Goal: 15 points
Total: 22 points

1. Prove the perpendicular bisectors of a triangle are concurrent. $(1)$

2. Prove the angle bisectors of a triangle are concurrent. $(1)$

3. Prove the medians of a triangle are concurrent. $(2)$

4. Prove the altitudes of a triangle are concurrent. $(2)$

5. Prove that the orthocenter, centroid, and the circumcenter of a triangle are collinear. $(\star 4)$

6. Consider $\triangle ABC$ with point $D$ on $BC.$ Let $M, N$ be the circumcenters of $\triangle ABD$ and $\triangle ACD,$ respectively. Let the circumcircles of $\triangle ACD$ and $\triangle MND$ intersect at $H \neq D.$ Prove $A, H, M$ are collinear. $(\star 6)$

7. In $\triangle ABC$ lines $CE$ and $AD$ are drawn so that $\frac{CD}{DB} = \frac{3}{1}$ and $\frac{AE}{EB} = \frac{3}{2}.$ Let $r = \frac{CP}{PE}$ where $P$ is the intersection point of $CE$ and $AD.$ Find $r.$ $(3)$

8. Let $CH$ be the altitude of acute $\triangle ABC.$ The points $X, Z, Y$ lie on lines $CA, CH, CB$ respectively in such a manner that $AX = AC, BY = BX,$ and $HZ = HC.$ Prove that $X, Y,$ and $Z$ are collinear. $(3)$