

Inductors and AC

Head dizzy, brain oscillate

Units

Voltage (V): Volts (V)

Current (I): Amperes (I)

Resistance (R): Ohms (Ω)

Capacitance (C): Farads (F)

Inductance (L): Henrys (H)

Magnetic flux (Φ): Webers (Wb)

Angular speed (ω): Radians/Second (1/s)

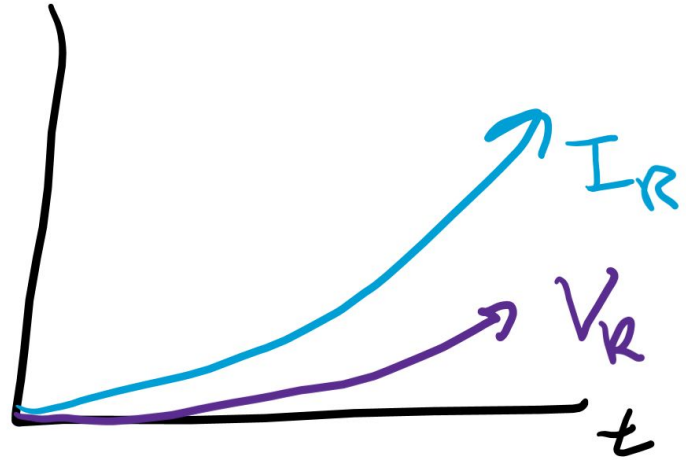
Reactance (X): Ohms (Ω)

Inductance (Z): Ohms (Ω)

All circuit components

Resistors

When current hits a resistor, a voltage is generated. There is no delay from when the current hits the resistor and when a voltage is generated.



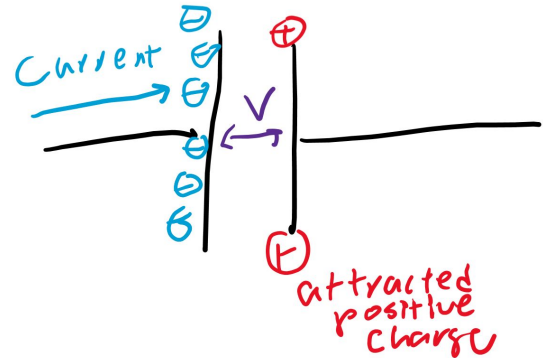
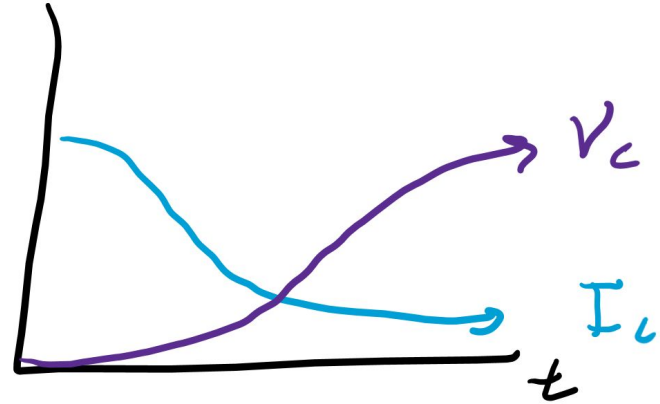
Ohm's law:

$$V \propto I$$

increasing V increases I

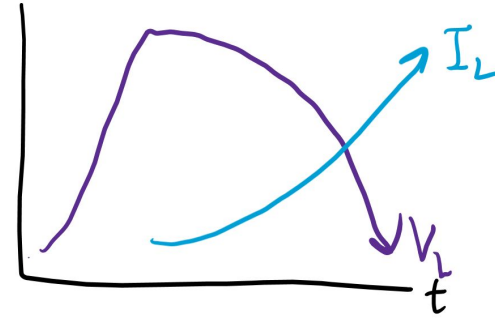
Capacitors

Current flows through the capacitor. After a little delay, the voltage slowly builds up, and the current stops flowing through the capacitor.



Inductors

Inductors have very high resistance, so current does not flow through it. Rather, a voltage builds up. After a little delay, since $I = V/R$, the current is strong enough to flow through the inductor.

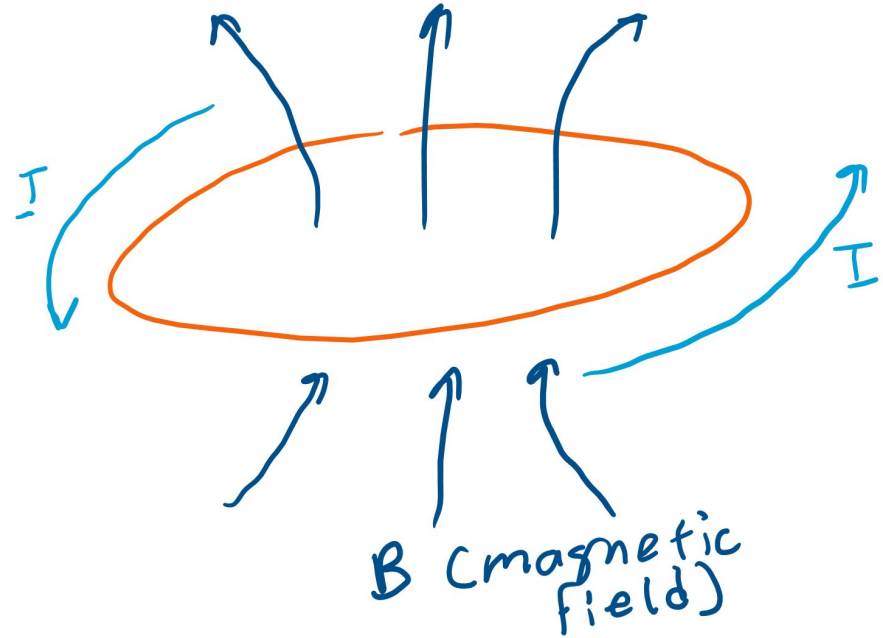


Induction

Electric current can induce a magnetic field

When current flows in a loop, a magnetic field can be generated.

This idea is fundamental behind electromagnetism.



Faraday's (and Lenz's) law

An EMF induces a magnetic field, and a changing magnetic field induces an EMF. This idea is Faraday's law.

The EMF opposes the change in magnetic flux and or the current. This idea is Lenz's law.

Faraday's Law

$$\mathcal{E} = - \frac{d\Phi_B}{dt}$$

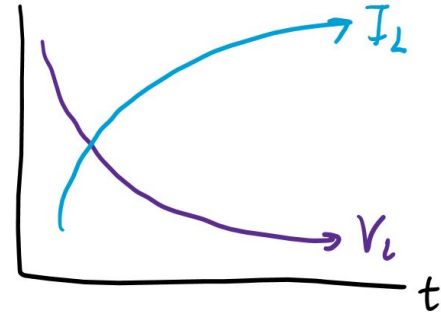
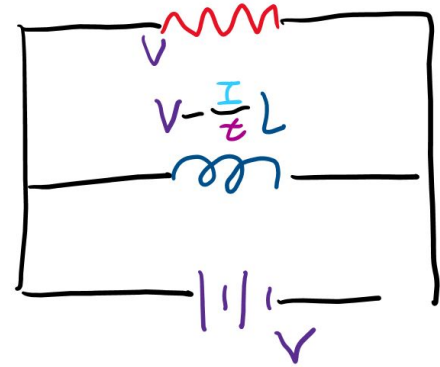
Annotations:

- \mathcal{E} : Voltage
- $-$: Lenz's law
- Φ_B : Amount of magnetic "stuff"
- t : time

Inductors

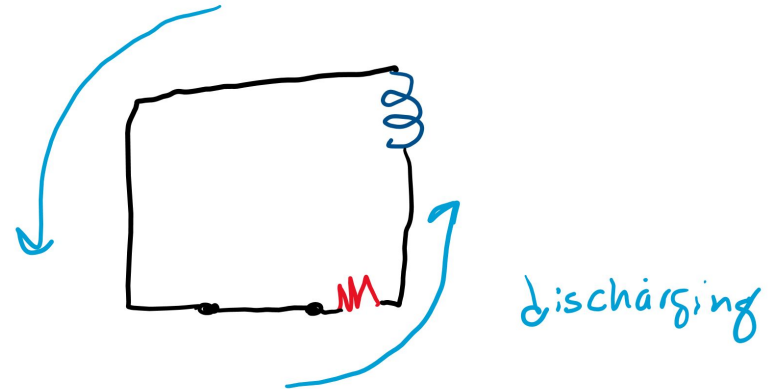
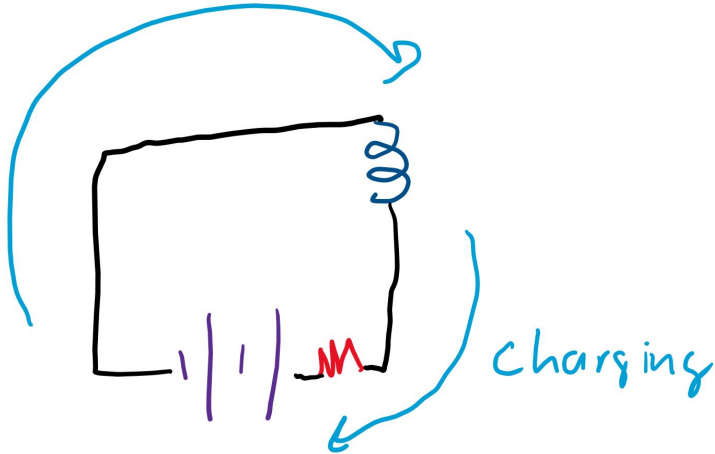
Voltage (EMF) on an inductor is measured as “inductance” times the change in current (the EMF opposes change in current).

Since $\text{EMF} = -L(dI/dt)$, total voltage decreases as current increases. That means as the voltage drops to zero, the current increases by less and less.



Discharging an inductor

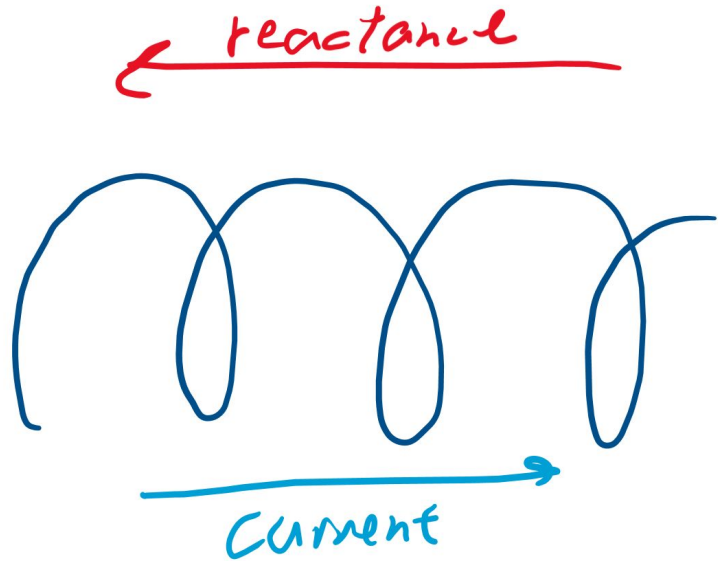
Like a capacitor, the current will go in the opposite direction when discharging.



Reactance

Where does the resistance go?

An inductor impedes current really well at first, but suddenly that impedance lowers. The part that “disappears” when an inductor starts moving is called “reactance.”



Inductive reactance

The faster current moves, the larger the EMF that opposes the current becomes, and the harder it is to charge an inductor.

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$$X_L \text{ (reactance)} = L\omega$$

$$V = L \frac{I}{t}$$

$$\frac{V}{I} = L \omega$$

$$X_L = L \omega$$

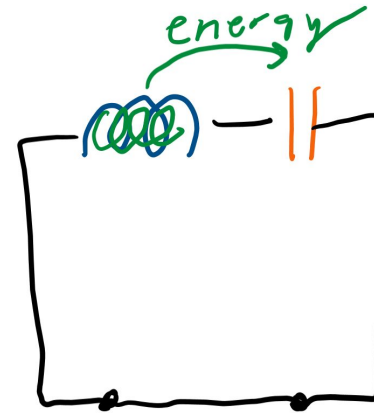
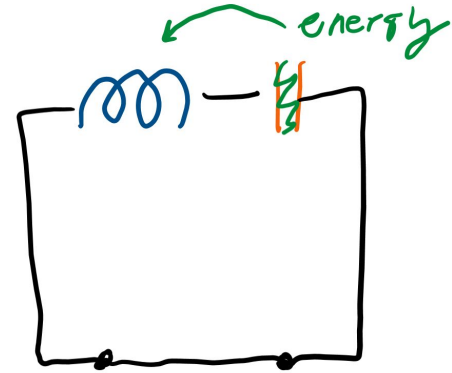
LC circuits

Start with L and C

Let's start by putting all the energy on the capacitor. What will happen after a long time to the energy on the capacitor?

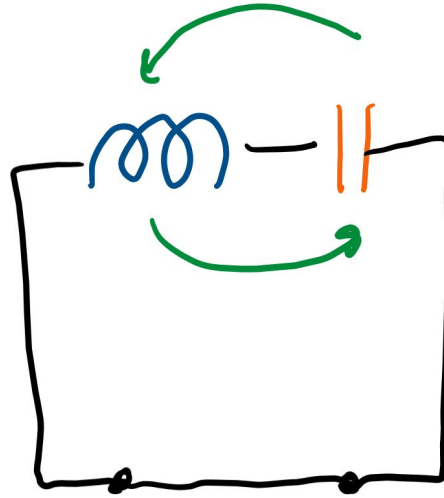
The capacitor should discharge energy into the inductor.

After a long time and the inductor is fully charged, what will happen to the energy on the inductor?



Oscillatory motion

We see that the current alternates from the capacitor to the inductor.
This is called “alternating current.”

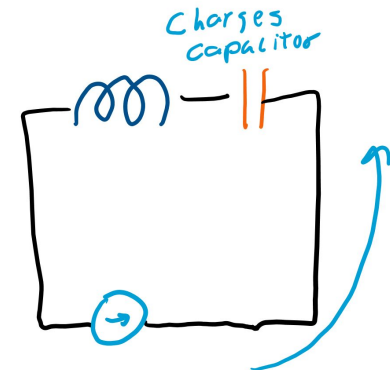
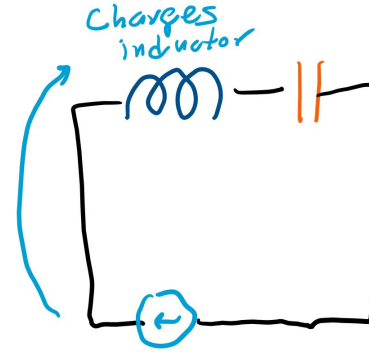


Alternating
direction

Introducing an alternating current source

Let's say that when the inductor is charging, we add energy to the inductor, and when the capacitor is charging, we add energy to the capacitor.

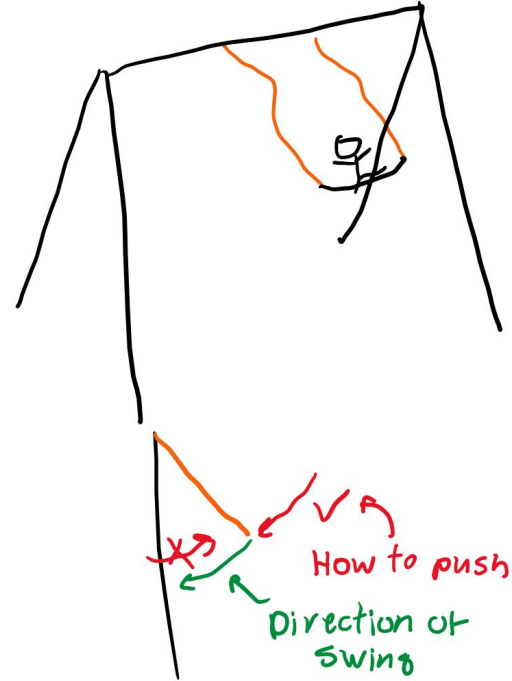
This requires us to alternate the direction we provide energy, but if we do it perfectly, we can increase the total amount of energy in the system.



AC source out of phase

What if we do it not perfectly? Then we're not charging it perfectly. Imagine we're pushing a swing. If we push the swing in the wrong direction, we lose energy!

Charging the inductor while the capacitor is gaining energy is pushing the swing in the wrong direction.



Capacitive reactance

When we're charging the capacitor, we actually have reactance since we use a current to induce a voltage.

The equation for this is $X_c = 1/(C\omega)$

Note that when the change in voltage is instantaneous (no inductor), no reactance exists.

$$V = CQ = CIt$$

$$\frac{V}{I} = Ct$$

$$X_c = \frac{1}{C\omega}$$

Impedance

Resistance and reactance

Resistance and reactance behave differently, with reactance being “perpendicular” to resistance.

Reactance, resistance, and the “hypotenuse” of the right triangle, impedance, have the relationship where impedance (Z) squared is resistance squared plus reactance squared.

