

A decorative graphic on the left side of the slide consisting of two overlapping parallelograms. The front one is blue and the back one is a light green. They are positioned diagonally, with the blue one partially covering the green one.

Planetary Motion

We're finally here!

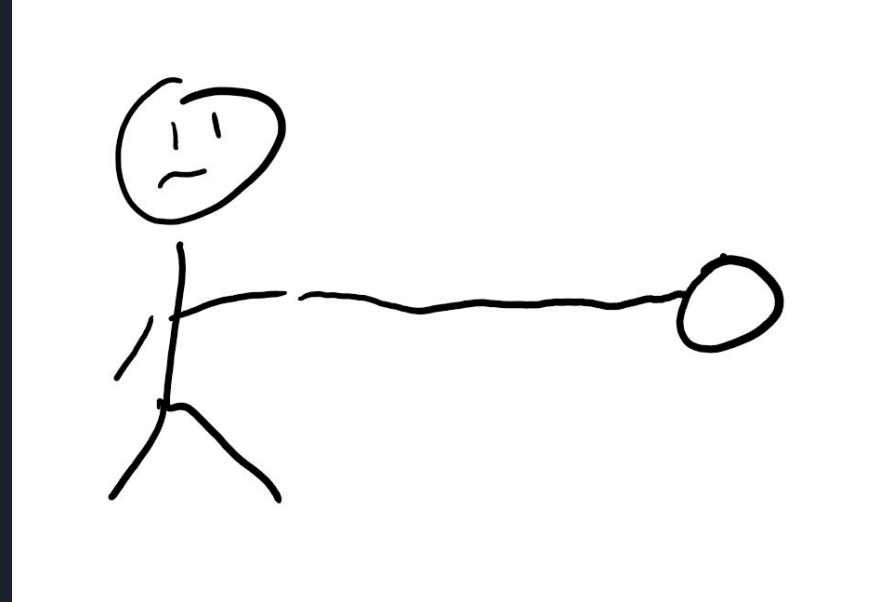
Centripetal Motion



Ball on a rope

When you swing some object tied on a rope, what path does it follow?

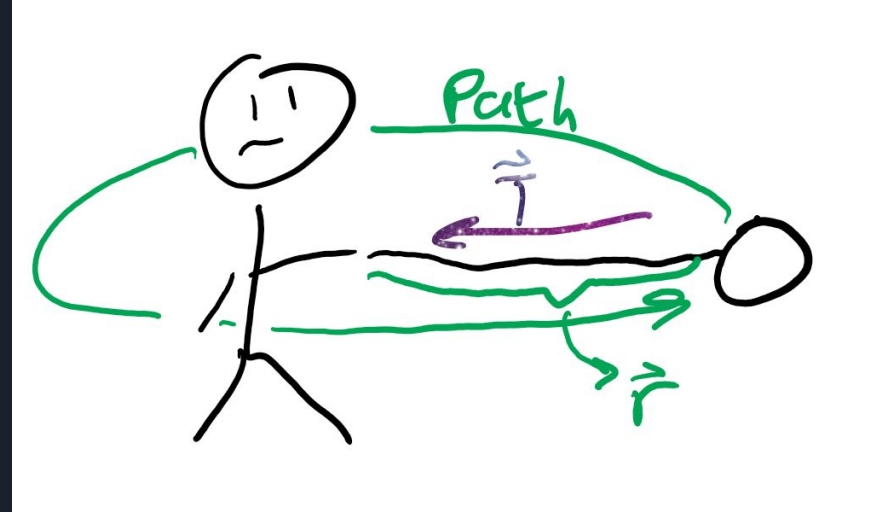
What direction is the force on the object?



Inward forces

Centri (center) petal
(seeking) forces are forces
that go toward the center.

These forces cause circular
motion.

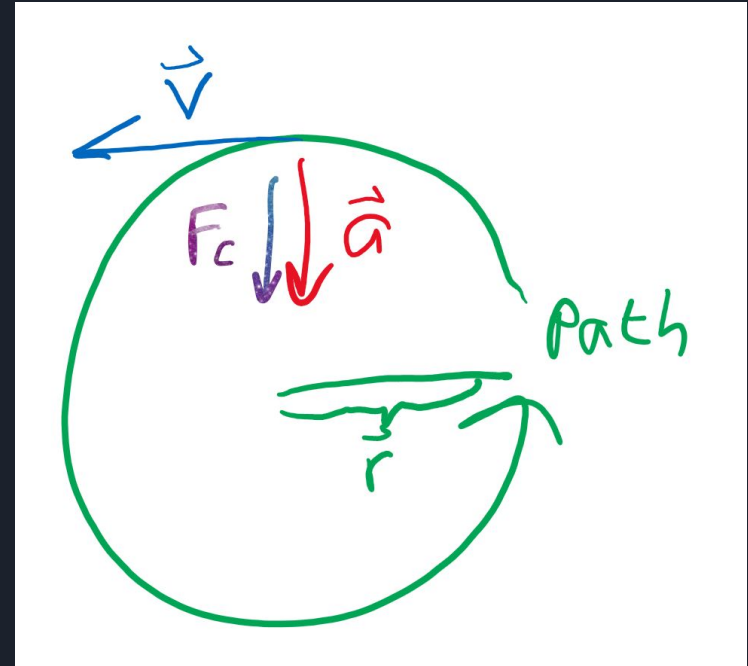


Vectors?

The force (and acceleration) are directed inward

The velocity is “tangent”/“touching” the circular path

The path is, well, I said it already, but it’s circular



Equation (uh oh spaget)

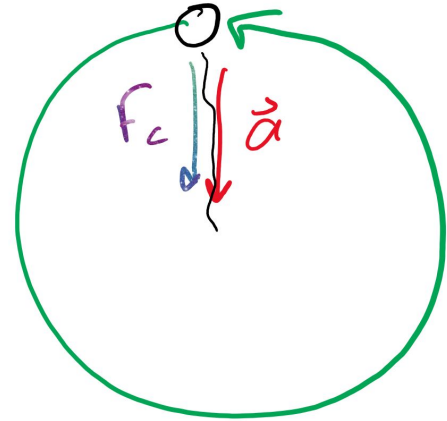
$$F_c = m v^2 / r$$

$$F = m a, \text{ so } a = v^2 / r$$

$$F = \frac{m v^2}{r}$$

$$F = m a$$

$$a = \frac{v^2}{r}$$



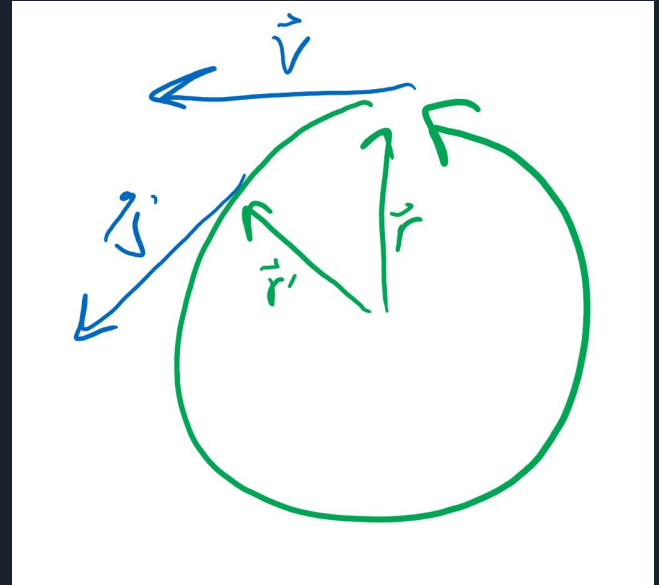
Proof?

Velocity is change in distance over time ($v = d / t$).

If a constant F_c is applied, velocity doesn't change.

The time it takes to move an entire circle (on left) is therefore d / v .

Distance traveled around the circle is $2 (\pi) r$, so $t = 2 (\pi) r / v$.



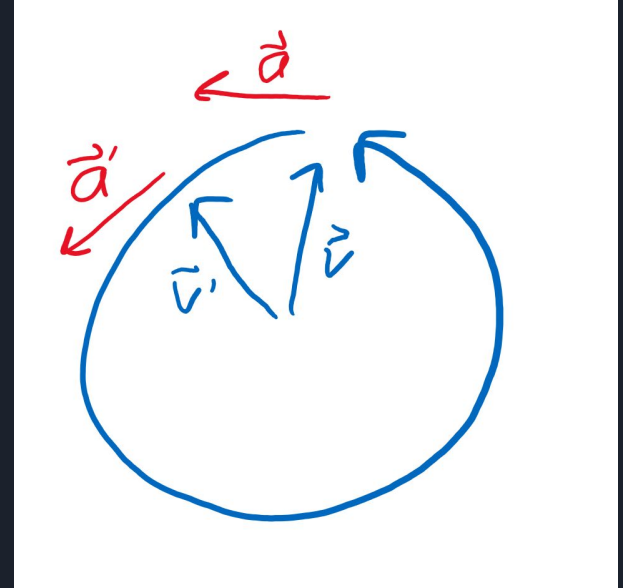
Proof (2 electric boogaloo)?

Velocity also “moves in a circle.”

Acceleration is change in velocity over time ($a = s / t$), so the time it takes to travel the full “velocity circle” is s / a .

The total “velocity” over the circle is $2 (\pi) v$, so $t = 2 (\pi) v / a$.

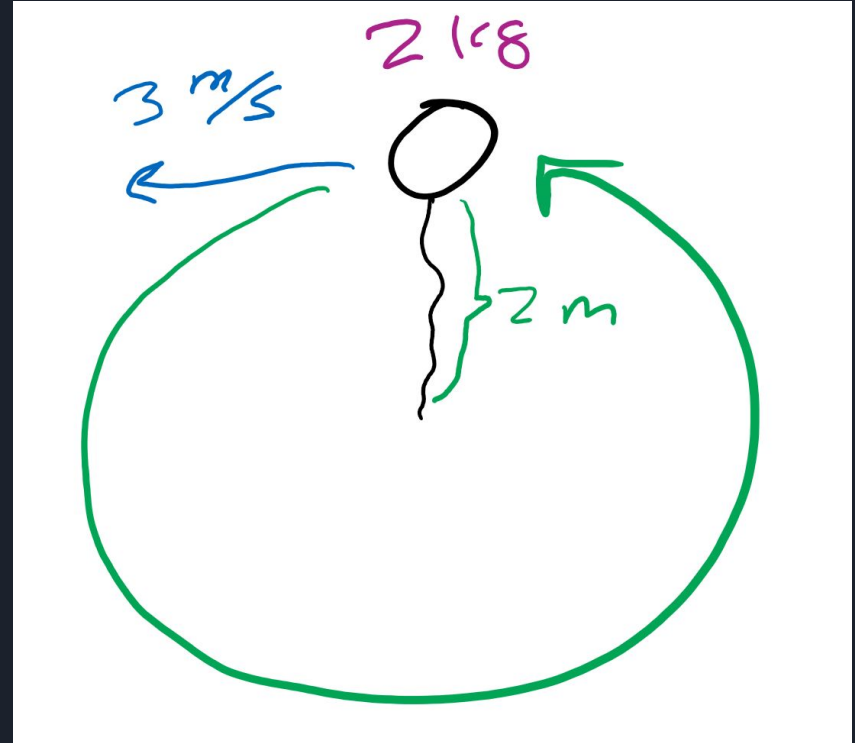
The two “t”s from the two slides are equal, so $2 (\pi) r / v = 2 (\pi) v / a$, so we get $a = v^2 / r$.



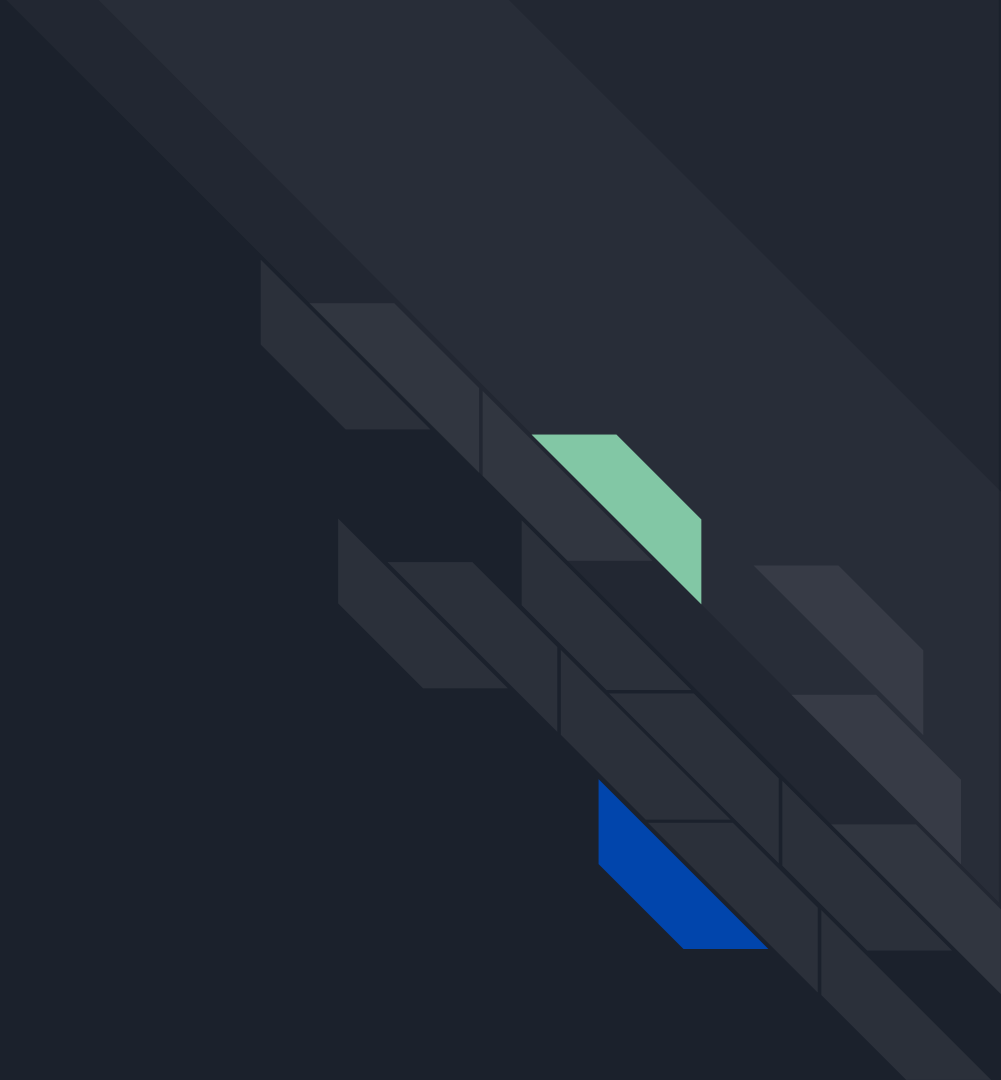
Review

What force do I need to keep a mass of 2 kg traveling at a speed of 3 m/s in a 2 m orbit.

What is the direction of the force?



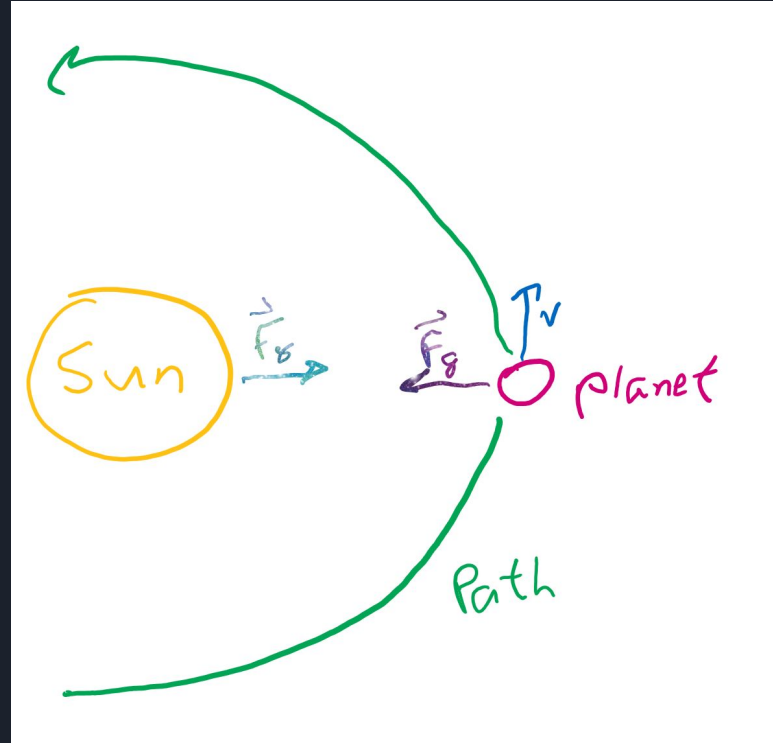
Gravitation



Forces in space

What forces act on planets?

Why are their orbits
“circular-ish?”



Newton's Universal Law of Gravitation

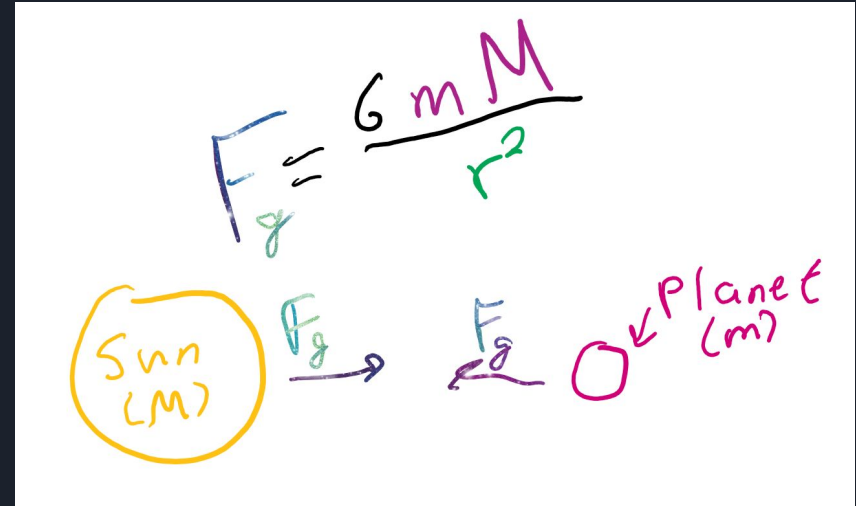
G (gravitational constant): $6.67 \times 10^{-11} \text{ N m}^2 / (\text{kg})^2$

m (mass of planet)

M (mass of sun)

r (distance between them)

Why is there only one equation for force for both bodies?

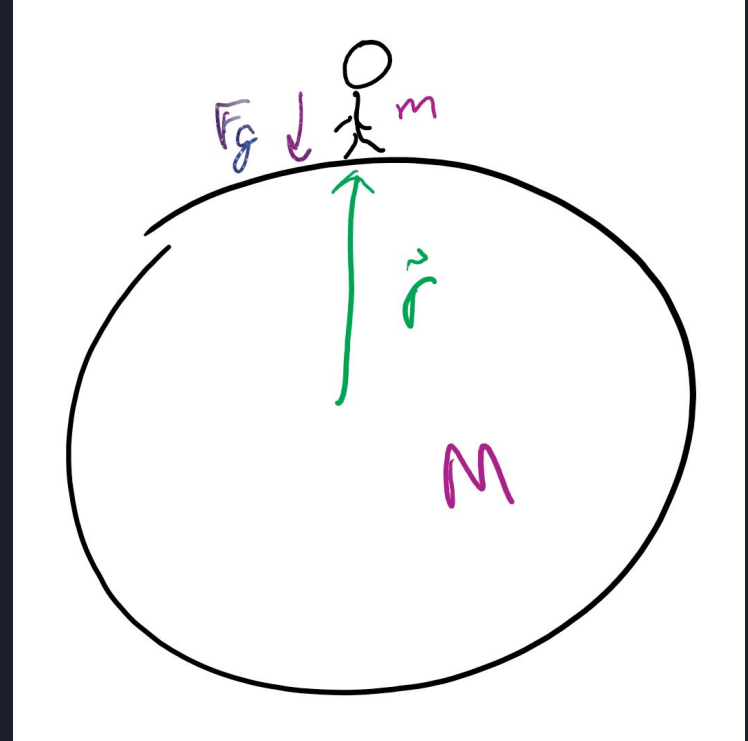


Works for any two bodies

We know our weight is m^*g ,
but how do we find g ?

$F_g = G m M / r^2$, and also
that $F_g = m a$.

That means $a = G M / r^2$.



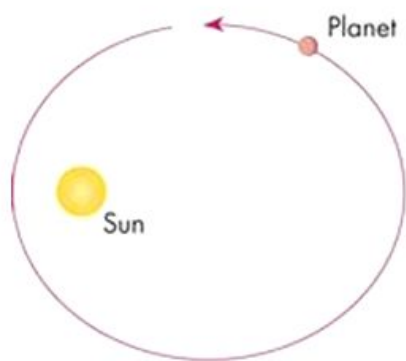
How planets really move



Kepler's 3 Laws

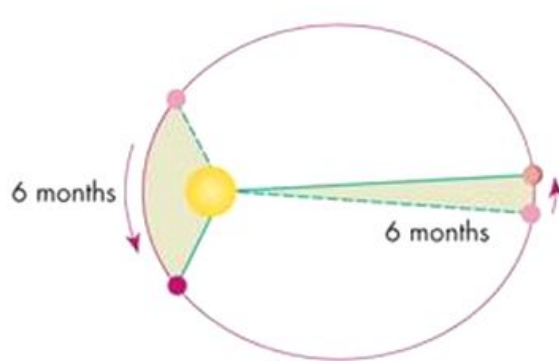
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Kepler's 3 Laws of Planetary Motion



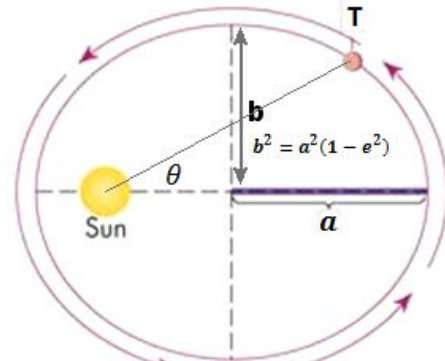
(1)

The orbits are ellipses



(2)

Equal areas in equal time



(3)

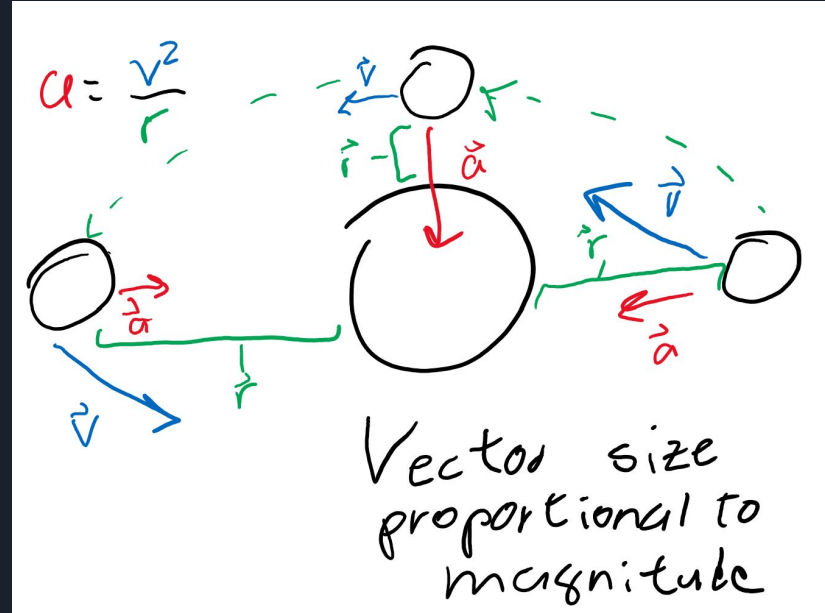
$T^2 \propto a^3$ T = time to complete orbit
 a = semi-major axis

Elliptical orbits?

Think of it like a spring.

If the planet approaches the other planet in a line, the planet will oscillate getting closer and further.

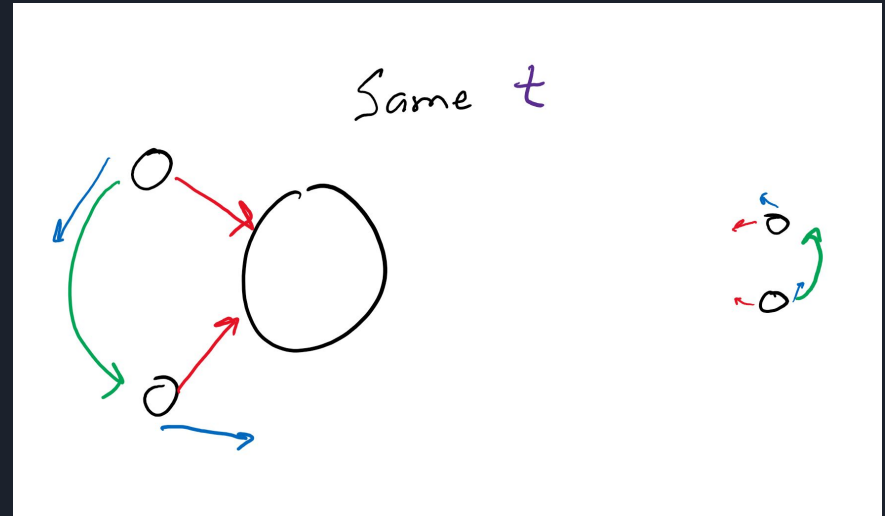
Unless the planet goes into orbit at the perpendicular, orbits will always be elliptical.



Closer means speedy-er

What happens to the force of gravity when the objects are closer?

How does this affect speed?



Longer radius, longer time

The further away you are, the longer it takes to orbit.

How is the force affected the further we are?

How does that affect velocity?

And how does that affect distance traveled?

$$\begin{aligned}\frac{GmM}{r^2} &= \frac{mv^2}{r} & GM &= \left(\frac{2\pi r}{t}\right)^2 r \\ GM &= v^2 r & GM &= \frac{4\pi^2 r^3}{t^2} \\ t &= \frac{2\pi r}{v} & GMt^2 &= 4\pi^2 r^3 \\ v &= \frac{2\pi r}{t}\end{aligned}$$



Review?

“Prove” Kepler’s first law

Second law?

Third law?

What is Newton’s Law of
Universal Gravitation?

How does it help us find the
gravitational acceleration?

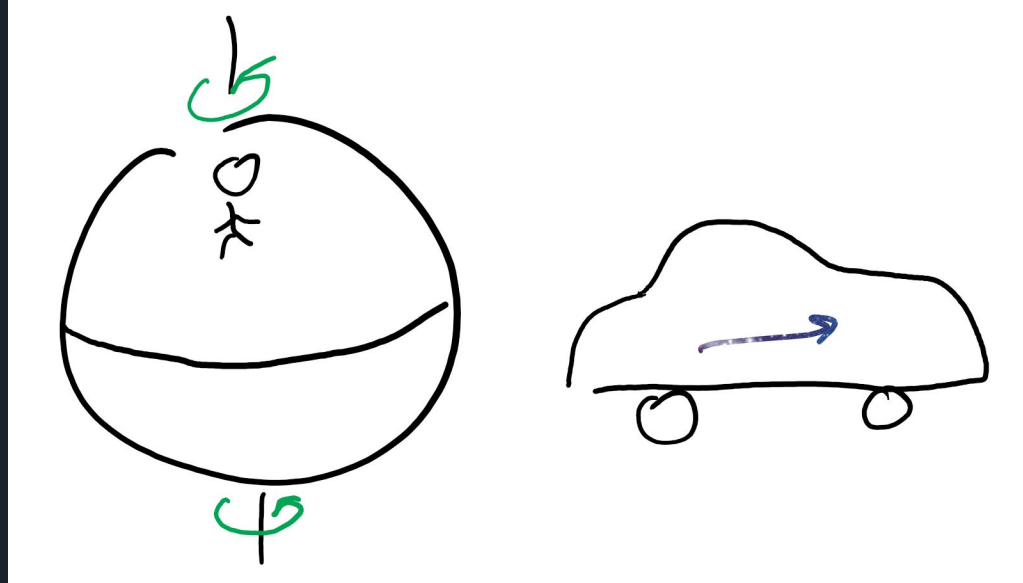
“Fictitious” forces



Experienced on an “non-inertial” frame

You are moving.

Aka, you have “inertia.”



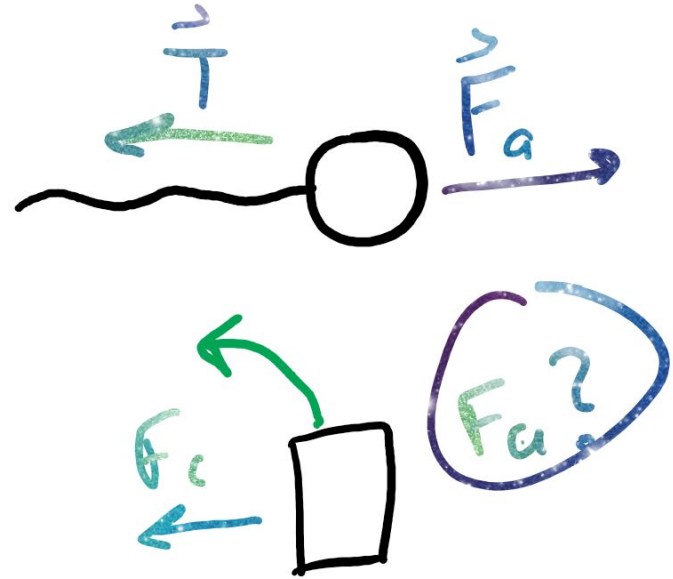
Centrifugal force

A result of inertia.

Proportional to F_c .

When you take a left turn on a car, what direction do you feel like you're being pulled in?

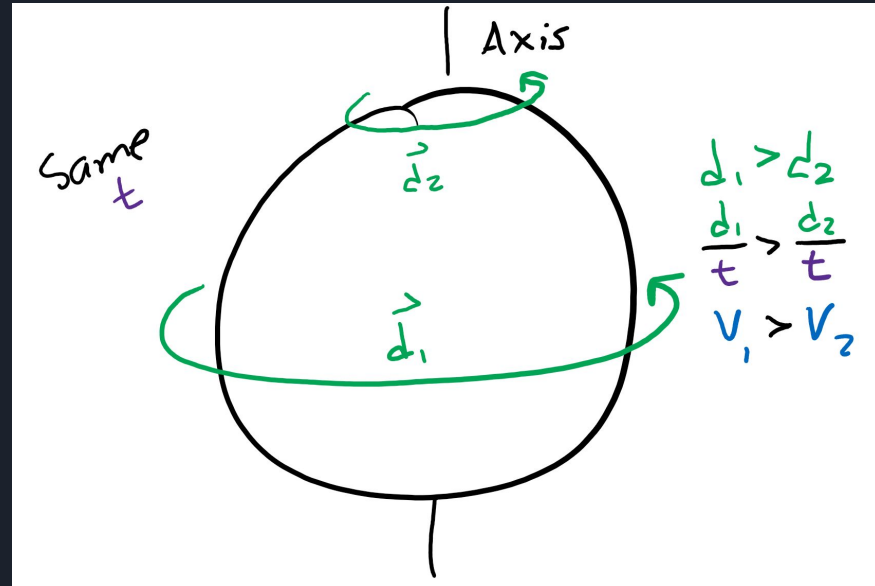
Does it feel like a force?



Why do planets bulge pt 1?

The equator spins a lot further than places by the poles.

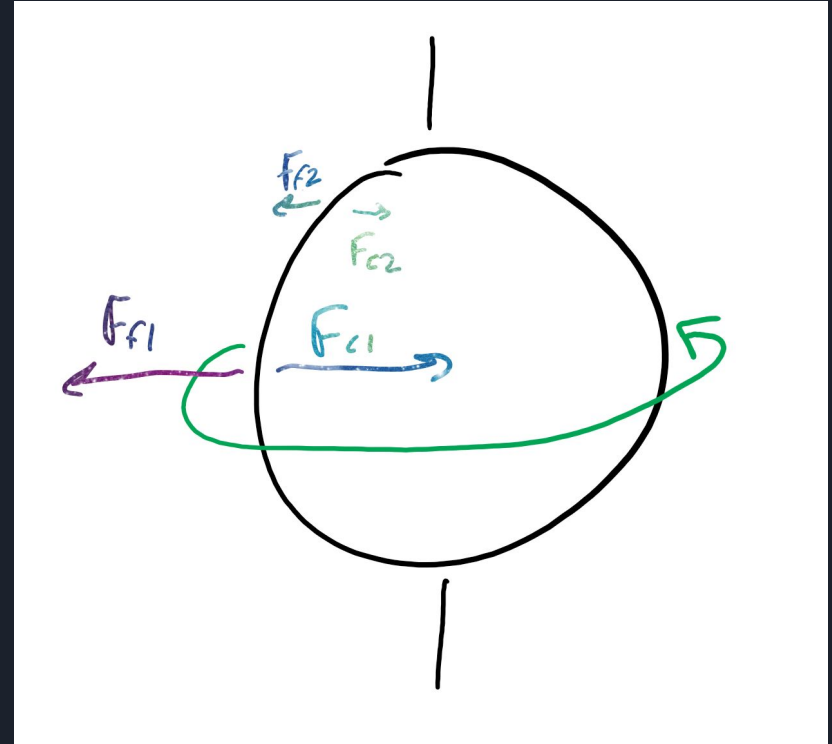
That means the velocity around the equator is much bigger.



Why do planets bulge pt 2?

The centripetal force is proportional to velocity, so the centripetal force along the equator is much bigger.

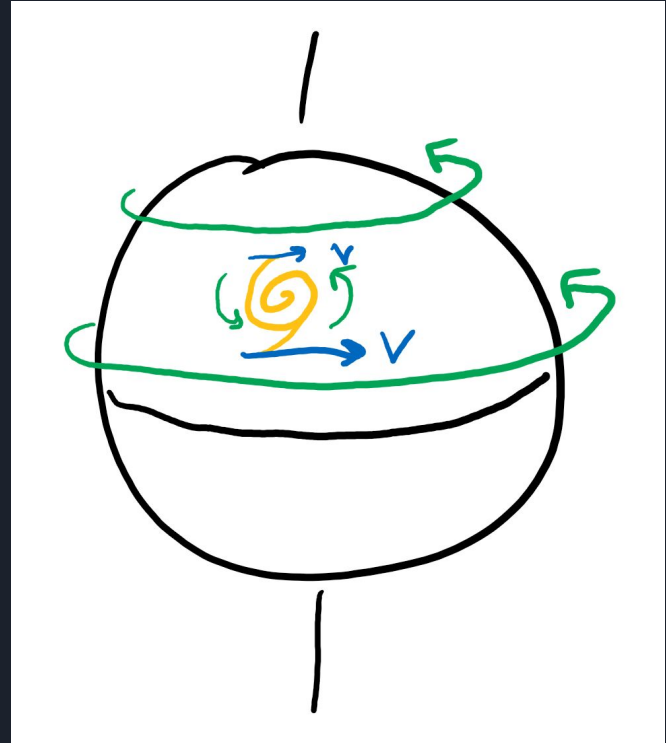
The centrifugal force is an “equal and opposite” force to the centripetal force, so the equator is pushed out a lot more than the poles.



Coriolis force

On the Earth, the velocity by the equator is larger than the velocity away from the equator.

This makes the hurricanes “spin.”

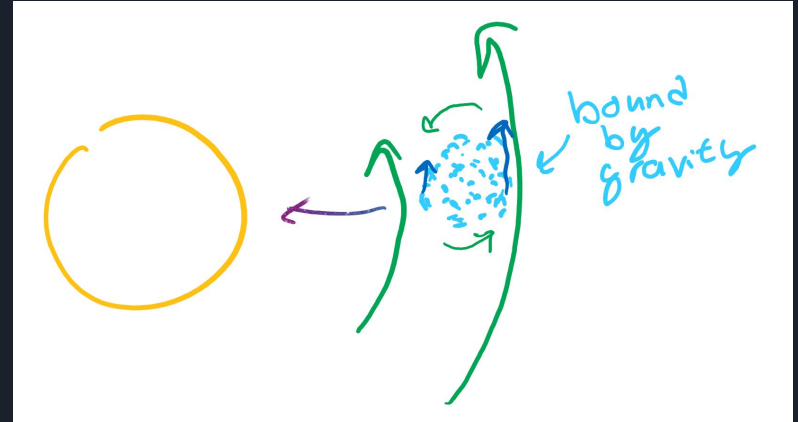


Why planets spin a certain way

Very similar to the reason hurricanes rotate.

When planets were particles, those further away have a larger velocity; thus, spin.

Why does this effect not cause Venus to spin the “right” way?





Fictitious forces?

When have you experienced the “centrifugal” force?

Coriolis force?

How do these forces factor into how our planets move?