# Project Part 4:

## Instructions:

- This project is due on May 28th before 11:59 pm.
- Form a team of 1,2,3,4 or 5 students and complete all the following questions.
- All graphs should have title and axes labels.
- You will submit a pdf file and all your MATLAB files on the Learning Hub in the project 4 folder.
- Your document should be clear and well organized.
- One document per team.
- Your pdf and MATLAB files should all contained the names of all team members at the top of the file in comments.

## Question: [30 Marks]:

This question is a continuation of the project 3. Please refer to the project 3 for the details about the kinematic equations of a differential drive robot.

Suppose that:

$$v_r(t) = 2 - 0.5t$$
$$v_l(t) = 1$$

Just like in part d) of project 3.

a. [5 marks]: Now write down the formulas for x(t) and y(t) but do not try to calculate the integrals inside of each formula yet. As you know from project 3, the two integrals we obtain for x(t) and y(t) cannot be calculated analytically. In the project 3, you needed to use a numerical integration method to calculate these two integrals. Here we will do something different and use our knowledge of MacLaurin series instead.
b. [5 marks]: Using properties of MacLaurin series, find the first 4 non-zero terms of the MacLaurin series for the function **INSIDE** the integral for x(t) and the function inside the integral for y(t).
c. [5 marks]: Calculate the error (not a bound) you are making using your series to approximate the value of the function at 1, 2, and 3.
d. [5 marks]: Calculate a bound on the error you are making using your series to approximate the value of the function at 1, 2, and 3. You might want to use Matlab or WolframAlpha to help out with calculating the different derivatives.
e. [5 marks]: We have to be careful here; the position function x(t) and y(t) are not the functions which are inside of the integrals, they are the integral of these functions. In order to obtain the position function x(t) and y(t) we need to integrate the two series found in part b. term by term. Do that.

f. [5 marks]: Now there is an interesting question that arises: what is happening with the error and the bound on the error ? The things we calculated in part c and d have something to do with the functions inside of the integrals and not about the integral of these functions. We need a way to calculate which kind of error we are making. I have used a software to calculate the values of x(t) and y(t) for the different times 1, 2, 3 seconds. You will assume that these values are the correct ones and use those to calculate the errors. Calculate the error you are making both for the x(t) and y(t) position for the different times 1, 2, and 3 by comparing the values you obtain by replacing t by 1, 2, and 3 in the integrals of your series and the values I calculated and provided in the table below.. Does it seem to be any relationship between these errors and the errors and bounds you found in part c and d?

| Functions/Time | 1 second | 2 seconds | 3 seconds |
|---|---|---|---|
| x(t) | 0.8578719116 | 0.5788243975 | 0.3438756633 |
| y(t) | 0.9066061753 | 1.983282853 | 2.816517512 |

BONUS QUESTIONS: (In a normal year, these 3 next questions are not bonuses but I thought I would give you all a bit of a break):

g. [5 marks]: Now that we have a way to calculate the error or at least approximate the error we would be able to use this to determine how many terms of the MacLaurin series we should actually keep. NOTE: you will most likely need a software to answer this question (either of Matlab, Wolfram Alpha, etc.) Figure out how many terms of the MacLaurin series you need to keep to approximate the position x(t) and y(t) for any point between t=0s to t=3s accurate to 0.01.

h. [5 marks]: Using your series with the appropriate number of terms obtained in part g, create a graph for the position of the robot in the x-y plane from t=0s to t=3s by varying the time by increment of 0.1s. Does this graph look similar to the graph you obtained in project 3? Compare both of them by putting them in the same figure.

i. [5 marks]: Compare the computation time of the method from project 3 to this method to find the x-position of the robot after 3 seconds, i.e., x(3). Which method is the fastest?