2017 试题答案

一、

$$\overline{x_1} = \frac{1}{6}\sum_{j=1}^{6} x_{1j} = \frac{1}{6}(370 + 420 + 450 + 490 + 500 + 450) = 446.7$$

$$\overline{x_2} = \frac{1}{6}\sum_{j=1}^{6} x_{2j} = \frac{1}{6}(490 + 380 + 400 + 390 + 500 + 410) = 428.3$$

$$\overline{x_3} = \frac{1}{6}\sum_{j=1}^{6} x_{3j} = \frac{1}{6}(330 + 340 + 400 + 380 + 470 + 360) = 380.0$$

$$\overline{x} = \frac{1}{3}\sum_{i=1}^{3} \overline{x}_i = \frac{1}{3}(446.7 + 428.3 + 380.0) = 418.3$$

$$\begin{aligned}Q_A &= 6\sum_{i=1}^{3}(\overline{x_i} - \overline{x})^2 \\ &= 6[(446.7 - 418.3)^2 + (428.3 - 418.3)^2 + (380 - 418.3)^2] = 14240.7\end{aligned}$$

$$\begin{aligned}Q_E &= \sum_{i=1}^{3}\sum_{j=1}^{6}(x_{ij} - \overline{x_i})^2 \\ &= (370 - 446.7)^2 + (420 - 446.7)^2 + (450 - 446.7)^2 \\ &+ (490 - 446.7)^2 + (500 - 446.7)^2 + (450 - 446.7)^2 \\ &+ (490 - 428.3)^2 + (380 - 428.3)^2 + (400 - 428.3)^2 \\ &+ (390 - 428.3)^2 + (500 - 428.3)^2 + (410 - 428.3)^2 \\ &+ (330 - 380)^2 + (340 - 380)^2 + (400 - 380)^2 \\ &+ (380 - 380)^2 + (470 - 380)^2 + (360 - 380)^2 = 38216.68\end{aligned}$$

$$S_A^2 = \frac{Q_A}{r - 1} = \frac{14240.7}{2} = 7120.35$$

$$S_E^2 = \frac{Q_E}{n - r} = \frac{38216.68}{15} = 2547.78$$

$$F = \frac{S_A^2}{S_E^2} = \frac{7120.35}{2547.78} = 2.79$$

$$F_\alpha(r - 1, n - r) = F_{0.01}(2,15) = 6.36$$

因为$F < F_{0.01}(2,15)$，所以在显著性水平$\alpha = 0.01$下可以认为，这 3 种不同的饲料配方对

小鸡增重没有显著差异。

二、

$\overline{x}_{11。} = 5.70,\quad \overline{x}_{12。} = 7.05,\quad \overline{x}_{13。} = 7.70,$

$\overline{x}_{21。} = 7.60,\quad \overline{x}_{22。} = 4.80,\quad \overline{x}_{23。} = 4.20,$

$\overline{x}_{31。} = 6.35,\quad \overline{x}_{32。} = 4.40,\quad \overline{x}_{33。} = 3.95,$

$\overline{x}_{1..} = 6.85,\quad \overline{x}_{2..} = 5.53,\quad \overline{x}_{3..} = 4.90,$

$\overline{x}_{.1.} = 6.55,\quad \overline{x}_{.2.} = 5.42,\quad \overline{x}_{.3.} = 5.28,$

$\overline{x} = 5.76$

$$Q_A = st\sum_{i=1}^{r}(\overline{x}_{i..} - \overline{x})^2$$

$$= 3 \times 2 \times [(6.85 - 5.76)^2 + (5.53 - 5.76)^2 + (4.90 - 5.76)^2] = 11.88,$$

$$Q_B = rt\sum_{j=1}^{s}(\overline{x}_{.j.} - \overline{x})^2$$

$$= 3 \times 2 \times [(6.55 - 5.76)^2 + (5.42 - 5.76)^2 + (5.28 - 5.76)^2] = 5.82,$$

$$Q_{A\times B} = t\sum_{i=1}^{r}\sum_{j=1}^{s}(\overline{x}_{ij.} - \overline{x}_{i..} - \overline{x}_{.j.} + \overline{x})^2$$

$$= 2 \times [(5.70 - 6.85 - 6.55 + 5.76)^2 + (7.05 - 6.85 - 5.42 + 5.76)^2$$

$$+ (7.70 - 6.85 - 5.28 + 5.76)^2 + (7.60 - 5.53 - 6.55 + 5.76)^2$$

$$+ (4.80 - 5.53 - 5.42 + 5.76)^2 + (4.20 - 5.53 - 5.28 + 5.76)^2$$

$$+ (6.35 - 4.90 - 6.55 + 5.76)^2 + (4.40 - 4.90 - 5.42 + 5.76)^2$$

$$+ (3.95 - 4.90 - 5.28 + 5.76)^2]$$

$$= 18.04.$$

$$Q_E = \sum_{i=1}^{r}\sum_{j=1}^{s}\sum_{k=1}^{t}(x_{ijk} - \overline{x}_{ij.})^2$$

$$= (5.6 - 5.7)^2 + (5.8 - 5.7)^2 + (6.9 - 7.05)^2 + (7.2 - 7.05)^2$$

$$+ (7.5 - 7.7)^2 + (7.9 - 7.7)^2 + (7.4 - 7.6)^2 + (7.8 - 7.6)^2$$

$$+(4.5-4.8)^2+(5.1-4.8)^2+(4.4+4.2)^2+(4.0-4.2)^2$$

$$+(6.2-6.35)^2+(6.5-6.35)^2+(4.2-4.4)^2+(4.6-4.4)^2$$

$$+(3.7-3.95)^2+(4.2-3.95)^2$$

$$=0.73.$$

$$S_A^2=\frac{Q_A}{r-1}=\frac{11.88}{2}=5.94,$$

$$S_B^2=\frac{Q_B}{s-1}=\frac{5.82}{2}=2.91,$$

$$S_{A\times B}^2=\frac{Q_{A\times B}}{(r-1)(s-1)}=\frac{18.04}{4}=4.51,$$

$$S_E^2=\frac{Q_E}{rs(t-1)}=\frac{0.73}{9}=0.08,$$

$$F_A=\frac{S_A^2}{S_E^2}=\frac{5.94}{0.08}=74.2,$$

$$F_B=\frac{S_B^2}{S_E^2}=\frac{2.91}{0.08}=36.4,$$

$$F_{A\times B}=\frac{S_{A\times B}^2}{S_E^2}=\frac{4.51}{0.08}=56.4,$$

$$F_\alpha(r-1,rs(t-1))=F_{0.05}(2,9)=4.62.$$

$$F_\alpha(s-1,rs(t-1))=F_{.0.05}(2,9)=4.62,$$

$$F_\alpha((r-1)(s-1),rs(t-1))=F_{0.05}(4,9)=3.63.$$

因为$F_A>F_{0.05}(2,9)$，$F_B>F_{0.05}(2,9)$，$F_{A\times B}>F_{0.05}(4,9)$，所以在显著性水平$\alpha=0.05$下可认为焊接时间、焊接温度及其交互作用对焊接点拉拔力均有显著影响。

三、

（1）从下面的试验值的散点图可以看出，试验点大致分布在一条直线附近。因此，该问题看作一元线性回归比较合适。

（2）经计算得

$$\overline{x} = 550，\overline{y} = 64.33，\overline{x^2} = 331666.67，$$
$$\overline{y^2} = 4421.67，\overline{x}^2 = 302500，\overline{xy} = 38250.$$

根据公式

$$\hat{\beta} = \frac{\overline{xy} - \overline{x}\cdot\overline{y}}{\overline{x^2} - \overline{x}^2} = \frac{38250 - 550\times 64.33}{331666.67 - 302500} = 0.1，$$

$$\overline{\alpha} = \overline{y} - \hat{\beta}\cdot\overline{x} = 64.33 - 0.1\times 550 = 9.33,$$

$$\hat{\sigma}^* = \sqrt{\frac{6}{6-2}\left[(\overline{y^2} - \overline{y}^2) - \hat{\beta}^2(\overline{x^2} - \overline{x}^2)\right]} = 3.54.$$

于是所求经验回归方程为 $y = 9.33 + 0.1x$。

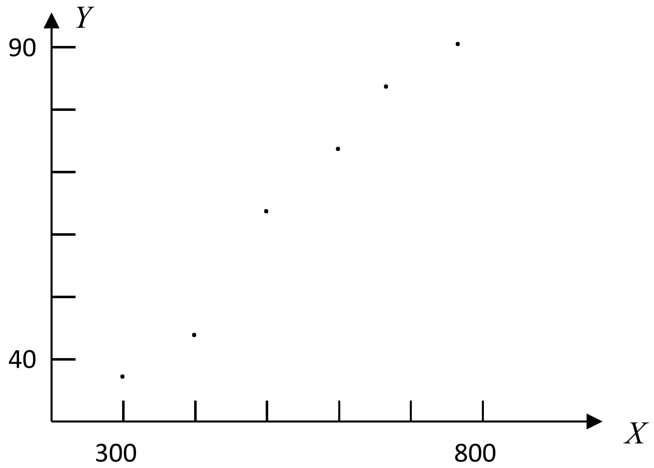
（3）根据经验公式，当 $x = 1200$ 时，Y 的预测值为 $y = 9.33 + 0.1\times 1200 = 129.33$。由 $\delta(x)$ 的表达式

$$\delta(1200) = t_{\frac{0.05}{2}}(6-2)\sqrt{1 + \frac{1}{6} + \frac{(1200 - 550)^2}{\sum_{i=3}^{8}(100i - 550)^2}} \times 3.54 = 35.23，$$

于是 Y 的置信度为95%的预测区间的下限、上限分别为

$$y_1 = 129.33 - 35.23 = 94.1，$$

$$y_2 = 129.33 + 35.23 = 164.56。$$



四、

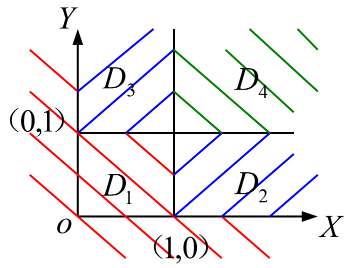
（1）$X(\frac{\pi}{3}) = \frac{1}{2}\cos\Phi + \frac{\sqrt{3}}{2}\sin\Phi$，$\Phi$ 取 0 和 $\frac{\pi}{2}$ 的概率分别为 $\frac{1}{3}$ 和 $\frac{2}{3}$，所以 $X(\frac{\pi}{3})$ 取 $\frac{1}{2}$

和 $\frac{\sqrt{3}}{2}$ 的概率分别为 $\frac{1}{3}$ 和 $\frac{2}{3}$。$X(\frac{\pi}{3})$ 的分布函数为

$$F(x;\frac{\pi}{3}) = \begin{cases} 0, & -\infty < x < \frac{1}{2} \\ \frac{1}{3}, & \frac{1}{2} \le x < \frac{\sqrt{3}}{2} \\ 1, & \frac{\sqrt{3}}{2} < x < +\infty \end{cases}.$$

$X(0) = \cos\Phi$, $X(\frac{\pi}{2}) = \sin\Phi$，$\Phi$ 取 0 和 $\frac{\pi}{2}$ 的概率分别为 $\frac{1}{3}$ 和 $\frac{2}{3}$，所以 $\left(X(0), X(\frac{\pi}{2})\right)$ 取 $(1,0)$ 和 $(0,1)$ 的概率分别为 $\frac{1}{3}$ 和 $\frac{2}{3}$。$\left(X(0), X(\frac{\pi}{2})\right)$ 的分布函数为

$$F(x, y;0, \frac{\pi}{2}) = \begin{cases} 0, & (x, y) \in D_1 \\ \frac{1}{3}, & (x, y) \in D_2 \\ \frac{2}{3}, & (x, y) \in D_3 \\ 1, & (x, y) \in D_4 \end{cases}$$



（2）均值函数为

$$\begin{aligned} m_X(t) &= E[\cos(t-\Phi)] \\ &= \frac{1}{3}\cos(t-0) + \frac{2}{3}\cos(t-\frac{\pi}{2}) \\ &= \frac{\cos t + 2\sin t}{3}. \end{aligned}$$

自相关函数为

$$\begin{aligned} R_X(t_1, t_2) &= E[\cos(t_1-\Phi)\cos(t_2-\Phi)] \\ &= \frac{1}{3}\cos(t_1-0)\cos(t_2-0) + \frac{2}{3}\cos(t_1-\frac{\pi}{2})\cos(t_2-\frac{\pi}{2}) \\ &= \frac{1}{3}(\cos t_1 \cos t_2 + 2\sin t_1 \sin t_2). \end{aligned}$$

五、

均值函数为

$$m_X(t) = E[A\cos t + B\sin t] = (EA)\cos t + (EB)\sin t$$

$$= 0\cos t + 0\sin t = 0 = m_X(\text{常数}),$$

自相关函数为

$$R_X(t, t+\tau) = E[A\cos t + B\sin t][A\cos(t+\tau) + B\sin(t+\tau)]$$

$$= (EA^2)\cos t\cos(t+\tau) + (EA)(EB)[\cos t\sin(t+\tau)$$

$$+ \sin t\cos(t+\tau)] + (EB^2)\sin t\sin(t+\tau)$$

$$= \frac{1}{3}[\cos t\cos(t+\tau) + \sin t\sin(t+\tau)] = \frac{1}{3}\cos\tau$$

$$= R_X(\tau)\,(\tau\text{的一元函数}).$$

所以此过程为平稳过程。

此过程的时间平均为

$$\langle X(t)\rangle = \underset{T\to+\infty}{l.i.m}\frac{1}{2T}\int_{-T}^{T}[A\cos t + B\sin t]dt$$

$$= A\underset{T\to+\infty}{l.i.m}\frac{\sin T}{T} = 0 = m_X,$$

即此过程具有数学期望的各态历经性。此过程的时间相关函数为

$$\langle X(t)X(t+\tau)\rangle = \underset{T\to+\infty}{l.i.m}\frac{1}{2T}\int_{-T}^{T}[A\cos t + B\sin t][A\cos(t+\tau) + B\sin(t+\tau)]dt$$

$$= \underset{T\to+\infty}{l.i.m}\frac{1}{2T}\int_{-T}^{T}[A^2\cos t\cos(t+\tau) + B^2\sin t\sin(t+\tau)]dt$$

$$+ \underset{T\to+\infty}{l.i.m}\frac{1}{2T}\int_{-T}^{T}AB[\cos t\sin(t+\tau) + \sin t\cos(t+\tau)]dt$$

$$= \frac{1}{2}(A^2 + B^2)\cos\tau,$$

由于A, B均在$[-1,1]$上服从均匀分布，$\frac{1}{2}(A^2 + B^2)$不可能以概率 1 等于$\frac{1}{3}$，所以此过程不具有相关函数的各态历经性。

六、

此 $R—C$ 电路为线性定常系统，它的脉冲响应函数为$h(t) = \begin{cases}\alpha e^{-\alpha t}, t \ge 0\\ 0, \quad t < 0\end{cases}$，频率响应函

数为$H(i\omega)=\dfrac{\alpha}{i\omega+\alpha}$,输入$X(t)$的普密度为$S_X(\omega=\dfrac{2\lambda}{\lambda^2+\omega^2}$。由定理2知,输出$Y(t)$的谱密度为

$$S_Y(\omega)=\left|H(i\omega)\right|^2S_X(\omega)=\frac{\alpha^2}{\omega^2+\alpha^2}\cdot\frac{2\lambda}{\omega^2+\lambda^2}$$
$$=\frac{\lambda\alpha}{\lambda^2-\alpha^2}\left[\frac{2\alpha}{\omega^2+\alpha^2}\right]-\frac{\alpha^2}{\lambda^2-\alpha^2}\left[\frac{2\lambda}{\omega^2+\lambda^2}\right]$$

两边取傅里叶逆变换得

$$R_Y(\tau)=\frac{\lambda\alpha}{\lambda^2-\alpha^2}\mathrm{e}^{-\alpha|\tau|}-\frac{\alpha^2}{\lambda^2-\alpha^2}e^{-\lambda|\tau|},\ -\infty<\tau<+\infty.$$

七、

（1）一步转移概率矩阵为

$$P=\begin{pmatrix}0&1/3&1/3&1/3\\1/3&0&1/3&1/3\\1/3&1/3&0&1/3\\1/3&1/3&1/3&0\end{pmatrix},$$

二步转移概率矩阵为

$$P(2)=P^2=\begin{pmatrix}0&1/3&1/3&1/3\\1/3&0&1/3&1/3\\1/3&1/3&0&1/3\\1/3&1/3&1/3&0\end{pmatrix}\begin{pmatrix}0&1/3&1/3&1/3\\1/3&0&1/3&1/3\\1/3&1/3&0&1/3\\1/3&1/3&1/3&0\end{pmatrix}$$
$$=\begin{pmatrix}1/3&2/9&2/9&2/9\\2/9&1/3&2/9&2/9\\2/9&2/9&1/3&2/9\\2/9&2/9&2/9&1/3\end{pmatrix}.$$

（2）二步转移概率矩阵的每个元素都大于零，所以此马氏链是遍历的。此马氏链又为有限马氏链，必存在极限分布。设极限分布为p_1, p_2,p_3, p_4。求方程组

$$\begin{pmatrix}-1&1/3&1/3&1/3\\1/3&-1&1/3&1/3\\1/3&1/3&-1&1/3\\1/3&1/3&1/3&-1\end{pmatrix}\begin{pmatrix}p_1\\p_2\\p_3\\p_4\end{pmatrix}=\begin{pmatrix}0\\0\\0\\0\end{pmatrix}$$

满足 $p_1+p_2+p_3+p_4=1$，$p_1>0, p_2>0, p_3>0, p_4>0$的解为

$$p_1 = p_2 = p_3 = p_4 = \frac{1}{4}.$$

（3）所求绝对概率

$$P\{X_2 = 1\} = \sum_{i=1}^{4} p_i^{(0)} p_{i1}(2) = \frac{1}{4}(\frac{1}{3} + \frac{2}{9} + \frac{2}{9} + \frac{2}{9}) = \frac{1}{4},$$

所求多维概率

$$\begin{aligned} P\{X_1 = 2，X_3 = 3，X_5 = 4\} &= \sum_{i=1}^{4} p_i^{(0)} p_{i2}(1) p_{23}(2) p_{34}(2) \\ &= \frac{1}{4} \times \frac{2}{9} \times \frac{2}{9}\left(\frac{1}{3} + 0 + \frac{1}{3} + \frac{1}{3}\right) \\ &= \frac{1}{81}. \end{aligned}$$