Linear Algebra : Determinants and Eigen Space Practice

Determinant of a Matrix

1) Evaluate the following determinants :

a) $\begin{vmatrix} 1 & 5 \\ 2 & 6 \end{vmatrix}$ b) $\begin{vmatrix} 3 & 5 \\ -2 & 4 \end{vmatrix}$ c) $\begin{vmatrix} -5 & 6 \\ -7 & -2 \end{vmatrix}$ d) $\begin{vmatrix} 2 & 6 \\ 1 & 3 \end{vmatrix}$

e) $\begin{vmatrix} -2 & 1 & 4 \\ 3 & 5 & -7 \\ 1 & 6 & 2 \end{vmatrix}$ f) $\begin{vmatrix} -1 & 1 & 2 \\ 3 & 0 & -5 \\ 1 & 7 & 2 \end{vmatrix}$ g) $\begin{vmatrix} 2 & -4 & 3 \\ 3 & 1 & 2 \\ 1 & 4 & -1 \end{vmatrix}$ h) $\begin{vmatrix} 2 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 4 & 0 \end{vmatrix}$

Determinant of a triangular and diagonal matrix

2) Use the properties of determinants to calculate the determinant of the following matrices

a) $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 5 & 0 & 0 \\ 1 & 2 & 3 & 0 \\ 5 & 7 & 9 & 2 \end{pmatrix}$ b) $A = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ c) $A = \begin{pmatrix} 2 & 1 & 3 & 5 & 7 \\ 0 & 3 & 7 & 11 & 2 \\ 0 & 0 & 4 & 9 & 1 \\ 0 & 0 & 0 & 4 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

Minor determinant and cofactor

3) Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 2 & 1 & 4 \end{pmatrix}$, calculate the indicated cofactors and minors determinants of A :

$m_{11}, m_{21}, m_{32}, c_{12}\ , c_{33},\ and\ c_{11}$

Inverse of a 2x2 matrix

4) Show that the following matrix are invertible, and find their inverse

a) $\begin{pmatrix} 7 & 5 \\ 3 & 2 \end{pmatrix}$ b) $\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ c) $\begin{pmatrix} 4 & 8 \\ 2 & 3 \end{pmatrix}$, d) $\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$

Inverse of a 3x3 matrix

5) Given a matrix M
1) Show that M is invertible
2) Calculate the adjoint matrix of M , Adj(M).
3) Calculate the inverse of M
In each case where M is:

a) $M = \begin{pmatrix} 2 & 5 & 5 \\ -1 & -1 & 0 \\ 2 & 4 & 3 \end{pmatrix}$ b) $M = \begin{pmatrix} 2 & 0 & 3 \\ 0 & 3 & 2 \\ -2 & 0 & -4 \end{pmatrix}$ c) $M = \begin{pmatrix} 2 & -3 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 2 \end{pmatrix}$ d) $M = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 2 & 1 & 4 \end{pmatrix}$

Solving a two linear equations with two unknowns using matrix form

6) Express the following linear systems of equations in matrix form, and solve it using the inverse matrix methods.

a) $\begin{cases} 5x + 7y = 3 \\ 2x + 4y = 1 \end{cases}$ b) $\begin{cases} 3x - 2y = 7 \\ -5x + 6y = -5 \end{cases}$ c) $\begin{cases} 4x + y = 6 \\ 5x + 2y = 7 \end{cases}$

Matrix Inverse by Row Reduced Echelon Form(RREF)

7) Compute the inverse of the matrix below using Row Reduce Echelon Form (RREF) method

a) $M=\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ b) $M=\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$ c) $M=\begin{pmatrix} 1 & 1 \\ 5 & 4 \end{pmatrix}$ d) $M=\begin{pmatrix} 1 & -1 & 2 \\ 2 & 0 & 3 \\ 1 & 0 & 1 \end{pmatrix}$, e) $M=\begin{pmatrix} 1 & -1 & 2 \\ 3 & -1 & 5 \\ 1 & 0 & 1 \end{pmatrix}$

f) $M=\begin{pmatrix} 1 & -1 & 0 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{pmatrix}$, g) $M=\begin{pmatrix} 1 & 1 & 2 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{pmatrix}$ h) $M=\begin{pmatrix} 1 & 3 & 2 \\ 3 & 9 & 5 \\ 4 & 11 & 6 \end{pmatrix}$

Least Square approximation Method

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System of equation with Cramer's Method

10) Solve the following systems of equations using Cramer's methods

a) $\begin{cases} 5x + 7y = 3 \\ 2x + 4y = 1 \end{cases}$ b) $\begin{cases} 3x - 2y = 7 \\ -5x + 6y = -5 \end{cases}$ c) $\begin{cases} 4x + y = 6 \\ 5x + 2y = 7 \end{cases}$

d) $\begin{cases} 2x + y = 4 \\ -3x + z = -8 \\ y + 2z = -3 \end{cases}$ e) $\begin{cases} 2x + y + z = 4 \\ -x + 2z = 2 \\ 3x + y + 3z = -2 \end{cases}$ f) $\begin{cases} x - 4y + z = 6 \\ 4x - y + 2z = -1 \\ 2x + 2y - 3z = -20 \end{cases}$

Linear Independence of vector using Determinant

11) Check whether the set of vectors below are linear dependent or linear independent

a) $\vec{v}_1 = (2,1)$, $\vec{v}_2 = (5,4)$
b) $\vec{v}_1 = (5,0)$, $\vec{v}_2 = (0,1)$
c) $\vec{v}_1 = (1,-1)$, $\vec{v}_2 = (4,5)$
d) $\vec{v}_1 = (1,1,0), \vec{v}_2 = (0,2,1), \vec{v}_3 = (0,0,1)$
e) $\vec{v}_1 = (4,0,0), \vec{v}_2 = (0,2,0), \vec{v}_3 = (0,0,3)$
f) $\vec{v}_1 = (1,1,0), \vec{v}_2 = (0,2,1), \vec{v}_3 = (1,3,1)$

Basis of a vector space using determinant

12) Is the set $S = \{\vec{v}_1, \vec{v}_2\}$ forming a basis for $\mathbb{R}^2$ in each case below?

a) $\vec{v}_1 = (2,1)$ and $\vec{v}_2 = (5,4)$
b) $\vec{v}_1 = (5,0)$, $\vec{v}_2 = (0,1)$
c) $\vec{v}_1 = (5,2)$, $\vec{v}_2 = (2,1)$
d) $\vec{v}_1 = (1,2)$, $\vec{v}_2 = (4,8)$

13) Is the set $S = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ forming a basis for $\mathbb{R}^3$ in each case below?

a) $\vec{v}_1 = (1,1,0), \vec{v}_2 = (0,2,1), \vec{v}_3 = (0,0,1)$
b) $\vec{v}_1 = (4,0,0), \vec{v}_2 = (0,2,0), \vec{v}_3 = (0,0,3)$
c) $\vec{v}_1 = (1,1,0), \vec{v}_2 = (0,2,1), \vec{v}_3 = (1,3,1)$
d) $\vec{v}_1 = (1,1,0), \vec{v}_2 = (0,2,1), \vec{v}_3 = (0,3,1)$

Characteristics Equation

14) Find the characteristic equation of the following matrices (no need to solve the equation)

a) M= $\begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$

b) M= $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

c) M= $\begin{pmatrix} 2 & 3 \\ 0 & -3 \end{pmatrix}$

d) M= $\begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix}$

e) $M = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix}$

f) $M = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 4 & 5 & 1 \end{pmatrix}$

g) $M = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

Eigen Values and Eigen Vectors

15) Find the eigen values and vectors of the following matrices

a) M=$\begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$

b) M=$\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

c) M=$\begin{pmatrix} 2 & 3 \\ 0 & -3 \end{pmatrix}$

d) M=$\begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix}$

e) $M = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix}$

f) $M = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 4 & 5 & 1 \end{pmatrix}$

g) $M = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

Cayley-Hamilton Theorem

16) Do the matrices below satisfy their characteristic equation?

a) M=$\begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$

b) M=$\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

c) M=$\begin{pmatrix} 2 & 3 \\ 0 & -3 \end{pmatrix}$

d) M=$\begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix}$

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