2020

JMC 10A

REMAINS OPEN UNTIL THE DUE DATE

****Administration On An Earlier Date Is Not Even Possible****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL. PLEASE READ THE MANUAL BEFORE FEBRUARY 2, 2016.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 10 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions for this contest during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet, or media of any type is a violation of the competition rules.*

The June/July Math Competitions are brought to you by:

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JAA JMC

July Mathematics Competitions

1st ANNUAL

JMC 10A

July Mathematics Competition 10A

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL THE TIMER STARTS.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer clearly, edits in submission will not be accepted.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have 75 minutes to complete the test.
9. When you finish the exam, sign your name in the space provided on the Answer Form.

The Committee on the June/July Mathematics Competitions (CJMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. Students who score well on this JMC 10 will be invited to take the annual American National Invitational Mathematics Examination (ANIME) on Thursday, March 69, 6969 or Wednesday, March 69, 6969.

1. What is the value of x satisfying $4^3 = \sqrt{x^4\sqrt{4^x}}$?

   (A) 1 **(B)** 2 **(C)** 4 **(D)** 8 **(E)** 12

2. Maya starts at a point and walks 5 meters north, 3 meters east, and n meters south. If Maya is now 5 meters from her original location, what is the sum of all possible n?

   (A) 0 **(B)** 1 **(C)** 4 **(D)** 9 **(E)** 10

3. Two integers a and b satisfy $3^a \cdot 4^b = 6^6 \cdot 9^b$. What is $a + b$?

   (A) 4 **(B)** 8 **(C)** 9 **(D)** 12 **(E)** 15

4. What is the sum of the digits of the least 3 digit integer divisor of $9! = 9 \cdot 8 \cdot 7 \cdots 1$?

   (A) 1 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 9

5. Five years ago, the average of Albert and Bessy's ages was the same as Bessy's current age. If Albert is 20 years old, how old is Bessy?

   (A) 5 **(B)** 10 **(C)** 15 **(D)** 20 **(E)** 30

6. Cars A and B, travelling at constant, different speeds, are headed directly from Austin to Boston and from Boston to Austin respectively. Car A leaves at 9:00 AM, and Car B leaves an hour later. If the two cars meet when Car A is $\frac{2}{3}$ of the way to Boston, and Car A arrives at Boston at 3:00 PM, when does Car B reach Austin?

   (A) 6:00 PM **(B)** 6:30 PM **(C)** 7:00 PM **(D)** 7:30 PM **(E)** 8:00 PM

7. Three kids stand in a row from left to right. Simultaneously, each kid randomly points left or right. What is the probability that no two adjacent kids point at each other?

   (A) $\frac{1}{8}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{3}{8}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{3}{4}$

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8. The operator $\tau(n)$ denotes the number of divisors of n. What is the sum of the digits of the smallest positive integer n that satisfies $\tau(\tau(n^2)) = 4$?

(A) 3 **(B)** 6 **(C)** 8 **(D)** 9 **(E)** 12

9. If γ is a root of $x^2 + x + 1$, what is the value of $(1 - \gamma)(\gamma + 2)$?

(A) -3 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 3

10. In regular hexagon $ABCDEF$ with sides of length 3, diagonal $\overline{AD}$ intersects segments $\overline{BE}$ and $\overline{CE}$ at points P and Q respectively. What is the area of triangle FPQ?

(A) $\frac{3\sqrt{3}}{8}$ **(B)** $\frac{3\sqrt{3}}{4}$ **(C)** 2 **(D)** $\frac{9\sqrt{3}}{8}$ **(E)** $\frac{9\sqrt{3}}{4}$

11. Jack has eight sticks of different lengths in the set $\{1, 2, 3, \ldots, 7, 8\}$. How many nonempty subsets of these eight sticks can Jack choose so the range of the lengths is at most 4 meters?

(A) 64 **(B)** 79 **(C)** 80 **(D)** 81 **(E)** 128

12. Alice, Albert, Bella, Bernie, Carol, and Carl are randomly split into two indistinguishable groups of three. What is the probability that no two people whose names start with the same letter are in the same group?

(A) $\frac{1}{10}$ **(B)** $\frac{1}{5}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{2}{5}$ **(E)** $\frac{1}{2}$

13. Triangle ABC exists in the coordinate plane such that $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$ have slopes equal to $1, \frac{1}{3}$ and -1 respectively. If $\overline{AB}$ has length $\frac{8}{3}$, what is the length of $\overline{BC}$?

(A) $\frac{2\sqrt{2}}{3}$ **(B)** $\frac{4}{3}$ **(C)** $\frac{4\sqrt{2}}{3}$ **(D)** 4 **(E)** $\frac{8\sqrt{2}}{3}$

14. Steven has 4 lit candles and each candle is blown out with a probability $\frac{1}{2}$. After he finishes blowing, he randomly selects a possibly empty subset out of all the candles. What is the probability his subset has at least one lit candle?

(A) $\frac{5}{16}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{175}{256}$ **(D)** $\frac{11}{16}$ **(E)** $\frac{3}{4}$

15. What is the sum of all positive integers b greater than 1 such that the base 10 numbers 13, 167, and 233 all have the same last digit when expressed in base b?

(A) 2 **(B)** 13 **(C)** 17 **(D)** 22 **(E)** 35

16. What is the value of

$$\frac{(2019+2020)(2020+2021)(2021+2019)+2019\cdot 2020\cdot 2021}{2019\cdot 2020+2020\cdot 2021+2021\cdot 2019}?$$

(A) 2019 **(B)** 2020 **(C)** 2021 **(D)** 3030 **(E)** 6060

17. Let triangle ABC be acute and point D be the altitude from point A to $\overline{BC}$. If $AD = 24$ and $BC = 32$, what is the distance between the midpoints of $\overline{BD}$ and $\overline{AC}$?

(A) 16 **(B)** 18 **(C)** 20 **(D)** 24 **(E)** 25

18. Harvey *transforms* a four-digit number by reversing the order of its digits, subtracting 1 from all digits that are 1 more than a multiple of 3, and adding 1 to all even digits in that order. Harvey obtains 7793 after transformation. How many distinct numbers could he have transformed?

(A) 4 **(B)** 6 **(C)** 12 **(D)** 16 **(E)** 24

19. Circle ω_A has center M and radius 5. Line ℓ intersects ω_A at A, B such that $AB = 7$. Point P is on line ℓ outside of ω_A with $PA = 9$ and P closer to A. Circle ω_B with diameter $\overline{PM}$ intersects ω_A at two points X and Y. If the length of $\overline{XY}$ can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers, what is $m+n$?

(A) 23 **(B)** 29 **(C)** 41 **(D)** 73 **(E)** 133

20. Consider triangle ABC with medians $\overline{BE}$ and $\overline{CF}$ that intersect at G. If $AG = BC = 8$ and $CG = 6$, what is the length of $\overline{GE}$?

(A) $\sqrt{3}$ **(B)** $\sqrt{7}$ **(C)** $2\sqrt{2}$ **(D)** $2\sqrt{7}$ **(E)** $4\sqrt{2}$

21. What is the base-10 sum of all positive integers such that, when expressed in binary, have 7 digits and have no two consecutive 1's?

(A) 667 **(B)** 921 **(C)** 963 **(D)** 1022 **(E)** 1411

22. What is the units digit of the remainder when $17^7 + 17^2 + 1$ is divided by 307^2?

(A) 2 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 8

23. For each positive integer n, let $s(n)$ denote the sum of the digits of n when written in base 10. An integer x is said to be tasty if

$$s(11x) = s(101x) = s(1001x) = \cdots = 18.$$

How many *tasty* integers are less than 10^8?

(A) 6435 **(B)** 8008 **(C)** 10000 **(D)** 11440 **(E)** 12870

24. Define $f(k)$ as the number of real solutions x to $x^2 - \lfloor kx \rfloor = 0$. What is the value of

$$f(1) + f(2) + f(3) + \cdots + f(25)?$$

(A) 73 **(B)** 74 **(C)** 75 **(D)** 98 **(E)** 100

25. What is the sum of the digits of the positive integer n such that the number

$$\frac{1!\cdot 2!\cdot 3!\cdots 2019!\cdot 2020!}{n!}$$

is a perfect square?

(A) 2 **(B)** 4 **(C)** 8 **(D)** 10 **(E)** 12