Matrix Practice(Linear Algebra)

Order of a Matrix

1. Find the order of the following matrices

a) $A=\begin{pmatrix}1&2&3\\2&5&7\\6&7&5\end{pmatrix}$ b) $A=\begin{pmatrix}1&1&3&5\\5&-1&6&2\\7&8&9&-2\\3&5&9&10\end{pmatrix}$ c) $A=\begin{pmatrix}2&-1\\1&3\end{pmatrix}$ d) $A=\begin{pmatrix}1&4&0&4\\1&8&3&1\end{pmatrix}$ e) $\begin{pmatrix}1\\2\\3\end{pmatrix}$

Trace of a Square Matrix

2. Calculate the trace , trace(A) , of the following matrices

a) $\mathrm{A}=\begin{pmatrix}1&2&3\\2&5&7\\6&7&5\end{pmatrix}$, b) $A=\begin{pmatrix}2&-1\\1&3\end{pmatrix}$ c) $A=\begin{pmatrix}1&1&3&5\\5&-1&6&2\\7&8&9&-2\\3&5&9&10\end{pmatrix}$ d) $A=\begin{pmatrix}1&7&5\\1&4&7\\1&7&-5\end{pmatrix}$

Transpose of a Matrix

3. Find the transpose of matrix A (A^T)

b) $A=\begin{pmatrix}1&5&7\\9&1&7\\0&7&1\end{pmatrix}$, b) $A=\begin{pmatrix}2&-1\\1&3\end{pmatrix}$ c) $A=\begin{pmatrix}1&1&3&5\\5&-1&6&2\\7&8&9&-2\\3&5&9&10\end{pmatrix}$ d) $A=\begin{pmatrix}1&2&3\\2&5&7\\6&7&5\end{pmatrix}$, e) $\begin{pmatrix}1\\2\\3\\4\end{pmatrix}$

Matrix Entry Value

4. Find the following entry value

a) $m_{12}, m_{22}, m_{34}, m_{44}, m_{14}$ and m_{33} for $M=\begin{pmatrix}1&1&3&5\\5&-1&6&2\\7&8&9&-2\\3&5&9&10\end{pmatrix}$

Columns and rows vectors

5. Write the column and row vector of

a) $\vec{v} = (2,1,3)$ b) $\vec{v} = (2,0,3,4)$ c) $\vec{v} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ d) $\vec{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Symmetric Matrix

6. Which of the following matrices are symmetric ?

a) $A = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$ b) $A = \begin{pmatrix} 2 & 6 \\ 6 & 3 \end{pmatrix}$ c) $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 6 & 7 & 5 \end{pmatrix}$, d) $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 0 \\ 3 & 0 & 5 \end{pmatrix}$ e) $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 7 \end{pmatrix}$

b) If $M = \begin{pmatrix} 2 & x & y & 7 \\ 0 & 4 & z & t \\ 1 & 0 & 1 & u \\ v & 6 & 8 & 5 \end{pmatrix}$ is a symmetric matrix , then what are the value of x, y ,z ,t ,u ,v ?

Diagonal matrix ,Triangular Matrix and Skew Symmetric Matrix

7. Which of the matrices are diagonal, upper , lower triangular matrix or skew symmetric?

a) $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & 5 \end{pmatrix}$ b) $B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 5 & 1 & 0 & 0 \\ 7 & 8 & 9 & 0 \\ 3 & 5 & 9 & 10 \end{pmatrix}$ c) $C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 7 \end{pmatrix}$ d) $D = \begin{pmatrix} 0 & -2 & 3 \\ 2 & 0 & -7 \\ -3 & 7 & 0 \end{pmatrix}$

b) Write the skew symmetric matric of $\vec{v}$=(1,2,3) $\vec{u}$=(0,2,-1) and $\vec{w}$=(4,-2,3)

Matrices Addition

8. Given the following matrices $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \\ 1 & 2 & 3 \end{pmatrix}$, find

a) $A + B$ b) $2B - A$ c) $B - B^t$ d) $2A - 3A$ e) $A^t + B$

Matrix Form of the Vector Dot Product

9. Write the matrix-dot product of the following vector-dot product

   a) $\vec{u}\bullet\vec{v}$ where $\vec{v}$=(1,2,3) $\vec{u}$=(0,2,-1)

   b) $\vec{u}\bullet\vec{v}$ where $\vec{v}$=(2,2,-1) $\vec{u}$=(3,2,-4)

Matrix Form of the Vector Cross Product

10. Write the matrix-cross product of the following vector-cross product

   a) $\vec{u}\times\vec{v}$ where $\vec{v}$=(1,2,3) $\vec{u}$=(0,2,-1)

   b) $\vec{u}\times\vec{v}$ where $\vec{v}$=(1,0,-1) $\vec{u}$=(2,1,-1)

Matrices Multiplication

11. Calculate the following matrix multiplication

a) $\begin{pmatrix}1&2&0\\0&5&1\\3&1&1\end{pmatrix}\begin{pmatrix}1&0&2\\3&2&1\\1&2&3\end{pmatrix}$ b) $\begin{pmatrix}2&-1\\1&3\end{pmatrix}\begin{pmatrix}2&6\\6&3\end{pmatrix}$ c) $\begin{pmatrix}1&0&3\\2&1&1\end{pmatrix}\begin{pmatrix}1&3\\1&4\\0&1\end{pmatrix}$ d) $\begin{pmatrix}1&1&3\\0&1&1\end{pmatrix}\begin{pmatrix}1&0\\3&1\\0&1\end{pmatrix}$

e) $\begin{pmatrix}1&0&1\\1&2&1\\4&1&1\end{pmatrix}\begin{pmatrix}2&1&2\\1&2&3\\1&2&3\end{pmatrix}$ f) $\begin{pmatrix}1&2&1\end{pmatrix}\begin{pmatrix}2\\0\\3\end{pmatrix}$ g) $\begin{pmatrix}2\\0\\3\end{pmatrix}\begin{pmatrix}1&2&1\end{pmatrix}$

Right and left Vector- Matrix multiplication

12. Let $A=\begin{pmatrix}2&1&0\\0&3&1\\3&1&0\end{pmatrix}$ and $B=\begin{pmatrix}2&1&3\\1&0&1\\2&1&0\end{pmatrix}$ $\vec{v}=\begin{pmatrix}1\\2\\1\end{pmatrix}$ $\vec{u}=\begin{pmatrix}0\\1\\3\end{pmatrix}$, $\vec{w}=\begin{pmatrix}3\\1\\2\end{pmatrix}$ calculate

   a) $A\cdot\vec{v}$ and $\vec{v}\cdot A$ b) $A\cdot\vec{u}$ and $\vec{u}\cdot A$ c) $B\cdot\vec{u}$ $\vec{u}\cdot B$ d) $B\cdot\vec{w}$ e) $\vec{v}\bullet\vec{u}^t$

System of Linear equations and Augmented Matrix.

13. Write the augmented matrix of the following systems of linear equations

a) $\begin{cases}x+2y=5\\2x-3y=-4\end{cases}$ b) $\begin{cases}x+2y=7\\5x-3y=9\end{cases}$ c) $\begin{cases}2x+3y=16\\2x-y=8\end{cases}$ d) $\begin{cases}3x+y=2\\2x+y=1\end{cases}$

e) $\begin{cases}x+y-5z=-3\\x+y+z=3\\7x-y+2z=8\end{cases}$ f) $\begin{cases}x+y+z=2\\x-3y+2z=-4\\5x-y+3z=8\end{cases}$ g) $\begin{cases}x+3y+z=4\\2x-y+2z=1\\3x-y+2z=3\end{cases}$ h) $\begin{cases}x+y-z=6\\2x+3y+z=7\\x-y+2z=-2\end{cases}$

Identifying a Row Echelon Form of a Matrix

14. Which of the matrices are in row echelon form ?

$$A=\begin{pmatrix}1&3&1\\0&0&0\\0&0&5\end{pmatrix},\ B=\begin{pmatrix}8&4&1\\0&3&1\\0&0&0\end{pmatrix},\ C=\begin{pmatrix}1&4&1\\0&1&2\\0&2&0\end{pmatrix},\ D=\begin{pmatrix}0&8&2\\2&3&4\\0&0&3\end{pmatrix}\quad E=\begin{pmatrix}1&4&2\\0&3&4\\0&0&2\end{pmatrix}$$

$$F=\begin{pmatrix}0&5&3&0&7\\0&0&5&5&3\\0&0&0&0&0\end{pmatrix}\quad G=\begin{pmatrix}1&3\\0&4\end{pmatrix}\quad H=\begin{pmatrix}0&0\\0&4\end{pmatrix}\quad I=\begin{pmatrix}1&0\\3&0\end{pmatrix}\quad J=\begin{pmatrix}1&2&3\\0&3&5\end{pmatrix}$$

Identifying Reduced Row Echelon Form of a Matrix (RREF)

15. Which of the matrices are in reduced row echelon form ?

$$A=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix},\ B=\begin{pmatrix}1&1&2\\0&1&1\\0&0&0\end{pmatrix},\ C=\begin{pmatrix}1&0&1\\0&1&0\\0&0&2\end{pmatrix},\ D=\begin{pmatrix}1&0&0\\0&0&0\\0&1&0\end{pmatrix}\quad E=\begin{pmatrix}2&0&0\\0&1&0\\0&0&1\end{pmatrix}$$

$$F=\begin{pmatrix}1&5&0&0&7\\0&0&1&5&3\\0&0&0&0&0\end{pmatrix}\quad G=\begin{pmatrix}1&3\\0&4\end{pmatrix}\quad H=\begin{pmatrix}1&0\\0&1\end{pmatrix}\quad I=\begin{pmatrix}1&0&3\\0&1&5\end{pmatrix}\quad J=\begin{pmatrix}1&0&0&0&7\\0&1&0&5&3\\0&0&1&0&0\end{pmatrix}$$

Computing Row Echelon Form of a Matrix

16. Convert the following matrices in row echelon form

a) $A=\begin{pmatrix}2&-1\\1&3\end{pmatrix}$ b) $A=\begin{pmatrix}2&6\\6&3\end{pmatrix}$ c) $A=\begin{pmatrix}1&2&3\\2&5&7\\6&7&5\end{pmatrix}$, d) $A=\begin{pmatrix}1&2&3\\2&5&0\\3&0&5\end{pmatrix}$ e) $A=\begin{pmatrix}1&2&1\\2&3&1\\4&7&3\end{pmatrix}$

Computing Reduced Row Echelon Form of a Matrix (RREF)

17. Convert the following matrices into reduced row echelon form (canonical form)

a) $\begin{pmatrix}1&2&5\\2&-3&-4\end{pmatrix}$ b) $\begin{pmatrix}1&2&7\\5&-3&9\end{pmatrix}$ c) $\begin{pmatrix}2&3&16\\2&-1&8\end{pmatrix}$ d) $\begin{pmatrix}3&1&2\\2&1&1\end{pmatrix}$

e) $\begin{pmatrix}1&1&-5&-3\\1&1&1&3\\7&-1&2&8\end{pmatrix}$ f) $\begin{pmatrix}1&1&1&2\\1&-3&2&-4\\5&-1&3&8\end{pmatrix}$ g) $\begin{pmatrix}1&3&1&4\\2&-1&2&1\\3&-1&2&3\end{pmatrix}$ h) $\begin{pmatrix}1&1&-1&6\\2&3&1&7\\1&-1&2&-2\end{pmatrix}$

Solution of System of Linear Equations Using Reduced Row Echelon Form(RREF)

18. Solve the system of linear equations using Reduced Row Echelon Form matrix

a) $\begin{cases} x+2y=5 \\ 2x-3y=-4 \end{cases}$ b) $\begin{cases} x+2y=7 \\ 5x-3y=9 \end{cases}$ c) $\begin{cases} 2x+3y=16 \\ 2x-y=8 \end{cases}$ d) $\begin{cases} 3x+y=2 \\ 2x+y=1 \end{cases}$

e) $\begin{cases} x+y-5z=-3 \\ x+y+z=3 \\ 7x-y+2z=8 \end{cases}$ f) $\begin{cases} x+y+z=2 \\ x-3y+2z=-4 \\ 5x-y+3z=8 \end{cases}$ g) $\begin{cases} x+3y+z=4 \\ 2x-y+2z=1 \\ 3x-y+2z=3 \end{cases}$ h) $\begin{cases} x+y-z=6 \\ 2x+3y+z=7 \\ x-y+2z=-2 \end{cases}$

Rank of a Matrix

19. Find the rank of the Matrix

a) $A=\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 0 \\ 3 & 0 & 5 \end{pmatrix}$ b) $A=\begin{pmatrix} 1 & 2 & 1 \\ 2 & 0 & 1 \\ 3 & 2 & 2 \end{pmatrix}$ c) $A=\begin{pmatrix} 1 & 1 & 2 & 3 \\ 2 & 6 & 1 & 1 \\ 3 & 4 & 3 & 4 \end{pmatrix}$ d) $A=\begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & -1 \\ 3 & 1 & -5 \end{pmatrix}$

Linear dependence using Matrix Echelon Form

20. Are $\vec{u}_1$, $\vec{u}_2$ *and* $\vec{u}_3$ linear independent or linear dependent in $\mathbb{R}^3$?

a) $\vec{u}_1=(1,2,5), \vec{u}_2=(2,4,1)$ *and* $\vec{u}_3=(1,1,2)$ b) $\vec{u}_1=(1,4,3), \vec{u}_2=(3,0,1)$ *and* $\vec{u}_3=(1,1,2)$

c) $\vec{u}_1=(1,1,1), \vec{u}_2=(1,2,0)$ *and* $\vec{u}_3=(0,-1,1)$ d) $\vec{u}_1=(1,1,1), \vec{u}_2=(1,2,0)$ *and* $\vec{u}_3=(0,-1,2)$

21. Are $\vec{u}_1$ *and* $\vec{u}_2$ linear independent or linear dependent in $\mathbb{R}^2$?

a) $\vec{u}_1=(1,2,)$ *and* $\vec{u}_2=(2,4)$ b) $\vec{u}_1=(2,8)$ *and* $\vec{u}_2=(2,5)$

22. Are u , v , *and* w linear independent or linear dependent in P_2 ?

a) $u=1-x$, $v=5-3x+2x^2$ *and* $w=1+3x-x^2$

b) $u=1+x+x^2$, $v=x+2x^2$ *and* $w=x^2$

Basis Using Matrix Reduced Row Echelon Form(RREF)

23. Do the set of vector $B=\{\vec{u}_1, \vec{u}_2\}$ form a basis of $\mathbb{R}^3$?

a) $\vec{u}_1=(2,8)$ *and* $\vec{u}_2=(2,5)$ b) $\vec{u}_1=(1,3)$ *and* $\vec{u}_2=(2,6)$

24. Does the set of vector $B = \{ \vec{u}_1 , \vec{u}_2 \ \vec{u}_3 \}$ form a basis of $\mathbb{R}^3$ each case below?

a) $\vec{u}_1 = (1,0,0), \vec{u}_2 = (1,1,0)$ *and* $\vec{u}_3 = (1,1,1)$ b) $\vec{u}_1 = (1,2,3), \vec{u}_2 = (2,0,1)$ *and* $\vec{u}_3 = (3,2,2)$

c) $\vec{u}_1 = (1,2,1), \vec{u}_2 = (1,7,-1)$ *and* $\vec{u}_3 = (2,1,3)$ d) $\vec{u}_1 = (1,2,1), \vec{u}_2 = (5,2,3)$ *and* $\vec{u}_3 = (3,2,2)$

25. Which of the following set of vectors are bases for P_2 ?

a) $u = 1 - x \,,\ v = 5 - 3x + 2x^2$ *and* $w = 1 + 3x - x^2$

b) $u = 1 + 2x + x^2 \,,\ v = 2 + x^2$ *and* $w = 3 + 2x + 2x^2$

c) $u = 1 + x + x^2 \,,\ v = x + 2x^2$ *and* $w = x^2$

d) $u = 1 - 2x + 3x^2 \,,\ v = 5 + 6x - x^2$ *and* $w = 3 + 2x + x^2$

Basis of a Matrix Row Space

26. Find the basis of row space of matrix A , dim(rowsp(A)) and rank(A)

a) $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 0 \\ 3 & 0 & 5 \end{pmatrix}$ b) $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 0 & 1 \\ 3 & 2 & 2 \end{pmatrix}$ c) $A = \begin{pmatrix} 1 & -1 & 2 \\ 2 & 6 & 1 \\ 3 & -4 & 3 \end{pmatrix}$ d) $A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & -1 \\ 3 & 1 & -5 \end{pmatrix}$

Basis of a Matrix Column Space

27. Find the basis of column space of matrix A , dim(colsp(A)) and rank(A)

a) $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 0 \\ 3 & 0 & 5 \end{pmatrix}$ b) $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 0 & 1 \\ 3 & 2 & 2 \end{pmatrix}$ c) $A = \begin{pmatrix} 1 & -1 & 2 \\ 2 & 6 & 1 \\ 3 & -4 & 3 \end{pmatrix}$ d) $A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & -1 \\ 3 & 1 & -5 \end{pmatrix}$

Basis of a Matrix Null Space

28. Find a basis for the null space of A and dim($Null(A)$)

a) $A = \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}$ b) $A = \begin{pmatrix} 1 & 3 \\ 2 & 6 \end{pmatrix}$ c) $A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 2 & 3 & 1 \end{pmatrix}$ d) $A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 5 \\ 2 & 3 & 8 \end{pmatrix}$ e) $A = \begin{pmatrix} 1 & 5 & 3 \\ 2 & 5 & 1 \end{pmatrix}$

Coordinate of a Vector and Matrix

29.Find the coordinate of $\vec{v}$ with respect to the basis $B=\{\vec{i},\vec{j},\vec{k}\}=\{(1,0,0),(0,1,0),(0,0,1)\}$

a) $\vec{v}=2\vec{i}+3\vec{j}-\vec{k}$ b) $\vec{v}=\vec{i}+\vec{j}-\vec{k}$ c) $\vec{v}=5\vec{i}-\vec{k}$

30.Find the coordinate of matrix A with respect $B=\left\{\begin{pmatrix}1&0\\0&0\end{pmatrix},\begin{pmatrix}0&1\\0&0\end{pmatrix},\begin{pmatrix}0&0\\1&0\end{pmatrix},\begin{pmatrix}0&0\\0&1\end{pmatrix}\right\}$

a) $A=\begin{pmatrix}2&2\\4&3\end{pmatrix}$ b) $A=\begin{pmatrix}1&2\\1&0\end{pmatrix}$ c) $A=\begin{pmatrix}0&4\\2&1\end{pmatrix}$ d) $A=\begin{pmatrix}3&-7\\2&4\end{pmatrix}$

31. Find the coordinate of P with respect to the given basis B

a) $p(x)=5-4x+7x^2+10x^3$ in $B=\{1,x,x^2,x^3\}$

b) $p(x)=-x+3x^2$ in $B=\{1,x,x^2,x^3\}$

c) $p(x)=-x+3x^2$ in $B=\{1,x,x^2\}$

d) $p(x)=2-x+7x^2$ in $B=\{1,x,x^2\}$

32.Calculate the coordinates of $\vec{u}$ with respect to the given basis B

a) Find the coordinate of $\vec{u}=(2,-3)$ with respect to $B=\{(1,1),(3,4)\}$

b) Find the coordinate of $\vec{u}=(8,7)$ with respect to $B=\{(1,2),(2,1)\}$

c) Find the coordinate of $\vec{u}=(-3,1)$ with respect to $B=\{(1,3),(2,1)\}$

d) Find the coordinate of $\vec{u}=(1,2)$ with respect to $B=\{(1,1),(3,4)\}$

Change of Basis and Transition Matrix

33. Consider the bases $S=\{\vec{i},\vec{j}\}=\{(1,0),(0,1)\}$ and $B=\{\vec{u}_1,\vec{u}_2\}=\{(1,2),(2,5)\}$

a) Find the transition matrix from S to B , $M_{B\leftarrow S}$

b) If $\vec{v}=(1,2)=\vec{i}+2\vec{j}$, calculate its coordinate in B, that is find $[\vec{v}]_B$

c) Find the transition matrix from B to S , $M_{S\leftarrow B}$

34.Consider the bases $S=\{\vec{i},\vec{j}\}=\{(1,0),(0,1)\}$ and $B=\{\vec{u}_1,\vec{u}_2\}=\{(1,3),(1,4)\}$

a) Find the transition matrix from S to B , $M_{B\leftarrow S}$

b) If $\vec{v}=(1,2)=\vec{i}+2\vec{j}$, calculate its coordinate in B, that is find $[\vec{v}]_B$

c) Find the transition matrix from B to S , $M_{S\leftarrow B}$

35. Consider the bases $B=\{\vec{u}_1,\vec{u}_2\}=\{(1,3),(1,4)\}$ and $B'=\{\vec{v}_1,\vec{v}_2\}=\{(1,2),(2,5)\}$

a) Find the transition matrix from B to B', $M_{B'\leftarrow B}$

b) If $[\vec{v}]_B=\begin{pmatrix}2\\5\end{pmatrix}$ in B, calculate its coordinate in B', that is find $[\vec{v}]_{B'}$

36. Consider the bases $B=\{\vec{u}_1,\vec{u}_2\}=\{(1,3),(1,4)\}$ and $B'=\{\vec{v}_1,\vec{v}_2\}=\{(1,2),(1,1)\}$

a) Find the transition matrix from B to B', $M_{B'\leftarrow B}$

b) If $[\vec{v}]_B=\begin{pmatrix}3\\1\end{pmatrix}$ in B, calculate its coordinate in B', that is find $[\vec{v}]_{B'}$