AST7939: Homework 3

Due at the beginning of class on Wednesday, March 23

Instructions: *To receive partial/full credit you must show your work or explain your answer thoroughly. Please circle your final answer to each problem if it is a number.*

The Spectrum of the Standard Thin Accretion Disk Model

1. In what region of the spectrum is most of the energy emitted from geometrically thin, optically thick accretion disks?

   We can calculate the temperature of an accretion disk using a slightly modified version of disk blackbody temperature relation we derived:

   $$T(R) = \left(\frac{3GM\dot{M}}{8\pi\sigma R_\star^3} \frac{R_\star^3}{R^3} \left[1 - \left(\frac{R_\star}{R} \right)^{\frac{1}{2}} \right] \right)^{\frac{1}{4}} \tag{1}$$

   where the equation has simply been multiplied by $\frac{R_\star^3}{R_\star^3}$, so that we include the radius of the source. We will then clump the constants together and define a characteristic temperature of the disk

   $$T_c = \left(\frac{3GM\dot{M}}{8\pi\sigma R_\star^3} \right)^{\frac{1}{4}} \tag{2}$$

   so that

   $$T(R) = T_c \left(\frac{R_\star}{R} \right)^{3/4} \left[1 - \left(\frac{R_\star}{R} \right)^{\frac{1}{2}} \right]^{\frac{1}{4}} \tag{3}$$

   (a) The temperature peaks at some temperature, $T_{\max}$. Then,

   $$T_{\max} = C_1 T_c \tag{4}$$

   at some radius,

   $$R_\star = C_2 R. \tag{5}$$

   Find C_1 and C_2.

   (b) Where does each object emit most of its radiation? Find T_{max} and λ_{max} for:

   i. a neutron star with $\dot{M} = 1.0 \times 10^{-9}$ $M_\odot$ yr^{-1}, $M \sim 2$ $M_\odot$ and $R \sim 10$ km

   ii. a white dwarf near the Chandrasekhar limit with $\dot{M} = 1.0 \times 10^{-10}$ $M_\odot$ yr^{-1} and $R \sim R_\oplus$

   iii. a 10 $M_\odot$ black hole with $\dot{M} = 1.0 \times 10^{-8}$ $M_\odot$ yr^{-1} and a 10^9 $M_\odot$ black hole with $\dot{M} = 10$ $M_\odot$ yr^{-1}

   iv. a protostar with $\dot{M} = 1.0 \times 10^{-4}$ $M_\odot$ yr^{-1}, $R \sim 5$ $R_\odot$ and $M \sim 0.5$ $M_\odot$

2. For a standard thin accretion disk with $R_{in} \ll R_{out}$, derive the spectral indices of the emitted intensity discussed in class.

3. Congratulations! Your observing proposal is a success! You have just been awarded time to image a cataclysmic variable (a short-period semi-detached binary consisting of an accreting white dwarf primary and (typically) a main-sequence secondary star), not only in the optical, but also in the FUV during the same 48 hour period (this must have happened in the past when there existed a telescope in space capable of imaging in the FUV). Previous low-quality observations suggest that the cataclysmic variable is observed edge-on so that the companion star can occult most of the disk. After you have acquired these observations, you will be able to construct high-quality light curves of the source for both wavelength regimes. Upon overplotting the light curves from the simultaneous observations (let's say at 110 nm and 720 nm), would you expect any difference in widths or depths between the two different wavelength regimes? As a part of your explanation, for the two wavelength regimes, overplot the surface brightness as a function of radius.