

A Rushed Test

Dennis Chen

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Instructions

You will have 65 minutes to complete 7 problems. A correct answer will be worth 1 point, and incorrect/black answers worth 0 points. Calculators are not allowed. Problems are roughly ordered by difficulty. Good luck!

Problems

1. If $a^2 + 8a + b^2 - 6b + c^2 - 10c + d^2 + 14d = 70$, find the sum of the minimum and maximum values $a^2 + b^2 + c^2 + d^2$ can take.
2. What is the smallest value of k such that there is no integer solution n to $\lfloor \frac{n^2}{36} \rfloor = k$?
3. Consider $\triangle ABC$ with $AB = 5$, $BC = 7$, and $CA = 4\sqrt{2}$. Let H be the foot of the altitude from A to BC . If P is a point on AC , find the minimum value of $BP + HP$.
4. Consider a number line with integers $-65, -64 \dots 62, 63$. Every second, a particle at the origin randomly moves to an adjacent integer. Find the expected amount of seconds for the particle to reach either -65 or 63 .
5. Find the remainder of $(1^3)(1^3 + 2^3)(1^3 + 2^3 + 3^3) \dots (1^3 + 2^3 + 3^3 \dots + 99^3)$ when divided by 101.
6. Consider circle O with diameter AB . Let T be on the circle such that $TA < TB$. Let the tangent line through T intersect AB at X and intersect the tangent line through B at Y . Let M be the midpoint of YB , and let XM intersect circle O at P and Q . If $XP = MQ$, find AT .
7. Suppose for some angle θ , $\cos \theta = \frac{1}{7}$. Find $\sum_{n=1}^{\infty} \frac{n \cos(n\theta)}{2^n}$.