

Lecture 4 Satellite orbits and clocks

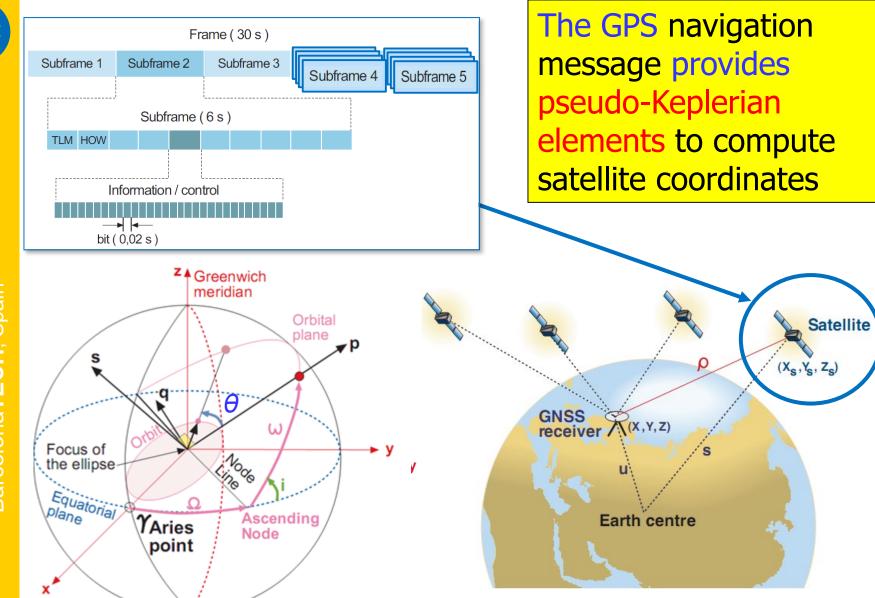
Professors: Dr. J. Sanz Subirana, Dr. J.M. Juan Zornoza and Dr. Adrià Rovira García

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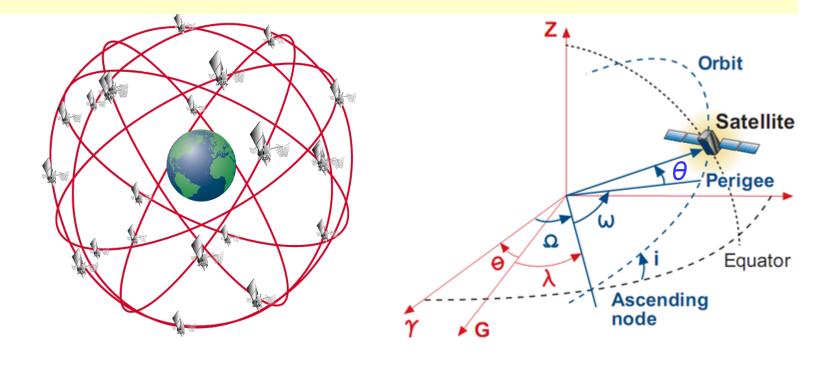
- 1. Elliptic orbit: Keplerian elements.
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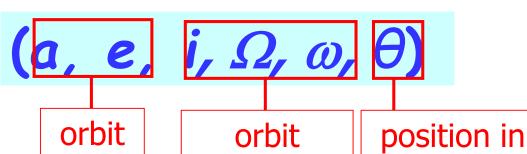




$(X, Y, Z, Vx, Vy, Vz) \rightarrow (a, e, i, \Omega, \omega, \theta)$

6 values are needed (X, Y, Z, Vx, Vy, Vz) to provide the position and velocity of a body. They can be mapped into the **six Keplerian elements** (a, e, i, Ω , ω , θ), which provides the "natural" representation of the orbit!



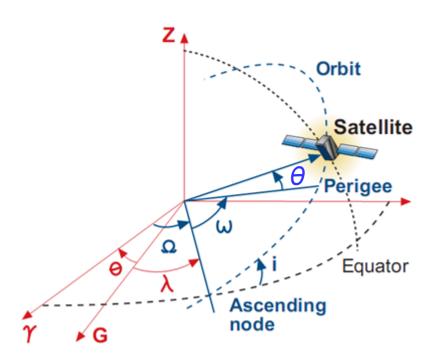


perigee

shape

orientation

the orbit



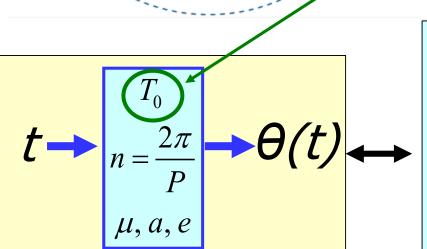
- inclination
- argument of perigee
- Ω arg. ascending node (Aries)
- arg. ascending node (Greenwich)
- true anomaly
- sidereal time
- vernal equinox
- Greenwich meridian

True anomaly $\theta(t)$

Fictitious body moving at velocity $n=2\pi/P=constant$

 \rightarrow Mean anomaly M(t)

 T_o : time of passage by satellite's perigee



b

Satellite

Earth

ae

Perigee

$$M(t) = n(t - T_0) = \sqrt{\frac{\mu}{a^3}}(t - T_0)$$

$$E(t) = M(t) + e \sin E(t)$$

$$\theta(t) = 2 \cdot atan \left[\sqrt{\frac{1+e}{1-e}} tan \left(\frac{E(t)}{2} \right) \right]$$

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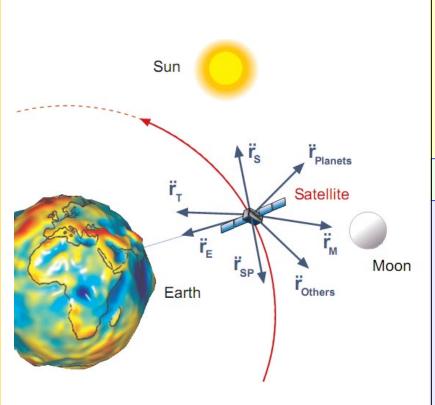


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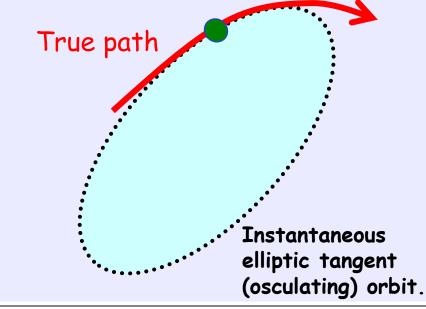


Due to the non-spherical nature of gravitational potential, the attraction of the Sun and Moon, the solar radiation pressure, etc., the true satellite path deviates from the elliptic orbit.



At any time an elliptical orbit tangent to the true path can be defined. This is the "osculating orbit", whose Keplerian elements vary with time "t":

a(t), e(t), i(t), $\Omega(t)$, $\omega(t)$, $\theta(t)$



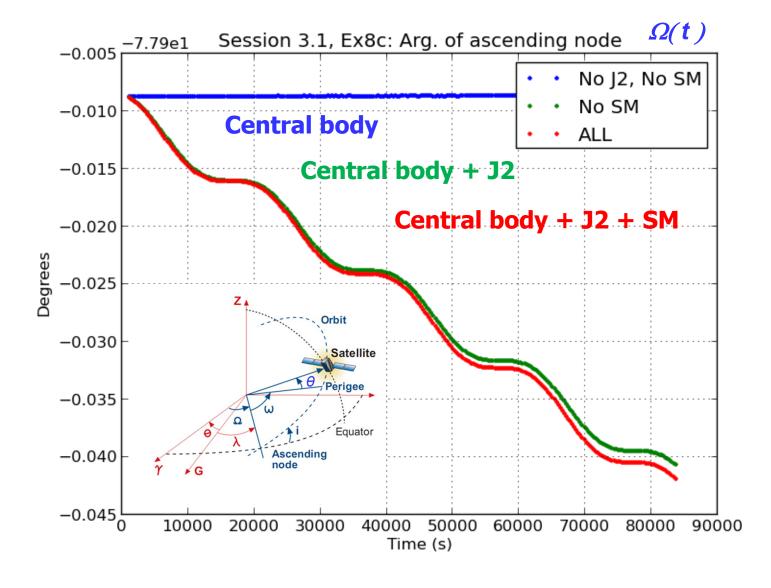


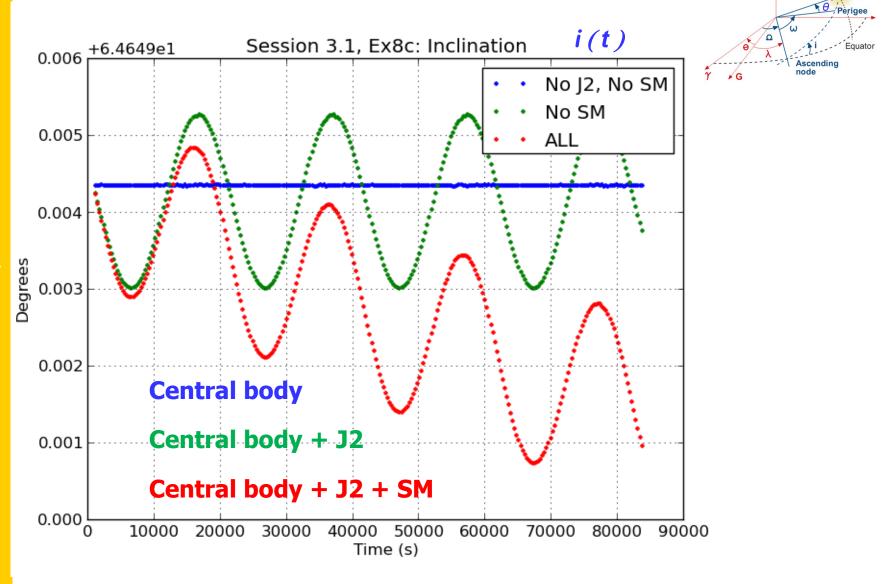
Different magnitudes of perturbation and their effects on GPS orbits

Perturbation	Acceleration	Orbital effect	
T CITUIDATION	(m/s ²)	in 3 hours	in 3 days
Central force	0.56		
(as a reference)			
J_2	$5 \cdot 10^{-5}$	2 km	14 km
Rest of the harmonics	$3 \cdot 10^{-7}$	50–80 m	100-1500 m
Solar + Moon grav.	$5 \cdot 10^{-6}$	5–150 m	1000–3000 m
Tidal effects	$1 \cdot 10^{-9}$	_	0.5–1.0 m
Solar rad. pressure	$1 \cdot 10^{-7}$	5–10 m	100–800 m

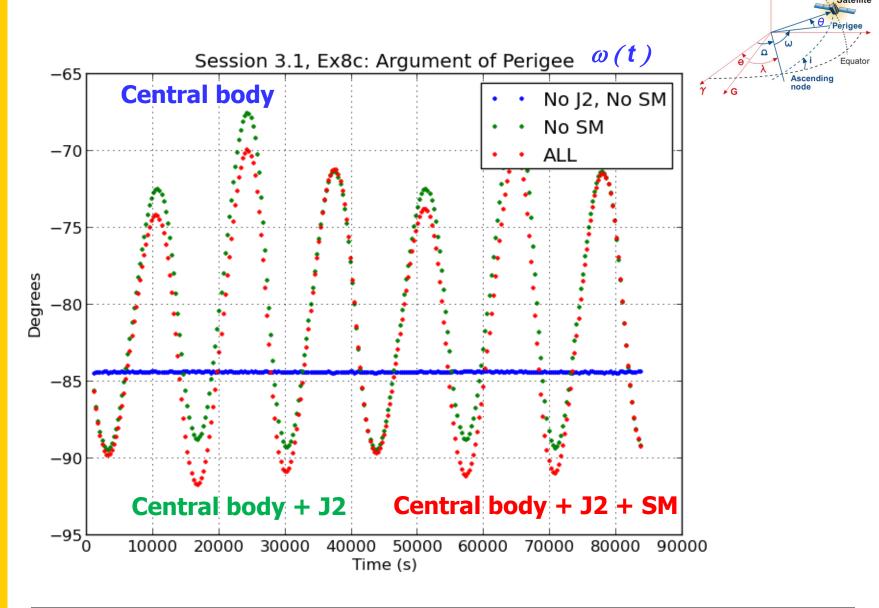
GLONASS Broadcast orbit integration terms

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Orbit



Orbit

Calculation of osculating orbital elements from position and velocity (rv2osc.f)

$$(x, y, z, v_x, v_y, v_z) \Rightarrow (a, e, i, \Omega, \omega, M)$$

$$c = r \times v \Rightarrow p = \frac{c^2}{\mu} \Rightarrow p$$

$$\frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2 a} \Rightarrow a$$

$$a = p \frac{1}{1 - e^2} \Rightarrow e$$

$$a = p \frac{1}{1 - a^2} \Rightarrow e$$

$$i = \arccos\left(\frac{c_z}{c}\right) \Rightarrow i$$

$$\Omega = \arctan\left(-\frac{c_x}{c_y}\right) \Rightarrow \Omega$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{R_3}(-\Omega)\mathbf{R_1}(-i)\mathbf{R_3}(-\omega) \begin{pmatrix} r\cos\theta \\ r\sin\theta \\ 0 \end{pmatrix} = r \begin{pmatrix} \cos\Omega\cdot\cos(\omega+\theta) - \sin\Omega\cdot\cos\mathrm{i}\cdot\sin(\omega+\theta) \\ \sin\Omega\cdot\cos(\omega+\theta) + \cos\Omega\cdot\cos\mathrm{i}\cdot\sin(\omega+\theta) \\ \sin\mathrm{i}\cdot\sin(\omega+\theta) \end{pmatrix} \Rightarrow (\omega+\theta)$$

$$r = \frac{p}{1 + e \cdot \cos \theta} \Rightarrow \omega, \theta$$

$$\tan\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E}{2}\right)$$

$$M = E - e \sin E \Rightarrow M$$



Calculation of position and velocity from osculating orbital elements (osc2rv.f)

$$(a, e, i, \Omega, \omega, t - T_0) \Rightarrow (x, y, z, v_x, v_y, v_z)$$

$$M = n(t - T_0) = \sqrt{\frac{\mu}{a^3}}(t - T_0) \Rightarrow M$$

$$M = E - e \sin E \implies E$$

$$r = a (1 - e \cos E) \Rightarrow r$$

$$\tan\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+e}{1-e}} \tan\left(\frac{E}{2}\right) \Rightarrow \theta$$

$$\begin{pmatrix} \mathbf{x} \\ \mathbf{y} \\ \mathbf{z} \end{pmatrix} = \mathbf{R} \begin{pmatrix} \mathbf{r} \cos \theta \\ \mathbf{r} \sin \theta \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{x} \\ \mathbf{y} \\ \mathbf{r} \sin \theta \end{pmatrix} = \mathbf{R} \begin{pmatrix} \mathbf{r} \cos \theta \\ \mathbf{r} \sin \theta \end{pmatrix} \qquad \begin{pmatrix} \mathbf{v}_{\mathbf{x}} \\ \mathbf{v}_{\mathbf{y}} \\ \mathbf{v}_{\mathbf{z}} \end{pmatrix} = \frac{na^2}{r} \{ \vec{\mathbf{Q}} \sqrt{1 - e^2} \cos E - \vec{\mathbf{P}} \sin E \}$$

Where:

$$\mathbf{R} = \mathbf{R}_3(-\Omega)\mathbf{R}_1(-i)\mathbf{R}_3(-\omega)$$

$$= \begin{pmatrix} \cos \Omega & -\sin \Omega & 0 \\ \sin \Omega & \cos \Omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos i & -\sin i \\ 0 & \sin i & \cos i \end{pmatrix} \begin{pmatrix} \cos \omega & -\sin \omega & 0 \\ \sin \omega & \cos \omega & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} P_x & Q_x & S_x \\ P_y & Q_y & S_y \\ P_z & Q_z & S_z \end{pmatrix} = \begin{bmatrix} \overrightarrow{\boldsymbol{P}} & \overrightarrow{\boldsymbol{Q}} & \overrightarrow{\boldsymbol{S}} \end{bmatrix}$$



Exercise: Orbital elements variation:

File 1995-10-18.eci contains the precise position and velocities of GPS satellites every 5 minutes for October 18th, 1995 in a Earth-Centred Inertial system (ECI) [from JPL/NASA server:

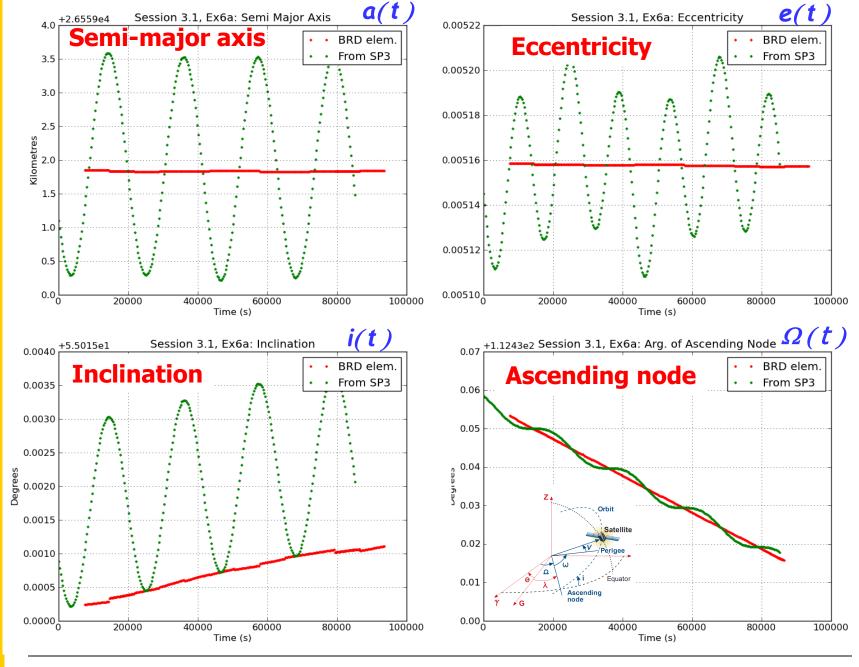
ftp://sideshow.jpl.nasa.gov/pub/gipsy_products

- a) Use program "rv2osc" to compute the instantaneous orbital eleme $(X, Y, Z, Vx, Vy, Vz) \rightarrow (a, e, i, \Omega, \omega, \theta)$
- b) Plot the orbital elements in function of time to show their variation: $a(t), e(t), i(t), \Omega(t), \omega(t), \theta(t)$
- c) Compare with the broadcast orbital elements

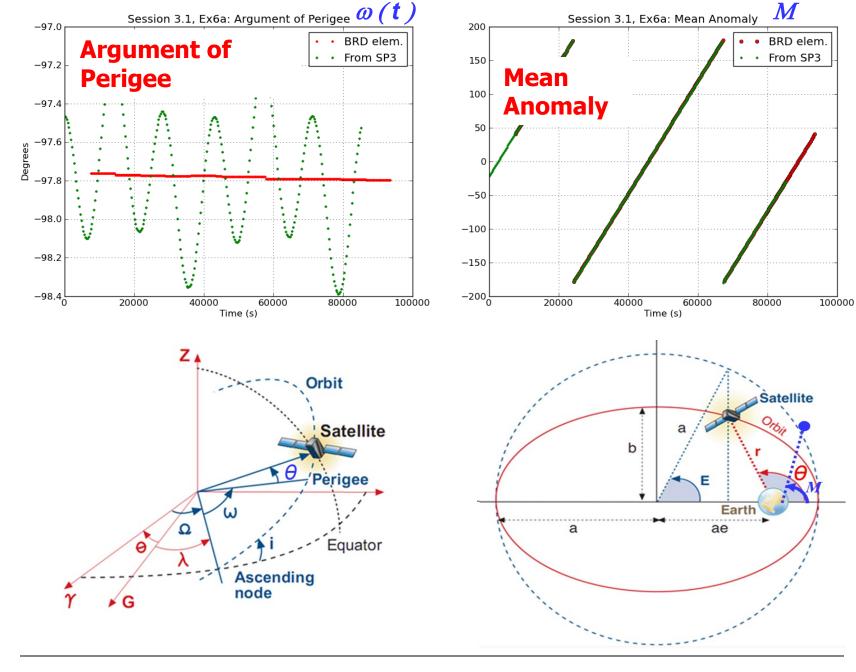
Solution:

- a) cat 1995-10-18.eci|rv2osc> orb.dat
- b) See the following plots









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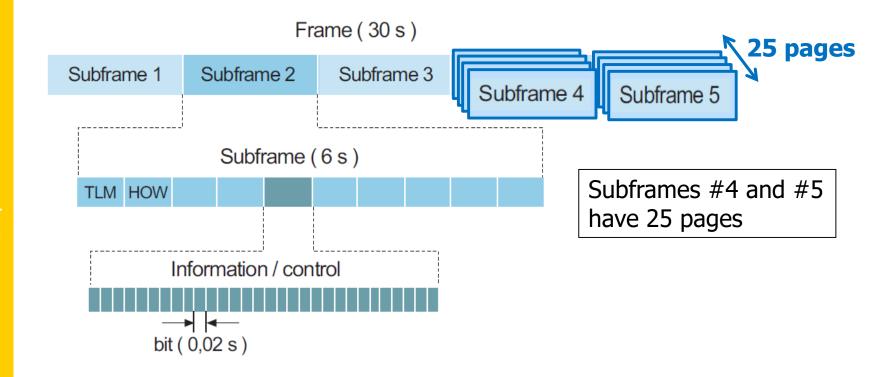


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GPS navigation message



One Master Frame includes All 25 pages of Subframes #4 and #5 \rightarrow 25 x 30s = 12.5 min

qAGE

Subframe 1 contains information about the parameters to be applied to **satellite clock** status for its correction. These values are polynomial coefficients that allow time onboard to be converted to GPS time. The subframe also contains information on satellite health condition.

Subframes 2 and 3 contain **satellite ephemerides**.

Subframe 4 provides **ionospheric model** parameters (in order to adjust for ionospheric refraction), UTC information, part of the almanac, and indications whether the A/S is activated or not (which transforms the P code into encrypted Y code).

Subframe 5 contains data from the **almanac** and on constellation status. It allows rapid identification of the satellite from which the signal comes. A total of 25 frames are needed to complete the almanac.

Ephemeris in navigation message

Parameter	Explanation
IODE	Series number of ephemerides data
t_{oe}	Ephemerides reference epoch
\sqrt{a}	Square root of semi-major axis
e	Eccentricity
M_o	Mean anomaly at reference epoch
ω	Argument of perigee
i_o	Inclination at reference epoch
Ω	Ascending node's right ascension
Δn	Mean motion difference
i	rate of inclination angle
$\overset{ullet}{\Omega}$	Rate of node's right ascension
c_{uc}, c_{us}	Latitude argument correction
c_{rc}, c_{rs}	Orbital radius correction
c_{ic}, c_{is}	Inclination correction

In order to calculate WGS84 satellite coordinates, you should apply de following algorithm [GPS/SPS-SS, table 2-15] (see in the book FORTRAN subroutine orbit.f)

RINEX ephemeris file

```
NAVIGATION DATA
                                         GPS
                                                              RINEX VERSION/ TYPE
XPRINT v1.1
                    gAGE
                                         00/08/17 09:31:37
                                                              PGM / RUN BY / DATE
gAGE BROADCAST EPHEMERIS FILE
                                                              COMMENT
   +1.7695E-08 +2.2352E-08 -1.1921E-07 -1.1921E-07
                                                              TON ALPHA
   +1.1878E+05 +1.4746E+05 -1.3107E+05 -3.2768E+05
                                                              ION BETA
                                                        1064 DELTA_UTC: AO,A1,T,W
   +1.955777406693E-08+1.598721155460E-14
                                             405504
    13
                                                              LEAP SECONDS
                                                              END OF HEADER
```

```
0 40.0+7.855705916882E-06+3.524291969370E-12+0.000000000000E+00
00
+1.01000000000E+02+6.50000000000E+01+5.456298524109E-09+5.530285585107E-01
+3.475695848465E-06+1.308503560722E-03+2.641230821609E-06+5.153678266525E+03
+2.088000000000E+05+1.117587089539E-08+7.472176136643E-01-1.862645149231E-09 T0E, \Omega
+9.412719852649E-01+3.163750000000E+02+1.125448382894E+00-8.826796182859E-09 io, \omega
+1.239337382719E-10+1.000000000000E+00+1.06400000000E+03+0.00000000000E+00
+4.0000000000E+00+0.0000000000E+00-4.190951585770E-09+6.13000000000E+02 TGD
0 0.0+1.636799424887E-06+0.00000000000E+00+0.0000000000E+00
+6.00000000000E+01+5.10000000000E+01+5.198073527168E-09-5.601816471398E-01
+2.635642886162E-06+6.763593177311E-03+2.468004822731E-06+5.153726325989E+03
+2.08800000000E+05+1.862645149231E-08+7.894129138508E-01+8.195638656616E-08
+9.487675576456E-01+3.229687500000E+02-2.409256713064E+00-8.734292400447E-09
+4.714481929846E-11+1.000000000000E+00+1.06400000000E+03+0.00000000000E+00
```



3.1. Computation of satellite coordinates from navigation message (orbit.f)

• Computation of t_k time since ephemerids reference epoch t_{oe} (t and t_{oe} are given in GPS seconds of week):

$$t_k = t - t_{oe}$$

• Computation of mean anomaly M_k for t_k ,

$$M_k = M_0 + \left(\frac{\sqrt{\mu}}{\sqrt{a^3}} + \Delta n\right) t_k$$

• Iterative resolution of Kepler's equation in order to compute eccentric anomaly E_k : $\boxed{M_k = E_k - e \sin E_k}$

• Calculation of true anomaly $\theta_{\mathbf{k}}$:

$$\theta_k = \arctan\left(\frac{\sqrt{1 - e^2} \sin E_K}{\cos E_k - e}\right)$$

• Computation of latitude argument u_k from perigee argument ω , true anomaly θ_k and corrections c_{uc} and c_{us} :

$$u_k = \omega + \theta_k + c_{uc}\cos 2(\omega + \theta_k) + c_{us}\sin 2(\omega + \theta_k)$$



• Computation of radial distance r_k taking into consideration corrections c_{rc} and c_{rs} :

$$r_k = a(1 - 2\cos E_k) + c_{rc}\cos 2(\omega + \theta_k) + c_{rs}\sin 2(\omega + \theta_k)$$

• Calculation of orbital plane inclination i_k from inclination i_o at reference epoch t_{oe} and corrections c_{ic} and c_{is} :

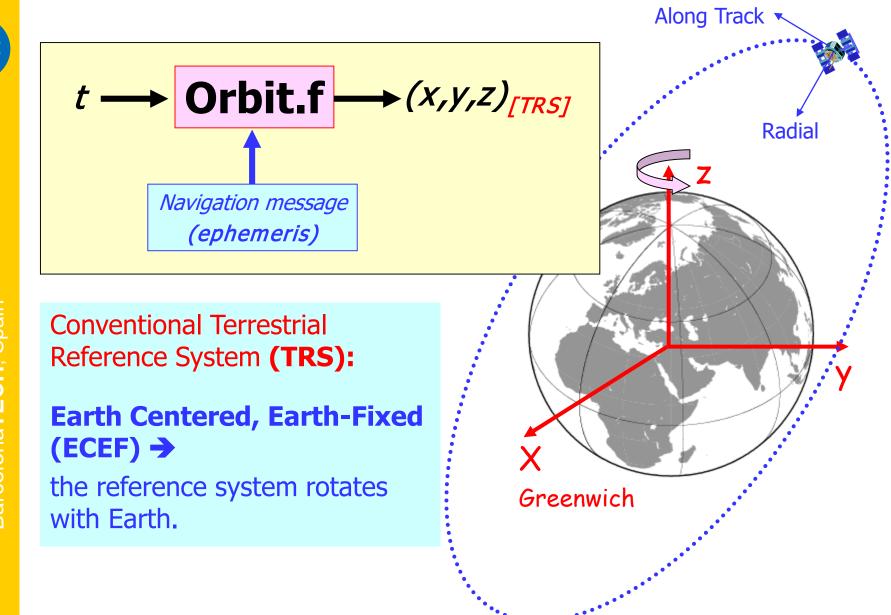
$$i_{k} = i_{0} + i \cdot t_{k} + c_{ic}\cos 2(\omega + \theta_{k}) + c_{is}\sin 2(\omega + \theta_{k})$$

• Computation of ascending node longitude λ_k (wrt Greenwich), from RAAN at start of GPS week Ω_{0} , corrected from apparent variation of sidereal time at Greenwich between start of week and reference time $t_k = t - t_{oe}$, and also corrected from change of ascending node longitude since reference epoch t_{oe} .

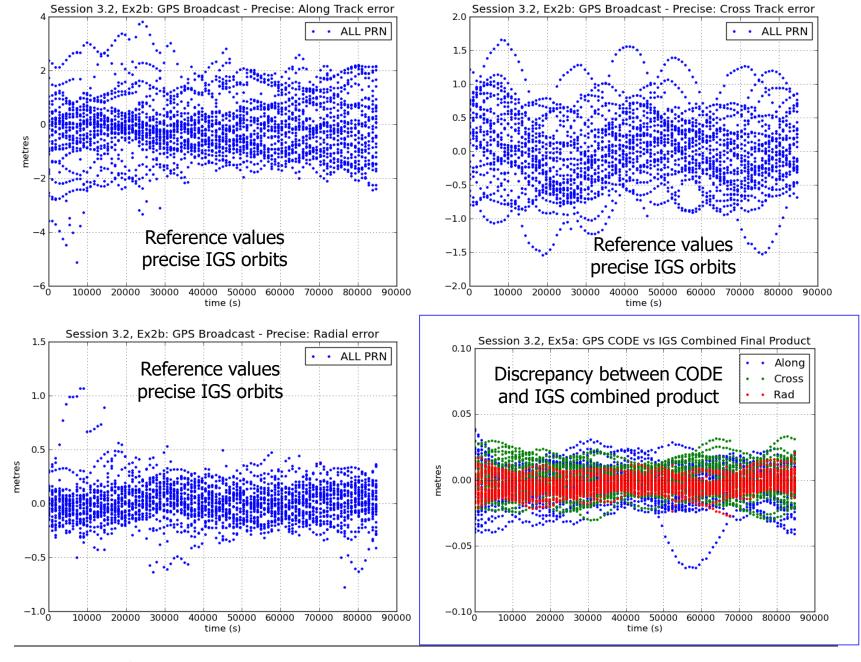
$$\lambda_k = \Omega_0 + (\dot{\Omega} - \omega_E)t_k - \omega_E t_{OE}$$

• Calculation of coordinates in Conventional Terrestrial System (CTS) applying three rotations (around u_k , i_k , λ_k):

$$\begin{bmatrix} X_k \\ Y_k \\ Z_k \end{bmatrix} = \mathbf{R_3}(-\lambda_k) \cdot \mathbf{R_1}(-i_k) \cdot \mathbf{R_3}(-u_k) \begin{bmatrix} r_k \\ 0 \\ 0 \end{bmatrix}$$







Antenna Phase Centre Correction has been applied



Session 3.2, Ex1c GPS Broadcast - Precise: Cross Track error [zoom] Session 3.2, Ex1c GPS Broadcast - Precise: ALong Track error [zoom] PRN02 PRN02 1.0 1.5 0.5 1.0 metres 0.0 0.5 -0.50.0 -1.0-1.5₀ -0.5^{L}_{0} 5000 10000 15000 20000 25000 15000 time (s) 30000 5000 10000 20000 25000 time (s) Zoom Session 3.2, Ex1c: GPS Broadcast - Precise: Radial error [zoon PRN02 -0.55-0.60-0.65metres -0.70 **Broadcast Orbit Updates** -0.75-0.80-0.85-0.90° 30000 5000 10000 15000 20000 25000 time (s)

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3.2 Computation of satellite coordinates from precise products.

Precise orbits for GPS satellites can be found on the International GNSS Service (IGS) server http://igscb.jpl.nasa.gov

Orbits are given by (x,y,z) coordinates with a sampling rate of 15 minutes. The satellite coordinates between epochs can be computed by polynomial **interpolation**. A 10th-order polynomial is enough for a centimetre level of accuracy with 15 min data.

$$P_{n}(x) = \sum_{i=1}^{n} y_{i} \frac{\prod_{j \neq i} (x - x_{j})}{\prod_{j \neq i} (x_{i} - x_{j})}$$

$$= y_{1} \frac{x - x_{2}}{x_{1} - x_{2}} \cdots \frac{x - x_{n}}{x_{1} - x_{n}} + \cdots$$

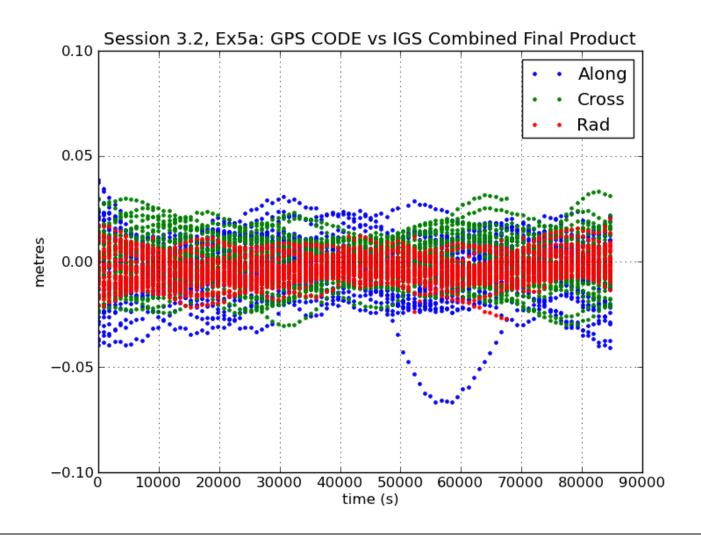
$$+ y_{i} \frac{x - x_{1}}{x_{i} - x_{1}} \cdots \frac{x - x_{i-1}}{x_{i} - x_{i-1}} \frac{x - x_{i+1}}{x_{i} - x_{i+1}} \cdots \frac{x - x_{n}}{x_{i} - x_{n}} + \cdots$$

$$+ y_{n} \frac{x - x_{1}}{x_{n} - x_{1}} \cdots \frac{x - x_{n-1}}{x_{n} - x_{n-1}}$$



IGS orbit and clock products (for PPP):

Discrepancy between CODE and IGS combined product.



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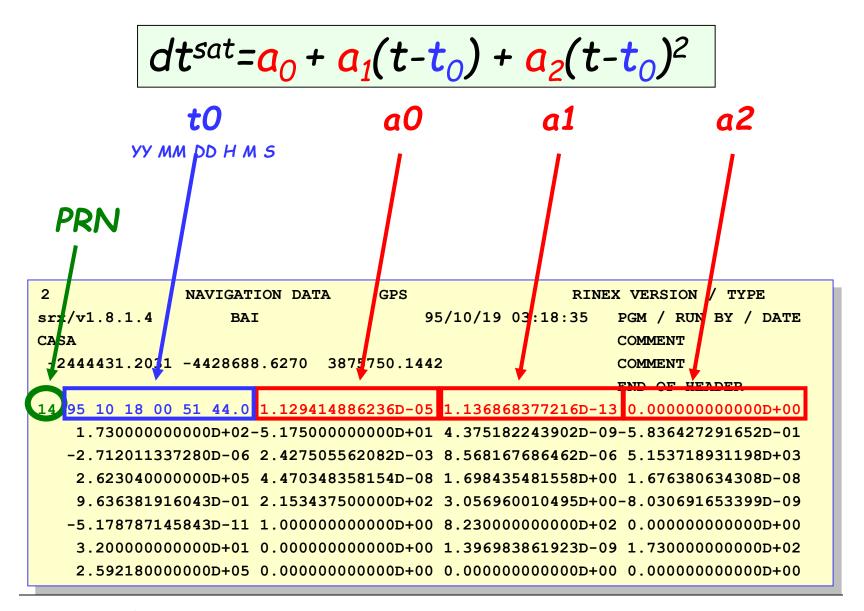
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GPS Satellite Clock computation: Broadcast message

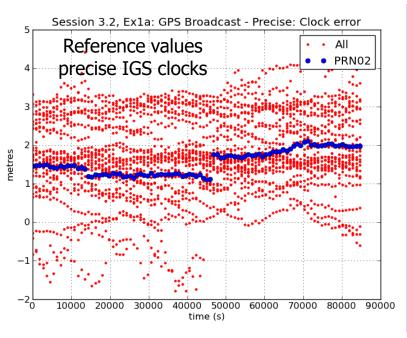


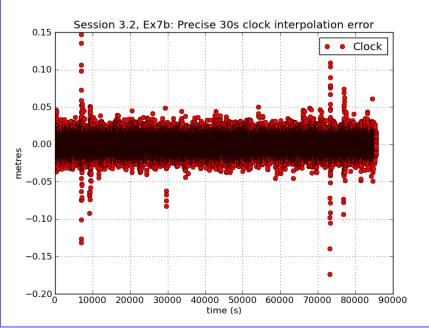
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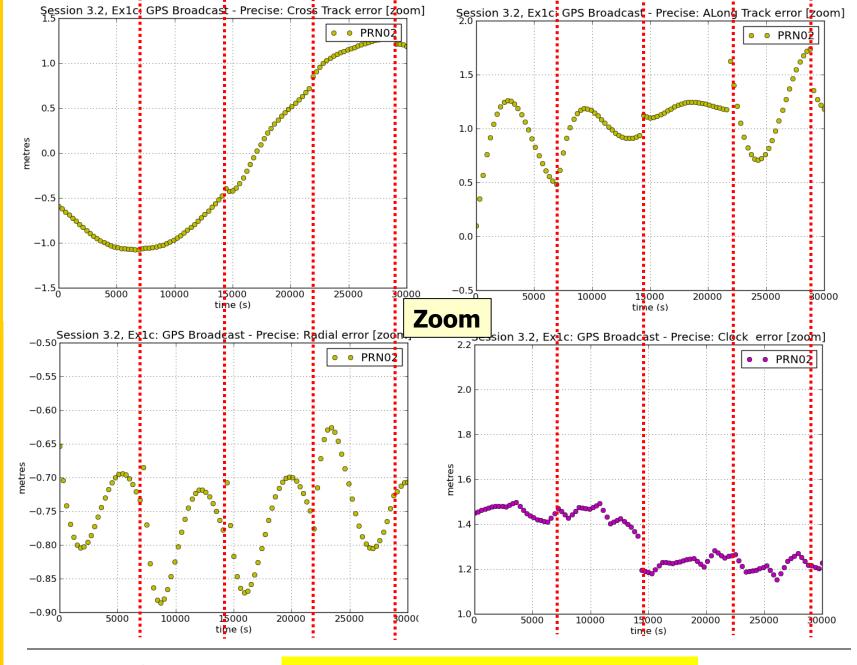
GPS Satellite Clock computation: Broadcast message

SA=off









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Computation of satellite clocks from precise products

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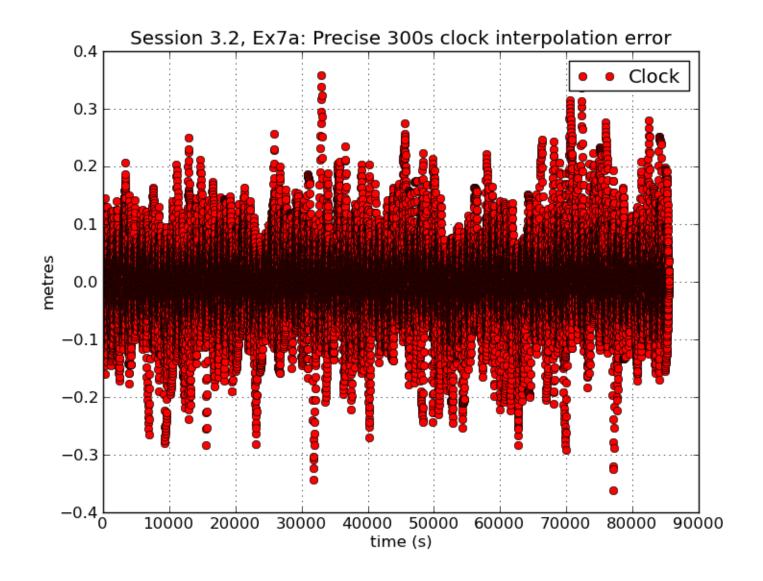
They are providing precise orbits and clock files with a sampling rate of 15 min (SP3 files), as well as precise clock files with a sample rate of 5 min and 30 s (CLK files).

Some centres also provide GPS satellite clocks with a 5 s sampling rate, like the les obtained from the Crustal Dynamics Data Information System (CDDIS) site.

Stable clocks with a sampling rate of 30 s or higher can be interpolated with a first-order polynomial to a few centimetres of accuracy. Clocks with a lower sampling rate should not be interpolated, because clocks evolve as random walk processes.

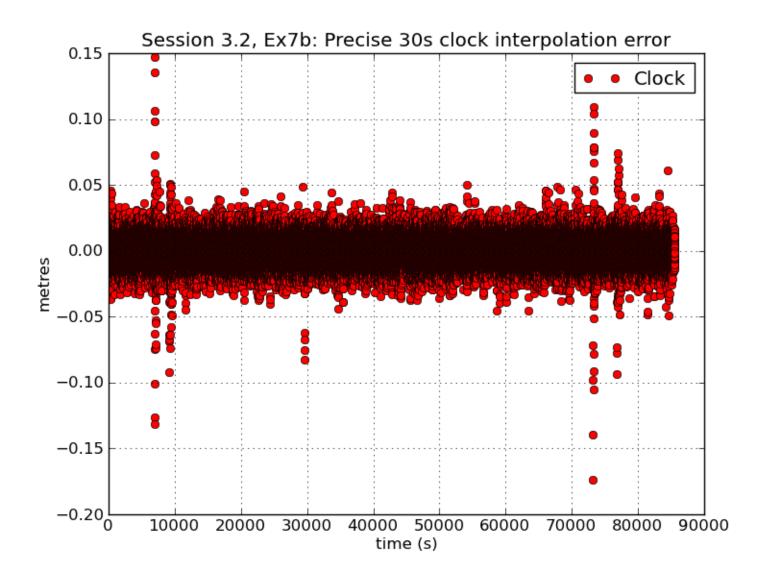


Precise Clock Interpolation: 300s samples





Precise Clock Interpolation: 30s samples



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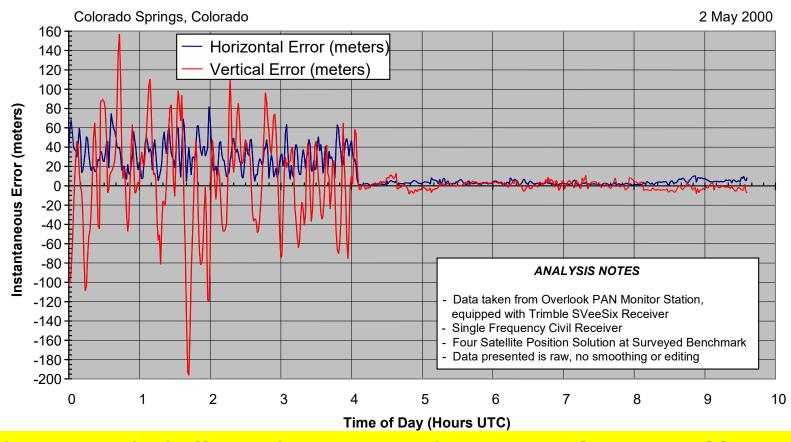
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Selective Availability (S/A): Intentional degradation of satellite clocks and broadcast ephemeris from 25/03/1990 to 02/05/2000.

GPS Before and After S/A was switched off

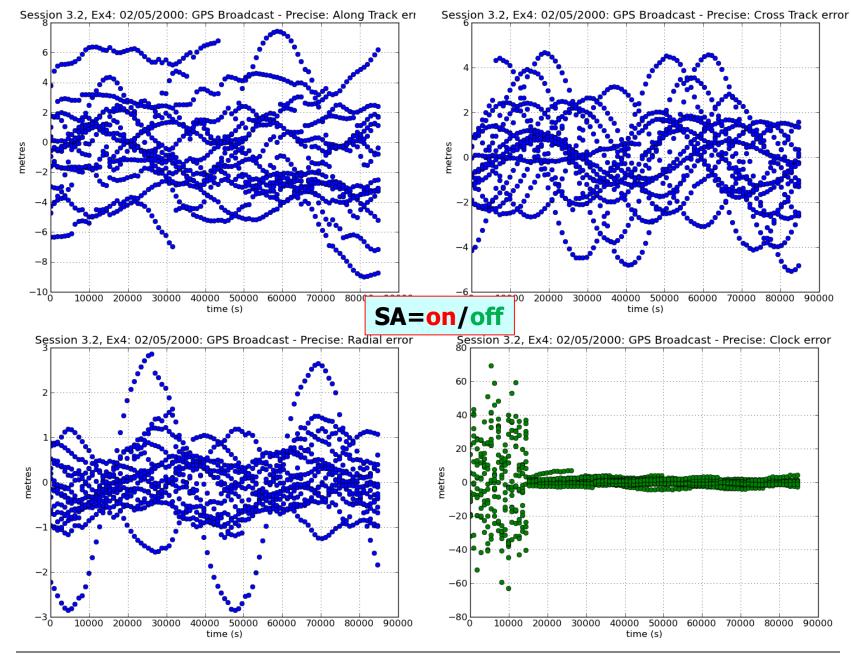


S/A was switched off at 2nd May 2000 and permanently removed in 2008

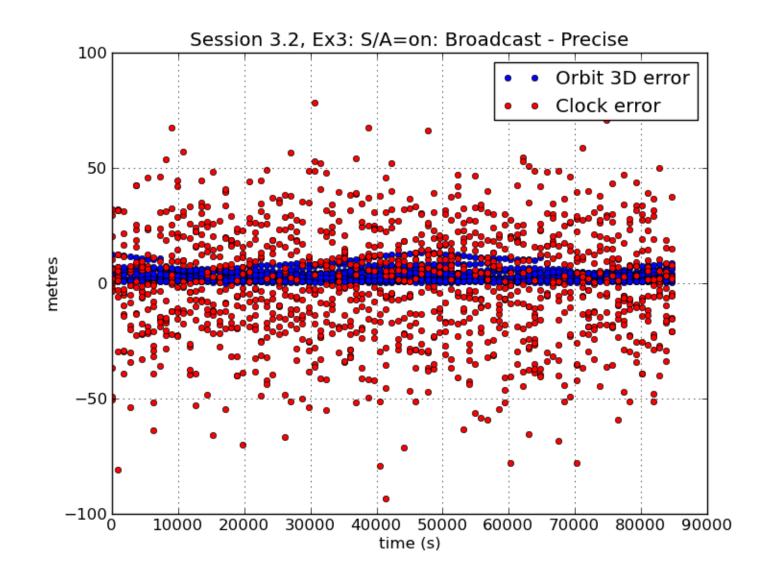
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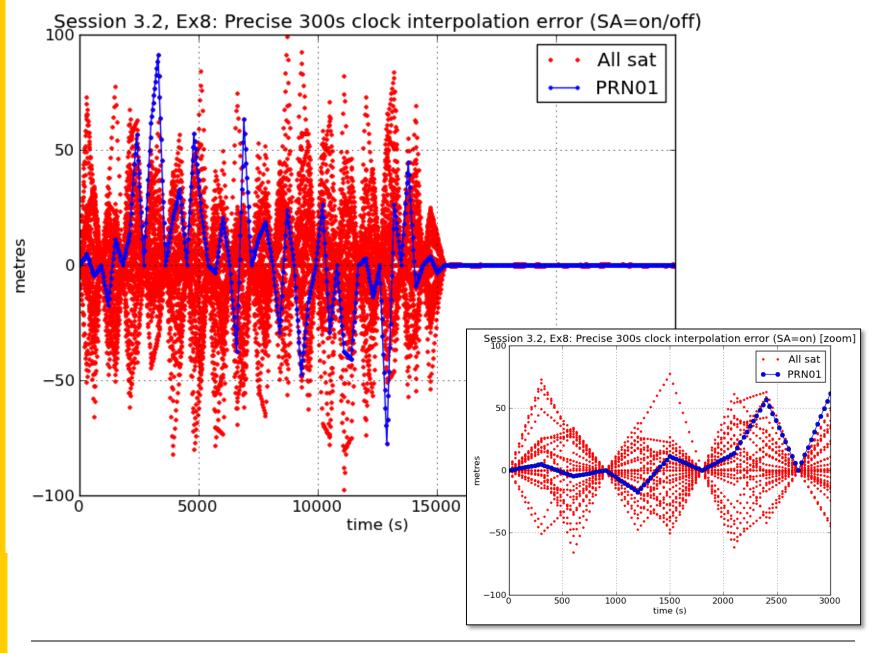














References

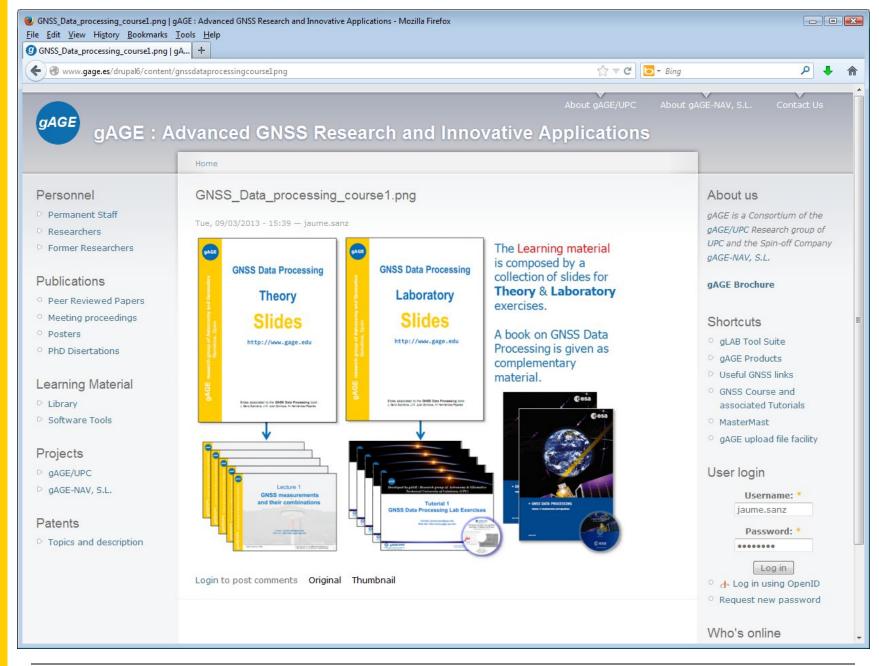
- [RD-1] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 1: Fundamentals and Algorithms. ESA TM-23/1. ESA Communications, 2013.
- [RD-2] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 2: Laboratory Exercises. ESA TM-23/2. ESA Communications, 2013.
- [RD-3] Pratap Misra, Per Enge. Global Positioning System. Signals, Measurements, and Performance. Ganga –Jamuna Press, 2004.
- [RD-4] B. Hofmann-Wellenhof et al. GPS, Theory and Practice. Springer-Verlag. Wien, New York, 1994.

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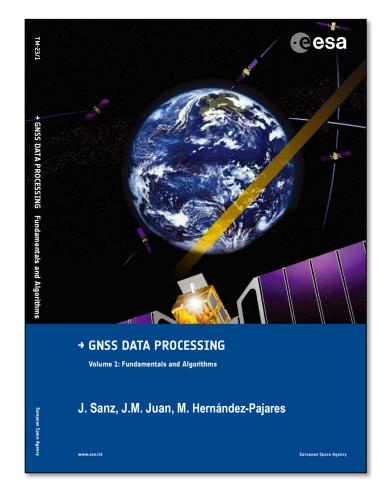


Thank you











GNSS Data Processing, Vol. 1: Fundamentals and Algorithms. GNSS Data Processing, Vol. 2: Laboratory exercises.