Vectors Practice (Linear Algebra) :

Norm, length or magnitude of a Vector

1. Calculate the length(norm) of the following vectors
   a) $\vec{u} = (1,0,1)$ b) $\vec{v} = (2,1,-2)$ c) $\vec{w} = (3,0,-4)$ d) $\vec{s} = (1,-1,1)$ e) $\vec{m} = (\frac{\sqrt{2}}{2},0,\frac{\sqrt{2}}{2})$

Normalized Vectors

2. Normalize the following vectors
   a) $\vec{u} = (1,0,1)$ b) $\vec{v} = (2,1,-2)$ c) $\vec{w} = (3,0,-4)$ d) $\vec{s} = (1,-1,1)$ e) $\vec{m} = (\frac{\sqrt{2}}{2},0,\frac{\sqrt{2}}{2})$

Vectors Direction

3. Find the direction and opposite direction of the vector
   a) $\vec{u} = (1,0,1)$ b) $\vec{v} = (2,1,-2)$ c) $\vec{w} = (3,0,-4)$ d) $\vec{s} = (1,-1,1)$ e) $\vec{m} = (\frac{\sqrt{2}}{2},0,\frac{\sqrt{2}}{2})$

4. Find the direction and speed of a car moving with velocity $\vec{v} = (\sqrt{3},0,1)\ m/s$
5. Find the direction and speed of a ball moving with velocity $\vec{v} = (-1,0,1)\ m/s$

Co-linear and Parallel Vectors

6. Given the vectors $\vec{P}$ and $\vec{Q}$, Show that they are collinear.
   a) $\vec{P}$ (1,2,0) and $\vec{Q}$ (3,6,0)
   b) $\vec{P}$ (2,0,-5) and $\vec{Q}$ $(8,0,-20)$
   c) $\vec{P}$ (-2,5,-3) and $\vec{Q}$ (12,-30,18)
   d) $\vec{P}$ (6,9,15) and $\vec{Q}$ (2,3,5)

Building a vector from 2 Vertices (points)

7. Find the vector between the 2 vertices and their distance
   a) $\vec{A}$ **(2,1,0)** , $\vec{B}$ **(1,1,1)**
   b) $\vec{A}$ **(3,0,4)** , $\vec{B}$ **(1,0,1)**
   c) $\vec{A}$ **(1,0,0)** , $\vec{B}$ **(1,1,0)**

8. What is the distance between Ann at $\vec{p}_1 = (2,5,4)$ and Paul at $\vec{p}_2 = (1,5,1)$?

Vectors Addition

9. Given the following vectors $\vec{A}$=(2,-5,1) and $\vec{B}$=(1,-2,-1) $\vec{C}$ =(1,1,0) , calculate

   a) $-2\vec{A} + 3\vec{B}$ **b)** $-\vec{A} + \vec{B}$ **c)** $-\vec{A} + 3\vec{B} + \vec{C}$ **d)** $-\vec{B} - \vec{C} + \vec{A}$ **e)** $-\vec{A} + \vec{B} + 2\vec{C}$

Dot Product of two Vectors

10.Calculate the dot product of $\vec{a}$ *and* $\vec{b}$, $\vec{a} \bullet \vec{b}$

a) $\vec{a}$=(2,-1, 3) and $\vec{b}$ = (0, 1 , 3) b) $\vec{a}$ = (1, -2, 0) and $\vec{b}$ = (-2, 4 , 0)
c) $\vec{a}$ = (0,-1, 3) and $\vec{b}$ = (0, 3 , 1) d) $\vec{a}$ =(3,-1, 4) and $\vec{b}$ = (1, 1 , 2)

Angle Between two Vectors

11.Calculate the angle between the two vectors $\vec{a}$ *and* $\vec{b}$

a) $\vec{a}$=(2,-1, 3) and $\vec{b}$ = (0, 1 , 3) b) $\vec{a}$ = (1, -2, 0) and $\vec{b}$ = (-2, 4 , 0)
c) $\vec{a}$ = (0,-1, 3) and $\vec{b}$ = (0, 3 , 1) d) $\vec{a}$ =(3,-1, 4) and $\vec{b}$ = (1, 1 , 2)

Type of Angle Between two Vectors

12.find the type of angle between without computing the angle

a) $\vec{A}$ (2,-1, 3) and $\vec{B}$ (0, 1 , 3)
b) $\vec{A}$ (1, -2, 0) and $\vec{B}$ (-2, 4 , 0)
c) $\vec{A}$ (0,-1, 3) and $\vec{B}$ (0, 3 , 1)
d) $\vec{A}$ (1,-1, 3) and $\vec{B}$ (1, 3 , 1)

Orthogonal or Perpendicular Vectors

13.Show that the two vectors $\vec{a}$ *and* $\vec{b}$ are orthogonal

a) $\vec{a}$=(2,-1, 3) and $\vec{b}$ = (0, 3 , 1) b) $\vec{a}$ = (-1, -2, 0) and $\vec{b}$ = (-2, 1 , 0)
c) $\vec{a}$ = (0,-1, 3) and $\vec{b}$ = (0, 3 , 1) d) $\vec{a}$ =(3,-1, 1) and $\vec{b}$ = (1, 1 , -2)

Components of a vector $\vec{a}$ onto a vector $\vec{b}$

14. Given $\vec{U}$ and $\vec{V}$, calculate $Comp_{\vec{v}}^{\vec{u}}$.

a) $\vec{U} = (1,2,1)$ and $\vec{V} = (1,1,1)$
b) $\vec{U} = 3\vec{i} - 2\vec{j} + \vec{k}$ and $\vec{V} = \vec{i} + 2\vec{j} - \vec{k}$
c) $\vec{U} = 5\vec{i} + \vec{j}$ and $\vec{V} = \vec{i} - \vec{k}$
d) $\vec{U} = (1,0,2)$ and $\vec{V} = (-2,3,1)$

Projection of a vector $\vec{a}$ onto a vector $\vec{b}$

15. Given $\vec{U}$ and $\vec{V}$, calculate $\mathrm{Pr}\,oj_{\vec{v}}^{\vec{u}}$.

a) $\vec{U} = (1,2,1)$ and $\vec{V} = (1,1,1)$
b) $\vec{U} = 3\vec{i} - 2\vec{j} + \vec{k}$ and $\vec{V} = \vec{i} + 2\vec{j} - \vec{k}$
c) $\vec{U} = 5\vec{i} + \vec{j}$ and $\vec{V} = \vec{i} - \vec{k}$
d) $\vec{U} = (1,0,2)$ and $\vec{V} = (-2,3,1)$

The Perpendicular Vector of $\vec{a}$ to a vector $\vec{b}$

16. Given two vectors $\vec{a}$ and $\vec{b}$, calculate the Perpendicular vector of $\vec{a}$ to $\vec{b}$, that is $\vec{a}_{\perp}$ using $\vec{a}_{\perp} = \vec{a} - proj_{\vec{b}}^{\vec{a}} = \vec{a} - (\vec{a} \bullet \hat{b})\hat{b}$

   a. a) $\vec{a}\,(2,-1, 3)$ and $\vec{b}$ $(0, 1, 3)$
   b. b) $\vec{a}\,(1, -2, 0)$ and $\vec{b}$ $(-2, 4, 0)$
   c. c) $\vec{a}\,(0,-1, 3)$ and $\vec{b}$ $(0, 3, 1)$

Cross Product of 2 Vectors

17. Calculate the cross product of $\vec{a}$ *and* $\vec{b}$, $\vec{a} \times \vec{b}$

   a) $\vec{a}=(2,-1, 3)$ and $\vec{b} = (0, 1, 3)$
   b) $\vec{a} = (1, -2, 0)$ and $\vec{b} = (-2, 4, 0)$
   c) $\vec{a} = (0,-1, 3)$ and $\vec{b} = (0, 3, 1)$
   d) $\vec{a} =(3,-1, 4)$ and $\vec{b} = (1, 1, 2)$

18. Simplify the following operations

   a) $(2\vec{i}) \times \vec{j} =$
   b) $(\vec{i} \times \vec{k}) \times (\vec{i} \times \vec{j}) =$
   c) $(\vec{i} \times \vec{i}) \bullet (\vec{i} \times \vec{j}) =$
   d) $\vec{k} \times (2\vec{i} - \vec{j}) =$
   e) $(\vec{i} + \vec{j}) \times (\vec{i} + 5\vec{k}) =$
   f) $\vec{i} \times (\vec{j} \times \vec{k}) =$
   g) $\vec{k} \bullet (\vec{j} \times \vec{k}) =$
   h) $(\vec{i} \times \vec{k}) \times (\vec{j} \times \vec{i}) =$

19. Find a third vector $\vec{C}$ orthogonal (perpendicular) to $\vec{A}$ and $\vec{B}$.

   a) $\vec{A}$ (2,1,1) and $\vec{B}$ (-1,2,2)
   b) $\vec{A}$ (1,0,1) and $\vec{B}$ (2,3,5).
   c) $\vec{A}$ (1,0,0) and $\vec{B}$ (0,1,0).
   d) $\vec{A} =(3,-1, 1)$ and $\vec{B} = (1, 1, -2)$

Vectors differentiation

20. calculate $\dfrac{d\vec{a}}{dt}$

   a) Given $\vec{a} = 3t^2\vec{i} + t^3\vec{j} - (t^2 - t^3)\vec{k}$
   b) Given $\vec{a} = 3t^2\vec{i} + 4t^3\vec{j} - 6t\vec{k}$,
   c) Given $\vec{a} = (t^2, \cos(t), 7)$
   d) Given $\vec{a} = (t,\ 4\ , -6t)$,

Partial Derivative of Vectors

21. Calculate the indicated partial derivative

   a) Given $\vec{u}(u_x, u_y, u_z) = (x + y^2, z + x, xz^2)$ find $\dfrac{\partial \vec{u}}{\partial x}, \dfrac{\partial \vec{u}}{\partial y}, \dfrac{\partial \vec{u}}{\partial z}, \dfrac{\partial^2 \vec{u}}{\partial x \partial y}$

   b) Given $\vec{u}(u_x, u_y, u_z) = (x^3 + y^2, zx, z^2 + y)$ find $\dfrac{\partial \vec{u}}{\partial x}, \dfrac{\partial \vec{u}}{\partial y}, \dfrac{\partial \vec{u}}{\partial z}, \dfrac{\partial^2 \vec{u}}{\partial x \partial y}$

   c) Given $\vec{u}(u_x, u_y, u_z) = (x^2 + y^2 + z^2, z, xz^2 + 2)$ find $\dfrac{\partial \vec{u}}{\partial x}, \dfrac{\partial \vec{u}}{\partial y}, \dfrac{\partial \vec{u}}{\partial z}, \dfrac{\partial^2 \vec{u}}{\partial x \partial y}$

Integration of Vectors

22. calculate the integral below

a) $\int_1^2 \vec{a}(t)dt$, given $\vec{a} = 3t^2\vec{i} + 4t^3\vec{j} - 6t\vec{k}$ b) $\int_1^2 \vec{a}(t)dt$, given $\vec{a} = t^2\vec{i} + 4t^3\vec{j} - \vec{k}$

c) $\int_0^{\pi} \vec{a}dt$, given $\vec{a} = (1, \cos(t), \sin(t))$ d) $\int_{-1}^{1} \vec{a}(t)dt$ Given $\vec{a} = 2t\vec{i} + \vec{k}$

Homogeneous System of linear Equations

23. Tell whether the systems of linear equations is the homogeneous

a) $\begin{cases} x + y - z = 0 \\ 2x + 3y + z = 0 \\ x - y + 2z = 0 \end{cases}$ b) $\begin{cases} x + 3y - z = 5 \\ x + 3y + 8z = 0 \\ x - y + 2z = 0 \end{cases}$ b) $\begin{cases} x + y - z = 1 \\ 3y + z = 0 \\ z = 0 \end{cases}$

Consistent and inconsistent System of linear Equations

24. Identify the inconsistence and consistence systems of equation

a) $\begin{cases} x + y - z = 1 \\ 2x + 3y + z = 6 \\ x - y + 2z = 2 \end{cases}$ b) $\begin{cases} x + 3y - z = 5 \\ x + 3y + 8z = 0 \\ 0z = 0 \end{cases}$ c) $\begin{cases} x + y - z = 1 \\ 3y + z = 0 \\ 0z = 4 \end{cases}$

Free Variables and leading unknowns (pivots)

25. Identify the free variables and the leading unknowns

a) $\begin{cases} x + y - z = 1 \\ 3y + z = 0 \end{cases}$ b) $\begin{cases} x + 3y - z + s - 2t = 5 \\ 2y + 8z + 2s + t = 4 \\ s + 2t = 1 \end{cases}$ c) $x + y - z = 1$

Gaussian Elimination

26. Solve the system of linear equations using Gaussian elimination

a) $\begin{cases} x + 2y = 4 \\ 2x + y = 5 \end{cases}$ b) $\begin{cases} x - 3y = -2 \\ 5x + y = 6 \end{cases}$ c) $\begin{cases} x + 3y = 8 \\ 3x + y = 16 \end{cases}$

d) $\begin{cases} x + y - z = 1 \\ 2x + 3y + z = 6 \\ x - y + 2z = 2 \end{cases}$ e) $\begin{cases} x + 3y - z = 7 \\ 2x + 3y + z = 8 \\ 3x - y + 2z = 1 \end{cases}$ f) $\begin{cases} x + y - z = 0 \\ 5x - 3y + z = 2 \\ 3x - 2y + z = 2 \end{cases}$

Subspace in $\mathbb{R}^n$

27.**D**

a) Is $W = \{\ (3x, 5y)\ \ : x \in \mathbb{R}\ and\ \ y \in \mathbb{R}\ \}$ subset of $\mathbb{R}^2$?

b) Is $W = \{\ (x, x+1)\ \ : x \in \mathbb{R}\ and\ \ y \in \mathbb{R}\ \}$ subset of $\mathbb{R}^2$?

c) Is $W = \{\ 10x\ \ : x \in \mathbb{R}\ \}$ subset of $\mathbb{R}$?

Linear Combination

28.Determine whether $\vec{w}$ is a linear combination of $\vec{u}$ and $\vec{v}$

a) $\vec{w} = (0,2)$, $\vec{u} = (1,3)\ and\ \vec{v} = (2,4)$

b) $\vec{w} = (3,0)$, $\vec{u} = (1,0)\ and\ \vec{v} = (0,2)$

c) $\vec{w} = (5,2)$, $\vec{u} = (1,0)\ and\ \vec{v} = (0,1)$

d) $\vec{w} = (1,2,0)$, $\vec{u} = (1,0,0)\ and\ \vec{v} = (0,1,0)$

Linear independence

29.Determine whether the vectors are linearly dependent or independent.

a) $\vec{a} = (1,3)\ and\ \vec{b} = (2,3)$

b) $\vec{a} = (6,4)\ and\ b = (12,8)$

c) $\vec{a} = (1,5)\ and\ b = (3,4)$

d) $\vec{a} = (1,1,0)$, $\vec{b} = (1,2,1)\ and\ \vec{c} = (1,1,1)$

e) $\vec{a} = (1,1,1)$, $\vec{b} = (1,2,0)\ and\ \vec{c} = (0,-1,1)$

f) $\vec{a} = (1,2,3)$, $\vec{b} = (3,2,9)\ and\ \vec{c} = (5,2,-1)$

g) $\vec{a} = (1,2,3)$, $\vec{b} = (3,2,1)\ and\ \vec{c} = (0,4,8)$

Basis of a Vector Space

30.Determine if the given set is a basis for the given set

a) $\vec{a} = (1,3)\ and\ \vec{b} = (2,3)$ for $\mathbb{R}^2$

b) $\vec{a} = (6,4)\ and\ b = (12,8)$ for $\mathbb{R}^2$

c) $\vec{a} = (1,5)\ and\ b = (3,4)$ for $\mathbb{R}^2$

d) $\vec{a} = (1,1,0)$, $\vec{b} = (1,2,1)\ and\ \vec{c} = (1,1,1)$ for $\mathbb{R}^3$

e) $\vec{a} = (1,1,1)$, $\vec{b} = (1,2,0)\ and\ \vec{c} = (0,-1,1)$ for $\mathbb{R}^3$

f) $\vec{a} = (1,2,3)$, $\vec{b} = (3,2,9)\ and\ \vec{c} = (5,2,-1)$ for $\mathbb{R}^3$

g) $\vec{a} = (1,2,3)$, $\vec{b} = (3,2,1)\ and\ \vec{c} = (0,4,8)$ for $\mathbb{R}^3$

Dimension of a Vector Space

31. Determine the dimension of the given subspaces bases

a) $B=\left\{\vec{a}\ ,\vec{b}\right\}=\{(1,3),(2,3)\}$

b) $B=\left\{\vec{a},\vec{b},\vec{c}\right\}=\{(1,1,0),(1,2,1),(1,1,1)\}$

c) $B=\left\{1,x,x^2,x^3,x^4\right\}$

d) $B=\left\{\vec{a},\vec{b},\vec{c},\vec{d}\right\}=\{(1,0,0,0),(0,2,0,0),(0,0,1,0),(0,0,0,3)\}$

Inner Product Space

32. If $\vec{a}=(2,1,2)$, $\vec{b}=(1,0,-1)$, $\vec{c}=(1,-1,1)$ Compute the following inner product

a) $\langle\vec{a},\vec{c}\rangle$, b) $\langle\vec{b},\vec{c}\rangle$, c) $\langle 5\vec{a}-2\vec{b},\vec{c}\rangle$, d) $\sqrt{\langle\vec{a},\vec{a}\rangle}$

33. **given** $f(x)=5x^2$ *and* $g(x)=x^3$ **with inner product** $\langle f,g\rangle=\int_{-1}^{1}f(x)g(x)dx$

Find $\langle f,g\rangle$, $\|f\|$ and normalized $f(x)$ that is $\hat{f}$

34. **given** $f(x)=x$ *and* $g(x)=x+2$ **with inner product** $\langle f,g\rangle=\int_{0}^{1}f(x)g(x)dx$

Find $\langle f,g\rangle$, $\|f\|$ and normalized $f(x)$ that is $\hat{f}$

35. **given** $f(x)=\cos(x)$ *and* $g(x)=\sin(x)$ **with inner product**

$\langle f,g\rangle=\int_{0}^{\frac{\pi}{2}} f(x)g(x)dx$

Find $\langle f,g\rangle$, $\|f\|$ and normalized $f(x)$ that is $\hat{f}$

Hint: $\cos^2(x)=\dfrac{1+\cos(2x)}{2}$

36. Given $p=1+2x+x^2+x^3$ and $q=1+5x^2+x^3$ compute $\langle p,q\rangle$

37. Given $p=1+2x-x^2+3x^3$ and $q=1+x-2x^2+4x^3$ compute $\langle p,q\rangle$