content
                     [DDL1-\lambda_1DDN1]=[Los_k - Los_06]*[baseline]
 in slide #31
cat DD_UPC1_UPC2_06_ALL.fixL1L2 | gawk 'BEGIN{g2r=atan2(1,1)/45}
                          {e1=$14*g2r;a1=$15*g2r;e2=$16*g2r;a2=$17*g2r;
  printf "%s %14.4f %8.4f %8.4f \n",
       $8-$18, -cos(e2)*sin(a2)+cos(e1)*sin(a1),
                  -cos(e2)*cos(a2)+cos(e1)*cos(a1), -sin(e2)+sin(e1)}'
 \begin{bmatrix} DDL_{1}^{6,03} - \lambda_{1}DDN_{1}^{6,03} \\ DDL_{1}^{6,07} - \lambda_{1}DDN_{1}^{6,07} \\ \vdots \end{bmatrix}
                                                 [DDL1-\lambda_1 DDN1] [ Los_k - Los_06]
                                                                    0.3398 -0.1028 0.0714
                                                                              0.5972 0.0691
                                                                              0.0227 0.2725
```

```
[Time], [DDL1-\lambda_1DDN1], [Los_k - Los_06]
```

The following sentence can be used:

b) Compute the Least Squares solution:

```
cat L1model.dat |LS > L1fix.pos
```



Plot the baseline estimation error:

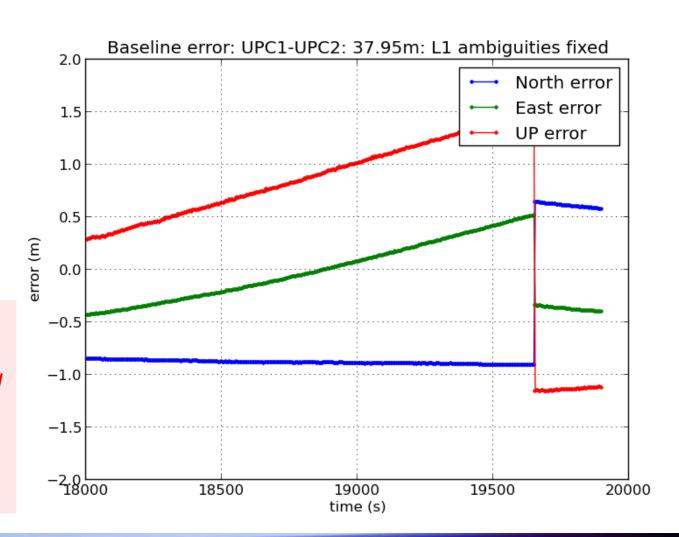
Note: An accurate estimate of baseline is:

Use this determination to assess the baseline vector estimation error.

L1 Baseline estimation error after fixing ambiguities

Questions:

- 1.- What is the expected accuracy when positioning with carrier after fixing ambiguities?
- 2.- Discuss why a trend and a discontinuity appears?





- ▲ In the previous exercise we have considered an implementation of differential positioning where the user estimates the baseline vector from the time-tagged measurements of the reference station.
- ▲ In the next exercises, we will consider the common implementation of Differential positioning, where the reference receiver coordinates are accurately known and used to compute range corrections for each tracked satellite in view. Then, the user applies these corrections to improve the positioning.
- ▲ Unlike in the previous implementation, the synchronism errors between the time-tagged measurements will be not critical in this approach, as the differential corrections vary slowly.

Using code DDL1 measurements, estimate the coordinates of receiver UPC2 taking UPC1 as a reference receiver.

Justify that the associated equations system is given by:

[DDL1-DDRho-
$$\lambda_1$$
DDN1]=[Los_k - Los_06]*[dr]

Notation

$$\begin{bmatrix} DDL_{1}^{6,03} - DD\rho^{6,03} - \lambda_{1}DDN_{1} \\ DDL_{1}^{6,07} - DD\rho^{6,07} - \lambda_{1}DDN_{1} \\ \vdots \\ DDL_{1}^{6,24} - DD\rho^{6,30} - \lambda_{1}DDN_{1} \end{bmatrix} = \begin{bmatrix} -(\hat{\boldsymbol{\rho}}^{3} - \hat{\boldsymbol{\rho}}^{6})^{T} \\ -(\hat{\boldsymbol{\rho}}^{7} - \hat{\boldsymbol{\rho}}^{6})^{T} \\ \vdots \\ -(\hat{\boldsymbol{\rho}}^{24} - \hat{\boldsymbol{\rho}}^{6})^{T} \end{bmatrix} \mathbf{dr}$$

$$\mathbf{dr} = \mathbf{r}_{IND3} - \mathbf{r}_{0,IND3}$$

$$DDL_1^{k j} \equiv \text{DDL1}(\text{involving satellites } j \text{ and } k)$$

$$\hat{\boldsymbol{\rho}}^k \equiv \text{Line-Of-Sight unit vector to satelite } k$$

$$\hat{\boldsymbol{\rho}}^k \equiv \left[\cos(El_k)\sin(Az_k) \quad \cos(El_k)\cos(Az_k) \quad \sin(El_k)\right]$$

Using code DDL1 measurements, estimate the coordinates of receiver UPC2 taking UPC1 as a reference receiver.

Justify that the associated equations system is given by:

[DDL1-DDRho-
$$\lambda_1$$
DDN1]=[Los_k - Los_06]*[dr]

Notation

$$\begin{bmatrix} DDL_{1}^{6,03} - DD\rho^{6,03} - \lambda_{1}DDN_{1} \\ DDL_{1}^{6,07} - DD\rho^{6,07} - \lambda_{1}DDN_{1} \\ \vdots \\ DDL_{1}^{6,24} - DD\rho^{6,30} - \lambda_{1}DDN_{1} \end{bmatrix} = \begin{bmatrix} -(\hat{\boldsymbol{\rho}}^{3} - \hat{\boldsymbol{\rho}}^{6})^{T} \\ -(\hat{\boldsymbol{\rho}}^{7} - \hat{\boldsymbol{\rho}}^{6})^{T} \\ \vdots \\ -(\hat{\boldsymbol{\rho}}^{24} - \hat{\boldsymbol{\rho}}^{6})^{T} \end{bmatrix} d\mathbf{r}$$

 $DDL_1^{k j} \equiv DDL1(\text{involving satellites } j \text{ and } k)$

$$DDL_{1}^{k j} - DD\rho^{k j} = D\left(L_{1,usr}^{k j} - \rho_{usr}^{k j}\right) - D\left(L_{1,ref}^{k j} - \rho_{ref}^{k j}\right)$$

$$= \left[\left(L_{1,usr}^{j} - \rho_{usr}^{j}\right) - \left(L_{1,usr}^{k} - \rho_{usr}^{k}\right)\right] - \left[\left(L_{1,ref}^{j} - \rho_{ref}^{j}\right) - \left(L_{1,ref}^{k} - \rho_{ref}^{k}\right)\right]$$

$$PRC_{L1,1}^{\ \ j} \equiv L_{1,ref}^{j} - \rho_{ref}^{j}$$
 Computed corrections broadcast by the reference station.



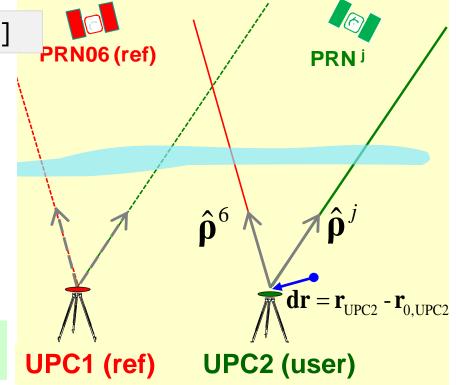
Using code DDL1 measurements, estimate the coordinates of receiver UPC2 taking UPC1 as a reference receiver.

Justify that the associated equations system is given by:

$$\begin{bmatrix} DDL1- DDRho-\lambda_1 DDN1 \end{bmatrix} = \begin{bmatrix} Los_k-Los_06 \end{bmatrix} * \begin{bmatrix} dr \end{bmatrix}$$

$$\begin{bmatrix} DDL_1^{6,03} - DD\rho^{6,03} - \lambda_1 DDN_1 \\ DDL_1^{6,07} - DD\rho^{6,07} - \lambda_1 DDN_1 \\ \vdots \\ DDL_1^{6,24} - DD\rho^{6,30} - \lambda_1 DDN_1 \end{bmatrix} = \begin{bmatrix} -(\hat{\boldsymbol{\rho}}^3 - \hat{\boldsymbol{\rho}}^6)^T \\ -(\hat{\boldsymbol{\rho}}^7 - \hat{\boldsymbol{\rho}}^6)^T \\ \vdots \\ -(\hat{\boldsymbol{\rho}}^{24} - \hat{\boldsymbol{\rho}}^6)^T \end{bmatrix} d\mathbf{r}$$

$$\hat{\boldsymbol{\rho}}^j \equiv \begin{bmatrix} \cos(El_j)\sin(Az_j), \cos(El_j)\cos(Az_j), \sin(El_j) \end{bmatrix}$$





Justify that the next sentence builds the navigation equations system

```
See file content
                                [DDL1-DDRho-\lambda_1DDN1]=[Los_k - Los_06]*[dr]
   in slide #43
cat DD_UPC1_UPC2_06_ALL.fixL1L2 | gawk 'BEGIN{g2r=atan2(1,1)/45}
                                      {e1=$14*g2r;a1=$15*g2r;e2=$16*g2r;a2=$17*g2r;
 printf "%s %14.4f %8.4f %8.4f \n",
 $6, $8-$11-$18, -cos(e2)*sin(a2)+cos(e1)*sin(a1),
                                -cos(e2)*cos(a2)+cos(e1)*cos(a1),-sin(e2)+sin(e1)}'> M.dat
                                                                     [DDL1-DDRho- \lambda_1 DDN1] [Los_k - Los_06]
\begin{bmatrix} DDL_{1}^{6,03} - DD\rho^{6,03} - \lambda_{1}DDN_{1}^{6,03} \\ DDL_{1}^{6,07} - DD\rho^{6,07} - \lambda_{1}DDN_{1}^{6,07} \\ \vdots \\ DDL_{1}^{6,24} - DD\rho^{6,24} - \lambda_{1}DDN_{1}^{6,24} \end{bmatrix} = \begin{bmatrix} -(\hat{\boldsymbol{\rho}}^{3} - \hat{\boldsymbol{\rho}}^{6})^{T} \\ -(\hat{\boldsymbol{\rho}}^{7} - \hat{\boldsymbol{\rho}}^{6})^{T} \\ \vdots \\ -(\hat{\boldsymbol{\rho}}^{24} - \hat{\boldsymbol{\rho}}^{6})^{T} \end{bmatrix}
                                                                                -3.3762
                                                                                                          0.3398 -0.1028 0.0714
                                                                                                                          0.5972 0.0691
                                                                                 4.3881
                                                                                                                          0.0227 0.2725
```

```
[Time], [DDL1- DDRho -\lambda_1 DDN1], [Los_k - Los_06]
```

The following sentence can be used

b) Compute the Least Squares solution

```
cat L1model.dat |LS > L1fix.pos
```



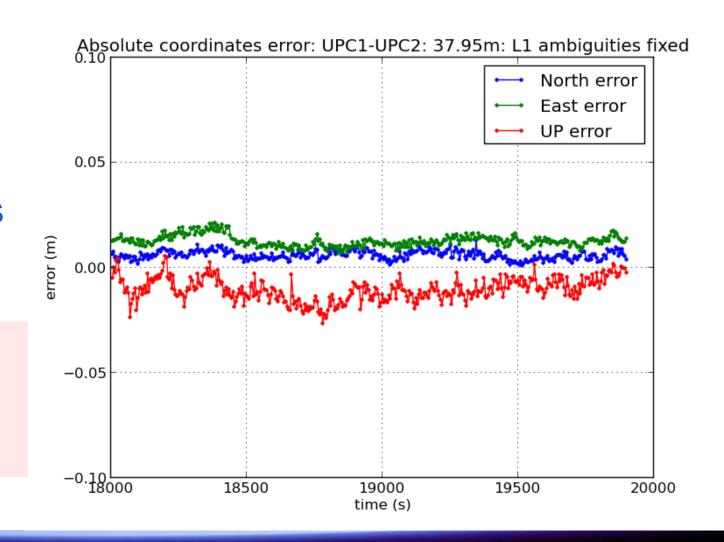
Plot the absolute positioning error:

```
graph.py -f L1fix.pos -x1 -y2- s.- -l "North error"
   -f L1fix.pos -x1 -y3 -s.- -l "East error"
   -f L1fix.pos -x1 -y4 -s.- -l "UP error"
   --yn -.1 --yx .1 --xl "time (s)" --yl "error (m)"
   -t "Absolute positioning error with DDL1"
```

L1 Differential positioning after fixing ambiguities

Questions:

Discuss why the results have improved, achieving centimetre level navigation.





Repeat the previous positioning, but with the DDL2 carrier

```
[Time], [DDL2- DDRho -\lambda_2 DDN2], [Los_k - Los_06]
```

The following sentence can be used

Compute the Least Squares solution

```
cat L2model.dat |LS > L2fix.pos
```



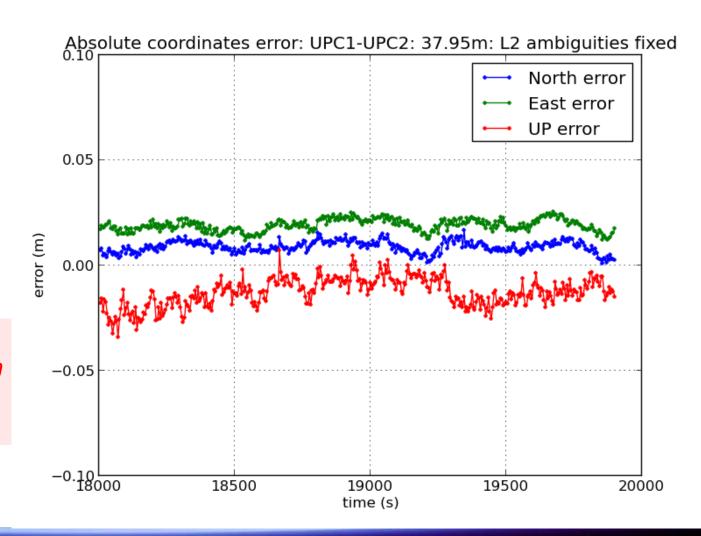
Plot the absolute positioning error:

```
graph.py -f L2fix.pos -x1 -y2- s.- -l "North error"
   -f L2fix.pos -x1 -y3 -s.- -l "East error"
   -f L2fix.pos -x1 -y4 -s.- -l "UP error"
   --yn -.1 --yx .1 --xl "time (s)" --yl "error (m)"
   -t "Absolute positioning error with DDL2"
```

L2 Differential positioning after fixing ambiguities

Questions:

Compare the results with the previous ones computed from DDL1.





Analyze the effect of a wrong ambiguity fix over a single satellite (e.g. PRN07)

Simulate an error of 1 cycle in DDN1 for satellite PRN07 and compute the navigation solution:

1 cycle is added to the DDN1 of satellite PRN07

The following sentence can be used

Compute the Least Squares solution

cat L1model.dat |LS > L1fix.pos



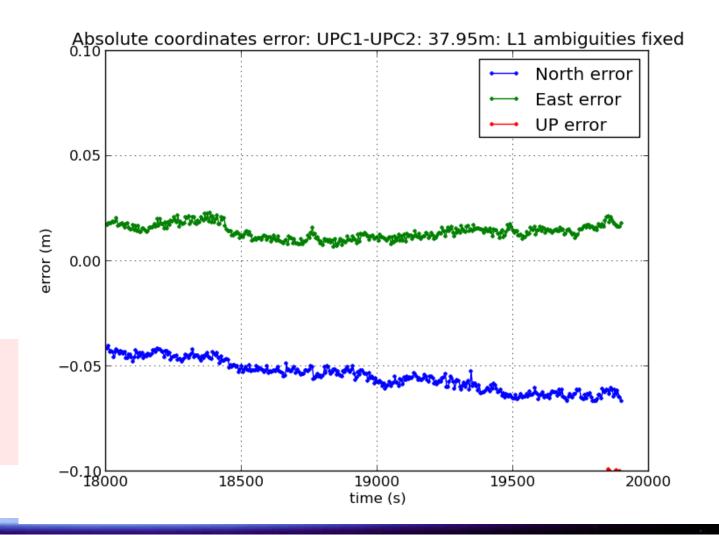
Plot the absolute positioning error:

```
graph.py -f L2fix.pos -x1 -y2- s.- -l "North error"
   -f L2fix.pos -x1 -y3 -s.- -l "East error"
   -f L2fix.pos -x1 -y4 -s.- -l "UP error"
   --yn -.1 --yx .1 --xl "time (s)" --yl "error (m)"
   -t "Absolute positioning error with a wrong ambiguity fix"
```

L1 Differential positioning with a wrong ambiguity fix on a single satellite

Questions:

Discuss the results.
What is the effect of the wrong fix?





B2.4. Differential positioning with DDL1 Effect of wrong ambiguity fix.

OVERVIEW

- ▲ Introduction: gLAB processing in command line.
- → Preliminary computations: Data files.
- ▲ Session A: Fixing DD ambiguities one at a time: UPC1-UPC2.
- ▲ **Session B:** Assessing the fixed ambiguities in navigation: Differential positioning of UPC1-UPC2 receivers.
- > Session C: Fixing DD ambiguities with LAMBDA method.

Note: UPC1-UPC2 receivers baseline: 37.95 metres.

Session C

Fixing DD ambiguities with LAMBDA method

(baseline: 37.95 metres)

Apply the LAMBDA method to fix the ambiguities.

Consider only the two epochs: $t_1 = 18000$ and $t_2 = 18015$.

Note:

To avoid the synchronization issues, consider the Differential Positioning using the computed differential corrections, instead of the time-tagged measurements.

That is, we are going to solve the following navigation equations systems:

1. Navigating with L1 carrier, to fix DDN1:

[DDL1-DDRho]=[Los_k-Los_06]*dr + [A]*[λ_1 *DDN1]

2. Navigating with L2 carrier, to fix DDN2:

[DDL2-DDRho]=[Los_k-Los_06]*dr + [A]*[λ_2 *DDN2]



Consider again the previous problem of estimating $\Delta \mathbf{r}$, a 3-vector of real numbers, and N a (K-1)-vector of integers, which are solution of

$$\mathbf{y} = \mathbf{G} \, \Delta \mathbf{r} + \lambda \, \mathbf{A} \, \mathbf{N} + \mathbf{v}$$

The solution comprises the following steps:

- 1. Obtain the float solution and its covariance matrix:
- 2. Find the integer vector **N** which minimizes the cost function

$$c(\mathbf{N}) = \|\mathbf{N} - \hat{\mathbf{N}}\|_{\mathbf{W}_{\hat{\mathbf{N}}}}^{2} = (\mathbf{N} - \hat{\mathbf{N}})^{T} \mathbf{W}_{\hat{\mathbf{N}}} (\mathbf{N} - \hat{\mathbf{N}})$$

$$\mathbf{W}_{\hat{\mathbf{N}}} = \mathbf{P}_{\hat{\mathbf{N}}}^{-1}$$

- <u>Decorrelation:</u> Using the **Z** transform, the ambiguity search space is reparametrized to decorrelate the float ambiguities.
- b) Integer ambiguities estimation (e.g. by rounding or by using sequential conditional least-squares adjustment, together with a discrete search strategy).
- c) Using the Z⁻¹ transform, the ambiguities are transformed to the original ambiguity space.
- 3. Obtain the 'fixed' solution $\Delta \mathbf{r}$, from the fixed ambiguities N.

$$\mathbf{y} - \lambda \mathbf{A} \mathbf{N} = \mathbf{G} \Delta \mathbf{r} + \mathbf{v}$$

C1. DDN1 ambiguity fixing: Differential positioning using computed differential corrections from a reference receiver.

Consider only the two epochs: $t_1 = 18000$ and $t_2 = 18015$.

The following procedure can be applied:

- 1. Build-up the navigation system.
- **2. Compute the FLOATED solution**, solving the equations system with octave. Assess the accuracy of the floated solution.
- 3. Apply the LAMBDA method to FIX the ambiguities. Compare the results with the solution obtained by rounding directly the floated solution and by rounding the solution after decorrelation.

1. Building-up the navigation system

$$[DDL1-DDRho] = [Los_k - Los_06]*[dr] + [A]*[lambda1*DDN1]$$

Notation

$$\begin{bmatrix} DDL_1^{6,03} - DD\rho^{6,03} \\ DDL_1^{6,07} - DD\rho^{6,07} \\ \vdots \\ DDL_1^{6,24} - DD\rho^{6,24} \end{bmatrix} = \begin{bmatrix} -(\hat{\boldsymbol{\rho}}^3 - \hat{\boldsymbol{\rho}}^6)^T \\ -(\hat{\boldsymbol{\rho}}^7 - \hat{\boldsymbol{\rho}}^6)^T \\ \vdots \\ -(\hat{\boldsymbol{\rho}}^{24} - \hat{\boldsymbol{\rho}}^6)^T \end{bmatrix} \mathbf{dr} + \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 DDN_1^{6,03} \\ \lambda_1 DDN_1^{6,07} \\ \vdots \\ \lambda_1 DDN_1^{6,24} \end{bmatrix}$$
Where the vector of unknowns $\underline{\mathbf{x}}$ includes the user coordinates and ambiguities

$$y = G x$$

The receiver was not moving (static) during the data collection. Thence, for each epoch we have the equations system:

$$\begin{bmatrix} DDL_{1}^{6,03}(t_{1}) - DD\rho^{6,03}(t_{1}) \\ DDL_{1}^{6,07}(t_{1}) - DD\rho^{6,07}(t_{1}) \\ \vdots \\ DDL_{1}^{6,24}(t_{1}) - DD\rho^{6,24}(t_{1}) \end{bmatrix} = \begin{bmatrix} -\left(\hat{\boldsymbol{\rho}}^{3}(t_{1}) - \hat{\boldsymbol{\rho}}^{6}(t_{1})\right)^{T} \\ -\left(\hat{\boldsymbol{\rho}}^{7}(t_{1}) - \hat{\boldsymbol{\rho}}^{6}(t_{1})\right)^{T} \\ \vdots \\ -\left(\hat{\boldsymbol{\rho}}^{30}(t_{1}) - \hat{\boldsymbol{\rho}}^{6}(t_{1})\right)^{T} \end{bmatrix} d\mathbf{r} + \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_{1} DDN_{1}^{6,03} \\ \lambda_{1} DDN_{1}^{6,07} \\ \vdots \\ \lambda_{1} DDN_{1}^{6,24} \end{bmatrix} \quad \mathbf{y}_{1} = \mathbf{G}_{1} \mathbf{X}$$

$$\mathbf{y}_{1} = \mathbf{y}_{1} = \mathbf{y}_{1}$$

$$\mathbf{y}_1 = \mathbf{G}_1 \ \mathbf{x}$$

G1:=G[t1]

$$\begin{bmatrix} DDL_{1}^{6,03}(t_{2}) - DD\rho^{6,03}(t_{2}) \\ DDL_{1}^{6,07}(t_{2}) - DD\rho^{6,07}(t_{2}) \\ \vdots \\ DDL_{1}^{6,24}(t_{2}) - DD\rho^{6,24}(t_{2}) \end{bmatrix} = \begin{bmatrix} -\left(\hat{\boldsymbol{\rho}}^{3}(t_{2}) - \hat{\boldsymbol{\rho}}^{6}(t_{2})\right)^{T} \\ -\left(\hat{\boldsymbol{\rho}}^{7}(t_{2}) - \hat{\boldsymbol{\rho}}^{6}(t_{2})\right)^{T} \\ \vdots \\ -\left(\hat{\boldsymbol{\rho}}^{30}(t_{2}) - \hat{\boldsymbol{\rho}}^{6}(t_{2})\right)^{T} \end{bmatrix} d\mathbf{r} + \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_{1} DDN_{1}^{6,03} \\ \lambda_{1} DDN_{1}^{6,07} \\ \vdots \\ \lambda_{1} DDN_{1}^{6,24} \end{bmatrix} \quad \mathbf{y}_{2} = \mathbf{G}_{2} \mathbf{X}$$

$$\mathbf{y}_2 = \mathbf{G}_2 \mathbf{x}$$

y2:=y[t2] G2:=G[t2]

 $[DDL1-DDRho]=[Los_k - Los_06]*[dr] + [A]*[lambda1*DDN1]$

In the previous computation we have not taken into account the correlations between the double differences of measurements. This is to $\mathbf{P}_{\mathbf{y}} = 2\sigma^{2} \begin{bmatrix} 2 & 1 & \cdots & 1 \\ 1 & 2 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & 2 \end{bmatrix}$ matrix will be used now, as the LAMBDA method will be applied to FIX the carrier ambiguities.

- a) Show that the covariance matrix of DDL1 is given by P_v
- b) Given the measurement vectors (y) and Geometry matrices (G) for two epochs

$$y1:=y[t1]$$
; $G1:=G[t1]$; Py $y2:=y[t2]$; $G2:=G[t2]$; Py

show that the user solution and covariance matrix can be computed as:

$$y = G x; W = P_y^{-1}$$

$$x = (G^TWG)^{-1}G^TWy$$

$$\mathbf{P} = (\mathbf{G}^{\mathrm{T}}\mathbf{W}\mathbf{G})^{-1}$$

P=inv(G1'*W*G1+G2'*W*G2);

$$x=P^*(G1'^*W^*y1+G2'^*W^*y2)$$
;

where: **W=inv(Py)**

The script MakeL1DifMat.scr builds the equations system

[DDL1-DDRho]=[Los_k- Los_06]*[dr] + [A]*[
$$\lambda_1$$
*DDN1]

for the two epochs required $t_1=18000$ and $t_2=18015$, using the input file **DD_UPC1_UPC2_06_ALL.dat** generated before.

Execute:

MakeL1DifMat.scr DD_UPC1_UPC2_06_ALL.dat 18000 18015

The **OUTPUT**

are the files M1.dat and M2.dat associated with each epoch.

Where:

the columns of files M.dat are the vector y (first column) and Matrix G (next columns)



2. Compute the FLOATED solution (solving the equations system).

The following procedure can be applied

```
octave
load M1.dat
load M2.dat

y1=M1(:,1);
G1=M1(:,2:12);

y2=M2(:,1);
G2=M2(:,2:12);
Py=(diag(ones(1,8))+ones(8))*2e-4;
W=inv(Py);
```

```
P=inv(G1'*W*G1+G2'*W*G2);
x=P*(G1'*W*y1+G2'*W*y2);

Solution
x(1:3)'
1.4216 -0.6058 0.4035
```

3. Applying the LAMBDA method to FIX the ambiguities.

Compare the results with the solution obtained by rounding the floated solution. The following procedure can be applied (justify the computations done)

```
octave

c=299792458;
f0=10.23e+6;
f1=154*f0;
lambda1=c/f1
   a=x(4:11)/lambda1;
   Q=P(4:11,4:11);
```

1. Rounding directly the floated solution

```
round(a)'
0 6 -6 6 6 -1 8 9
```

2.- Rounding the decorrelated floated solution

```
[Qz,Zt,Lz,Dz,az,iZ] = decorrel (Q,a);
afix=iZ*round(az);
2  1  2  -1  4  7  1  4
```

3.- Decorrelation and integer LS search solution

```
[Qz,Zt,Lz,Dz,az,iZ] = decorrel (Q,a);
[azfixed,sqnorm] = lsearch (az,Lz,Dz,2);
afixed=iZ*azfixed;
sqnorm(2)/sqnorm(1)
ans = 3.10696822814451
afixed(:,1)'
2  1  2  -1  4  7  1  4
```

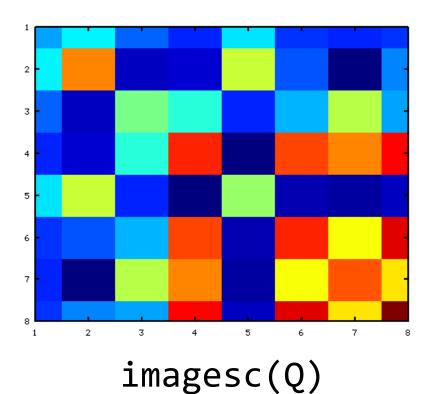
Questions:

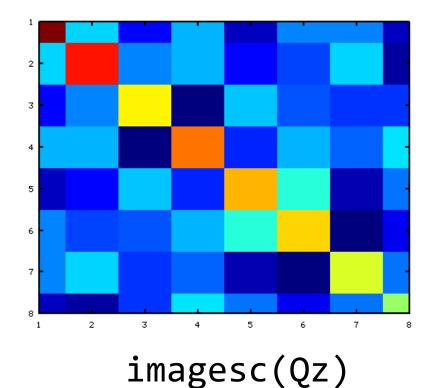
- 1. Can the ambiguities be well fixed?
- 2. Is the test resolutive?
- 3. Compare the fixed ambiguities with those obtained in the previous exercises when fixing the ambiguities one at a time. Are the same results found?
- 4. What is the elapsed time to needed fix the ambiguities? And in the previous exercise when fixing the ambiguities one at a time?
- 5. The values found for the ambiguities are the same than in the previous case?

@ J. Sanz & J.M. Juan

C2 Checking the **Z**-transform matrix

a. Using the Octave/MATLAB program sentence **imagesc** display the covariance matrix of ambiguities before and after the decorrelation with the Z-matrix.





C2 Checking the **Z**-transform matrix

b.- Show the content of the integer matrix Z

Note: The previous routines computes its transpose (Zt). Then: **Z=Zt'**.

C2 Checking the **Z**-transform matrix

c.- Compute by hand the transformed covariance matrix **Qz**:

d.- Compute the decorrelated ambiguities az:

```
Z*a
67.19877600816902
-27.00815309344809
-52.80792522348074
49.18410614456196
-53.87737144457776
-11.70730035212100
18.08081880749826
4.09968790147667
```

C2 Checking the Z-transform matrix

e.- Round-off the decorrelated ambiguities:

```
Nz=round(Z*a)
67 -27 -53 49 -54 -12 18 4
```

f.- Apply the inverse transform to these values:

g. - Compare the previous results with the direct rounding of initial ambiguities "a":

```
round(a)
0 -6 6 6 -1 12 8 9
```

C3. DDN2 ambiguity fixing: Differential positioning using computed differential corrections from a reference receiver.

Consider only the two epochs: $t_1 = 18000$ and $t_2 = 18015$.

The following procedure can be applied, as in the previous case:

- 1. Build-up the navigation system.
- **2. Compute the FLOATED solution**, solving the equations system with octave. Assess the accuracy of the floated solution.
- 3. Apply the LAMBDA method to FIX the ambiguities. Compare the results with the solution obtained by rounding directly the floated solution and by rounding the solution after decorrelation.

1. Building-up the navigation system

$$[DDL2-DDRho] = [Los_k - Los_06]*[dr] + [A]*[lambda2*DDN2]$$

Notation

$$\begin{bmatrix} DDL_{2}^{6,03} - DD\rho^{6,03} \\ DDL_{2}^{6,07} - DD\rho^{6,07} \\ \vdots \\ DDL_{2}^{6,24} - DD\rho^{6,24} \end{bmatrix} = \begin{bmatrix} -(\hat{\boldsymbol{\rho}}^{3} - \hat{\boldsymbol{\rho}}^{6})^{T} \\ -(\hat{\boldsymbol{\rho}}^{7} - \hat{\boldsymbol{\rho}}^{6})^{T} \\ \vdots \\ -(\hat{\boldsymbol{\rho}}^{24} - \hat{\boldsymbol{\rho}}^{6})^{T} \end{bmatrix} \mathbf{dr} + \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_{2} DDN_{2}^{6,03} \\ \lambda_{2} DDN_{2}^{6,07} \\ \vdots \\ \lambda_{2} DDN_{2}^{6,24} \end{bmatrix}$$
Where the vector of unknowns $\underline{\mathbf{x}}$ includes the user coordinates and ambiguities

$$y = G x$$

The receiver was not moving (static) during the data collection. Thence, for each epoch we have the equations system:

$$\begin{bmatrix} DDL_{2}^{6,03}(t_{1}) - DD\rho^{6,03}(t_{1}) \\ DDL_{2}^{6,07}(t_{1}) - DD\rho^{6,07}(t_{1}) \\ \vdots \\ DDL_{2}^{6,24}(t_{1}) - DD\rho^{6,24}(t_{1}) \end{bmatrix} = \begin{bmatrix} -\left(\hat{\boldsymbol{\rho}}^{3}(t_{1}) - \hat{\boldsymbol{\rho}}^{6}(t_{1})\right)^{T} \\ -\left(\hat{\boldsymbol{\rho}}^{7}(t_{1}) - \hat{\boldsymbol{\rho}}^{6}(t_{1})\right)^{T} \\ \vdots \\ -\left(\hat{\boldsymbol{\rho}}^{30}(t_{1}) - \hat{\boldsymbol{\rho}}^{6}(t_{1})\right)^{T} \end{bmatrix} d\mathbf{r} + \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_{2} DDN_{2}^{6,03} \\ \lambda_{2} DDN_{2}^{6,07} \\ \vdots \\ \lambda_{2} DDN_{2}^{6,24} \end{bmatrix} \quad \mathbf{y}_{1} = \mathbf{G}_{1} \mathbf{X}$$

$$\mathbf{y}_1 = \mathbf{G}_1 \mathbf{x}$$

$$y1:=y[t1]$$

$$G1:=G[t1]$$

$$\begin{bmatrix} DDL_{2}^{6,03}(t_{2}) - DD\rho^{6,03}(t_{2}) \\ DDL_{2}^{6,07}(t_{2}) - DD\rho^{6,07}(t_{2}) \\ \vdots \\ DDL_{2}^{6,24}(t_{2}) - DD\rho^{6,24}(t_{2}) \end{bmatrix} = \begin{bmatrix} -\left(\hat{\boldsymbol{\rho}}^{3}(t_{2}) - \hat{\boldsymbol{\rho}}^{6}(t_{2})\right)^{T} \\ -\left(\hat{\boldsymbol{\rho}}^{7}(t_{2}) - \hat{\boldsymbol{\rho}}^{6}(t_{2})\right)^{T} \\ \vdots \\ -\left(\hat{\boldsymbol{\rho}}^{30}(t_{2}) - \hat{\boldsymbol{\rho}}^{6}(t_{2})\right)^{T} \end{bmatrix} d\mathbf{r} + \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_{2} DDN_{2}^{6,03} \\ \lambda_{2} DDN_{2}^{6,07} \\ \vdots \\ \lambda_{2} DDN_{2}^{6,24} \end{bmatrix}$$

$$\mathbf{y}_{2} = \mathbf{G}_{2} \mathbf{X}$$

$$\mathbf{y}_{2} = \mathbf{y}_{2} \mathbf{y}_{$$

$$\mathbf{y}_2 = \mathbf{G}_2 \mathbf{x}$$

G2:=G[t2]

In the previous sessions A and B we have not taken into account the correlations between the double differences of measurements. This is to $\mathbf{P}_{\mathbf{y}} = 2\sigma^{2} \begin{bmatrix} 2 & 1 & \cdots & 1 \\ 1 & 2 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & 2 \end{bmatrix}$ matrix will be used now, as the LAMBDA method will be applied to FIX the carrier ambiguities.

- a) Show that the covariance matrix of DDL2 is given by P_v
- b) Given the measurement vectors (y) and Geometry matrices (G) for two epochs

show that the user solution and covariance matrix can be computed as:

$$\mathbf{y} = \mathbf{G} \mathbf{x}; \quad \mathbf{W} = \mathbf{P}_{\mathbf{y}}^{-1}$$

$$\mathbf{x} = (\mathbf{G}^{\mathrm{T}} \mathbf{W} \mathbf{G})^{-1} \mathbf{G}^{\mathrm{T}} \mathbf{W} \mathbf{y}$$

$$\mathbf{P} = (\mathbf{G}^{\mathrm{T}}\mathbf{W}\mathbf{G})^{-1}$$

P=inv(G1'*W*G1+G2'*W*G2);

$$x=P^*(G1'^*W^*y1+G2'^*W^*y2)$$
;

where: **W=inv(Py)**

@ J. Sanz & J.M. Juan

The script MakeL2DifMat.scr builds the equations system

[DDL2-DDRho]=[Los_k- Los_06]*[dr] + [A]*[
$$\lambda_2$$
*DDN2]

for the two epochs required $t_1=18000$ and $t_2=18015$, using the input file **DD_UPC1_UPC2_06_ALL.dat** generated before.

Execute:

MakeL2DifMat.scr DD_UPC1_UPC2_06_ALL.dat 18000 18015

The **OUTPUT**

are the files M1.dat and M2.dat associated with each epoch.

Where:

the columns of files M.dat are the vector y (first column) and Matrix G (next columns)



2. Computing the FLOATED solution (solving the equations system).

The following procedure can be applied

```
octave
load M1.dat
load M2.dat

y1=M1(:,1);
G1=M1(:,2:12);

y2=M2(:,1);
G2=M2(:,2:12);
Py=(diag(ones(1,8))+ones(8))*2e-4;
W=inv(Py);
```

```
P=inv(G1'*W*G1+G2'*W*G2);
x=P*(G1'*W*y1+G2'*W*y2);

Solution
x(1:3)'
0.1442 -0.5154 0.5568
```

3. Applying the LAMBDA method to FIX the ambiguities.

Compare the results with the solution obtained by rounding the floated solution. The following procedure can be applied (justify the computations done)

```
octave

c=299792458;
f0=10.23e+6;
f2=120*f0;
lambda2=c/f2
  a=x(4:11)/lambda2;
  Q=P(4:11,4:11);
```

1. Rounding directly the floated solution

```
round(a)'
-1 -2 1 2 1 -2 3 -1
```

2.- Rounding the decorrelated floated solution

```
[Qz,Zt,Lz,Dz,az,iZ] = decorrel (Q,a);
afix=iZ*round(az);
-1  1 -1  2  2  -1  1  0
```

3.- Decorrelation and integer LS search solution

```
[Qz,Zt,Lz,Dz,az,iZ] = decorrel (Q,a);
[azfixed,sqnorm] = lsearch (az,Lz,Dz,2);
afixed=iZ*azfixed;
sqnorm(2)/sqnorm(1)
ans = 3.54056715815950
afixed(:,1)'
-1  1  -1  2  2  -1  1  0
```

Questions:

- 1. Can the ambiguities be well fixed?
- 2. Is the test resolutive?
- 3. Compare the fixed ambiguities with those obtained in the previous exercises when fixing the ambiguities one at a time. Are the same results found?
- 4. What is the elapsed time to needed fix the ambiguities? And in the previous exercise when fixing the ambiguities one at a time?
- 5. The values found for the ambiguities are the same than in the previous case?

C4. Estimate the baseline vector between UPC1 and UPC2 receivers using the L1 carrier measurements of file (DD_UPC1_UPC2_06_ALL.dat).

Consider only the two epochs used in the previous exercise: $t_1=14500$ and $t_2=14515$

The following procedure can be applied, as in the previous case:

- 1. Build-up the navigation system.
- **2. Compute the FLOATED solution**, solving the equations system with octave. Assess the accuracy of the floated solution.
- 3. Apply the LAMBDA method to FIX the ambiguities. Compare the results with the solution obtained by rounding directly the floated solution and by rounding the solution after decorrelation.

C4.1 Estimate the baseline vector between UPC1 and UPC2 receivers using the L1 carrier measurements of file (DD_UPC1_UPC2_06_ALL.dat).

Notation (for each epoch t)

$$\begin{bmatrix} DDL_{1}^{6,03} \\ DDL_{1}^{6,07} \\ \vdots \\ DDL_{1}^{6,24} \end{bmatrix} = \begin{bmatrix} -(\hat{\boldsymbol{\rho}}^{3} - \hat{\boldsymbol{\rho}}^{6})^{T} \\ -(\hat{\boldsymbol{\rho}}^{7} - \hat{\boldsymbol{\rho}}^{6})^{T} \\ \vdots \\ -(\hat{\boldsymbol{\rho}}^{24} - \hat{\boldsymbol{\rho}}^{6})^{T} \end{bmatrix} \mathbf{r} + \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_{1} DDN_{1}^{6,03} \\ \lambda_{1} DDN_{1}^{6,07} \\ \vdots \\ \lambda_{1} DDN_{1}^{6,24} \end{bmatrix}$$
Where the vector of unknowns $\underline{\mathbf{x}}$ includes the user coordinates and ambiguities

$$y = G x$$

and ambiguities

The receiver was not moving (static) during the data collection. Therefore, for each epoch we have the equations system:

$$\begin{bmatrix} DDL_{1}^{6,03}(t_{1}) \\ DDL_{1}^{6,07}(t_{1}) \\ \vdots \\ DDL_{1}^{6,24}(t_{1}) \end{bmatrix} = \begin{bmatrix} -\left(\hat{\boldsymbol{\rho}}^{3}(t_{1}) - \hat{\boldsymbol{\rho}}^{6}(t_{1})\right)^{T} \\ -\left(\hat{\boldsymbol{\rho}}^{7}(t_{1}) - \hat{\boldsymbol{\rho}}^{6}(t_{1})\right)^{T} \\ \vdots \\ -\left(\hat{\boldsymbol{\rho}}^{24}(t_{1}) - \hat{\boldsymbol{\rho}}^{6}(t_{1})\right)^{T} \end{bmatrix} \mathbf{r} + \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_{1} DDN_{1}^{6,03} \\ \lambda_{1} DDN_{1}^{6,07} \\ \vdots \\ \lambda_{1} DDN_{1}^{6,24} \end{bmatrix}$$

$$\mathbf{y}_{1} = \mathbf{G}_{1} \mathbf{x}$$

$$\mathbf{y}_{1} = \mathbf{y}_{1} = \mathbf{G}_{1} \mathbf{x}$$

$$\mathbf{y}_{1} = \mathbf{G}_{1} \mathbf{x}$$

$$\begin{bmatrix} DDL_{1}^{6,03}(t_{2}) \\ DDL_{1}^{6,07}(t_{2}) \\ \vdots \\ DDL_{1}^{6,24}(t_{2}) \end{bmatrix} = \begin{bmatrix} -\left(\hat{\boldsymbol{\rho}}^{3}(t_{2}) - \hat{\boldsymbol{\rho}}^{6}(t_{2})\right)^{T} \\ -\left(\hat{\boldsymbol{\rho}}^{7}(t_{2}) - \hat{\boldsymbol{\rho}}^{6}(t_{2})\right)^{T} \\ \vdots \\ -\left(\hat{\boldsymbol{\rho}}^{24}(t_{2}) - \hat{\boldsymbol{\rho}}^{6}(t_{2})\right)^{T} \end{bmatrix} \mathbf{r} + \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_{1} DDN_{1}^{6,03} \\ \lambda_{1} DDN_{1}^{6,07} \\ \vdots \\ \lambda_{1} DDN_{1}^{6,24} \end{bmatrix}$$

$$\mathbf{y}_{2} = \mathbf{G}_{2} \mathbf{X}$$

$$\mathbf{y}_{2} = \mathbf{y}_{1}^{2} \mathbf{y}_{2}$$

$$\mathbf{y}_{2} = \mathbf{G}_{2} \mathbf{x}$$
 $y_{2} = \mathbf{y}[t_{2}]$
 $g_{2} = \mathbf{G}[t_{2}]$

In the previous sessions A and B we have not taken into account the correlations between the double differences of measurements. This matrix will be used now, as the LAMBDA method will be applied to FIX the carrier ambiguities.

to $\mathbf{P}_{\mathbf{y}} = 2\sigma^{2} \begin{bmatrix} 2 & 1 & \cdots & 1 \\ 1 & 2 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & 2 \end{bmatrix}$

- a) Show that the covariance matrix of DDL1 is given by P_v
- b) Given the measurement vectors (y) and Geometry matrices (G) for two epochs y1:=y[t1]; G1:=G[t1]; Py

y2:=y[t2] ; G2:=G[t2] ; Py

show that the user solution and covariance matrix can be computed as:

$$y = G x; W = P_y^{-1}$$

$$\mathbf{x} = (\mathbf{G}^{\mathrm{T}}\mathbf{W}\mathbf{G})^{-1}\mathbf{G}^{\mathrm{T}}\mathbf{W}\mathbf{y}$$

$$\mathbf{P} = (\mathbf{G}^{\mathrm{T}}\mathbf{W}\mathbf{G})^{-1}$$

P=inv(G1'*W*G1+G2'*W*G2);

$$x=P^*(G1'^*W^*y1+G2'^*W^*y2)$$
;

where: W=inv(Py)

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The script MakeL1Bs1Mat.scr builds the equations system

[DDL1]=[Los_k- Los_06]*[baseline] + [A]*[
$$\lambda_1$$
*DDN1]

for the two epochs required $t_1=18000$ and $t_2=18015$, using the input file DD UPC1 UPC2 06 ALL.dat generated before.

Execute:

MakeL1BslMat.scr DD UPC1 UPC2 06 ALL.dat 18000 18015

The **OUTPUT**

are the files M1.dat and M2.dat associated with each epoch.

Where:

the columns of files M.dat are the vector y (first column) and Matrix G (next columns)



1. Computing the FLOATED solution (solving the equations system). The following procedure can be applied:

```
octave
load M1.dat
load M2.dat
y1=M1(:,1);
G1=M1(:,2:12);
y2=M2(:,1);
G2=M2(:,2:12);
Py=(diag(ones(1,8))+ones(8))*2e-4;
W=inv(Py);
```

```
P=inv(G1'*W*G1+G2'*W*G2);
x=P*(G1'*W*y1+G2'*W*y2);
x(1:3)'
-24.5735 -27.1121 3.0021
bsl enu =[-27.4170 -26.2341 -0.0304]
x(1:3)'-bsl enu
ans= 2.84348 -0.87798 3.03248
```

3. Applying the LAMBDA method to FIX the ambiguities.

Compare the results with the solution obtained by rounding the floated solution. The following procedure can be applied (justify the computations done)

```
octave

c=299792458;
f0=10.23e+6;
f1=154*f0;
lambda1=c/f1
  a=x(4:11)/lambda1;
  Q=P(4:11,4:11);
```

1. Rounding directly the floated solution

```
round(a)'
-4 -20 13 11 -14 19 10 16
```

2.- Rounding the decorrelated floated solution

```
[Qz,Zt,Lz,Dz,az,iZ] = decorrel (Q,a);
afix=iZ*round(az);
-1 -12 18 2 -4 8 9 4
```

3.- Decorrelation and integer LS search solution

```
[Qz,Zt,Lz,Dz,az,iZ] = decorrel (Q,a);
[azfixed,sqnorm] = lsearch (az,Lz,Dz,2);
afixed=iZ*azfixed;
sqnorm(2)/sqnorm(1)
ans = 1.22717645070483
afixed(:,1)'
-1 -12 18 2 -4 8 9 4
```

Questions:

- 1. Can the ambiguities be well fixed?
- 2. Is the test resolutive?
- 3. Compare the fixed ambiguities with those obtained in the previous exercises when fixing the ambiguities one at a time. Are the same results found?
- 4. What is the elapsed time to needed fix the ambiguities? And in the previous exercise when fixing the ambiguities one at a time?
- 5. The values found for the ambiguities are the same than in the previous case?

C5. Estimate the baseline vector between UPC1 and UPC2 receivers using the L2 carrier measurements of file (DD_UPC1_UPC2_06_ALL.dat).

Consider only the two epochs used in the previous exercise: $t_1=14500$ and $t_2=14515$

The following procedure can be applied, as in the previous case:

- 1. Build-up the navigation system.
- **2. Compute the FLOATED solution**, solving the equations system with octave. Assess the accuracy of the floated solution.
- 3. Apply the LAMBDA method to FIX the ambiguities. Compare the results with the solution obtained by rounding directly the floated solution and by rounding the solution after decorrelation.

C5.1 Estimate the baseline vector between UPC1 and UPC2 receivers using the L2 carrier measurements of file (DD_UPC1_UPC2_06_ALL.dat).

$$[DDL2] = [Los_k - Los_06]*[baseline] + [A]*[lambda2*DDN2]$$

Notation (for each epoch t)

$$\begin{bmatrix} DDL_{2}^{6,03} \\ DDL_{2}^{6,07} \\ \vdots \\ DDL_{2}^{6,24} \end{bmatrix} = \begin{bmatrix} -(\hat{\boldsymbol{\rho}}^{3} - \hat{\boldsymbol{\rho}}^{6})^{T} \\ -(\hat{\boldsymbol{\rho}}^{7} - \hat{\boldsymbol{\rho}}^{6})^{T} \\ \vdots \\ -(\hat{\boldsymbol{\rho}}^{24} - \hat{\boldsymbol{\rho}}^{6})^{T} \end{bmatrix} \mathbf{r} + \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_{2} DDN_{2}^{6,03} \\ \lambda_{2} DDN_{2}^{6,07} \\ \vdots \\ \lambda_{2} DDN_{2}^{6,07} \end{bmatrix}$$
 Where the vector of unknowns $\underline{\mathbf{x}}$ includes the user coordinates and ambiguities

$$y = G x$$

The receiver was not moving (static) during the data collection. Therefore, for each epoch we have the equations system:

$$\begin{bmatrix} DDL_{2}^{6,03}(t_{1}) \\ DDL_{2}^{6,07}(t_{1}) \\ \vdots \\ DDL_{2}^{6,24}(t_{1}) \end{bmatrix} = \begin{bmatrix} -\left(\hat{\boldsymbol{\rho}}^{3}(t_{1}) - \hat{\boldsymbol{\rho}}^{6}(t_{1})\right)^{T} \\ -\left(\hat{\boldsymbol{\rho}}^{7}(t_{1}) - \hat{\boldsymbol{\rho}}^{6}(t_{1})\right)^{T} \\ \vdots \\ -\left(\hat{\boldsymbol{\rho}}^{24}(t_{1}) - \hat{\boldsymbol{\rho}}^{6}(t_{1})\right)^{T} \end{bmatrix} \mathbf{r} + \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_{2} DDN_{2}^{6,03} \\ \lambda_{2} DDN_{2}^{6,07} \\ \vdots \\ \lambda_{2} DDN_{2}^{6,24} \end{bmatrix}$$

$$\mathbf{y}_{1} = \mathbf{G}_{1} \mathbf{x}$$

$$\mathbf{y}_{1} = \mathbf{y}_{1} = \mathbf{G}_{1} \mathbf{x}$$

$$\mathbf{y}_{1} = \mathbf{G}_{1} \mathbf{x}$$

$$y_1 = G_1 x$$

y1:=y[t1]
G1:=G[t1]

$$\begin{bmatrix} DDL_{2}^{6,03}(t_{2}) \\ DDL_{2}^{6,07}(t_{2}) \\ \vdots \\ DDL_{2}^{6,24}(t_{2}) \end{bmatrix} = \begin{bmatrix} -(\hat{\boldsymbol{\rho}}^{3}(t_{2}) - \hat{\boldsymbol{\rho}}^{6}(t_{2}))^{T} \\ -(\hat{\boldsymbol{\rho}}^{7}(t_{2}) - \hat{\boldsymbol{\rho}}^{6}(t_{2}))^{T} \\ \vdots \\ -(\hat{\boldsymbol{\rho}}^{24}(t_{2}) - \hat{\boldsymbol{\rho}}^{6}(t_{2}))^{T} \end{bmatrix} \mathbf{r} + \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_{2} DDN_{2}^{6,03} \\ \lambda_{2} DDN_{2}^{6,07} \\ \vdots \\ \lambda_{2} DDN_{2}^{6,24} \end{bmatrix}$$

$$\mathbf{y}_{2} = \mathbf{G}_{2} \mathbf{X}$$

$$\mathbf{y}_{2} = \mathbf{y}_{2} \mathbf$$

$$y_2 = G_2 X$$

 $y_2 = y[t_2]$
 $g_2 = g[t_2]$

In the previous sessions A and B we have not taken into account the correlations between the double differences of measurements. This matrix will be used now, as the LAMBDA method will be applied to FIX the carrier ambiguities.

- to $\mathbf{P_y} = 2\sigma^2 \begin{bmatrix} 2 & 1 & \cdots & 1 \\ 1 & 2 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & 2 \end{bmatrix}$ a) Show that the covariance matrix of DDL2 is given by P_v
- b) Given the measurement vectors (y) and Geometry matrices (G) for two epochs y1:=y[t1]; G1:=G[t1]; Py y2:=y[t2]; G2:=G[t2]; Py

show that the user solution and covariance matrix can be computed as:

$$\mathbf{y} = \mathbf{G} \mathbf{x}; \quad \mathbf{W} = \mathbf{P}_{\mathbf{y}}^{-1}$$

 $\mathbf{x} = (\mathbf{G}^{\mathrm{T}} \mathbf{W} \mathbf{G})^{-1} \mathbf{G}^{\mathrm{T}} \mathbf{W} \mathbf{y}$

$$\mathbf{y} = \mathbf{G} \mathbf{x}; \quad \mathbf{W} = \mathbf{P}_{\mathbf{y}}^{-1}$$

$$\mathbf{x} = \mathbf{G}^{\mathrm{T}} \mathbf{W} \mathbf{G}^{-1} \mathbf{G}^{\mathrm{T}} \mathbf{W} \mathbf{y}$$

$$\mathbf{x} = \mathbf{G}^{\mathrm{T}} \mathbf{W} \mathbf{G}^{-1} \mathbf{G}^{\mathrm{T}} \mathbf{W} \mathbf{y}$$
where: $\mathbf{W} = \mathbf{I} \mathbf{v} (\mathbf{G} + \mathbf{v} \mathbf{v})$

 $\mathbf{P} = (\mathbf{G}^{\mathrm{T}} \mathbf{W} \mathbf{G})^{-1}$

The script MakeL1Bs1Mat.scr builds the equations system

[DDL2]=[Los_k- Los_06]*[baseline] + [A]*[
$$\lambda_2$$
*DDN2]

for the two epochs required $t_1=18000$ and $t_2=18015$, using the input file DD UPC1 UPC2 06 ALL.dat generated before.

Execute:

MakeL2BslMat.scr DD UPC1 UPC2 06 ALL.dat 18000 18015

The **OUTPUT**

are the files M1.dat and M2.dat associated with each epoch.

Where:

the columns of files M.dat are the vector y (first column) and Matrix G (next columns)



1. Computing the FLOATED solution (solving the equations system).

The following procedure can be applied:

```
octave
load M1.dat
load M2.dat

y1=M1(:,1);
G1=M1(:,2:12);

y2=M2(:,1);
G2=M2(:,2:12);
Py=(diag(ones(1,8))+ones(8))*2e-4;
W=inv(Py);
```

```
P=inv(G1'*W*G1+G2'*W*G2);
x=P*(G1'*W*y1+G2'*W*y2);

x(1:3)'
-25.85097 -27.02162 3.15538
bsl_enu =[-27.4170 -26.2341 -0.0304]

x(1:3)'-bsl_enu
ans= 1.5660 -0.7875 3.18578
```

3. Applying the LAMBDA method to FIX the ambiguities.

Compare the results with the solution obtained by rounding the floated solution. The following procedure can be applied (justify the computations done)

```
octave
c=299792458;
f0=10.23e+6;
f2=120*f0;
lambda2=c/f2
  a=x(4:11)/lambda2;
Q=P(4:11,4:11);
```

1. Rounding directly the floated solution

```
round(a)'
-5 -13 6 7 -9 4 4 4
```

2.- Rounding the decorrelated floated solution

```
[Qz,Zt,Lz,Dz,az,iZ] = decorrel (Q,a);
afix=iZ*round(az);
3  2  9  -27  13  -32  -13  -35
```

3.- Decorrelation and integer LS search solution

```
[Qz,Zt,Lz,Dz,az,iZ] = decorrel (Q,a);
[azfixed,sqnorm] = lsearch (az,Lz,Dz,2);
afixed=iZ*azfixed;
sqnorm(2)/sqnorm(1)
ans = 1.00508811343751
afixed(:,1)'
-3 7 -6 13 -3 19 -3 24
```

Questions:

- 1. Can the ambiguities be well fixed?
- 2. Is the test resolutive?
- 3. Compare the fixed ambiguities with those obtained in the previous exercises when fixing the ambiguities one at a time. Are the same results found?
- 4. What is the elapsed time to needed fix the ambiguities? And in the previous exercise when fixing the ambiguities one at a time?
- 5. The values found for the ambiguities are the same than in the previous case?

Thanks for your attention

Acknowledgements

- To the University of Delft for the MATLAB files of LAMBDA method.
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