Lecture 05

Date: 16 Jan 2024

Longest Increasing Subsequence. (LIS)

Input:- $A[1 \ldots n]$

Output:- Length of the longest increasing subsequence

$3 \quad 2 \quad 5 \quad 7 \quad 4 \quad 6$

$1 \quad 2 \quad 3 \quad 4 \quad 5$

$m[i] :=$ length of the LIS ending at $A[i]$.
(include $A[i]$.

$m[1] = 1 \qquad m[2] = 1 \qquad m[3] = 2, \quad m[4] = 3 \quad m[5] = 2$

$m[6] = 3.$

$$m[1] = 1$$

$$m[i] = 1 + \max_{\substack{j<i \\ A[j] \leq A[i]}} \{m[j]\} \quad i > 1$$

$$A[1] \; A[\,] \; \boxed{} \; - \; A[i-1] \; A[i]$$

$$A[5] \leq A[i]$$

$$\sum_{i=2}^{n} (i-1) = O(n^2)$$

Aim:- $O(n \log n)$

$d[i]$ = smallest element at which an increasing sequence of length i ends.

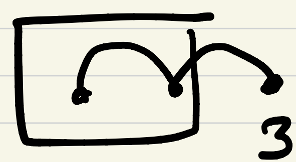
$$A \quad \underbrace{3, \; 2, \; 5,}\; 7 \,|\, 4$$

$d[1] = 2$ | 2
$d[2] = 5$ | $\boxed{4}$
$d[3] = 7$ | 7
$d[4] = \infty$ | ∞

Claim:- $\boxed{d[1] \leq d[2] \leq \cdots \leq d[n].}$

Why?

$d[2] = d[3] = 3$



Suppose $d[i] > d[i+1]$ ——— $*$

Assume, $d[i]$ & $d[i+1]$ are defined.

$$\longleftarrow i+1 \xrightarrow{\;x\;} d[i+1] = a_j \qquad \begin{array}{cccc} x_1 & x_2 \cdots & x_i & x_{i+1} \\ & & \shortparallel & = a_j \\ & & x & \end{array}$$

$$x \leq a_j$$

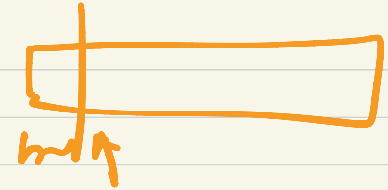
at x we have a seq of length i.

$$d[i] \leq x \leq a_j \underset{*}{<} d[i]$$

$\Rightarrow d[i] < d[i]$ contradiction.

Initialization

Set:- $d[1] = a_1, \ d[2] = \infty, \ \ldots, \ d[n] = \infty$



The algorithm has $n-1$ stages
$s = 2, \ldots, n$

At stage s we make changes to d s.t.
for $A[1 \ldots s]$, $d[1], \ldots, d[s]$ are correct.

At stage s find i s.t

$$[\, d[1] \le d[2] \le \cdots \le d[i] < A[s] \le d[i+1] \le \cdots \le d[s-1] \,]$$

By binary search $O(\log n)$.

$$[\text{A case } i = s-1 \quad d[1] \le \cdots \le d[\ell-1] < A[s] \,]$$

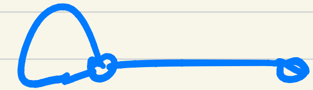
$$d[i+1] \leftarrow A[s]$$

$$d[\ell] \leftarrow A_s.$$

Runtime $O(n \log n)$

Graph:-

Simple, undirected graph.



Def^n:- A graph is a pair (V, E) where $E \subseteq \mathcal{P}_2(V)$.

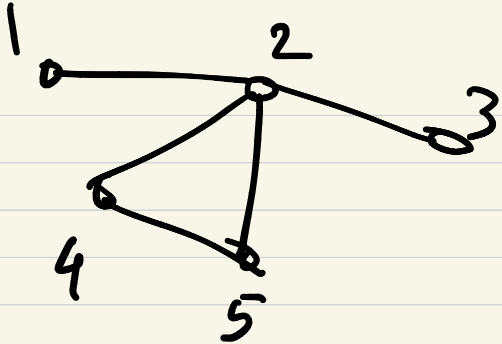
$\mathcal{P}_2$.

$\mathcal{P}_2(V)$ = all 2-element subsets of V.

$V = \{1, 2, 3, 4\}$

$\mathcal{P}_2(V) = \{\{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}\}$

Q: $|V| = 10$, $|\mathcal{P}_2(V)| = \binom{10}{2}$



(1,2)

$V = \{1, \dots, 5\}$

$E = \{\{1,2\}, \{2,3\}, \{2,4\}, \{2,5\}, \{4,5\}\}$

Adj matrix M

$$M[i][j] = \begin{cases} 0 & \{i,j\} \notin E \\ 1 & \text{o/w} \end{cases}$$

Adj list

$L[1] \rightarrow \boxed{2}$

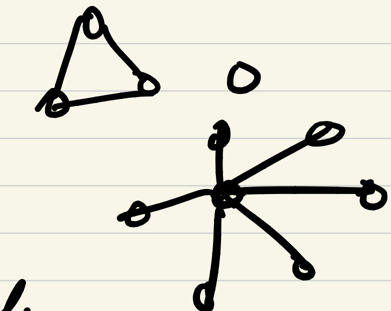
$L(2) \rightarrow \boxed{1 \mid} \rightarrow \boxed{3 \mid} - \boxed{}$ - - -

$\vdots$

$L[5]$

Q:- Given M how to find $|E|$?

Q:- Given M how do we check if G is a tree?



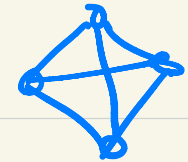
→ Count edges

Check if it is $n-1$

[HW] ← → There should be no cycle. → ?

→ Check if it is connected. [DFS or BFS]

Complete graph K_n

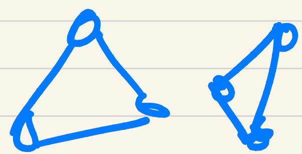


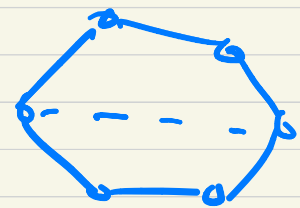
Q:- Given M for G, how to check if G is a complete graph?

Q:- - - - - - - if G is a cycle.

→ deg 2

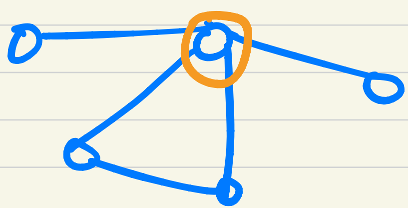
→ connected

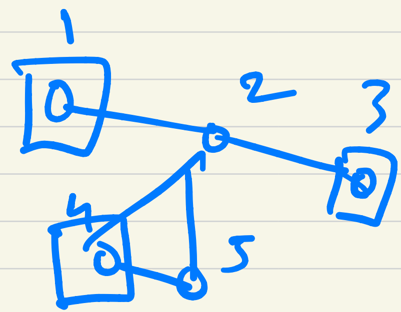




Independent Set (IS)

Def^n: Let $G=(V,E)$ be a graph. A subset $I \subseteq V$ is called an IS if $\forall\, u,v \in I \quad \{u,v\} \notin E$.





$\{1,3,4\}$