Vector and Matrix Differential Calculus Practice

Converting from Rectangular Coordinate to Polar Coordinate

1) Change $\vec{p}(3,\sqrt{3})$ from rectangular coordinate to cylindrical to coordinates
2) Change $\vec{p}(1,\sqrt{3})$ from rectangular coordinate to cylindrical to coordinates
3) Change $\vec{p}(-1,-1)$ from rectangular coordinate to cylindrical to coordinates

Converting from Polar Coordinate to Rectangular Coordinate

4) Change $\vec{p}=\left(\frac{7\pi}{4},3\sqrt{2}\right)$ from polar coordinate to rectangular coordinates
5) Change $\vec{p}=\left(\frac{\pi}{4},4\right)$ from polar coordinate to rectangular coordinates
6) Change $\vec{p}=\left(\frac{\pi}{3},1\right)$ from polar coordinate to rectangular coordinates

Converting from Cylindrical Coordinate to rectangular Coordinate

7) Change $\vec{p}$ (3, π /2 ,1) from cylindrical to rectangular coordinates.
8) Change $\vec{p}$ (4, π /6 ,2) from cylindrical to rectangular coordinates
9) Change $\vec{p}$ (1, π /4 ,5) from cylindrical to rectangular coordinates

Converting from rectangular Coordinate to Cylindrical Coordinate

10) Change $\vec{p}$ (1 ,1,1) from rectangular to cylindrical coordinates.
11) Change $\vec{p}\left(\frac{1}{2},\frac{\sqrt{3}}{2},5\right)$ from rectangular to cylindrical coordinates.
12) Change $\vec{p}\left(\sqrt{2},\sqrt{2},3\right)$ from rectangular to cylindrical coordinates.

Converting from Spherical Coordinate to rectangular Coordinate

13) Change $\vec{p}$ (1, π /4 , π) from spherical to rectangular coordinates.
14) Change $\vec{p}$ (3, π /3 , π /4) from spherical to rectangular coordinates.
15) Change $\vec{p}$ (5, π /2 , π) from spherical to rectangular coordinates.

Converting from rectangular Coordinate to Spherical Coordinate

16) Change $\vec{p}$ (1, 1, $\sqrt{2}$) from rectangular to spherical coordinates.

17) Change $\vec{p}\left(\frac{\sqrt{3}}{4},\frac{1}{2},\frac{\sqrt{3}}{2}\right)$ from rectangular to spherical coordinates.

18) Change $\vec{p}(1,1,0)$ from rectangular to spherical coordinates

Converting from Spherical Coordinate to Cylindrical Coordinate

19) Change $\vec{p}$ (4, π /4 , π /3) from spherical to cylindrical coordinates.

20) Change $\vec{p}=\left(2\sqrt{2},\frac{\pi}{4},\frac{\pi}{3}\right)$ from spherical to cylindrical coordinates.

21) Change $\vec{p}\left(\sqrt{2},\frac{\pi}{2},\frac{\pi}{4}\right)$ from spherical to cylindrical coordinates.

Converting from Cylindrical Coordinate to Spherical Coordinate

22) Change $\vec{p}$ (1, π /2 ,1) from cylindrical to spherical coordinates.

23) Change $\vec{p}\left(\sqrt{6},\frac{\pi}{4},\sqrt{2}\right)$ from cylindrical to spherical coordinates.

24) Change $\vec{p}$ (1, π /4 ,5) from cylindrical to spherical coordinates

Computing the Gradient of Scalar field

25) Given the scalar field $f(x,y,z)=x^2y\ +xz+y^2$ Calculate $\overrightarrow{grad}(f)=\vec{\nabla}f$.

26) Given the scalar field $f(x,y,z)=x^2+y^2+z^2+2$ Calculate $\overrightarrow{grad}(f)=\vec{\nabla}f$

27) Given the scalar field $f(x,y,z)=x+3y+5z+2$ Calculate $\overrightarrow{grad}(f)=\vec{\nabla}f$

Computing Curl of a Vector Field

28) Given the vector field $\vec{u}(u_x,u_y,u_z)=x^2y\cdot\vec{i}+(zy)\vec{j}-z^2\vec{k}$ calculate $curl\,\vec{u}=\vec{\nabla}\times\vec{u}$.

29) Given the vector field $\vec{u}(u_x,u_y,u_z)=x^2\cdot\vec{i}+z^2\vec{j}-xy^3\vec{k}$ calculate $curl\,\vec{u}=\vec{\nabla}\times\vec{u}$

30) Given the scalar field $\vec{u}(u_x,u_y,u_z)=x\cdot\vec{i}+z\vec{j}-x\vec{k}$ calculate $curl\,\vec{u}=\vec{\nabla}\times\vec{u}$

Computing the Divergence of a Vector Field

31) Given the vector field $\vec{u}(u_x,u_y,u_z) = x^2 y \cdot \vec{i} + (zy)\vec{j} - z^2\vec{k}$ calculate $div\vec{u} = \vec{\nabla} \cdot \vec{u}$

32) Given the vector field $\vec{u}(u_x,u_y,u_z) = x^2 \cdot \vec{i} + z^2\vec{j} - xy^3\vec{k}$ calculate $div\vec{u} = \vec{\nabla} \cdot \vec{u}$

33) Given the scalar field $\vec{u}(u_x,u_y,u_z) = x \cdot \vec{i} + z\vec{j} - x\vec{k}$ calculate $div\vec{u} = \vec{\nabla} \cdot \vec{u}$

Computing the Laplacian of a Scalar Field

34) Given the scalar field $f(x,y,z) = x^2 y + xz + y^2$ compute the Laplacian $\vec{\nabla}^2 f = \dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} + \dfrac{\partial^2 f}{\partial z^2}$

35) Given the scalar field $f(x,y,z) = x^2 + y^2 + z^2 + 2$ compute the Laplacian $\vec{\nabla}^2 f = \dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} + \dfrac{\partial^2 f}{\partial z^2}$

36) Given the scalar field $f(x,y,z) = zx + 3x^3y^2 + 2xz^2$ compute the Laplacian $\vec{\nabla}^2 f = \dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} + \dfrac{\partial^2 f}{\partial z^2}$

Computing the Laplacian of a Vector Field

37) Given the vector field $\vec{u}(u_x,u_y,u_z) = (3x^2 y,\ zy^2,\ 3z^2)$ find Laplacian of $\vec{u}$, $\vec{\nabla}^2\vec{u}$.

38) Given the vector field $\vec{u}(u_x,u_y,u_z) = (x^2 + y,\ x + zy^2,\ z^2)$ $\vec{p}$ find Laplacian of $\vec{u}$, $\vec{\nabla}^2\vec{u}$.

39) Given the vector field $\vec{u}(u_x,u_y,u_z) = (3x^2 y,\ zy^2,\ 3z^2)$ find Laplacian of $\vec{u}$, $\vec{\nabla}^2\vec{u}$.

Derivative of a vector with respect to a vector.

40) Calculate $\dfrac{\partial\vec{w}}{\partial\vec{u}}$, if $\vec{w} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$ where $\begin{cases} w_1 = 2x + 3y \\ w_2 = 7x + 5y \end{cases}$ with $\vec{u} = \begin{pmatrix} x \\ y \end{pmatrix}$

41) Calculate $\dfrac{\partial\vec{w}}{\partial\vec{u}}$, if $\vec{w} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$ where $\begin{cases} w_1 = x - y + z \\ w_2 = x + 2y - z \end{cases}$ with $\vec{u} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

42) Calculate $\dfrac{\partial\vec{w}}{\partial\vec{u}}$, if $\vec{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ where $\begin{cases} w_1 = xy + z \\ w_2 = x - y^2 + z \\ w_3 = 2x + y + xz \end{cases}$ with $\vec{u} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

Derivative of a scalar s with respect to a vector (Jacobian)

43) if $s = s\left(\vec{x}\right) = (x_1+1)^2 + x_2^{\ 2} + (x_3+2)^2$ where $\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$, calculate $\dfrac{\partial s}{\partial \vec{x}}$.

44) $f = f\left(\vec{x}\right) = x + y + z$ where $\vec{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, calculate $\dfrac{\partial f}{\partial \vec{x}}$.

45) $g = \left(\vec{x}\right) = x + xy + z^2$ where $\vec{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, calculate $\dfrac{\partial g}{\partial \vec{x}}$.

Quadric Forms

46) Express the quadric form $f(x,y) = x^2 + 6xy + 2y^2$ in matrix form, $f(\vec{x}) = \vec{x}^T \cdot A \cdot \vec{x}$,

and calculate $\dfrac{\partial f(\vec{x})}{\partial \vec{x}} = \dfrac{\partial}{\partial \vec{x}}\left(\vec{x}^T A \vec{x}\right)$

47) Express the quadric form $f(x,y) = 5x^2 + 2xy + 2y^2$ in matrix form, $f(\vec{x}) = \vec{x}^T \cdot A \cdot \vec{x}$,

And calculate $\dfrac{\partial f(\vec{x})}{\partial \vec{x}} = \dfrac{\partial}{\partial \vec{x}}\left(\vec{x}^T A \vec{x}\right)$

48) Express the quadric form $f(x,y,z) = 3x^2 + 8xy + 6xz + y^2 + 6yz + 3z^2$ in matrix form, $f(\vec{x}) = \vec{x}^T \cdot A \cdot \vec{x}$,

And calculate $\dfrac{\partial f(\vec{x})}{\partial \vec{x}} = \dfrac{\partial}{\partial \vec{x}}\left(\vec{x}^T A \vec{x}\right)$.

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