Homework Three

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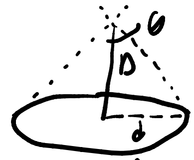


$1\,AU = 1.496\times10^{11}\,m$

$1\,pc = 3.08\times10^{16}\,m$

1. $m_V = 0.71,\ M_V = 0.14$

a) distance & parallax

$m - M = 5\log D - 5 + \cancel{A + BC}$ ignore

$\log D = \dfrac{0.71 - 0.14 + 5}{5}$

$\boxed{D = 13\,pc}$

$\theta = \dfrac{d}{D} = \dfrac{1AU}{13pc}$

$\dfrac{1AU}{13pc}\cdot\dfrac{1pc}{3.08\times10^{16}m}\cdot\dfrac{1.496\times10^{11}m}{1AU}\cdot\dfrac{180^0}{\pi\,rad}\cdot\dfrac{3600''}{1^0}$

$\boxed{\theta = 0.077''}$

b) $f_{bol} = 1.5\times10^{-5}\,erg\,cm^{-2}\,s^{-1}$ $L_\odot = 4\times10^{33}\,erg\,s^{-1}$

$f = \dfrac{L}{4\pi D^2} \Rightarrow L = 4\pi D^2 f = 4\pi(4\times10^{19}cm)^2(1.5\times10^{-5}erg\,cm^{-2}s^{-1})$

$D = 13pc\cdot\dfrac{3.08\times10^{16}m}{1pc}\cdot\dfrac{100cm}{1m} = 4\times10^{19}cm$

$L = \dfrac{3.02\times10^{35}erg\,s^{-1}}{4\times10^{33}erg\,s^{-1}}\cdot L_\odot \Rightarrow \boxed{L = 75.5\,L_\odot}$

c) $T_{eff} = 5300\,K$

$\lambda_{max}T_{eff} = 0.28979\,cm\,K\cdot\dfrac{1m}{100cm}\cdot\dfrac{10^9nm}{1m}$

$\lambda_{max} = \dfrac{0.28979\cdot\frac{10^9}{100}\,nm\,K}{5300K} \Rightarrow \boxed{\lambda_{max} = 546.8nm}$

d) $g = 700\,cm\,s^{2}$ $g = \dfrac{GM}{r^2}$ $R_\odot = 6.96\times10^{10}cm$ $G = 6.673\times10^{-8}cm^3g^{-1}s^{-2}$

$\dfrac{L}{L_\odot} = \left(\dfrac{R}{R_\odot}\right)^2\left(\dfrac{T_{eff}}{T_\odot}\right)^4 \Rightarrow 75.5 = \left(\dfrac{R}{R_\odot}\right)^2\left(\dfrac{5300}{5777}\right)^4$

$\boxed{R_* = 10.3\,R_\odot} = 7.185\times10^{11}cm$

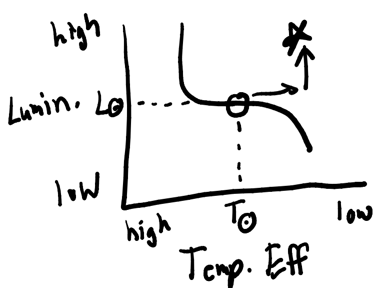
$700 = \dfrac{(6.673\times10^{-8})M}{(7.185\times10^{11})^2} \Rightarrow M = 5.42\times10^{33}g\cdot\dfrac{1M_\odot}{1.989\times10^{33}g}$

$\boxed{M_* = 2.7\,M_\odot}$

e) Is Capella a main-sequence ★?

$T_\odot = 5777K$ $L_\odot = L_\odot$

e) Is Capella a main-sequence ★?

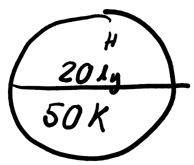


$T_\odot = 5777\,K \quad L_\odot = L_\odot$

$T_* = 5300 \quad L_* = 75 L_\odot$

No, Capella is not a main-sequence star. It is brighter than the Sun, yet has a lower effective temperature, and its radius must be much larger than the Sun making it a past main-sequence star.

2.



$\rho = 1\times10^{4}\ H\ cm^{-3}$

$\mu = 0.77$

a) $\rho = \frac{M}{V} \Rightarrow M = \rho V$

$V = \frac{4}{3}\pi R^3 \qquad m_H = 1.67\times10^{-24}\,g$

$R = 10\,ly \cdot \frac{0.3066\,pc}{1\,ly} \cdot \frac{3.08\times10^{18}\,cm}{pc} \qquad R = 9.44\times10^{18}\,cm$

$\rho = \frac{1\times10^{4}\ H}{cm^3} \cdot \frac{1.67\times10^{-24}\,g}{1\ H} = 1.67\times10^{-20}\,g/cm^3$

$M = (1.67\times10^{-20}\,\frac{g}{cm^3})(\frac{4}{3}\pi)(9.44\times10^{18}\,cm)^3$

$= 5.89\times10^{37}\,g \cdot \frac{1\,M_\odot}{1.989\times10^{33}\,g} = \boxed{29{,}616\,M_\odot}$

b) Thermal Energy $U = \frac{3}{2}NkT,\ N = \frac{M}{\mu m_H}$ Gravitational Energy $\Omega = -\frac{3}{5}\frac{GM^2}{R}$

$U = \frac{3}{2}\frac{(5.89\times10^{37}\,g)}{(0.77)(1.67\times10^{-24}\,g)}(1.381\times10^{-16}\,erg\,K^{-1})(50K) = 4.7\times10^{49}\,erg$

$\Omega = \frac{-3}{5}\frac{(6.673\times10^{-8}\,cm^3 g^{-1} s^{-2})(5.89\times10^{37}\,g)^2}{(9.44\times10^{18}\,cm)} = -1.47\times10^{49}\,erg$

$$\Omega = \frac{-3}{5}\frac{(6.673\times10^{-8}\,cm^3 g^{-1} s^{-2})(3.84\times10\ g)}{(9.44\times10^{18}\,cm)} = 1.47\times10\ erg$$

$u = 4.7\times10^{49}$ erg, $\Omega = -1.47\times10^{49}$ erg $\qquad M_u = m_H$

c) $2u < |\Omega|$ Gravitational Instability

$2\times 4.7\times10^{49}$ erg $> |-1.47\times10^{49}$ erg$|$

$2u > |\Omega| \Rightarrow$ the cloud is unstable and will collapse

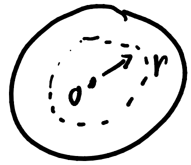
d) $$M_J = \left(\frac{3k_BT}{3/5\,G\mu m_H}\right)^{3/2}\left(\frac{3}{4\pi\rho}\right)^{1/2} \qquad \rho = 1.67\times10^{-20}\,g/cm^3 \quad \mu = 0.77$$

$$M_J = \left(\frac{5(1.381\times10^{-16}\,erg\,K^{-1})(50K)}{(6.67\times10^{-8}\,cm^3 g^{-1} s^{-2})(0.77)(1.67\times10^{-24}\,g)}\right)^{3/2}\left(\frac{3}{4\pi(1.67\times10^{-20}\frac{g}{cm^3})}\right)^{1/2}$$

$$M_J = 9.64\times10^{35}\,g = 482.3\,M_\odot$$

$M = 29{,}616\,M_\odot > M_J$ ∴ Cloud is gravitationally unstable and will collapse.

3. $\rho(r) = \rho_c\left[1-\left(\frac{r}{R_\star}\right)^2\right]$



a) $dM(r) = 4\pi r^2 \rho(r)\,dr$

$$\int_0^M dM(r) = 4\pi\rho_c\int_0^r\left(r^2 - \frac{r^4}{R_\star^2}\right)dr$$

$$M(r) = 4\pi\rho_c\left(\frac{r^3}{3} - \frac{r^5}{5R_\star^2}\right) \Rightarrow M(r) = \cancel{4\pi}\cdot\frac{15M_\star}{2\,\cancel{8\pi}R_\star^3}\left(\frac{r^3}{3} - \frac{r^5}{5R_\star^2}\right)$$

$$\boxed{M(r) = \frac{15M_\star}{2}\left(\frac{r^3}{3R_\star^3} - \frac{r^5}{5R_\star^5}\right)}$$

$$M_\star = 4\pi\rho_c\left(\frac{R_\star^3}{3} - \frac{R_\star^3}{5}\right)$$

$15\,M_\star$

$$M_\star = 4\pi\rho_c\left(\frac{R_\star^3}{3} - \frac{R_\star^3}{5}\right)$$

$$M_\star = 4\pi\rho_c R_\star^3 \cdot \frac{2}{15} \Rightarrow \rho_c = \frac{15 M_\star}{8\pi R_\star^3}$$

$$\rho(r) = \rho_c\left[1 - \frac{r^2}{R_\star^2}\right]$$

b) $P(R_\star) = 0$, find P_c

$$\int_{P_c}^{0} dP(r) = \int_0^{R_\star} \frac{-GM(r)\rho(r)}{r^2}\,dr = \int_0^{R_\star} \frac{-G}{r^2} \cdot 4\pi\left(\frac{15 M_\star}{8\pi R_\star^3}\right)^2\left(\frac{r^3}{3} - \frac{r^5}{5R_\star^2}\right)dr$$

$$P_c = 4\pi G\left(\frac{15 M_\star}{8\pi R_\star^3}\right)^2 \int_0^{R_\star}\left(\frac{r}{3} - \frac{r^3}{5R_\star^2}\right)dr$$

$$P_c = 4\pi G\left(\frac{15 M_\star}{8\pi R_\star^3}\right)^2\left(\frac{R_\star^2}{6} - \frac{R_\star^2}{20}\right) = \frac{7}{15}\pi G R_\star^2\left(\frac{15 M_\star}{8\pi R_\star^3}\right)^2$$

$$\boxed{P_c = \frac{105}{64}\frac{GM_\star^2}{\pi R_\star^4}}$$

c) Find central temperature

$$P_{gas} = \frac{\rho k_B T}{\mu m_H} = P_c = \frac{105}{64}\frac{GM_\star^2}{\pi R_\star^4}$$

$$R_\star^4 = \frac{105}{64}\frac{GM_\star^2}{\pi} \cdot \frac{\mu m_H}{\rho_c k_B T_c} = \frac{105}{64} \cdot \frac{GM_\star^{\cancel{2}}}{\cancel{\pi}} \cdot \frac{\mu m_H}{k_B T_c} \cdot \frac{8\cancel{\pi} R_\star^3}{15 \cancel{M_\star}}$$

$$\frac{\overset{7}{\cancel{105}}}{\underset{8}{\cancel{64}}} \cdot \frac{\cancel{8}}{\cancel{15}}$$

$$\boxed{R_\star = \frac{7}{8}\frac{GM_\star \mu m_H}{k_B T_c}}$$

$$T_c = \frac{7}{8}\frac{GM_\star \mu m_H}{k_B R_\star}$$

$$\frac{1}{\mu} = 2X + \frac{3}{4}Y + \frac{1}{2}Z$$

$X = 0.70$, $Y = 0.28$, $Z = 0.02$

$$\frac{1}{\mu} = 2(0.7) + \frac{3}{4}(0.28) + \frac{1}{2}(0.02)$$

$$\frac{1}{\mu} = 1.62 \Rightarrow \mu = 0.617$$

$G = 6.673\times10^{-8}\ cm^3\,g^{-1}\,s^{-2}$

$M_\odot = 1.989\times10^{33}\ g$

$\ldots{}^{-8})(1.989\times10^{33})(0.617)(1.67\times10^{-24})$

$M_\odot = 1.989 \times 10^{33}$ g
$R_\odot = 6.960 \times 10^{10}$ cm
$m_H = 1.67 \times 10^{-24}$ g
$k_B = 1.381 \times 10^{-16}$ erg K^{-1}

$$T_c = \left(\frac{7}{8}\right) \frac{(6.673 \times 10^{-8})(1.989 \times 10^{33})(0.617)(1.67 \times 10^{-24})}{(1.38) \times 10^{-16})(6.96 \times 10^{10})}$$

$$T_c = 1.24 \times 10^7 \text{ K} \lessapprox (T_{pp} > 1.3 \times 10^7 \text{ K})$$

The core temperature calculated is just a little too low for pp-chain fusion, but is so close that it might be a rounding error somewhere.

d) The two pressures at play in stars are gas pressure and radiation pressure.

$$P_{rad} = \frac{aT^4}{3} \qquad a = \frac{\pi^2 k^4}{45 c^3 \hbar^3} = 7.565 \times 10^{-15} \text{ [cgs]} \qquad \rho_c = \frac{15 M_*}{8\pi R_*^3}$$

$$P_{gas} = \frac{\rho k_B T}{\mu m_H} \qquad P_{tot} = P_{gas} + P_{rad} \qquad \frac{P_{rad}}{P_{gas} + P_{rad}} = ?$$

$$P_{gas} = \frac{15 M_* k_B T_c}{\mu m_H 8\pi R_*^3} = \left(\frac{15}{8\pi}\right) \frac{(1.989 \times 10^{33})(1.381 \times 10^{-16})(1.24 \times 10^7)}{(0.617)(1.67 \times 10^{-24})(6.96 \times 10^{10})}$$

$$P_{gas} = 2.8 \times 10^{37} \qquad P_{rad} = \frac{(7.565 \times 10^{-15})(1.24 \times 10^7)^4}{3} = 6.07 \times 10^{13}$$

$$\boxed{\frac{P_{rad}}{P_{tot}} = 2.13 \times 10^{-24}}$$

Gas pressure dominates in the core.

4. Questions about stellar evolution

a) Why does a $0.8 M_\odot$ * into a Red Giant while a $0.2 M_\odot$ * does not?

A $0.8 M_\odot$ star has a core that is separate from its outer envelope where it burns hydrogen into Helium. As the more massive star evolves past the main sequence the temperature of the core Increases, causing to burn helium, the temperature and turn the star into a Red Giant.

As the more massive [illegible] the temperature of the core increases. [illegible] to burn helium, the temperature of the core increases to turn the star into a Red Giant.
the envelope to expand and turn the star into a Red Giant.
For a $0.2 M_\odot$ star, the whole star is convective, meaning that there is no distinct core. Any helium made is cycled through the star instead of kept in a dense core, meaning that the star will gradually burn through its entire envelope supply of hydrogen and never turn into a Red Giant.

b) A He core flash happens at the tip of the Red Giant Branch, and will happen to the $0.8 M_\odot$ star not the $0.2 M_\odot$ star. A helium flash happens when the core of a Red Giant star becomes completely degenerate helium. Since degenerate pressure is supporting the core, an increase of temperature does not cause the core to expand. When the core reaches the helium burning temperature, it undergoes a runaway thermonuclear reaction as the helium burning increases the core temperature, which accelerates the rate of burning.

c) $1 M_\odot$ star's evolution: stellar structure, R and T_{eff} properties, dominant internal power source.

(i) Hayashi Track $\quad R > R_\odot$, $T_{eff} < T_\odot$

A star following the Hayashi track is completely convective until it forms a radiative core, at which point it will move onto the Henyey track. The
[illegible] star contracts

convective until ... onto the Henyey track. The point it will move onto the Henyey track. The temperature stays constant as the star contracts with a decreasing radius. Its internal power source is the gravitational energy it converts to thermal energy.

(ii) Main Sequence $R = R_\odot$, $T_{eff} = T_\odot$

A star on the main sequence maintains a constant radius and effective temperature. It has a convective envelope and radiative core, where the star is powered by hydrogen fusion with the pp-chain.

(iii) Red Giant Branch $R > R_\odot$, $T_{eff} < T_\odot$

A star on the Red Giant Branch will have an expanding convective envelope and a contracting helium core. The effective temperature of the star decreases as the star expands. It is <u>not</u> burning helium yet, instead it burns hydrogen in a shell around the solid helium core, which contracts and increases in temperature.

(iv) Horizontal Branch/Red Clump $R > R_\odot$, $T_{eff} \gtrsim T_\odot$

After the Helium flash at the end of the RGB, the star now has a core that is burning helium via the 3α process and shell burning H in the CNO cycle. The star's envelope contracts slightly, and the effective temperature increases.

temperature increases.

(v) Asymptotic Giant Branch $R > R_\odot$, $T_{eff} < T_\odot$

The star forms a dense core made of Carbon and Oxygen which is fed mass by shell burning. Both hydrogen and helium burning occur in shells around the core and power the star. The core contracts, but the convective envelope expands, making the effective temperature decrease. The star is losing mass as shell burning turns on and off.

(vi) White Dwarf $R < R_\odot$, $T_{eff} > T_\odot$

A white dwarf is what is left after the AGB star has shed its envelope, exposing the hot, degenerate core to the vaccuum of space. The star's source of energy is thermal as it slowly cools down and contracts over time. The star is very small, around the size of Earth, but has a high temperature from when it was actively burning.