# Pedagogy

*Dennis Chen, suggestions by Alex Zheng*

Preliminary - Lesson 0

## § 1 Resources

### § 1.1 Dennis Chen

I highly recommend you read the essays on my website. My favorites are Sort, Where to Start? and Brewing and Flooding. Most of the handouts on my website aren't going to be of much help at this level, but I think the preview of my geometry book will be of interest.

Recommended reads for parents are AoPS Introduction to Geometry, "Starting Costs" in Escape Velocity, and Whims.

### § 1.2 Elsewhere

I recommend Evan Chen and Paul Graham's essays. Where to Start? has more extensive recommendations, especially for math.

## § 2 Pedagogy

### § 2.1 Intuition

One class of problems are those that are easy to verify the solution to, but said solution is hard to find. Having deep intuition and familiarity with common factorizations, configurations, and so on will help solve these problems, even if these types of things never appear in the official solution.

**Example 1 (AIME II 2019/1)** Two different points, $C$ and $D$, lie on the same side of line $AB$ so that $\triangle ABC$ and $\triangle BAD$ are congruent with $AB = 9$, $BC = AD = 10$, and $CA = DB = 17$. The intersection of these two triangular regions has area $\frac{m}{n}$, where $m$ and $n$ are relatively prime positive integers. Find $m + n$.

**Walkthrough:**

1. The numbers 10 and 17 should make you very, very suspicious there are Pythagorean Triples at play here.
2. Figure out how the triangles are configured.
3. Finish with similar triangles.

**Example 2 (AIME I 2015/3)** There is a prime number $p$ such that $16p+1$ is the cube of a positive integer. Find $p$.

**Walkthrough:**

1. Let $16p + 1 = a^3$.
2. Subtract 1 from both sides and factor.
3. $p$ is odd, because it is a prime not equal to 2.
4. Figure out what $a$ *feels like* it should be.
5. Find $p$ and verify that it is indeed prime for the value of $a$ you plugged in.

## § 2.2 Learning Competition Math

Above all, I want to emphasize the following: **Don't only use one teacher or class as a source**. And be wary of people who claim their class is the *key* to acing the AMCs or whatever. If you only take away one thing from all four weeks of this class, *this needs to be it.*

For most people, there are roughly three stages between the transition from computational to olympiad math contests in the United States.

1. Dump a lot of information (mostly about Algebra/Combo/NT) into your head.
2. Learn geometry, and *internalize* it.
3. Stay organized and have a clear mental map of what you're doing at all times.

With this in mind, remember that you can't expect to immediately gain knowledge in any one subject. You'll learn a lot of new theory on every subject, but you have to take the time to remember and really incorporate what you learn into your head before moving on.

## § 2.3 Problem Solving

When doing problems on your own time, there are a few general steps to take:

1. Try the problem.
   (a) If you can solve it, great! Make sure to realize *why* and *how* you solved the problem. Remember the techniques for future use.
   (b) If you can't solve the problem, look for a hint. If there isn't a specific hint, go to the solution, maybe read the one or two lines from where you're stuck, in order to get an idea of what to do next.
2. Try the problem, again.
   (a) If you can solve the problem now, this is still good! Again, make sure to realize *why* and *how* you solved the problem. Keep in mind where you were stuck and why you couldn't think of the next step. Incorporate these techniques for a time to come.
   (b) If you **still** can't solve the problem, **now** is when you can read the solution, or if there isn't one, ask someone who you believe could solve the problem.

3. Read the solution, even if you solved the problem. If there are multiple solutions, at least **skim all of them**. You might not be able to understand some of the more advanced solutions now, but at least it'll be in the back of your head for later. It's important to see what you missed, and if there's a better to way to do a problem than the method you constructed. If you didn't solve the problem, **remember why you couldn't solve it**.

4. With the solution(s) in mind, try the problem again with what you have learned. Now that you can solve it, internalize what you learned for later.

The takeaway from this is that whether you solved the problem or not, had the quickest method or the slowest, it's important that you not only **read the solution** but **remember the ones that you didn't come up with**, and always try the problem again. Never give up on a problem until you're sure that you couldn't have solved it.

On the other hand, everything should be done in moderation - some people give up too infrequently. (See Section 4 of Math Contest Platitudes, v3 by Evan Chen.)

## § 2.4 Typesetting in LaTeX

I highly recommend you keep your writeups for the homework sets in this class. You're going to be spending a lot of time on them, so you might as well make sure they're somewhere you can reference easily in the future. I recommend learning how to use LaTeX. A short 30-minute tutorial is linked here. A good online compiler for beginners is Overleaf - it requires little technical knowledge.

Just google commands as you need them. Don't start by reading a reference or whatever, you'll pick stuff up as you go. StackExchange is sufficient for pretty much every question you have. If you have any specific questions (how to set up a style file, why isn't asymptote working, how to use olympiad.asy), feel free to email me.

Why bother learning how to use LaTeX and Asymptote when something like Microsoft Word theoretically works? It's because of accessibility, portability, and editability. You have more options with what to do with it. If you want to include a solution in another document, you can just copy-paste it. And most importantly, **if you want to change something**, you can do it very easily. It also gives a good output.

## § 2.5 Competitions and Ego

Here's a secret I think few people will tell you overtly: Basically the only competition that matters is the AMC 10/12 and AIME. This is a good thing, because you should be proud of your performance in HMMT/PUMaC/ARML/random local or miscellaneous competitions, but you won't have to care too much if you don't do so well. This is the same for MATHCOUNTS. I learned this much too late - so I'll tell it to you upfront, MATHCOUNTS doesn't matter.

Take other contests for fun and to learn, and don't worry about scoring well or the contest at all. For easier contests, like MATHCOUNTS Open this year, I went in with the explicit intention of finding problems to put on my handouts. This made it much more fun and I think I learned more that way.

## § 2.6 Burnout

Have a life. Do other things too, and don't feel guilty about doing unproductive things. The yardstick isn't "How much time am I wasting each day?" or "How much time am I spending a day not doing math?". It's "How much time am I using well each day?" Hang out with your friends and join fun clubs. Not only is this helpful to lead a healthy life, I also believe that it's **better for learning math**. When you start to get bored, you can find a nice way to wrap things up for the day. As the title suggests, spending all of your time on math leads to burnout. Burnout is losing the motivation to do math problems, and it's hard to continue to work on anything without motivation.

I've found that writing materials and contributing to mock tests by writing problems is a much better way to keep interest than just spending all your time solving problems. Fun fact: A big reason this class exists was because a couple of students didn't have anything better to do over the summer!