Practice: Lines and Planes in 3D Space

A) Write the equations of the Ray and line segment passing through the point $\vec{s}_0$ and parallel to the vector $\vec{u}$ in each case:

1) $\vec{s}_0=(1,5,-3)$ and $\vec{u}=(1,2,3)$
2) $\vec{s}_0=(1,0,-1)$ and $\vec{u}=(1,0,2)$

B) Describe the following line , line segment or ray

1) L1: $\begin{cases} x=1 \\ y=3+t \\ z=1-t \end{cases} \quad t\in\mathbb{R}$

2) L1: $\begin{cases} x=5+2t \\ y=3+t \\ z=2-3t \end{cases} \quad 0\le t\le 1$

3) L1: $\begin{cases} x=5 \\ y=-3-2t \\ z=4t \end{cases} \quad t\ge 0$

C) Show that the 2 given rays , L1 and L2 , are parallel

1) L1 : $\begin{cases} x=5+t \\ y=-3-2t \\ z=4t \end{cases} \quad t\ge 0$ and L2: $\begin{cases} x=2+2t \\ y=5-4t \\ z=3+8t \end{cases} \quad t\ge 0$

2) L1: $\begin{cases} x=1-3t \\ y=5-t \\ z=3+t \end{cases} \quad t\ge 0$ and L2: $\begin{cases} x=2+15t \\ y=1+5t \\ z=10-5t \end{cases} \quad t\ge 0$

D) Show that the 2 given rays , L1 and L2 , are perpendicular

1) L1 : $\begin{cases} x=5+t \\ y=-3-2t \\ z=4t \end{cases} \quad t\ge 0$ and L2: $\begin{cases} x=2+12t \\ y=5-4t \\ z=3-5t \end{cases} \quad t\ge 0$

2) L1: $\begin{cases} x=1-3t \\ y=5-t \\ z=3+t \end{cases} \quad t\ge 0$ and L2: $\begin{cases} x=2+t \\ y=1-t \\ z=1+2t \end{cases} \quad t\ge 0$

3) L1 : $\vec{s}_0=(1,5,-3)$ and $\vec{u}=(1,2,0)$ and L2 : $\vec{s}_0=(0,1,-3)$ and $\vec{u}=(-12,6,1)$

E) Calculate the angle between the lines or line segments.

2) $\begin{cases} x=1 \\ y=3+t \\ z=1-t \end{cases} \quad 0\le t\le 1$ and $\begin{cases} x=5+2t \\ y=3+t \\ z=2-3t \end{cases} \quad 0\le t\le 1$

Lab: Planes in 3D space

A) Describe the following planes(find reference point and normal vector)

1) $4x+3y-2z-10=0$
2) $-x+2y+z=14$
3) $x-y+7=0$

B) Write the equation of the plane with normal $\vec{N}$ and reference point $\vec{A}$

1) $\vec{N}=(1,2,3)$ $\vec{A}=(0,1,-2)$
2) $\vec{N}=(2,-1,5)$ $\vec{A}=(1,2,-1)$
3) $\vec{N}=(1,-5,0)$ $\vec{A}$ $(1,-2,0)$

C) Describe the location(coplanar ,positive or negative half-space) of point $\vec{A}$ relative to the plane P in each case

1) P: $x-y+z-4=0$ and $\vec{A}$ $(1,2,1)$
2) P: $-x+2y+z=14$ and $\vec{A}$ $(-7,0,7)$
3) P: $2x+y-5z+2=0$ and $\vec{A}$ $(2,1,-2)$

D) Write the equation of a plane given its 3 vertices $\vec{A}$, $\vec{B}$, $\vec{C}$

1) $\vec{A}=(1,0,1)$ $\vec{B}=(2,1,-1)$ $\vec{C}=(0,1,2)$
2) $\vec{A}=(1,2,3)$ $\vec{B}=(1,1,1)$ $\vec{C}=(5,2,4)$

E) Find the projection of the point A onto the plane P in the direction of the plane normal vector

1) P: $x-y+z+4=0$ and $\vec{A}$ $(1,2,1)$
2) P: $-x+2y+z=1$ and $\vec{A}$ $(2,1,3)$
3) P: $2x+y-5z+2=0$ and $\vec{A}$ $(2,1,-2)$

F) Find the distance between the plane P and the point $\vec{A}$

1) $x+2y-z+1=0$ $\vec{A}=(1,1,1)$
2) $x-y+z-4=0$ $\vec{A}=(1,2,1)$
3) $3x+2y+z=7$ $\vec{A}=(1,2,1)$

G) Find the parametric equations for the line of intersection of the planes

1) P1 : $-3x+2y+z=-5$ and P2: $7x+3y-2z=-2$
2) P1 : $-x+2y+z=14$ and P2 : $2x+y=3$

H) Given the two planes P1 and P2, Find the angle θ between the two planes

1) P1: $x-y+z-4=0$ and P2 : $x-y+2z=0$
2) P1: $2x-y+3z=1$ and P2 : $x-y+2z=0$

I) Given the two planes P1 and P2, Show that they are perpendicular.

1) P1: $x-2y+4z=7$ and p2 : $2x-5y-3z=1$
2) P1: $2x+y=3$ and p2 : $5x-10y-3z=1$

J) Given the two planes P1 and P2, Show that they are parallel.

1) P1: $x-2y+4z=7$ and p2 : $-2x+4y-8z=1$
2) P1: $2x+y-z=3$ and p2 : $6x+3y-3z=1$