Goal: 47
Total: 61

1. Graph $3i + 4$ and graph the circle centered at the origin that contains it. (1)

2. Find the magnitude and argument of $\frac{\sqrt{3}}{3} + i$. (1)

3. Prove that $|x||y| = |xy|$. (1)

4. Find $\sin(\frac{\pi}{3}) + \sin(\frac{2\pi}{3}) + \sin(\pi) + \cos(\pi) + \cos(\frac{2\pi}{3}) + \cos(\frac{\pi}{3})$. (2)

5. Prove that $|z| + |w| \geq |z + w|$ with equality when z, w have equivalent arguments. (This means that their angle when expressed in polar form is identical, or that the line through z, w passes the origin.) (2)

6. Prove that $\cosh^2(\theta) - \sinh^2(\theta) = 1$. (2)

7. Simplify $\sin(\frac{2\pi}{n}) + \sin(2\frac{2\pi}{n}) + ... + \sin(n\frac{2\pi}{n})$. (3)

8. Create a polynomial with integer coefficients that has a root of $3 + \sqrt[4]{2}$. (3)

9. Create a polynomial with integer coefficients with a root of $\sqrt{3} + \sqrt[4]{2}$. ($\star$ 6)

10. Prove that $31|5^{31} + 5^{17} + 1$. (3)

11. Prove that $x^2 + x + 1|x^{31} + x^{17} + 1$. (5)

12. Prove that for positive integers a, b, c, $x^2 + x + 1|x^{3a} + x^{3b+1} + x^{3c+2}$. (5)

13. Find $\sin(\frac{\pi}{2} + i\frac{2\pi}{3})$. (3)

14. Find all possible values of i^i. (3)

15. Find all possible values of $(-1)^i$. (2)

16. Find the form all values of $\ln(\frac{e^3\sqrt{3}}{2} + i\frac{e^3}{2})$ can take. (4)

17. Find $\Re(\log_2(2 + 2\sqrt{3}i))$. (4)

18. Find $\Im((\cos 12^\circ + i \sin 12^\circ + \cos 48^\circ + i \sin 48^\circ)^6)$. (5)

19. Evaluate $\sum_{n=0}^{\infty} \frac{\cos(n\theta)}{2^n}$ where $\cos(\theta) = \frac{1}{5}$. ($\star$ 6)