# Quaternion Practice

1)Given 2 quaternions, $q_1$ and $q_2$, calculate:

1) $\tilde{q}_1, \tilde{q}_2$
2) $\mathrm{N}(q_1), N(q_2)$
3) $q_1^{-1}, q_2^{-1}$
4) $q_1 + q_2$
5) $q_1 \bullet q_2$
6) $q_1 q_2$

In each case:

A: $q_1 = [2,1,3,1]$ $\quad q_2 = [1,1,0,2]$
B: $q_1 = [3,2,1,1]$ $\quad q_2 = [2,2,1,0]$
C: $q_1 = [-2,-1,1,3]$ $\quad q_2 = [5,0,0,1]$
D: $q_1 = [2,-1,1,0]$ $\quad q_2 = [-1,3,-4,1]$

2)
Answer the following ten questions letting $q_0$ be the quaternion representing a rotation of 180° about the Z-axis and $q_1$ the quaternion representing a rotation of 120° about an axis parallel to the vector <1, 1, -1>.

(a) Construct the quaternions $q_0$ and $q_1$.
(b) Determine the sum of $q_0$ and $q_1$.
(c) Determine the conjugate of $q_0$.
(d) Determine $q_2 = s\, q_0$ if $s = 2$.
(e) Determine the norm of $q_2$ from the previous question.
(f) Determine $q_0 \bullet q_1$.
(g) Determine $q_0\, q_1$.
(h) Convert $q_0$ to a rotation matrix.
(i) Rotate the vector s = <1, 0, 0> with the quaternion $q_0$.

3) Given the following quaternions $q_0 = [2, 3, 2, 1]$ and $q_1 = [3, 2, -2, 0]$
(a) Determine $q_0 + q_1$
(b) Determine $q_0\, q_1$
(c) Determine $q_0 \bullet q_1$
(d) Determine the inverse $q_0$ . The inverse of a quaternion q is defined as

$$q^{-1} = \frac{\tilde{q}}{N(q)}$$

Where $\tilde{q}$ is the conjugate of q and N(q) its norm.

(e) Determine the angle between the 2 quaternions . $\cos\theta = \dfrac{q_0 \bullet q_1}{\sqrt{N(q_0)} \bullet \sqrt{N(q_1)}}$

$q_0 q_1 = [a, \overrightarrow{v_0}][b, \overrightarrow{v_1}] = [a.b \; - \overrightarrow{v_0} \bullet \overrightarrow{v_1} \;,\; a\overrightarrow{v_1} \;+\; b\overrightarrow{v_0} \;+\; \overrightarrow{v_0} \times \overrightarrow{v_1} \;]$

4)Given the following quaternions $q_0$ =[2, 0 , -1 , 2] and $q_1$ = [ 3 , 2 , 0 , 3]

(a) Determine $q_0 + q_1$
(b) Determine $q_0\, q_1$
(c) Determine $q_0 \bullet q$
(d) Determine the inverse $q_0$ . The inverse of a quaternion q is defined as

$$q^{-1} = \frac{\tilde{q}}{N(q)}$$

Where $\tilde{q}$ is the conjugate of q and N(q) its norm.

(e) Determine the angle between the 2 quaternions .

$$\cos\theta = \frac{q_0 \bullet q_1}{\sqrt{N(q_0)} \bullet \sqrt{N(q_1)}}$$

$$q_0 q_1 = [a, \overrightarrow{v_0}][b, \overrightarrow{v_1}] = [a.b \; - \overrightarrow{v_0} \bullet \overrightarrow{v_1} \;,\; a\overrightarrow{v_1} \;+\; b\overrightarrow{v_0} \;+\; \overrightarrow{v_0} \times \overrightarrow{v_1} \;]$$

5) The quaternion representation of a rotation of angle $\theta$ about an arbitrary axis spanned by the vector $\vec{v}$ is $q = [\cos(\frac{\theta}{2}), \hat{v} \bullet \sin(\frac{\theta}{2})]$ where $\hat{v}$ =normalized vector of $\vec{v}$ .

Let $q_0$ be the quaternion representing a rotation of 90° about the Y-axis and $q_1$ the quaternion representing a rotation of 180° about an axis parallel to the vector < 1, 1, 0>.

(a) Construct the quaternion $q_0$.
(b) Construct the quaternion $q_1$.
(c) Determine the conjugate of $q_1$.
(d) Determine the angle $\theta$ between the two quaternion . $\theta = \cos^{-1}(q_0 \bullet q_1)$

6) The quaternion representation of a rotation of angle $\theta$ about an arbitrary axis spanned by the vector $\vec{v}$ is $q = [\cos(\frac{\theta}{2}), \hat{v} \bullet \sin(\frac{\theta}{2})]$ where $\hat{v}$ =normalized vector of $\vec{v}$ .

Let $q_0$ be the quaternion representing a rotation of 90° about the Z-axis and $q_1$ the quaternion representing a rotation of 180° about an axis parallel to the vector < -1, 0, 1>.

(a) Construct the quaternions $q_0$.
(b) Construct the quaternions $q_1$.
(c) Determine the conjugate of $q_1$ .
(d) Determine the angle $\theta$ between the two quaternions . $\theta = \cos^{-1}(q_0 \bullet q_1)$