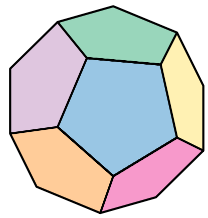
Sample NARML



Credits

This sample was created with the help of many people over months of work. The MAC would like to thank the following for their contributions to the contest:

- Problem writers: dchenmathcounts, skyscraper, youyanli, Depsilon0, and vvluo, who wrote the problems that appeared on the final contest.
- Problem proposers: innumerateguy, freeman66, vvluo, Radio2, and smartatmath, who proposed problems to the contest.
- Test-solvers: nukelauncher, bob, carol, for their suggestions and advice for the contest.

Rules

The rules, procedures, and format of this mock ARML follow.

- The Individual Rounds consist of five rounds, each with two questions. One point is awarded for each question answered correctly for a total of 10 points possible per person.
- Answers should be sent via the submission portal here.
- The only acceptable aids are a pencil or pen, an eraser, blank paper, a straightedge, and a compass. In particular, graph paper and protractors are not permitted.
- Calculators will not be allowed on any part of the ARML contest.
- During the Individual round there is to be absolutely no communication between students once the questions are handed out. This includes communication between students who have already finished the contest. The only exception is in the official discussion forum set up by the MAC.

Problems 1 and 2

Problem 1

Triangle ABC exists in the coordinate plane such that AB, AC, and BC have slopes equal to $1, \frac{1}{3}$ and -1 respectively. If AB has length $\frac{8}{3}$, compute the length of BC.

Problem 2

Steven has 4 lit candles and each candle is blown out with a probability $\frac{1}{2}$. After he finishes blowing, he randomly selects a possibly empty subset out of all the candles. Compute the probability his subset has at least one lit candle.

MAC

Problems 3 and 4

Problem 3

Compute the smallest positive integer k such that no integer n satisfies $\lfloor \frac{n^2}{36} \rfloor = k$.

Problem 4

Consider parallelogram $ABCD$ with $AB = 7$, $BC = 6$. Let the angle bisector of $\angle DAB$ intersect BC at X and CD at Y. Let the line through X parallel to BD intersect AD at Q. If $QY = 6$, find $\cos \angle DAB$.

Problems 5 and 6

Problem 5

For each positive integer n, let $s(n)$ denote the sum of the digits of n when written in base 10. An integer x is said to be tasty if

$$s(11x) = s(101x) = s(1001x) = \cdots = 18.$$

Compute the number of tasty integers less than 10^8.

Problem 6

Compute $\sum\limits_{a=1}^{\infty} \frac{32a}{16a^4+24a^2+25}$.

Problems 7 and 8

Problem 7

Consider unit circle O with diameter AB. Let T be on the circle such that $TA < TB$. Let the tangent line through T intersect AB at X and intersect the tangent line through B at Y. Let M be the midpoint of YB, and let XM intersect circle O at P and Q. If $XP = MQ$, compute AT.

Problem 8

Determine all positive integers n such that

$$\frac{1! \cdot 2! \cdot 3! \cdots 2019! \cdot 2020!}{n!}$$

is a perfect square.

MAC

Problems 9 and 10

Problem 9

Compute the sum of all odd integers n such that $\frac{1}{n}$ expressed in base 8 is a repeating decimal with period 4.

Problem 10

Compute the maximal value of k such that $(x+1)^4 \geq kx^3$ for all x.

MAC