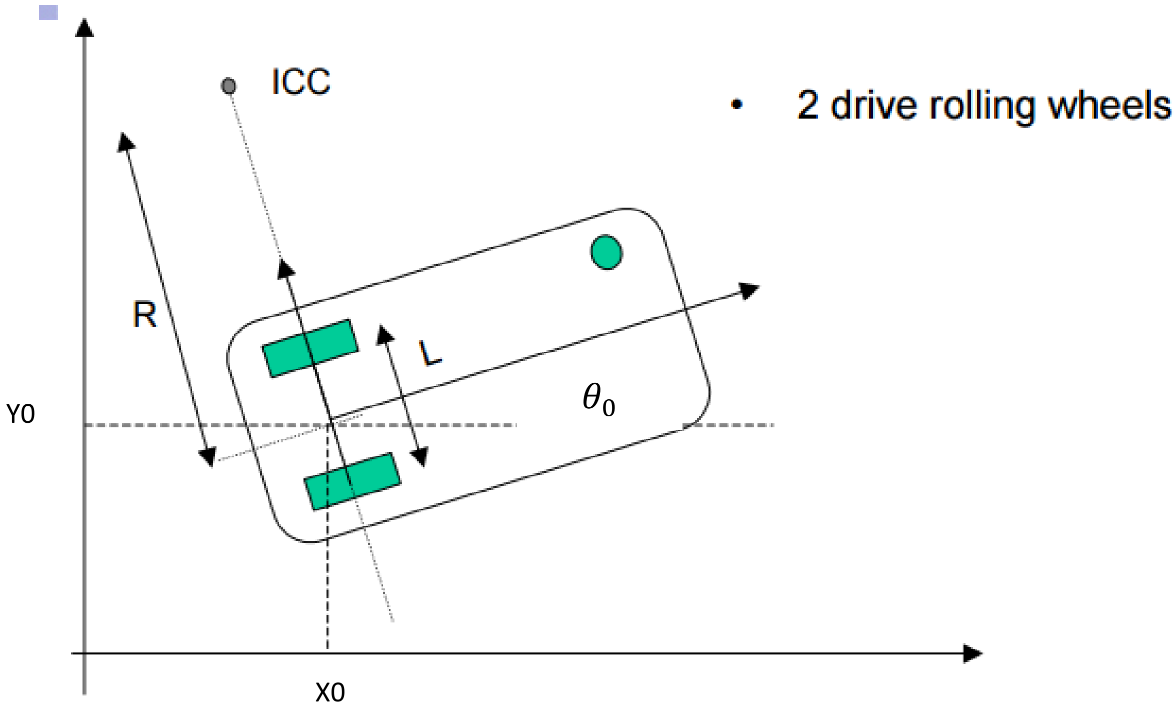
Project 3

Instructions:

- This project is due on May 1^{st} before 11:59 pm.
- Form a team of 1,2,3,4 or 5 students and complete all the following questions.
- All graphs should have title and axes labels.
- You will submit a pdf file and all your MATLAB files on the Learning Hub in the project 3 folder.
- Your document should be clear and well organized.
- One document per team.
- Your pdf and MATLAB files should all contained the names of all team members at the top of the file in comments.

Question: [60 Marks]: Many very popular mobile robot platform use a kinematic called differential drive. The concept is that there are two big wheels at the front (or the back) of the robot that can rotate at different velocities. At the back (or the front) of the robot there are either one or two swivel wheels.

Refer to the figure and the equations on the next page that describe the kinematics equations for a differential drive robot.



Let: L= 0.5m

$v_r(t)$: linear velocity of the right wheel

$v_l(t)$: linear velocity of the left wheel

Then we get the following:

$$v(t) = \frac{v_r(t) + v_l(t)}{2}$$

$$\omega(t) = \frac{v_r(t) - v_l(t)}{L}$$

$$\theta(t) = \theta_0 + \int_0^t \omega(z)\, dz$$

Using these 3 equations we can obtain parametric equations for the x and y position of the robot as a function of time:

$$x(t) = x_0 + \int_0^t v(z) \cos\big(\theta(z)\big)\, dz$$

$$y(t) = y_0 + \int_0^t v(z) \sin\big(\theta(z)\big)\, dz$$

To make our life easier, we will just assume (unless otherwise mentioned) that the robot starts at the origin pointing towards the right. This means that:

$$x_0 = y_0 = \theta_0 = 0$$

The way to control the robot is by giving the linear velocity of the right wheel and the linear velocity of the left wheel. Now let's try to answer some questions.

a. **[10 marks]:** Suppose that

$$v_r(t) = 1\ m/s$$
$$v_l(t) = 1\ m/s$$

Show that the trajectory of the robot in the x-y plane is a straight line by finding the function x(t) and y(t) and plot the trajectory of the robot in the x-y plane for 5 seconds.

b. **[10 marks]:** Suppose that

$$v_r(t) = 1\ m/s$$
$$v_l(t) = 1\ m/s$$
$$\theta_0 = 30°$$

Should the trajectory still be a straight line? Find the function x(t) and y(t) and plot the trajectory of the robot in the x-y plane for t=0 to t=5 seconds to verify.

c. **[10 marks]:** Suppose that

$$v_r(t) = 2\ m/s$$
$$v_l(t) = 1\ m/s$$

What do you think the trajectory of the robot will be? Find the function x(t) and y(t) and plot the trajectory of the robot in the x-y plane for t=0 to t=3 seconds.

d. **[30 marks]:** So far the examples above have only dealt with cases where the two linear velocities of the wheels are constant. In this example, we will attempt to do the same for the case where one of them is a function of time. Suppose that:

$$v_r(t) = 2 - 0.5t$$
$$v_l(t) = 1$$

Now write down the formulas for x(t) and y(t) but do not try to calculate the integrals inside of each formula yet.
Now if you have done this properly, the integrals you obtained cannot actually be calculated analytically, i.e., there is no close formulas for these two integrals.

So a differential drive robot even with fairly simple linear wheel velocity can have non-integrable position functions. This means that we will have to use numerical integration to find the trajectory of the robot.

1) In a first step, you need to create a code to do numerical integration. You can code either the midpoint method, the trapezoid method or the Simpson's method. Before you decide which method you will use make sure that the assumptions for the error term are satisfied. To test your code, use it to calculate the following integral

$$\int_3^8 x^2 + 3x \, dx$$

2) Using your code, calculate the x(4) and y(4), i.e., the x and y position of the robot after 4 seconds.
3) Create a plot in Matlab of the robot's trajectory for t=0 to t=4 seconds. To do so, find the x(i) and y(i) position of the robot by increment of 0.1 s and use them to create the x-y plane plot.