GEN 242: Linear Algebra

Chapter 5: Lines and Planes

Solutions Guide

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Answers

Lines

A. Parametric equations for rays and lines:

1. $\vec{s}_0 = (1,5,-3)$ and $\vec{u} = (1,2,3)$

$$\boxed{L: \begin{cases} x = 1 + t \\ y = 5 + 2t \\ z = -3 + 3t \end{cases}, t \geq 0}$$ and $$\boxed{L: \begin{cases} x = 1 + t \\ y = 5 + 2t \\ z = -3 + 3t \end{cases}, 0 \leq t \leq 1}$$

2. $\vec{s}_0 = (1,0,-1)$ and $\vec{u} = (1,0,2)$

$$\boxed{L: \begin{cases} x = 1 + t \\ y = 0 \\ z = -1 + 2t \end{cases}, t \geq 0}$$ and $$\boxed{L: \begin{cases} x = 1 + t \\ y = 0 \\ z = -1 + 2t \end{cases}, 0 \leq t \leq 1}$$

B. Descriptions:

1. $L: \begin{cases} x = 1 \\ y = 3 + t \\ z = 1 - t \end{cases}, t \in \mathbb{R} \rightarrow$ Line A originates at point $\vec{s}_0 = (1,3,1)$ and is parallel to vector $\vec{v} = (0,1,-1)$.

2. $L: \begin{cases} x = 5 + 2t \\ y = 3 + t \\ z = 2 - 3t \end{cases} \; 0 \leq t \leq 1 \rightarrow$ Line segment B originates at point $\vec{s}_0 = (5,3,2)$ and is parallel to vector $\vec{v} = (2,1,-3)$.

3. $L: \begin{cases} x = 5 \\ y = -3 - 2t \\ z = 4t \end{cases}, t \geq 0 \rightarrow$ Ray C originates at point $\vec{s}_0 = (5,-3,0)$ and is parallel to vector $\vec{v} = (0,-2,4)$.

C. Parallel rays:

1. $L_1: \begin{cases} x = 5 + t \\ y = -3 - 2t \\ z = 4t \end{cases}, t \geq 0$ and $L_2: \begin{cases} x = 2 + 2t \\ y = 5 - 4t \\ z = 3 + 8t \end{cases}, t \geq 0$

$\boxed{\vec{v}_2 = 2\vec{v}_1 = k\vec{v}_1 : k \in \mathbb{R} \rightarrow \vec{v}_1 \parallel \vec{v}_2 \rightarrow L_1 \parallel L_2}$

2. $L_1: \begin{cases} x = 1 - 3t \\ y = 5 - t \\ z = 3 + t \end{cases}, t \geq 0$ and $L_2: \begin{cases} x = 2 + 15t \\ y = 1 + 5t \\ z = 10 - 5t \end{cases}, t \geq 0$

$\boxed{\vec{v}_2 = -5\vec{v}_1 = k\vec{v}_1 : k \in \mathbb{R} \rightarrow \vec{v}_1 \parallel \vec{v}_2 \rightarrow L_1 \parallel L_2}$

D. Perpendicular rays:

1. $L_1: \begin{cases} x = 5 + t \\ y = -3 - 2t \\ z = 4t \end{cases}, t \geq 0$ and $L_2: \begin{cases} x = 2 + 12t \\ y = 5 - 4t \\ z = 3 - 5t \end{cases}, t \geq 0$

$\boxed{\vec{v}_1 \cdot \vec{v}_2 = 12 + 8 + (-20) = 0 \rightarrow \vec{v}_1 \perp \vec{v}_2 \rightarrow L_1 \perp L_2}$

2. $L_1: \begin{cases} x = 1 - 3t \\ y = 5 - t \\ z = 3 + t \end{cases}, t \geq 0$ and $L_2: \begin{cases} x = 2 + t \\ y = 1 - t \\ z = 1 + 2t \end{cases}, t \geq 0$

$\boxed{\vec{v}_1 \cdot \vec{v}_2 = -3 + 1 + 2 = 0 \rightarrow \vec{v}_1 \perp \vec{v}_2 \rightarrow L_1 \perp L_2}$

3. $L_1: \vec{s}_{0,1} = (1,5,-3), \vec{u}_1 = (1,2,0)$ and $L_2: \vec{s}_{0,2} = (0,1,-3), \vec{u}_2 = (-12,6,1)$

$\boxed{\vec{u}_1 \cdot \vec{u}_2 = -12 + 12 + 0 = 0 \rightarrow \vec{u}_1 \perp \vec{u}_2 \rightarrow L_1 \perp L_2}$

E. $L_1: \begin{cases} x = 1 \\ y = 3 + t \\ 1 - t \end{cases}, 0 \leq t \leq 1$ and $L_2: \begin{cases} x = 5 + 2t \\ y = 3 + t \\ 2 - 3t \end{cases}, 0 \leq t \leq 1 \rightarrow \boxed{\theta \approx 40.9°}$

Planes

A. Planes, reference point and normal vector:

> For Question A, answers involve an arbitrary choice of possible $\vec{p}_0$ values.
>
> You can check different $\vec{p}_0$ values by substituting them back into the plane equation.

1. $4x + 3y - 2z - 10 = 0 \rightarrow \boxed{\begin{array}{c}\vec{n} = (4,3,-2) \\ \vec{p}_0 = (0,0,-5)\end{array}}$

2. $-x + 2y + z = 14 \rightarrow \boxed{\begin{array}{c}\vec{n} = (-1,2,1) \\ \vec{p}_0 = (0,7,0)\end{array}}$

3. $x - y + 7 = 0 \rightarrow \boxed{\begin{array}{c}\vec{n} = (1,-1,0) \\ \vec{p}_0 = (0,7,0)\end{array}}$

B. Plane equations:

1. $\vec{N} = (1,2,3), \vec{A} = (0,1,-2) \rightarrow \boxed{P_a: x + 2y + 3z + 4 = 0}$

2. $\vec{N} = (2,-1,5), \vec{A} = (1,2,-1) \rightarrow \boxed{P_b: 2x - y + 5z + 5 = 0}$

3. $\vec{N} = (1,-5,0), \vec{A} = (1,-2,0) \rightarrow \boxed{P_c: x - 5y - 11 = 0}$

C. Point's location relative to plane:

1. $P: x - y + z - 4 = 0$ and $\vec{A} = (1,2,1) \rightarrow$ Point A is behind plane P.

2. $P: -x + 2y + z = 14$ and $\vec{A} = (-7,0,7) \rightarrow$ Point A is on plane P.

3. $P: 2x + y - 5z + 2 = 0$ and $\vec{A} = (2,1,-2) \rightarrow$ Point A is in front of plane P.

D. Plane equations from point triads:

1. $\vec{A} = (1,0,1), \vec{B} = (2,1,-1), \vec{C} = (0,1,2) \rightarrow \boxed{P: 3x + y + 2z - 5 = 0}$

2. $\vec{A} = (1,2,3), \vec{B} = (1,1,1), \vec{C} = (5,2,4) \rightarrow \boxed{P: -x - 8y + 4z + 5 = 0}$

E. Point projection on plane:

1. $P: x - y + z + 4 = 0$ and $\vec{A} = (1,2,1) \rightarrow \boxed{\vec{q} = \left(-\frac{1}{3}, \frac{10}{3}, -\frac{1}{3}\right)}$

2. $P: -x + 2y + z = 1$ and $\vec{A} = (2,1,3) \rightarrow \boxed{\vec{q} = \left(\frac{7}{3}, \frac{1}{3}, \frac{8}{3}\right)}$

3. $P: 2x + y - 5z + 2 = 0$ and $\vec{A} = (2,1,-2) \rightarrow \boxed{\vec{q} = \left(\frac{13}{15}, \frac{13}{30}, \frac{5}{6}\right)}$

F. Distance between plane and point:

1. $x + 2y - z + 1 = 0$ and $\vec{A} = (1,1,1) \rightarrow \boxed{D = \frac{3}{\sqrt{6}} \approx 1.2}$

2. $x - y + z - 4 = 0$ and $\vec{A} = (1,2,1) \rightarrow \boxed{D = \frac{4}{\sqrt{3}} \approx 2.3}$

3. $3x + 2y + z = 7$ and $\vec{A} = (1,2,1) \rightarrow \boxed{D = \frac{1}{\sqrt{14}} \approx 0.3}$

G. Plane intersection:

1. $P_1: -3x + 2y + z = -5$ and $P_2: 7x + 3y - 2z = -2$

$$\boxed{L: \begin{cases} x = -7t \\ y = -\dfrac{12}{7} + t \\ z = -\dfrac{11}{7} - 23t \end{cases}, t \in \mathbb{R}}$$

2. $P_1: -x + 2y + z = 14$ and $P_2: 2x + y = 3$

$$\boxed{L: \begin{cases} x = -t \\ y = 3 + 2t \\ z = 8 - 5t \end{cases}, t \in \mathbb{R}}$$

Answers to Question G include an arbitrary choice of possible $\vec{s}_0$ values, so the constants in your parametric equations may vary.

You can check different $\vec{s}_0$ values by substituting them in both original plane equations.

H. Angle between planes:

1. $P_1: x - y + z - 4 = 0$ and $P_2: x - y + 2z = 0 \rightarrow \boxed{\theta \approx 19.5°}$

2. $P_1: 2x - y + 3z = 1$ and $P_2: x - y + 2z = 0 \rightarrow \boxed{\theta \approx 10.9°}$

I. Perpendicular planes:

1. $P_1 : x - 2y + 4z = 7$ and $P_2 : 2x - 5y - 3z = 1$

$$\boxed{\vec{n}_1 \cdot \vec{n}_2 = 2 + 10 + (-12) = 0 \rightarrow \vec{n}_1 \perp \vec{n}_2 \rightarrow P_1 \perp P_2}$$

2. $P_1 : 2x + y = 3$ and $P_2 : 5x - 10y - 3z = 1$

$$\boxed{\vec{n}_1 \cdot \vec{n}_2 = 10 + (-10) + 0 = 0 \rightarrow \vec{n}_1 \perp \vec{n}_2 \rightarrow P_1 \perp P_2}$$

J. Parallel planes:

1. $P_1 : x - 2y + 4z = 7$ and $P_2 : -2x + 4y - 8z = 1$

$$\boxed{\vec{n}_2 = -2\vec{n}_1 = k\vec{n}_1, k \in \mathbb{R} \rightarrow \vec{n}_1 \parallel \vec{n}_2 \rightarrow P_1 \parallel P_2}$$

2. $P_1 : 2x + y - z = 3$ and $P_2 : 6x + 3y - 3z = 1$

$$\boxed{\vec{n}_2 = 3\vec{n}_1 = k\vec{n}_1, k \in \mathbb{R} \rightarrow \vec{n}_1 \parallel \vec{n}_2 \rightarrow P_1 \parallel P_2}$$

Solutions

Lines in 3D Space

Problem A

For each of the following points and vectors, write equations for a ray and a line segment that originates at the given point and is parallel to the given vector.

$$L:\begin{cases} x = s_{0,x} + u_x t \\ y = s_{0,y} + u_y t \\ z = s_{0,z} + u_z t \end{cases}$$ with $t \geq 0$ for a ray and $0 \leq t \leq 1$ for a line segment.

A.1 $\vec{s}_0 = (1,5,-3)$ and $\vec{u} = (1,2,3)$

$$L:\begin{cases} x = (1) + (1)t \\ y = (5) + (2)t \quad ,t \geq 0 \\ z = (-3) + (3)t \end{cases}$$

$$L:\begin{cases} x = (1) + (1)t \\ y = (5) + (2)t \quad ,0 \leq t \leq 1 \\ z = (-3) + (3)t \end{cases}$$

$$\boxed{L:\begin{cases} x = 1 + t \\ y = 5 + 2t \quad ,t \geq 0 \\ z = -3 + 3t \end{cases}}$$

$$\boxed{L:\begin{cases} x = 1 + t \\ y = 5 + 2t \quad ,0 \leq t \leq 1 \\ z = -3 + 3t \end{cases}}$$

A.2 $\vec{s}_0 = (1,0,-1)$ and $\vec{u} = (1,0,2)$

$$L:\begin{cases} x = (1) + (1)t \\ y = (0) + (0)t \quad ,t \geq 0 \\ z = (-1) + (2)t \end{cases}$$

$$L:\begin{cases} x = (1) + (1)t \\ y = (0) + (0)t \quad ,0 \leq t \leq 1 \\ z = (-1) + (2)t \end{cases}$$

$$\boxed{L:\begin{cases} x = 1 + t \\ y = 0 \quad ,t \geq 0 \\ z = -1 + 2t \end{cases}}$$

$$\boxed{L:\begin{cases} x = 1 + t \\ y = 0 \quad ,0 \leq t \leq 1 \\ z = -1 + 2t \end{cases}}$$

Problem B

For each of the given parametric equations, describe the associated line, line segment, or ray:

B.1 $L_1: \begin{cases} x = 1 \\ y = 3 + t, t \in \mathbb{R} \\ z = 1 - t \end{cases}$

$L: \begin{cases} x = s_{0,x} + v_x t \\ y = s_{0,y} + v_y t \\ z = s_{0,z} + v_z t \end{cases}$

$L_1: \begin{cases} x = (1) + (0)t \\ y = (3) + (1)t \\ z = (1) + (-1)t \end{cases}$

$\vec{s}_0 = (1,3,1)$

$\vec{v} = (0,1,-1)$

$t \in \mathbb{R}$ means this is a line.

Line A originates at point $\vec{s}_0 = (1,3,1)$ and is parallel to vector $\vec{v} = (0,1,-1)$.

B.2 $L_2: \begin{cases} x = 5 + 2t \\ y = 3 + t, 0 \leq t \leq 1 \\ z = 2 - 3t \end{cases}$

$L: \begin{cases} x = s_{0,x} + v_x t \\ y = s_{0,y} + v_y t \\ z = s_{0,z} + v_z t \end{cases}$

$L_1: \begin{cases} x = (5) + (2)t \\ y = (3) + (1)t \\ z = (2) + (-3)t \end{cases}$

$\vec{s}_0 = (5,3,2)$

$\vec{v} = (2,1,-3)$

$0 \leq t \leq 1$ means this is a line segment.

Line segment B originates at point $\vec{s}_0 = (5,3,2)$ and is parallel to vector $\vec{v} = (2,1,-3)$.

B.3 $L_1: \begin{cases} x = 5 \\ y = -3 - 2t, t \geq 0 \\ z = 4t \end{cases}$

$L: \begin{cases} x = s_{0,x} + v_x t \\ y = s_{0,y} + v_y t \\ z = s_{0,z} + v_z t \end{cases}$

$L_1: \begin{cases} x = (5) + (0)t \\ y = (-3) + (-2)t \\ z = (0) + (4)t \end{cases}$

$\vec{s}_0 = (5,-3,0)$

$\vec{v} = (0,-2,4)$

$t \geq 0$ means this is a ray.

Ray C originates at point $\vec{s}_0 = (5,-3,0)$ and is parallel to vector $\vec{v} = (0,-2,4)$.

Problem C

For each of the following ray pairs, show that each pair is parallel:

Find the parallel vectors, then try to factor. If one vector is a multiple of the other, the vectors are parallel. If the vectors are parallel, their corresponding rays are parallel.

C.1 $L_1:\begin{cases} x = 5 + t \\ y = -3 - 2t \\ z = 4t \end{cases}, t \geq 0$ and $L_2:\begin{cases} x = 2 + 2t \\ y = 5 - 4t \\ z = 3 + 8t \end{cases}, t \geq 0$

$$L:\begin{cases} x = s_{0,x} + v_x t \\ y = s_{0,y} + v_y t \\ z = s_{0,z} + v_z t \end{cases}$$

$$L_1:\begin{cases} x = 5 + (1)t \\ y = -3 + (-2)t \\ z = (4)t \end{cases} \qquad L_2:\begin{cases} x = 2 + (2)t \\ y = 5 + (-4)t \\ z = 3 + (8)t \end{cases}$$

$$\vec{v}_1 = (1, -2, 4) \qquad \vec{v}_2 = (2, -4, 8)$$

$$\vec{v}_2 = 2(1, -2, 4)$$

$$\vec{v}_2 = 2\vec{v}_1$$

$$\vec{v}_2 = k\vec{v}_1 : k \in \mathbb{R} \rightarrow \vec{v}_1 \parallel \vec{v}_2 \rightarrow L_1 \parallel L_2$$

C.2 $L_1:\begin{cases} x = 1 - 3t \\ y = 5 - t \\ z = 3 + t \end{cases}, t \geq 0$ and $L_2:\begin{cases} x = 2 + 15t \\ y = 1 + 5t \\ z = 10 - 5t \end{cases}, t \geq 0$

$$L:\begin{cases} x = s_{0,x} + v_x t \\ y = s_{0,y} + v_y t \\ z = s_{0,z} + v_z t \end{cases}$$

$$L_1:\begin{cases} x = 1 + (-3)t \\ y = 5 + (-1)t \\ z = 3 + (1)t \end{cases} \qquad L_2:\begin{cases} x = 2 + (15)t \\ y = 1 + (5)t \\ z = 10 + (-5)t \end{cases}$$

$$\vec{v}_1 = (-3, -1, 1) \qquad \vec{v}_2 = (15, 5, -5)$$

$$\vec{v}_2 = -5(-3, -1, 1)$$

$$\vec{v}_2 = -5\vec{v}_1$$

$$\vec{v}_2 = k\vec{v}_1 : k \in \mathbb{R} \rightarrow \vec{v}_1 \parallel \vec{v}_2 \rightarrow L_1 \parallel L_2$$

Problem D

For each of the following ray pairs, show that each pair is perpendicular:

Find parallel vectors, find the dot product between them. If the result is 0, the vectors are perpendicular. If the vectors are perpendicular, their corresponding rays are perpendicular.

D.1 Show that $L_1: \begin{cases} x = 5 + t \\ y = -3 - 2t \\ z = 4t \end{cases}, t \geq 0$ and $L_2: \begin{cases} x = 2 + 12t \\ y = 5 - 4t \\ z = 3 - 5t \end{cases}, t \geq 0$ are perpendicular.

$$L: \begin{cases} x = s_{0,x} + v_x t \\ y = s_{0,y} + v_y t \\ z = s_{0,z} + v_z t \end{cases}$$

$$L_1: \begin{cases} x = 5 + (1)t \\ y = -3 + (-2)t \\ z = (4)t \end{cases} \qquad L_2: \begin{cases} x = 2 + (12)t \\ y = 1 + (-4)t \\ z = 10 + (-5)t \end{cases}$$

$$\vec{v}_1 = (1, -2, 4) \qquad \vec{v}_2 = (12, -4, -5)$$

$$\vec{v}_1 \cdot \vec{v}_2 = v_{1,x}v_{2,x} + v_{1,y}v_{2,y} + v_{1,z}v_{2,z}$$

$$\vec{v}_1 \cdot \vec{v}_2 = (1)(12) + (-2)(-4) + (4)(-5)$$

$$\vec{v}_1 \cdot \vec{v}_2 = 12 + 8 + (-20)$$

$$\vec{v}_1 \cdot \vec{v}_2 = 0$$

$$\vec{v}_1 \perp \vec{v}_2$$

$$L_1 \perp L_2$$

D.2 Show that $L_1: \begin{cases} x = 1 - 3t \\ y = 5 - t \\ z = 3 + t \end{cases}, t \geq 0$ and $L_2: \begin{cases} x = 2 + t \\ y = 1 - t \\ z = 1 + 2t \end{cases}, t \geq 0$ are perpendicular.

$$L: \begin{cases} x = s_{0,x} + v_x t \\ y = s_{0,y} + v_y t \\ z = s_{0,z} + v_z t \end{cases}$$

$$L_1: \begin{cases} x = 1 + (-3)t \\ y = 5 + (-1)t \\ z = 3 + (1)t \end{cases} \qquad L_2: \begin{cases} x = 2 + (1)t \\ y = 1 + (-1)t \\ z = 1 + (2)t \end{cases}$$

$$\vec{v}_1 = (-3, -1, 1) \qquad \vec{v}_2 = (1, -1, 2)$$

$$\vec{v}_1 \cdot \vec{v}_2 = v_{1,x} v_{2,x} + v_{1,y} v_{2,y} + v_{1,z} v_{2,z}$$

$$\vec{v}_1 \cdot \vec{v}_2 = (-3)(1) + (-1)(-1) + (1)(2)$$

$$\vec{v}_1 \cdot \vec{v}_2 = -3 + 1 + 2$$

$$\vec{v}_1 \cdot \vec{v}_2 = 0$$

$$\vec{v}_1 \perp \vec{v}_2$$

$$L_1 \perp L_2$$

D.3 Show that $L_1: \vec{s}_{0,1} = (1,5,-3), \vec{u}_1 = (1,2,0)$ and $L_2: \vec{s}_{0,2} = (0,1,-3), \vec{u}_2 = (-12,6,1)$ are perpendicular.

$$\vec{u}_1 \cdot \vec{u}_2 = u_{1,x} u_{2,x} + u_{1,y} u_{2,y} + u_{1,z} u_{2,z}$$

$$\vec{u}_1 \cdot \vec{u}_2 = (1)(-12) + (2)(6) + (0)(1)$$

$$\vec{u}_1 \cdot \vec{u}_2 = -12 + 12 + 0$$

$$\vec{u}_1 \cdot \vec{u}_2 = 0$$

$$\vec{u}_1 \perp \vec{u}_2$$

$$L_1 \perp L_2$$

Problem E

Given line segments $L_1: \begin{cases} x = 1 \\ y = 3 + t \\ 1 - t \end{cases}, 0 \le t \le 1$ and $L_2: \begin{cases} x = 5 + 2t \\ y = 3 + t \\ 2 - 3t \end{cases}, 0 \le t \le 1$, find the angle between them.

Find the parallel vectors, then use the definition of dot product to find the angle between them. That angle will also be the angle between the line segments.

$$L: \begin{cases} x = s_{0,x} + v_x t \\ y = s_{0,y} + v_y t \\ z = s_{0,z} + v_z t \end{cases}$$

$$L_1: \begin{cases} x = 1 + (0)t \\ y = 3 + (1)t \\ z = 1 + (-1)t \end{cases} \qquad L_2: \begin{cases} x = 5 + (2)t \\ y = 3 + (1)t \\ z = 2 + (-3)t \end{cases}$$

$$\vec{v}_1 = (0,1,-1) \qquad \vec{v}_2 = (2,1,-3)$$

$$\vec{v}_1 \cdot \vec{v}_2 = \|\vec{v}_1\| \cdot \|\vec{v}_2\| \cdot \cos(\theta)$$

$$\cos(\theta) = \frac{\vec{v}_1 \cdot \vec{v}_2}{\|\vec{v}_1\| \cdot \|\vec{v}_2\|}$$

$$\theta = \cos^{-1}\left(\frac{\vec{v}_1 \cdot \vec{v}_2}{\|\vec{v}_1\| \cdot \|\vec{v}_2\|}\right)$$

$$\theta = \cos^{-1}\left(\frac{v_{1,x}v_{2,x} + v_{1,y}v_{2,y} + v_{1,z}v_{2,z}}{\sqrt{(v_{1,x})^2 + (v_{1,y})^2 + (v_{1,z})^2} \cdot \sqrt{(v_{2,x})^2 + (v_{2,y})^2 + (v_{2,z})^2}}\right)$$

$$\theta = \cos^{-1}\left(\frac{(0)(2) + (1)(1) + (-1)(-3)}{\sqrt{(0)^2 + (1)^2 + (-1)^2} \cdot \sqrt{(2)^2 + (1)^2 + (-3)^2}}\right)$$

$$\theta = \cos^{-1}\left(\frac{0 + 1 + 3}{\sqrt{0 + 1 + 1} \cdot \sqrt{4 + 1 + 9}}\right)$$

$$\theta = \cos^{-1}\left(\frac{4}{\sqrt{2} \cdot \sqrt{14}}\right)$$

$$\theta = \cos^{-1}\left(\frac{4}{\sqrt{28}}\right)$$

$$\boxed{\theta \approx 40.9^\circ}$$

Planes in 3D Space

Problem A

For each of the following planes, find its normal vector and a reference point:

$$P: n_x x + n_y y + n_z z + d = 0$$
$$d = -\vec{n} \cdot \vec{p}_0$$

A.1 Given $P: 4x + 3y - 2z - 10 = 0$, find its normal vector and a reference point.

$$(4)x + (3)y + (-2)z + (-10) = 0$$
$$\boxed{\vec{n} = (4,3,-2)}$$

$$d = -\vec{n} \cdot \vec{p}_0$$
$$-10 = -\left(n_x p_{0,x} + n_y p_{0,y} + n_z p_{0,z}\right)$$
$$10 = n_x p_{0,x} + n_y p_{0,y} + n_z p_{0,z}$$
$$10 = (4)p_{0,x} + (3)p_{0,y} + (-2)p_{0,z}$$
$$10 = 4p_{0,x} + 3p_{0,y} - 2p_{0,z}$$

Let $p_{0,x} = p_{0,y} = 0.$

We could choose any values for $p_{0,x}$ and $p_{0,y}$.

$$10 = 4(0) + 3(0) - 2p_{0,z}$$
$$10 = 0 + 0 - 2p_{0,z}$$
$$10 = -2p_{0,z}$$
$$p_{0,z} = \frac{10}{-2}$$
$$p_{0,z} = -5$$
$$\boxed{\vec{p}_0 = (0,0,-5)}$$

A.2 Given $P: -x + 2y + z = 14$, find its normal vector and a reference point.

$-x + 2y + z - 14 = 0$

$(-1)x + (2)y + (1)z + (-14) = 0$

$\boxed{\vec{n} = (-1,2,1)}$

$d = -\vec{n} \cdot \vec{p}_0$

$-14 = -\left(n_x p_{0,x} + n_y p_{0,y} + n_z p_{0,z}\right)$

$14 = n_x p_{0,x} + n_y p_{0,y} + n_z p_{0,z}$

$14 = (-1)p_{0,x} + (2)p_{0,y} + (1)p_{0,z}$

$14 = -p_{0,x} + 2p_{0,y} + p_{0,z}$

Let $p_{0,x} = p_{0,z} = 0$.

We could choose any values for $p_{0,x}$ and $p_{0,z}$.

$14 = -(0) + 2p_{0,y} + (0)$

$14 = 0 + 2p_{0,y} + 0$

$14 = 2p_{0,y}$

$p_{0,y} = \frac{14}{2}$

$p_{0,y} = 7$

$\boxed{\vec{p}_0 = (0,7,0)}$

A.3 Given, $P: x - y + 7 = 0$, find its normal vector and a reference point.

$(1)x + (-1)y + (0)z + (7) = 0$

$\boxed{\vec{n} = (1, -1, 0)}$

$d = -\vec{n} \cdot \vec{p}_0$

$7 = -\left(n_x p_{0,x} + n_y p_{0,y} + n_z p_{0,z}\right)$

$-7 = n_x p_{0,x} + n_y p_{0,y} + n_z p_{0,z}$

$-7 = (1)p_{0,x} + (-1)p_{0,y} + (0)p_{0,z}$

$-7 = p_{0,x} - 1p_{0,y} + 0$

$-7 = p_{0,x} - 1p_{0,y}$

Let $p_{0,x} = p_{0,z} = 0$.

We could choose any values for $p_{0,x}$ and $p_{0,z}$.

$-7 = (0) - p_{0,y}$

$-7 = 0 - p_{0,y}$

$-7 = -p_{0,y}$

$p_{0,y} = \dfrac{-7}{-1}$

$p_{0,y} = 7$

$\boxed{\vec{p}_0 = (0,7,0)}$

Problem B

For each of the following pairs of normal vector ($\vec{N}$) and reference point ($\vec{A}$), write the equation of the corresponding plane:

$$P: n_x x + n_y y + n_z z + d = 0$$

$$d = -\vec{n} \cdot \vec{p}_0$$

B.1 Given $\vec{N} = (1,2,3)$ and $\vec{A} = (0,1,-2)$, write the equation of the corresponding plane.

$$P: n_x x + n_y y + n_z z + d = 0$$

$$P: (1)x + (2)y + (3)z + d = 0$$

$$P: x + 2y + 3z + d = 0$$

$$d = -\vec{N} \cdot \vec{A}$$

$$d = -\left(n_x a_x + n_y a_y + n_z a_z\right)$$

$$d = -[(1)(0) + (2)(1) + (3)(-2)]$$

$$d = -[0 + 2 + (-6)]$$

$$d = -(-4)$$

$$d = 4$$

$$\boxed{P: x + 2y + 3z + 4 = 0}$$

B.2 Given, $\vec{N} = (2,-1,5)$ $\vec{A} = (1,2,-1)$, write the equation of the corresponding plane.

$$P: n_x x + n_y y + n_z z + d = 0$$

$$P: (2)x + (-1)y + (5)z + d = 0$$

$$P: 2x - y + 5z + d = 0$$

$$d = -\vec{N} \cdot \vec{A}$$

$$d = -\left(n_x a_x + n_y a_y + n_z a_z\right)$$

$$d = -[(2)(1) + (-1)(2) + (5)(-1)]$$

$$d = -[2 + (-2) + (-5)]$$

$$d = -(-5)$$

$$d = 5$$

$$\boxed{P: 2x - y + 5z + 5 = 0}$$

B.3 Given $\vec{N} = (1, -5, 0)$ and $\vec{A} = (1, -2, 0)$, write the equation of the corresponding plane.

$P: n_x x + n_y y + n_z z + d = 0$

$P: (1)x + (-5)y + (0)z + d = 0$

$P: x - 5y + 0 + d = 0$

$P: x - 5y + d = 0$

$d = -\vec{N} \cdot \vec{A}$

$d = -\left(n_x a_x + n_y a_y + n_z a_z\right)$

$d = -[(1)(1) + (-5)(-2) + (0)(0)]$

$d = -(1 + 10 + 0)$

$d = -(11)$

$d = -11$

$\boxed{P: x - 5y - 11 = 0}$

Problem C

For each of the following pairs of plane and point, describe the point's location relative to the plane:

Find the dot product of the plane's normal vector and the given point, then add the plane's d value.

$\vec{n} \cdot \vec{A} + d = 0$	Coplanar.
$\vec{n} \cdot \vec{A} + d < 0$	Behind the plane.
$\vec{n} \cdot \vec{A} + d > 0$	In front of the plane.

C.1 Given, $P: x - y + z - 4 = 0$ and $\vec{A} = (1,2,1)$, where is $\vec{A}$ relative to P?

$P: n_x x + n_y y + n_z z + d = 0$

$P: (1)x + (-1)y + (1)z + (-4) = 0$

$\vec{n} = (1,-1,1)$

$d = -4$

$\vec{n} \cdot \vec{A} + d = n_x A_x + n_y A_y + n_z A_z + d$

$\vec{n} \cdot \vec{A} + d = (1)(1) + (-1)(2) + (1)(1) + (-4)$

$\vec{n} \cdot \vec{A} + d = 1 + (-2) + 1 - 4$

$\vec{n} \cdot \vec{A} + d = -4$

$\vec{n} \cdot \vec{A} + d < 0$

Point $\vec{A}$ is behind plane P.

C.2 Given $P: -x + 2y + z = 14$ and $\vec{A} = (-7,0,7)$, where is $\vec{A}$ relative to P?

$P: n_x x + n_y y + n_z z + d = 0$

$P_b: -x + 2y + z - 14 = 0$

$P_b: (-1)x + (2)y + (1)z + (-14) = 0$

$\vec{n} = (-1,2,1)$

$d = -14$

$\vec{n} \cdot \vec{A} + d = n_x A_x + n_y A_y + n_z A_z + d$

$\vec{n} \cdot \vec{A} + d = (-1)(-7) + (2)(0) + (1)(7) + (-14)$

$\vec{n} \cdot \vec{A} + d = 7 + 0 + 7 - 14$

$\vec{n} \cdot \vec{A} + d = 0$

Point $\vec{A}$ is on plane P.

C.3 Given $P: 2x + y - 5z + 2 = 0$ and $\vec{A} = (2,1,-2)$, where is $\vec{A}$ relative to P?

$P: n_x x + n_y y + n_z z + d = 0$

$P_c: (2)x + (1)y + (-5)z + (2) = 0$

$\vec{n} = (2,1,-5)$

$d = 2$

$\vec{n} \cdot \vec{A} + d = n_x A_x + n_y A_y + n_z A_z + d$

$\vec{n} \cdot \vec{A} + d = (2)(2) + (1)(1) + (-5)(-2) + (2)$

$\vec{n} \cdot \vec{A} + d = 4 + 1 + 10 + 2$

$\vec{n} \cdot \vec{A} + d = 17$

$\vec{n} \cdot \vec{A} + d > 0$

Point A is in front of plane P.

Problem D

D.1 Given vertices $\vec{A} = (1,0,1), \vec{B} = (2,1,-1), \vec{C} = (0,1,2)$, write the equation of the corresponding plane.

$\overrightarrow{AB} = \vec{B} - \vec{A}$

$\overrightarrow{AB} = (2,1,-1) - (1,0,1)$

$\overrightarrow{AB} = (2-1,1-0,-1-1)$

$\overrightarrow{AB} = (1,1,-2)$

$\overrightarrow{AC} = \vec{C} - \vec{A}$

$\overrightarrow{AC} = (0,1,2) - (1,0,1)$

$\overrightarrow{AC} = (0-1,1-0,2-1)$

$\overrightarrow{AC} = (-1,1,1)$

$$\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC}$$

$$\vec{n} = (1,1,-2) \times (-1,1,1)$$

$$\vec{n} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & -2 \\ -1 & 1 & 1 \end{vmatrix}$$

$$\vec{n} = \begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix} \hat{\imath} - \begin{vmatrix} 1 & -2 \\ -1 & 1 \end{vmatrix} \hat{\jmath} + \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} \hat{k}$$

$$\vec{n} = [(1)(1) - (1)(-2)]\hat{\imath} - [(1)(1) - (-1)(-2)]\hat{\jmath} + [(1)(1) - (-1)(1)]\hat{k}$$

$$\vec{n} = [1 - (-2)]\hat{\imath} - [1 - 2]\hat{\jmath} + [1 - (-1)]\hat{k}$$

$$\vec{n} = (1+2)\hat{\imath} - (1-2)\hat{\jmath} + (1+1)\hat{k}$$

$$\vec{n} = 3\hat{\imath} - (-1)\hat{\jmath} + 2\hat{k}$$

$$\vec{n} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$$

$$\vec{n} = (3,1,2)$$

Let $\vec{p}_0 = \vec{A} = (1,0,1)$.

$$d = -\vec{n} \cdot \vec{p}_0$$

$$d = -[(3)(1) + (1)(0) + (2)(1)]$$

$$d = -(3 + 0 + 2)$$

$$d = -(5)$$

$$d = -5$$

$$P: n_x x + n_y y + n_z z + d = 0$$

$$P: (3)x + (1)y + (2)z + (-5) = 0$$

$$\boxed{P: 3x + y + 2z - 5 = 0}$$

D.2 Given vertices $\vec{A} = (1,2,3), \vec{B} = (1,1,1), \vec{C} = (5,2,4)$, write the equation of the corresponding plane.

$\overrightarrow{AB} = \vec{B} - \vec{A}$

$\overrightarrow{AB} = (1,1,1) - (1,2,3)$

$\overrightarrow{AB} = (1-1, 1-2, 1-3)$

$\overrightarrow{AB} = (0,-1,-2)$

$\overrightarrow{AC} = \vec{C} - \vec{A}$

$\overrightarrow{AC} = (5,2,4) - (1,2,3)$

$\overrightarrow{AC} = (5-1, 2-2, 4-3)$

$\overrightarrow{AC} = (4,0,1)$

$\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC}$

$\vec{n} = (1,1,-2) \times (-1,1,1)$

$$\vec{n} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & -1 & -2 \\ 4 & 0 & 1 \end{vmatrix}$$

$$\vec{n} = \begin{vmatrix} -1 & -2 \\ 0 & 1 \end{vmatrix} \hat{\imath} - \begin{vmatrix} 0 & -2 \\ 4 & 1 \end{vmatrix} \hat{\jmath} + \begin{vmatrix} 0 & -1 \\ 4 & 0 \end{vmatrix} \hat{k}$$

$\vec{n} = [(-1)(1) - (0)(-2)]\hat{\imath} - [(0)(1) - (4)(-2)]\hat{\jmath} + [(0)(0) - (4)(-1)]\hat{k}$

$\vec{n} = (-1 - 0)\hat{\imath} - [0 - (-8)]\hat{\jmath} + [0 - (-4)]\hat{k}$

$\vec{n} = (-1 - 0)\hat{\imath} - (0 + 8)\hat{\jmath} + (0 + 4)\hat{k}$

$\vec{n} = (-1)\hat{\imath} - (8)\hat{\jmath} + (4)\hat{k}$

$\vec{n} = -\hat{\imath} - 8\hat{\jmath} + 4\hat{k}$

$\vec{n} = (-1,-8,4)$

Let $\vec{p}_0 = \vec{A} = (1,2,3)$.

$d = -\vec{n} \cdot \vec{p}_0$

$d = -[(-1)(1) + (-8)(2) + (4)(3)]$

$d = -[-1 + (-16) + 12]$

$d = -(-5)$

$d = 5$

$P: n_x x + n_y y + n_z z + d = 0$

$P: (-1)x + (-8)y + (4)z + (5) = 0$

$\boxed{P: -x - 8y + 4z + 5 = 0}$

Problem E

E.1 Given $P: x - y + z + 4 = 0$ and point $\vec{A} = (1,2,1)$, find the projection of $\vec{A}$ onto P in the direction of the plane's normal vector.

$$P: n_x x + n_y y + n_z z + d = 0$$

$$P: (1)x + (-1)y + (1)z + (4) = 0$$

$$\vec{n} = (1,-1,1)$$

$$d = 4$$

$$\vec{q} = \vec{A} - \frac{\vec{n} \cdot \vec{A} + d}{\vec{n} \cdot \vec{n}} \cdot \vec{n}$$

$$\vec{q} = (1,2,1) - \frac{[(1)(1) + (-1)(2) + (1)(1)] + (4)}{(1)(1) + (-1)(-1) + (1)(1)} \cdot (1,-1,1)$$

$$\vec{q} = (1,2,1) - \frac{[1 + (-2) + 1] + 4}{1 + 1 + 1} \cdot (1,-1,1)$$

$$\vec{q} = (1,2,1) - \frac{0 + 4}{3} \cdot (1,-1,1)$$

$$\vec{q} = (1,2,1) - \frac{4}{3} \cdot (1,-1,1)$$

$$\vec{q} = (1,2,1) - \left(\frac{4}{3}, -\frac{4}{3}, \frac{4}{3}\right)$$

$$\vec{q} = \left(\frac{3}{3} - \frac{4}{3}, \frac{6}{3} - \left(-\frac{4}{3}\right), \frac{3}{3} - \frac{4}{3}\right)$$

$$\boxed{\vec{q} = \left(-\frac{1}{3}, \frac{10}{3}, -\frac{1}{3}\right)}$$

E.2 Given $P\!: -x + 2y + z = 1$ and point $\vec{A} = (2,1,3)$, find the projection of $\vec{A}$ onto P in the direction of the plane's normal vector.

$P\!: n_x x + n_y y + n_z z + d = 0$

$P\!: -x + 2y + z - 1 = 0$

$P\!: (-1)x + (2)y + (1)z + (-1) = 0$

$\vec{n} = (-1,2,1)$

$d = -1$

$$\vec{q} = \vec{A} - \frac{\vec{n} \cdot \vec{A} + d}{\vec{n} \cdot \vec{n}} \cdot \vec{n}$$

$$\vec{q} = (2,1,3) - \frac{[(-1)(2) + (2)(1) + (1)(3)] + (-1)}{(-1)(-1) + (2)(2) + (1)(1)} \cdot (-1,2,1)$$

$$\vec{q} = (2,1,3) - \frac{(-2 + 2 + 3) - 1}{1 + 4 + 1} \cdot (-1,2,1)$$

$$\vec{q} = (2,1,3) - \frac{3 - 1}{6} \cdot (-1,2,1)$$

$$\vec{q} = (2,1,3) - \frac{2}{6} \cdot (-1,2,1)$$

$$\vec{q} = (2,1,3) - \frac{1}{3} \cdot (-1,2,1)$$

$$\vec{q} = (2,1,3) - \left(-\frac{1}{3}, \frac{2}{3}, \frac{1}{3}\right)$$

$$\vec{q} = \left(\frac{6}{3} - \left(-\frac{1}{3}\right), \frac{3}{3} - \frac{2}{3}, \frac{9}{3} - \frac{1}{3}\right)$$

$$\boxed{\vec{q} = \left(\frac{7}{3}, \frac{1}{3}, \frac{8}{3}\right)}$$

E.3 Given $P_c: 2x + y - 5z + 2 = 0$ and point $\vec{A} = (2,1,-2)$, find the projection of $\vec{A}$ onto P in the direction of the plane's normal vector.

$$P: n_x x + n_y y + n_z z + d = 0$$

$$P: (2)x + (1)y + (-5)z + (2) = 0$$

$$\vec{n} = (2,1,-5)$$

$$d = 2$$

$$\vec{q} = \vec{A} - \frac{\vec{n} \cdot \vec{A} + d}{\vec{n} \cdot \vec{n}} \cdot \vec{n}$$

$$\vec{q} = (2,1,-2) - \frac{[(2)(2) + (1)(1) + (-5)(-2)] + (2)}{(2)(2) + (1)(1) + (-5)(-5)} \cdot (2,1,-5)$$

$$\vec{q} = (2,1,-2) - \frac{(4 + 1 + 10) + 2}{4 + 1 + 25} \cdot (2,1,-5)$$

$$\vec{q} = (2,1,-2) - \frac{15 + 2}{30} \cdot (2,1,-5)$$

$$\vec{q} = (2,1,-2) - \frac{17}{30} \cdot (2,1,-5)$$

$$\vec{q} = (2,1,-2) - \left(\frac{34}{30}, \frac{17}{30}, -\frac{85}{30}\right)$$

$$\vec{q} = \left(\frac{60}{30} - \frac{34}{30}, \frac{30}{30} - \frac{17}{30}, \frac{-60}{30} - \left(-\frac{85}{30}\right)\right)$$

$$\vec{q} = \left(\frac{26}{30}, \frac{13}{30}, \frac{25}{30}\right)$$

$$\boxed{\vec{q} = \left(\frac{13}{15}, \frac{13}{30}, \frac{5}{6}\right)}$$

Problem F

F.1 Given $P: x + 2y - z + 1 = 0$ and point $\vec{A} = (1,1,1)$, find the distance between them.

$P: n_x x + n_y y + n_z z + d = 0$

$P: (1)x + (2)y + (-1)z + (1) = 0$

$\vec{n} = (1,2,-1)$

$d = 1$

$$D = \frac{|\vec{n} \cdot \vec{A} + d|}{\|\vec{n}\|}$$

$$D = \frac{|(1)(1) + (2)(1) + (-1)(1) + (1)|}{\sqrt{(1)^2 + (2)^2 + (-1)^2}}$$

$$D = \frac{|1 + 2 + (-1) + (1)|}{\sqrt{1 + 4 + 1}}$$

$$D = \frac{|3|}{\sqrt{6}}$$

$$\boxed{D = \frac{3}{\sqrt{6}} \approx 1.2}$$

F.2 Given $P: x - y + z - 4 = 0$ and point $\vec{A} = (1,2,1)$, find the distance between them.

$P: n_x x + n_y y + n_z z + d = 0$

$P: (1)x + (-1)y + (1)z + (-4) = 0$

$\vec{n} = (1,-1,1)$

$d = -4$

$$D = \frac{|\vec{n} \cdot \vec{A} + d|}{\|\vec{n}\|}$$

$$D = \frac{|(1)(1) + (-1)(2) + (1)(1) + (-4)|}{\sqrt{(1)^2 + (-1)^2 + (1)^2}}$$

$$D = \frac{|1 + (-2) + 1 - 4|}{\sqrt{1 + 1 + 1}}$$

$$D = \frac{|-4|}{\sqrt{3}}$$

$$\boxed{D = \frac{4}{\sqrt{3}} \approx 2.3}$$

F.3 Given $P: 3x + 2y + z = 7$ and point $\vec{A} = (1,2,1)$, find the distance between them.

$$P: n_x x + n_y y + n_z z + d = 0$$

$$P: 3x + 2y + z - 7 = 0$$

$$P: (3)x + (2)y + (1)z + (-7) = 0$$

$$\vec{n} = (3,2,1)$$

$$d = -7$$

$$D = \frac{\left|\vec{n} \cdot \vec{A} + d\right|}{\|\vec{n}\|}$$

$$D = \frac{|(3)(1) + (2)(2) + (1)(1) + (-7)|}{\sqrt{(3)^2 + (2)^2 + (1)^2}}$$

$$D = \frac{|3 + 4 + 1 - 7|}{\sqrt{9 + 4 + 1}}$$

$$D = \frac{|1|}{\sqrt{14}}$$

$$\boxed{D = \frac{1}{\sqrt{14}} \approx 0.3}$$

Problem G

G.1 Given $P_1: -3x + 2y + z = -5$ and $P_2: 7x + 3y - 2z = -2$, find the parametric equations for their line of intersection.

$P: n_x x + n_y y + n_z z + d = 0$

$P_1: -3x + 2y + z + (5) = 0$

$\vec{n}_1 = (-3,2,1)$

$d_1 = 5$

$P_2: 7x + 3y - 2z + 2 = 0$

$P_2: (7)x + (3)y + (-2)z + (2) = 0$

$\vec{n}_2 = (7,3,-2)$

$d_2 = 2$

$\vec{v} = \vec{n}_1 \times \vec{n}_2$

$\vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -3 & 2 & 1 \\ 7 & 3 & -2 \end{vmatrix}$

$\vec{v} = \begin{vmatrix} 2 & 1 \\ 3 & -2 \end{vmatrix} \hat{\imath} - \begin{vmatrix} -3 & 1 \\ 7 & -2 \end{vmatrix} \hat{\jmath} + \begin{vmatrix} -3 & 2 \\ 7 & 3 \end{vmatrix} \hat{k}$

$\vec{v} = [(2)(-2) - (3)(1)]\hat{\imath} - [(-3)(-2) - (7)(1)]\hat{\jmath} + [(-3)(3) - (7)(2)]\hat{k}$

$\vec{v} = (-4 - 3)\hat{\imath} - (6 - 7)\hat{\jmath} + (-9 - 14)\hat{k}$

$\vec{v} = (-7)\hat{\imath} - (-1)\hat{\jmath} + (-23)\hat{k}$

$\vec{v} = -7\hat{\imath} + \hat{\jmath} - 23\hat{k}$

$\vec{v} = (-7,1,-23)$

The intersection of the planes is the set of simultaneous solutions for the system. But the two equations each have three unknowns, so we assume an arbitrary value for one of the unknowns and solve the resulting system.

Let $x = 0$.

$$\begin{cases} 2y + z = -5 \\ 3y - 2z = -2 \end{cases}$$

$$z = -2y - 5$$

$$3y - 2(-2y - 5) = -2$$

$$3y + 4y + 10 = -2$$

$$7y = -2 - 10$$

$$7y = -12$$

$$y = -\frac{12}{7}$$

$$z = -2\left(-\frac{12}{7}\right) - 5$$

$$z = \frac{24}{7} - \frac{35}{7}$$

$$z = -\frac{11}{7}$$

$$\vec{s}_0 = \left(0, -\frac{12}{7}, -\frac{11}{7}\right)$$

$$L = \vec{s}_0 + \vec{v}t, t \in \mathbb{R}$$

$$L: \begin{cases} x = (0) + (-7)t \\ y = \left(-\frac{12}{7}\right) + (1)t \\ z = \left(-\frac{11}{7}\right) + (-23)t \end{cases}, t \in \mathbb{R}$$

$$\boxed{L: \begin{cases} x = -7t \\ y = -\frac{12}{7} + t \\ z = -\frac{11}{7} - 23t \end{cases}, t \in \mathbb{R}}$$

G.2 Given $P_1: -x + 2y + z = 14$ and $P_2: 2x + y = 3$, find the parametric equations for their line of intersection.

$P: n_x x + n_y y + n_z z + d = 0$

$P_1: -x + 2y + z - 14 = 0$	$P_2: 2x + y - 3 = 0$
$P_1: (-1)x + (2) + (1)\, z + (-14) = 0$	$P_2: (2)x + (1)y + (0)z + (-3) = 0$
$\vec{n}_1 = (-1,2,1)$	$\vec{n}_2 = (2,1,0)$
$d_1 = -14$	$d_2 = -3$

$\vec{v} = \vec{n}_1 \times \vec{n}_2$

$\vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 2 & 1 \\ 2 & 1 & 0 \end{vmatrix}$

$\vec{v} = \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} \hat{\imath} - \begin{vmatrix} -1 & 1 \\ 2 & 0 \end{vmatrix} \hat{\jmath} + \begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix} \hat{k}$

$\vec{v} = [(2)(0) - (1)(1)]\hat{\imath} - [(-1)(0) - (2)(1)]\hat{\jmath} + [(-1)(1) - (2)(2)]\hat{k}$

$\vec{v} = (0 - 1)\hat{\imath} - (0 - 2)\hat{\jmath} + (-1 - 4)\hat{k}$

$\vec{v} = (-1)\hat{\imath} - (-2)\hat{\jmath} + (-5)\hat{k}$

$\vec{v} = -\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$

$\vec{v} = (-1,2,-5)$

The intersection of the planes is the set of simultaneous solutions for the system. But the two equations each have three unknowns, so we assume an arbitrary value for one of the unknowns and solve the resulting system.

Let $x = 0$.

$\begin{cases} 2y + z = 14 \\ \quad\ \ y = 3 \end{cases}$

$2(3) + z = 14$

$6 + z = 14$

$z = 14 - 6$

$z = 8$

$\vec{s}_0 = (0,3,8)$

$$L = \vec{s}_0 + \vec{v}t, t \in \mathbb{R}$$

$$L: \begin{cases} x = (0) + (-1)t \\ y = (3) + (2)t \\ z = (8) + (-5)t \end{cases}, t \in \mathbb{R}$$

$$\boxed{L: \begin{cases} x = -t \\ y = 3 + 2t \\ z = 8 - 5t \end{cases}, t \in \mathbb{R}}$$

Problem H

H.1 Given $P_1: x - y + z - 4 = 0$ and $P_2: x - y + 2z = 0$, find the angle between them.

$P: n_x x + n_y y + n_z z + d = 0$

$P_1: (1)x + (-1)y + (1)z - 4 = 0$

$\vec{n}_1 = (1, -1, 1)$

$P_2: (1)x + (-1)y + (2)z = 0$

$\vec{n}_2 = (1, -1, 2)$

$$\vec{n}_1 \cdot \vec{n}_2 = \|\vec{n}_1\| \cdot \|\vec{n}_2\| \cdot \cos(\theta)$$

$$\cos(\theta) = \frac{\vec{n}_1 \cdot \vec{n}_2}{\|\vec{n}_1\| \cdot \|\vec{n}_2\|}$$

$$\theta = \cos^{-1}\left(\frac{\vec{n}_1 \cdot \vec{n}_2}{\|\vec{n}_1\| \cdot \|\vec{n}_2\|}\right)$$

$$\theta = \cos^{-1}\left[\frac{(1)(1) + (-1)(-1) + (1)(2)}{\sqrt{(1)^2 + (-1)^2 + (1)^2} \cdot \sqrt{(1)^2 + (-1)^2 + (2)^2}}\right]$$

$$\theta = \cos^{-1}\left(\frac{1 + 1 + 2}{\sqrt{1 + 1 + 1} \cdot \sqrt{1 + 1 + 4}}\right)$$

$$\theta = \cos^{-1}\left(\frac{4}{\sqrt{3} \cdot \sqrt{6}}\right)$$

$$\theta = \cos^{-1}\left(\frac{4}{\sqrt{18}}\right)$$

$$\boxed{\theta \approx 19.5^\circ}$$

H.2 Given $P_1: 2x - y + 3z = 1$ and $P_2: x - y + 2z = 0$, find the angle between them.

$P: n_x x + n_y y + n_z z + d = 0$

$P_1: 2x - y + 3z - 1 = 0$

$P_1: (2)x + (-1)y + (3)z - 1 = 0$

$\vec{n}_1 = (2, -1, 3)$

$P_2: (1)x + (-1)y + (2)z = 0$

$\vec{n}_2 = (1, -1, 2)$

$$\theta = \cos^{-1}\left(\frac{\vec{n}_1 \cdot \vec{n}_2}{\|\vec{n}_1\| \cdot \|\vec{n}_2\|}\right)$$

$$\theta = \cos^{-1}\left[\frac{(2)(1) + (-1)(-1) + (3)(2)}{\sqrt{(2)^2 + (-1)^2 + (3)^2} \cdot \sqrt{(1)^2 + (-1)^2 + (2)^2}}\right]$$

$$\theta = \cos^{-1}\left(\frac{2 + 1 + 6}{\sqrt{4 + 1 + 9} \cdot \sqrt{1 + 1 + 4}}\right)$$

$$\theta = \cos^{-1}\left(\frac{9}{\sqrt{14} \cdot \sqrt{6}}\right)$$

$$\theta = \cos^{-1}\left(\frac{9}{\sqrt{84}}\right)$$

$$\boxed{\theta \approx 10.9^\circ}$$

Problem I

I.1 Given $P_1: x - 2y + 4z = 7$ and $P_2: 2x - 5y - 3z = 1$, show they are perpendicular.

$P: n_x x + n_y y + n_z z + d = 0$

$P_1: x - 2y + 4z - 7 = 0$

$P_1: (1)x + (-2)y + (4)z - 7 = 0$

$\vec{n}_1 = (1, -2, 4)$

$P_2: 2x - 5y - 3z - 1 = 0$

$P_2: (2)x + (-5)y + (-3)z - 1 = 0$

$\vec{n}_2 = (2, -5, -3)$

$\vec{n}_1 \cdot \vec{n}_2 = (1)(2) + (-2)(-5) + (4)(-3)$

$\vec{n}_1 \cdot \vec{n}_2 = 2 + 10 + (-12)$

$\vec{n}_1 \cdot \vec{n}_2 = 0$

$\vec{n}_1 \perp \vec{n}_2$

$P_1 \perp P_2$

I.2 Given $P_1: 2x + y = 3$ and $P_2: 5x - 10y - 3z = 1$, show they are perpendicular.

$P: n_x x + n_y y + n_z z + d = 0$

$P_1: 2x + y - 3 = 0$

$P_1: (2)x + (1)y + (0)z - 3 = 0$

$\vec{n}_1 = (2, 1, 0)$

$P_2: 5x - 10y - 3z - 1 = 0$

$P_2: (5)x + (-10)y + (-3)z - 1 = 0$

$\vec{n}_2 = (5, -10, -3)$

$\vec{n}_1 \cdot \vec{n}_2 = (2)(5) + (1)(-10) + (0)(-3)$

$\vec{n}_1 \cdot \vec{n}_2 = 10 + (-10) + 0$

$\vec{n}_1 \cdot \vec{n}_2 = 0$

$\vec{n}_1 \perp \vec{n}_2$

$P_1 \perp P_2$

Problem J

J.A Given $P_1: x - 2y + 4z = 7$ and $P_2: -2x + 4y - 8z = 1$, show they are parallel.

$P: n_x x + n_y y + n_z z + d = 0$

$P_1: x - 2y + 4z - 7 = 0$

$P_1: (1)x + (-2)y + (4)z - 7 = 0$

$\vec{n}_1 = (1, -2, 4)$

$P_2: -2x + 4y - 8z - 1 = 0$

$P_2: (-2)x + (4)y + (-8)z - 1 = 0$

$\vec{n}_2 = (-2, 4, -8)$

$\vec{n}_2 = -2(1, -2, 4)$

$\vec{n}_2 = -2\vec{n}_1$

$\vec{n}_2 = k\vec{n}_1, k \in \mathbb{R}$

$\vec{n}_1 \parallel \vec{n}_2$

$P_1 \parallel P_2$

J.2 Given $P_1: 2x + y - z = 3$ and $P_2: 6x + 3y - 3z = 1$, show they are parallel.

$P: n_x x + n_y y + n_z z + d = 0$

$P_1: 2x + y - z - 3 = 0$

$P_1: (2)x + (1)y + (-1)z - 3 = 0$

$\vec{n}_1 = (2, 1, -1)$

$P_2: 6x + 3y - 3z - 1 = 0$

$P_2: (6)x + (3)y + (-3)z - 1 = 0$

$\vec{n}_2 = (6, 3, -3)$

$\vec{n}_2 = 3(2, 1, -1)$

$\vec{n}_2 = 3\vec{n}_1$

$\vec{n}_2 = k\vec{n}_1, k \in \mathbb{R}$

$\vec{n}_1 \parallel \vec{n}_2$

$P_1 \parallel P_2$

END