

Branching Processes

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1 Binary Branching Process

1.1 General Properties

In the following, a binary branching process is simulated, starting with an initial single ancestor. An element can produce with branching probability p_{branch} either two (in which it itself is one of its own descendants) or zero offspring. This process is repeated for every element in the next generation and the ones thereafter. Figure 1 shows simulations of the population per generation of binary branching processes with different branching probabilities.

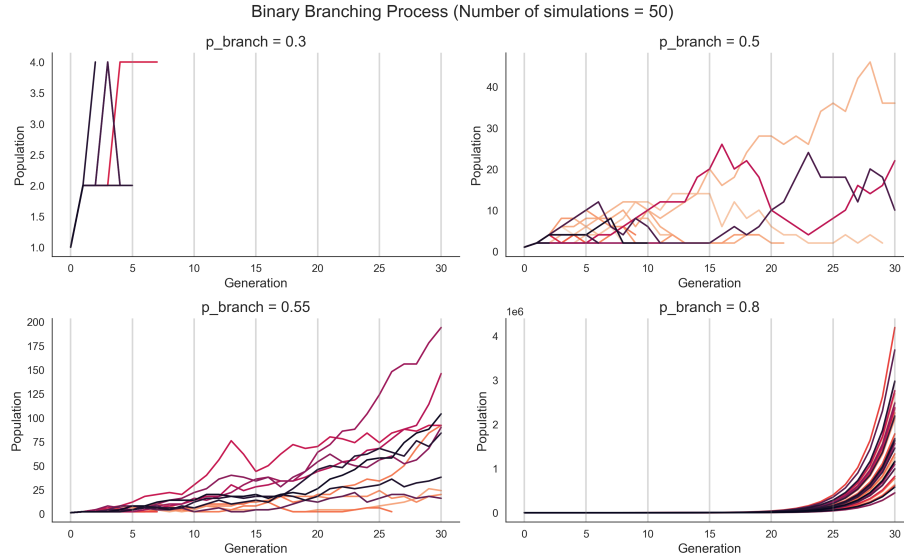


Figure 1: Simulation of binary branching processes with branching probabilities 0.3, 0.5, 0.55 and 0.8. Population is the number of elements per generation. Different scalings of the vertical axes were applied.

One can see that none or only a few processes survive to the last generation for branching probabilities below $p_{branch} = 0.5$ whereas an exponential growth is present for $p_{branch} > 0.5$. $p_c = 0.5$, therefore, poses the critical value for the branching probability. As expected, the average number of descendants remains constant at the critical state. The system is in a supercritical state for $p_{branch} > p_c$ and in a subcritical state for $p_{branch} < p_c$, as shown in Figure 2. The total size over time is shown in Figure 3.

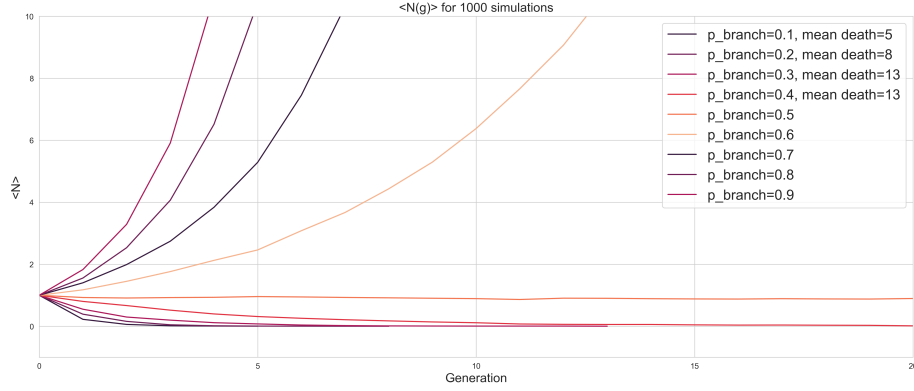


Figure 2: Mean Number of elements per generation. The average last surviving generations for $p_{branch}=0.1/0.2/0.3/0.4$ are 5/8/13/13

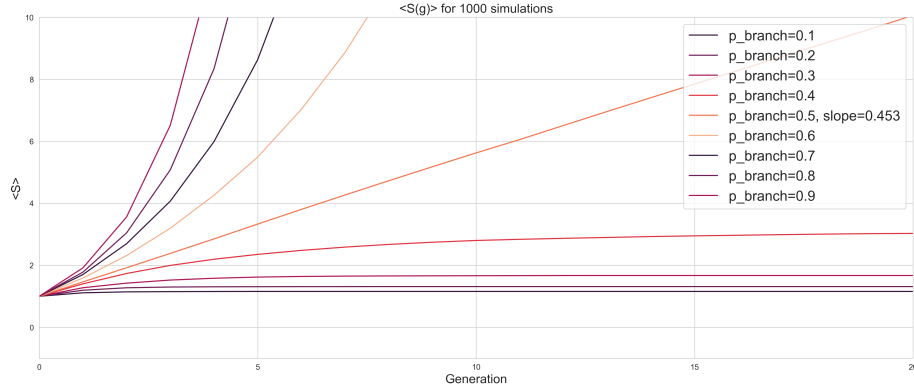


Figure 3: Mean size per generation. The total mean size increases linearly with a slope of 0.453 when p_{branch} is at its critical value of 0.5.

2 Appendix

Code: <https://github.com/Spaceferkel/Branching.git>

3 Paper review

4 Project ideas