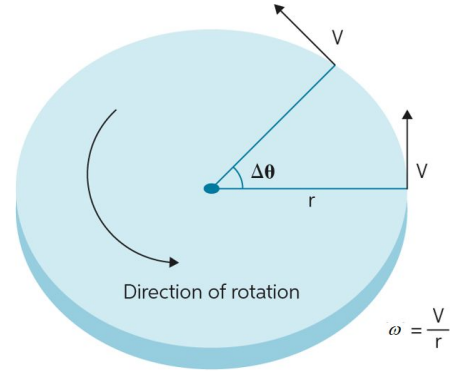

Rotational Motion

Circle movement

Rotating Objects?

- Wheel
- Disk
- Sphere
- Lots of things can rotate!



Variables

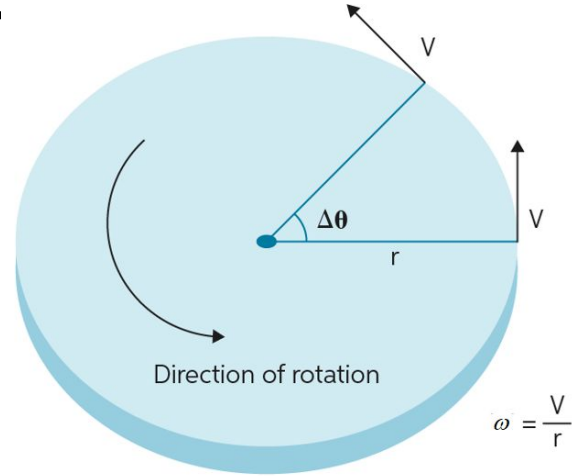
Angular displacement (θ): Change in Angle

Angular velocity (ω): Change in angle of time

Angular acceleration (α): Change of angular velocity over time

Radius (r): How far away something is

Moment of Inertia (I): like mass but for rotating objects



These are Awfully Similar to Kinematics...

- Everything has a linear equivalent!
- Kinematic equations still apply
- A horizontally mounted wheel is initially at rest, and then begins to accelerate constantly until it has reached an angular velocity 10π after 5 complete revolutions. What was the angular acceleration of the wheel?

Equations

$$1. \omega = \omega_0 + \alpha t$$

$$2. \theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$3. \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$4. \theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$$

More Kinematic and Rotational Connections

- $r \cdot \Delta\theta = \Delta x$

- $r \cdot \omega = v$

- $r \cdot \alpha = a$
 - Units?
 - Radians!

- What is the acceleration of a point a distance of 3m away from the rotational point and an angular acceleration of 3 rad/s²?

$$v = \omega r$$

v is linear speed, ω is angular speed, and r is radius

The linear speed is proportional to the angular speed and the radius.

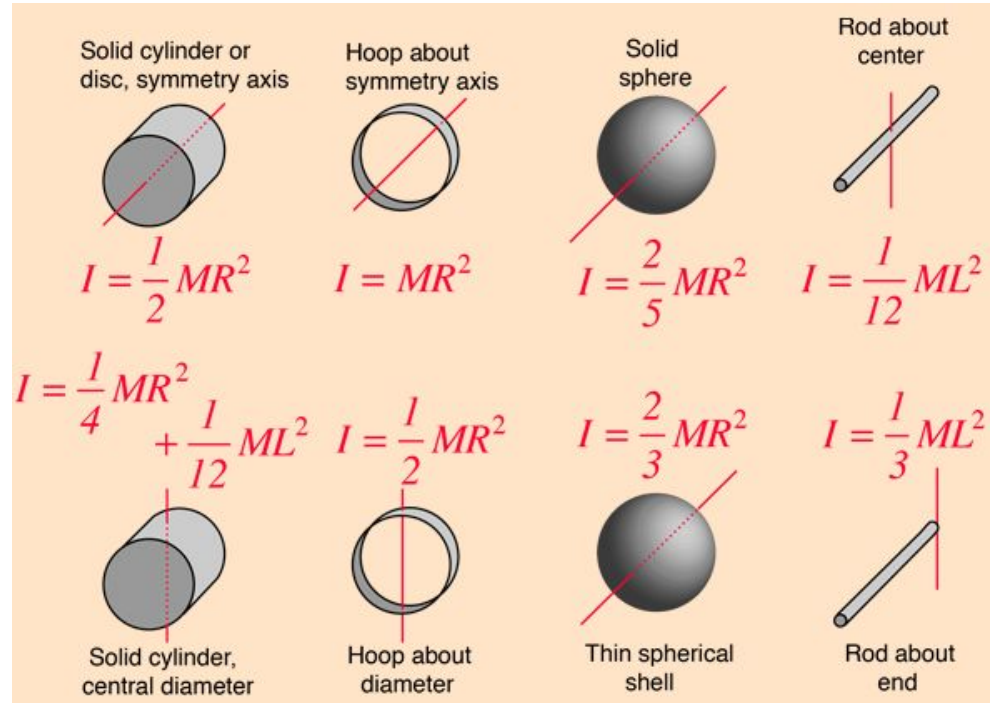
$$a_t = \alpha r$$

a_t is tangential acceleration

The tangential acceleration is proportional to the angular acceleration and the radius.

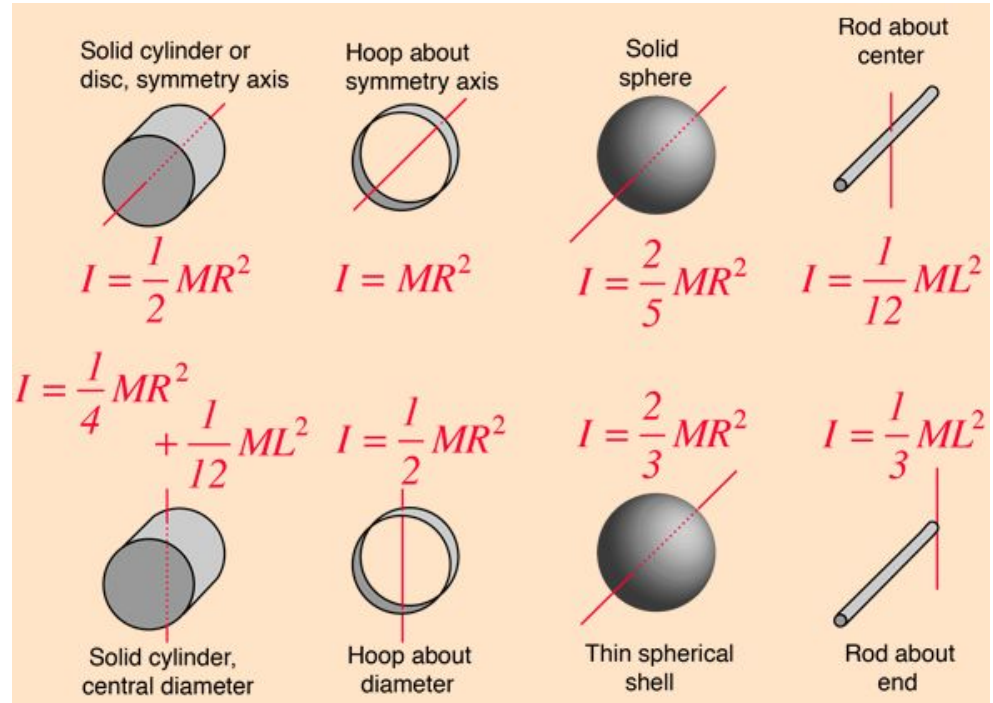
Moment of Inertia

- Rotational Mass
- Equations depend on shape
 - Why it is harder to rotate an unfilled hoop
 - <https://www.youtube.com/watch?v=CHQOctEvtTY>
 - Units?



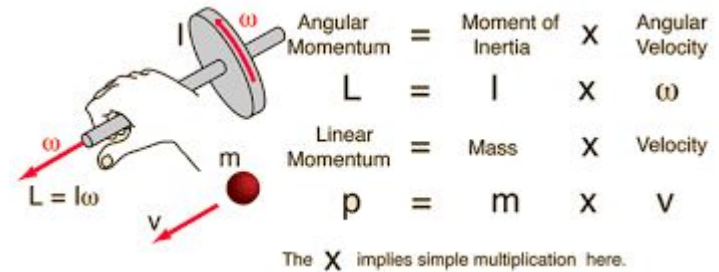
Practice Question

- What is the moment of inertia of a sphere with mass 6kg and radius 5m?
- What about a solid cylinder?
- Or a spherical shell?



Angular Momentum

- Momentum has a rotational equivalent!
- Angular momentum (L)
 - Momentum is usually $p=mv$
 - Rotational equivalents
 - Also $L=mvr$ for a point
- Why is this useful?
 - Conservation of angular momentum!
 - Angular Impulse...
 - Torque!

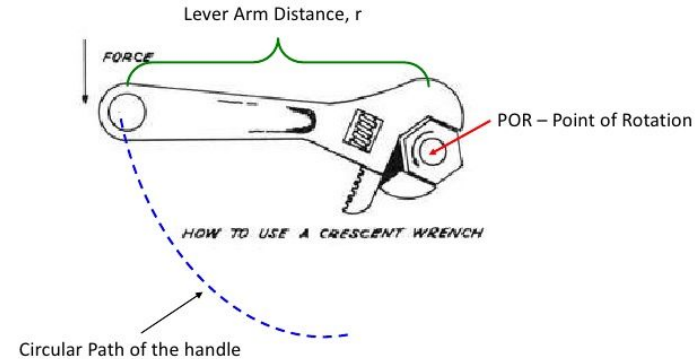


Torque

- Force has a rotational equivalent!
- Torque (τ)
 - Rotational Force
 - Force is usually $F=ma$
 - Rotational equivalents?
 - Torque is also $\tau = Fd = I\alpha$
 - Torques cause rotation!
 - Units?
- Rotational Impulse
 - Linear impulse, Ft
 - Rotational equivalents?

What is Torque?

Torque is defined as the Force that is applied TANGENT to the circle and at some lever arm distance causing rotation around a specific point.



Practice

A figure skater spins at a rate of 4 revolutions/second with her arms outstretched. Her rotational inertia is 2 kg m^2 . When she pulls her arms in, her rotational inertia is 0.64 kg m^2 .

What is her initial angular momentum?

What will be her angular velocity after her arms are pulled in?

A flywheel is a solid disk with a mass of 10 kg and a radius of 2 meters. What torque is necessary to accelerate the flywheel from 20 revolutions/minute to 50 revolutions/minute in 10 seconds?

$$(I = \frac{1}{2} * m * r^2)$$

Energy

- Rotational motion has energy!
 - Linear Kinetic energy is $\frac{1}{2}mv^2$
 - Rotational equivalents?
 - What is the rotational energy of a wheel with moment of inertia of $3 \text{ kg}\cdot\text{m}^2$ and angular velocity of 2 rad/s ?
-

Centripetal Acceleration

- <https://www.youtube.com/watch?v=bpFK2VCRHUs>

- $$a_c = \frac{v^2}{r}$$

- Units?
 - What is the centripetal acceleration of an object with a velocity of 6 m/s rotating at a radius of 3m?
-

Review Videos

<https://www.youtube.com/watch?v=fmXFWi-WfyU>

<https://www.youtube.com/watch?v=b-HZ1SZPaQw>
