

# Savannah Gramze

$$1. \frac{1}{2} \ddot{\vec{r}} = 2(T - T_s) + \vec{w}$$

$$T_s = 4\pi R^3 P_s \quad \omega = \sqrt{\frac{GM}{R}}$$

$$T = \frac{3}{2} M C_s^2 \quad \rho = \text{const.}$$

$$\ddot{\vec{r}} = 0 \quad a = \frac{-3}{5} \quad \text{for uniform cloud}$$

$$0 = 2 \left[ \frac{3}{2} M C_s^2 - 4\pi R^3 P_s \right] - \frac{3}{5} \frac{GM^2}{R}$$

$$0 = 3 M C_s^2 - 8\pi R^3 P_s - \frac{3}{5} \frac{GM^2}{R}$$

$$8\pi R^3 P_s = 3 M C_s^2 - \frac{3}{5} \frac{GM^2}{R}$$

$$P_s = \frac{3M}{8\pi} \left( \frac{C_s^3}{R^3} - \frac{1}{5} \frac{GM}{R^4} \right)$$

$$2. \frac{dP_s}{dR} = 0 = \frac{d}{dR} \left[ C_s^2 R^{-3} - \frac{1}{5} GM R^{-4} \right]$$

$$0 = -3C_s^2 R^{-4} + \frac{4}{5} GM R^{-5}$$

$$3C_s^2 R^{-4} = \frac{4}{5} GM R^{-5}$$

$$3C_s^2 R = \frac{4}{5} GM$$

$$R = \frac{4}{15} \frac{GM}{C_s^2}$$

$$P_s = \frac{3}{8} \frac{M}{\pi} \left[ C_s^2 \left( \frac{15C_s^2}{4GM} \right)^3 - \frac{1}{5} GM \left( \frac{15C_s^2}{4GM} \right)^4 \right]$$

$$P_{s,\max} = 1.57 \frac{C_s^8}{G^3 M^2}$$

$$3. -\frac{1}{\rho} \frac{d}{dr} \rho = \frac{d}{dr} \phi$$

$$\rho = \rho C_s^2 \quad \phi = \text{grav. potential}$$

$$\rho_c = \rho(r=0)$$

$$\phi = 0 \quad \text{at } r=0$$

$$\frac{d}{dx} (\ln y(x)) = \frac{1}{y} \frac{dy}{dx}$$

$$-\frac{1}{\rho} \frac{d}{dr} (\rho C_s^2) = \frac{d}{dr} (\phi)$$

$$-\underbrace{\frac{1}{\rho} \frac{d}{dr} (\rho C_s^2)}_1 \frac{dr}{r} = \frac{d}{dr} (\phi) dr$$

$$\hookrightarrow C_s^2 \frac{1}{\rho} \frac{dp}{dr} = C_s^2 \frac{d}{dr} (\ln \rho(r)) = \frac{d}{dr} (\phi) dr$$

$$\int \frac{dp}{c_s^2} \frac{1}{p} \frac{dp}{dr} = c_s^2 \frac{d}{dr} (\ln p(r)) = \frac{d}{dr} (\phi) dr$$

$$-c_s^2 \int \frac{d}{dr} (\ln p(r)) dr = \int \frac{d}{dr} (\phi) dr$$

$$-c_s^2 \ln(p(r)) + A = \phi \quad \therefore \phi = c_s^2 \ln\left(\frac{-p(r)}{p_c}\right)$$

$$-c_s^2 \ln(p_c) + A = 0$$

$$\frac{\phi}{c_s^2} = \ln\left(\frac{p}{p_c}\right)$$

$$A = +c_s^2 \ln(p_c)$$

$$\frac{p}{p_c} = e^{\frac{\phi}{c_s^2}} \Rightarrow \boxed{p = p_c e^{-\phi/c_s^2}}$$

$$4. \quad \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d\psi}{dr} \right) = 4\pi G p \quad \psi = \phi/c_s^2$$

$$q = \frac{r}{r_0} \quad \text{or} \quad r = q r_0 \quad \frac{d\psi}{dr} = \frac{d\psi}{dq} \frac{dq}{dr}$$

$$\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d\psi}{dr} \right) = 4\pi G p_c e^{-\psi/c_s^2}$$

$$\frac{dq}{dr} = \frac{d}{dr} \left( \frac{r}{r_0} \right) = \frac{1}{r_0}$$

$$\frac{c_s^2}{r^2} \frac{d}{dr} \left( r^2 \frac{d\psi}{dr} \right) = 4\pi G p_c e^{-\psi}$$

$$\frac{c_s^2}{r_0^2} \frac{1}{q^2} \frac{d}{dq} \left( q^2 \frac{d\psi}{dq} \right) = 4\pi G p_c e^{-\psi}$$

$$\frac{1}{q^2} \frac{d}{dq} \left( q^2 \frac{d\psi}{dq} \right) = \frac{4\pi G p_c r_0^2}{c_s^2} e^{-\psi} \rightarrow r_0^2 = \frac{c_s^2}{4\pi G p_c}$$

$$\therefore \boxed{\frac{1}{q^2} \frac{d}{dq} \left( q^2 \frac{d\psi}{dq} \right) = e^{-\psi}} \quad \checkmark$$

$$5. \quad \frac{1}{q^2} \frac{d}{dq} \left( q^2 \frac{d\psi}{dq} \right) = e^{-\psi}$$

$$a) \quad \psi' = \frac{d\psi}{dq}$$

$$\frac{1}{q^2} \frac{d}{q} \left( q^2 \psi' \right) = e^{-\psi}$$

$$\frac{1}{\xi^2} \frac{d}{\xi} (\xi^\gamma \Psi') = c$$

$$\frac{1}{\xi^2} \left( 2\xi^\gamma \Psi' + \xi^2 \frac{d\Psi'}{d\xi} \right) = e^{-\Psi}$$

$$\frac{d\Psi'}{d\xi} = e^{-\Psi} - \frac{2}{\xi} \Psi' \quad \begin{cases} \Psi = 0 & \frac{d\Psi}{d\xi} = 0 \\ \Psi = 0 = \frac{\Psi}{r_0} \end{cases}$$

$$\Psi(\xi) = a_0 \xi + a_1 \xi^2 + a_2 \xi^3 + a_3 \xi^4 + O(4)$$

$$\Psi(\xi) = a_2 \xi^2 + a_3 \xi^3 + O(4)$$

$$\underline{\Psi' = \frac{d\Psi}{d\xi}} = 2a_2 \xi + 3a_3 \xi^2 + O(3)$$

$$\begin{aligned} b) \underline{\frac{d\Psi'}{d\xi}} &= e^{a_2 \xi^2 + a_3 \xi^3 + O(4)} - \frac{2}{\xi} [2a_2 \xi + 3a_3 \xi^2 + O(3)] \\ &= e^{a_2 \xi^2 + a_3 \xi^3 + O(4)} - 4a_2 - 6a_3 \xi + O(2) \end{aligned}$$

$$e^x = 1 + x + \frac{x^2}{2} + \dots$$

$$e^{a_2 \xi^2 + a_3 \xi^3 + O(4)} = 1 + a_2 \xi^2 + a_3 \xi^3 + O(4) + (a_2 \xi^2 + a_3 \xi^3 + O(4))^2$$

$$\underline{\frac{d\Psi'}{d\xi}} = 1 + a_2 \xi^2 + a_3 \xi^3 - 4a_2 - 6a_3 \xi + O(2)$$

$$\underline{\frac{d\Psi'}{d\xi}} = 1 - 4a_2 - 6a_3 \xi + O(2)$$

$$\underline{\frac{d\Psi'}{d\xi}} = \underline{\frac{d}{d\xi} \left( 2a_2 \xi + 3a_3 \xi^2 + O(3) \right)}$$

$$= 2a_2 + 6a_3 \xi + O(2)$$

$$2a_2 + 6a_3 \xi + O(2) = 1 - 4a_2 - 6a_3 \xi + O(2)$$

$$6a_2 + 12a_3 \xi + O(2) = 1 + O(2)$$

$$a_2 = 1/6 \quad a_3 = 0$$

$$a_2 = \frac{1}{6} \quad a_3 = 0$$

$$\therefore \Psi(q) = \frac{q^2}{6} + O(4)$$

so,

$$\Psi' = \frac{d\Psi}{dq} = \frac{q}{3} + O(3)$$

$$\frac{1}{q^2} \frac{d}{dq} \left( q^2 \frac{d\Psi}{dq} \right) = e^{-\Psi}$$

C. Mass enclosed:

$$M = 4\pi \int_0^R \rho r^2 dr \Rightarrow M = \frac{C_s^4}{\sqrt{4\pi G P_s}} \left( e^{-\Psi/2} q^2 \frac{d\Psi}{dq} \right) \Big|_{q_s} \quad r_0^2 = \frac{C_s^2}{4\pi G p_c}$$

$$q_s = \frac{R}{r_0}, \quad dR = R dq_s \quad \frac{P}{P_c} = \frac{P}{P_c} = e^{-\Psi} \quad \underline{P_s = P_s C_s^2} \quad \Psi = \phi/c^2 \quad \frac{P}{P_c} = \frac{P}{P_c} = e^{-\Psi}$$

$$R = q_s r_0 \quad P_c = P_s e^{\Psi}$$

$$M = 4\pi \int_0^{q_s r_0} \rho q^2 r_0^3 dq = 4\pi p_c r_0^3 \int_0^{q_s} e^{-\Psi} q^2 dq \quad r_0 = \frac{C_s}{\sqrt{4\pi G p_c}}$$

$$e^{-\Psi} q^2 = \frac{d}{dq} \left( q^2 \frac{d\Psi}{dq} \right)$$

$$M = 4\pi p_c r_0^3 \int_0^{q_s} \frac{d}{dq} \left( q^2 \frac{d\Psi}{dq} \right) dq = 4\pi p_c r_0^3 \left( q^2 \frac{d\Psi}{dq} \right) \Big|_{q_s} \quad C_s = \sqrt{P_s}$$

$$= \frac{4\pi p_c C_s^3}{(4\pi G p_c)^{3/2}} \left( q^2 \frac{d\Psi}{dq} \right) \Big|_{q_s} \cdot \frac{C_s}{\sqrt{P_s}} = \frac{C_s^4}{\sqrt{4\pi G^3 p_c P_s / P_s}} \left( q^2 \frac{d\Psi}{dq} \right) \Big|_{q_s}$$

$$M = \frac{C_s^4}{\sqrt{4\pi G^3 p_c^4 P_s / P_s}} \left( q^2 \frac{d\Psi}{dq} \right) \Big|_{q_s} \Rightarrow \boxed{M = \frac{C_s^4}{\sqrt{4\pi G^3 P_s^2 p / P_s}} \left( e^{-\Psi/2} q^2 \frac{d\Psi}{dq} \right)}$$

$$\star 7. m = \frac{M \sqrt{G^3 P_s}}{C_s^4} \quad \text{Coding}$$

$$P_s = \frac{1}{G^3} \frac{m^2 C_s^8}{M^2}$$

$$\star 8. P_{s,\max} = \frac{1}{G^3} \frac{m_{\max}^2 C_s^8}{M^2}$$

$$P_s = P_s C_s^2 \quad P_s/k_B = 3 \times 10^5 K \text{ cm}^{-3}$$

$$T = 10K, M = 2.3$$

$$\cancel{\text{Ex 8.}} \quad p_{s,\max} = \frac{1}{G^3} \frac{m_{\max} c_s}{M^2}$$

is vs is  
T = 10K, m = 2.3

$$c_s = \sqrt{k_B T / m}$$

$$\cancel{\text{Ex 9.}} \quad M_{BE} = m_{\max} \frac{c_s^4}{\sqrt{G^3 P_s}} = 1.18 \frac{c_s^4}{\sqrt{G^3 P_s}}$$

$$P_s / k_B = 10^7 \text{ K cm}^{-3}$$

$$T = 50K$$

$$M_{BE} = 1.18 \frac{(k_B T / m)^2}{\sqrt{G^3 P_s}}$$

$$\cancel{\text{Ex 10.}} \quad r_0 = \frac{c_s}{\sqrt{4\pi G_s \rho_c}}$$

11.  $\cancel{\text{Ex}}$  coding

12.  $\cancel{\text{Ex}}$  coding

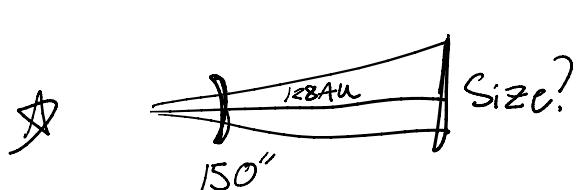
13.  $\cancel{\text{Ex}}$  refers to 7

B68 = BE sphere

$$q_{\max} = 6.9$$

Is the core stable? Refers part 7 for max stable dimension mass

14. B68  $r = 150''$  @  $d = 128 \text{ AU}$   $T = 10.5K$



$$\text{Central density? } q = r/r_0 \quad (q_{\max} = r_{\max}/r_0)$$

15. Compare surface pressure  $\star$   $r_0 = \frac{c_s}{\sqrt{4\pi G_s \rho_c}}$   $P/P_c = 0.06$

$$r_0^2 4\pi G_s \rho_c = c_s^2$$

$$\rho_c = \frac{c_s^2}{4\pi G r_0^2}$$