Problem Set 3: IMF

1. The IMF is the function describing how many stars are born at each mass.
$$\xi \equiv \frac{dN}{dM} \tag{1}$$
There are several common parametrizations. We adopt $M_{max} = 120\ \mathbf{M}_\odot$ as the default for all.

   (a) Salpeter (you will find different M_{min} in use in different locations in the literature; we adopt 0.01 here to simplify comparisons):
$$\xi = M^{-2.35}, M_{min} = 0.01\mathbf{M}_\odot \tag{2}$$

   (b) Kroupa (eqn 2 of 2001MNRAS.322..231K):
$$\xi = M^{-\alpha} \begin{cases} \alpha = 0.3, & 0.01 \le M/\mathbf{M}_\odot < 0.08 \\ \alpha = 1.3, & 0.08 \le M/\mathbf{M}_\odot < 0.5 \\ \alpha = 2.3, & 0.08 \le M/\mathbf{M}_\odot < 0.5 \end{cases}$$

   (c) Chabrier (Eqn 18 of 2003PASP..115..763C):
$$\begin{cases} \xi(\log M) \equiv \frac{dN}{d\log M} = A\exp\left[(\log M - \log M_c)^2/2\sigma^2\right] & \\ A = 0.086, M_c = 0.22, \sigma = 0.57 & M \le 1\mathbf{M}_\odot \\ \xi(\log M) = AM^{-\Gamma} & \\ \Gamma = 1.3, A = 4.43 & M > \mathbf{M}_\odot \end{cases}$$

   You will perform calculations using these equations to infer properties of stellar populations. For each distribution, compute:

   (a) What is the average mass?

   (b) What is the average mass of stars with $M > 8\mathbf{M}_\odot$?

   (c) What is the ratio of the number of high-mass to low-mass stars? (use M=8 $\mathbf{M}_\odot$ as the dividing mass)

   (d) What is the ratio of the *mass* of high-mass to low-mass stars?

   (e) Do these numbers change if you change M_{max} to 100 $\mathbf{M}_\odot$? To 1000 $\mathbf{M}_\odot$?

   (f) Do these numbers change if you change M_{min} to 0.03 $\mathbf{M}_\odot$? To 0.3 $\mathbf{M}_\odot$?

   (g) For a cluster of 1000 stars, how many would you expect to be $M > 8\mathbf{M}_\odot$ (able to go supernova)?

   (h) In an 'optimal distribution function', the cluster mass to maximum star mass is fixed by defining $\int_{M_{max,cl}}^{M_{max}} \xi dM = 1$, where M_{max} is the maximum possible mass for a star and $M_{max,cl}$ is the most massive star in the cluster. From this definition, determine how many stars must be in a cluster to form one 10 $\mathbf{M}_\odot$ star or one 100 $\mathbf{M}_\odot$ star.

(i) What *mass* of cluster is required to produce a star of that mass?

(j) In a probability distribution function, there is only a fix likelihood of forming a star of a given mass. What is the minimum cluster mass required to have a $> 95\%$ ($> 63.21\%$) probability of forming at least one ≥ 100 $\mathbf{M}_\odot$ star?
Recall that the likelihood of rolling at least one six after 100 rolls is equal to one minus the likelihood of rolling *no* sixes in 100 rolls i.e., $P(\geq 1$⚅$) = 1 - \left(\frac{5}{6}\right)^{100}$.

(k) Compare the results from the ODF and the PDF for the presence of a $M > 100\mathbf{M}_\odot$ star. How can you interpret the difference?

(l) What is the expected light-to-mass ratio of each distribution assuming $L = \frac{M}{\mathbf{M}_\odot}^3$?

(m) The IMF is the *system* IMF. If we are primarily interested in the luminosity of a system, splitting the mass between multiple stars can make a big difference. If we assume every star system consists of an equal-mass binary, what is the effect on the L/M ratio? Is this a reasonable approximation to the multiplicity fraction?

(n) For an ODF, the effective maximum stellar mass can be smaller. If all star-forming events in a galaxy occur in Taurus-like star-forming regions, with $M_* = 100\mathbf{M}_\odot$, what is the maximum mass? What is the resulting L/M ratio? Recall that this is the *maximum* the L/M will be in such a galaxy.

You may choose to use external resources, like the `imf` package, but it's a good idea to try to solve some of these - at least the simple Salpeter IMF - by hand.