一、解：属于单因素方差分析问题，水平数 $r=3$，重复试验次数 $n_1=n_2=n_3=5$，即样本数 $n=15$.

	$T_i=\sum_{j=1}^{5}X_{ij}$	$T_i^2=(\sum_{j=1}^{5}X_{ij})^2$	$\sum_{j=1}^{5}X_{ij}^2$
玻璃	33.37	1113.5569	222.71141
金	33.397	1115.359609	223.071951
铂	33.32	1110.2224	222.044516
Σ	$\sum_{i=1}^{3}\sum_{j=1}^{5}X_{ij}=100.087$	$\sum_{i=1}^{3}T_i^2=3339.138909$	$\sum_{i=1}^{3}\sum_{j=1}^{5}X_{ij}^2=667.827877$

$$Q_T^2=\sum_{i=1}^{r}\sum_{j=1}^{n_i}(X_{ij}-\bar{X})^2=\sum_{i=1}^{r}\sum_{j=1}^{n_i}X_{ij}^2-\frac{1}{n}\Big(\sum_{i=1}^{r}\sum_{j=1}^{n_i}X_{ij}\Big)^2=667.827877-\cancel{3339.138909}\,(100.087)^2\times\frac{1}{15}=0.0007057$$

$$Q_A^2=\sum_{i=1}^{r}n_i(\bar{X}_i-\bar{X})^2=\frac{1}{n_i}\sum_{i=1}^{r}\Big(\sum_{j=1}^{n_i}X_{ij}\Big)^2-\frac{1}{n}\Big(\sum_{i=1}^{r}\sum_{j=1}^{n_i}X_{ij}\Big)^2=\frac{1}{5}\times3339.138909-\frac{1}{15}\times(100.087)^2=0.0006105$$

$$Q_E^2=Q_T^2-Q_A^2=0.0000952$$

当 $\alpha=0.05$ 时，方差分析如下：

方差来源	离差平方和	自由度	均方离差平方和	F比值 $=S_A^2/S_E^2$
组间	$Q_A^2=0.0006105$	$r-1=2$	$S_A^2=Q_A^2/2=0.00030525$	$=38.44458438$
组内	$Q_E^2=0.0000952$	$n-r=12$	$S_E^2=Q_E^2/12=0.00000794$	
总和	$Q_T^2=0.0007057$	$n-1=14$		

∵ F比值 $=38.44>F_{0.05}(2,12)=3.89$ 故有显著影响.

二.解：本题属于两因素重复试验的方差分析问题，由条件知 $\overset{时段}{r}=2$，$\overset{路段}{s}=2$，$t=5$，$n=20$，由所给数据列表计算如下：

B / A	B_1	B_2	$T_{i..}=\sum_{j=1}^{2}\sum_{k=1}^{5}x_{ijk}$	$T_{i..}^2=(\sum_{j=1}^{2}\sum_{k=1}^{5}x_{ijk})^2$	$\sum_{j=1}^{2}\sum_{k=1}^{5}x_{ijk}^2$
A_1	26, 24, 27, 25, 26 (128), 16384	20, 19, 20, 23, 22 (104), 10816	232	53824	5456
A_2	17, 22, 18, 21, 17 (95), 9025	14, 13, 17, 16, 12 (72), 5184	167	27889	2881
$T_{.j.}=\sum_{i=1}^{2}\sum_{k=1}^{5}x_{ijk}$	128+95=223	104+72=176	$T=\sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{5}x_{ijk}=399$	$\sum_{i=1}^{2}T_{i..}^2=\boxed{81713}$	$\sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{5}x_{ijk}^2=8337$
$T_{.j.}^2=(\sum_{i=1}^{2}\sum_{k=1}^{5}x_{ijk})^2$	49729 ~~16384+9025=25409~~	30976 ~~10816+5184=30976~~	$\sum_{j=1}^{2}T_{.j.}^2=\boxed{80705}$		
$\sum_{i=1}^{2}(\sum_{k=1}^{5}x_{ijk})^2$	16384+9025=25409	10816+5184=16000	$\sum_{j=1}^{2}\sum_{i=1}^{2}(\sum_{k=1}^{5}x_{ijk})^2=25409+16000=\boxed{41409}$		

其中，$T_{.j.}=\sum_{i=1}^{2}\sum_{k=1}^{5}x_{ijk}$，$T_{i..}=\sum_{j=1}^{2}\sum_{k=1}^{5}x_{ijk}$，$T=\sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{5}x_{ijk}$，

$$Q_T^2=\sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{5}(x_{ijk}-\bar{x})^2=\sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{5}x_{ijk}^2-\frac{T^2}{n}=8337-\frac{1}{20}\times(399)^2=376.95$$

$$Q_A^2=\sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{5}(\bar{X}_{i..}-\bar{X})^2=st\sum_{i=1}^{2}(\bar{X}_{i..}-\bar{X})^2=\frac{1}{st}\sum_{i=1}^{2}T_{i..}^2-\frac{T^2}{n}=\frac{1}{10}\times\boxed{81713}-\frac{1}{20}\times 399^2=211.25$$

$$Q_B^2=\sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{5}(\bar{X}_{.j.}-\bar{X})^2=rt\sum_{j=1}^{2}(\bar{X}_{.j.}-\bar{X})^2=\frac{1}{rt}\sum_{j=1}^{2}T_{.j.}^2-\frac{T^2}{n}=\frac{1}{10}\times\boxed{80705}-\frac{1}{20}\times 399^2=110.45$$

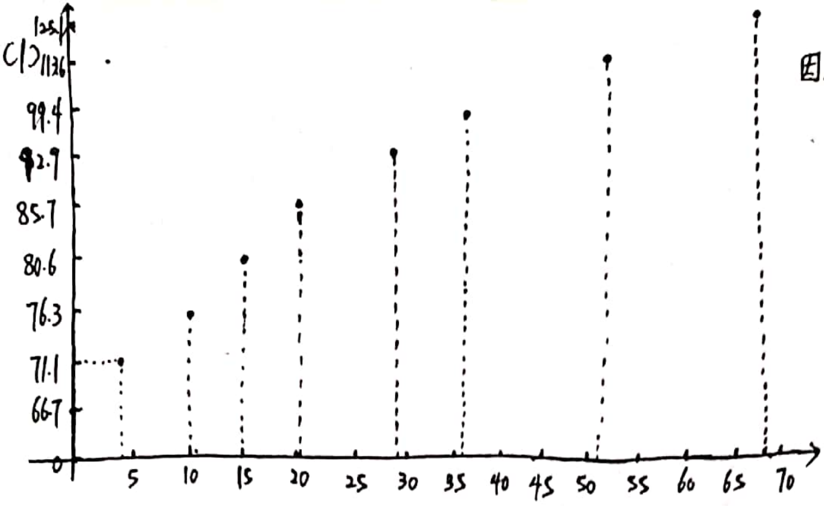
$$Q_E^2=\sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{5}(X_{ijk}-\bar{X}_{ij})^2=\sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{5}X_{ijk}^2-\frac{1}{t}\sum_{i=1}^{2}\sum_{j=1}^{2}(\sum_{k=1}^{5}X_{ijk})^2=8337-\frac{1}{5}\times\boxed{41409}=55.2$$

$Q_{A\times B}^2=Q_T^2-Q_A^2-Q_B^2-Q_E^2=376.95-211.25-110.45-55.2=0.05$，于是得方差分析表如下：

方差来源	离差平方和	自由度	均方离差平方和	F比值	显著性
因素A	$Q_A^2=211.25$	$r-1=1$	$S_A^2=Q_A^2/1=211.25$	$S_A^2/S_E^2=61.23188$	高度显著
因素B	$Q_B^2=110.45$	$s-1=1$	$S_B^2=Q_B^2/1=110.45$	$S_B^2/S_E^2=32.01449$	高度显著
交互作用A×B	$Q_{A\times B}^2=0.05$	$(r-1)\cdot(s-1)=1$	$S_{A\times B}^2=Q_{A\times B}^2/1=0.05$	$S_{A\times B}^2/S_E^2=0.01449$	无显著
误差E	$Q_E^2=55.2$	$rs(t-1)=16$	$S_E^2=Q_E^2/16=3.45$		
总和	$Q_T^2=376.95$	$rst-1=19$			

当 $\alpha=0.05$ 时，查表得 $F_{0.05}(1,16)=4.49<61.23188$，$F_{0.05}(1,16)=4.49<32.01449$，$F_{0.05}(1,16)=4.49>0.01449$

三.解：(1)



因此,能直观的认为硝酸钠的溶解重量y对水温x的回归是线性的.

(2)

	x_t	y_t	x_t^2	y_t^2	$x_t y_t$
	0	66.7	0	4448.89	0
	4	71.1	16	5055.21	284.4
	10	76.3	100	5821.69	763
	15	80.6	225	6496.36	1209
	21	85.7	441	7344.49	1799.7
	29	92.9	841	8630.41	2694.1
	36	99.4	1296	9880.36	3578.4
	51	113.6	2601	12904.96	5793.6
	68	125.1	4624	15650.01	8506.8
$\sum_{t=1}^{9}$	234	811.4	10144	76232.38	24627

$\bar{x}=\frac{1}{9}\sum_{t=1}^{9}x_t=\frac{1}{9}\times 234=26$

$\bar{y}=\frac{1}{9}\sum_{t=1}^{9}y_t=\frac{1}{9}\times 811.4=90.16$

$L_{xx}=\sum_{t=1}^{9}(x_t-\bar{x})^2=\sum_{t=1}^{9}x_t^2-n\bar{x}^2$

$=10144-9\times(26)^2=4060$

$L_{xy}=\sum_{t=1}^{9}x_t y_t-9\bar{x}\bar{y}=24627-9\times 26\times 90.16=3531.36$

$\hat{\beta}_1=\frac{L_{xy}}{L_{xx}}=\frac{3531.36}{4060}=0.87$

$\hat{\beta}_0=\bar{y}-\hat{\beta}_1\bar{x}=90.16-0.87\times 26=67.54$

故:回归方程: $\hat{y}=\hat{\beta}_0+\hat{\beta}_1 x=67.54+0.87x$

(3) 当$x=80$时, y的预测值为 $\hat{y}=67.54+0.87\times 80=137.14$

$L_{yy}=\sum_{t=1}^{9}y_t^2-9\bar{y}^2=76232.38-9\times(90.16)^2=3072.95$

$Q_E=L_{yy}-\hat{\beta}_1 L_{xy}=3072.95-0.87\times 3531.36=0.6668$, $\hat{\sigma}^2=\frac{Q_E}{n-2}=\frac{0.6668}{7}=0.095$

$$T=\frac{y-\hat{y}}{\sqrt{\frac{Q_E}{n-2}\left[1+\frac{1}{n}+\frac{(x-\bar{x})^2}{L_{xx}}\right]}}\sim t_{(n-2)}=t_{(7)}$$

给定置信度$1-\alpha=0.95$时, 令$P\{|T|\le t_{0.025}(7)\}=1-\alpha=0.95$

$\because t_{\frac{\alpha}{2}}(n-2)\cdot\sqrt{\hat{\sigma}^2\left[1+\frac{1}{n}+\frac{(x-\bar{x})^2}{L_{xx}}\right]}=t_{0.025}(7)\cdot\sqrt{0.095\times\left[1+\frac{1}{9}+\frac{(80-26)^2}{4060}\right]}=t_{0.025}(7)\times 0.03=2.3646\times 0.03=0.0709$

$\therefore$ 当$x=80$时, 置信度为95%的y的预测区间为: $\left[\hat{y}-t_{0.025}(7)\cdot\sqrt{\hat{\sigma}^2\left[1+\frac{1}{n}+\frac{(x-\bar{x})^2}{L_{xx}}\right]},\ \hat{y}+t_{0.025}(7)\cdot\sqrt{\hat{\sigma}^2\left[1+\frac{1}{n}+\frac{(x-\bar{x})^2}{L_{xx}}\right]}\right]$

$=[137.0691, 137.2109]$

四.解: (1)

A	0	$\frac{1}{2}$	1
P	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

$X(\frac{\pi}{3})=\frac{1}{2}A+\frac{\sqrt{3}}{2}(1-A)$

$X(\frac{\pi}{3})$的分布律为

$X(\frac{\pi}{3})$	$\frac{1}{2}$	$\frac{1+\sqrt{3}}{4}$	$\frac{\sqrt{3}}{2}$
P	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

当A=0时, $X(\frac{\pi}{3})=\frac{\sqrt{3}}{2}$, $P\{A=0\}=\frac{1}{3}$

当$A=\frac{1}{2}$时, $X(\frac{\pi}{3})=\frac{1}{4}+\frac{\sqrt{3}}{4}$, $P\{A=\frac{1}{2}\}=\frac{1}{3}$

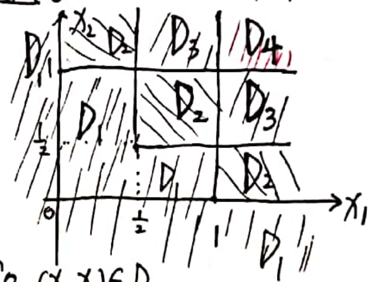
当A=1时, $X(\frac{\pi}{3})=\frac{1}{2}$, $P\{A=1\}=\frac{1}{3}$

一维分布函数: $F(x;\frac{\pi}{3})=\begin{cases}0, & x<\frac{1}{2}\\ \frac{1}{3}, & \frac{1}{2}\le x<\frac{1+\sqrt{3}}{4}\\ \frac{2}{3}, & \frac{1+\sqrt{3}}{4}\le x<\frac{\sqrt{3}}{2}\\ 1, & \frac{\sqrt{3}}{2}\le x\end{cases}$

$F(x_1,x_2;0,\frac{\pi}{2})=P\{A\le x_1, 1-A\le x_2\}$.

~~(A)~~ $X(0)=A$, $X(\frac{\pi}{2})=1-A$, 二维随机变量$(A,1-A)$的分布律为:

$(A,1-A)$	$(0,1)$	$(\frac{1}{2},\frac{1}{2})$	$(1,0)$
P	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$



$F(x_1,x_2;0,\frac{\pi}{2})=P\{A\le x_1, 1-A\le x_2\}=\begin{cases}0, & (x_1,x_2)\in D_1\\ \frac{1}{3}, & (x_1,x_2)\in D_2\\ \frac{2}{3}, & (x_1,x_2)\in D_3\\ 1, & (x_1,x_2)\in D_4\end{cases}$

(2) $m_X(t)=E[X(t)]=E[A\cos t+(1-A)\sin t]=\frac{1}{3}\sin t+\frac{1}{3}[\frac{1}{2}\cos t+\frac{1}{2}\sin t]+\frac{1}{3}\cos t$

$=\frac{1}{2}\sin t+\frac{1}{2}\cos t$

A	0	$\frac{1}{2}$	1
$X(t)$	$\sin t$	$\frac{1}{2}\cos t+\frac{1}{2}\sin t$	$\cos t$
P	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

$R_X(t_1,t_2)=\frac{1}{3}\sin t_1\cdot\sin t_2+\frac{1}{3}\times[(\frac{1}{2}\cos t_1+\frac{1}{2}\sin t_1)\times(\frac{1}{2}\cos t_2+\frac{1}{2}\sin t_2)]+\frac{1}{3}\cos t_1\cdot\cos t_2$

$=\frac{1}{3}\sin t_1\cdot\sin t_2+\frac{1}{3}\times\frac{1}{4}(\cos t_1+\sin t_1)(\cos t_2+\sin t_2)+\frac{1}{3}\cos t_1\cdot\cos t_2$

$=\frac{1}{3}\sin t_1\cdot\sin t_2+\frac{1}{12}\cos t_1\cdot\cos t_2+\frac{1}{12}\cos t_1\cdot\sin t_2+\frac{1}{12}\sin t_1\cdot\cos t_2+\frac{1}{12}\sin t_1\cdot\sin t_2+\frac{1}{3}\cos t_1\cdot\cos t_2$

$=\frac{5}{12}\sin t_1\cdot\sin t_2+\frac{5}{12}\cos t_1\cdot\cos t_2+\frac{1}{12}\cos t_1\cdot\sin t_2+\frac{1}{12}\sin t_1\cdot\cos t_2$

$=\frac{5}{12}\cos(t_1-t_2)+\frac{1}{12}\sin(t_1+t_2)$

注: $\cos(a+b)=\cos a\cos b-\sin a\sin b$　　$\sin(a\pm b)=\sin a\cos b\pm\cos a\sin b$.

$\cos(a-b)=\cos a\cos b+\sin a\sin b$

五.解 (1) $E(A)=\frac{1}{2}$，$D(A)=\frac{1}{12}$ $f(\Phi)=\begin{cases}\frac{1}{2\pi}, & \chi\in(0,2\pi)\\ 0, & \end{cases}$ $E(A^2)=\frac{1}{12}+(\frac{1}{2})^2=\frac{1}{3}$

$$m_X(t)=E[X(t)]=E[A\cos(\omega_0 t+\Phi)]=\frac{1}{2}\int_0^{2\pi}\frac{1}{2\pi}\cos(\omega_0 t+\Phi)d\Phi=0$$

$$R_X(t_1,t_2)=E[X_1(t)\cdot X_2(t)]=E[A\cos(\omega_0 t_1+\Phi)\cdot A\cos(\omega_0 t_2+\Phi)]$$

$$=E[\frac{1}{2}A^2(\cos(\omega_0 t_1+\omega_0 t_2+2\Phi)+\cos(\omega_0 t_1-\omega_0 t_2))]$$

$$=E[\frac{1}{2}A^2\cos(\omega_0 t_1+\omega_0 t_2+2\Phi)]+E[\frac{1}{2}A^2\cdot\cos(\omega_0 t_1-\omega_0 t_2)]$$

$$=\frac{1}{6}\int_0^{2\pi}\frac{1}{2\pi}\cos(\omega_0 t_1+\omega_0 t_2+2\Phi)d\Phi+\frac{1}{6}\int_0^{2\pi}\frac{1}{2\pi}\cos(\omega_0 t_1-\omega_0 t_2)d\Phi$$

$$=\frac{1}{6}\times\frac{1}{2\pi}\times 2\pi\cdot\cos(\omega_0 t_1-\omega_0 t_2)=\frac{1}{6}\cos\omega_0(t_1-t_2)=\frac{1}{6}\cos\omega_0(t_2-t_1)$$

$\therefore R_X(\tau)=\frac{1}{6}\cos(\omega_0\tau)$

又 $\because m_X(t)$ 为常数，$R_X(t_1,t_2)$ 只与 τ 有关

$\therefore X(t)$ 具有平稳性.

(2) $\langle X(t)\rangle=\lim\limits_{T\to\infty}\frac{1}{T}\int_0^T A\cos(\omega_0 t+\Phi)dt=\lim\limits_{T\to\infty}\frac{A}{\omega_0 T}\sin(\omega_0 t+\Phi)\Big|_0^T=\lim\limits_{T\to\infty}\frac{A}{\omega_0 T}\cdot[\sin(\omega_0 T+\Phi)-\sin(\omega_0\cdot 0+\Phi)]=0$

$\because m_X(t)=0$，$\langle X(t)\rangle=0$，$m_X(t)=\langle X(t)\rangle=0$

$\therefore X(t)$ 具有数学期望的各态历经性.

$$\langle X(t)\cdot X(t+\tau)\rangle=\lim_{T\to\infty}\frac{1}{T}\int_0^T A^2\cos(\omega_0 t+\Phi)\cdot\cos(\omega_0(t+\tau)+\Phi)dt$$

$$=\lim_{T\to\infty}\frac{A^2}{T}[\int_0^T\cos(\omega_0(2t+\tau)+2\Phi)dt+\int_0^T\cos(\omega_0\tau)dt=\lim_{T\to\infty}\frac{A^2}{T}\cdot T\cdot\cos(\omega_0\tau)$$

$$=A^2\cdot\cos(\omega_0\tau)\neq R_X(\tau)$$

$\therefore X(t)$ 不具有相关函数的各态历经性.

六.解: (1) ∵ $R_X(\tau)=1+\frac{1}{2}\cos\tau$, $S_X(\omega)=\int_{-\infty}^{+\infty}e^{-i\tau\omega}\cdot R_X(\tau)d\tau$

∴ $S_X(\omega)=\int_{-\infty}^{+\infty}e^{-i\tau\omega}\cdot[1+\frac{1}{2}\cos\tau]d\tau=2\pi\delta(\omega)+\frac{\pi}{2}[\delta(\omega+1)+\delta(\omega-1)]$

频率响应函数 $H(i\omega)=\frac{\alpha}{i\omega+\alpha}$, $\alpha=\frac{1}{RC}$

又 $S_Y(\omega)=|H(i\omega)|^2\cdot S_X(\omega)=\frac{\alpha^2}{\omega^2+\alpha^2}\cdot[2\pi\delta(\omega)+\frac{\pi}{2}\delta(\omega+1)+\frac{\pi}{2}\delta(\omega-1)]$

$=2\pi\delta(\omega)+\frac{\alpha^2}{\alpha^2+1}\cdot\frac{\pi}{2}[\delta(\omega+1)+\delta(\omega-1)]$

(2) $R_Y(\tau)=\mathcal{F}^{-1}[S_Y(\omega)]=1+\frac{\alpha^2}{2(\alpha^2+1)}\cdot\cos\tau$. $\alpha=\frac{1}{RC}$

七.解: (1)
$$P^2=\begin{bmatrix}0&\frac13&\frac13&\frac13\\\frac13&0&\frac13&\frac13\\\frac13&\frac13&0&\frac13\\\frac13&\frac13&\frac13&0\end{bmatrix}\cdot\begin{bmatrix}0&\frac13&\frac13&\frac13\\\frac13&0&\frac13&\frac13\\\frac13&\frac13&0&\frac13\\\frac13&\frac13&\frac13&0\end{bmatrix}=\begin{bmatrix}\frac13&\frac29&\frac29&\frac29\\\frac29&\frac13&\frac29&\frac29\\\frac29&\frac29&\frac13&\frac29\\\frac29&\frac29&\frac29&\frac13\end{bmatrix}$$

由于二步转移概率矩阵$P(2)$的元素全大于零，所以此马氏链是遍历的.

$$\begin{bmatrix}\frac13-1&\frac29&\frac29&\frac29\\\frac29&\frac13-1&\frac29&\frac29\\\frac29&\frac29&\frac13-1&\frac29\\\frac29&\frac29&\frac29&\frac13-1\end{bmatrix}\cdot\begin{bmatrix}p_1\\p_2\\p_3\\p_4\end{bmatrix}=0\Rightarrow\begin{cases}-\frac23p_1+\frac29p_2+\frac29p_3+\frac29p_4=0 & ①\\\frac29p_1-\frac23p_2+\frac29p_3+\frac29p_4=0 & ②\\\frac29p_1+\frac29p_2-\frac23p_3+\frac29p_4=0 & ③\\\frac29p_1+\frac29p_2+\frac29p_3-\frac23p_4=0 & ④\end{cases}\Rightarrow\begin{cases}③-①\Rightarrow p_1=p_3\\④-②\Rightarrow p_2=p_4\end{cases}$$

满足$p_1+p_2+p_3+p_4=1$, $p_j>0(j=1,2,3,4)$的解为: $p_1=p_2=p_3=p_4=\frac14$

(2) 绝对概率: $P\{X_2=3\}=p_3^{(2)}=\sum_{i=1}^{4}p_i^{(0)}\cdot p_{i3}(2)=\sum_{i=1}^{4}P\{X_0=i\}\cdot p_{i3}(2)$

$=\frac14\cdot\sum_{i=1}^{4}p_{i3}(2)=\frac14[\frac29+\frac29+\frac13+\frac29]=\frac14\times1=\frac14$

多维概率: $P\{X_1=2,X_3=1,X_5=4\}=\sum_{i=1}^{4}p_i^{(0)}\cdot p_{i2}(1)\cdot p_{21}(3-1)\cdot p_{14}(5-3)$

$=p_i^{(0)}\cdot p_{21}(2)\cdot p_{14}(2)\cdot\sum_{i=1}^{4}p_{i2}(1)=\frac14\times\frac29\times\frac29\times(\frac13+0+\frac13+\frac13)=\frac{1}{81}$

注:

$P\{X_{(\underset{\downarrow 2}{n})}=\underset{\downarrow 3}{j}\}=\underset{\downarrow 3}{p_j}^{(n)}$, $\underset{\downarrow 3}{p_j}^{(n)}=\sum_{i\in E}p_i^{(0)}\cdot \underset{\downarrow 3}{p_{ij}}(n)$ ⇒ 绝对概率

$P\{X(\underset{\downarrow 1}{n_1})=\underset{\downarrow 2}{i_1},X(\underset{\downarrow 3}{n_2})=\underset{\downarrow 1}{i_2},\dots,X(\underset{\downarrow 5}{n_m})=\underset{\downarrow 4}{i_m}\}=\sum_{i\in E}p_i^{(0)}\cdot p_{i\underset{\downarrow 2}{i_1}}(\underset{\downarrow 1}{n_1})\cdot p_{\underset{\downarrow 2}{i_1}\underset{\downarrow 1}{i_2}}(\underbrace{n_2-n_1}_{3-1})\cdots p_{\underset{\downarrow 1}{i_{m-1}}\underset{\downarrow 4}{i_m}}(\underbrace{n_m-n_{m-1}}_{5-3})$

$=p_i^{(0)}\cdot p_{i_1i_2}(n_2-n_1)\cdots p_{i_{m-1}i_m}(n_m-n_{m-1})\cdot\sum_{i\in E}p_{ii_1}(n_1)$ ⇒ 多维概率