HW Three — Savannah Gramze

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1.
$$T(R) = \left(\frac{3GM\dot{M}}{8\pi\sigma R_*^3}\ \frac{R_*^3}{R^3}\left[1-\left(\frac{R_*}{R}\right)^{1/2}\right)^{1/4}$$

$$T_c = \left(\frac{3GM\dot{M}}{8\pi\sigma R_*^3}\right)^{1/4}$$

$$T(R) = T_c\left(\frac{R_*}{R}\right)^{3/4}\left[1-\left(\frac{R_*}{R}\right)^{1/2}\right]^{1/4} = T_c\left[\frac{R_*^3}{R^3} - \frac{R_*^{1/2}}{R^{1/2}}\frac{R_*^3}{R^3}\right]^{1/4} = T_c\left[\left(\frac{R_*}{R}\right)^3 - \left(\frac{R_*}{R}\right)^{7/2}\right]^{1/4}$$

$T_{max} = C_1 T_c \qquad R_* = C_2 R$

$$\frac{\partial T}{\partial R} = \frac{\partial}{\partial R}\left(T_c\left[\left(\frac{R_*}{R}\right)^3-\left(\frac{R_*}{R}\right)^{7/2}\right]^{1/4}\right)$$

$$\begin{array}{c|c} (\ \ )^{1/4} & \frac{1}{4}(\ \ )^{-3/4} \\ \hline R_*^3R^{-3} - R_*^{7/2}R^{-7/2} & -3\frac{R_*^3}{R^4}+\frac{7}{2}\frac{R_*^{7/2}}{R^{9/2}} \end{array}$$

$$0 = T_c\frac{1}{4}\left(\left(\frac{R_*}{R}\right)^3-\left(\frac{R_*}{R}\right)^{7/2}\right)^{-3/4}\left(-3\frac{R_*^3}{R^4}+\frac{7}{2}\frac{R_*^{7/2}}{R^{9/2}}\right) \qquad 0 = \left(-3\frac{R_*^3}{R^4}+\frac{7}{2}\frac{R_*^{7/2}}{R^{9/2}}\right)$$

$$3\frac{R_*^3}{R^4} = \frac{7}{2}\frac{R_*^{7/2}}{R^{9/2}} \Rightarrow R^{1/2} = \frac{7}{6}R_*^{1/2} \Rightarrow R = R_*\left(\frac{7}{6}\right)^2,\ R_* = R\left(\frac{6}{7}\right)^2$$

$$\boxed{\begin{array}{l} C_1 = 0.488 \\ C_2 = (6/7)^2 \end{array}}$$

$$T_{max} = T_c\left(\left(\frac{6}{7}\right)^2\right)^{3/4}\left[1-\left(\frac{6}{7}\right)\right]^{1/4} = 0.488\,T_c$$

b) Wein's Law $\lambda_{max} = \frac{0.0029}{T_{max}}$ Km

i) Neutron Star

$T_c = \left(\frac{3GM\dot{M}}{8\pi\sigma R_*^3}\right)^{1/4}$

$\dot{M} = 1.0\times10^{-9}\ M_\odot\ yr^{-1}$

$M \sim 2M_\odot \qquad R\sim 10km$ Note: See Jupyter notebook for calculations

$T_{max} = 6.68\times10^6\ K$

$\lambda_{max} = 4.34$ Å

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ii) White Dwarf near Chandrasekhar Limit

$M = 1.4\ M_{sun}$

$T_{max} = 4.82 \times 10^4\ K$

$\lambda_{max} = 601.65\ \text{Å}$

iii) Stellar BH, $R = \frac{2GM}{c^2}$

$T_{max} = 7.888 \times 10^6\ K$

$\lambda_{max} = 3.67\ \text{Å}$

SMBH

$T_{max} = 1.403 \times 10^5\ K$

$\lambda_{max} = 206.58\ \text{Å}$

iv) Protostar

$T_{max} = 5866.8\ K$

$\lambda_{max} = 4939.29\ \text{Å}$

2. $T(R) = T_c \left(\frac{R_*}{R}\right)^{3/4}\left(1 - \left(\frac{R_*}{R}\right)^{1/2}\right)^{1/4} = \left[\left(\frac{R_*}{R}\right)^3 - \left(\frac{R_*}{R}\right)^{7/2}\right]^{1/4}$

$q = -3/4,\ q = 7/8$

$$B_\nu = \frac{2h\nu^3}{c} \frac{1}{\exp(h\nu/kT) - 1}$$

$$F_\nu = \frac{4\pi\nu^3 h \cos i}{c^2 d^2} \int \frac{R\,dR}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$

$T \propto R^q \qquad A = \frac{4\pi h \cos i}{c^2 d^2}$

$$F_\nu = A\nu^3 \int \frac{R\,dR}{\exp\left(\frac{h\nu}{kR^q}\right) - 1}$$

$x = \frac{h\nu}{kR^q} \Rightarrow R^q = \frac{h\nu}{kx} \Rightarrow R = \left(\frac{h\nu}{kx}\right)^{1/q}$

$dx = -q\frac{h\nu}{k} R^{-q-1}\,dR$

$dR = \left(\frac{-qh\nu}{k}\right)^{-1} R^{q+1}\,dx$

$$dR = \left(\frac{-qh\nu}{t_k}\right) R \quad dx$$

$$\left(\frac{h\nu}{kx}\right)^{1/q} \left(\frac{-qh\nu}{k}\right)^{-1} \left(\frac{h\nu}{kx}\right)^{\frac{q+1}{q}} dx$$

$$F_\nu = A_0 \nu^3 \nu^{-2/q} \int \frac{x}{e^x - 1} dx$$

$$F_\nu \propto \nu^{3-\frac{2}{q}} \Rightarrow \boxed{\alpha = 3 - \frac{2}{q}}$$

So, what is q? $\frac{3}{4} < q < 7/8$

$q = \frac{3}{4} \Rightarrow \alpha = 1/3$

$q = \frac{7}{8} \Rightarrow \alpha = 5/7$

3, Two regiemes, optical and UV

Surface Brightness as a function of radius

Disk Blackbody Spectrum $\rightarrow$ Surface Brightness

Expect the light curves in optical to show a larger disk than the FUV disk, as the FUV radiation is concentrated in a smaller area. The center of the disk tends to be the hottest due to the velocity of Keplarian orbits close to the White Dwarf, and friction due to the viscosity of the disk.

See Jupyter Notebook for Plot