Homework One

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1. $x^2+y^2+z^2-c^2t^2=0$ $\quad$ $x'^2+y'^2+z'^2-c^2t'^2=0$

$y=y'$ $\quad$ $x-vt=0$

$z=z'$ $\quad$ $x'=0$

$x'=A(x-vt)$ so that when $A\sim 1$, $x'=x-vt$

relative velocities

$x=A(x'+vt')$ $\quad$ $x'-vt'=0$, $x=0$

$x=A(Ax-Avt+vt')$

$x=A^2x-A^2vt+Avt'$

$Avt'=x(1-A^2)+A^2vt$

$t'=\frac{x}{Av}(1-A^2)+At$

$x=ct$

$x'=ct'$

$x'=A(x-vt)$

$x=A(x'+vt')$

$ct=x=A(ct'+vt')$

$x=At'(c+v)$

$$x=A\left[\frac{x}{Av}(1-A^2)+A\frac{x}{c}\right]\left[c+v\right]$$

$$x=x\left[\frac{(1-A^2)}{v}+\frac{A}{c}\right]\left[c+v\right]$$

$$1=\frac{c}{v}(1-A^2)+A+(1-A^2)+A\frac{v}{c}$$

$$0=A+A\frac{v}{c}+\frac{c}{v}-\frac{c}{v}A^2-A^2$$

$$\Rightarrow A=\gamma=\sqrt{\frac{1}{1-v^2/c^2}}=\sqrt{\frac{1}{1-\beta^2}}$$

$$\Rightarrow A = \gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-\beta^2}}$$

$$0 = \frac{c}{v} + A\left(1 + \frac{v}{c} - \frac{c}{v}A - 1\right)$$

$$t' = \frac{x}{Av}(1-A^2) + At$$

$$t' = \frac{x}{Av} - \frac{xA}{v} + At = A\left(\frac{x}{A^2 v} - \frac{x}{v} + t\right)$$

$$= A\left[t - x\left(\frac{A^2-1}{A^2 v}\right)\right]$$

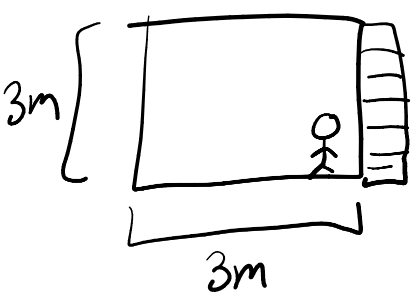
$$\frac{[A^2-1]}{A^2 v} = \frac{\left[\frac{1}{1-\beta^2} - 1\right]}{v/1-\beta^2} \cdot \frac{1-\beta^2}{1-\beta^2}$$

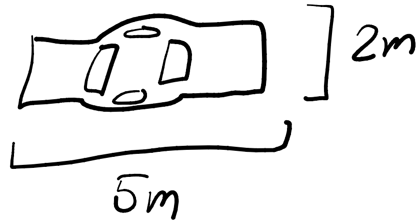
$$= \frac{1-1+\beta^2}{v} = \frac{\beta^2}{v} = \frac{v^2}{vc^2} = \frac{v}{c^2}$$

$$\boxed{\begin{aligned} t' &= \gamma\left[t - \frac{xv}{c^2}\right] \\ x' &= \gamma\left[x - vt\right] \end{aligned}}$$

2.

$v = \frac{4}{5}c$





Length Contraction & Time Dilation

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$$

Frame x' Frame x $v = \frac{4}{5}c$

$$L = \frac{L'}{\gamma} \qquad L' = \text{proper length} = \text{size in rest frame}$$

t = time in moving frame

$$L = L'\sqrt{1-\frac{v^2}{c^2}} \qquad \Delta t = \gamma\Delta t'$$

t' = proper time

t = time in moving

$L = L'\sqrt{1 - v^2/c^2}$ $\qquad$ $\Delta t = \gamma \Delta t'$ $\qquad$ t' = proper time

$L = 5m\sqrt{1-\left(\frac{4}{5}\right)^2} = 3m$ $\qquad$ $L = 3m\sqrt{1-(4/5)^2} = 1.8m$

a) The car enters the garage, they shut it as the car goes in and open the back door, and the car drives out unharmed.

b) From the frame of the car, they pass through the front door, drive in, the back door opens, they drive through, and then the front door closes.

c) From the frame of the garage attendant, the car is 3m in length due to length contraction, and so fits in the garage.
There is a difference in the time of the front and back doors closing/opening between the frames due to time dilation.

d) $v = \frac{2}{3}c$ $\qquad$ $L' = 5m$ $\qquad$ $L = \frac{L'}{\gamma} = 5m\sqrt{1-(2/3)^2} = 3.73m$

The car is too long in the frame of the garage attendant to fit in the garage. The driver should keep to a speed of $\frac{4}{5}c$ or higher to not crash into the back door.