Careful!

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MQU

We discuss common mistakes that happen in math competitions, also known as "sillies." Included are stupid "problems" with stupid solutions.

1 Pitfalls

I believe that you fix your "stupider" mistakes first. In practice, this means that people who say they "silly" a lot are often the ones who are losing a significant amount of points to them. In order of descending stupidity, common mistakes people make:

1. Read the problem wrong.
2. Make arithmetic/algebra errors.
3. In the AIME - forgetting to simplify fractions before doing $m+n$. (Particularly infamous this year is AIME I 2020/6, with $\frac{480}{39} \to 519$.) Or just forget to simplify answers at all.
4. For problems in different bases, remember that the base must be greater than the value of the largest digit. (For example, 658_7 is absurd because $7 \leq 8$.)
5. "Find all..." or "How many..." problems are two-part: First, you must find all of the things that work **and verify they do**, then you must verify no other work.
6. Pointwise trap for functional equations, in particular. For example, for $f(x)^2 = x^2$, the solutions are NOT $f(x) = x$ and $f(x) = -x$. It is possible for $f(1) = 1$ and $f(2) = -2$. In practice this will not happen, but **you have to check that it doesn't.**

2 Don't Do This

I don't think the problems I'm about to present have some sort of intrinsic condition that you mess up (which makes them kind of boring). But I messed these up, and from the looks of it, several other people have done the exact same thing. So I think they could be valuable - if you have already seen these problems/don't really care about arithmetic mistakes as opposed to fundamental ones, feel free to skip.

Example 2.1: AMC 8 2018/3

Students Arn, Bob, Cyd, Dan, Eve, and Fon are arranged in that order in a circle. They start counting: Arn first, then Bob, and so forth. When the number contains a 7 as a digit (such as 47) or is a multiple of 7 that person leaves the circle and the counting continues. Who is the last one present in the circle?

Here's how you mess it up: There are 5 answer choices, so there are clearly 5 people in the circle.

This cost me a bid for a perfect score. Grr dumb problem.

Example 2.2: AMC 10A 2020/7

The 25 integers from -10 to 14, inclusive, can be arranged to form a 5-by-5 square in which the sum of the numbers in each row, the sum of the numbers in each column, and the sum of the numbers along each of the main diagonals are all the same. What is the value of this common sum?

The main idea is pretty obvious: the sum of one of the rows is $\frac{1}{5}$ the sum of all the numbers, which is $-10 + -9 + \cdots + 14$. The way you screw this up is by forgetting the tens digit and thinking $11 + 12 + 13 + 14 = 10$. Oops.

3 Problems

On the page below, I detail common mistakes (so if you don't know what you did wrong, you can learn).

1. Let a, b, c, d, e be pairwise relatively prime non-negative integers. Find the minimum value $a+b+c+d+e$ can take.

2. (AMC 10B 2019/10) In a given plane, points A and B are 10 units apart. How many points C are there in the plane such that the perimeter of $\triangle ABC$ is 50 units and the area of $\triangle ABC$ is 100 square units?

3. (AMC 10B 2020/12) The decimal representation of
$$\frac{1}{20^{20}}$$
consists of a string of zeros after the decimal point, followed by a 9 and then several more digits. How many zeros are in that initial string of zeros after the decimal point?

4. (AIME II 2016/2) There is a 40% chance of rain on Saturday and a 30% of rain on Sunday. However, it is twice as likely to rain on Sunday if it rains on Saturday than if it does not rain on Saturday. The probability that it rains at least one day this weekend is $\frac{a}{b}$, where a and b are relatively prime positive integers. Find $a+b$.

5. (AMC 12B 2019/14) Let S be the set of all positive integer divisors of $100,000$. How many numbers are the product of two distinct elements of S?

6. (AIME I 2020/5) Six cards numbered 1 through 6 are to be lined up in a row. Find the number of arrangements of these six cards where one of the cards can be removed leaving the remaining five cards in either ascending or descending order.

7. (AIME II 2017/9) A special deck of cards contains 49 cards, each labeled with a number from 1 to 7 and colored with one of seven colors. Each number-color combination appears on exactly one card. Sharon will select a set of eight cards from the deck at random. Given that she gets at least one card of each color and at least one card with each number, the probability that Sharon can discard one of her cards and still have at least one card of each color and at least one card with each number is $\frac{p}{q}$, where p and q are relatively prime positive integers. Find $p+q$.

8. (AIME I 2020/11) For integers a, b, c and d, let $f(x) = x^2 + ax + b$ and $g(x) = x^2 + cx + d$. Find the number of ordered triples (a, b, c) of integers with absolute values not exceeding 10 for which there is an integer d such that $g(f(2)) = g(f(4)) = 0$.

9. (PAMO 2018) Find all functions $f : \mathbb{Z} \to \mathbb{Z}$ such that
$$(f(x+y))^2 = f(x^2) + f(y^2)$$
for all $x, y \in \mathbb{Z}$.

4 Common Mistakes

1. The problem never says "distinct."
2. First, you must find all of the things that work **and verify they do**...
3. If you're getting 27, the 0 to the left of the decimal place doesn't count. There are also a dozen of other ways to screw this up and this is just generally tricky.
4. The chance that it rains on Sunday given that it doesn't rain on Saturday is **not** 30%. That refers to the overall probability.
5. What divisors don't work?
6. The most common method is the "insertion" method (where you have a list of 5 numbers and insert the sixth to satisfy the requirement). But what about the cases where one valid arrangement can be produced by more than one insertion?
7. You can't assign the non-unique color to the non-unique number both times.
8. What about $f(2) = f(4)$?
9. What if $f(0) = 2$? Watch out for pointwise trap here!