一、解：属单因素方差分析问题： $r=4$ ，$n_1=n_2=n_3=n_4=5$ ， $n=20$.

对于A_1→ $T_{i\cdot}=\sum_{j=1}^{5}X_{ij}=5.2+6.3+4.9+6.8+3.2=26.4$

$T_{i\cdot}^2=(\sum_{j=1}^{5}X_{ij})^2=(26.4)^2=696.96$

$\sum_{j=1}^{5}X_{ij}^2=5.2^2+6.3^2+4.9^2+6.8^2+3.2^2=147.22$

对于A_2→ $T_{i\cdot}=\sum_{j=1}^{5}X_{ij}=7.4+8.3+5.9+4.9+6.5=33$

$T_{i\cdot}^2=(\sum_{j=1}^{5}X_{ij})^2=(33)^2=1089$

$\sum_{j=1}^{5}X_{ij}^2=7.4^2+8.3^2+5.9^2+4.9^2+6.5^2=224.72$

对于A_4→ $T_{i\cdot}=\sum_{j=1}^{5}X_{ij}=12.3+9.4+7.8+8.5+10.8=48.8$

$T_{i\cdot}^2=(\sum_{j=1}^{5}X_{ij})^2=(48.8)^2=2381.44$

$\sum_{j=1}^{5}X_{ij}^2=12.3^2+9.4^2+7.8^2+8.5^2+10.8^2=489.38$

	$T_{i\cdot}=\sum_{j=1}^{5}X_{ij}$	$T_{i\cdot}^2$	$\sum_{j=1}^{5}X_{ij}^2$
A_1	26.4	696.96	147.22
A_2	33	1089	~~224.72~~
A_3	31.5	992.25	220.03 ~~224.72~~
A_4	48.8	2381.44	489.38 ~~220.03~~
Σ	$\sum_{i=1}^{4}T_{i\cdot}=139.7$	$\sum_{i=1}^{4}T_{i\cdot}^2=5159.65$	$\sum_{i=1}^{4}\sum_{j=1}^{5}X_{ij}^2=1081.35$

~~$T_{i\cdot总}=26.4+33+48.8=$~~

对于A_3→ $T_{i\cdot}=\sum_{j=1}^{5}X_{ij}=3.9+6.4+7.9+4.1+9.2=31.5$ ，$T_{i\cdot}^2=992.25$，$\sum_{j=1}^{5}X_{ij}^2=220.03$

$T_{i总}=\sum_{i=1}^{4}T_{i\cdot}=26.4+33+31.5+48.8=139.7$

$T_{i总}^2=\sum_{i=1}^{4}T_{i\cdot}^2=696.96+1089+992.25+2381.44=5159.65$

$\sum_{i=1}^{4}\sum_{j=1}^{5}X_{ij}^2=147.22+224.72+220.03+489.38=1081.35$

$Q_T^2=\sum_{i=1}^{r}\sum_{j=1}^{n_i}(X_{ij}-\overline{X})^2=\sum_{i=1}^{r}\sum_{j=1}^{n_i}X_{ij}^2-\frac{1}{n}(\sum_{i=1}^{r}\sum_{j=1}^{n_i}X_{ij})^2=1081.35-\frac{1}{20}\times(139.7)^2=1081.35-975.8045=105.5455$

$Q_A^2=\sum_{i=1}^{r}n_i(\overline{X_{i\cdot}}-\overline{X})^2=\frac{1}{n_i}\sum_{i=1}^{r}(\sum_{j=1}^{n_i}X_{ij})^2-\frac{1}{n}(\sum\sum X_{ij})^2=\frac{1}{5}\times5159.65-\frac{1}{20}\times139.7^2=1031.93-975.8045=561.25$

$Q_E^2=Q_T^2-Q_A^2=105.5455-561.25=49.42$

当$\alpha=0.01$时，方差分析如下：

方差来源	离差平方和	自由度	均方离差平方和	F比值	$F_{0.01}(3,16)=5.29$
组间	$Q_A^2=56.1255$	$r-1=3$	$S_A^2=Q_A^2/3=18.7085$	$S_A^2/S_E^2=6.0567$	
组内	$Q_E^2=49.42$	$n-r=16$	$S_E^2=Q_E^2/16=3.08875$		
总和	$Q_T^2=105.5455$	$n-1=19$			

由于 $F=6.0567>F_{0.01}(3,16)=5.29$ 故有显著差异

二、解：本题属于两因素重复试验的方差分析问题，由条件已知 $r=3$, $s=3$, $t=2$, $n=18$.

浮选剂 施肥

$A \backslash B$	B_1	B_2	B_3	$T_{i..}=\sum_{j=1}^{3}\sum_{k=1}^{2}X_{ijk}$	$T_{i..}^2=(\sum_{j=1}^{3}\sum_{k=1}^{2}X_{ijk})^2$	$\sum_{j=1}^{3}\sum_{k=1}^{2}X_{ijk}^2$
A_1	43, 39 (82), 6724	37, 29 (66), 4356	58, 42 (100), 10000	248	61504	10708
A_2	47, 53 (100), 10000	41, 30 (71), 5041	46, 60 (106), 11236	277	76729	13315
A_3	38, 42 (80), 6400	48, 47 (95), 9025	56, 41 (97), 9409	272	73984	12538
$T_{.j.}=\sum_{i=1}^{3}\sum_{k=1}^{2}X_{ijk}$	282	232	303 ⇒	$\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{2}X_{ijk}=797$	$\sum_{i=1}^{3}T_{i..}^2=212217$	$\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{2}X_{ijk}^2=\boxed{36561}$
$T_{.j.}^2=(\sum_{i=1}^{3}\sum_{k=1}^{2}X_{ijk})^2$	68644	53824	91809 ⇒	$\sum_{j=1}^{3}T_{.j.}^2=\boxed{214277}$		
$\sum_{i=1}^{3}(\sum_{k=1}^{2}X_{ijk})^2$	23124	18422	30645 ⇒	$\sum_{j=1}^{3}\sum_{i=1}^{3}(\sum_{k=1}^{2}X_{ijk})^2=72191$		

其中，$T_{.j.}=\sum_{i=1}^{3}\sum_{k=1}^{2}X_{ijk}$，$T_{i..}=\sum_{j=1}^{3}\sum_{k=1}^{2}X_{ijk}$，$T=\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{2}X_{ijk}$，

$$Q_T^2=\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{2}(X_{ijk}-\overline{X})^2=\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{2}X_{ijk}^2-\frac{T^2}{n}=\boxed{36561}-\frac{1}{18}\times 797^2=1271.61$$

$$Q_A^2=\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{2}(\overline{X}_{i..}-\overline{X})^2=st\sum_{i=1}^{3}(\overline{X}_{i..}-\overline{X})^2=\frac{1}{st}\sum_{i=1}^{3}T_{i..}^2-\frac{T^2}{n}=\frac{1}{6}\times 212217-\frac{1}{18}\times 797^2=80.11$$

$$Q_B^2=\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{2}(\overline{X}_{.j.}-\overline{X})^2=rt\sum_{j=1}^{3}(\overline{X}_{.j.}-\overline{X})^2=\frac{1}{rt}\sum_{j=1}^{3}T_{.j.}^2-\frac{T^2}{n}=\frac{1}{6}\times\boxed{214277}-\frac{1}{18}\times 797^2=423.44$$

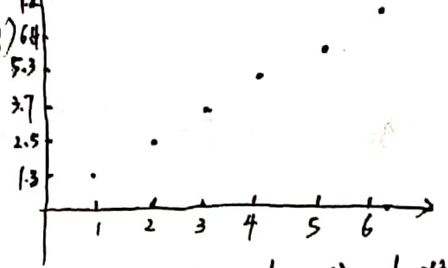
$$Q_E^2=\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{2}(X_{ijk}-\overline{X}_{ij.})^2=\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{2}X_{ijk}^2-\frac{1}{t}\sum_{i=1}^{3}\sum_{j=1}^{3}(\sum_{k=1}^{2}X_{ijk})^2=\boxed{36561}-\frac{1}{2}\times 72191=465.5$$

$$Q_{A\times B}^2=Q_T^2-Q_A^2-Q_B^2-Q_E^2=1271.61-80.11-423.44-465.5=302.56$$

方差来源	离差平方和	自由度	均方离差平方和	F比值	显著性
因素A	$Q_A^2=80.11$	$r-1=2$	$S_A^2=Q_A^2/2=40.055$	$S_A^2/S_E^2=0.774$	不显著
因素B	$Q_B^2=423.44$	$s-1=2$	$S_B^2=Q_B^2/2=211.72$	$S_B^2/S_E^2=4.094$	不显著
交互作用A×B	$Q_{A\times B}^2=302.56$	$(r-1)\cdot(s-1)=4$	$S_{A\times B}^2=Q_{A\times B}^2/4=75.64$	$S_{A\times B}^2/S_E^2=1.4625$	不显著
误差E	$Q_E^2=\cancel{423.44}$ 465.5	$rs(t-1)=9$	$S_E^2=Q_E^2/9=\cancel{70.488}$ 51.72		
总和	$Q_T^2=1271.61$	$rst-1=17$			

当 $\alpha=0.05$ 时，查表得 $F_{0.05}(2,9)=4.26>0.774$，$F_{0.05}(2,9)=4.26>4.094$，$F_{0.05}(4,9)=3.63>1.4625$

三、解：(1)



可以看作是一元线性回归模型

(2)

x_t	y_t	x_t^2	y_t^2	$x_t y_t$
1	1.3	1	1.69	1.3
2	2.5	4	6.25	5
3	3.7	9	13.69	11.1
4	5.3	16	28.09	21.2
5	6.4	25	40.96	32
6	7.2	36	51.84	43.2
Σ 21	26.4	91	142.52	113.8

$\bar{x}=\frac{1}{n}\sum_{t=1}^{6}x_t=\frac{1}{6}\times 21=3.5$

$\bar{y}=\frac{1}{n}\sum_{t=1}^{6}y_t=\frac{1}{6}\times 26.4=4.4$

$L_{xx}=\sum_{t=1}^{6}(x_t-\bar{x})^2=\sum_{t=1}^{6}x_t^2-n\bar{x}^2$

$=91-6\times(3.5)^2=17.5$

$L_{xy}=\sum_{t=1}^{6}x_t y_t-6\bar{x}\bar{y}=113.8-6\times 3.5\times 4.4=21.4$

$\hat{\beta}_1=\frac{L_{xy}}{L_{xx}}=\frac{21.4}{17.5}=1.22$， $\hat{\beta}_0=\bar{y}-\hat{\beta}_1\bar{x}$

$=4.4-1.22\times 3.5=0.1$

$\therefore\ \hat{y}=\hat{\beta}_0+\hat{\beta}_1 x=0.13+1.22x$

(3) 当 $x=10$ 时，y 的预测值 $\hat{y}=0.13+1.22\times 10=12.33$.

$L_{yy}=\sum_{t=1}^{6}y_t^2-6\bar{y}^2=142.52-6\times 4.4^2=26.36$

$Q_E^2=L_{yy}-\hat{\beta}_1 L_{xy}=26.36-1.22\times 21.4=0.252$， $\hat{\sigma}^2=\frac{Q_E^2}{n-2}=\frac{0.252}{4}=0.063$

$$T=\frac{y-\hat{y}}{\sqrt{\frac{Q_E^2}{n-2}\left[1+\frac{1}{n}+\frac{(x-\bar{x})^2}{L_{xx}}\right]}}\sim t_{(n-2)}=t_{(4)}$$

给定置信度 $1-\alpha=0.95$ 时，令 $P\{|T|\le t_{0.025}(4)\}=1-\alpha=0.95$

$\because\ t_{\frac{\alpha}{2}}(n-2)\sqrt{\hat{\sigma}^2\left[1+\frac{1}{n}+\frac{(x-\bar{x})^2}{L_{xx}}\right]}=t_{0.025}(4)\sqrt{0.063\times\left[1+\frac{1}{6}+\frac{(10-3.5)^2}{17.5}\right]}=t_{0.025}(4)\times 0.47=2.7764\times 0.47=1.305$

$\therefore$ 当 $x=10$ 时，置信度为 95% 的 y 的预测区间为：$\left[\hat{y}-t_{0.025}(4)\sqrt{\hat{\sigma}^2\left[1+\frac{1}{n}+\frac{(x-\bar{x})^2}{L_{xx}}\right]},\ \hat{y}+t_{0.025}(4)\sqrt{\hat{\sigma}^2\left[1+\frac{1}{n}+\frac{(x-\bar{x})^2}{L_{xx}}\right]}\right]$

$=[11.025,\ 13.635]$

四.解：(1) 构造函数 $X(t)=\cos(t-\Phi)$，$-\infty<t<\infty$

Φ	0	$\frac{\pi}{2}$
P	$\frac{1}{2}$	$\frac{1}{2}$

$X(0)=\cos(-\Phi)=\cos\Phi$

$X(0)$ 的分布律为：

$X(0)$	1	0
P	$\frac{1}{2}$	$\frac{1}{2}$

当 $\Phi=0$ 时，$X(0)=1$，$P\{\Phi=0\}=\frac{1}{2}$

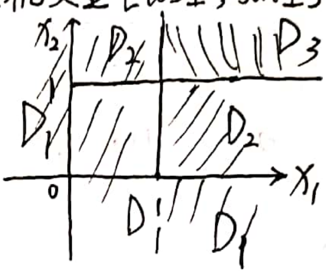
当 $\Phi=\frac{\pi}{2}$ 时，$X(0)=0$，$P\{\Phi=\frac{\pi}{2}\}=\frac{1}{2}$

一维分布函数：

$$F(x,0)=\begin{cases}0, & x<0\\ \frac{1}{2}, & 0\le x<1\\ 1, & 1\le x\end{cases}$$

$X(0)=\cos\Phi$，$X(\frac{\pi}{2})=\sin\Phi$，二维随机变量 $[\cos\Phi,\sin\Phi]$ 的分布律为：

$(\cos\Phi,\sin\Phi)$	$(1,0)$	$(0,1)$
P	$\frac{1}{2}$	$\frac{1}{2}$



$$F(x_1,x_2;0,\tfrac{\pi}{2})=P\{\cos\Phi\le x_1,\sin\Phi\le x_2\}=\begin{cases}0, & (x_1,x_2)\in D_1\\ \frac{1}{2}, & (x_1,x_2)\in D_2\\ 1, & (x_1,x_2)\in D_3\end{cases}$$

(2) $m_X(t)=E[X(t)]=E[\cos(t-\Phi)]=\frac{1}{2}\cos t+\frac{1}{2}\sin t$

$=\frac{1}{2}(\sin t+\cos t)$

Φ	0	$\frac{\pi}{2}$
$X(t)$	$\cos t$	$\sin t$
P	$\frac{1}{2}$	$\frac{1}{2}$

$R_X(t_1,t_2)=\frac{1}{2}\cos t_1\cdot\cos t_2+\frac{1}{2}\sin t_1\cdot\sin t_2=\frac{1}{2}\cos(t_1-t_2)$

注：$\cos(a\pm b)=\cos a\cos b\mp\sin a\sin b$

$\sin(a\pm b)=\sin a\cos b\pm\cos a\sin b$

五. 解：(1) $E(U)=\frac{b-a}{2}=\frac{1-0}{2}=\frac{1}{2}$， $D(U)=\frac{(b-a)^2}{12}=\frac{1}{12}$， $E(U^2)=D(U)+E^2(U)=\frac{1}{12}+\frac{1}{4}=\frac{1}{3}$

$m_X(t)=E[X(t)]=E[U+\cos(2\pi t+\Phi)]=E(U)+E[\cos(2\pi t+\Phi)]=\frac{1}{2}+\int_0^{2\pi}\frac{1}{2\pi}\cos(2\pi t+\Phi)d\Phi=\frac{1}{2}$

$R_X(t_1,t_2)=E[X_1(t)\cdot X_2(t)]=E[(U+\cos(2\pi t_1+\Phi))\cdot(U+\cos(2\pi t_2+\Phi))]$

$=E[U^2+U\cdot\cos(2\pi t_2+\Phi)+U\cdot\cos(2\pi t_1+\Phi)+\cos(2\pi t_1+\Phi)\cdot\cos(2\pi t_2+\Phi)]$

$=E(U^2)+E[U\cos(2\pi t_2+\Phi)]+E[U\cos(2\pi t_1+\Phi)]+E[\cos(2\pi t_1+2\pi t_2+2\Phi)+\cos(2\pi t_1-2\pi t_2)]\times\frac{1}{2}$

$=\frac{1}{3}+\frac{1}{2}\int_0^{2\pi}\frac{1}{2\pi}\cdot\cos(2\pi t_2+\Phi)d\Phi+\frac{1}{2}\int_0^{2\pi}\frac{1}{2\pi}\cos(2\pi t_1+\Phi)d\Phi+\frac{1}{2}\int_0^{2\pi}\frac{1}{2\pi}\cos(2\pi t_1+2\pi t_2+2\Phi)d\Phi$

$+\frac{1}{4\pi}\int_0^{2\pi}\cos(2\pi t_1-2\pi t_2)d\Phi$ $\cos\alpha\cdot\cos\beta=\frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$

$=\frac{1}{3}+\frac{2\pi}{4\pi}\cdot\cos(2\pi t_1-2\pi t_2)=\frac{1}{3}+\frac{1}{2}\cos 2\pi(t_2-t_1)$

$\therefore R_X(\tau)=\frac{1}{3}+\frac{1}{2}\cos(2\pi\tau)$

又$\because$数学期望是常数，相关函数仅与时间τ有关

$\therefore X(t)$具有平稳性

$\lim\limits_{T\to\infty}\frac{1}{T}\int_0^{2\pi}(1-\frac{\tau}{2T})\cdot[R_X(\tau)-m_X^2]d\tau=\lim\limits_{T\to\infty}\frac{1}{T}\int_0^{2\pi}(1-\frac{\tau}{2T}]\cdot[\frac{1}{3}+\frac{1}{2}\cos(2\pi\tau)-\frac{1}{4}]d\tau$

$=\lim\limits_{T\to\infty}\frac{1}{T}\int_0^{2\pi}(1-\frac{\tau}{2T})\cdot[\frac{1}{12}+\frac{1}{2}\cos(2\pi\tau)]d\tau=\lim\limits_{T\to\infty}\frac{1}{T}\int_0^{2\pi}(1-\frac{\tau}{2T})\times\frac{1}{12}d\tau+\lim\limits_{T\to\infty}\frac{1}{T}\int_0^{2\pi}(1-\frac{\tau}{2T})\cdot\frac{1}{2}\cos(2\pi\tau)d\tau$

$=0+0=0$

$\therefore\langle X(t)\rangle=m_X\Rightarrow X(t)$具有数学期望的各态历经性.

$Y(t)=\int_0^{+\infty}X(t-\lambda)\cdot h(\lambda)d\lambda$， $R_Y(t,t+\tau)=E[Y(t)\cdot Y(t+\tau)]=\int_0^{+\infty}\int_0^{+\infty}R_X(\lambda_2-\lambda_1-\tau)h(\lambda_1)h(\lambda_2)d\lambda_1 d\lambda_2$

$\therefore Y(t)$具有平稳性.

(2) $S_X(\omega)=\int_{-\infty}^{+\infty}e^{-i\tau\omega}\cdot R_X(\tau)d\tau=\int_{-\infty}^{+\infty}e^{-i\tau\omega}[\frac{1}{3}+\frac{1}{2}\cos(2\pi\tau)]d\tau$

$\therefore S_X(\omega)=\frac{1}{3}\times 2\pi\delta(\omega)+\frac{1}{2}\cdot\pi[\delta(\omega-2\pi)+\delta(\omega+2\pi)]$

又频率响应函数为：$H(i\omega)=\frac{\alpha}{i\omega+\alpha}$， $\alpha=\frac{1}{RC}$.

$\therefore S_Y(\omega)=|H(i\omega)|^2\cdot S_X(\omega)=\frac{\alpha}{\alpha^2+\omega^2}\times[\frac{2\pi}{3}\delta(\omega)+\frac{\pi}{2}\delta(\omega-2\pi)+\frac{\pi}{2}\delta(\omega+2\pi)]$

$=\frac{2\pi}{3}\delta(\omega)+\frac{\alpha}{\alpha^2+4\pi^2}[\frac{\pi}{2}\delta(\omega-2\pi)+\frac{\pi}{2}\delta(\omega+2\pi)]$

$f(\omega)\cdot\delta(\omega)=f(0)\cdot\delta(\omega)$

$f(\omega)\cdot\delta(\omega-\omega_0)=f(\omega_0)\cdot\delta(\omega-\omega_0)$

六、解：(1) 一步转移概率矩阵为：

$$P=\begin{bmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{bmatrix}$$

二步转移概率矩阵为：

$$P(2)=P^2=\begin{bmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{bmatrix}\cdot\begin{bmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{bmatrix}=\begin{bmatrix} \frac{4}{9} & \frac{1}{9} & \frac{1}{3} & \frac{1}{9} \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{6} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{9} & \frac{4}{9} & \frac{1}{9} \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{6} & \frac{1}{3} \end{bmatrix}$$

(2) 由于二步转移概率矩阵 $P(2)$ 的元素全大于零，所以此马氏链是遍历的。又因为该马氏链是有限马氏链，所以存在极限分布 p_1, p_2, p_3, p_4.

$$\begin{bmatrix} \frac{4}{9}-1 & \frac{1}{6} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{9} & \frac{1}{3}-1 & \frac{1}{9} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{6} & \frac{4}{9}-1 & \frac{1}{6} \\ \frac{1}{9} & \frac{1}{3} & \frac{1}{9} & \frac{1}{3}-1 \end{bmatrix}\cdot\begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix}=0 \Rightarrow \begin{cases} -\frac{5}{9}p_1+\frac{1}{6}p_2+\frac{1}{3}p_3+\frac{1}{6}p_4=0 & ① \\ \frac{1}{9}p_1+(-\frac{2}{3})p_2+\frac{1}{9}p_3+\frac{1}{3}p_4=0 & ② \\ \frac{1}{3}p_1+\frac{1}{6}p_2-\frac{5}{9}p_3+\frac{1}{6}p_4=0 & ③ \\ \frac{1}{9}p_1+\frac{1}{3}p_2+\frac{1}{9}p_3+(-\frac{2}{3})p_4=0 & ④ \end{cases} \Rightarrow \begin{cases} ③-① \Rightarrow p_1=p_3 \\ ④-② \Rightarrow p_2=p_4 \end{cases}$$

满足 $p_1+p_2+p_3+p_4=1$，$p_j>0\ (j=1,2,3,4)$ 的解为：$p_1=\frac{3}{10}=p_3$，$p_2=p_4=\frac{1}{5}$.

(3) 绝对概率：$P\{X_2=1\}=p_1^{(2)}=\sum_{i=1}^{4}p_i^{(0)}\cdot p_{i1}(2)=\sum_{i=1}^{4}P\{X_0=i\}\cdot p_{i1}(2)=\frac{1}{4}\times(\frac{4}{9}+\frac{1}{6}+\frac{1}{3}+\frac{1}{6})=\frac{5}{18}$

多维概率：$P\{X_1=1, X_3=2, X_5=3\}=\sum_{i=1}^{4}p_i(0)\cdot p_{i1}(1)\cdot p_{12}(3-1)\cdot p_{23}(5-3)$

$$=\sum_{i=1}^{4}p_i(0)\cdot p_{i1}(1)\cdot p_{12}(2)\cdot p_{23}(2)=p_i(0)\cdot p_{12}(2)\cdot p_{23}(2)\cdot\sum_{i=1}^{4}p_{i1}(1)$$

$$=\frac{1}{4}\times\frac{1}{9}\times\frac{1}{6}\times[0+\frac{1}{2}+\frac{1}{3}+\frac{1}{2}]=\frac{1}{4\times9\times6}\times\frac{4}{3}=\frac{1}{9\times6\times3}=\frac{1}{172}$$

注：$P\{X(n)=i\}=p_i^{(n)}$　　$p_j^{(n)}=\sum_{i\in E}p_i^{(0)}\cdot p_{ij}(n)$ ⇒ 绝对概率

$$P\{X(n_1)=i_1, X(n_2)=i_2, \ldots, X(n_m)=i_m\}=\sum_{i\in E}p_i^{(0)}\cdot p_{ii_1}(n_1)\,p_{i_1i_2}(n_2-n_1)\cdots p_{i_{m-1}i_m}(n_m-n_{m-1})$$

$$=p_i^{(0)}\cdot p_{i_1i_2}(n_2-n_1)\cdots p_{i_{m-1}i_m}(n_m-n_{m-1})\cdot\sum_{i\in E}p_{ii_1}(n_1)$$ ⇒ 多维概率