In-Class Activity #7

Vancouver Summer Program 2017

Algorithms and the Internet

(Paths in a grid) Given is a grid. Count all shortest paths along the grid from $(0, 0)$ to (a, b). Assume that $(0, 0)$ is the bottom left corner of the grid, and that a, b are nonnegative integers. The grids we consider are two dimensional and can be viewed as a graph (see figure).

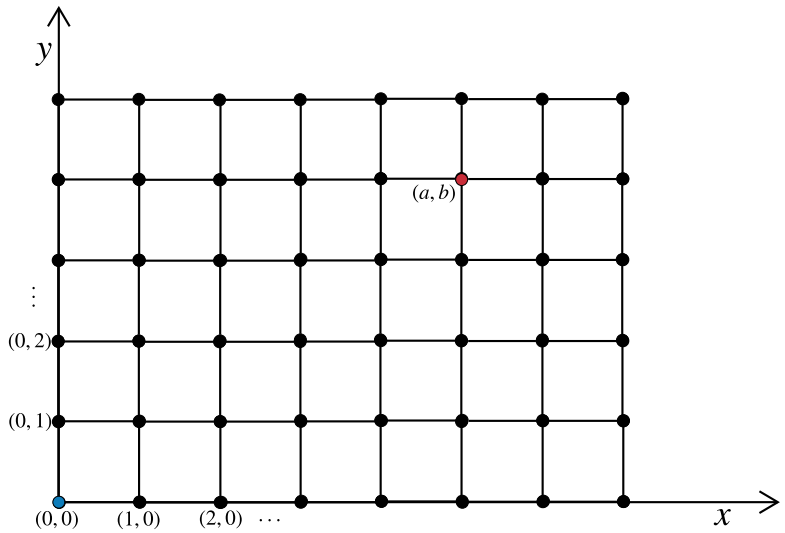

Figure 1: Grid (2D Integer Lattice)

1. Using a combinatorial argument, derive an equation in terms of a and b for the number of shortest paths between $(0, 0)$ and (a, b).

2. Devise a iterative algorithm (i.e., bottom-up, DP) that counts the number of paths between $(0, 0)$ and (a, b). [You may want first to derive a recursive algorithm, then add memoization to it, then examine the recursion tree to observe redundant computation and then convert the memoization-based recursive algorithm into an iterative one that uses less memory compared to the recursive memoization procedure.]

3. Now some grid points are blocked by some obstacles. Count all paths that go from $(0, 0)$ to (a, b) in $O(ab)$ steps and avoid all obstacles. If a point has an obstacle then as soon as you reach the point you cannot go anywhere else except to go back.