Homework 2 Corrections

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1, a) $T = T_0 \left(\frac{r}{r_0}\right)^{-q}$ $\quad F_\nu = \frac{2\pi \cos\theta}{d^2} \int_{R_{min}}^{R_{max}} B_\nu(T(r))\, r\,dr$

$$B(T) = \frac{2h\nu^3}{c^2}\left(e^{h\nu/kT} - 1\right)^{-1} \Rightarrow B(r) = \frac{2h\nu^3}{c^2}\left(e^{h\nu/kT_0 (r/r_0)^{-q}} - 1\right)^{-1}$$

$$F_\nu = \frac{2\pi\cos\theta}{d^2}\,\frac{2h\nu^3}{c^2}\int\left(\exp\left(\frac{h\nu}{kT_0}\left(\frac{r}{r_0}\right)^{q}\right) - 1\right)^{-1} r\,dr$$

$$u^q = \frac{h\nu}{kT_0}\left(\frac{r}{r_0}\right)^q \Rightarrow u = \left(\frac{h\nu}{kT_0}\right)^{1/q}\frac{r}{r_0} \Rightarrow r = u r_0 \left(\frac{kT_0}{h\nu}\right)^{1/q} \Rightarrow dr = r_0\left(\frac{kT_0}{h\nu}\right)^{1/q} du$$

$$F_\nu = \frac{2\pi\cos\theta}{d^2}\,\frac{2h\nu^3}{c^2}\, r_0^2 \left(\frac{kT_0}{h\nu}\right)^{2/q}\int\frac{u\,du}{e^{-u^q} - 1}$$

$F_\nu \propto \nu^{3-2/q}$ $\quad \nu F_\nu \propto \nu^{4-2/q}$

b) $\log \nu F_\nu = \left(4 - \frac{2}{q}\right)\log\nu$ $\quad T \propto r^{-3/4} \Rightarrow q = 3/4$

$$\frac{d\log\nu F_\nu}{d\log\nu} = \alpha_{IR} = \left(4 - \frac{2}{q}\right)$$

$q = 3/4$

$\alpha_{IR} = 4 - 8/3 = 4/3$

$$\boxed{\alpha_{IR} = 4/3}$$