



# **Performance of Processor**

# Processor Performance - Terminologies

- Clock Rate CR
- Cycle Count CC
- Cycle Time CT
- Cycles Per Instructions  
CPI

# Machine Clock Rate

- Clock Rate (CR) (MHz, GHz) is inverse of Clock Cycle (CC) time (clock period)

$$CC = 1 / CR$$



10 nsec clock cycle => 100 MHz clock rate

5 nsec clock cycle => 200 MHz clock rate

2 nsec clock cycle => 500 MHz clock rate

1 nsec clock cycle => 1 GHz clock rate

500 psec clock cycle => 2 GHz clock rate

250 psec clock cycle => 4 GHz clock rate

200 psec clock cycle => 5 GHz clock rate

Clock Rate (CR) is measured in MHz, GHz

# Computer Clock

- The clock rate is the inverse of the clock cycle time.
- ie, Clock Rate =  $1/\text{Clock Cycle Time}$
- The clock cycle time is the amount of time for one clock period to elapse (e.g. 5 ns).
- Question
- If a computer has a clock cycle time of 5 ns, What is the clock rate?

**your answer ?**

# Computer Clock

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- Clock Rate =  $1/\text{Clock Cycle Time}$
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- Question
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$$\frac{1}{5 \times 10^{-9} \text{ sec}} = 200 \text{ MHz}$$

# Processor Performance Metrics

- **Execution time:** It is the time taken to finish a task
- **Response time:** the time between the start and the completion of a task (in time units)
- **Throughput:** the total amount of tasks done in a given time period (in number of tasks per unit of time)
- **Example: Car assembly factory:**
  - 4 hours to produce a car (response time)
  - 6 cars per hour produced (throughput)

# Defining (Speed) Performance

- Normally interested in reducing
  - Response time ( execution time) – the time between the start and the completion of a task
  - **Important to individual users**
  - Thus, to maximize performance, need to minimize execution time

$$\text{performance}_x = 1 / \text{execution\_time}_x$$

If X is n times faster than Y, then

$$\frac{\text{performance}_x}{\text{performance}_y} = \frac{\text{execution\_time}_y}{\text{execution\_time}_x} = n$$

# How many times faster is machine A?

- Problem:
- machine A runs a program in 20 seconds
- machine B runs the same program in 25 seconds
- **how many times faster is machine A than machine B??**

If X is n times faster than Y, then

**your answer ?**

$$\frac{\text{performance}_x}{\text{performance}_y} = \frac{\text{execution\_time}_y}{\text{execution\_time}_x} = n$$



# How many times faster is machine A?

- Problem:
- machine A runs a program in 20 seconds
- machine B runs the same program in 25 seconds
- **how many times faster is machine A?**

$$\frac{25}{20} = 1.25$$

# Performance Factors

- Want to distinguish elapsed time and the time spent on our task
- CPU execution time (CPU time) – time the CPU spends working on a task
  - Does not include time waiting for I/O or running other programs

$$\begin{array}{l} \text{CPU execution time} \\ \text{for a program} \end{array} = \begin{array}{l} \# \text{ CPU clock cycles} \\ \text{for a program} \end{array} \times \text{clock cycle time}$$

or

$$\begin{array}{l} \text{CPU execution time} \\ \text{for a program} \end{array} = \frac{\# \text{ CPU clock cycles for a program}}{\text{clock rate}}$$

- Can improve performance by reducing either the length of the clock cycle or the number of clock cycles required for a program

# Performance Equation

- Our basic performance equation is then

$$\text{CPU time} = \text{Instruction\_count} \times \text{CPI} \times \text{clock\_cycle}$$

or

$$\text{CPU time} = \frac{\text{Instruction\_count} \times \text{CPI}}{\text{clock\_rate}}$$

# Cycles Per Instruction (CPI)

- Computing the CPI is done by looking at the different types of instructions and their individual cycle counts

$$\text{CPI} = \sum_{i=1}^n (\text{CPI}_i \times \text{IC}_i)$$

- Where  $\text{IC}_i$  is the count (percentage) of the number of instructions of class  $i$  executed
- $\text{CPI}_i$  is the (average) number of clock cycles per instruction for that instruction class
- $n$  is the number of instruction classes

# Calculating CPI

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The table below indicates frequency of all instruction types executed in a “typical” program and, from the reference manual, we are provided with a number of cycles per instruction for each type.

Instruction Type	Frequency	Cycles
ALU instruction	50%	4
Load instruction	30%	5
Store instruction	5%	4
Branch instruction	15%	2

$$\text{CPI} = 0.5 \cdot 4 + 0.3 \cdot 5 + 0.05 \cdot 4 + 0.15 \cdot 2 = 4 \text{ cycles/instruction}$$

# MIPS

- A common measure of performance for a processor is the rate at which instructions are executed, expressed as Millions of Instructions Per Second (MIPS), referred to as the **MIPS rate**.
- We can express the MIPS rate in terms of the clock rate and CPI as follows:

$$\text{MIPS rate} = \frac{I_c}{T \times 10^6} = \frac{f}{CPI \times 10^6}$$

# MIPS Example

- Consider the execution of a program which results in the execution of 2 million instructions on a 400-MHz processor.
- The instruction mix and the CPI for each instruction type are given below based on the result of a program trace experiment:

Instruction Type	CPI	Instruction Mix
Arithmetic and logic	1	60%
Load/store with cache hit	2	18%
Branch	4	12%
Memory reference with cache miss	8	10%

$$CPI = 0.6 + (2 \times 0.18) + (4 \times 0.12) + (8 \times 0.1) = 2.24.$$

$$\text{MIPS rate is } (400 \times 10^6) / (2.24 \times 10^6) \approx 178.$$

## MFLOPS

- Floating point performance is expressed as millions of floating-point operations per second (MFLOPS), defined as follows:

$$\text{MFLOPS rate} = \frac{\text{Number of executed floating-point operations in a program}}{\text{Execution time} \times 10^6}$$



# Problem 1

- Assume that # of instructions in the program is 1,000,000,000. Suppose we have two implementations of the same instruction set architecture (ISA). For some program,
  - Machine A has a clock cycle time of 10 ns. and a CPI of 2.0
  - Machine B has a clock cycle time of 20 ns. and a CPI of 1.2 □
- **Which machine is faster for this program, and by how much?**

## Problem 2

- CPU clock rate is 1 MHz [Program takes 45 million cycles to execute] What's the CPU time?
- CPU clock rate is 500 MHz [Program takes 45 million cycles to execute] What's the CPU time?

# Problem 3

- You are on the design team for a new processor. The clock of the processor runs at 200 MHz. The following table gives instruction frequencies for Benchmark B, as well as how many cycles the instructions take, for the different classes of instructions. Calculate the CPI and MIPS.

Instruction Type	Frequency	Cycles
Loads & Stores	30%	6 cycles
Arithmetic Instructions	50%	4 cycles
All Others	20%	3 cycles

# Problem 4

Question: Calculate CPI and MIPS for a CPU with 200MHz frequency which is executing a benchmark program with following instruction mix

Instruction Category	Percentage of occurrence	No. of cycles per instruction
• ALU	38	1
• Load & Store	15	3
• Branch	42	4
• Others	5	5

# Problem 5

- Suppose a program (or a program task) takes 1 billion instructions to execute on a processor running at 2 GHz. Suppose also that 50% of the instructions execute in 3 clock cycles, 30% execute in 4 clock cycles, and 20% execute in 5 clock cycles. **What is the execution time for the program or task?**
- Note: We have the instruction count:  $10^9$  instructions. The clock time can be computed quickly from the clock rate to be  $0.5 \times 10^{-9}$  seconds. So we only need to compute clocks per instruction as an effective value:

# Problem 6

A benchmark program is run on a 40 MHz processor. The executed program consists of 100,000 instruction executions, with the following instruction mix and clock cycle count:

Instruction Type	Instruction Count	Cycles per Instruction
Integer arithmetic	45.000	1
Data transfer	32.000	2
Floating point	15.000	2
Control transfer	8000	2

Determine the effective *CPI*, MIPS rate, and execution time for this program.

# Problem 7

- We want to compare the computers R1 and R2, which differ that R1 has the machine instructions for the floating point operations, while R2 has not (FP operations are implemented in the software using several non-FP instructions). Both computers have a clock frequency of 400 MHz. In both we perform the same program, which has the following mixture of commands:
  - a) Calculate the MIPS for the computers R1 and R2.
  - b) Calculate the CPU program execution time on the computers R1 and R2, if there are 12000 instructions in the program?

Type the command	Dynamic Share of instructions in program ( $p_i$ )	Instruction duration (Number of clock periods CPI)	
		R1	R2
FP addition	16%	6	20
FP multiplication	10%	8	32
FP division	8%	10	66
Non - FP instructions	66%	3	3

## Problem 8

- The clock of the processor runs at 200 MHz with 4.4 Cycles per instruction. Compute the MIPS processor speed for the benchmark in millions of instructions per second?



# Problem 9

- Suppose that we are considering developing a parallel program to improve on an existing sequential program and that we determine that 20% of the execution time of the sequential program is spent in inherently sequential code. (We have to inspect the code to determine this.) The remaining code can be parallelized, although we do not as yet know how many processors would be optimal. What is the maximum possible speedup that could be obtained if we were to develop a parallel version that used ten processors?

# Problem 10

- Suppose that we know that fraction of inherently sequential computation is 0.12 in the problem of interest. What is the least number of processors that we need to use to obtain a speedup of 5.0?



# Problem 11

# QUESTION

Our favorite program runs in 10 seconds on computer A, which has a 2 GHz clock. We are trying to help a computer designer build a computer, B, which will run this program in 6 seconds. The designer has determined that a substantial increase in the clock rate is possible, but this increase will affect the rest of the CPU design, causing computer B to require 1.2 times as many clock cycles as computer A for this program. What clock rate should we tell the designer to target?

Let's first find the number of clock cycles required for the program on A:

$$\text{CPU time}_A = \frac{\text{CPU clock cycles}_A}{\text{Clock rate}_A}$$

$$10 \text{ seconds} = \frac{\text{CPU clock cycles}_A}{2 \times 10^9 \frac{\text{cycles}}{\text{second}}}$$

$$\text{CPU clock cycles}_A = 10 \text{ seconds} \times 2 \times 10^9 \frac{\text{cycles}}{\text{second}} = 20 \times 10^9 \text{ cycles}$$

CPU time for B can be found using this equation:

$$\text{CPU time}_B = \frac{1.2 \times \text{CPU clock cycles}_A}{\text{Clock rate}_B}$$

$$6 \text{ seconds} = \frac{1.2 \times 20 \times 10^9 \text{ cycles}}{\text{Clock rate}_B}$$

$$\text{Clock rate}_B = \frac{1.2 \times 20 \times 10^9 \text{ cycles}}{6 \text{ seconds}} = \frac{0.2 \times 20 \times 10^9 \text{ cycles}}{\text{second}} = \frac{4 \times 10^9 \text{ cycles}}{\text{second}} = 4 \text{ GHz}$$

To run the program in 6 seconds, B must have twice the clock rate of A.



# Problem 12

Suppose we have two implementations of the same instruction set architecture. Computer A has a clock cycle time of 250 ps and a CPI of 2.0 for some program, and computer B has a clock cycle time of 500 ps and a CPI of 1.2 for the same program. Which computer is faster for this program and by how much?

# QUESTION

We know that each computer executes the same number of instructions for the program; let's call this number  $I$ . First, find the number of processor clock cycles for each computer:

$$\text{CPU clock cycles}_A = I \times 2.0$$

$$\text{CPU clock cycles}_B = I \times 1.2$$

Now we can compute the CPU time for each computer:

$$\begin{aligned}\text{CPU time}_A &= \text{CPU clock cycles}_A \times \text{Clock cycle time} \\ &= I \times 2.0 \times 250 \text{ ps} = 500 \times I \text{ ps}\end{aligned}$$

Likewise, for B:

$$\text{CPU time}_B = I \times 1.2 \times 500 \text{ ps} = 600 \times I \text{ ps}$$

Clearly, computer A is faster. The amount faster is given by the ratio of the execution times:

$$\frac{\text{CPU performance}_A}{\text{CPU performance}_B} = \frac{\text{Execution time}_B}{\text{Execution time}_A} = \frac{600 \times I \text{ ps}}{500 \times I \text{ ps}} = 1.2$$

We can conclude that computer A is 1.2 times as fast as computer B for this program.





# Problem 13

# QUESTION

A compiler designer is trying to decide between two code sequences for a particular computer. The hardware designers have supplied the following facts:

	CPI for each instruction class		
	A	B	C
CPI	1	2	3

For a particular high-level language statement, the compiler writer is considering two code sequences that require the following instruction counts:

Code sequence	Instruction counts for each instruction class		
	A	B	C
1	2	1	2
2	4	1	1

Which code sequence executes the most instructions? Which will be faster? What is the CPI for each sequence?

Sequence 1 executes  $2 + 1 + 2 = 5$  instructions. Sequence 2 executes  $4 + 1 + 1 = 6$  instructions. Therefore, sequence 1 executes fewer instructions.

We can use the equation for CPU clock cycles based on instruction count and CPI to find the total number of clock cycles for each sequence:

$$\text{CPU clock cycles} = \sum_{i=1}^n (\text{CPI}_i \times C_i)$$

This yields

$$\text{CPU clock cycles}_1 = (2 \times 1) + (1 \times 2) + (2 \times 3) = 2 + 2 + 6 = 10 \text{ cycles}$$

$$\text{CPU clock cycles}_2 = (4 \times 1) + (1 \times 2) + (1 \times 3) = 4 + 2 + 3 = 9 \text{ cycles}$$

So code sequence 2 is faster, even though it executes one extra instruction. Since code sequence 2 takes fewer overall clock cycles but has more instructions, it must have a lower CPI. The CPI values can be computed by

$$\text{CPI} = \frac{\text{CPU clock cycles}}{\text{Instruction count}}$$

$$\text{CPI}_1 = \frac{\text{CPU clock cycles}_1}{\text{Instruction count}_1} = \frac{10}{5} = 2.0$$

$$\text{CPI}_2 = \frac{\text{CPU clock cycles}_2}{\text{Instruction count}_2} = \frac{9}{6} = 1.5$$



# Problem 14

# QUESTION

Consider three different processors P1, P2, and P3 executing the same instruction set. P1 has a 3 GHz clock rate and a CPI of 1.5. P2 has a 2.5 GHz clock rate and a CPI of 1.0. P3 has a 4.0 GHz clock rate and has a CPI of 2.2.

- Which processor has the highest performance expressed in instructions per second?
- If the processors each execute a program in 10 seconds, find the number of cycles and the number of instructions.
- We are trying to reduce the execution time by 30% but this leads to an increase of 20% in the CPI. What clock rate should we have to get this time reduction?

(a)

We must find instructions per second, which we will denote by IPS. We must find the formula which will use only the clock rate and CPI of a processor. From the formula in the book, we have that

$$\text{CPU time} = \frac{\text{instructions} \times \text{CPI}}{\text{clock rate}}$$

Therefore,

$$\frac{\text{instructions}}{\text{CPU time}} = \frac{\text{clock rate}}{\text{CPI}}$$

Now notice that  $\text{IPS} = \frac{\text{instructions}}{\text{CPU time}}$ , so

$$\text{IPS} = \frac{\text{clock rate}}{\text{CPI}}$$

Therefore, since  $1 \text{ GHz} = 10^9 \text{ Hz}$ ,

$$\begin{aligned}\text{IPS}_1 &= \frac{\text{clock rate}_1}{\text{CPI}_1} = \frac{3 \text{ GHz}}{1.5} = 2 \times 10^9 \\ \text{IPS}_2 &= \frac{\text{clock rate}_2}{\text{CPI}_2} = \frac{2.5 \text{ GHz}}{1} = 2.5 \times 10^9 \\ \text{IPS}_3 &= \frac{\text{clock rate}_3}{\text{CPI}_3} = \frac{4 \text{ GHz}}{2.2} = 1.82 \times 10^9\end{aligned}$$

So, [processor 2](#) has the highest performance in instructions per second.

(b)

Here CPU time = 10 seconds. From (a) we already know that

$$\text{instructions} = \text{IPS} \times \text{CPU time}$$

Also, we have that

$$\text{CPU time} = \frac{\text{clock cycles}}{\text{clock rate}}$$

Therefore,

$$\text{clock cycles} = \text{CPU time} \times \text{clock rate}$$

Also recall that  $1 \text{ GHz} = 10^9 \text{ Hz}$ . Now we find both informations for every processor:

$$\text{instructions}_1 = \text{IPS}_1 \times \text{CPU time}_1 = 2 \times 10^9 \times 10 = 2 \times 10^{10}$$

$$\text{clock cycles}_1 = \text{CPU time}_1 \times \text{clock rate}_1 = 10 \times 3 \text{ GHz} = 10 \times 3 \times 10^9 = 3 \times 10^{10}$$

$$\text{instructions}_2 = \text{IPS}_2 \times \text{CPU time}_2 = 2.5 \times 10^9 \times 10 = 2.5 \times 10^{10}$$

$$\text{clock cycles}_2 = \text{CPU time}_2 \times \text{clock rate}_2 = 10 \times 2.5 \text{ GHz} = 10 \times 2.5 \times 10^9 = 2.5 \times 10^{10}$$

$$\text{instructions}_3 = \text{IPS}_3 \times \text{CPU time}_3 = 1.82 \times 10^9 \times 10 = 1.82 \times 10^{10}$$

$$\text{clock cycles}_3 = \text{CPU time}_3 \times \text{clock rate}_3 = 10 \times 4 \text{ GHz} = 10 \times 4 \times 10^9 = 4 \times 10^{10}$$

We can also write this in the form of a table so that it is easier to read:

Processor	Instructions	Clock cycles
1	$2 \times 10^{10}$	$3 \times 10^{10}$
2	$2.5 \times 10^{10}$	$2.5 \times 10^{10}$
3	$1.82 \times 10^{10}$	$4 \times 10^{10}$



(c)

First of all, we have that

$$\text{Execution time} = \frac{\text{clock cycles}}{\text{clock rate}}$$

Also,

$$\text{clock cycles} = \text{instructions} \times \text{CPI}$$

Therefore,

$$\text{Execution time} = \frac{\text{instructions} \times \text{CPI}}{\text{clock rate}}$$

Now, we want to find  $\text{clock rate}_{\text{new}}$  such that

$$\text{Execution time}_{\text{new}} = 0.7 \text{ Execution time}_{\text{old}},$$

which yields

$$\frac{\text{instructions}_{\text{new}} \times \text{CPI}_{\text{new}}}{\text{clock rate}_{\text{new}}} = 0.7 \cdot \frac{\text{instructions}_{\text{old}} \times \text{CPI}_{\text{old}}}{\text{clock rate}_{\text{old}}}$$

Notice that  $\text{instructions}_{\text{new}} = \text{instructions}_{\text{old}}$ , so

$$\frac{\text{CPI}_{\text{new}}}{\text{clock rate}_{\text{new}}} = 0.7 \cdot \frac{\text{CPI}_{\text{old}}}{\text{clock rate}_{\text{old}}}$$

Now use that  $\text{CPI}_{\text{new}} = 1.2 \text{ CPI}_{\text{old}}$  to get

$$\frac{1.2}{\text{clock rate}_{\text{new}}} = \frac{0.7}{\text{clock rate}_{\text{old}}}$$

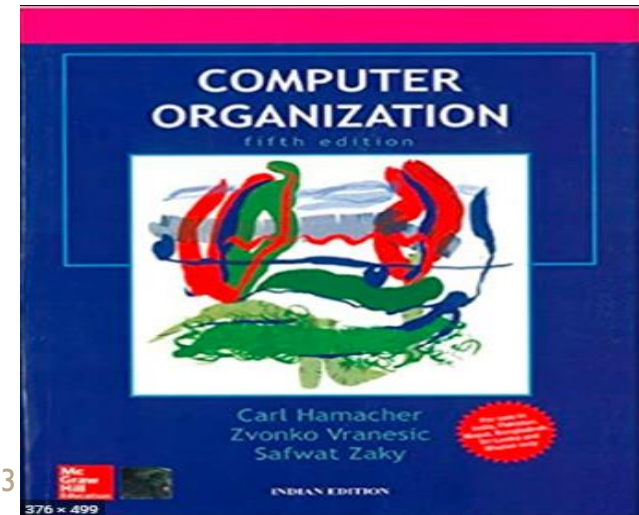
Finally, rearranging we get that

$$\text{clock rate}_{\text{new}} = \frac{1.2}{0.7} \times \text{clock rate}_{\text{old}} = 1.71 \times \text{clock rate}_{\text{old}}$$

Therefore, the clock rate must increase by approximately 71%.

# Text Book(s)

- David A. Patterson and . John L. Hennessy  
—Computer Organization and Design-The  
Hardware/Software Interfacell 5th edition,  
Morgan Kaufmann, 2011.
- Carl Hamacher, Zvonko Vranesic, Safwat Zaky,  
Computer organization, Mc Graw Hill, Fifth  
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6 March 2023

# Reference Books

- W. Stallings, Computer organization and architecture, Prentice-Hall, 8th edition, 2009

