Gravity Gradient Passive Control

Trade Study Number – A0006

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Purpose of Trade Study (Forsee)

The purpose of TS A0006 is to explore the possibility of using gravity gradient stabilization for periods when the payload is offline. This is to introduce power savings while the payload is offline and reduce the loads on the chosen active attitude control system.

Investigation

Derivations of EOM

Expressing the gravity gradient torque in a body fixed frame for a spacecraft in LEO, we know that the gravity gradient torque is given by:

$$\tau_{gg}^2 = \frac{3\,\mu}{|r^2|^5}\,\hat{r}^2\,J^2 r^2$$

Where
$\hat{r}$ = A vector from Earth's center to the spacecraft
μ = Earth's gravitational constant parameter
and ^^{2} denotes the body frame of the satellite and ^^{5} denotes a function to the fifth power.

Note: In the orbit frame, $r^1 = \begin{bmatrix} 0 \\ 0 \\ d \end{bmatrix}$, where d is the distance from the center of the earth

Assuming that the rotation matrix from the orbit frame to the body frame is:

$$R_2^1 = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

And that the moment of inertia is:

$$J^2 = \begin{bmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_3 \end{bmatrix}$$

Noting that the orbit angular velocity, n, satisfies:

$$n = \sqrt{\frac{\mu}{d^3}}$$

Explicitly computing the vector τ_{gg}^2 in terms of n, J and A and assuming the small angle approximation, we get:

$$\tau_{gg}^2 = 3\,n^2 \begin{bmatrix} A_{32}A_{33}(J_3 - J_2) \\ A_{31}A_{33}(J_1 - J_3) \\ A_{31}A_{32}(J_2 - J_1) \end{bmatrix} \dashrightarrow \mathit{Small\ Angle\ Approx} \;\; = -3\,n^2 \begin{bmatrix} \theta_1\,(J_3 - J_2) \\ -\theta_2\,(J_1 - J_3) \\ 0 \end{bmatrix}$$

The linearized equations of motion can be rewritten and approximated such that:

$$\begin{aligned} -3\,n^2(J_2 - J_3)\,\theta_1 &= J_1\dot{\omega}_1 - (J_2 - J_3)(-n\,\omega_3) \\ 3\,n^2(J_3 - J_1)\,\theta_2 &= J_2\dot{\omega}_2 \\ 0 &= J_3\dot{\omega}_3 - (J_1 - J_2)(-n\,\omega_1) \end{aligned}$$

Several steps were skipped in the following derivation. Listed below are the resulting EOM.

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & n & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -n & 0 & 0 & 0 & 0 & 1 \\ \frac{(J_3 - J_2)3\,n^2}{J_1} & 0 & 0 & 0 & 0 & \frac{(J_3 - J_2)\,n}{J_1} \\ 0 & \frac{(J_3 - J_1)3\,n^2}{J_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{(J_2 - J_1)\,n}{J_3} & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} + \begin{bmatrix} 0 \\ n \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

These equations of motion consider only gravity gradient stabilization for a spacecraft in LEO.

Simulation

Using the equations of motion derived in the previous section, a simulation was created to examine the motion of the satellite. Since the structures team have yet to provide a detailed CAD along with the related moment of inertias, two different arbitrary moment of inertias were tested for this trade study with the expectation of going back and revising with the actual number for the Fire-LOC satellites. The tested moment of inertia tensors tested are as follows:

Assuming a homogeneous 6U CubeSat with mass = 7.98kg	**From a previous 6U cubesat**

`J1 = 0.0665;`
`J2 = 0.0865;`
`J3 = 0.0333;`
	`J1 = 0.1614;`
	`J2 = 0.1854;`
	`J3 = 0.1397;`

Lastly, each case was tested with the expected initial conditions and with an extreme case of initial conditions. We first test with parameters that we would expect after the active attitude control is turned off. Since the payload requires a pointing accuracy of +- 0.0699deg while active, the initial conditions were [in deg & deg/second]:

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \begin{bmatrix} 0.115 \\ 0.115 \\ 0.115 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

For the extreme case, the following initial conditions were used. [in deg & deg/second]

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \begin{bmatrix} 5.75 \\ 5.75 \\ 4 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

As shown in later sections, both cases showed capability for gravity gradience being a viable option for passive control. It is important to note however that these simulations show a timespan much larger than is expected. The areas of interest are the first 34minutes (2040 seconds) since that is roughly the amount of time the satellite will be out of its scanning region.

One notable difference when looking at the first 2000seconds is the peaks between the homogenous 6U and the arbitrary heritage 6U case. We see that for the homogenous case, the peak angle for any axis is much less than for the heritage case. This establishes a precedence on the design of the craft so that the resulting moment of inertia makes using gravity gradient viable.

It should be noted that test cases involving varying angular velocities were also observed but not documented.

Homogenous 6U Case

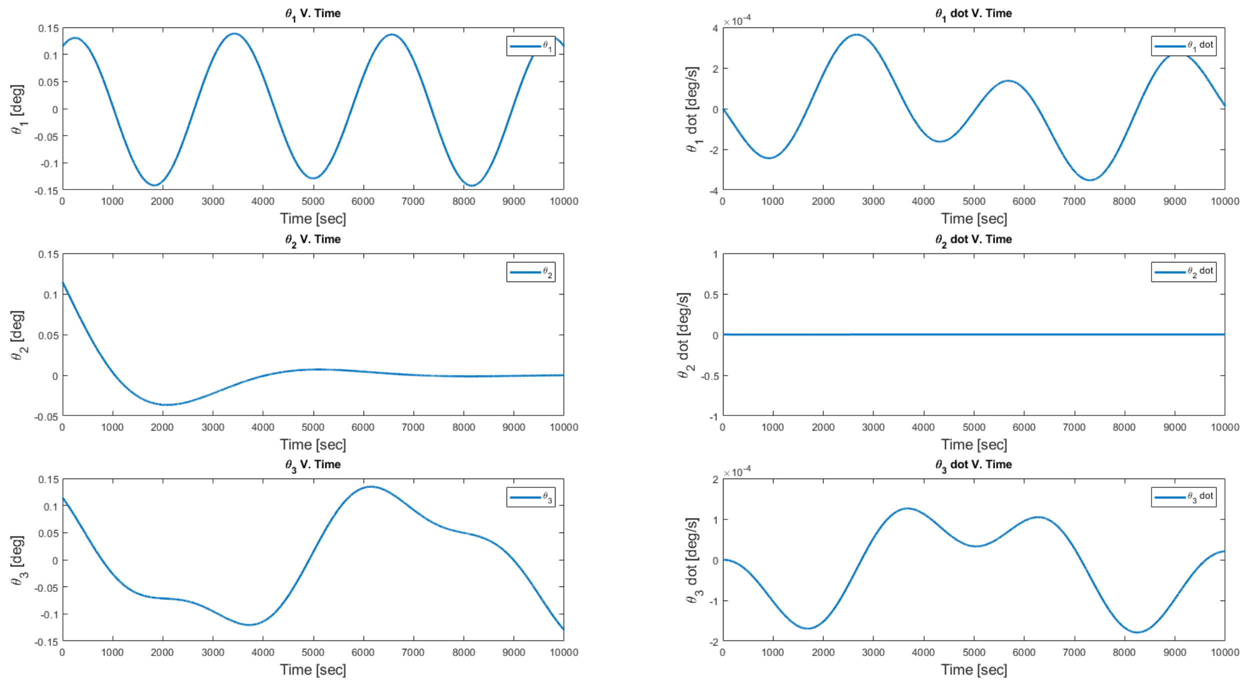

Figure 1: Realistic Initial Conditions for a homogenous 6u cubesat

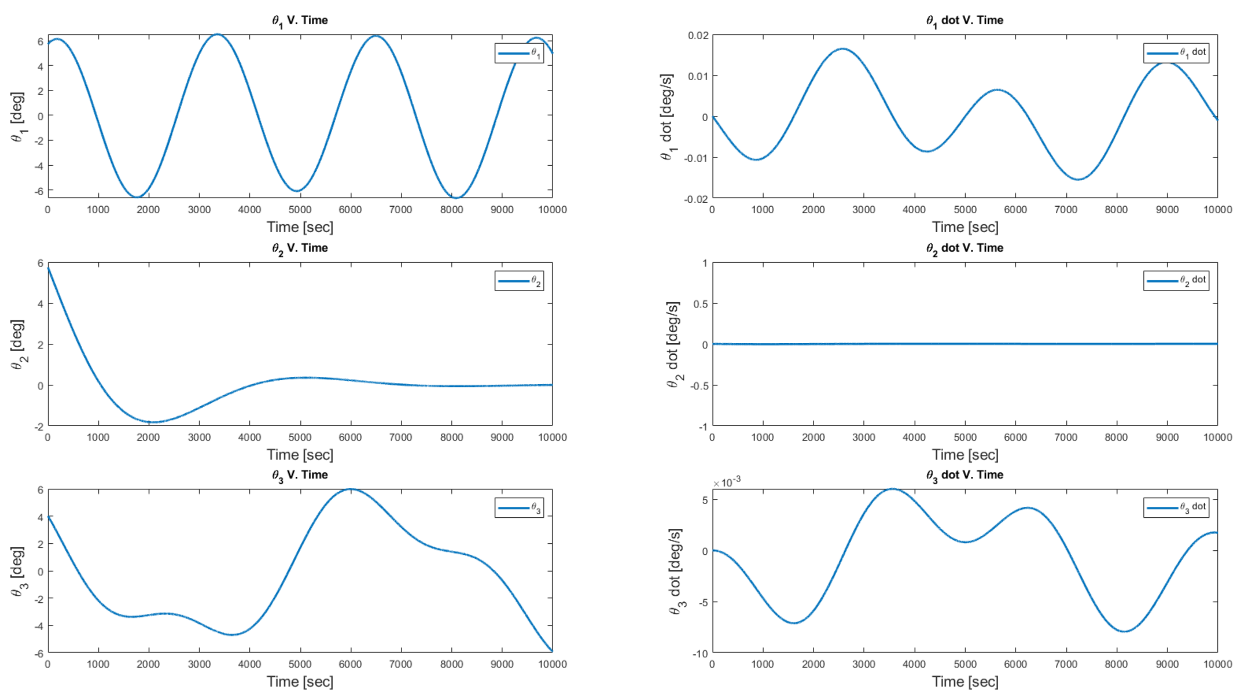

Figure 2: Extreme Initial Conditions for a homogenous 6u cubesat

Arbitrary Heritage 6U Case

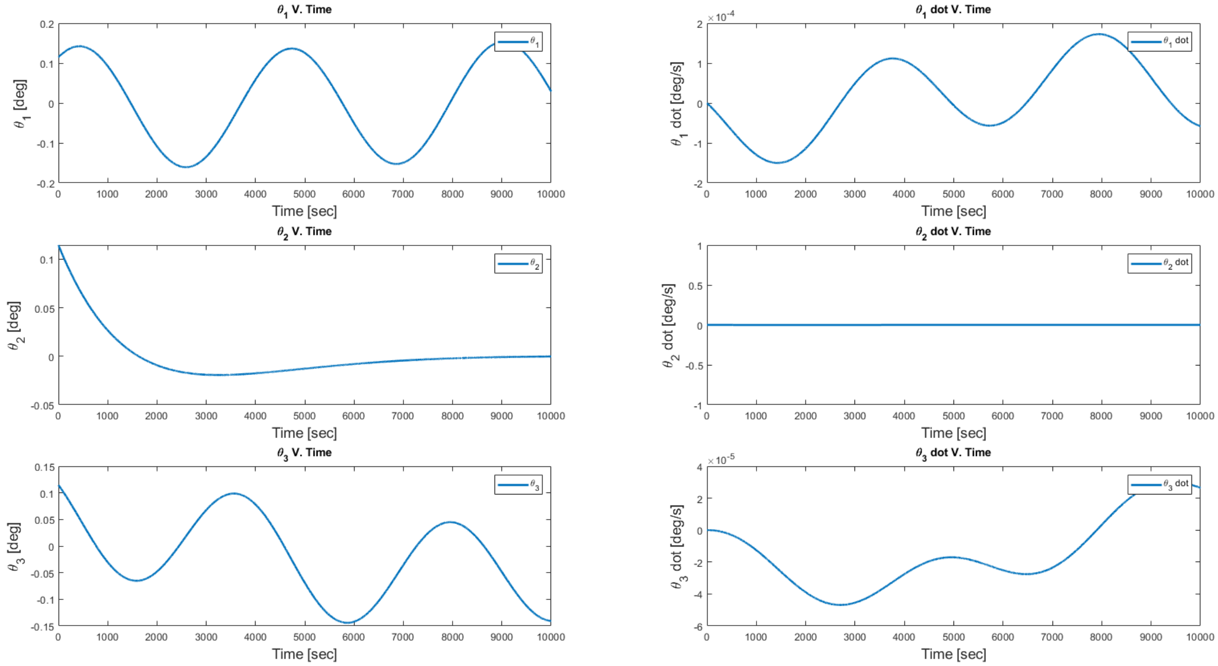

Figure 3: Realistic Initial Conditions for an arbitrary heritage 6U cubesat

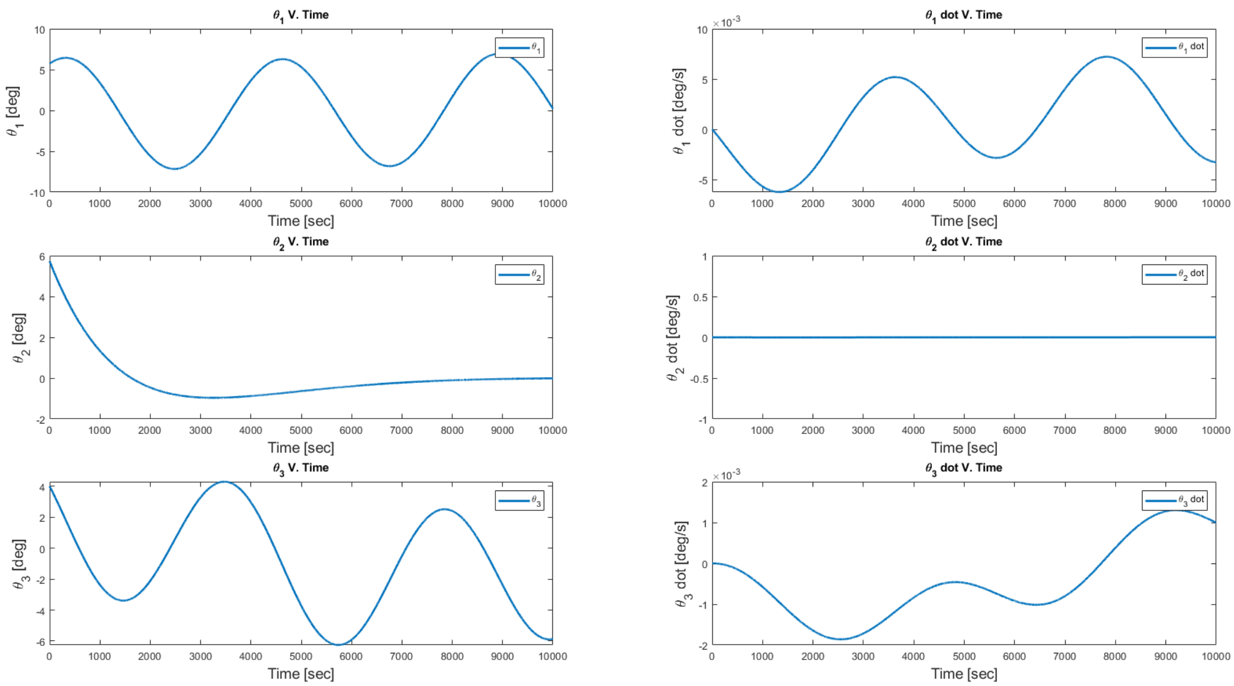

Figure 4: Extreme Initial Conditions for an arbitrary heritage 6U cubesat

Record

It was shown that the design of the craft effects the resulting moment of inertia which greatly effects the ability to employ gravity gradient stabilization on the off-modes of satellite operation. If we observe a large disturbance from the satellite within the first 2000 seconds as was observed for the heritage case, gravity gradience would not be a profitable option. This is because the power cost for returning to the desired pointing angle when the payload turns back on would negate the cost savings if the craft swayed greatly from the desired pointing angle. Larger swaying angles results is more power needed to return to that power when payload is back on. Criteria for designing the spacecraft with optimal moment of inertia for gravity gradient stabilization with examined in TS A0007.

Execute

Signatures:

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