GEN 242: Linear Algebra

Chapter 6: Collision

Solutions Guide

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Table of Contents

Answers

Sphere-to-Sphere Collision

1. $S_1: (x+3)^2 + (y-5)^2 + (z+4)^2 = 1$ and $S_2 = (x+1)^2 + (y-2)^2 + (z-2)^2 = 4$:

   a. $\boxed{\vec{c}_1 = (-3,5,-4)}$

   b. $\boxed{\vec{c}_2 = (-1,2,2)}$

   c. $\boxed{\|\overrightarrow{c_1c_2}\| = 7}$

   d. $r_1 + r_2 = 3 < \|\overrightarrow{c_1c_2}\|$, so no collision.

2. $S_1: (x-1)^2 + (y-2)^2 + (z-3)^2 = 4$ and $S_2 = (x-5)^2 + (y-5)^2 + (z-3)^2 = 1$:

   a. $\boxed{\vec{c}_1 = (1,2,3)}$

   b. $\boxed{\vec{c}_2 = (5,5,3)}$

   c. $\boxed{\|\overrightarrow{c_1c_2}\| = 5}$

   d. $r_1 + r_2 = 3 < \|\overrightarrow{c_1c_2}\|$, so no collision.

Ray-Plane Collision

1. $P: x + 2y + z - 10 = 0$ and $L: \begin{cases} x = 1 \\ y = 3 + t\,, t \geq 0 \\ z = 1 - t \end{cases} \rightarrow$ collision at $t = 2$.

2. $P: -2x + 3y + z + 15 = 0$ and $L: \begin{cases} x = 1 + 4t \\ y = 5 + 2t\,, \geq 0 \\ z = 1 - 3t \end{cases} \rightarrow$ collision at $t = 5.8$.

Sphere-to-Plane Collision

1. $P: \frac{2}{3}x + \frac{1}{3}y + \frac{2}{3}z + 1 = 0$ and $S: (x-1)^2 + (y-2)^2 + (z-3)^2 = 9$ and $\vec{v}_s = (1,0,1)$:

   $\hat{n} \cdot \vec{c} + d = \frac{13}{3} > 0 \rightarrow$ sphere is in front of plane.

   $\vec{n} \cdot \vec{v} = \frac{4}{3} > 0 \rightarrow$ since sphere is in front of plane, sphere is moving away from plane.

   No collision.

2. $P\colon \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}y + \frac{\sqrt{3}}{3}z + 1 = 0$ and $S\colon (x-1)^2 + (y+1)^2 + z^2 = 1$ and $\vec{v}_s = (-2,0,1)$:

   $\hat{n} \cdot \vec{c} + d = 1 > 0 \rightarrow$ sphere is in front of plane

   $\vec{n} \cdot \vec{v} = -\frac{\sqrt{3}}{3} < 0 \rightarrow$ since sphere is in front of plane, sphere is moving toward plane

   $|\hat{n} \cdot \vec{c} + d| = 1 = r \rightarrow$ sphere has collided with (is touching) plane

Ray-to-Sphere Collision

1. $S\colon (x+1)^2 + (y-2)^2 + (z-3)^2 = 1$ and $L\colon \begin{cases} x = 1 \\ y = 3 + t, t \geq 0\colon \\ z = 1 - t \end{cases}$

   No collision.

2. $S\colon (x+1)^2 + (y-2)^2 + (z-3)^2 = 1$ and $L\colon \begin{cases} x = 1 \\ y = 3 + t, t \geq 0\colon \\ z = 1 - t \end{cases}$

   No collision.

Solutions

Sphere-to-Sphere Collision

Problem 1

Given spheres $S_1: (x+3)^2 + (y-5)^2 + (z+4)^2 = 1$ and
$S_2 = (x+1)^2 + (y-2)^2 + (z-2)^2 = 4$:

1.a Find the components of the center of sphere S_1.

$$S = (x - c_x)^2 + \left(y - c_y\right)^2 + (z - c_z)^2 = r^2$$

$$S_1: (x+3)^2 + (y-5)^2 + (z+4)^2 = 1$$

$$S_1: \left(x - (-3)\right)^2 + \left(y - (5)\right)^2 + \left(z - (-4)\right)^2 = 1$$

$$\boxed{\vec{c}_1 = (-3,5,-4)}$$

1.b Find the components of the center of sphere S_2.

$$S = (x - c_x)^2 + \left(y - c_y\right)^2 + (z - c_z)^2 = r^2$$

$$S_2 = (x+1)^2 + (y-2)^2 + (z-2)^2 = 4$$

$$S_2: \left(x - (-1)\right)^2 + \left(y - (2)\right)^2 + \left(z - (2)\right)^2 = 1$$

$$\boxed{\vec{c}_2 = (-1,2,2)}$$

1.c Calculate the distance between the two centers.

$$\|\overrightarrow{c_1 c_2}\| = \sqrt{\left(c_{2,x} - c_{1,x}\right)^2 + \left(c_{2,y} - c_{1,y}\right)^2 + \left(c_{2,z} - c_{1,z}\right)^2}$$

$$\|\overrightarrow{c_1 c_2}\| = \sqrt{\left((-1) - (-3)\right)^2 + \left((2) - (5)\right)^2 + \left((2) - (-4)\right)^2}$$

$$\|\overrightarrow{c_1 c_2}\| = \sqrt{(-1+3)^2 + (2-5)^2 + (2+4)^2}$$

$$\|\overrightarrow{c_1 c_2}\| = \sqrt{(2)^2 + (-3)^2 + (6)^2}$$

$$\|\overrightarrow{c_1 c_2}\| = \sqrt{4 + 9 + 36}$$

$$\|\overrightarrow{c_1 c_2}\| = \sqrt{49}$$

$$\boxed{\|\overrightarrow{c_1 c_2}\| = 7}$$

1.d Determine the possibility of a collision though computation.

$r_1 = \sqrt{1}$ $r_2 = \sqrt{4}$

$r_1 = 1$ $r_2 = 2$

$r_1 + r_2 = 1 + 2$

$r_1 + r_2 = 3$

$3 < 7$

$r_1 + r_2 < \|\overrightarrow{c_1 c_2}\|$

The distance between the spheres' centers is further than the combined radii. Therefore, there is no collision.

Problem 2

Given spheres $S_1: (x-1)^2 + (y-2)^2 + (z-3)^2 = 4$ and $S_2 = (x-5)^2 + (y-5)^2 + (z-3)^2 = 1$:

2.a Find the components of the center of sphere S_1.

$S = (x - c_x)^2 + \left(y - c_y\right)^2 + (z - c_z)^2 = r^2$

$S_1: (x-1)^2 + (y-2)^2 + (z-3)^2 = 4$

$S_1: \left(x-(1)\right)^2 + \left(y-(2)\right)^2 + \left(z-(3)\right)^2 = 4$

$\boxed{\vec{c}_1 = (1,2,3)}$

2.b Find the components of the center of sphere S_2.

$S = (x - c_x)^2 + \left(y - c_y\right)^2 + (z - c_z)^2 = r^2$

$S_2 = (x-5)^2 + (y-5)^2 + (z-3)^2 = 1$

$S_2: \left(x-(5)\right)^2 + \left(y-(5)\right)^2 + \left(z-(3)\right)^2 = 1$

$\boxed{\vec{c}_2 = (5,5,3)}$

2.c Calculate the distance between the two centers.

$$\|\overrightarrow{c_1c_2}\| = \sqrt{\left(c_{2,x} - c_{1,x}\right)^2 + \left(c_{2,y} - c_{1,y}\right)^2 + \left(c_{2,z} - c_{1,z}\right)^2}$$

$$\|\overrightarrow{c_1c_2}\| = \sqrt{\left((5) - (1)\right)^2 + \left((5) - (2)\right)^2 + \left((3) - (3)\right)^2}$$

$$\|\overrightarrow{c_1c_2}\| = \sqrt{(5-1)^2 + (5-2)^2 + (3-3)^2}$$

$$\|\overrightarrow{c_1c_2}\| = \sqrt{(4)^2 + (3)^2 + (0)^2}$$

$$\|\overrightarrow{c_1c_2}\| = \sqrt{16 + 9 + 0}$$

$$\|\overrightarrow{c_1c_2}\| = \sqrt{25}$$

$$\boxed{\|\overrightarrow{c_1c_2}\| = 5}$$

2.d Determine the possibility of a collision though computation.

$r_1 = \sqrt{4}$ $r_2 = \sqrt{1}$

$r_1 = 2$ $r_2 = 1$

$$r_1 + r_2 = 2 + 1$$

$$r_1 + r_2 = 3$$

$$3 < 5$$

$$r_1 + r_2 < \|\overrightarrow{c_1c_2}\|$$

The distance between the spheres' centers is further than the combined radii. Therefore, there is no collision.

Ray-Plane Collision

Problem 1

Given plane $P: x + 2y + z - 10 = 0$ and ray $L: \begin{cases} x = 1 \\ y = 3 + t \\ z = 1 - t \end{cases}$, determine through computation the possibility of a collision.

$$P: n_x x + n_y y + n_z z + d = 0$$
$$P: x + 2y + z - 10 = 0$$
$$P: (1)x + (2)y + (z) + (-10) = 0$$
$$\vec{n} = (1,2,1)$$
$$d = -10$$

$$L: \vec{s}(t) = \vec{s}_0 + \vec{v}t, t < 0$$
$$L: \begin{cases} x = s_{0,x} + v_x t \\ y = s_0, y + v_y t \\ z = s_0, z + v_z t \end{cases}$$
$$L: \begin{cases} x = 1 \\ y = 3 + t \\ z = 1 - t \end{cases}$$
$$L: \begin{cases} x = 1 + 0t \\ y = 3 + 1t \\ z = 1 + (-1)t \end{cases}$$
$$\vec{s}_0 = (1,3,1)$$
$$\vec{v} = (0,1,-1)$$

$$\hat{n} = \frac{\vec{n}}{||\vec{n}||}$$
$$\hat{n} = \frac{\vec{n}}{\sqrt{(n_x)^2 + (n_y)^2 + (n_z)^2}}$$
$$\hat{n} = \frac{(1,2,1)}{\sqrt{(1)^2 + (2)^2 + (1)^2}}$$
$$\hat{n} = \frac{(1,2,1)}{\sqrt{1 + 4 + 1}}$$
$$\hat{n} = \frac{(1,2,1)}{\sqrt{6}}$$
$$\hat{n} = \left(\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$$

$$\hat{n} \cdot \vec{s}_0 + d = \left(\frac{1}{\sqrt{6}}\right)(1) + \left(\frac{2}{\sqrt{6}}\right)(3) + \left(\frac{1}{\sqrt{6}}\right)(1) + (-10)$$

$$\hat{n} \cdot \vec{s}_0 = \frac{1}{\sqrt{6}} + \frac{6}{\sqrt{6}} + \frac{1}{\sqrt{6}} - \frac{10}{\sqrt{6}}$$

$$\hat{n} \cdot \vec{s}_0 = \frac{-2}{\sqrt{6}}$$

$$\hat{n} \cdot \vec{s}_0 < 0$$

The ray's origin is behind the plane.

$$\vec{n} \cdot \vec{v} = n_x v_x + n_y v_y + n_z v_z$$

$$\vec{n} \cdot \vec{v} = (1)(0) + (2)(1) + (1)(-1)$$

$$\vec{n} \cdot \vec{v} = 0 + 2 + (-1)$$

$$\vec{n} \cdot \vec{v} = 1$$

$$\vec{n} \cdot \vec{v} > 0$$

Possible collision.

$$t = -\frac{\vec{n} \cdot \vec{s}_0 + d}{\vec{n} \cdot \vec{v}}$$

$$t = -\frac{n_x s_{0,x} + n_y s_{0,y} + n_z s_{0,z} + (-10)}{1}$$

$$t = -\frac{(1)(1) + (2)(3) + (1)(1) + (-10)}{1}$$

$$t = -(1 + 6 + 1 - 10)$$

$$t = -(-2)$$

$$t = 2$$

$$t > 0$$

Collision at $t = 2$.

Problem 2

Given plane $P\colon -2x + 3y + z + 15 = 0$ and ray $L\colon \begin{cases} x = 1 + 4t \\ y = 5 + 2t \\ z = 1 - 3t \end{cases}$, determine through computation the possibility of a collision.

$$P\colon n_x x + n_y y + n_z z + d = 0$$
$$P\colon -2x + 3y + z + 15 = 0$$
$$P\colon (-2)x + (3)y + (1)z + (15) = 0$$
$$\vec{n} = (-2,3,1)$$
$$d = 15$$

$$L\colon \vec{s}(t) = \vec{s}_0 + \vec{v}t, t < 0$$
$$L\colon \begin{cases} x = s_{0,x} + v_x t \\ y = s_0, y + v_y t \\ z = s_0, z + v_z t \end{cases}$$
$$L\colon \begin{cases} x = 1 + 4t \\ y = 5 + 2t \\ z = 1 - 3t \end{cases}$$
$$L\colon \begin{cases} x = 1 + 4t \\ y = 5 + 2t \\ z = 1 + (-3)t \end{cases}$$
$$\vec{s}_0 = (1,5,1)$$
$$\vec{v} = (4,2,-3)$$

$$\hat{n} = \frac{\vec{n}}{\|\vec{n}\|}$$
$$\hat{n} = \frac{\vec{n}}{\sqrt{(n_x)^2 + \left(n_y\right)^2 + (n_z)^2}}$$
$$\hat{n} = \frac{(-2,3,1)}{\sqrt{(-2)^2 + (3)^2 + (1)^2}}$$
$$\hat{n} = \frac{(-2,3,1)}{\sqrt{4 + 9 + 1}}$$
$$\hat{n} = \frac{(-2,3,1)}{\sqrt{14}}$$
$$\hat{n} = \left(-\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}\right)$$

$$\hat{n} \cdot \vec{s}_0 + d = \left(-\frac{2}{\sqrt{14}}\right)(1) + \left(\frac{3}{\sqrt{14}}\right)(5) + \left(\frac{1}{\sqrt{14}}\right)(1) + (15)$$

$$\hat{n} \cdot \vec{s}_0 = -\frac{2}{\sqrt{14}} + \frac{15}{\sqrt{14}} + \frac{1}{\sqrt{14}} + 15$$

$$\hat{n} \cdot \vec{s}_0 = \frac{14}{\sqrt{14}} + 15$$

$$\hat{n} \cdot \vec{s}_0 \approx 18.7$$

$$\hat{n} \cdot \vec{s}_0 > 0$$

The ray's origin is in front of the plane.

$$\vec{n} \cdot \vec{v} = n_x v_x + n_y v_y + n_z v_z$$

$$\vec{n} \cdot \vec{v} = (-2)(4) + (3)(2) + (1)(-3)$$

$$\vec{n} \cdot \vec{v} = -8 + 6 + (-3)$$

$$\vec{n} \cdot \vec{v} = -5$$

$$\vec{n} \cdot \vec{v} < 0$$

Possible collision.

$$t = -\frac{\vec{n} \cdot \vec{s}_0 + d}{\vec{n} \cdot \vec{v}}$$

$$t = -\frac{n_x s_{0,x} + n_y s_{0,y} + n_z s_{0,z} + (-10)}{1}$$

$$t = -\frac{(-2)(1) + (3)(5) + (1)(1) + (15)}{-5}$$

$$t = \frac{-2 + 15 + 1 + 15}{5}$$

$$t = \frac{29}{5} = 5.8$$

$$t > 0$$

Collision at $t = 5.8$ s.

Sphere-to-Plane Collision

Problem 1

Given plane $P: \frac{2}{3}x + \frac{1}{3}y + \frac{2}{3}z + 1 = 0$, sphere $S: (x-1)^2 + (y-2)^2 + (z-3)^2 = 9$, sphere velocity $\vec{v} = (1,0,1)$ m/s, and reference point $A = (0,3,0)$, determine the possibility of collision through computation.

$$P: n_x x + n_y y + n_z z + d = 0$$

$$P: \frac{2}{3}x + \frac{1}{3}y + \frac{2}{3}z + 1 = 0$$

$$P: \left(\frac{2}{3}\right)x + \left(\frac{1}{3}\right)y + \left(\frac{2}{3}\right)z + (1) = 0$$

$$\vec{n} = \left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$$

$$d = 1$$

$$S: (x - c_x)^2 + \left(y - c_y\right)^2 + (z - c_z)^2 = r^2$$

$$S: \left(x - (1)\right)^2 + \left(y - (2)\right)^2 + \left(z - (3)\right)^2 = (3)^2$$

$$\vec{c} = (1,2,3)$$

$$r = 3$$

Location of sphere relative to plane:

$$\hat{n} \cdot \vec{c} + d = ?$$

$$\hat{n} = \frac{\vec{n}}{\|\vec{n}\|}$$

$$\hat{n} = \frac{\vec{n}}{\sqrt{(n_x)^2 + \left(n_y\right)^2 + (n_z)^2}}$$

$$\hat{n} = \frac{\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)}{\sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2}}$$

$$\hat{n} = \frac{\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)}{\sqrt{\frac{4}{9} + \frac{1}{9} + \frac{4}{9}}}$$

$$\hat{n} = \frac{\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)}{\sqrt{\frac{9}{9}}}$$

$$\hat{n} = \frac{\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)}{\sqrt{1}}$$

$$\hat{n} = \frac{\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)}{1}$$

$$\hat{n} = \left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$$

$$\hat{n} \cdot \vec{c} + d = \left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right) \cdot (1,2,3) + 1$$

$$\hat{n} \cdot \vec{c} + d = \left(\frac{2}{3}\right)(1) + \left(\frac{1}{3}\right)(2) + \left(\frac{2}{3}\right)(3) + 1$$

$$\hat{n} \cdot \vec{c} + d = \frac{2}{3} + \frac{2}{3} + 2 + 1$$

$$\hat{n} \cdot \vec{c} + d = \frac{13}{3} \approx 4.3$$

$$\hat{n} \cdot \vec{c} + d > 0$$

The sphere is in front of the plane.

Direction of sphere's motion:

$$\vec{n} \cdot \vec{v} = n_x v_x + n_y v_y + n_z v_z$$

$$\vec{n} \cdot \vec{v} = \left(\frac{2}{3}\right)(1) + \left(\frac{1}{3}\right)(0) + \left(\frac{2}{3}\right)(1)$$

$$\vec{n} \cdot \vec{v} = \frac{2}{3} + 0 + \frac{2}{3}$$

$$\vec{n} \cdot \vec{v} = \frac{4}{3}$$

$$\vec{n} \cdot \vec{v} > 0$$

The sphere is moving away from the plane.

No collision.

Problem 2

Given plane $P: \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}y + \frac{\sqrt{3}}{3}z + 1 = 0$, sphere $S: (x-1)^2 + (y+1)^2 + z^2 = 1$, sphere velocity $\vec{v} = (-2,0,1)$ m/s, and reference point $A = (0,0,-\sqrt{3})$, determine the possibility of collision through computation.

$$P: n_x x + n_y y + n_z z + d = 0$$

$$P: \left(\frac{\sqrt{3}}{3}\right)x + \left(\frac{\sqrt{3}}{3}\right)y + \left(\frac{\sqrt{3}}{3}\right)z + (1) = 0$$

$$\vec{n} = \left(\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$$

$$d = 1$$

$$S: (x-c_x)^2 + \left(y-c_y\right)^2 + (z-c_z)^2 = r^2$$

$$S: \left(x-(1)\right)^2 + \left(y-(-1)\right)^2 + \left(z-(0)\right)^2 = (1)^2$$

$$\vec{c} = (1,-1,0)$$

$$r = 1$$

Location of sphere relative to plane:

$$\hat{n} \cdot \vec{c} + d = ?$$

$$\hat{n} = \frac{\vec{n}}{\|\vec{n}\|}$$

$$\hat{n} = \frac{\vec{n}}{\sqrt{(n_x)^2 + \left(n_y\right)^2 + (n_z)^2}}$$

$$\hat{n} = \frac{\left(\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)}{\sqrt{\left(\frac{\sqrt{3}}{3}\right)^2 + \left(\frac{\sqrt{3}}{3}\right)^2 + \left(\frac{\sqrt{3}}{3}\right)^2}}$$

$$\hat{n} = \frac{\left(\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)}{\sqrt{\frac{3}{9} + \frac{3}{9} + \frac{3}{9}}}$$

$$\hat{n} = \frac{\left(\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)}{\sqrt{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}}}$$

$$\hat{n} = \frac{\left(\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)}{\sqrt{\frac{3}{3}}}$$

$$\hat{n} = \frac{\left(\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)}{\sqrt{1}}$$

$$\hat{n} = \frac{\left(\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)}{1}$$

$$\hat{n} = \left(\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$$

$$\hat{n} \cdot \vec{c} + d = \left(\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right) \cdot (1, -1, 0) + 1$$

$$\hat{n} \cdot \vec{c} + d = \left(\frac{\sqrt{3}}{3}\right)(1) + \left(\frac{\sqrt{3}}{3}\right)(-1) + \left(\frac{\sqrt{3}}{3}\right)(0) + 1$$

$$\hat{n} \cdot \vec{c} + d = \frac{\sqrt{3}}{3} + \left(-\frac{\sqrt{3}}{3}\right) + 0 + 1$$

$$\hat{n} \cdot \vec{c} + d = 1$$

$$\hat{n} \cdot \vec{c} + d > 0$$

The sphere is in front of the plane.

Direction of sphere's motion:

$$\vec{n} \cdot \vec{v} = n_x v_x + n_y v_y + n_z v_z$$

$$\vec{n} \cdot \vec{v} = \left(\frac{\sqrt{3}}{3}\right)(-2) + \left(\frac{\sqrt{3}}{3}\right)(0) + \left(\frac{\sqrt{3}}{3}\right)(1)$$

$$\vec{n} \cdot \vec{v} = -\frac{2\sqrt{3}}{3} + 0 + \frac{\sqrt{3}}{3}$$

$$\vec{n} \cdot \vec{v} = -\frac{\sqrt{3}}{3}$$

$$\vec{n} \cdot \vec{v} < 0$$

Because the sphere is in front of the plane, this negative result means the sphere is moving toward the plane.

Proximity:

$$|\hat{n} \cdot \vec{c} + d| = |1|$$

$$|\hat{n} \cdot \vec{c} + d| = 1$$

$$|\hat{n} \cdot \vec{c} + d| = r$$

The distance between the plane and the sphere's center is equal to the sphere's radius.

Collision.

Ray-to-Sphere Collision

Problem 1

Given sphere S: $(x+1)^2 + (y-2)^2 + (z-3)^2 = 1$ and ray L: $\begin{cases} x = 1 \\ y = 3 + t \\ z = 1 - t \end{cases}$, determine the possibility of collision through computation.

If collision occurs, then at least one point on the ray satisfies the sphere equation:

$$\big((1)+1\big)^2 + \big((3+t)-2\big)^2 + \big((1-t)-3\big)^2 = 1$$
$$(1+1)^2 + (3+t-2)^2 + (1-t-3)^2 = 1$$
$$(2)^2 + (t+1)^2 + (-t-2)^2 = 1$$
$$4 + (t^2+2t+1) + (t^2+4t+4) = 1$$
$$4 + t^2 + 2t + 1 + t^2 + 4t + 4 - 1 = 0$$
$$4 + t^2 + 2t + 1 + t^2 + 4t + 4 - 1 = 0$$
$$t^2 + t^2 + 2t + 4t + 4 + 4 + 1 - 1 = 0$$
$$2t^2 + 6t + 8 = 0$$
$$t^2 + 3t + 4 = 0$$
$$t = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(4)}}{2(1)}$$
$$t = \frac{-3 \pm \sqrt{9-16}}{2}$$
$$t = \frac{-3 \pm \sqrt{-7}}{2}$$

No real solutions.

No collision.

Problem 2

Given sphere S: $(x-1)^2+(y+1)^2+z^2=9$ and segment L: $\begin{cases} x = 1+4t \\ y = 5+2t \\ z = 1-3t \end{cases}$, determine the possibility of collision through computation.

If collision occurs, then at least one point on the segment satisfies the sphere equation:

$$\big((1+4t)-1\big)^2+\big((5+2t)+1\big)^2+(1-3t)^2=9$$

$$(1+4t-1)^2+(5+2t+1)^2+(1-3t)^2=9$$

$$(4t)^2+(2t+6)^2+(-3t+1)^2=9$$

$$16t^2+(4t^2+24t+36)+(9t^2-6t+1)=9$$

$$16t^2+4t^2+24t+36+9t^2-6t+1-9=0$$

$$16t^2+4t^2+9t^2+24t-6t+36+1-9=0$$

$$29t^2+18t+28=0$$

$$t=\frac{-29\pm\sqrt{(18)^2-4(29)(28)}}{2(29)}$$

$$t=\frac{-29\pm\sqrt{324-3248}}{58}$$

$$t=\frac{-29\pm\sqrt{-2924}}{58}$$

No real answers.

No collision.

END