

Lecture 5

Code pseudorange modelling

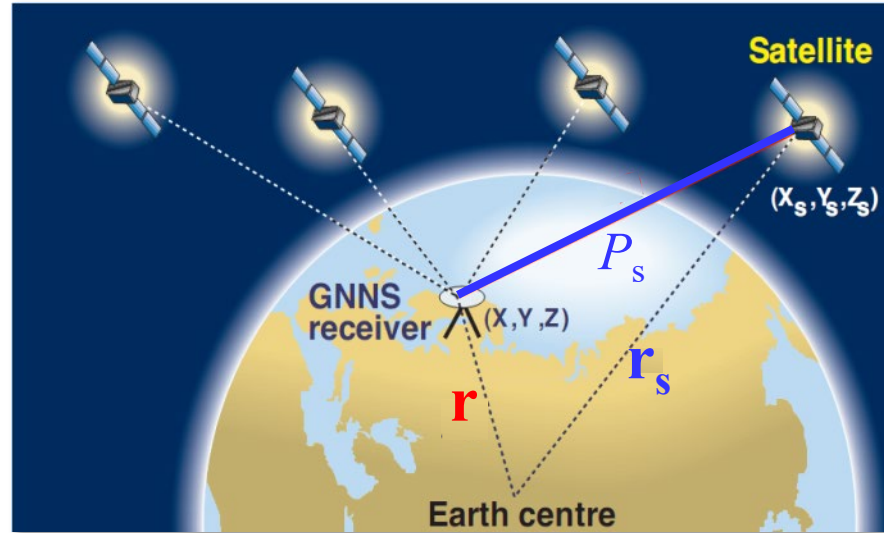
Professors: Dr. J. Sanz Subirana, Dr. J.M. Juan Zornoza
and Dr. Adrià Rovira García

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Measurements modelling and error sources

1. Introduction: Linear model and Prefit-residual
2. Code measurements modelling
3. Example of computation of modelled pseudorange

Introduction: Linear model and Prefit-residuals



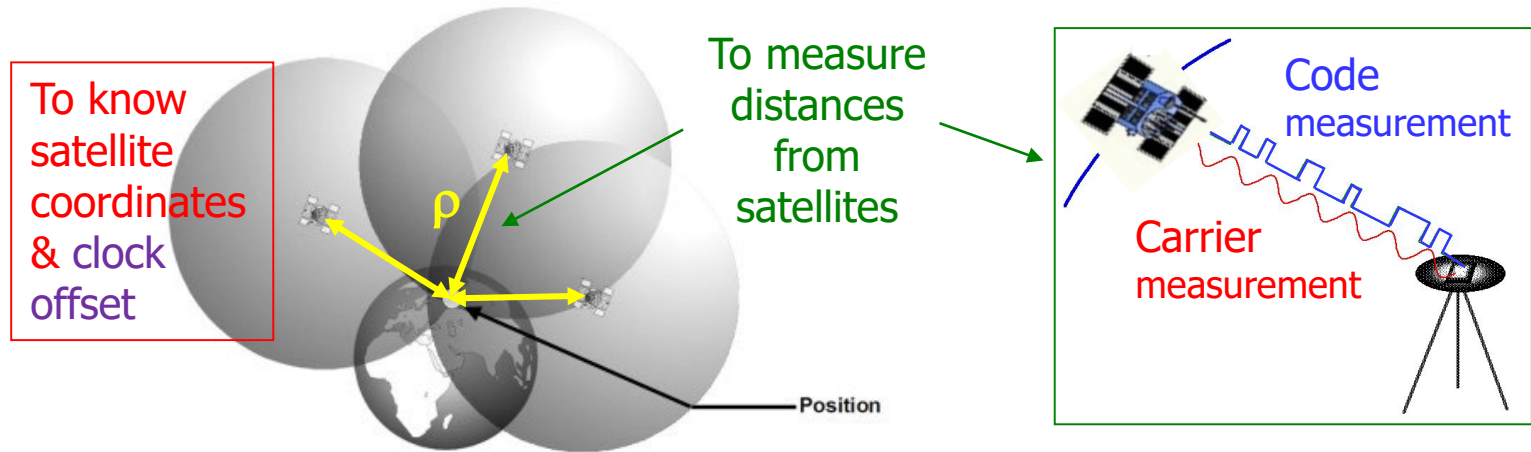
Input:

- **Pseudoranges** (receiver-satellite j): P_s
- **Navigation message**. In particular:
 - **Satellites position** when transmitting signal: $\mathbf{r}_s = (x_s, y_s, z_s)$
 - **Offsets of satellite clocks**: dt_s
(satellites = 1, 2, ..., n) ($n \geq 4$)

Unknowns:

- **Receiver position**: $\mathbf{r} = (x, y, z)$
- **Receiver clock offset**: dT

GNSS positioning concept

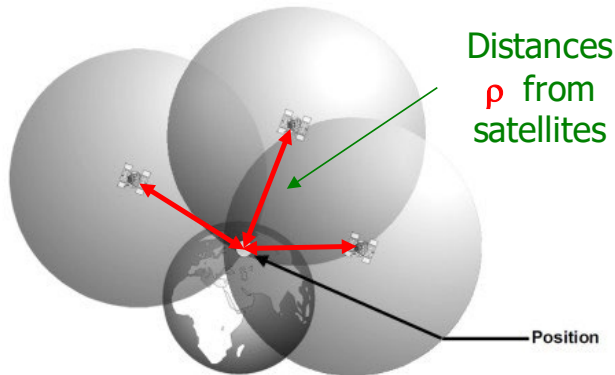


This picture is from <https://gpsfleettrackingexpert.wordpress.com>

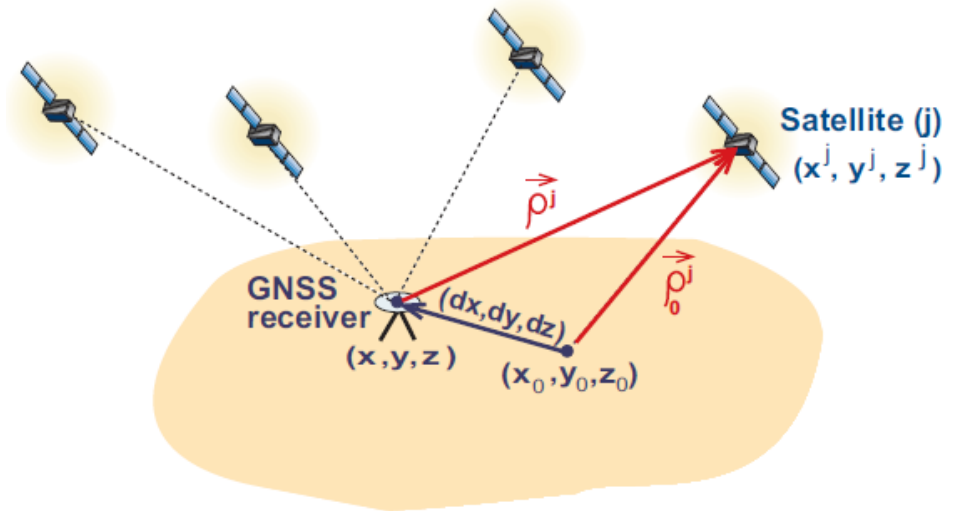
- GNSS uses technique of “**triangulation**” to find user location
- To “**triangulate**” a GNSS receiver needs:
 - **To know the satellite coordinates** and clock synchronism errors:
 ➔ Satellites broadcast orbits parameters and clock offsets.
 - **To measure distances from satellites**:
 ➔ This is done measuring the **traveling time** of radio signals:
 (“Pseudo-ranges”: **Code** and **Carrier** measurements)
 - ➔ Measurements must be corrected by several error sources:
 Atmospheric propagation, relativity, clock offsets, instrumental delays...

$$C1_{rec}^{sat} = \rho_{rec}^{sat} + c \cdot (dt_{rec} - dt^{sat}) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + K_{1rec} + TGD^{sat} + \varepsilon_1$$

Figure 6.1: Geometric concept of GNSS positioning: Equations



This picture is from <https://gpsfleettrackingexpert.wordpress.com>



Then, linearising the satellite–receiver geometric range

$$\rho^j(x, y, z) = \sqrt{(x^j - x)^2 + (y^j - y)^2 + (z^j - z)^2}$$

gives, for the approximate solution $\mathbf{r}_0 = (x_0, y_0, z_0)$,

$$\rho^j = \boxed{\rho_0^j} + \frac{x_0 - x^j}{\rho_0^j} dx + \frac{y_0 - y^j}{\rho_0^j} dy + \frac{z_0 - z^j}{\rho_0^j} dz$$

with $dx = x - x_0$, $dy = y - y_0$, $dz = z - z_0$

$$C1_{rec}^{sat}[\text{modelled}] = \boxed{\rho_{rec,0}^{sat}} - c \left(d\bar{t}^{sat} + \Delta rel^{sat} \right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

For each satellite in view

Iono+Tropo+TGD...

$$C1_{rec}^{sat} = \rho_{rec}^{sat} + c \cdot (dt_{rec} - dt^{sat}) + \sum \delta_k + \varepsilon$$

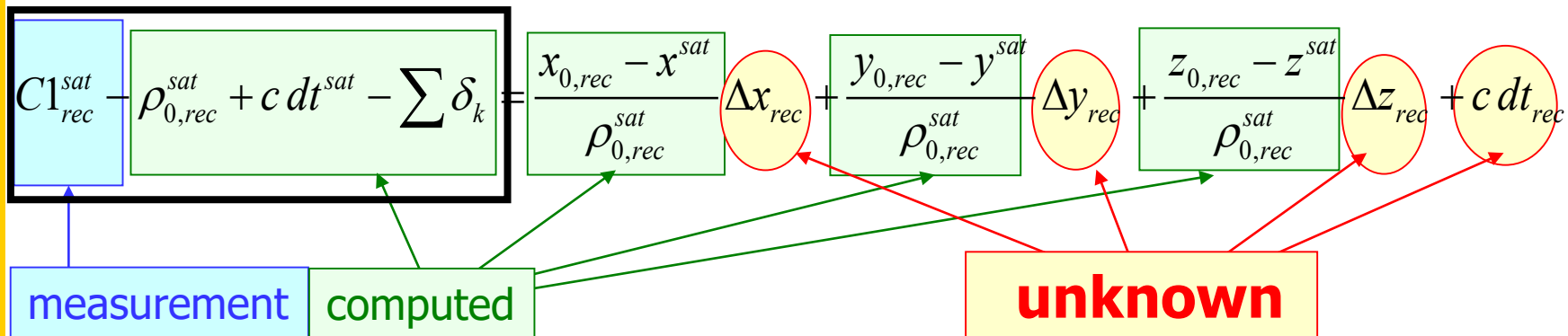
Linearising ρ around an 'a priori' receiver position $(x_{0,rec}, y_{0,rec}, z_{0,rec})$

$$= \rho_{0,rec}^{sat} + \frac{x_{0,rec} - x^{sat}}{\rho_{0,rec}^{sat}} \Delta x_{rec} + \frac{y_{0,rec} - y^{sat}}{\rho_{0,rec}^{sat}} \Delta y_{rec} + \frac{z_{0,rec} - z^{sat}}{\rho_{0,rec}^{sat}} \Delta z_{rec} + c(dt_{rec} - dt^{sat}) + \sum \delta_k$$

where:

$$\Delta x_{rec} = x_{rec} - x_{0,rec} \quad ; \quad \Delta y_{rec} = y_{rec} - y_{0,rec} \quad ; \quad \Delta z_{rec} = z_{rec} - z_{0,rec}$$

Prefit-residuals (Prefit)



$$\rho_{0,rec}^{sat} = \sqrt{\left(x^{sat} - x_{0,rec}\right)^2 + \left(y^{sat} - y_{0,rec}\right)^2 + \left(z^{sat} - z_{0,rec}\right)^2}$$

Of course, receiver coordinates $(x_{rec}, y_{rec}, z_{rec})$ are not known (they are the target of this problem). But, we can always assume that an “approximate position $(x_{0,rec}, y_{0,rec}, z_{0,rec})$ is known”.

Thence, the navigation problem will consist on:

- 1.- To start from an approximate value for the receiver position $(x_{0,rec}, y_{0,rec}, z_{0,rec})$ e.g. the Earth's centre to linearise the equations.
- 2.- With the pseudorange measurements and the navigation equations, compute the correction $(\Delta x_{rec}, \Delta y_{rec}, \Delta z_{rec})$ to have improved estimates: $(x_{rec}, y_{rec}, z_{rec}) = (x_{0,rec}, y_{0,rec}, z_{0,rec}) + (\Delta x_{rec}, \Delta y_{rec}, \Delta z_{rec})$
- 3.- Linearise the equations again, about the new improved estimates, and iterate until the change in the solution estimates is sufficiently small.

The estimates converges quickly. Generally in two to four iterations, even if starting from the Earth's centre.

For all satellites in view

$$\begin{bmatrix} Prefit^1 \\ Prefit^2 \\ \dots\dots \\ Prefit^n \end{bmatrix}$$

$$= \begin{bmatrix} \frac{x_{0,rec} - x^{sat1}}{\rho_{0,rec}^{sat1}} & \frac{y_{0,rec} - y^{sat1}}{\rho_{0,rec}^{sat1}} & \frac{z_{0,rec} - z^{sat1}}{\rho_{0,rec}^{sat1}} & 1 \\ \frac{x_{0,rec} - x^{sat2}}{\rho_{0,rec}^{sat2}} & \frac{y_{0,rec} - y^{sat2}}{\rho_{0,rec}^{sat2}} & \frac{z_{0,rec} - z^{sat2}}{\rho_{0,rec}^{sat2}} & 1 \\ \dots\dots\dots & \dots\dots\dots & \dots\dots\dots & \dots\dots\dots \\ \frac{x_{0,rec} - x^{satn}}{\rho_{0,rec}^{satn}} & \frac{y_{0,rec} - y^{satn}}{\rho_{0,rec}^{satn}} & \frac{z_{0,rec} - z^{satn}}{\rho_{0,rec}^{satn}} & 1 \end{bmatrix}$$

$$\begin{bmatrix} \Delta x_{rec} \\ \Delta y_{rec} \\ \Delta z_{rec} \\ c dt_{rec} \end{bmatrix}$$

Observations
(measured-modelled)

Unknowns

Measurements modelling:

Prefit residual is the difference between measured and modeled pseudorange:

$$Prefit_{rec}^{sat} = C1_{rec}^{sat} [\text{measured}] - C1_{rec}^{sat} [\text{modelled}]$$

where:

$$C1_{rec}^{sat} [\text{modelled}] = \rho_{rec,0}^{sat} - c \left(\boxed{d\bar{t}^{sat} + \Delta rel^{sat}} \right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

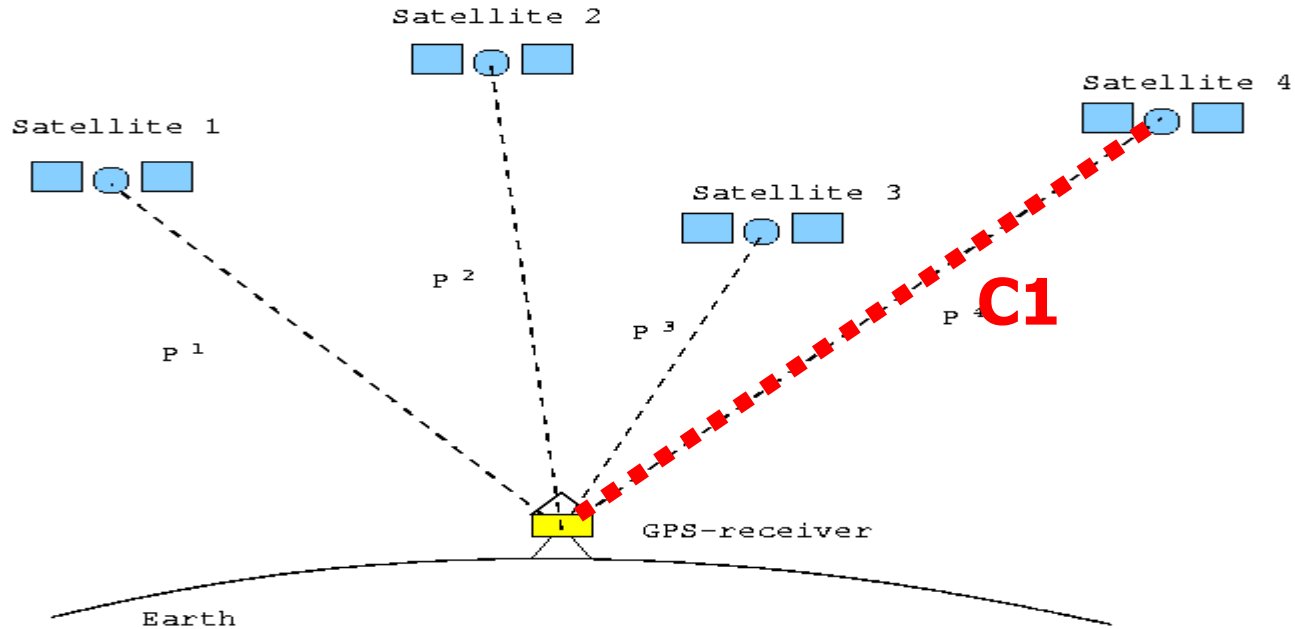
$$\boxed{dt^{sat}}$$

Contents

Measurements modelling and error sources

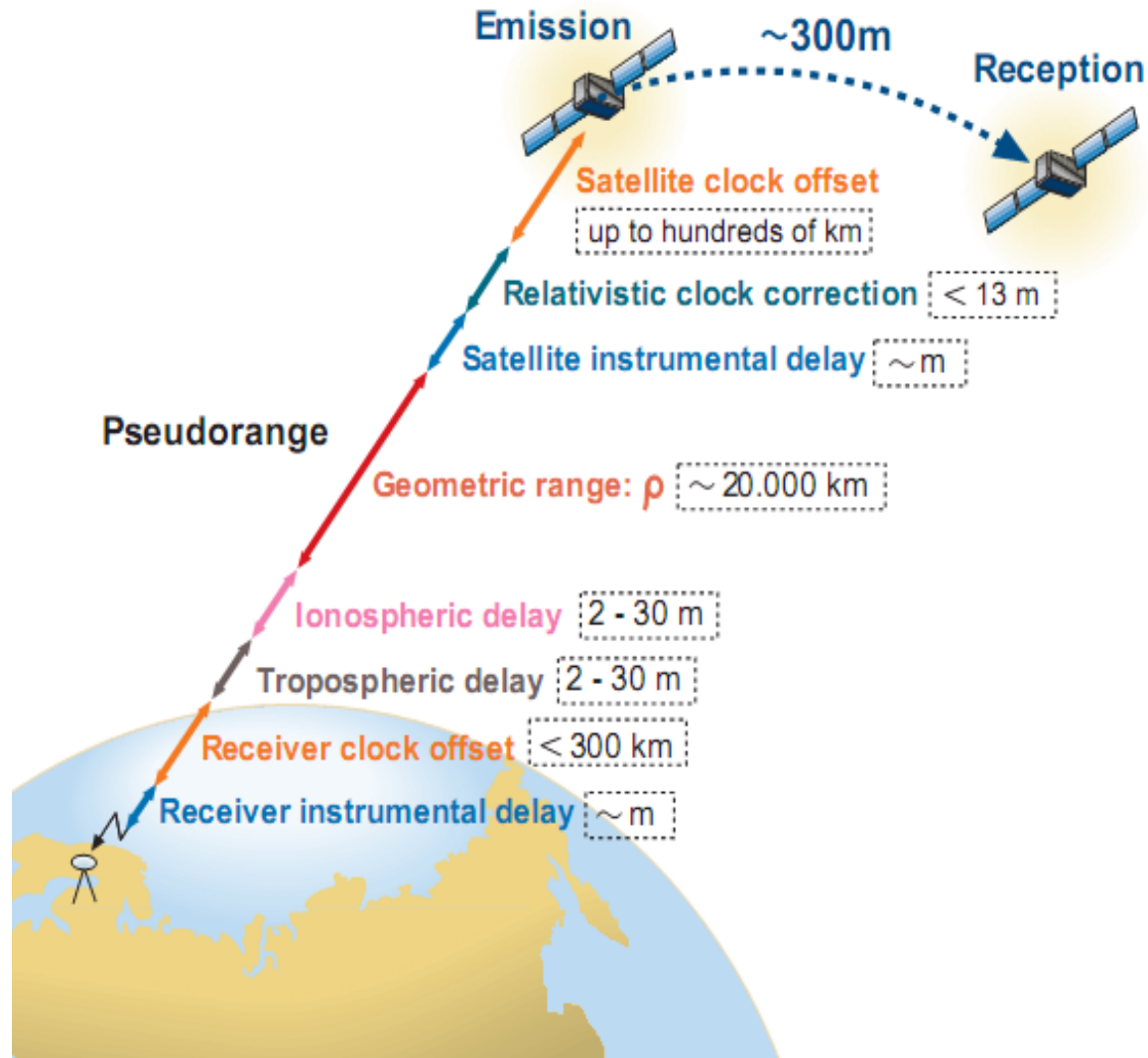
1. Introduction: Linear model and Prefit-residual
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Code Pseudorange modeling



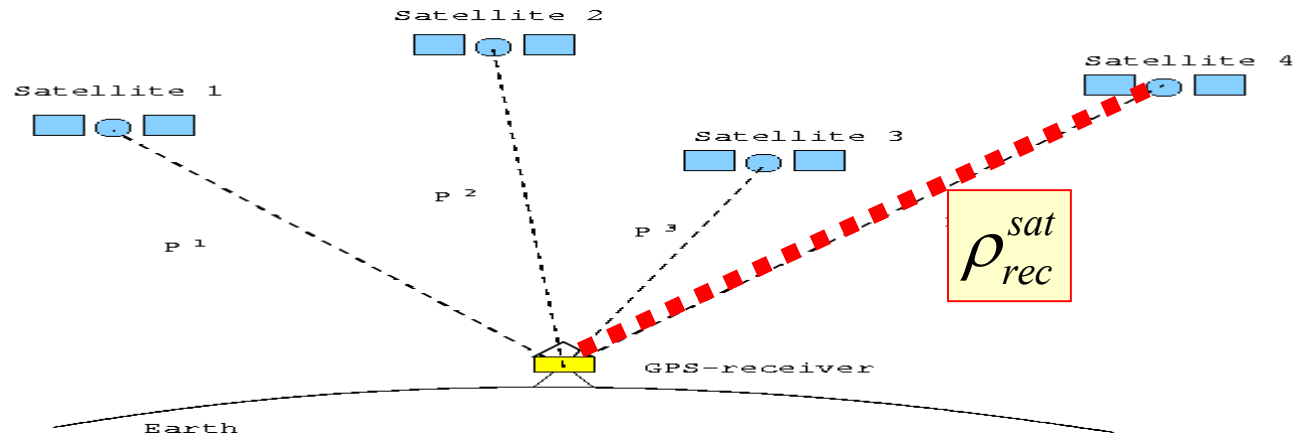
The pseudorange modeling is based in the GPS Standard Positioning Service Signal Specification (GPS/SPS-SS).

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{rec,0}^{sat} - c \left(d\bar{t}^{sat} + \Delta rel^{sat} \right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$



$$C1_{rec}^{sat}[\text{modelled}] = \rho_{rec,0}^{sat} - c \left(d\bar{t}^{sat} + \Delta rel^{sat} \right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

Geometric range



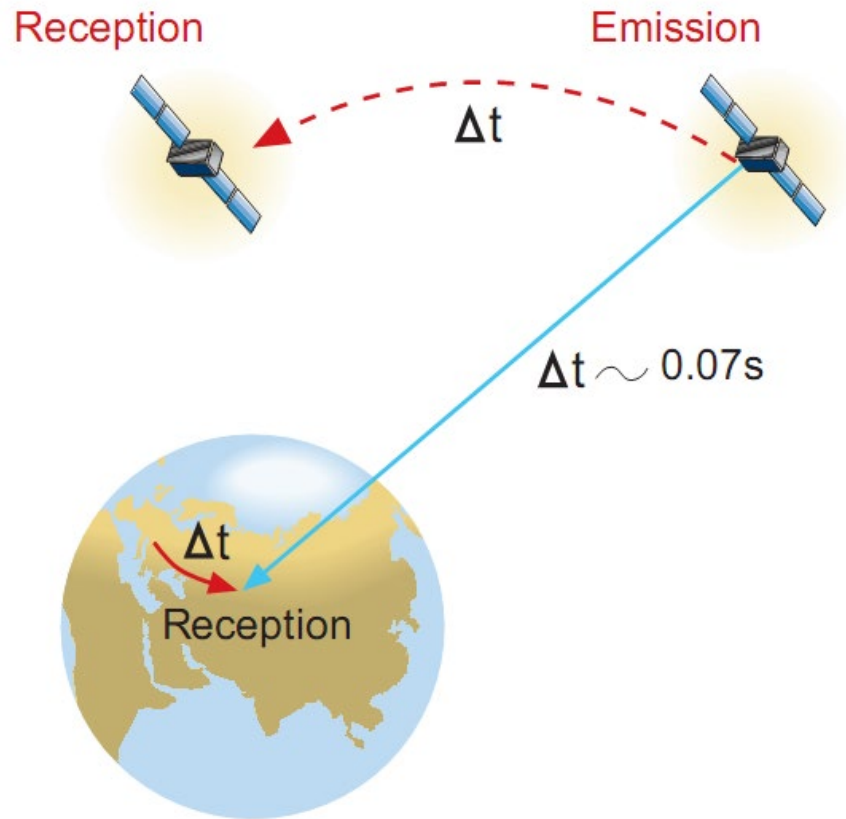
Euclidean distance between satellite coordinates at emission time and receiver coordinates at reception time.

$$\rho_{0,rec}^{sat} = \sqrt{\left(x^{sat} - x_{0,rec}\right)^2 + \left(y^{sat} - y_{0,rec}\right)^2 + \left(z^{sat} - z_{0,rec}\right)^2}$$

Of course, receiver coordinates are not known (is the target of this problem). But

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{rec,0}^{sat} - c \left(d\bar{t}^{sat} + \Delta rel^{sat} \right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

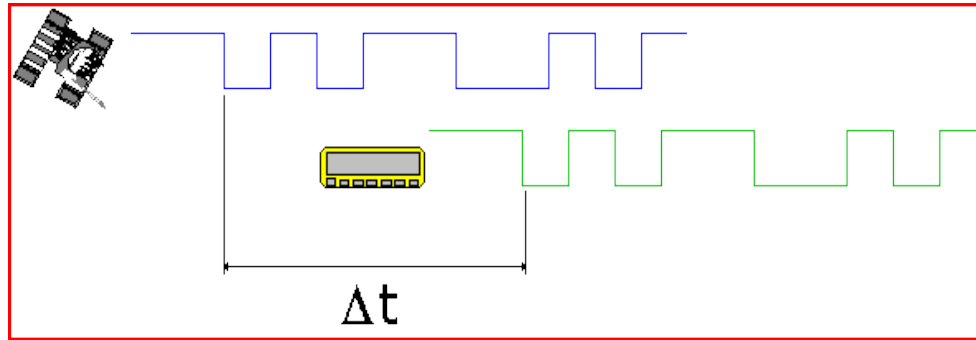
Satellite coordinates **at emission time** (rec2ems.f)



- The GPS signal travels from **satellite coordinates at emission time** (T_{emis}) to **receiver coordinates at reception time** (T_{recep}).
- The satellite can move several hundreds of meters from T_{emis} to T_{recep} .

The receiver time-tags are given at reception time and in the receiver clock time.

An algorithm is needed to compute the satellite coordinates at **emission time** "in the GPS system time" from **reception time** in the receiver time tags.



The satellite clock offset dt^S can be computed from the navigation message

$$C1 = c \Delta t = c [t_R(T_{recep}) - t^S(T_{emis})]$$

Pseudorange measurements link the “emission time (T_{emis})” in satellite clock (t^S) with reception time (T_{recep}) in receiver clock (t_R) (receiver time tags).

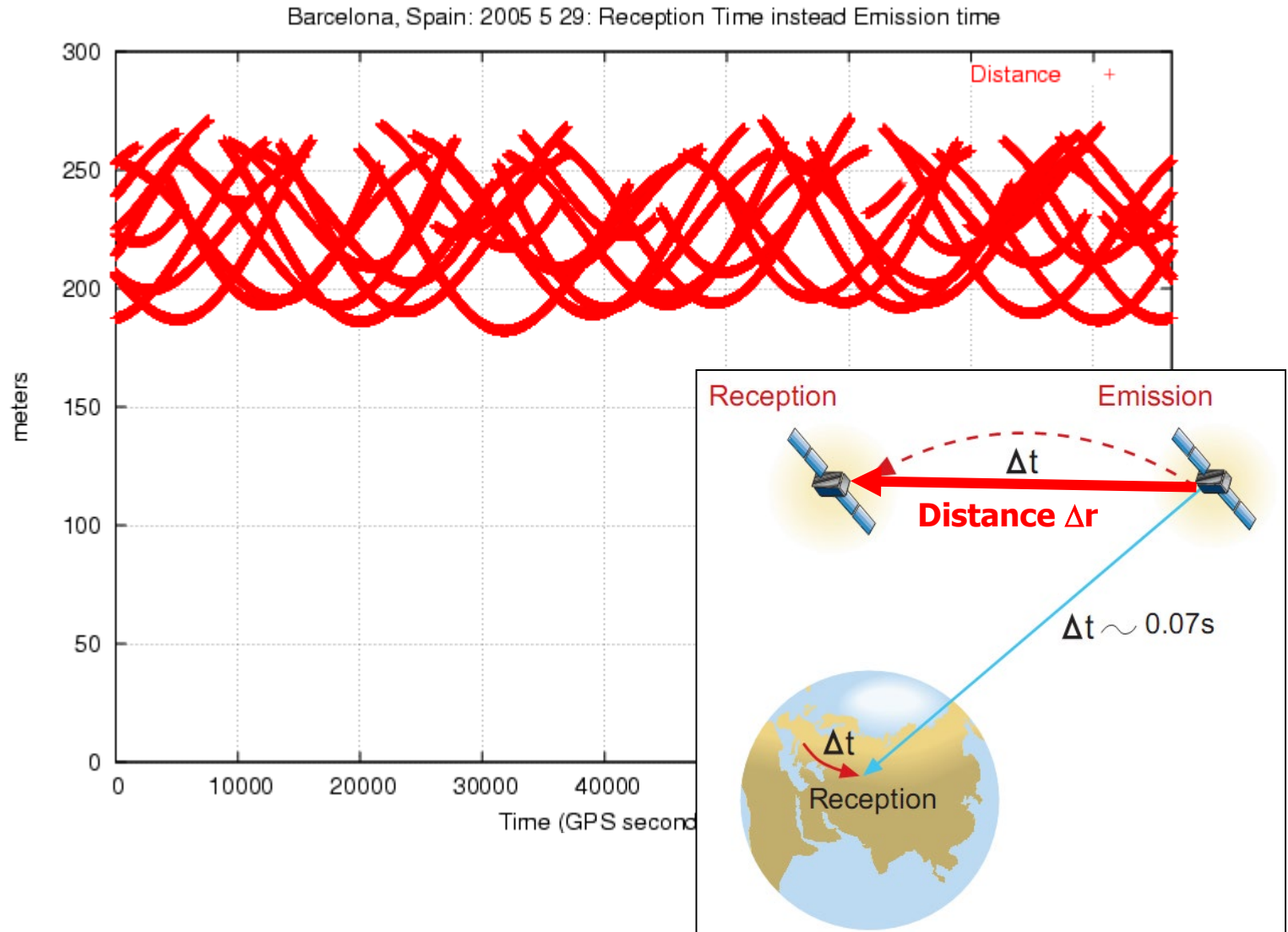
Thence, the emission time in the satellite clock is:

$$t^S(T_{emis}) = t_R(T_{recep}) - C1/c$$

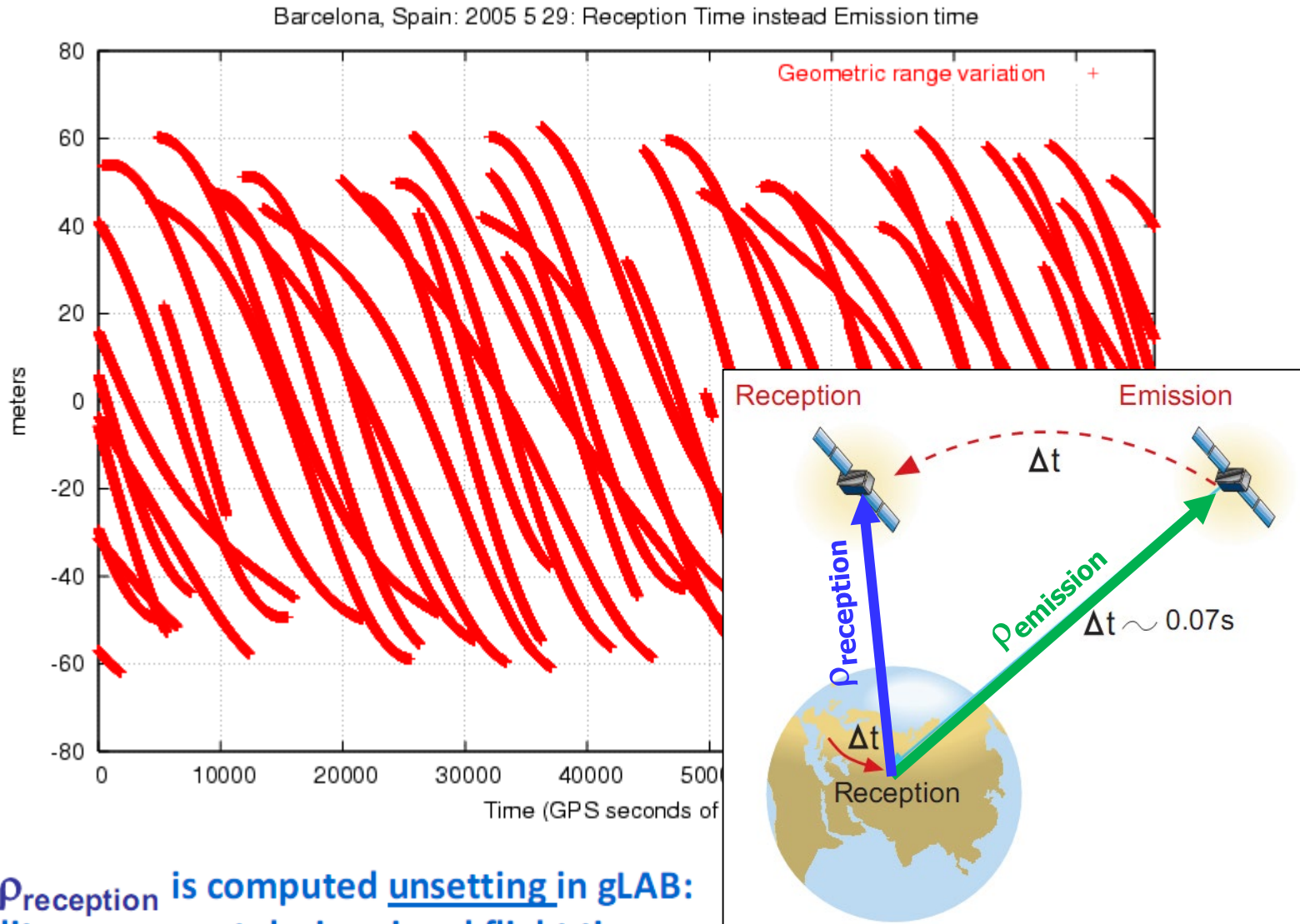
Finally, since $dt^S = t^S - T$ is the time offset between satellite clock (t^S) and **GPS system time** (T), thence:

$$T_{emis} = t^S(T_{emis}) - dt^S = t_R(T_{recep}) - C1/c - dt^S$$

Distance: Δr



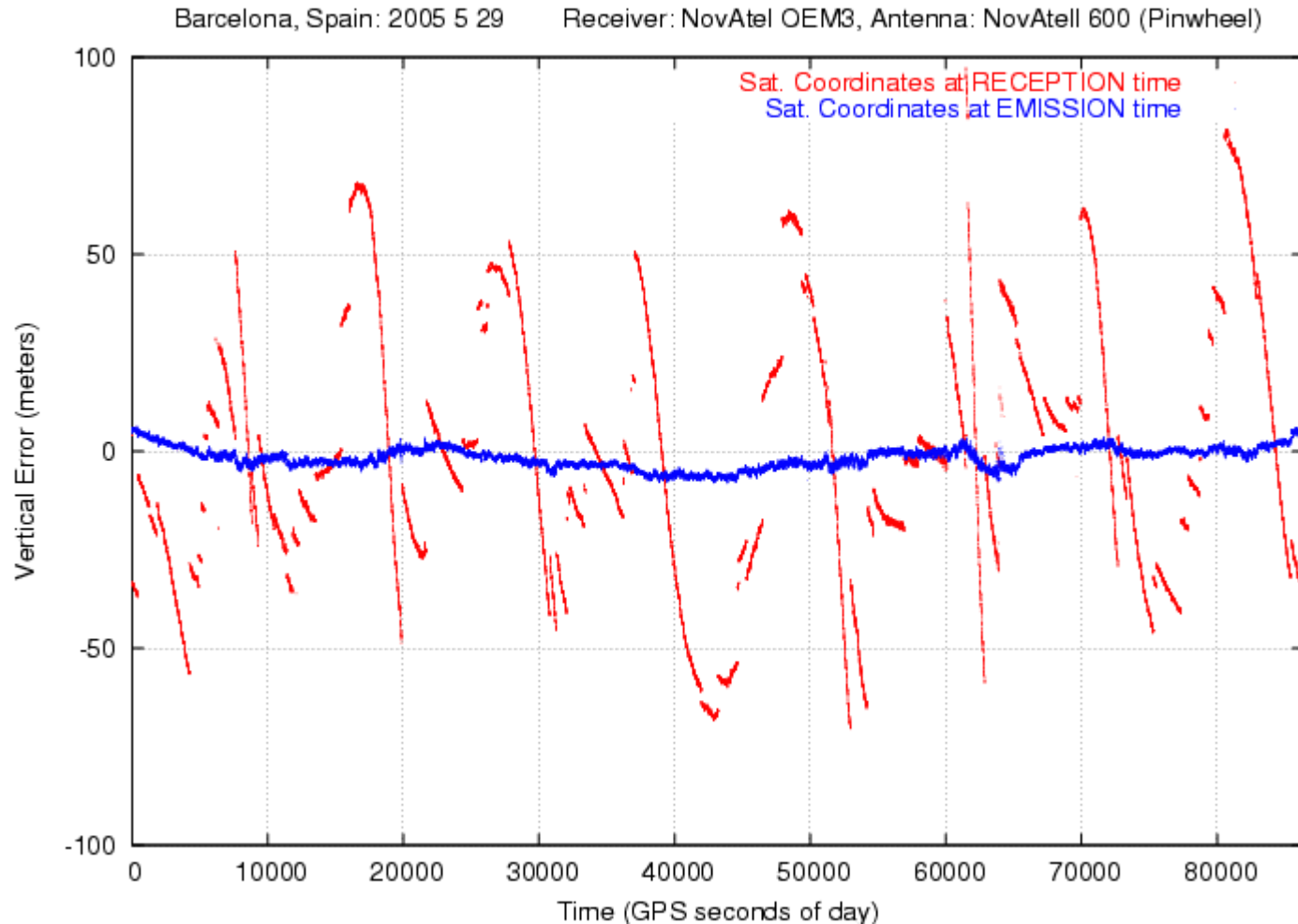
Variation in range: $\Delta\rho = \rho_{\text{emission}} - \rho_{\text{reception}}$



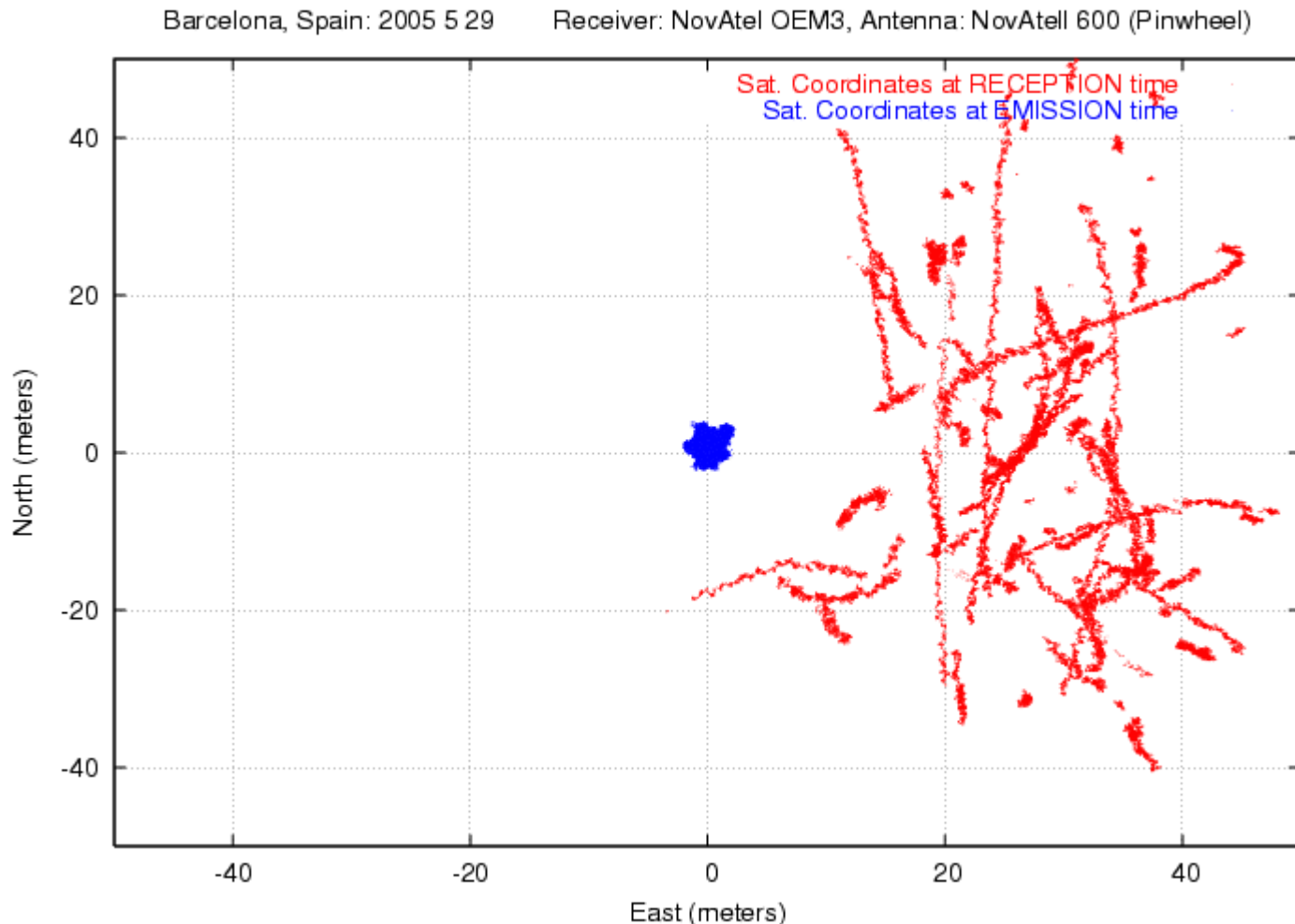
Note: $\rho_{\text{reception}}$ is computed unsettling in gLAB:

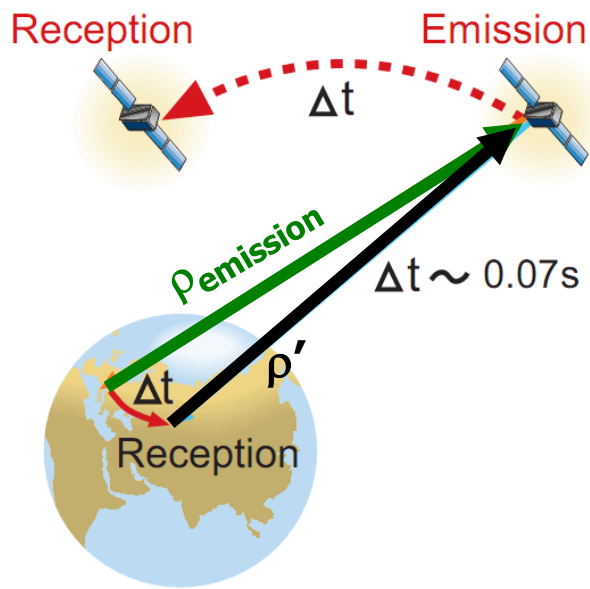
- Satellite movement during signal flight time.
- Earth rotation during signal flight time.

Vertical error comparison



Horizontal error comparison





Coordinates computation at emission time

vided by the GPS/SPS-SS (**orbit.f**) computes
nates in an Earth-Fixed reference frame. To
satellite coordinates

See **rec2ems.f**

ne, the following algorithm can be applied:
time-tags, compute emission time in GPS system

time:

$$T_{\text{emis}} = t_R(T_{\text{recep}}) - (C1/c + dt^S)$$

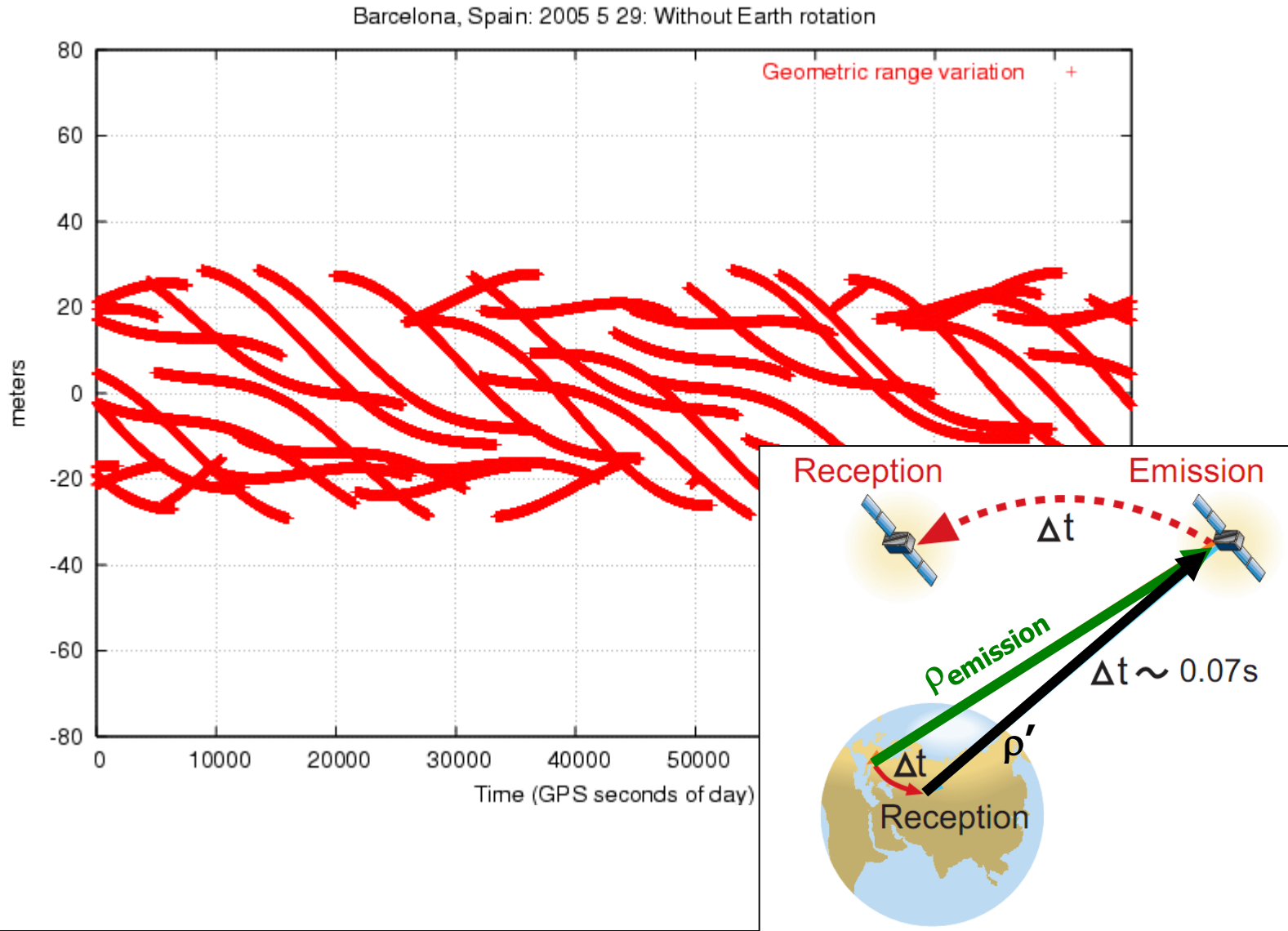
2. Compute satellite coordinates at emission time T_{emis}

$$T_{\text{emis}} \rightarrow [\text{orbit}] \rightarrow (X^{\text{sat}}, Y^{\text{sat}}, Z^{\text{sat}})_{\text{CTS}[\text{emission}]}$$

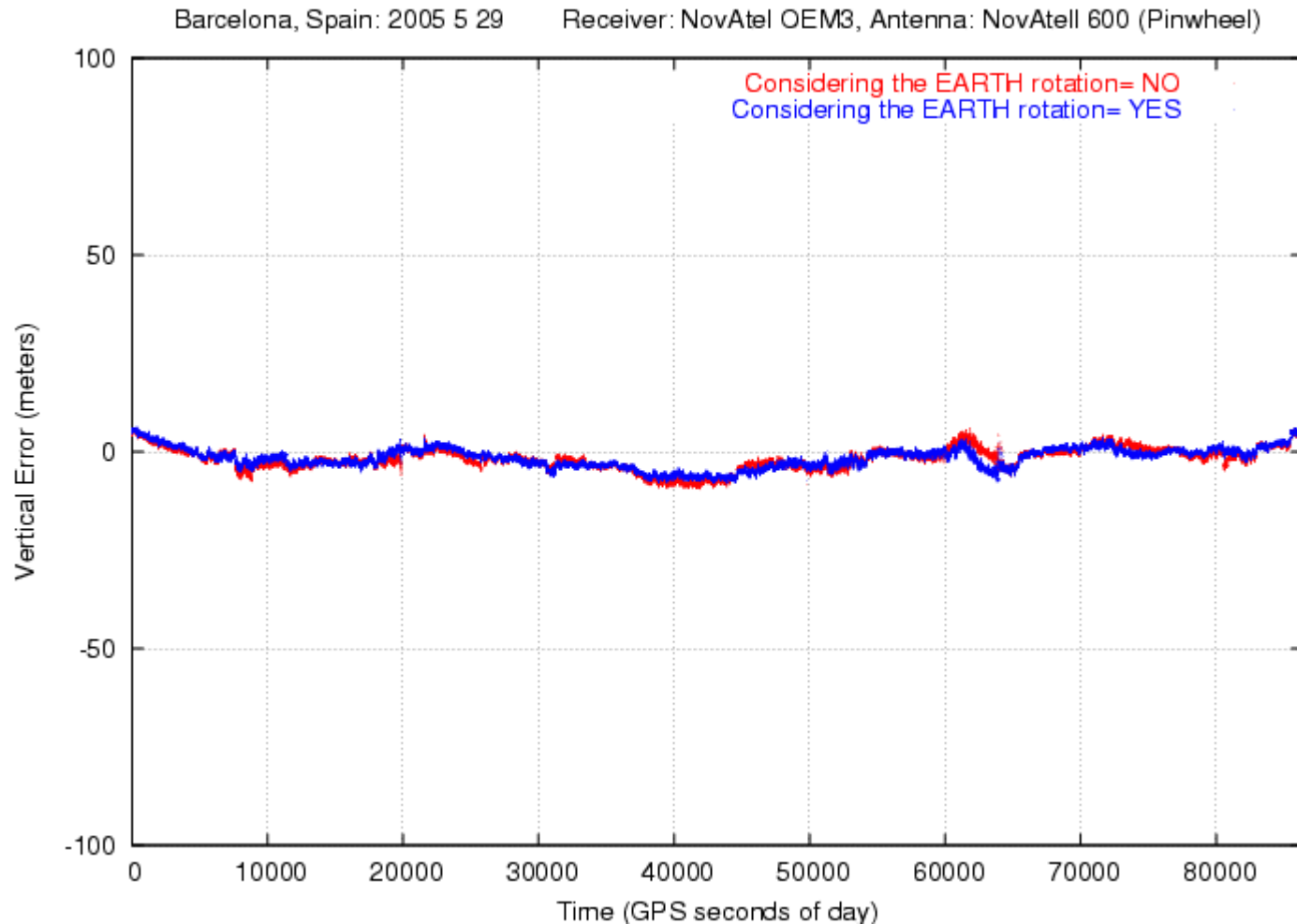
3. Account for Earth rotation during traveling time from emission to reception " Δt " (*CTS reference system at reception time is used to build the navigation equations*).

$$(X^{\text{sat}}, Y^{\text{sat}}, Z^{\text{sat}})_{\text{CTS}[\text{reception}]} = R_3(\omega_E \Delta t) \cdot (X^{\text{sat}}, Y^{\text{sat}}, Z^{\text{sat}})_{\text{CTS}[\text{emission}]}$$

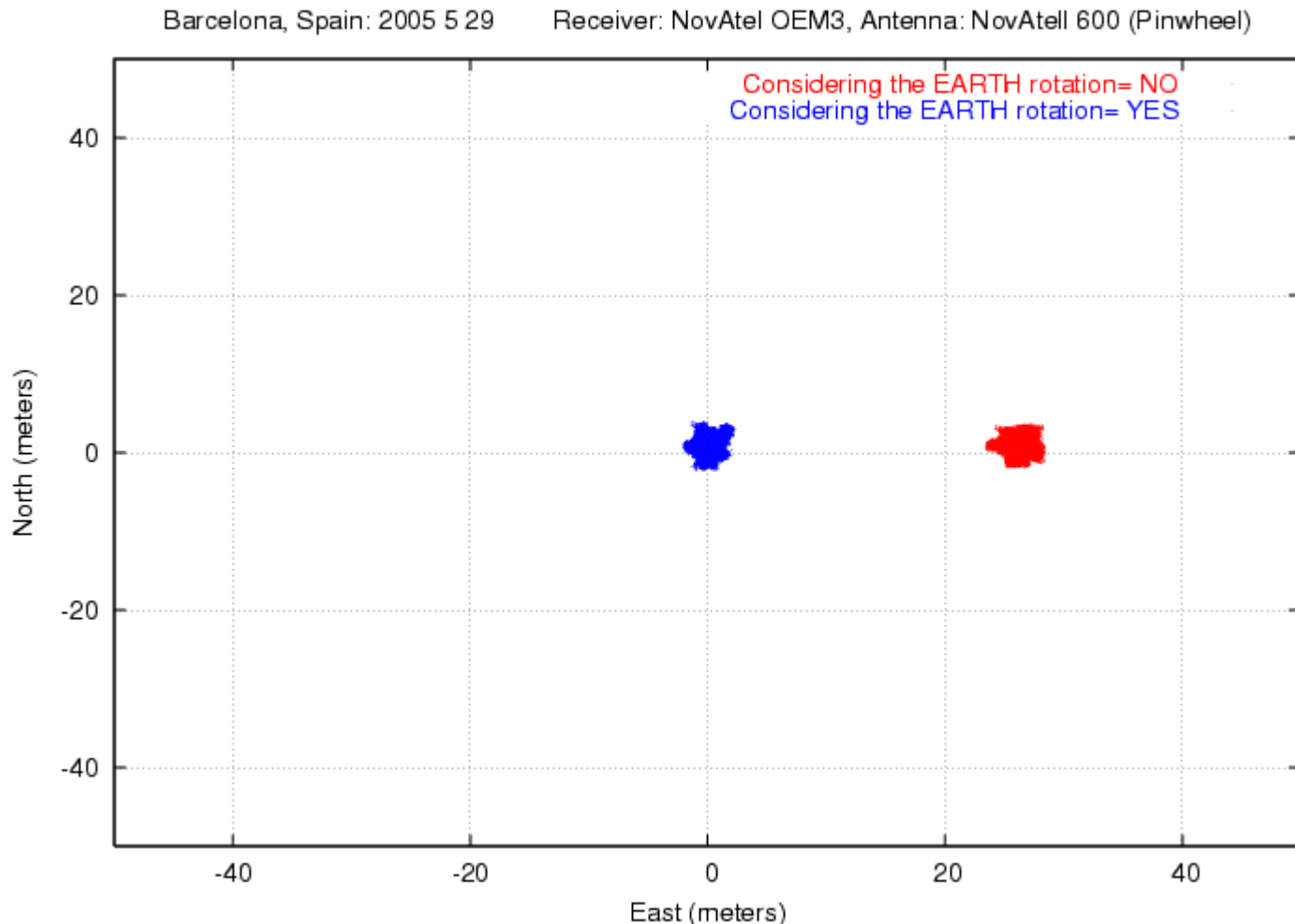
Variation in range: $\Delta\rho = \rho' - \rho_{\text{emission}}$



Vertical error comparison



Horizontal error comparison



Satellite and receiver clock offsets

- They are time-offsets between satellite/receiver time and GPS system time (provided by the ground control segment):
 - The receiver clock offset (dt_{rec}) is estimated together with receiver coordinates.
 - Satellite clock offset (dt^{sat}) may be computed from navigation message **plus a Relativistic clock correction**

$$dt^{sat} = a_0 + a_1(t - t_0) + a_2(t - t_0)^2 + \Delta rel^{sat}$$

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{rec,0}^{sat} - c \left(\underbrace{dt^{sat} + \Delta rel^{sat}}_{dt^{sat}} \right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

$$a_0 + a_1(t-t_0) + a_2(t-t_0)^2$$

PRN (green arrow pointing to 14)

t0 (blue arrow pointing to 95 10 18 00 51 44.0)
 YY MM DD H M S

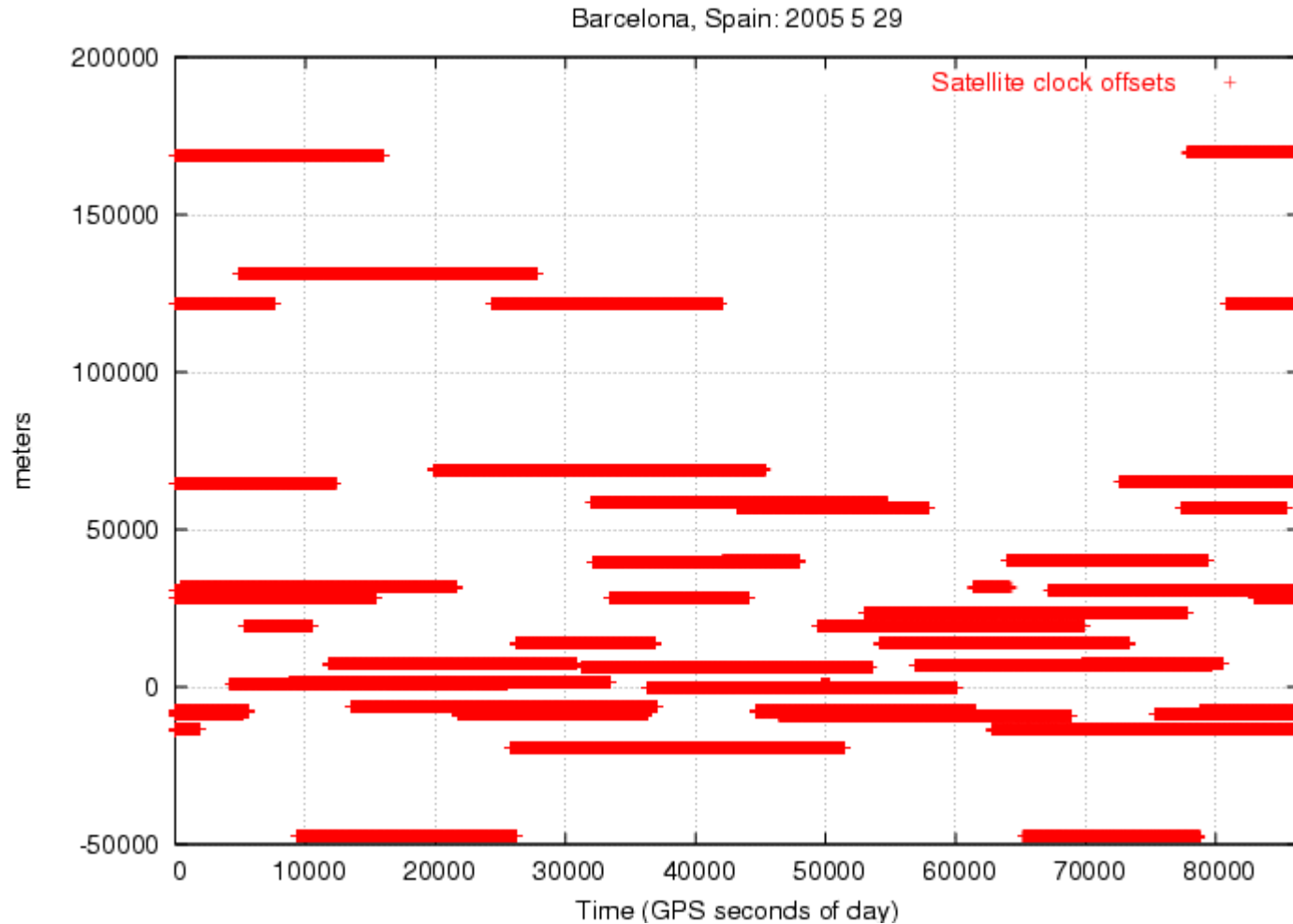
a0 (red arrow pointing to 1.129414886236D-05)

a1 (red arrow pointing to 1.136868377216D-13)

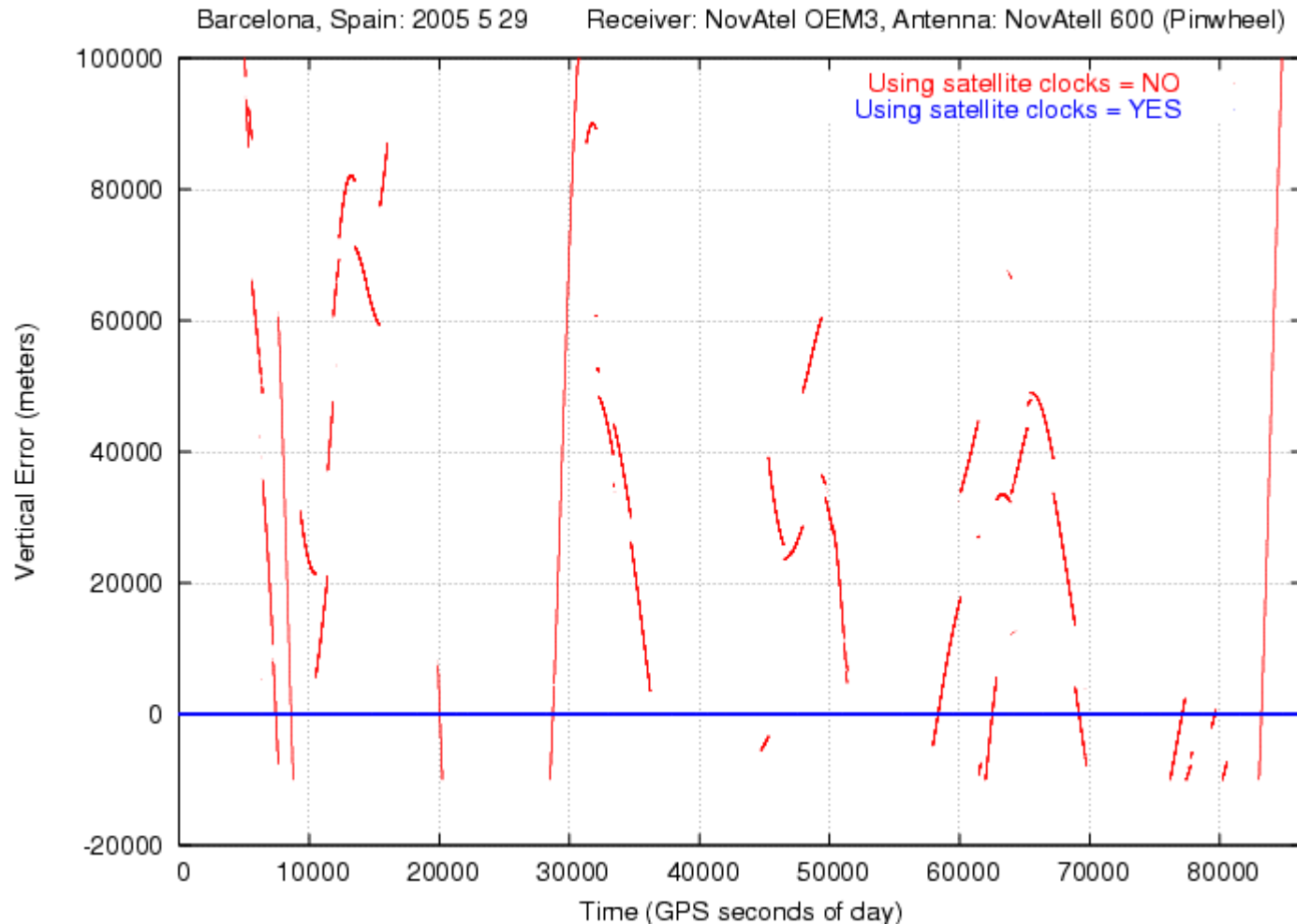
a2 (red arrow pointing to 0.000000000000D+00)

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CASA			COMMENT	
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END OF HEADER				
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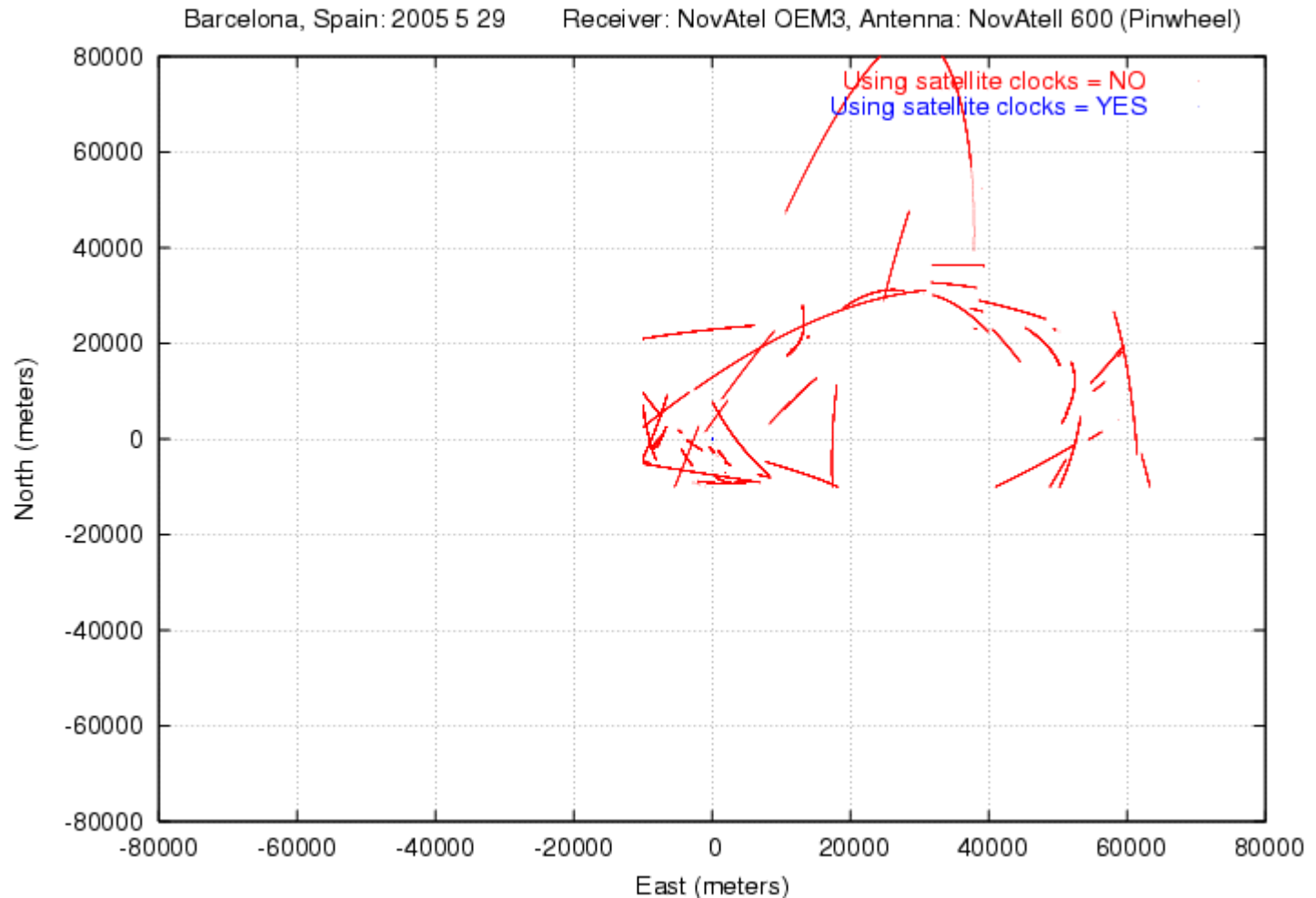
Range variation: satellite clocks



Vertical error comparison



Horizontal error comparison



Relativistic clock correction (Δ_{rel})

- A constant component depending only on nominal value of satellite's orbit major semi-axis, being corrected modifying satellite's clock oscillator frequency*:

$$\frac{f'_0 - f_0}{f_0} = \frac{1}{2} \left(\frac{v}{c} \right)^2 + \frac{\Delta U}{c^2} = -4.464 \cdot 10^{-10}$$

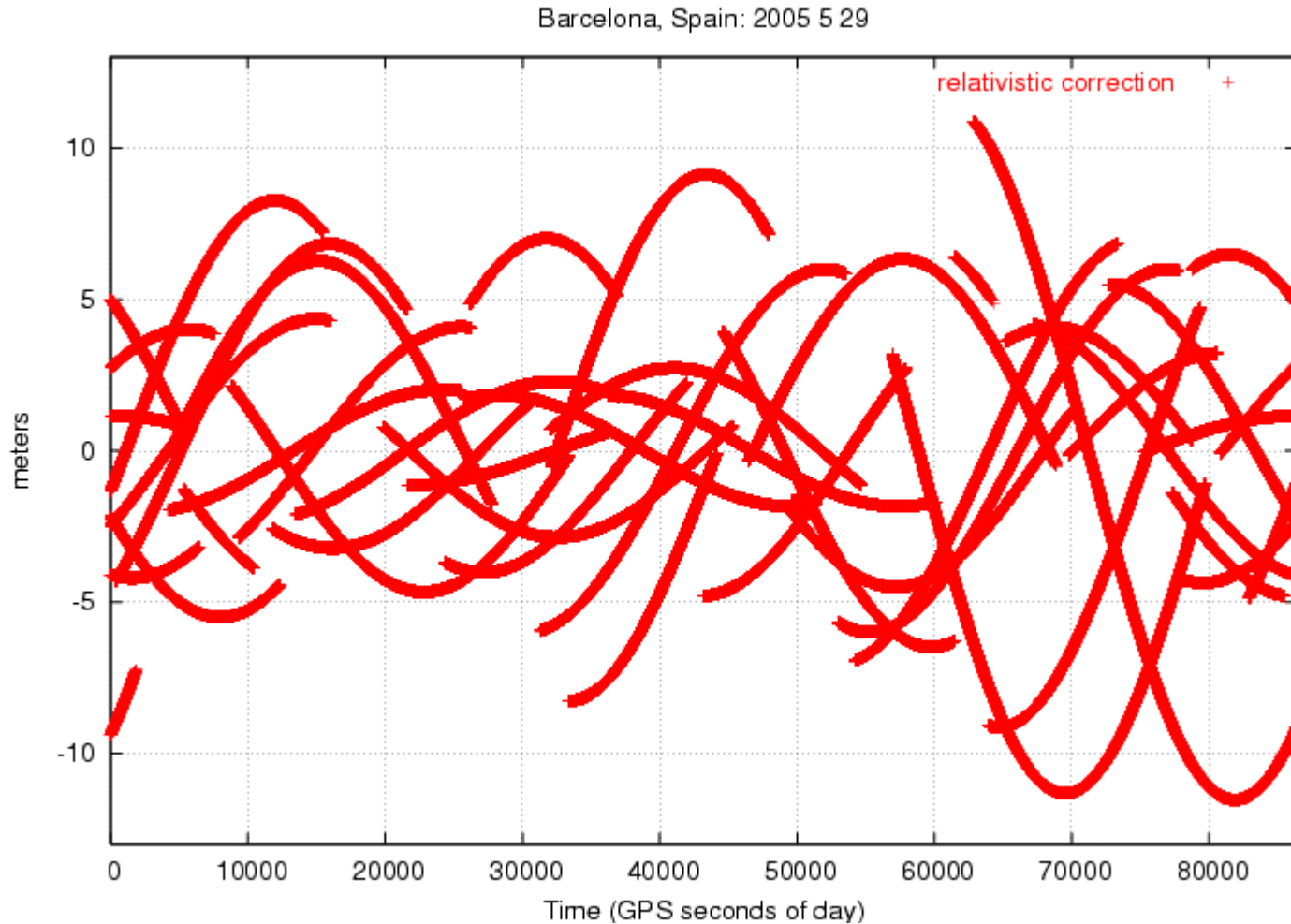
- A periodic component due to orbit eccentricity (to be corrected by user receiver):

$$\Delta_{rel} = -2 \frac{\sqrt{\mu a}}{c^2} e \sin(E) = -2 \frac{\mathbf{r} \cdot \mathbf{v}}{c^2} \text{ (seconds)}$$

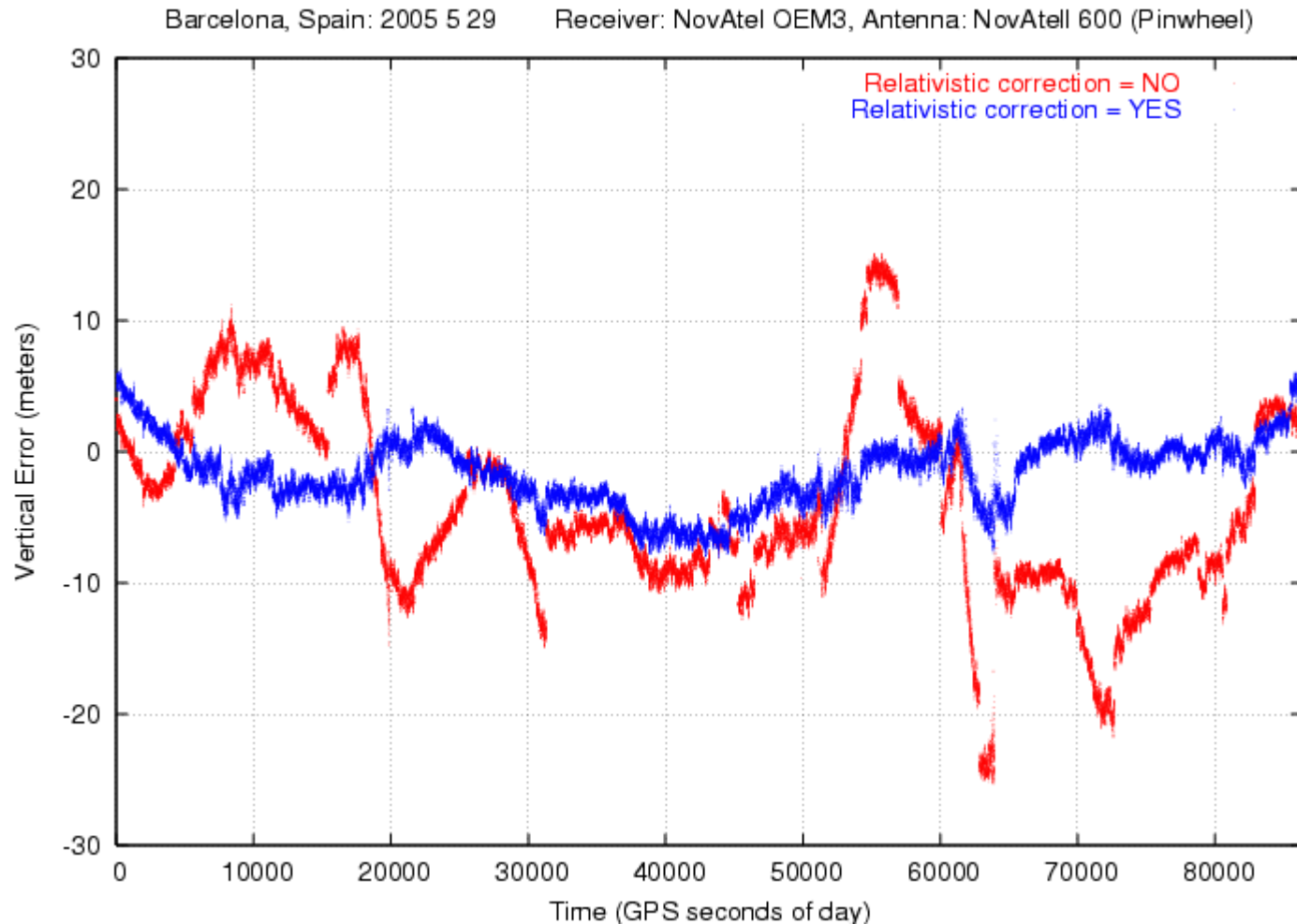
Being $\mu = 3.986005 \cdot 10^{14} \text{ (m}^3/\text{s}^2\text{)}$ Earth gravity constant, $c = 299792458 \text{ (m/s)}$ light speed in vacuum, a is orbit's major semi-axis, e is its eccentricity, E is satellite's eccentric anomaly, and r and v are satellite's geocentric position and speed in an inertial system.

*being $f_0 = 10.23 \text{ MHz}$, we have $\Delta f = 4.464 \cdot 10^{-10} \cdot f_0 = 4.57 \cdot 10^{-3} \text{ Hz}$
so satellite should use $f'_0 = 10.22999999543 \text{ MHz}$.

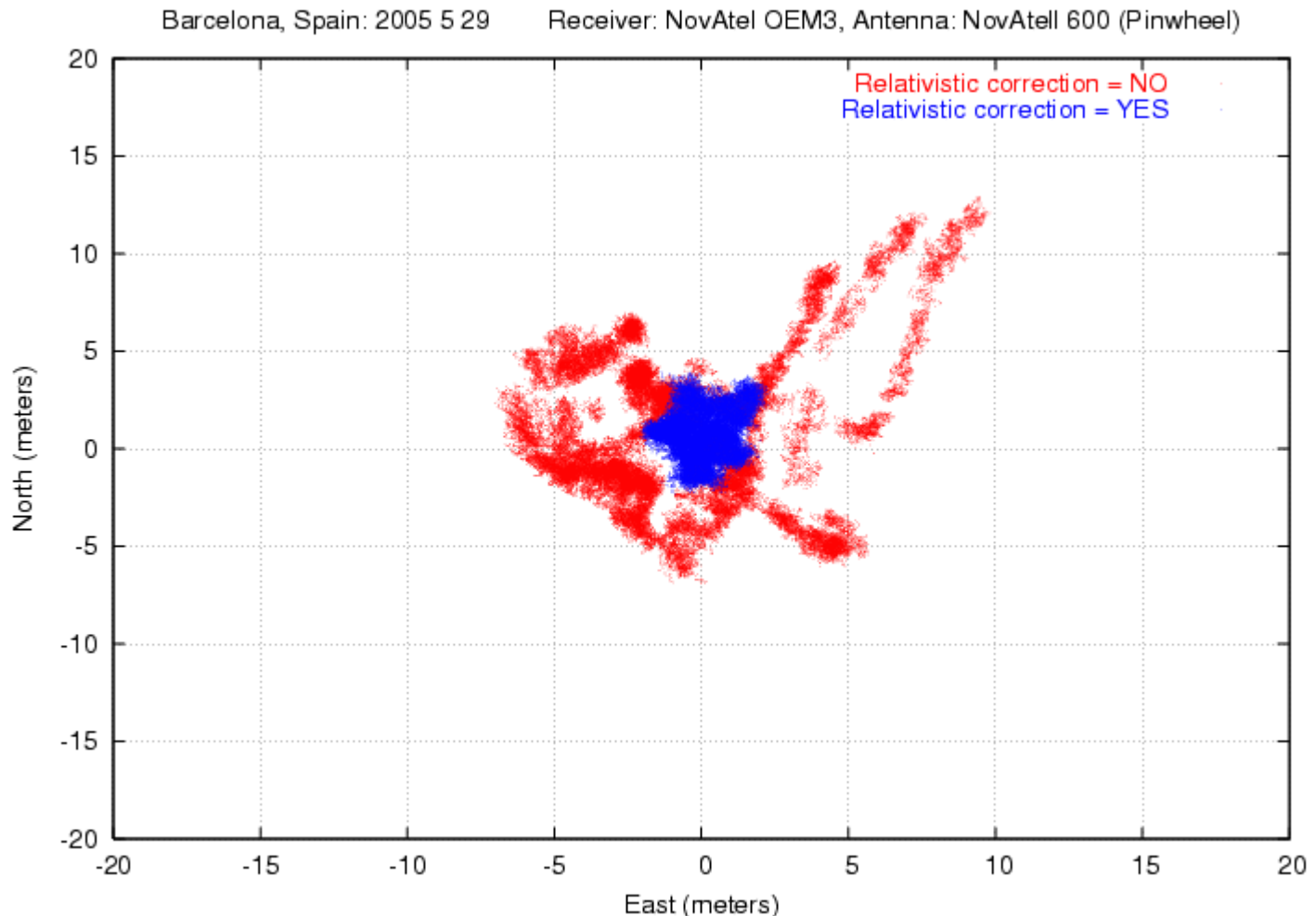
Range variation: relativistic correction



Vertical error comparison



Horizontal error comparison



Ionospheric Delay $Ion_{f \text{ rec}}^{sat}$

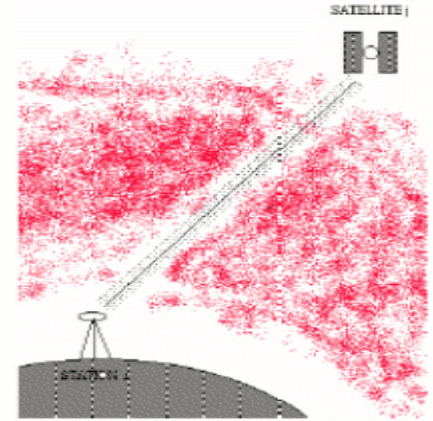
The ionosphere extends from about 60 km in height until more than 2000 km, with a sharp electron density maximum at around 350 km. The ionosphere delays code and advances carrier by the same amount.

The ionospheric delay depends on signal frequency as given by:

$$Ion_{1 \text{ rec}}^{sat} = \frac{40.3}{f_1^2} I$$

Where I is number of electrons per area unit in the direction of observation, or STEC (*Slant Total Electron Content*)

$$I = \int_{rec}^{sat} N_e ds$$



- For two-frequency receivers, it may be cancelled (99.9%) using ionosphere-free combination

$$LC = \frac{f_1^2 L1 - f_2^2 L2}{f_1^2 - f_2^2}$$

- For one-frequency receivers, it may be corrected (about 60%) using Klobuchar model (defined in GPS/SPS-SS), whose parameters are sent in navigation message. (See program klob.f)

$$C1_{rec}^{sat} [\text{modelled}] = \rho_{0,rec}^{sat} - c \left(d\bar{t}^{sat} + \Delta rel^{sat} \right) + Trop_{rec}^{sat} + \boxed{Ion_{1 \text{ rec}}^{sat}} + TGD^{sat}$$

Ionospheric models used by the GNSSs

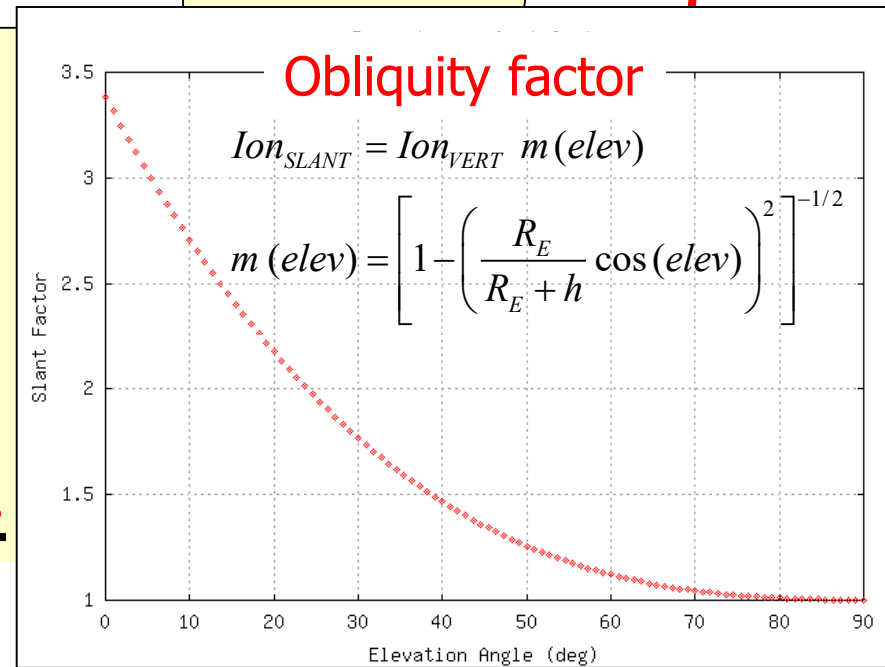
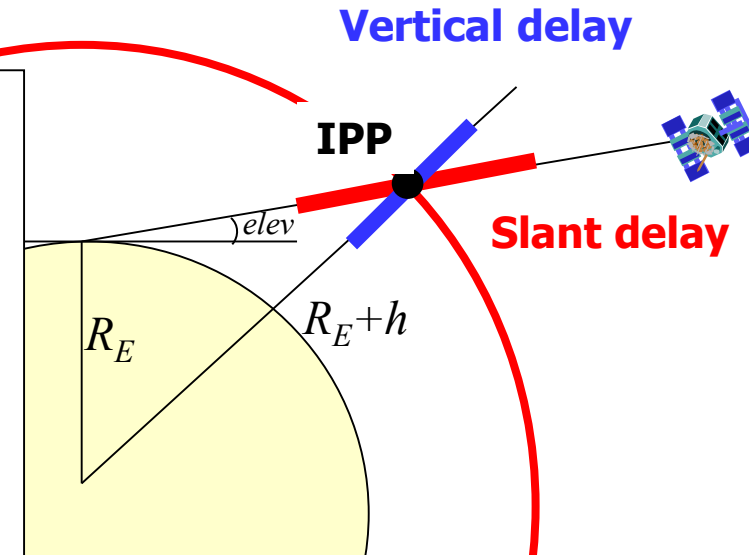
GPS	Klobuchar model
GLONASS	No ionospheric model is broadcasted
BeiDou	Klobuchar model (with layer height at 375 km instead of 350 km)
Galileo	NeQuick model

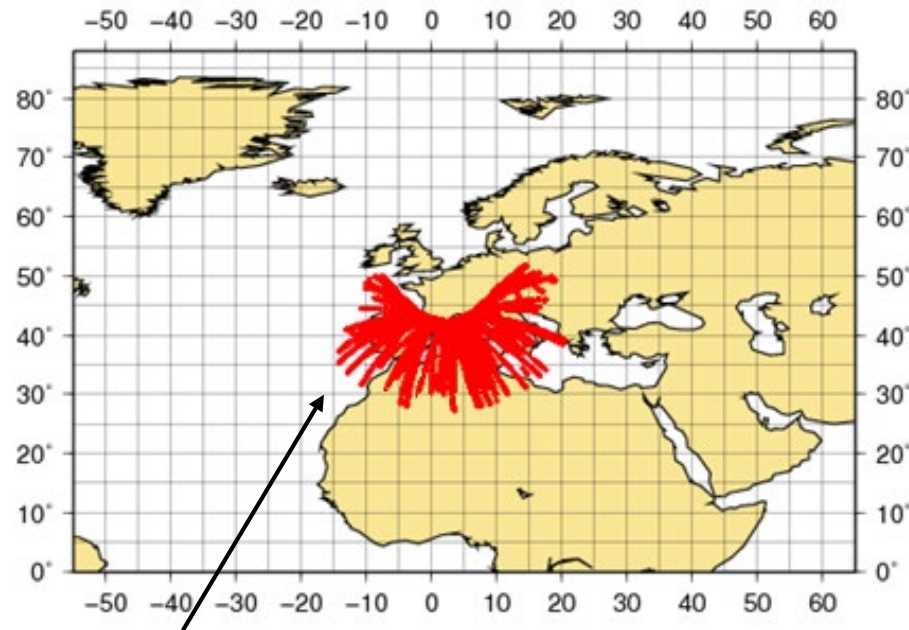
Klobuchar model (klob.f)

It was designed to minimize user computational complexity.

- Minimum user computer storage
- Minimum number of coefficients transmitted on satellite-user link
- At least 50% overall RMS ionospheric error reduction worldwide.

- It is assumed that the electron content is concentrated in a thin layer at 350 km in height.
- The **slant delay** is computed from the **vertical delay** at the Ionospheric Pierce Point (IPP), multiplying by the **obliquity factor**.





IPPs trajectories
for a receiver in
Barcelona, Spain

Vertical Delay

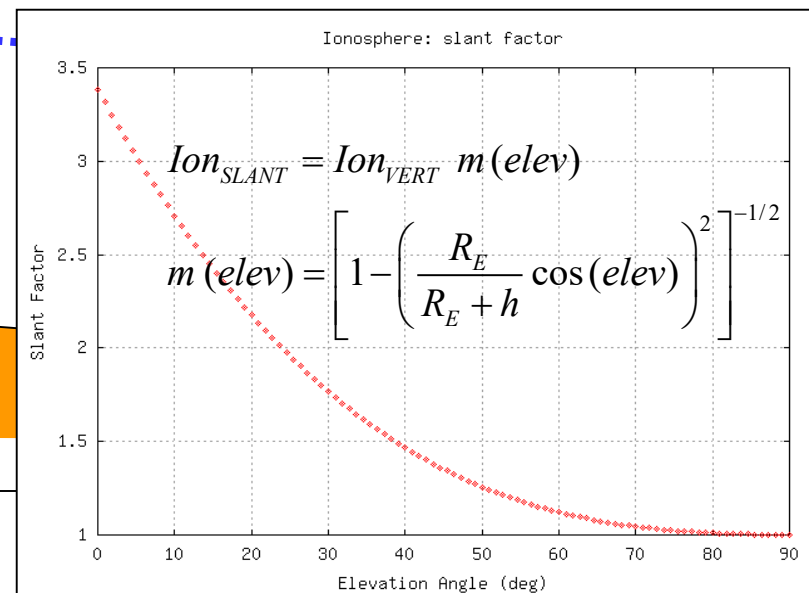
IPP

IONOSPHERIC PIERCE POINTS (IPP)

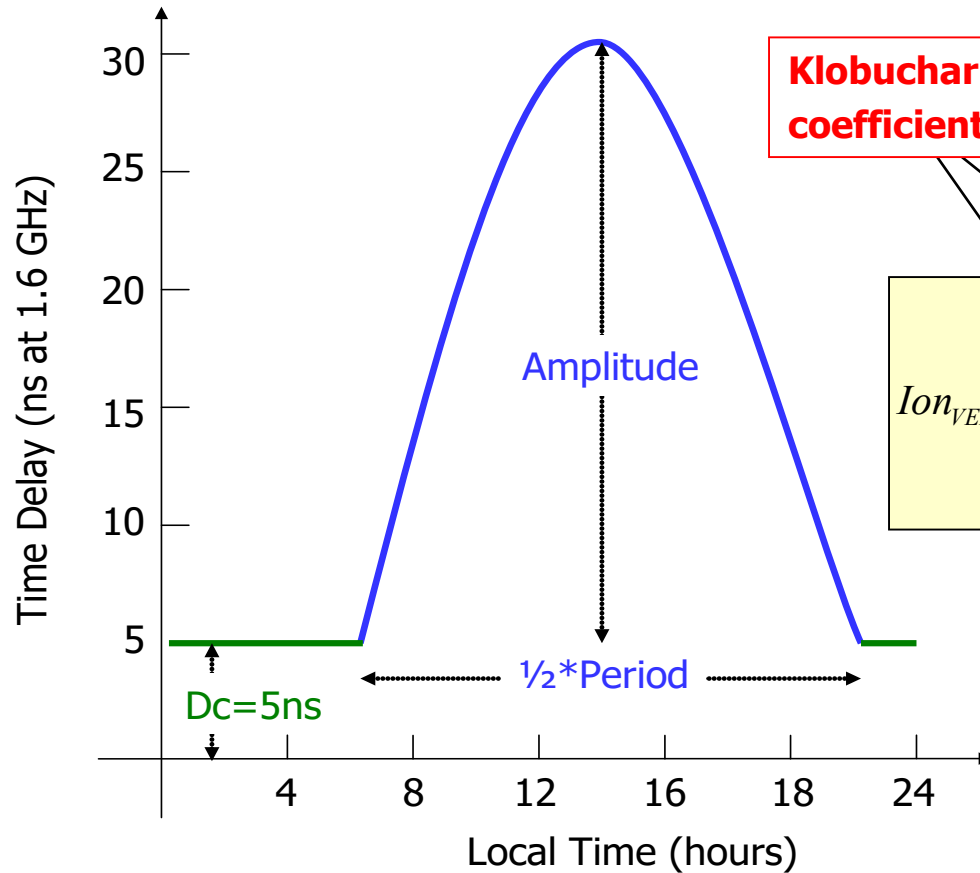
Slant Delay

Ionospheric Layer
(350 km in height)

www.gage.upc.edu



Klobuchar model



Klobuchar coefficients

$$Ion_{VERT} = \begin{cases} DC + A \cos \left[\frac{2\pi(t - \Phi)}{P} \right] & (day) \\ DC & ; \text{ if } \left[\frac{2\pi(t - \Phi)}{P} \right] > \frac{\pi}{2} \quad (night) \end{cases}$$

Being:

$$A = \sum_{n=0}^3 \alpha_n \phi^n ; \quad P = \sum_{n=0}^3 \beta_n \phi^n$$

$\phi = \text{Geomagnetic Latitude}$

$$Ion_{SLANT} = Ion_{VERT} m(elev)$$

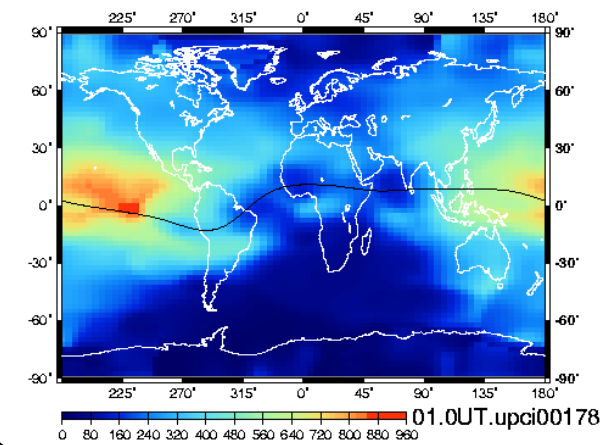
$$m(elev) = \left[1 - \left(\frac{R_E}{R_E + h} \cos(elev) \right)^2 \right]^{-1/2}$$

Where:

DC = 5 ns

$\Phi = 14$ (ctt. phase offset)

t = Local Time

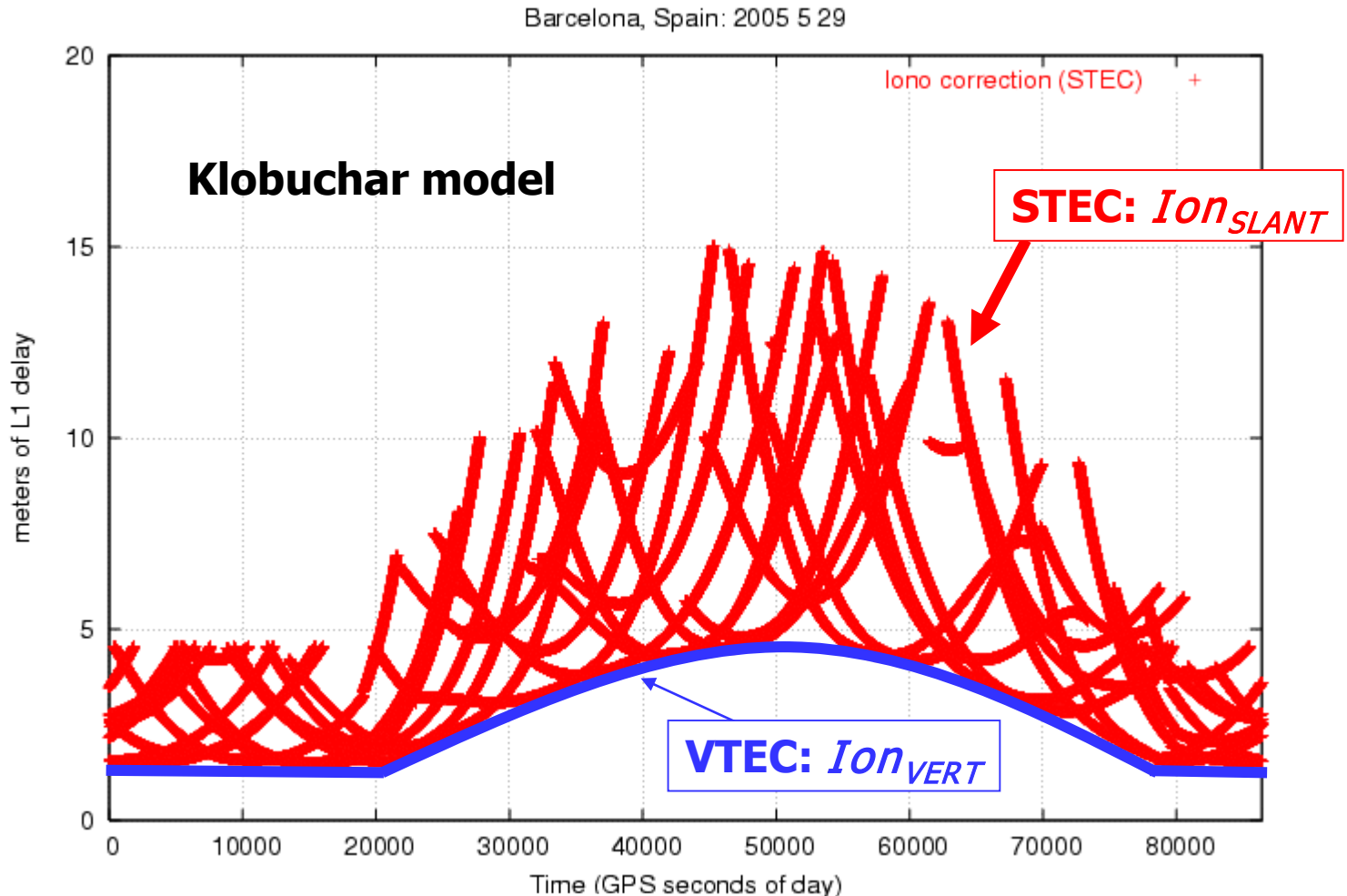


$(\text{time}, r_{\text{sta}}, r^{\text{sat}}, \alpha_0, \alpha_1, \alpha_2, \alpha_3, \beta_0, \beta_1, \beta_2, \beta_3) \rightarrow [\text{Klob}] \rightarrow \text{Iono}$

$elev, \phi$

2 NAVIGATION DATA										RINEX VERSION / TYPE									
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0.191000000000D+03-0.106250000000D+01 0.487163149444D-08-0.123716752769D+01																			
-0.540167093277D-07 0.476544268895D-02 0.713579356670D-05 0.515433833885D+04																			
0.172800000000D+06-0.260770320892D-07-0.850753478531D+00 0.763684511185D-07																			
0.957259887797D+00 0.241437500000D+03-0.167990552187D+01-0.823998608564D-08																			
0.174650132022D-09 0.100000000000D+01 0.100200000000D+04 0.000000000000D+00																			
0.320000000000D+02 0.000000000000D+00 0.465661287308D-09 0.191000000000D+03																			
0.172800000000D+06 0.000000000000D+00 0.000000000000D+00 0.000000000000D+00																			

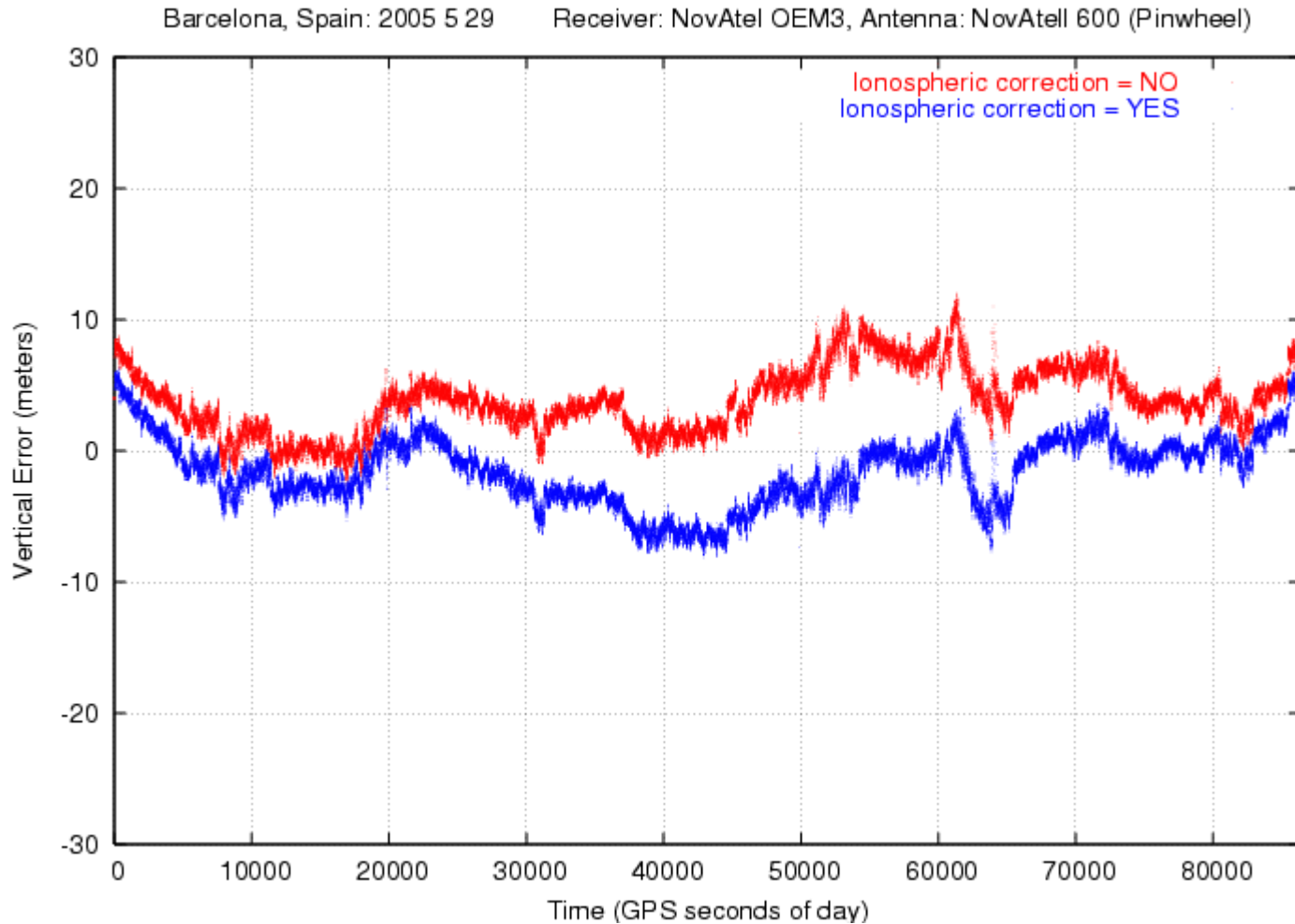
Range variation: Ionospheric correction



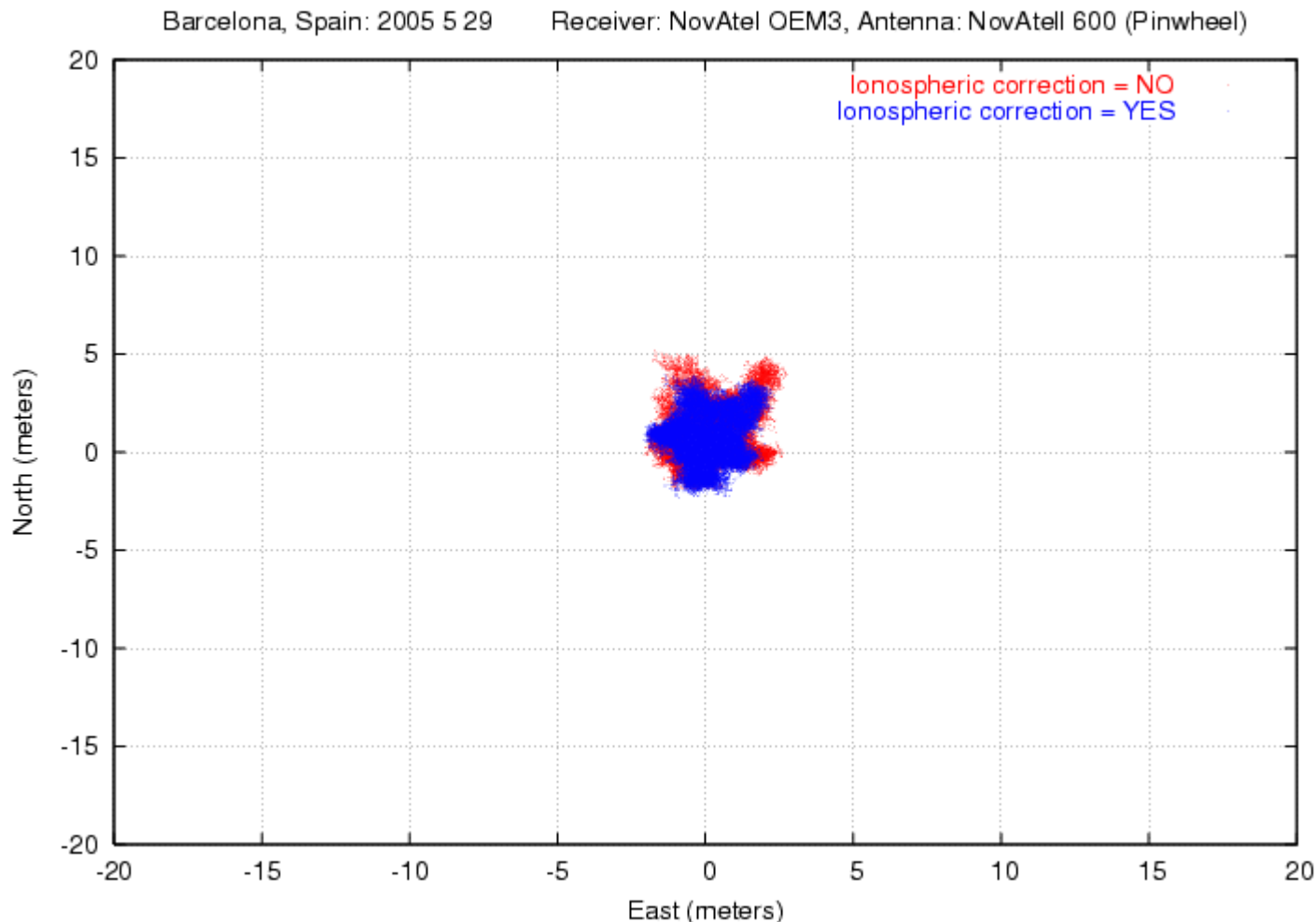
$$Ion_{SLANT} = Ion_{VERT} m(elev)$$

$$m(elev) = \left[1 - \left(\frac{R_E}{R_E + h} \cos(elev) \right)^2 \right]^{-1/2}$$

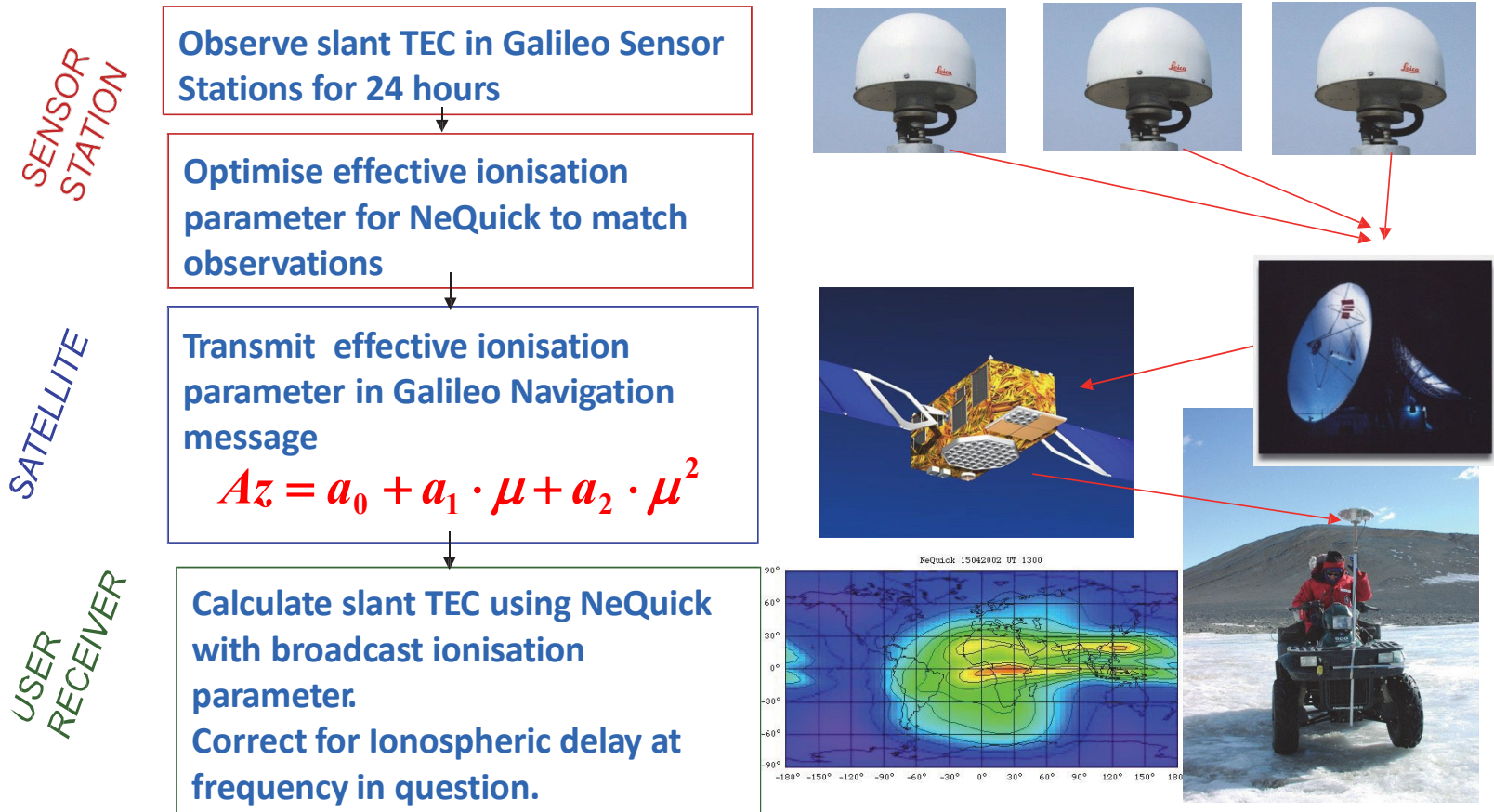
Vertical error comparison



Horizontal error comparison

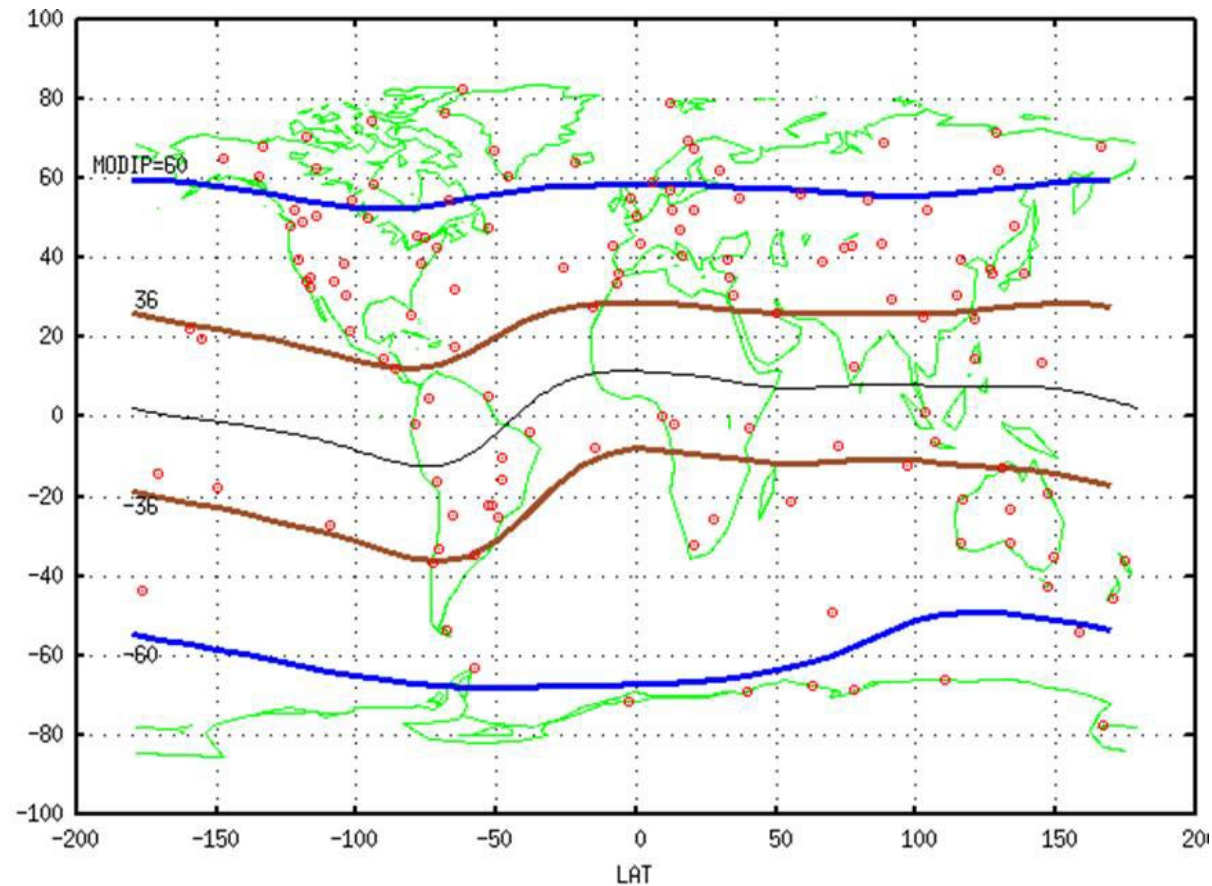


Galileo Single Frequ. Ionospheric Corr. Algo. (NeQuick model)



μ is the Modified DIP latitude (**MODIP**)

MODIP bands



MODied DIP latitude (MODIP) μ ,

$$\tan \mu = \frac{I}{\sqrt{\cos \varphi}}$$

with I the true magnetic inclination, or **dip** in the ionosphere (usually at 300 km), and φ the geographic latitude of the receiver.

Ionospheric models performance comparison

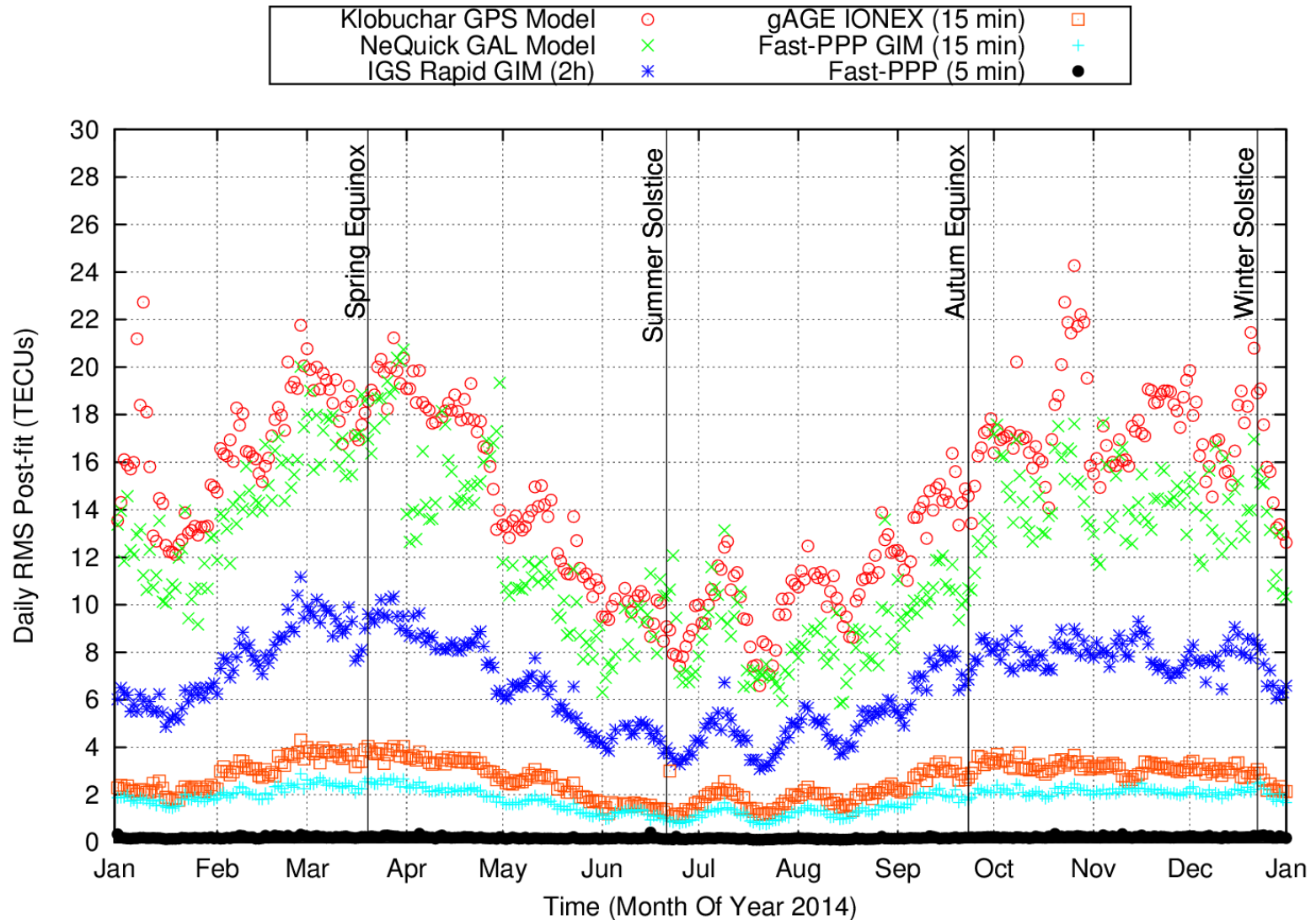
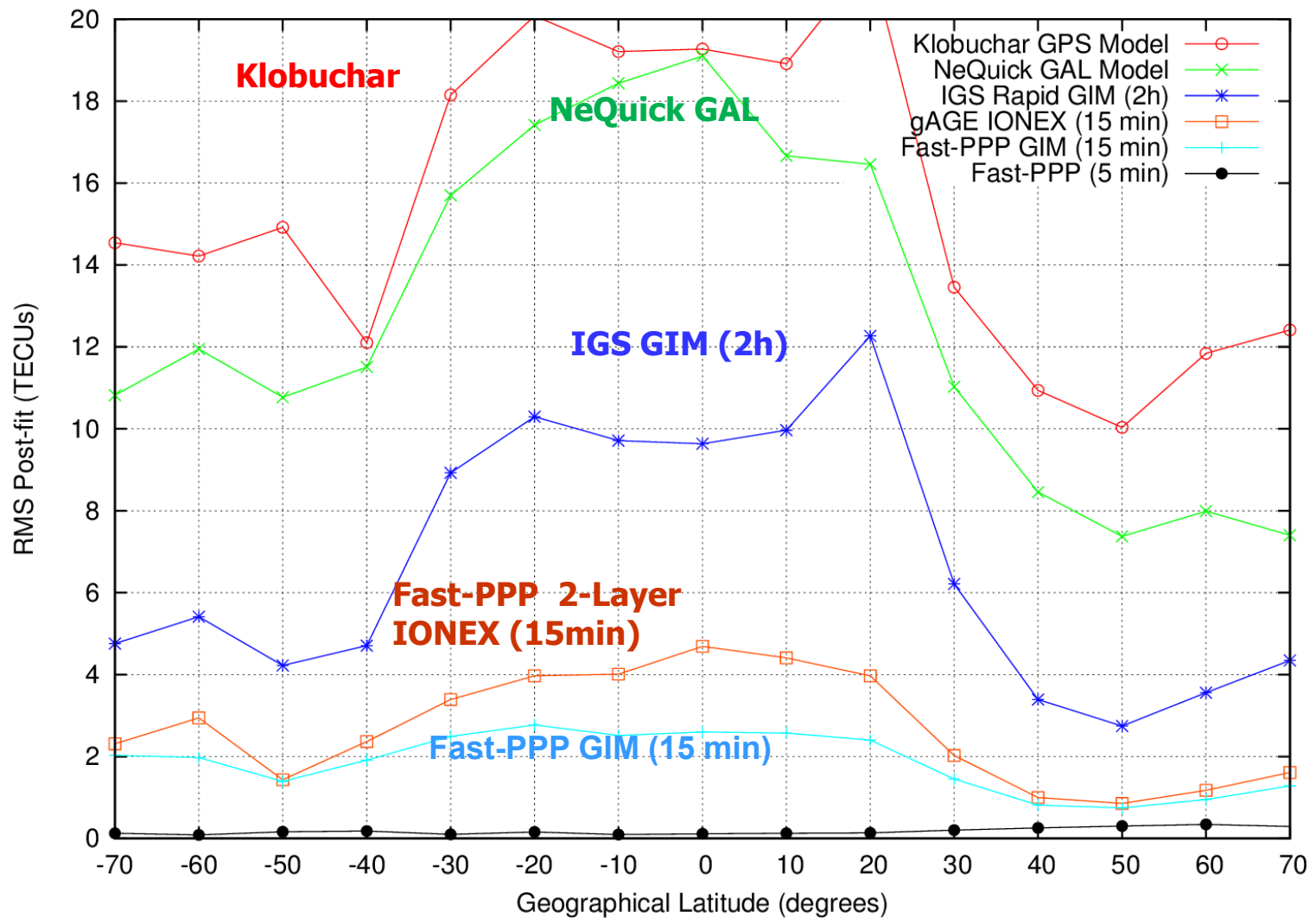


Figure from [RD-5]

where $1 \text{ TECU} = 10^{16} \frac{e^-}{m^2} = 16.24 \text{ cm at } L_1$

Ionospheric models performance comparison



Picture from [RD-5]

Tropospheric Delay

Troposphere is the atmospheric layer comprised between Earth's surface and an altitude of about 60 km.

The tropospheric delay does not depend on frequency and affects both the code and carrier phases in the same way. It can be modeled (about 90%) as:

- d_{dry} corresponds to the vertical delay of the dry atmosphere (basically oxygen and nitrogen in hydrostatical equilibrium)
→ It can be modeled as an **ideal gas**.
- d_{wet} corresponds to the vertical delay of the wet component (water vapor) → **difficult to model**.

A simple model is:

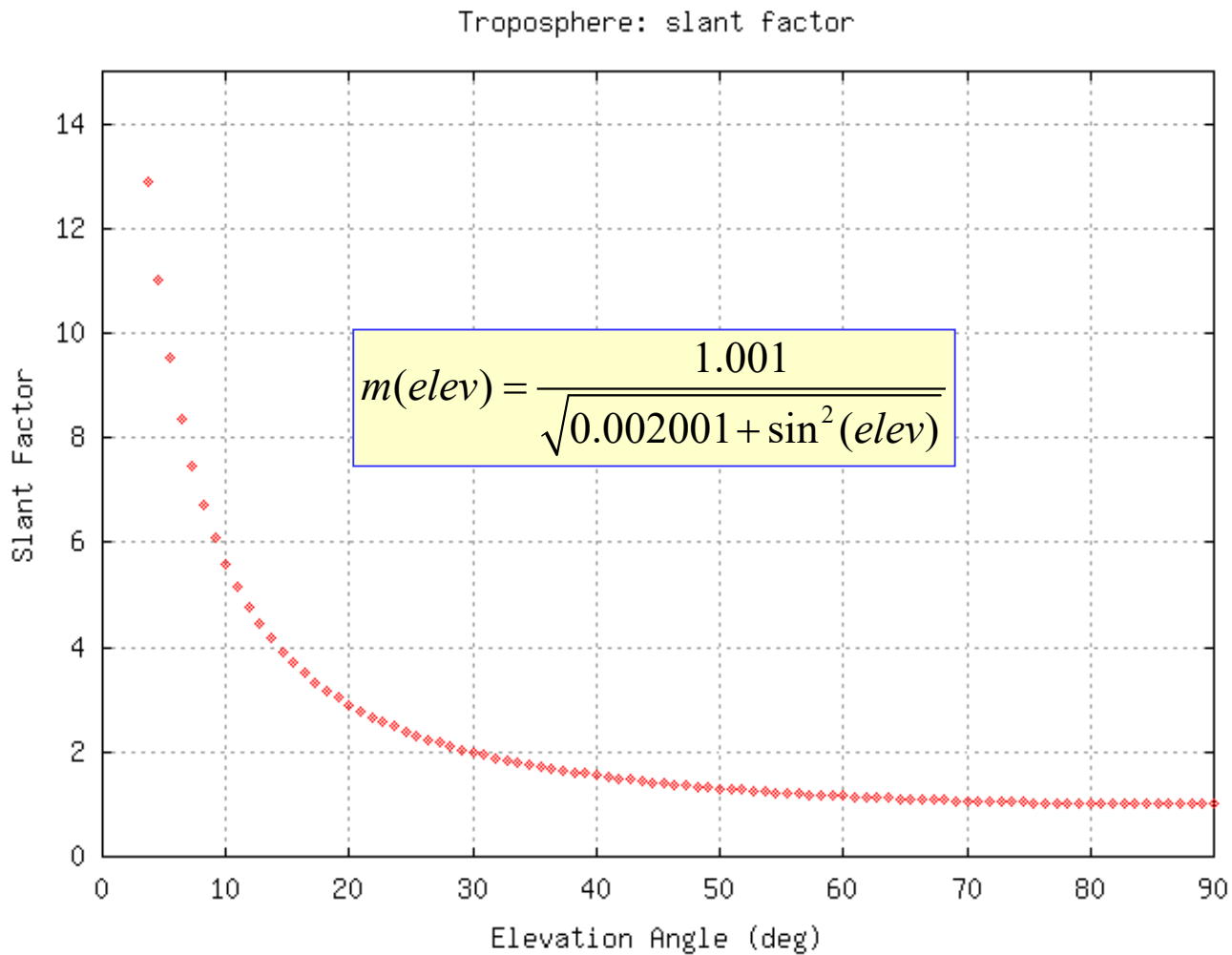
$$Trop_{rec}^{sat} = (d_{dry} + d_{wet}) \cdot m(elev)$$

$$m(elev) = \frac{1.001}{\sqrt{0.002001 + \sin^2(elev)}}$$

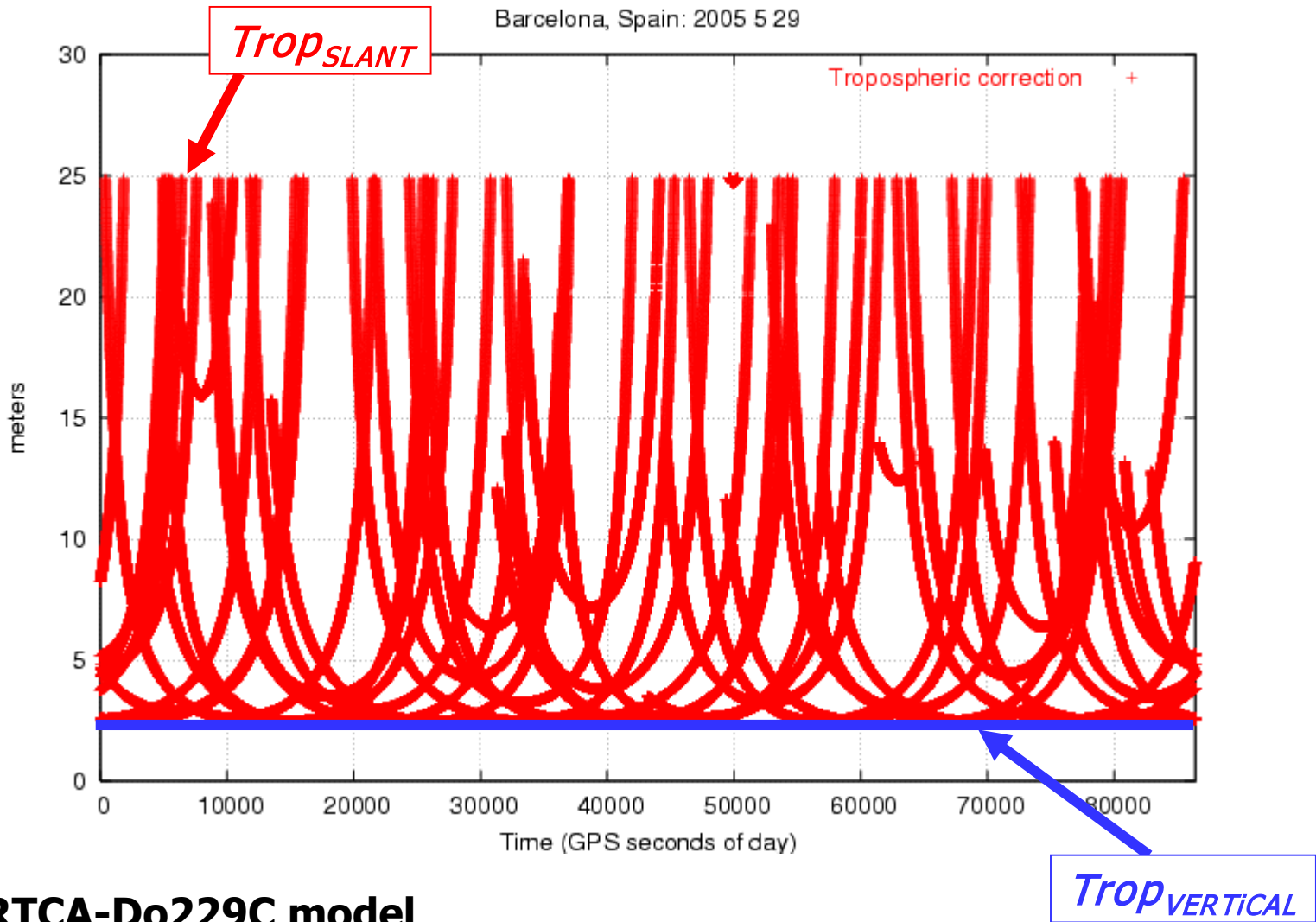
$$d_{dry} = 2.3 \exp(-0.116 \cdot 10^{-3} H) \text{ meters}$$

$$d_{wet} = 0.1m \quad [H : \text{height over the sea level}]$$

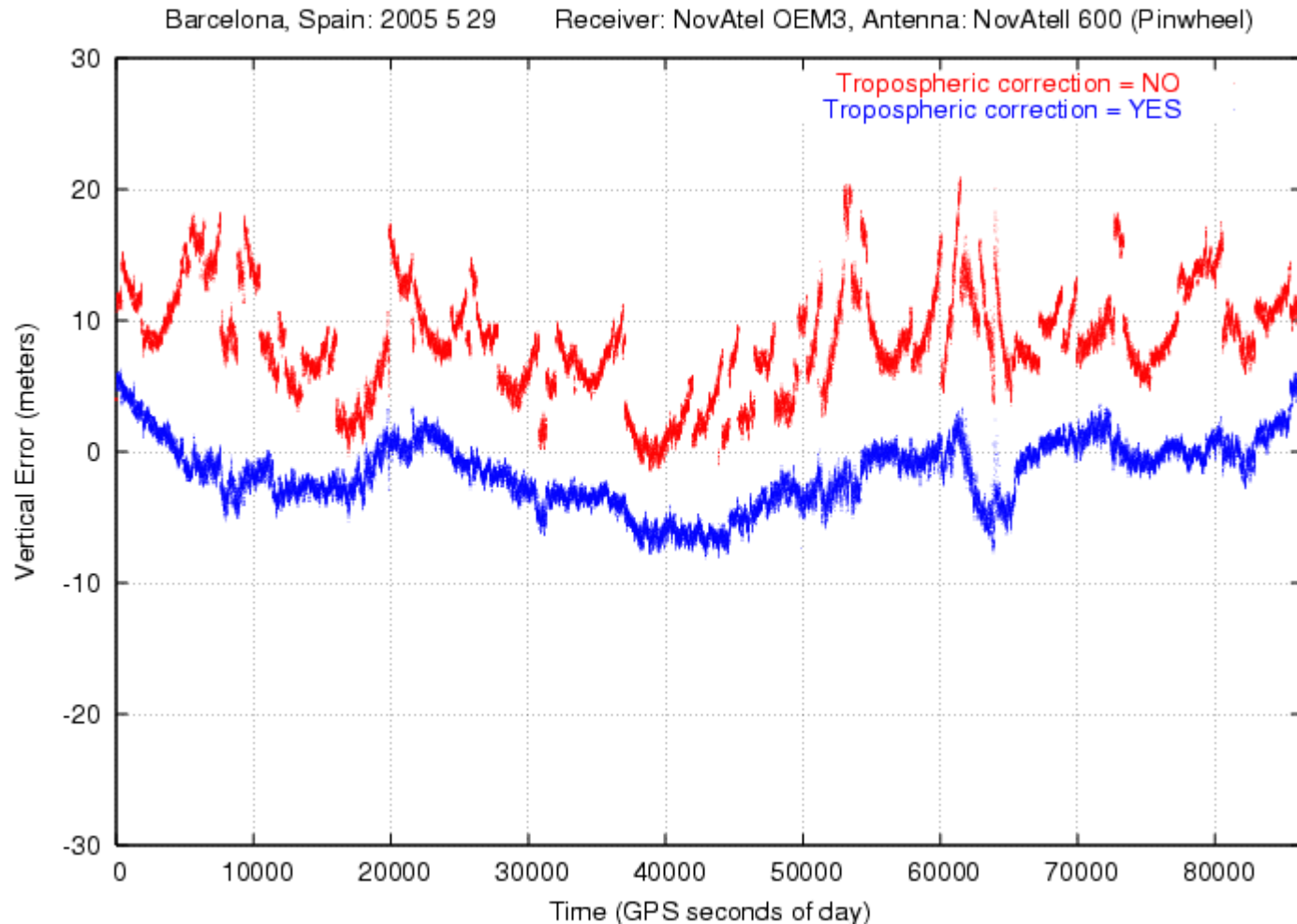
$$Cl_{rec}^{sat} [\text{modelled}] = \rho_{0,rec}^{sat} - c \left(d\bar{t}^{sat} + \Delta rel^{sat} \right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$



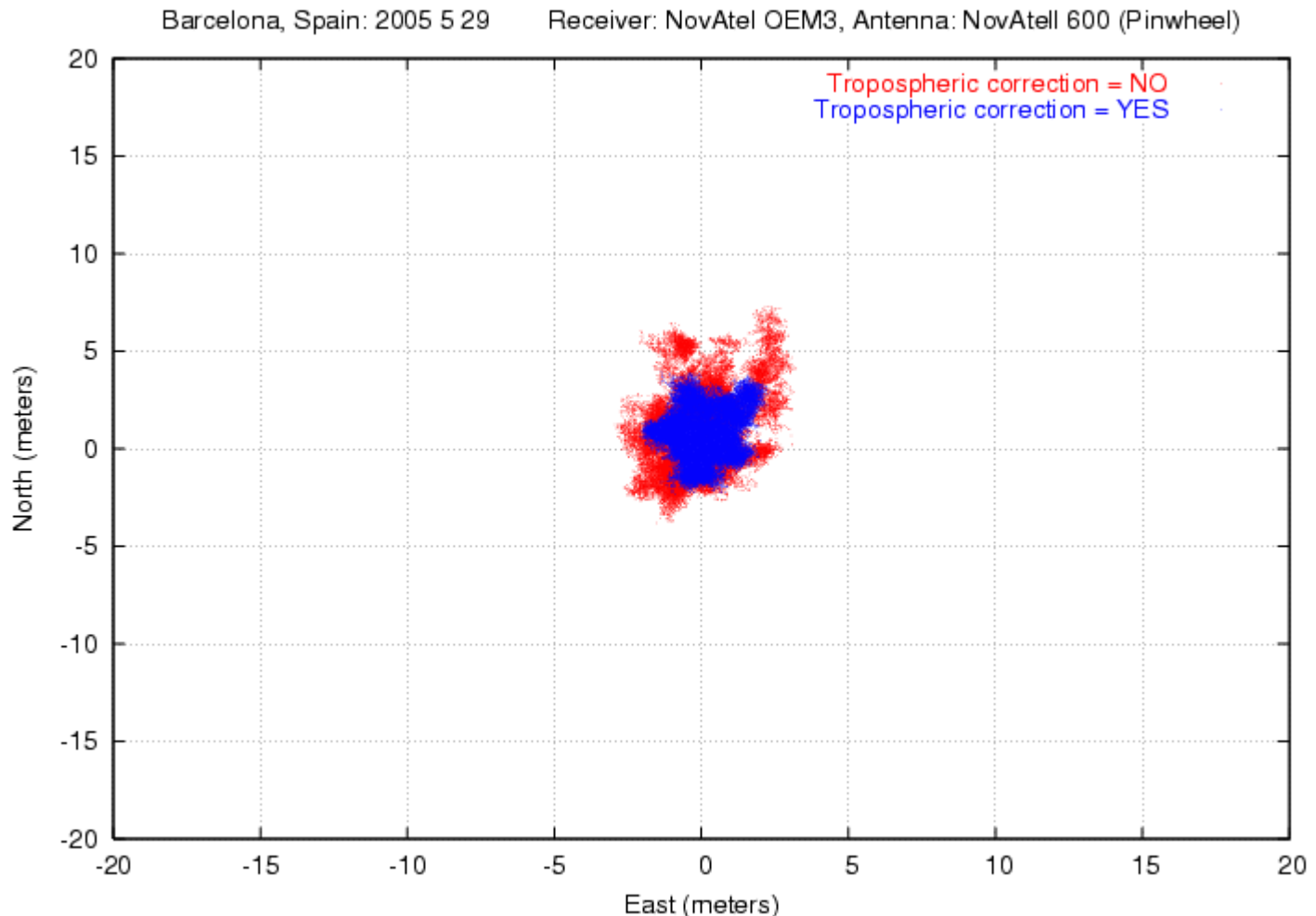
Range variation: Tropospheric correction



Vertical error comparison



Horizontal error comparison



Instrumental Delays

Some sources for these delays are antennas, cables, as well as several filters used in both satellites and receivers.

They are composed by a delay corresponding to satellite and other to receiver, depending on frequency:

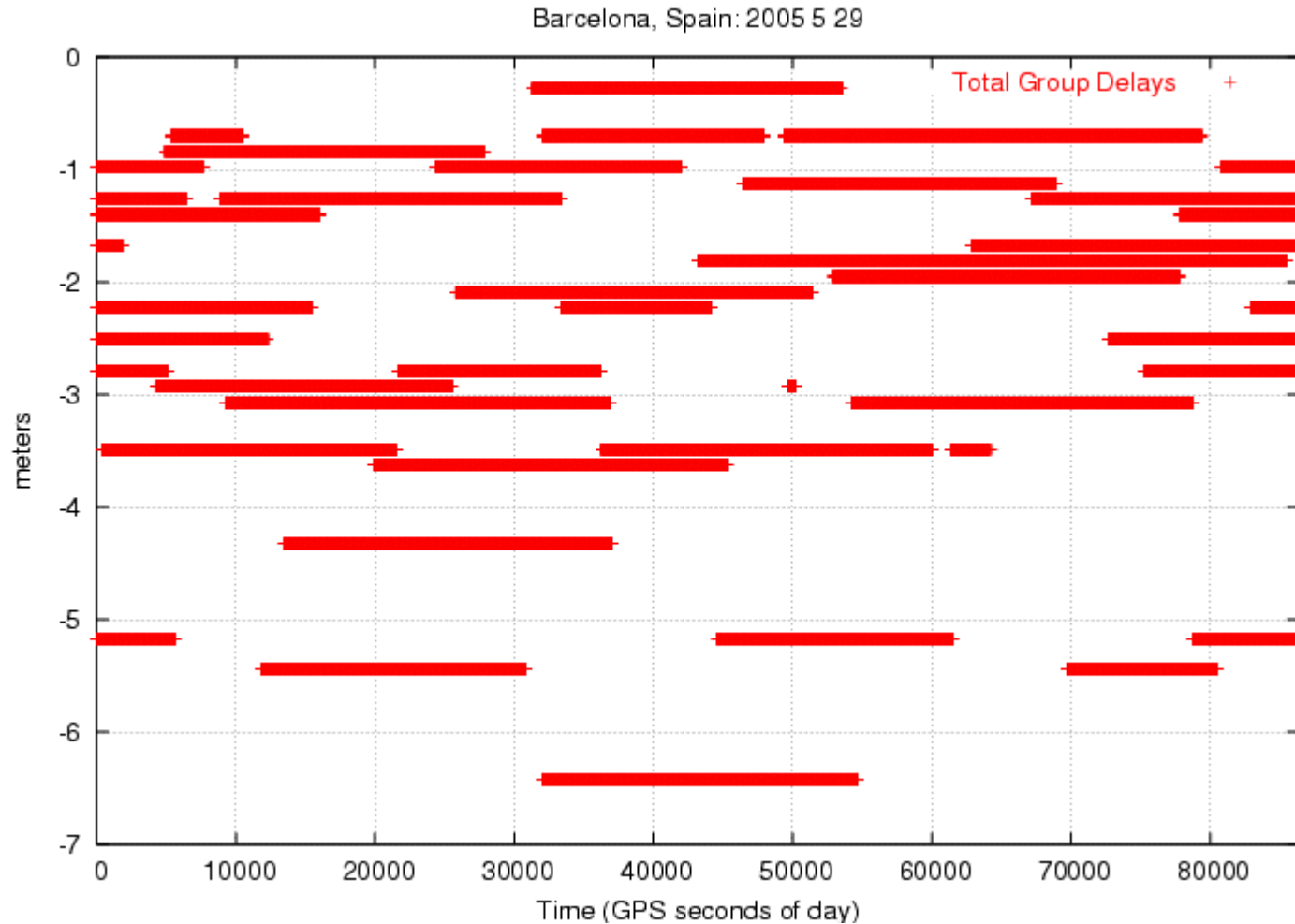
$$\begin{aligned} K_{1,rec}^{sat} &= K_{1,rec} + TGD^{sat} \\ K_{2,rec}^{sat} &= K_{2,rec} + \frac{f_1^2}{f_2^2} TGD^{sat} \end{aligned}$$

- $K_{1,rec}$ may be assumed as zero (included in the receiver clock offset).
- TGD^{sat} is transmitted in satellite's navigation message (*Total Group Delay*).

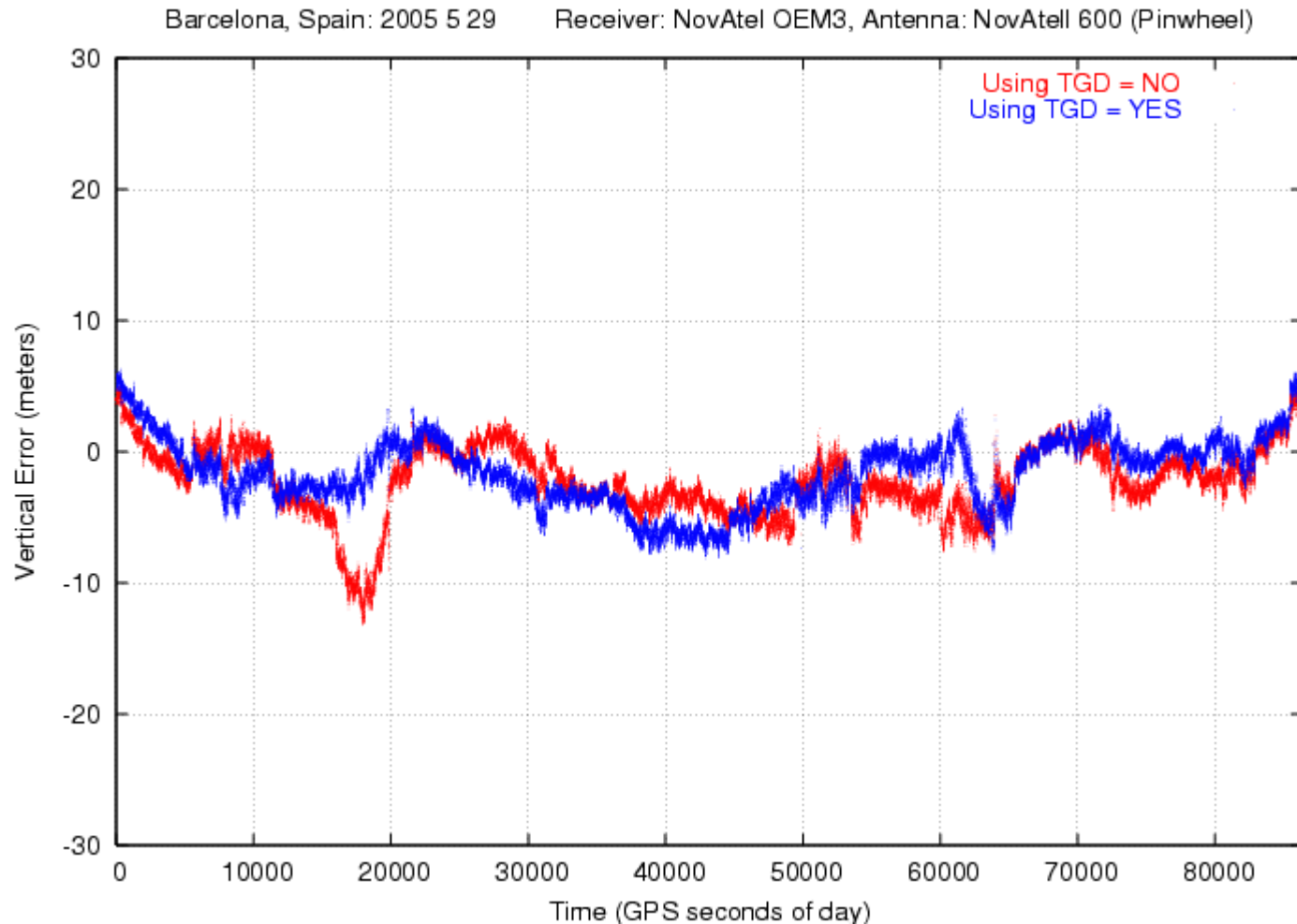
According to ICD GPS-2000, the GPS control segment monitors satellite timing, so TGDs cancels when using the ionosphere-free combination. That is why we have that particular equation for K_2 .

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c \left(d\bar{t}^{sat} + \Delta rel^{sat} \right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

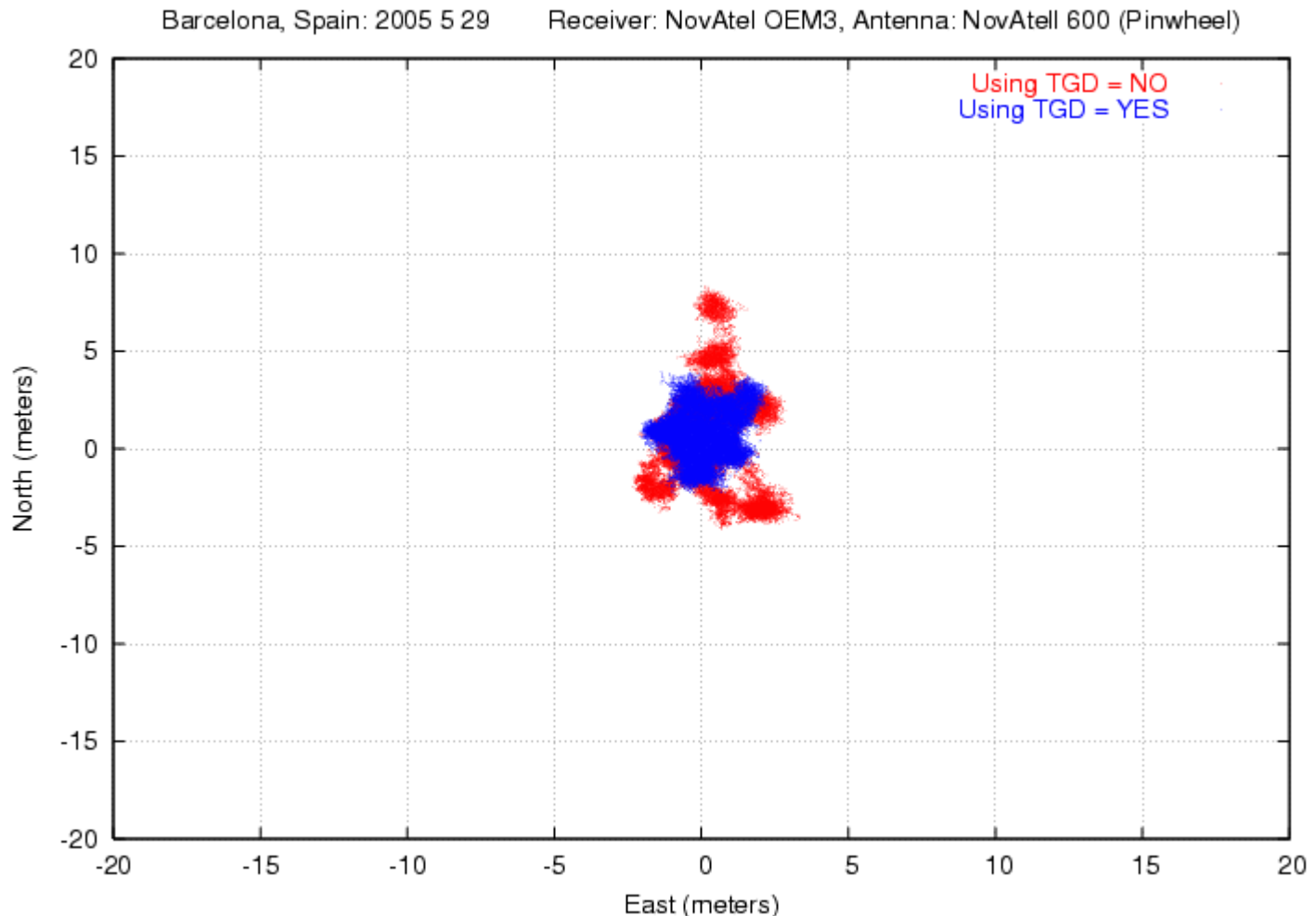
Range variation: Instrumental delays (TGD)



Vertical error comparison



Horizontal error comparison



Measurement noise (thermal noise)

Antispoofing (A/S):

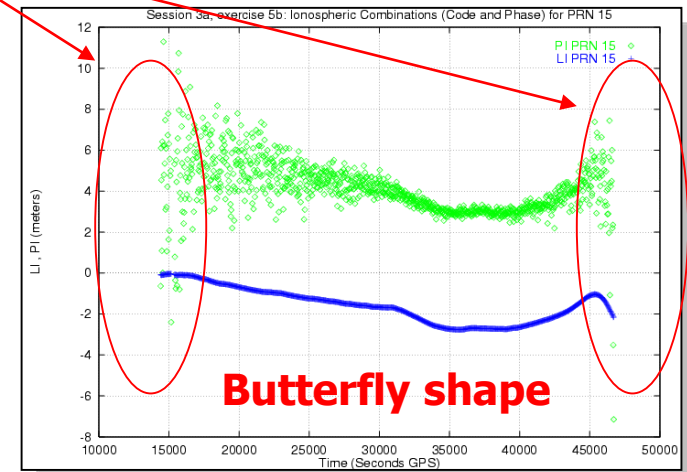
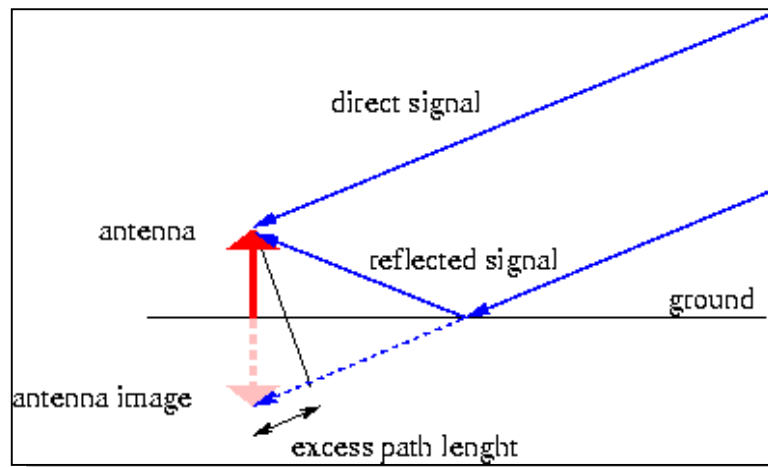
The code **P** is encrypted to **Y**.
 → Only the code **C** at frequency **L1** is available.

Wavelength (chip-length)	σ noise (1% of λ) [*]	Main characteristics
Code measurements		
300 m	3 m	<u>Unambiguous</u> but noisier
30 m	30 cm	
30 m	30 cm	
Phase measurements		
19.05 cm	2 mm	<u>Precise</u> but ambiguous
24.45 cm	2 mm	

[*] codes may be smoothed with the phases in order to reduce noise
 (i.e., **C1** smoothed with **L1** → 50 cm noise)

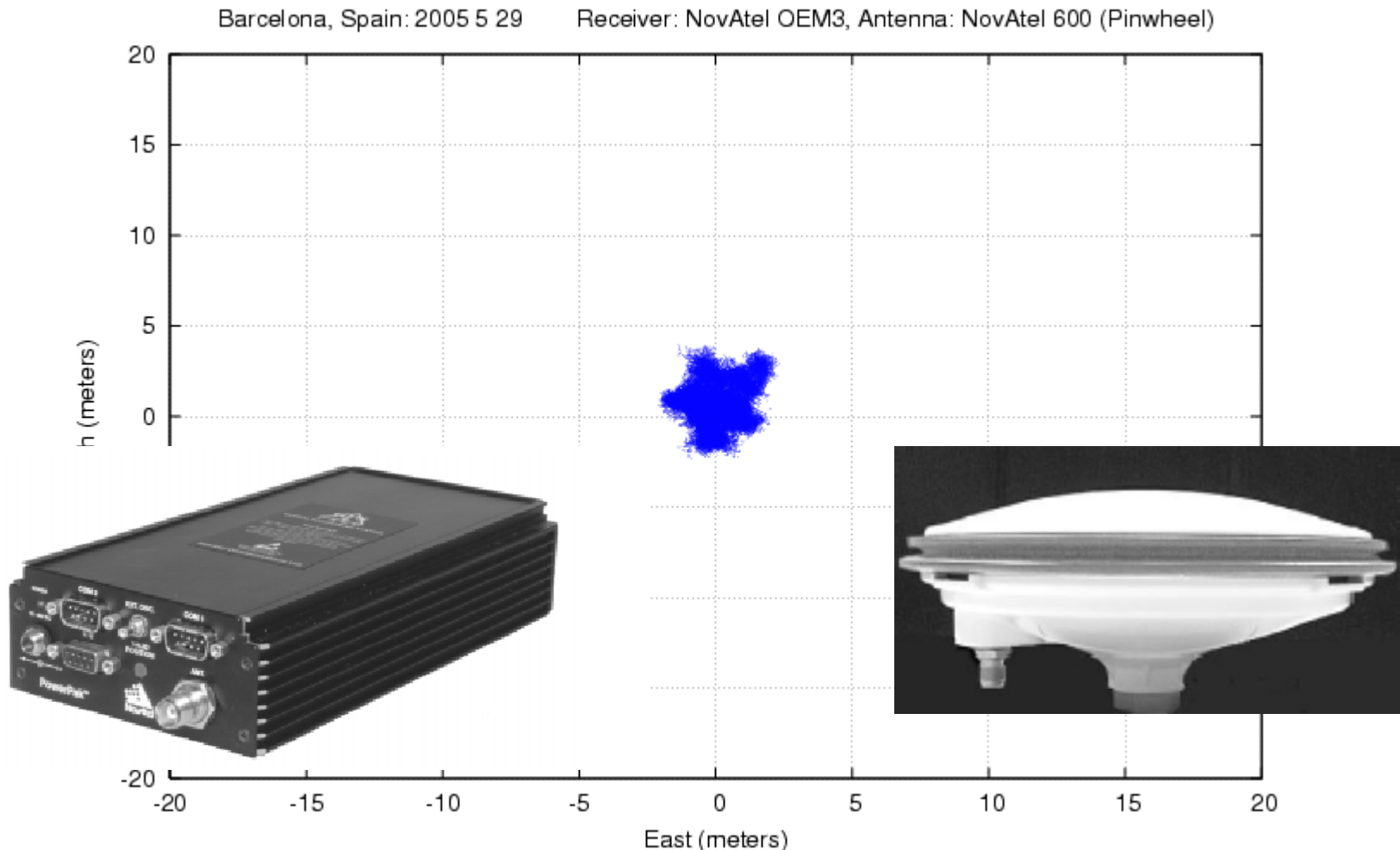
Multipath

- One or more reflected signals reach the antenna in addition to the direct signal. Reflective objects can be Earth surface (ground and water), buildings, trees, hills, etc.
- It affects both code and carrier phase measurements, and it is more important at low elevation angles.



- **Code:** up to 1.5 chip-length → up to 450m for C1 [theoretically]
Typically: less than 2-3 m.
- **Phase:** up to $\lambda/4$ → up to 5 cm for L1 and L2 [theoretically]
Typically: less than 1 cm

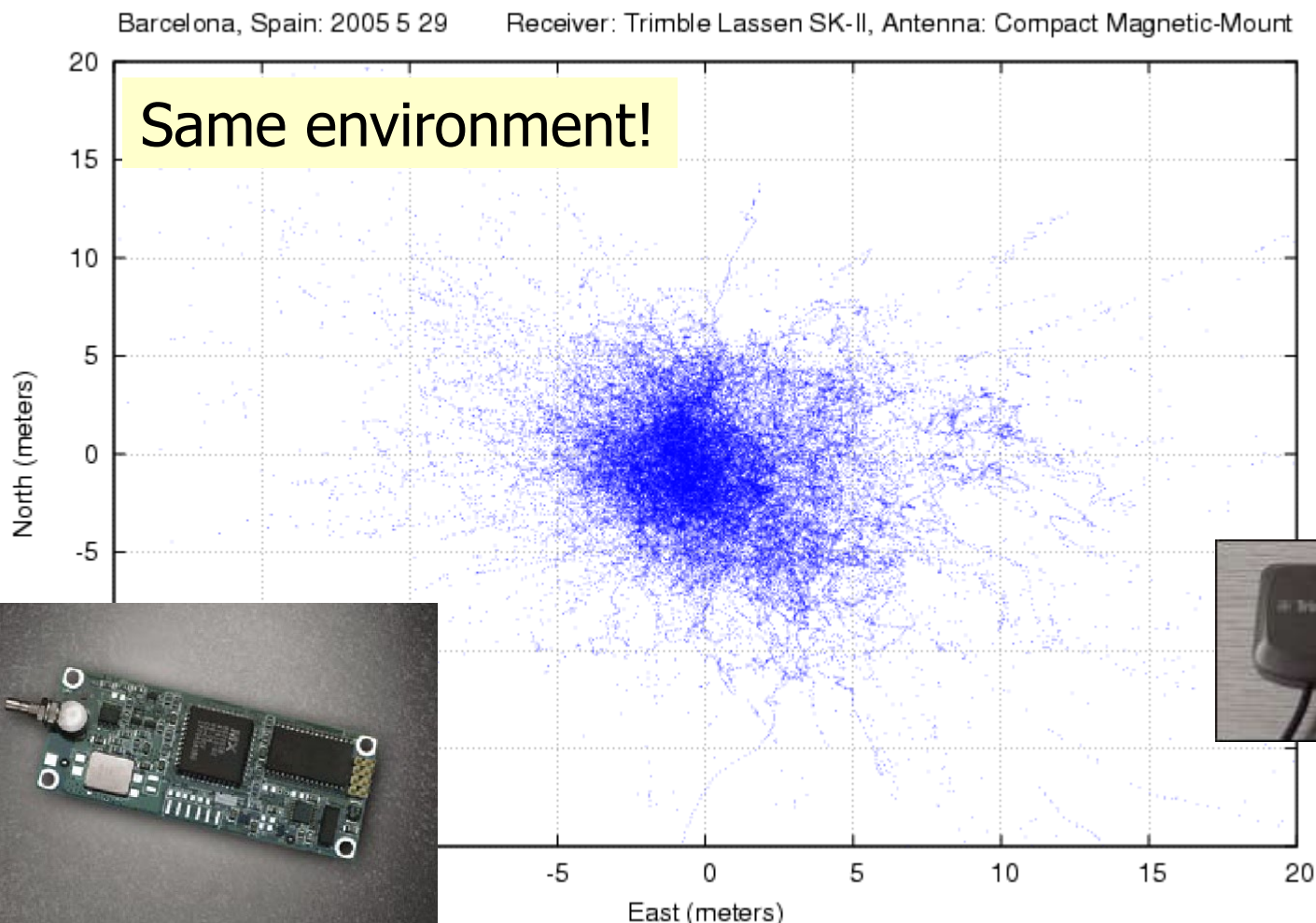
Receiver and multipath noise



GPS standalone (C1 code)

10,000 €

Receiver noise and multipath



GPS standalone (C1 code)

100 €

Contents

Measurements modelling and error sources

1. Introduction: Linear model and Prefit-residual
2. Code measurements modelling
3. Example of computation of modelled pseudorange

Example of Computation of modeled pseudorange

Using data of files **gage2860.98o** and **brdc2860.98n**, compute "by hand" the modeled pseudorange for satellite PRN 14 at t=38230 sec (10h37m10s).

$$Cl_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c \left(d\bar{t}^{sat} + \Delta rel^{sat} \right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

Follow these steps:

See also exercise 5, Session 5.2 in [RD-2]

1. Select orbital elements closer to $t = 38230$ s
2. Compute satellite clock offset
3. Compute satellite-receiver aprox. geometric range
 - 3.1 *Compute emission time from receiver (reception) time-tags and code pseudorange.*
 - 3.2 *Compute satellite coordinates at emission time*
 - 3.3 *Compute approximate geometric range.*
4. Compute satellite Instrumental delay (TGD):
5. Compute relativistic satellite clock correction
6. Compute tropospheric delay
7. Compute ionospheric delay
8. Compute modeled pseudorange from previous values:

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c \left(d\bar{t}^{sat} + \Delta rel^{sat} \right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

- Selection of orbital elements:** From file **brdc2860.98n**, select the **last transmitted** navigation message block **before instant $t=38230$ s (10h37m10s)**.

Transmission time:

979 208818 → 10h 0m 18s

PRN 14 98 10 13 12 0 0 +5.65452501178E-06 +9.09494701773E-13 +0.000000000000E+00
 +1.280000000000E+02 -6.100000000000E+01 +4.38125402624E-09 +8.198042513605E-01
 -3.31364572048E-06 +1.09227513894E-03 +5.67547976971E-06 +5.153795101166E+03
 +2.160000000000E+05 -6.33299350738E-08 }52E+00 -3.725290298462E-09
 +9.73658001335E-01 +2.74031250000E+02 }83E+00 -8.081050495434E-09
GPS sec of week +1.000000000000E+00 **GPS week** +9.790000000000E+02 +0.000000000000E+00
 +0.000000000000E+00 -2.32830643654E-09 +1.280000000000E+02
+2.088180000000E+05 +0.000000000000E+00 +0.000000000000E+00 +0.000000000000E+00

2. Satellite clock offset computation: From file

brdc2860.98n, compute satellite clock offset at time $t=3830$ s for PRN14:

	t_0	a_0	a_1	a_2
14	98 10 13 12 0 0	+5.65452501178E-06	+9.09494701773E-13	+0.000000000000E+00
	+1.280000000000E+02	-6.100000000000E+01	+4.38125402624E-09	+8.198042513605E-01
	-3.31364572048E-06	+1.09227513894E-03	+5.67547976971E-06	+5.153795101166E+03
	+2.160000000000E+05	-6.33299350738E-08	+1.00409621952E+00	-3.725290298462E-09
	+9.73658001335E-01	+2.74031250000E+02	+2.66122811383E+00	-8.081050495434E-09
	-1.45720352451E-10	+1.00000000000E+00	+9.79000000000E+02	+0.000000000000E+00
	+3.200000000000E+01	+0.00000000000E+00	-2.32830643654E-09	+1.280000000000E+02
	+2.08818000000E+05	+0.00000000000E+00	+0.00000000000E+00	+0.000000000000E+00

$$t_0 = 12h \ 0m \ 0s = 43200 \ s$$

$$d\bar{t}^{sat} = a_0 + a_1(t-t_0) + a_2(t-t_0)^2 = 5.65 \cdot 10^{-6} \text{ s}$$

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c \left(d\bar{t}^{sat} + \Delta rel^{sat} \right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

3. Satellite-receiver geometric range computation:

Use the following values (4789031, 176612, 4195008) as approximate coordinates.

3.1: Emission time computation from receiver time-tag and code pseudorange:

$$T_{\text{emis}} = t_R(T_{\text{recep}}) - (C1/c + dt^{\text{sat}})$$

Measurement
file gage2860.98o



Pseudorange *C1* at receiver time-tag
t=38230sec: *C1*= 23585247.703 m

Ephemeris file
brdc2860.98n



Satellite clock offset at t=38230 sec
 $dt^{\text{sat}} = 5.65 \cdot 10^{-6}$ sec (see previous results)

Thence, the emission time in GPS system time is:

$$\begin{aligned} T_{\text{emis}} &= 38230 - (23585247.703/c + 5.65 \cdot 10^{-6}) \\ &= 38230 - (0.07867 + 5.65 \cdot 10^{-6}) \\ &= \mathbf{38229.921 \text{ sec (10h37m09s921).}} \end{aligned}$$

(where $c=299792458 \text{ m/s}$)

Note:

From RINEX measurement file **gage2860.98o**, select the **C1** pseudorange measurement at receiver time-tag for **PRN14**:

PRN 14

$t = 38230 \text{ sec} = 10\text{h } 37\text{m } 10\text{s}$

4	L1	L2	C1	P2	# / TYPES OF OBSERV
98 10 13 10 37	10.0000000	0	5G18G14G16G 4G19		
5007753.999		0.000	20143892.105		0.000
-220595.001		0.000	23585247.703		0.000
1305085.999		0.000	23146887.826		0.000
6246118.999		0.000	20798091.711		0.000
-19853878.999		0.000	22235319.057		0.000

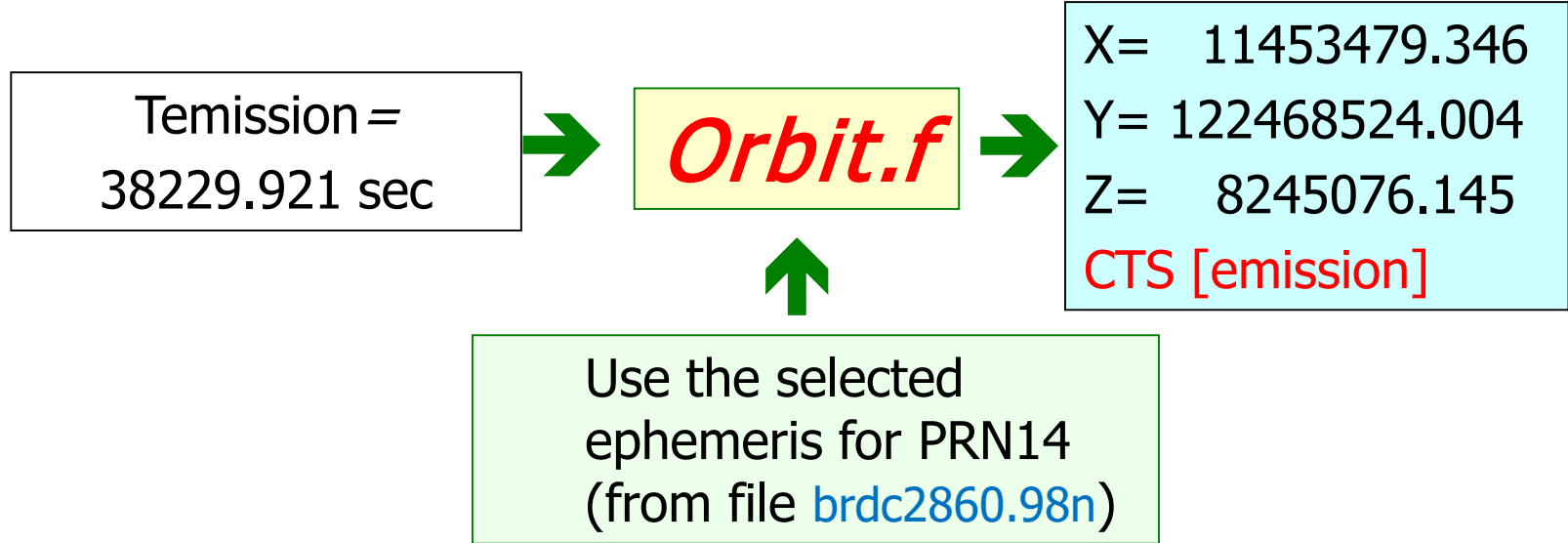
Thence:

Measurement
file **gage2860.98o**



Pseudorange **C1** at receiver time-tag
 $t=38230\text{sec}$: **C1= 23585247.703 m**

3.2: Satellite coordinates at emission time pseudorange:



The previous coordinates are given in an Earth-Centered-Earth-Fixed reference frame (CTS) at $T_{\text{emission}} = 38229.921$ s. This reference frame rotates by an amount " $\omega_E \Delta t$ " during traveling time $\Delta t = T_{\text{reception}} - T_{\text{emission}}$.

$$(X^{\text{sat}}, Y^{\text{sat}}, Z^{\text{sat}})_{\text{CTS}[\text{reception}]} = R_3(\omega_E \Delta t) \cdot (X^{\text{sat}}, Y^{\text{sat}}, Z^{\text{sat}})_{\text{CTS}[\text{emission}]}$$

$$(X^{\text{sat}}, Y^{\text{sat}}, Z^{\text{sat}})_{\text{CTS[reception]}} = R_3(\omega_E \Delta t) \cdot (X^{\text{sat}}, Y^{\text{sat}}, Z^{\text{sat}})_{\text{CTS[emission]}}$$

$$\begin{pmatrix} 11453350.377 \\ 122468589.797 \\ 8245076.145 \end{pmatrix}_{\text{CTS[reception]}} = \begin{pmatrix} \cos(\omega_E \Delta t) & \sin(\omega_E \Delta t) & 0 \\ -\sin(\omega_E \Delta t) & \cos(\omega_E \Delta t) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 11453479.346 \\ 122468524.004 \\ 8245076.145 \end{pmatrix}_{\text{CTS[emission]}}$$

$$\omega_E \Delta t = -5.74 \cdot 10^{-6} \text{ rad.} \quad (\text{where } \omega_E = 7.2921151467 \cdot 10^{-5} \text{ rad / sec})$$

$$\Delta t = -\frac{\rho_{0, \text{rec}}^{\text{sat}}}{c} = -0.079 \text{ sec}$$

$$\rho_{0, \text{rec}}^{\text{sat}} = \sqrt{(x^{\text{sat}} - x_{0, \text{rec}})^2 + (y^{\text{sat}} - y_{0, \text{rec}})^2 + (z^{\text{sat}} - z_{0, \text{rec}})^2} \approx 23616673.3 \text{ m}$$

$$(x, y, z)^{\text{satellite}} \approx (11453479, 122468524, 8245076)$$

$$(x_0, y_0, z_0)_{\text{receiver}} \approx (4789031, 176612, 4195008)$$

An approximate value
is enough to compute
 Δt .

Note: Both satellite and receiver coordinates must be given in the same reference system!

→ the CTS[reception] will be used to build navigation equations.

3.2: Geometric range computation

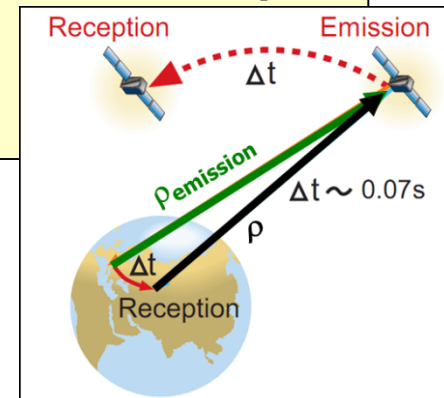
The geometric range between **satellite coordinates at emission time** and the “approximate position of the receiver” at reception time (*both coordinates given in the same reference system [for instance the CTS system at reception time]*) is computed by:

$$\rho_{0,receiver}^{satellite} = \sqrt{(x^{sat} - x_{0,rec})^2 + (y^{sat} - y_{0,rec})^2 + (z^{sat} - z_{0,rec})^2} = 23616699.124m$$

$$(x, y, z)^{satellite} = (11453350.2771, 22468589.7975, 8245076.1448)_{CTS[reception]}$$

$$(x_0, y_0, z_0)_{receiver} = (4789031, 176612, 4195008)_{CTS[reception]}$$

“Approximate” receiver coordinates at reception time.



$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c \left(d\bar{t}^{sat} + \Delta rel^{sat} \right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

4. Time Group Delay (TGD) or Satellite Instrumental delay.

→ From file brdc2860.98n, compute the TGD for PRN14:

PRN

TGD (in sec)

14	98	10	13	12	0	0	+5.65452501178E-06	+9.09494701773E-13	+0.000000000000E+00
							+1.28000000000E+02	-6.10000000000E+01	+4.38125402624E-09
							+8.198042513605E-01	-3.31364572048E-06	+1.09227513894E-03
							+5.67547976971E-06	+5.153795101166E+03	+2.16000000000E+05
							-6.33299350738E-08	+1.00409621952E+00	-3.725290298462E-09
							+9.73658001335E-01	+2.74031250000E+02	+2.66122811383E+00
							-8.081050495434E-09	-1.45720352451E-10	+1.00000000000E+00
							+9.79000000000E+02	+0.00000000000E+00	+3.20000000000E+01
							+0.00000000000E+00	-2.32830643654E-09	+1.28000000000E+02
							+2.08818000000E+05	+0.00000000000E+00	+0.00000000000E+00

$$\text{TGD} = -2.32830643654\text{E-09} * c = -0.69801 \text{ m}$$

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c \left(d\bar{t}^{sat} + \Delta rel^{sat} \right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

5. Relativistic clock correction:

gAGE

PRN

e

sqrt(a)

```

14 98 10 13 12 0 0 +5.65452501178E-06 +9.09494701773E-13 +0.000000000000E+00
+1.280000000000E+02 -6.100000000000E+01 +4.38125102624E-09 +8.198042513605E-01
-3.31364572048E-06 +1.09227513894E-03 +5.67547976971E-06 +5.153795101166E+03
+2.160000000000E+05 -6.33299350738E-08 +1.00409621952E+00 -3.725290298462E-09
+9.73658001335E-01 +2.74031250000E+02 +2.66122811383E+00 -8.081050495434E-09
-1.45720352451E-10 +1.000000000000E+00 +9.790000000000E+02 +0.000000000000E+00
+3.200000000000E+01 +0.000000000000E+00 -2.32830643654E-09 +1.280000000000E+02
+2.088180000000E+05 +0.000000000000E+00 +0.000000000000E+00 +0.000000000000E+00
    
```

**Temission =
38229.921 s**



Orbit.f



E = 0.095 rad
(eccentric anomaly)

$$\Delta rel^{sat} = -2 \frac{\sqrt{\mu a}}{c^2} e \sin(E) = -2.3 \cdot 10^{-10} \text{ s}$$

$$\mu = 3.986005 \cdot 10^{14} \text{ m}^3 \text{ s}^{-2}$$

$$c = 299792458 \text{ m s}^{-1}$$

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c \left(d\bar{t}^{sat} + \Delta rel^{sat} \right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

6. Tropospheric correction

$$Trop_{rec}^{sat} = (d_{dry} + d_{wet}) m(elev) = 6.76m$$

$$d_{dry} = 2.3 e^{-0.116 \cdot 10^{-3} H} = 2.3m$$

$$d_{wet} = 0.1m$$

$$m(elev) = \frac{1.001}{\sqrt{0.002001 + \sin^2(elev)}}$$

See klob.f

$$elev = 20.57 \frac{\pi}{180} = 0.359rad$$

$$H = 160m \quad (\text{height over the ellipsoid})$$

$$(x, y, z)_{rec} \rightarrow [car2geo] \rightarrow (Lon, Lat, H)_{rec}$$

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c \left(\overline{dt}^{sat} + \Delta rel^{sat} \right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

7. Ionospheric correction

(time, r_{sta} , r^{sat} , $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \beta_0, \beta_1, \beta_2, \beta_3$) \rightarrow [Klob] \rightarrow Iono=10.26m

2	NAVIGATION DATA	GPS	RINEX VERSION/ TYPE
XPRINT v1.1	gAGE	00/06/04 17:36:23	PGM / RUN BY / DATE
gAGE BROADCAST EPHEMERIS FILE			COMMENT
$+1.9558E-08$ $+0.0000E+00$ $-1.1921E-07$ $+0.0000E+00$			ION ALPHA
$+1.2288E+05$ $-1.6384E+04$ $-2.6214E+05$ $+1.9661E+05$			ION BETA
$-8.381903171539E-09$ $-1.421085471520E-14$ 405504			979 DELTA.UTC: A0,A1,T,W
12			LEAP SECONDS
			END OF HEADER

$t = 38230 \text{ sec}$

$(x, y, z)^{satellite} = (11453350.2771, 22468589.7975, 8245076.1448)_{CTS[reception]}$

$(x_0, y_0, z_0)_{receiver} = (4789031, 176612, 4195008)_{CTS[reception]}$

Approximate values for receiver or satellite coordinates are enough

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c \left(d\bar{t}^{sat} + \Delta rel^{sat} \right) + Trop_{rec}^{sat} + \boxed{Ion_{1rec}^{sat}} + TGD^{sat}$$

7. Compute the modeled pseudorange.

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{rec,0}^{sat} - c \left(d\bar{t}^{sat} + \Delta rel^{sat} \right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

$$\rho_{0,rec}^{sat} = 23616699.124 \text{ m}$$

$$c d\bar{t}^{sat} = 5.65 \cdot 10^{-6} c = 1693.828 \text{ m}$$

$$c \Delta rel^{sat} = -2.33 \cdot 10^{-10} c = -0.071 \text{ m}$$

$$Trop_{rec}^{sat} = 6.760 \text{ m}$$

$$Ion_{1rec}^{sat} = 10.260 \text{ m}$$

$$TGD^{sat} = -0.698 \text{ m}$$



$$C1_{rec}^{sat}[\text{modelled}] = 23615021.689 \text{ m}$$

Prefit residual:

Is the difference between measured and modeled pseudorange

$$\text{Pref}_{rec}^{sat} = C1_{rec}^{sat} - C1[\text{mod}]_{rec}^{sat} = \rho_{rec}^{sat} - \rho_{0,rec}^{sat} + c dt_{rec} + K_{1rec} + \varepsilon$$

In the previous example (PRN14 at $t = 38230$ s):

$$\text{Pref} = 23585247.703 - 23615021.689 = -29773.986 \text{ m}$$

Previously calculated

From measurement file

References

- [RD-1] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 1: Fundamentals and Algorithms. ESA TM-23/1. ESA Communications, 2013.
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The Learning material is composed by a collection of slides for **Theory & Laboratory** exercises.

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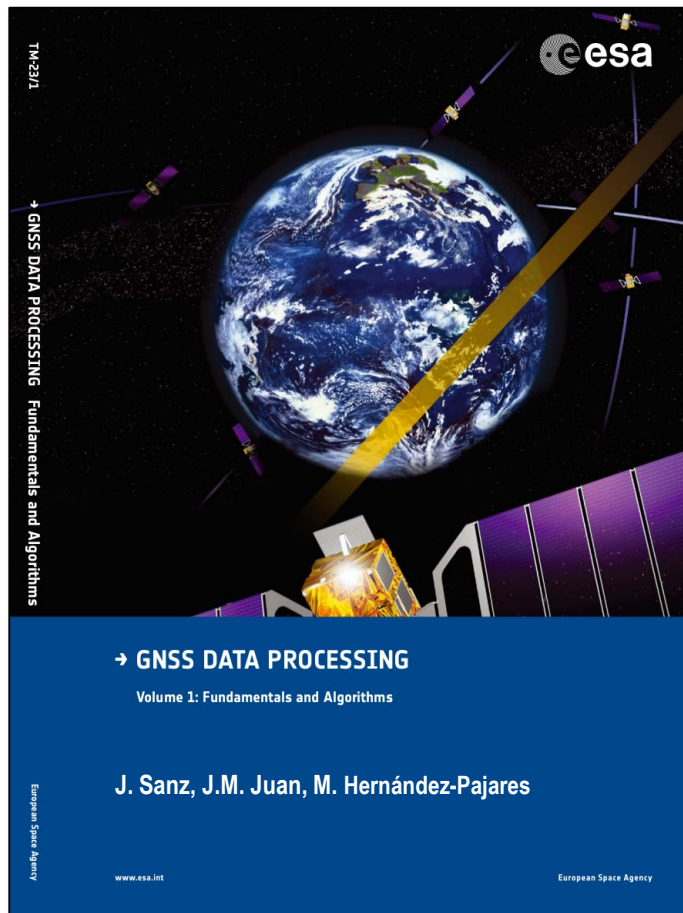
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