

CHAP 09 Computer Arithmetic

ALU (Arithmetic and Logic Unit)

- does calculations
- handles integers, may handle floating point num,
- May be separate FPU (math's coprocessor)
- May be on chip separate FPU



□ Integer Representation — no sign, no period

41 → 00101001

□ Sign-Magnitude — left most bit is sign bit

+18 → 00010010

-18 → 10010010

Problem → need to consider sign, magnitude in arithmetic

→ two representation of 0

+0 → 0000 0000

-0 → 1000 0000

Two's Complement

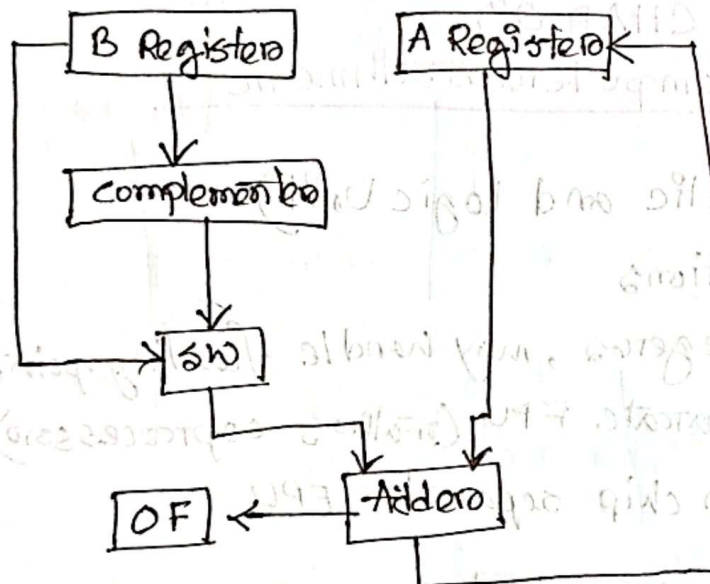
+3 → 0000 0011

-3 → 1111 1101

+0 → 0000 0000

-0 → 0000 0000

Hardware for addition



$$a - b = a + (-b)$$

Multiplication — complex

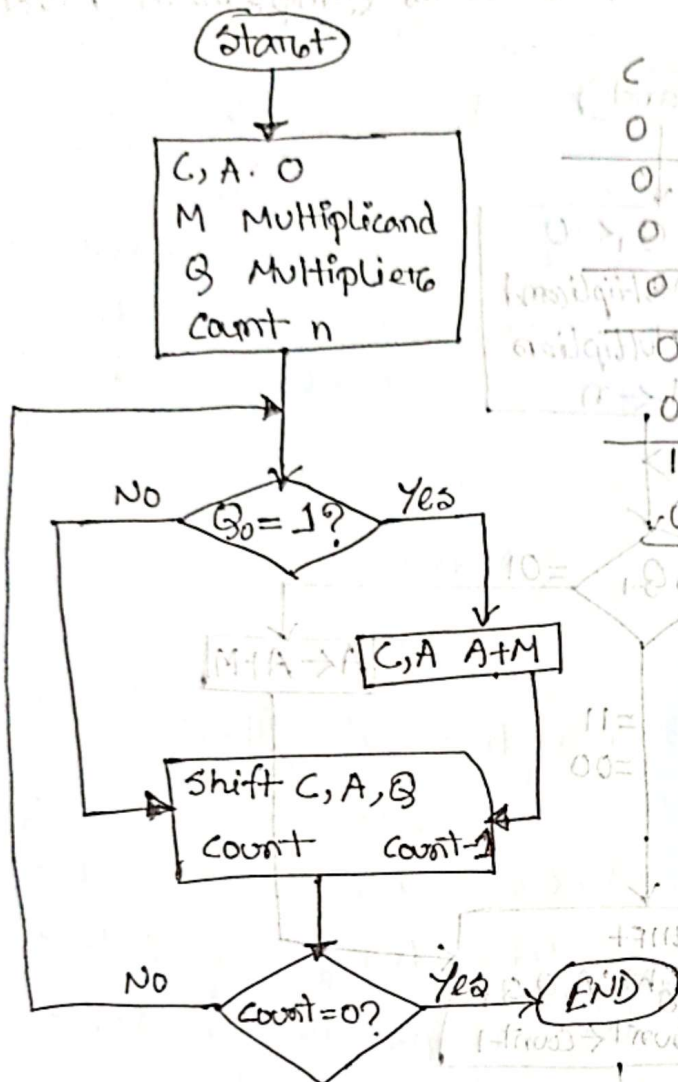
$$\begin{array}{r}
 1011 \text{ Multiplicand (11)} \\
 \times 1101 \text{ Multiplier (13)} \\
 \hline
 1011 \\
 0000 \\
 1011 \\
 1011 \\
 \hline
 10001111 \text{ (sum)}
 \end{array}$$

Partial Product

(143 = 11 × 13)

Multiplication of Unsigned Binary Integers

Flowchart for unsigned Binary Multiplication



C	A	Q ₂ Q ₁	M	
0	0000	1101	1011	
0	1011	1101	1011	ADD
0	0101	1110	1011	SHIFT
0	0010	1111	1011	SHIFT ($Q_0 = 0$)
0	1101	1111	1011	ADD ($A + M$)
0	0110	1111	1011	SHIFT
1	0001	1111	1011	ADD
0	1000	1111	1011	SHIFT

(7x3)

A

Q

Q₋₁

M

0000

0011

0

0111

0001

1001

0011

0

0111

(10)

1100

1001

1

0111

1110

0100

1

0111

11

0101

0100

1

0111

0010

1010

0

0111

0001

0101

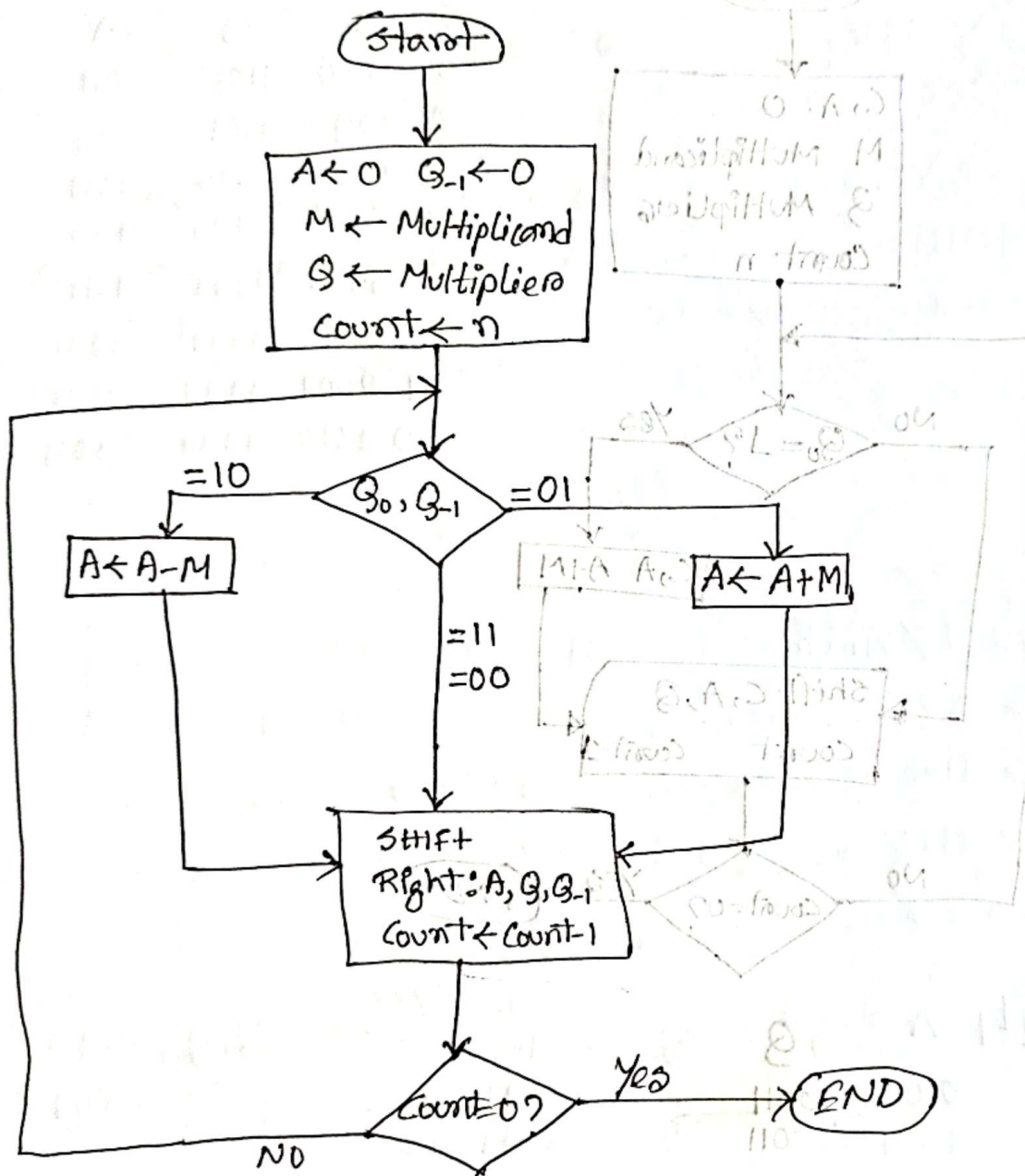
0

0111

00

$Q_0, Q_{-1} \rightarrow 10 (A \leftarrow A - M)$
 $\rightarrow 01 (A \leftarrow A + M)$

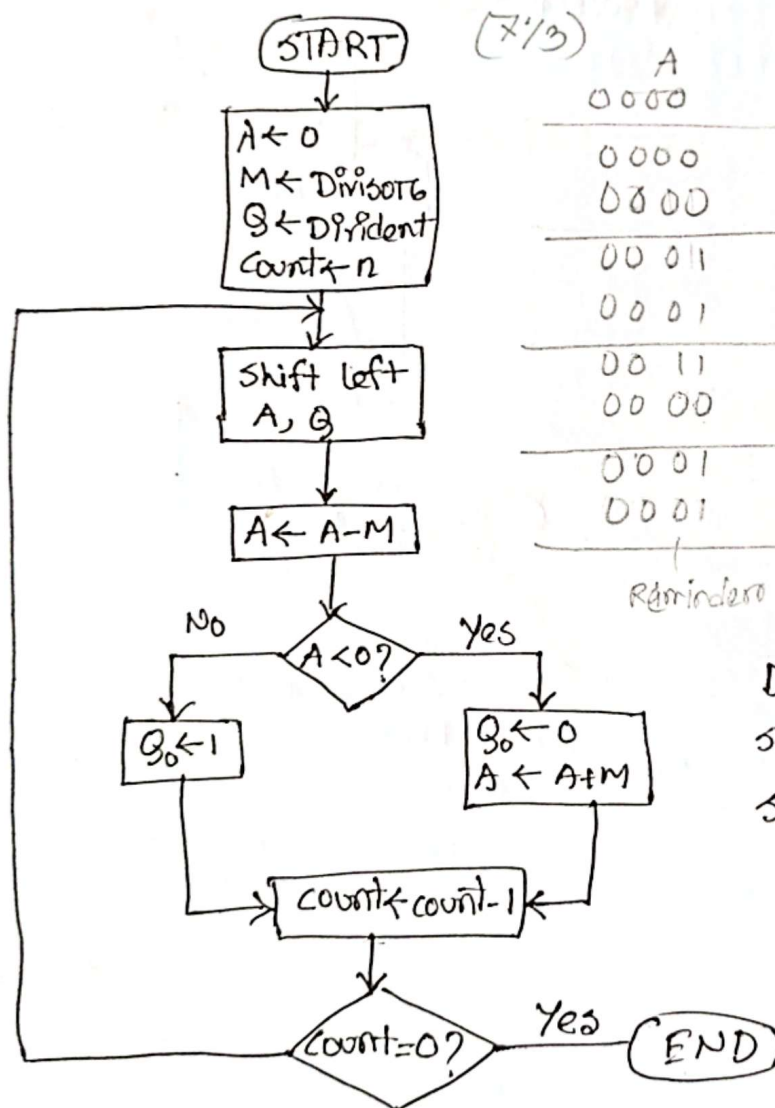
Booth's Algorithm for two's complement Multiplication



Division

$$\begin{array}{r}
 \text{Divisor} \rightarrow 1011 \overline{) 00001101} \\
 \underline{10010011} \\
 001110 \\
 \underline{1011} \\
 001111 \\
 \underline{1011} \\
 0100
 \end{array}$$

Quotient: 00001101
 Divident: 10010011
 Partial Remainders: 001110, 001111
 Remainder: 0100



A	Q	
0000	0111	
0000	1110	A - M = 1101
0000	1110	Q ← previous
0001	1100	
0001	1100	
0011	1000	
0000	1001	
0001	0010	
0001	0010	

Remainders: 1101, 1001, 0010, 0010
Quotient: 0111

$$\begin{aligned}
 D &= Q \times V + R \\
 \text{sign}(R) &= \text{sign}(D) \\
 \text{sign}(Q) &= \text{sign}(D) \times \text{sign}(V)
 \end{aligned}$$