Using Algebraic Geometry

With 0 Figures

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Contents

Preface

At the time of writing this, I am starting my PhD at The Ohio State University. Currently a large part of my interests in algebra are about algorithms as they relate to polynomials and algebraic geometry. I've been doing a bunch of problems from *Ideals, Varieties, and Algorithms* [CLO15]. However, it seems that *Using Algebraic Geometry* [CLO05] moves through the material faster as it assumes you know more algebra. So I've moved onto working through this books as well as trying to comprehend Sturmfel's *Algorithms in Invariant Theory* [Str08].

Chapter 1

Introduction

1.1 Polynomials and Ideals

Exercise 1.1 (CLO05 1.1.1):

1. Show that $x^2 \in \langle x - y^2, xy \rangle$ in $k[x, y]$.
2. Show that $\langle x - y^2, xy, y^2 \rangle = \langle x, y^2 \rangle$.
3. Is $\langle x - y^2, xy \rangle = \langle x^2, xy \rangle$? Why or why not?

Proof:

1. We have that $x(x - y^2) + y(xy) = x^2 - xy^2 + xy^2 = x^2$.
2. It suffices to check for generators. We have that $x + (-1)(y^2) = x - y^2$, $y(x) = xy$, and $y^2 = y^2$ showing that $\langle x - y^2, xy, y^2 \rangle \subseteq \langle x, y^2 \rangle$. Then $x - y^2 + y^2 = x$ and $y^2 = y^2$ shows the reverse containment and overall the ideals are equal.
3. We already know from 1. that x^2 lives in $\langle x - y^2, xy \rangle$. Since $xy = xy$, we overall have that $\langle x^2, xy \rangle \subseteq \langle x - y^2, xy \rangle$. It remains to check if $x - y^2 \in \langle x^2, xy \rangle$. However, notice that every element of $\langle x^2, xy \rangle$ is divisible by x while $x - y^2$ is clearly not divisible by x. Thus $x - y^2 \notin \langle x^2, xy \rangle$ and the two ideals are not equal.

$\square$

Bibliography

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