

Equation for Success: Mathematical Modeling in Group Formation

Quantitative Management Modeling (BA-64018-005)

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Abstract

This assignment delves into the art of mathematical optimization to strategically form student groups with the overarching aim of maximizing success in a collaborative class project. With a class of 15 students organized into 5 groups of 3, the assignment identifies three key factors influencing project success. Utilizing R's randomization function, realistic data is generated for these factors, and a mathematical model is formulated. The model's objective is to enhance the potential success of each group, considering the constraints of group size and the identified success-affecting factors (GPA, Technical Skill, Teamwork). The project culminates in the implementation of the model in R, providing valuable insights into the optimal composition of groups and offering recommendations for future endeavors in group formation. This study bridges the gap between mathematical precision and the nuanced dynamics of collaborative success, showcasing the potential of optimization techniques in educational settings.

Objective

The primary goal of the project is to leverage mathematical optimization techniques such as linear programming to strategically form groups of students for a class project, with the overarching objective of maximizing the potential success of each group. This involves dividing 15 students into 5 groups, each comprising 3 members. The project aims to go beyond random grouping by considering three identified factors that influence student contributions to project success, which are GPA, Technical Skill, and Teamwork. Through the formulation and solution of a mathematical model in R, the project seeks to find the optimal composition of groups based on these factors, ultimately providing insights and recommendations for creating effective and successful student collaborations in educational settings.

Data and Variables

The attributes of the students that play a role in their grouping for the project, influencing the composition of different teams are described as follows,

- ✓ **GPA:** The average GPA so far shows how serious a student is about their studies and how well they might work on the group project. It also indicates their performance as a graduate student. GPA is a crucial factor for project success because it tells us about the student's overall quality.

- ✓ **Technical Skill:** Technical skill refers to a person's proficiency and capability in applying specific knowledge and expertise to perform tasks or solve problems within a particular field. In the context of a project, technical skills become a critical factor for success. Individuals with strong technical skills bring a level of competence and efficiency to the project, enabling them to contribute effectively to its completion. In a collaborative setting, having team members with diverse and complementary technical skills can enhance the overall capability of the group. This diversity ensures that the team is well-equipped to tackle various aspects of the project, from problem-solving to implementation. Moreover, a balance of technical skills within the group can mitigate potential gaps or challenges that may arise during the project's execution.
- ✓ **Teamwork:** Teamwork is a crucial element influencing the success of a project. It refers to how well individuals collaborate, communicate, and coordinate their efforts towards a common goal within a group setting. In the context of a project, effective teamwork is often a determining factor for achieving optimal outcomes.

No of students: We have 15 students represents by S_j , where $j = 1$ to 15

No of Groups: We have 5 groups representing by G_i , where $i = 1$ to 5. Also, each group has 3 students.

Data Collection / Generation Process

The data was created using R's randomization function ("runif") for three specific factors. These factors, namely GPA, Technical Skill, and Teamwork, were identified as influencing student contributions to project success. The weight assigned to each factor in contributing to success is as follows: GPA 40%, Technical Skill 30%, and Teamwork 30%. To create the objective function, we computed the weighted average of these three factors. Additionally, we calculated the average for each factor, which will be used later when formulating constraints. The table below presents the data points for GPA, Technical Skill, and Teamwork, along with the weighted average for each student and the overall average.

```
GPA <- round(runif(num_rows, min = 2.5, max = 4.0),2)
gpa_data <- data.frame(GPA = GPA)

Technical_skill_scale <- round(runif(num_rows, min = 4, max = 10),0)
gpa_data <- data.frame(Technical_skill_scale = Technical_skill_scale)

Teamwork_scale <- round(runif(num_rows, min = 4, max = 10),0)
gpa_data <- data.frame(Teamwork_scale = Teamwork_scale)
```

Figure 1: Randomization

Table 1: Random Data for all 3 factors

Student	GPA Value	Technical skill scale Value	Teamwork scale value	Weighted Avg
1	2.93	9	10	6.87
2	3.68	5	9	5.67
3	3.11	4	8	4.84
4	3.82	6	9	6.03
5	3.91	10	4	5.76
6	2.57	9	7	5.83
7	3.29	8	9	6.42
8	3.84	8	5	5.44
9	3.33	10	6	6.13
10	3.18	8	5	5.17
11	3.94	8	5	5.48
12	3.18	7	6	5.17
13	3.52	8	6	5.61
14	3.36	6	6	4.94
15	2.65	5	5	4.06
Avg	3.35	7	7	-

Model Building

The model's goal is to enhance the likelihood of success for each group in a class project, considering three factors. Therefore, we utilized the weighted average value to construct the objective function. The objective function is displayed below.

$$\begin{aligned}
 \text{MaxZ} = & 6.87G_1S_1 + 5.67G_1S_2 + 4.84G_1S_3 + 6.03G_1S_4 + 5.76G_1S_5 + 5.83G_1S_6 + 6.42G_1S_7 \\
 & + 5.44G_1S_8 + 6.13G_1S_9 + 5.17G_1S_{10} + 5.48G_1S_{11} + 5.17G_1S_{12} + 5.61G_1S_{13} \\
 & + 4.94G_1S_{14} + 4.06G_1S_{15} \\
 & + 6.87G_2S_1 + 5.67G_2S_2 + 4.84G_2S_3 + 6.03G_2S_4 + 5.76G_2S_5 + 5.83G_2S_6 + 6.42G_2S_7 \\
 & + 5.44G_2S_8 + 6.13G_2S_9 + 5.17G_2S_{10} + 5.48G_2S_{11} + 5.17G_2S_{12} + 5.61G_2S_{13} \\
 & + 4.94G_2S_{14} + 4.06G_2S_{15} \\
 & + 6.87G_3S_1 + 5.67G_3S_2 + 4.84G_3S_3 + 6.03G_3S_4 + 5.76G_3S_5 + 5.83G_3S_6 + 6.42G_3S_7 \\
 & + 5.44G_3S_8 + 6.13G_3S_9 + 5.17G_3S_{10} + 5.48G_3S_{11} + 5.17G_3S_{12} + 5.61G_3S_{13} \\
 & + 4.94G_3S_{14} + 4.06G_3S_{15} \\
 & + 6.87G_4S_1 + 5.67G_4S_2 + 4.84G_4S_3 + 6.03G_4S_4 + 5.76G_4S_5 + 5.83G_4S_6 + 6.42G_4S_7 \\
 & + 5.44G_4S_8 + 6.13G_4S_9 + 5.17G_4S_{10} + 5.48G_4S_{11} + 5.17G_4S_{12} + 5.61G_4S_{13} \\
 & + 4.94G_4S_{14} + 4.06G_4S_{15} \\
 & + 6.87G_5S_1 + 5.67G_5S_2 + 4.84G_5S_3 + 6.03G_5S_4 + 5.76G_5S_5 + 5.83G_5S_6 + 6.42G_5S_7 \\
 & + 5.44G_5S_8 + 6.13G_5S_9 + 5.17G_5S_{10} + 5.48G_5S_{11} + 5.17G_5S_{12} + 5.61G_5S_{13} \\
 & + 4.94G_5S_{14} + 4.06G_5S_{15}
 \end{aligned}$$

In addition, constraints have been formulated based on all the decision variables. These constraints are categorized into Group Constraint, Student constraint, Factor 1 (GPA) constraint, Factor 2 (technical skill) constraint and Factor 3 (teamwork) constraint. Group constraint containing five constraints, and fifteen constraints for individual students. The conditions include ensuring each group consists of three students and that each student is assigned to only one group. Additionally, factor constraints have been established using the average of each factor, resulting in five constraints for each factor.

Group Constraints: -

Group1_Students:

$$G_1S_1 + G_1S_2 + G_1S_3 + G_1S_4 + G_1S_5 + G_1S_6 + G_1S_7 + G_1S_8 + G_1S_9 + G_1S_{10} + G_1S_{11} + G_1S_{12} + G_1S_{13} + G_1S_{14} + G_1S_{15} = 3$$

Group2_Students:

$$G_2S_1 + G_2S_2 + G_2S_3 + G_2S_4 + G_2S_5 + G_2S_6 + G_2S_7 + G_2S_8 + G_2S_9 + G_2S_{10} + G_2S_{11} + G_2S_{12} + G_2S_{13} + G_2S_{14} + G_2S_{15} = 3$$

Group3_Students:

$$G_3S_1 + G_3S_2 + G_3S_3 + G_3S_4 + G_3S_5 + G_3S_6 + G_3S_7 + G_3S_8 + G_3S_9 + G_3S_{10} + G_3S_{11} + G_3S_{12} + G_3S_{13} + G_3S_{14} + G_3S_{15} = 3$$

Group4_Students:

$$G_4S_1 + G_4S_2 + G_4S_3 + G_4S_4 + G_4S_5 + G_4S_6 + G_4S_7 + G_4S_8 + G_4S_9 + G_4S_{10} + G_4S_{11} + G_4S_{12} + G_4S_{13} + G_4S_{14} + G_4S_{15} = 3$$

Group5_Students:

$$G_5S_1 + G_5S_2 + G_5S_3 + G_5S_4 + G_5S_5 + G_5S_6 + G_5S_7 + G_5S_8 + G_5S_9 + G_5S_{10} + G_5S_{11} + G_5S_{12} + G_5S_{13} + G_5S_{14} + G_5S_{15} = 3$$

Student Constraints: -

Student1:

$$G_1S_1 + G_2S_1 + G_3S_1 + G_4S_1 + G_5S_1 = 1$$

Student2:

$$G_1S_2 + G_2S_2 + G_3S_2 + G_4S_2 + G_5S_2 = 1$$

Student3:

$$G_1S_3 + G_2S_3 + G_3S_3 + G_4S_3 + G_5S_3 = 1$$

Student4:

$$G_1S_4 + G_2S_4 + G_3S_4 + G_4S_4 + G_5S_4 = 1$$

Student5:

$$G_1S_5 + G_2S_5 + G_3S_5 + G_4S_5 + G_5S_5 = 1$$

Student6:

$$G_1S_6 + G_2S_6 + G_3S_6 + G_4S_6 + G_5S_6 = 1$$

Student7:

$$G_1S_7 + G_2S_7 + G_3S_7 + G_4S_7 + G_5S_7 = 1$$

Student8:

$$G_1S_8 + G_2S_8 + G_3S_8 + G_4S_8 + G_5S_8 = 1$$

Student9:

$$G_1S_9 + G_2S_9 + G_3S_9 + G_4S_9 + G_5S_9 = 1$$

Student10:

$$G_1S_{10} + G_2S_{10} + G_3S_{10} + G_4S_{10} + G_5S_{10} = 1$$

Student11:

$$G_1S_{11} + G_2S_{11} + G_3S_{11} + G_4S_{11} + G_5S_{11} = 1$$

Student12:

$$G_1S_{12} + G_2S_{12} + G_3S_{12} + G_4S_{12} + G_5S_{12} = 1$$

Student13:

$$G_1S_{13} + G_2S_{13} + G_3S_{13} + G_4S_{13} + G_5S_{13} = 1$$

Student14:

$$G_1S_{14} + G_2S_{14} + G_3S_{14} + G_4S_{14} + G_5S_{14} = 1$$

Student15:

$$G_1S_{15} + G_2S_{15} + G_3S_{15} + G_4S_{15} + G_5S_{15} = 1$$

Factor constraints: -

For Factor GPA:

Group1_Factor1:

$$\begin{aligned} &2.93G_1S_1 + 3.68G_1S_2 + 3.11G_1S_3 + 3.82G_1S_4 + 3.91G_1S_5 + 2.57G_1S_6 + 3.29G_1S_7 \\ &+ 3.84G_1S_8 + 3.33G_1S_9 + 3.18G_1S_{10} + 3.94G_1S_{11} + 3.18G_1S_{12} \\ &+ 3.52G_1S_{13} + 3.36G_1S_{14} + 2.65G_1S_{15} \geq 3.35 \end{aligned}$$

Group2_Factor1:

$$2.93G_2S_1 + 3.68G_2S_2 + 3.11G_2S_3 + 3.82G_2S_4 + 3.91G_2S_5 + 2.57G_2S_6 + 3.29G_2S_7 \\ + 3.84G_2S_8 + 3.33G_2S_9 + 3.18G_2S_{10} + 3.94G_2S_{11} + 3.18G_2S_{12} \\ + 3.52G_2S_{13} + 3.36G_2S_{14} + 2.65G_2S_{15} \geq 3.35$$

Group3_Factor1:

$$2.93G_3S_1 + 3.68G_3S_2 + 3.11G_3S_3 + 3.82G_3S_4 + 3.91G_3S_5 + 2.57G_3S_6 + 3.29G_3S_7 \\ + 3.84G_3S_8 + 3.33G_3S_9 + 3.18G_3S_{10} + 3.94G_3S_{11} + 3.18G_3S_{12} \\ + 3.52G_3S_{13} + 3.36G_3S_{14} + 2.65G_3S_{15} \geq 3.35$$

Group4_Factor1:

$$2.93G_4S_1 + 3.68G_4S_2 + 3.11G_4S_3 + 3.82G_4S_4 + 3.91G_4S_5 + 2.57G_4S_6 + 3.29G_4S_7 \\ + 3.84G_4S_8 + 3.33G_4S_9 + 3.18G_4S_{10} + 3.94G_4S_{11} + 3.18G_4S_{12} \\ + 3.52G_4S_{13} + 3.36G_4S_{14} + 2.65G_4S_{15} \geq 3.35$$

Group5_Factor1:

$$2.93G_5S_1 + 3.68G_5S_2 + 3.11G_5S_3 + 3.82G_5S_4 + 3.91G_5S_5 + 2.57G_5S_6 + 3.29G_5S_7 \\ + 3.84G_5S_8 + 3.33G_5S_9 + 3.18G_5S_{10} + 3.94G_5S_{11} + 3.18G_5S_{12} \\ + 3.52G_5S_{13} + 3.36G_5S_{14} + 2.65G_5S_{15} \geq 3.35$$

For Factor Technical Skill:

Group1_Factor2:

$$9G_1S_1 + 5G_1S_2 + 4G_1S_3 + 6G_1S_4 + 10G_1S_5 + 9G_1S_6 + 8G_1S_7 + 8G_1S_8 + 10G_1S_9 \\ + 8G_1S_{10} + 8G_1S_{11} + 7G_1S_{12} + 8G_1S_{13} + 6G_1S_{14} + 5G_1S_{15} \geq 7$$

Group2_Factor2:

$$9G_2S_1 + 5G_2S_2 + 4G_2S_3 + 6G_2S_4 + 10G_2S_5 + 9G_2S_6 + 8G_2S_7 + 8G_2S_8 + 10G_2S_9 \\ + 8G_2S_{10} + 8G_2S_{11} + 7G_2S_{12} + 8G_2S_{13} + 6G_2S_{14} + 5G_2S_{15} \geq 7$$

Group3_Factor2:

$$9G_3S_1 + 5G_3S_2 + 4G_3S_3 + 6G_3S_4 + 10G_3S_5 + 9G_3S_6 + 8G_3S_7 + 8G_3S_8 + 10G_3S_9 \\ + 8G_3S_{10} + 8G_3S_{11} + 7G_3S_{12} + 8G_3S_{13} + 6G_3S_{14} + 5G_3S_{15} \geq 7$$

Group4_Factor2:

$$9G_4S_1 + 5G_4S_2 + 4G_4S_3 + 6G_4S_4 + 10G_4S_5 + 9G_4S_6 + 8G_4S_7 + 8G_4S_8 + 10G_4S_9 + 8G_4S_{10} + 8G_4S_{11} + 7G_4S_{12} + 8G_4S_{13} + 6G_4S_{14} + 5G_4S_{15} \geq 7$$

Group5_Factor2:

$$9G_5S_1 + 5G_5S_2 + 4G_5S_3 + 6G_5S_4 + 10G_5S_5 + 9G_5S_6 + 8G_5S_7 + 8G_5S_8 + 10G_5S_9 + 8G_5S_{10} + 8G_5S_{11} + 7G_5S_{12} + 8G_5S_{13} + 6G_5S_{14} + 5G_5S_{15} \geq 7$$

For Factor Teamwork:

Group1_Factor3:

$$10G_1S_1 + 9G_1S_2 + 8G_1S_3 + 9G_1S_4 + 4G_1S_5 + 7G_1S_6 + 9G_1S_7 + 5G_1S_8 + 6G_1S_9 + 5G_1S_{10} + 5G_1S_{11} + 6G_1S_{12} + 6G_1S_{13} + 6G_1S_{14} + 5G_1S_{15} \geq 7$$

Group2_Factor3:

$$10G_2S_1 + 9G_2S_2 + 8G_2S_3 + 9G_2S_4 + 4G_2S_5 + 7G_2S_6 + 9G_2S_7 + 5G_2S_8 + 6G_2S_9 + 5G_2S_{10} + 5G_2S_{11} + 6G_2S_{12} + 6G_2S_{13} + 6G_2S_{14} + 5G_2S_{15} \geq 7$$

Group3_Factor3:

$$10G_3S_1 + 9G_3S_2 + 8G_3S_3 + 9G_3S_4 + 4G_3S_5 + 7G_3S_6 + 9G_3S_7 + 5G_3S_8 + 6G_3S_9 + 5G_3S_{10} + 5G_3S_{11} + 6G_3S_{12} + 6G_3S_{13} + 6G_3S_{14} + 5G_3S_{15} \geq 7$$

Group4_Factor3:

$$10G_4S_1 + 9G_4S_2 + 8G_4S_3 + 9G_4S_4 + 4G_4S_5 + 7G_4S_6 + 9G_4S_7 + 5G_4S_8 + 6G_4S_9 + 5G_4S_{10} + 5G_4S_{11} + 6G_4S_{12} + 6G_4S_{13} + 6G_4S_{14} + 5G_4S_{15} \geq 7$$

Group5_Factor3:

$$10G_5S_1 + 9G_5S_2 + 8G_5S_3 + 9G_5S_4 + 4G_5S_5 + 7G_5S_5 + 9G_5S_7 + 5G_5S_8 + 6G_5S_9 + 5G_5S_{10} + 5G_5S_{11} + 6G_5S_{12} + 6G_5S_{13} + 6G_5S_{14} + 5G_5S_{15} \geq 7$$

Non-negativity of the decision variables: -

$$G_iS_j \geq 0 \text{ where } i = 1,2,3,4,5 \text{ and } j = 1,2,\dots,15$$

Approach

The task is to form groups for a class project, aiming to allocate 15 students into five groups of three members each. Integer Programming (IP) helps us do this by ensuring that the number of students in each group is a whole number. This is useful because students can't be divided into fractions. IP makes the process clear and efficient, especially since each group must have exactly three students. It's like making sure everyone fits perfectly into groups without any parts left over. This way, when we use Integer Programming, we make decisions easily and precisely, matching the nature of how students come together in a class.

We formulated the linear programming by setting the objective function for 75 decision variables, set the objective sense to max, Added group constraint, student constraint and factors constraint to the LP problem. Created vector RHS containing right hand side value. For 2 constraints. Below is the code to solve integer programming and result.

Setting type for the variables to 'Integer'.

```
set.type(lp,1:75,type = "integer")
lp
```

Model name:
a linear program with 75 decision variables and 35 constraints

Solving the LP problem using the specified constraints and objective function.

```
solve(lp)
```

```
[1] 0
```

Retrieving the optimal objective value of the LP problem.

```
get.objective(lp)
```

```
[1] 83.42
```

Retrieving the values of the decision variables at the optimal solution.

```
get.variables(lp)
```

```
[1] 0 0 0 0 0 1 1 0 0 0 0 0 1 0 0 1 0 0 1 0 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 0 1
[39] 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 1 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 1
```

Retrieving the values of the constraints at the optimal solution.

```
get.constraints(lp)
```

```
[1] 3.00 3.00 3.00 3.00 3.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
[13] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 9.38 10.08 11.46 9.72
[25] 9.67 25.00 25.00 21.00 21.00 19.00 22.00 25.00 19.00 17.00 17.00
```

Figure 2: Linear Programming

Finding and Conclusion

This project utilizes mathematical optimization techniques, specifically linear programming, to strategically form student groups for class projects. By considering factors such as GPA, Technical Skill, and Teamwork, the goal is to maximize the success potential of each group. Each group is assigned a specific success probability, representing the likelihood of achieving optimal outcomes. Below table shows the optimal solution for each group:

Table 2: Optimal Solution

Students	Group				
	1	2	3	4	5
1		6.87			
2			5.67		
3					4.84
4		6.03			
5					5.76
6	5.83				
7	6.42				
8			5.44		
9		6.13			
10				5.17	
11			5.48		
12				5.17	
13	5.61				
14				4.94	
15					4.06

Group Success Analysis:

Group 1:

Group 1 falls within the moderate range of overall success with a chance of success at 21%. This positioning can be considered moderate as it reflects a balanced distribution of individual contributions. While cohesive, the group's overall chance of success is neither the highest nor the lowest among the analyzed groups.

Group 2:

Group 2 stands out as the best-performing group with the highest overall chance of success at 23%. The superior overall success potential can be attributed to the well-balanced composition of individual contributions within the group. This balanced dynamic positions Group 2 as the most promising team in the collaborative context.

Group 3:

Group 3 maintains a competitive chance of success at 20%, placing it within the moderate range. The visual representation suggests a well-distributed set of individual contributions, contributing to the group's balanced nature. While not the highest, Group 3's overall success potential remains competitive.

Group 4:

With an overall chance of success at 18%, Group 4 is positioned within the moderate range. The group displays a balanced distribution of individual contributions, contributing to its competitive stance. However, among the analyzed groups, Group 4 exhibits the lowest overall chance of success, categorizing it as the least promising in the collaborative landscape.

Group 5:

Group 5 shares an overall chance of success at 18%, positioning it within the moderate range. However, despite its competitiveness, Group 5 exhibits the widest range of individual contributions (33%, 39%, and 28%), introducing variability that might present challenges for cohesive collaboration. This diversity in contributions places Group 5 as the least promising group among the analyzed teams.

Graphs accompanying this information provide a visual breakdown of each group's overall success rates and individual member contributions for a clearer understanding of their respective dynamics.

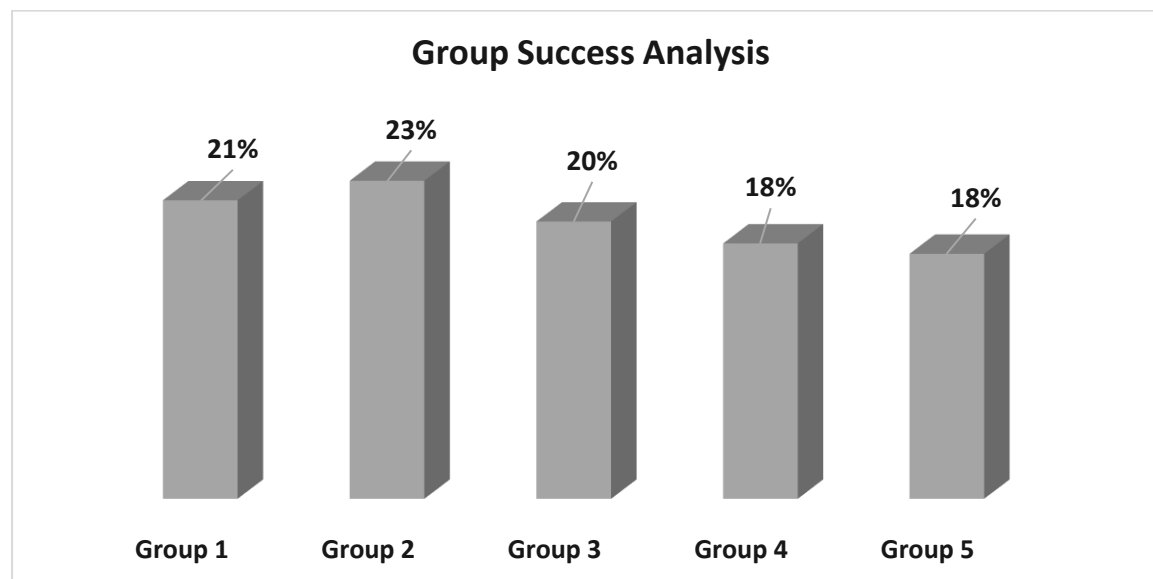


Figure 3: Group Success Analysis

Let's dive into how each student in Groups 1 to 5 brings their unique strengths to the table, as reflected in the percentages that showcase their contributions.

Student Contributions Analysis:

Group 1 Contributions:

In Group 1, each member contributes distinctively, with percentages of 33%, 36%, and 31%. The pie chart visually outlines these contributions, offering a clear representation of the unique strengths brought by each member to the collaborative table.

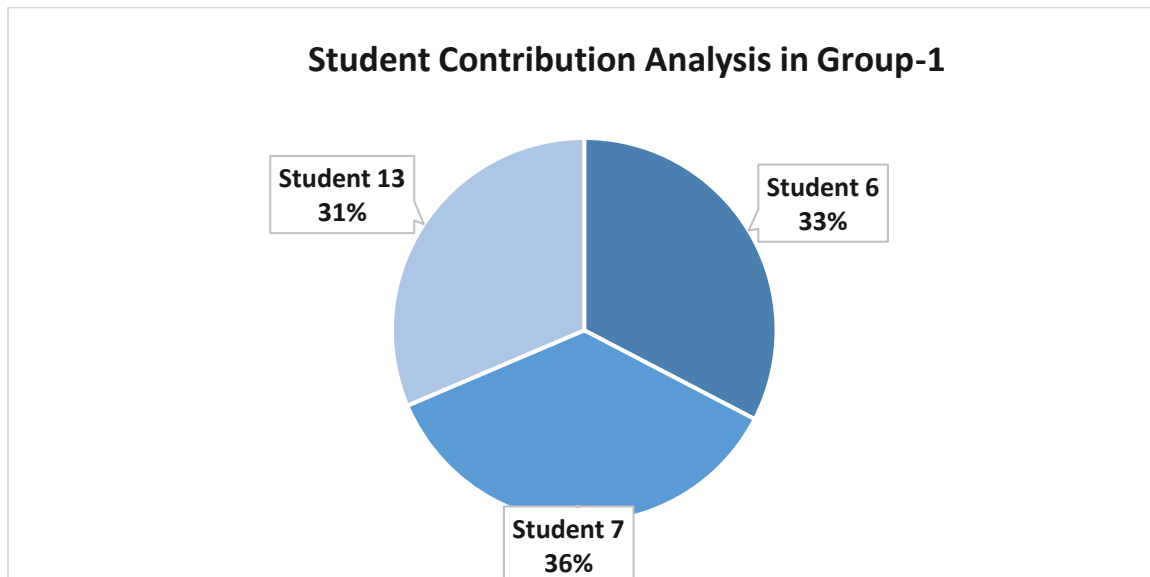


Figure 4: Group 1 - Student Contribution Analysis

Group 2 Contributions:

Group 2 members contribute harmoniously, with percentages of 36%, 32%, and 32%. The pie chart visually emphasizes the balanced composition, showcasing the individual strengths that contribute to the group's overall success potential.

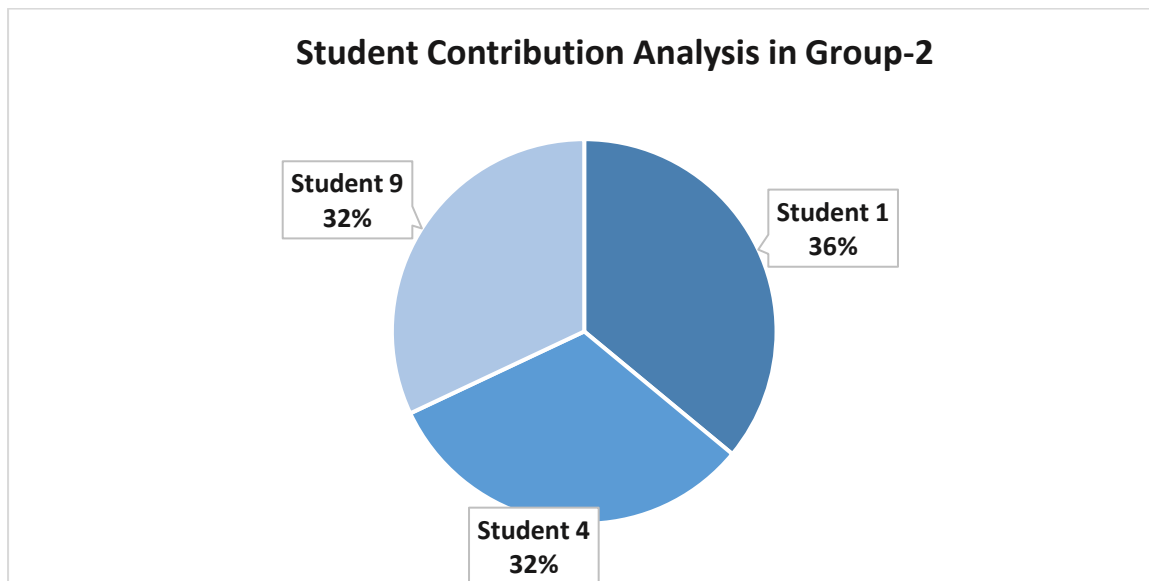


Figure 5: Group 2 - Student Contribution Analysis

Group 3 Contributions:

The members of Group 3 exhibit a consistent distribution, contributing percentages of 34%, 33%, and 33%. The visual representation in the pie chart illustrates the complementary strengths of each member, providing insight into their collective collaborative potential.

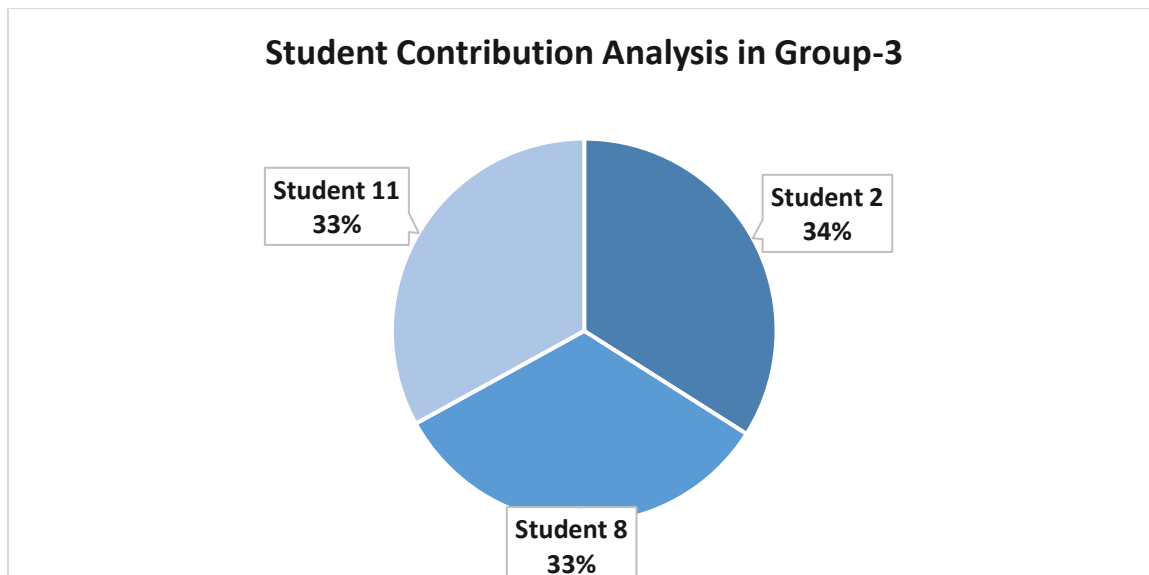


Figure 6: Group 3 - Student Contribution Analysis

Group 4 Contributions:

In Group 4, individual contributions are well-balanced, with percentages of 34%, 34%, and 32%. The pie chart visually depicts the cohesive dynamics within the group, highlighting the combined strengths contributing to their overall chance of success.

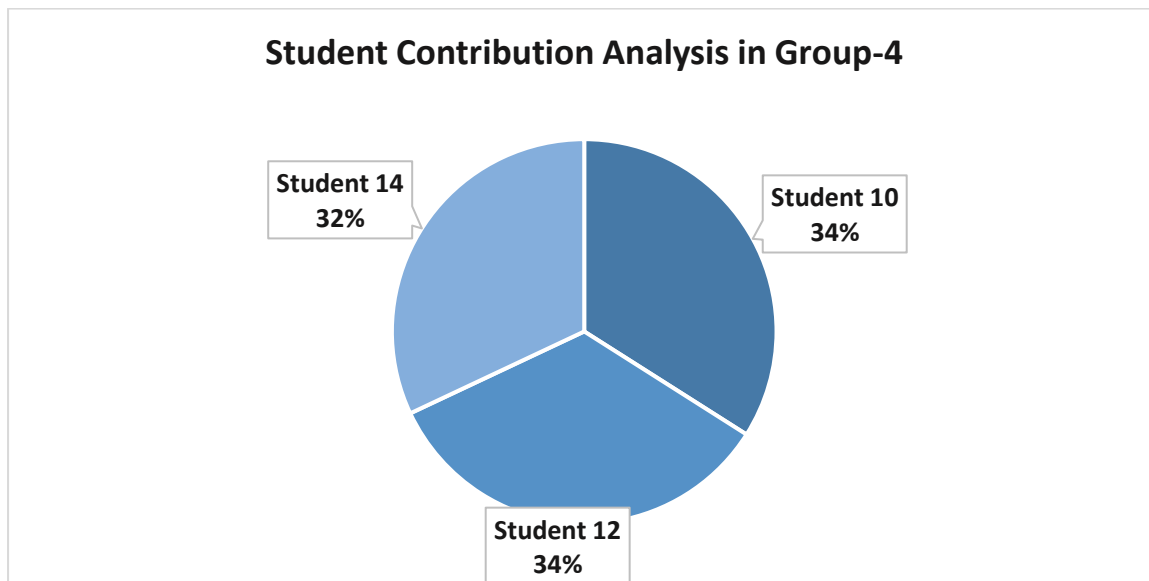


Figure 7: Group 4 - Student Contribution Analysis

Group 5 Contributions:

Group 5 members present a range of individual contributions, with percentages of 33%, 39%, and 28%. The pie chart visually captures the diversity within the group, offering a nuanced understanding of the distinct strengths each member brings to the collaborative effort.

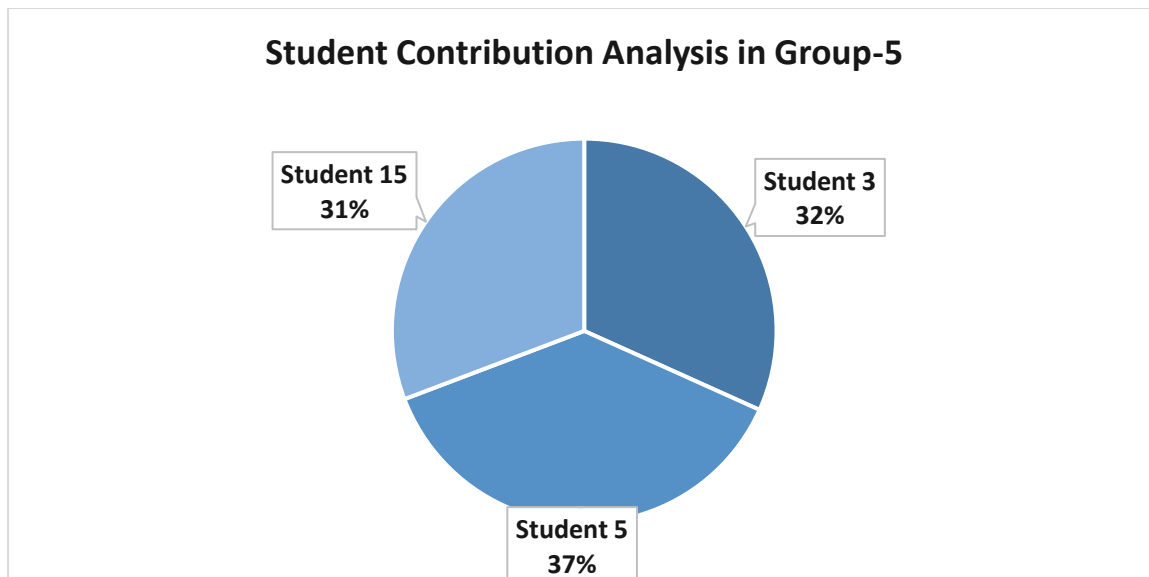


Figure 8: Group 5 - Student Contribution Analysis

In essence, this comprehensive analysis showcases how mathematical optimization, coupled with an understanding of individual contributions, shapes the collaborative landscape of student groups. As we navigate the varied success probabilities and unique strengths within Groups 1 to 5, it becomes evident that the project's goal of maximizing success potential relies on a delicate balance of diverse talents and cohesive dynamics.

References:

- Building, Complexica. “Integer Programming.” *Supply & Demand Optimisation*, www.complexica.com/narrow-ai-glossary/integer-programming#:~:text=Integer%20programming%20is%20an%20optimisation,while%20minimising%20cost%20and%20resources. Accessed 9 Dec. 2023.
- “Integer Programming.” *Integer Programming - an Overview / ScienceDirect Topics*, www.sciencedirect.com/topics/mathematics/integer-programming. Accessed 9 Dec. 2023.
- Sign in to Your Account*, kent.instructure.com/courses/70509/pages/module-11-introduction?module_item_id=3215100. Accessed 9 Dec. 2023.