Makeup Homework

Vancouver Summer Program 2017
Algorithms and the Internet
Due: August 12, 2017, by 11:59 p.m.

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1. (Dinner Time!) Josh, the program coordinator of VSP 2017, is organizing an end-of-term dinner gathering for all VSP students. Josh has n students to invite. However, Josh believes that Facebook is the true model of friendship, so he prepared a list of pairs of students who are Facebook friends. He wants to invite as many students as possible, but subject to the following constraints: at the dinner, each student should have at least *five* students with whom they are friends on Facebook, and *five* other students with whom they are not Facebook friends. Give an efficient algorithm that takes as input the list of all n VSP students and the list of pairs who are Facebook friends and returns the best choice of dinner invitees. Analyze the running time of your algorithm.

2. In some other planet far far away, where life also happens to exist, there are three types of organisms, A, B and C. Life evolves on this planet according to the following rules (and only these rules):

   - When an organism of type A and an organism of type B mate, they both become type C organisms.
   - When an organism of type A and an organism of type C mate, they both become type B organisms.
   - When an organism of type B and an organism of type C mate, they both become type A organisms.

   When we earthlings detected this planet, there were 10 type-A organisms, 11 type-B organisms and 111 type-C organisms. Is it possible that life on this alien planet evolves to have only one organism type?

3. Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$ be a generic polynomial of degree n. Is it possible to select $a_0, a_1, a_2, \ldots, a_n$ such that a_i is an integer and $a_0 = \pm 1$ such that $f(x)$ only emits primes for $x \in \mathbb{N}$. ($\mathbb{N}$ is the set of natural numbers.) n is some arbitrary natural number. You must prove the correctness of your answer.