

ES-215 Assignment 1

Birudugadda Srivibhav (22110050)

Computer Science And Engineering, Indian Institute Of Technology Gandhinagar

Note: All Codes can be found here : <https://github.com/Sparky1743/ES-215-Assignments>

Question -1

Implement a program(s) to list the first 50 fibonacci numbers preferably in C/C++

a) Using Recursion:

The program consists of two main functions: `fibonacci_recursive` and `n_fibonacci_recursive`.

```
long long fibonacci_recursive(int n) {
    if (n == 0) return 0;
    if (n == 1) return 1;
    return fibonacci_recursive(n - 1) + fibonacci_recursive(n - 2);
}

vector <long long> n_fibonacci_recursive(int n){
    vector <long long> n_fibonacci_numbers;
    for (int i = 1; i < n + 1; i++){
        n_fibonacci_numbers.push_back(fibonacci_recursive(i));
    }
    return n_fibonacci_numbers;
}
```

`fibonacci_recursive(int n):`

This is a recursive function that takes an integer `n` as input and returns the `n`-th Fibonacci number.

The function checks for the base cases:

- 1) If `n` is 0, it returns 0.
- 2) If `n` is 1, it returns 1.

For all other values of `n`, it returns the sum of the Fibonacci numbers of the two preceding positions:
`fibonacci_recursive(n - 1) + fibonacci_recursive(n - 2)`.

`n_fibonacci_recursive(int n):`

This function generates the first `n` Fibonacci numbers. It uses a vector to store the Fibonacci numbers.

A loop iterates from 1 to `n`, calling `fibonacci_recursive(i)` for each iteration and appending the result to the vector.

The vector of Fibonacci numbers is returned.

b) Using Loops

The program consists of two main functions: `fibonacci_iterative` and `n_fibonacci_iterative`.

```
long long fibonacci_iterative(long long n) {
    if (n == 0) return 0;
    if (n == 1) return 1;

    long long n_minus_one = 1;
    long long n_minus_two = 0;
    long long n_th_term;

    for (int i = 2; i <= n; i++) {
        n_th_term = n_minus_one + n_minus_two;
        n_minus_two = n_minus_one;
        n_minus_one = n_th_term;
    }

    return n_th_term;
}

vector<long long> n_fibonacci_iterative(int n){
    vector<long long> n_fibonacci_numbers;
    for (int i = 1; i < n + 1; i++){
        n_fibonacci_numbers.push_back(fibonacci_iterative(i));
    }
    return n_fibonacci_numbers;
}
```

`fibonacci_iterative(long long n):`

This function computes the n -th Fibonacci number using an iterative loop. It handles the base cases for $n = 0$ and $n = 1$ by returning 0 and 1, respectively. For $n > 1$, the function initializes two variables, `n_minus_one` and `n_minus_two`, to represent the two preceding Fibonacci numbers. The loop starts from 2 and iterates up to n , updating the variables to calculate the current Fibonacci number, `n_th_term`, without recalculating any Fibonacci number.

`n_fibonacci_iterative(int n):`

This function generates the first n Fibonacci numbers using the `fibonacci_iterative` function. A vector is used to store the Fibonacci numbers. A loop runs from 1 to n , calling `fibonacci_iterative(i)` for each iteration and appending the result to the vector. The vector containing the Fibonacci sequence is returned.

c) Using Recursion and memoization

The program consists of the following key component:

```
long long fibo_recursive_memoized(int n, vector<long long> &memo) {
    if (n == 0) return 0;
    if (n == 1) return 1;
    if (memo[n] != -1) return memo[n];

    memo[n] = fibo_recursive_memoized(n - 1, memo) + fibo_recursive_memoized(n - 2, memo);
    return memo[n];
}
```

fibo_recursive_memoized(int n, vector<long long> &memo):

This function calculates the n-th Fibonacci number recursively, with memoization to avoid redundant calculations.

Base cases are checked first:

- 1) If n is 0, it returns 0.
- 2) If n is 1, it returns 1.

The function checks if the result for the current n is already stored in the memo vector. If so, it returns the stored value to avoid recalculating it. If the result is not memoized, the function computes it recursively and stores the result in the memo vector.

d) Using Loops and memoization

The program consists of the following key component:

```
long long fibo_iterative_memoized(int n, vector<long long> &memo) {
    if (n == 0) return 0;
    if (n == 1) return 1;

    memo[0] = 0;
    memo[1] = 1;

    for (int i = 2; i <= n; i++) {
        memo[i] = memo[i - 1] + memo[i - 2];
    }

    return memo[n];
}
```

fibo_iterative_memoized(int n, vector<long long> &memo):

This function computes the n-th Fibonacci number using an iterative approach while storing each result in a memoization vector to prevent redundant calculations.

Base cases are immediately handled:

- 1) If n is 0, the function returns 0.
- 2) If n is 1, the function returns 1.

The memo vector is initialized such that memo[0] equals 0 and memo[1] equals 1. A loop iterates from 2 to n, filling the memo vector with the Fibonacci values, ensuring that each value is computed only once.

Time Computation:

```
struct timespec start, end;

clock_gettime(CLOCK_PROCESS_CPUTIME_ID, &start);

n_fibo_recursive(50);

clock_gettime(CLOCK_PROCESS_CPUTIME_ID, &end);

// time taken
long long seconds = end.tv_sec - start.tv_sec;
long long nanoseconds = end.tv_nsec - start.tv_nsec;
if (nanoseconds < 0) {
    seconds--;
    nanoseconds += 1000000000L;
}
```

The execution time for each of the four Fibonacci implementations was measured using the `clock_gettime` function with the `CLOCK_PROCESS_CPUTIME_ID` setting to get the CPU time. This method provides a high-resolution measure of time, recorded in seconds and nanoseconds. By capturing the time just before and after the function execution, I calculated the elapsed time in nanoseconds to ensure precise measurement. Using this consistent timing method across all implementations allows for a fair comparison of their performance, highlighting differences in speed due to their respective algorithmic efficiency and optimization techniques.

Time taken:

- 1) recursion: 282.695997200 seconds
- 2) loops: 0.000008000 seconds
- 3) recursive memoized: 0.000001000 seconds
- 4) loop memoized: 0.000000500 seconds

The first 50 fibonacci numbers are:

1 1 2 3 5 8 13 21 34 55 89 144 233 377 610 987 1597 2584 4181 6765 10946 17711 28657 46368 75025
121393 196418 317811 514229 832040 1346269 2178309 3524578 5702887 9227465 14930352 24157817
39088169 63245986 102334155 165580141 267914296 433494437 701408733 1134903170 1836311903
2971215073 4807526976 7778742049 12586269025

Speed up = (Execution time of baseline algorithm) / (Execution time of improved algorithm)

$$S = T_{\text{baseline}} / T_{\text{improved}}$$

- 1) loops implementation is 35,336,999.65 times faster than recursive implementation
- 2) recursive memoized implementation is 282,695,997.2 times faster than recursive implementation
- 3) loops memoized implementation is 565,391,994.4 times faster than recursive implementation

Question -2

Write a simple Matrix Multiplication program for a given NxN matrix in any two of your preferred Languages from the following listed buckets, where N is iterated through the set of values 64, 128, 256, 512 and 1024. N can either be hardcoded or specified as input.

Assumptions: The experiments were conducted with matrix sizes of N=64, 128, 256, 512, and 1024, using two data types: integer and double. Timing measurements were taken in several ways: system time was recorded with `CLOCK_MONOTONIC` in C++ and `time.time()` in Python to capture the total elapsed wall-clock time; CPU time was measured using `CLOCK_PROCESS_CPUTIME_ID` in C++ and `time.process_time()` in Python to reflect the time the CPU spent actively executing the program; and matrix multiplication time was specifically measured to assess the efficiency of the core algorithm. A simple three-loop approach was used for matrix multiplication in both C++ and Python implementations, focusing the performance evaluation on the core algorithm.

a) implementation of integer and double matrix multiplication

Integer implementation for cpp:

```
// Function to multiply integer matrices
void multiplyIntegerMatrices(const vector<vector<int>>& matrix1, const vector<vector<int>>& matrix2, vector<vector<int>>& result, int dimension) {
    for (int i = 0; i < dimension; i++) {
        for (int j = 0; j < dimension; j++) {
            result[i][j] = 0;
            for (int k = 0; k < dimension; k++) {
                result[i][j] += matrix1[i][k] * matrix2[k][j];
            }
        }
    }
}
```

Double implementation for cpp:

```
// Function to multiply double matrices
void multiplyFloatingPointMatrices(const vector<vector<double>>& matrix1, const vector<vector<double>>& matrix2, vector<vector<double>>& result, int dimension) {
    for (int i = 0; i < dimension; i++) {
        for (int j = 0; j < dimension; j++) {
            result[i][j] = 0.0;
            for (int k = 0; k < dimension; k++) {
                result[i][j] += matrix1[i][k] * matrix2[k][j];
            }
        }
    }
}
```

Integer implementation for python:

```
def multiply_matrices_int(matrix_a, matrix_b, size):
    result_matrix = [[0] * size for _ in range(size)]
    for row in range(size):
        for col in range(size):
            for k in range(size):
                result_matrix[row][col] += matrix_a[row][k] * matrix_b[k][col]
    return result_matrix
```

Double implementation for python:

```
def multiply_matrices_double(matrix_a, matrix_b, size):  
    result_matrix = [[0.0] * size for _ in range(size)]  
    for row in range(size):  
        for col in range(size):  
            for k in range(size):  
                result_matrix[row][col] += matrix_a[row][k] * matrix_b[k][col]  
    return result_matrix
```

Time calculation:

For CPP:

Dimension	Type	System Time (s)	CPU Time (s)	Multiplication Time (s)
64	Integer	0.002548	0.002518	0.0024871
64	Floating-Point	0.0027406	0.002526	0.0025714
128	Integer	0.0197397	0.015625	0.019626
128	Floating-Point	0.0192835	0.015625	0.0190888
256	Integer	0.1674563	0.140625	0.1670953
256	Floating-Point	0.1617419	0.140625	0.1611423
512	Integer	1.2892412	1.09375	1.2881428
512	Floating-Point	1.3998034	1.234375	1.3974659
1024	Integer	11.3875216	10.5	11.3832001
1024	Floating-Point	22.4779611	19.921875	22.4692746

For Python:

Dimension	Type	System Time (s)	CPU Time (s)	Multiplication Time (s)
64	Integer	0.010989428	0.00921532	0.010942102
64	Double	0.02064085	0.015625	0.02053053
128	Integer	0.13636899	0.09375	0.135789633
128	Double	0.162477493	0.140625	0.162437431
256	Integer	1.125142097	0.9375	1.123148216
256	Double	1.392737627	1.203125	1.392357934
512	Integer	10.35852647	9.484375	10.33232647
512	Double	14.3518312	11.265625	14.3488369
1024	Integer	104.8778133	92.890625	104.8537605
1024	Double	173.6508904	148.125	173.6268759

b) Meat time refers to matrix multiplication time

so proportion = matrix multiplication time / total system time

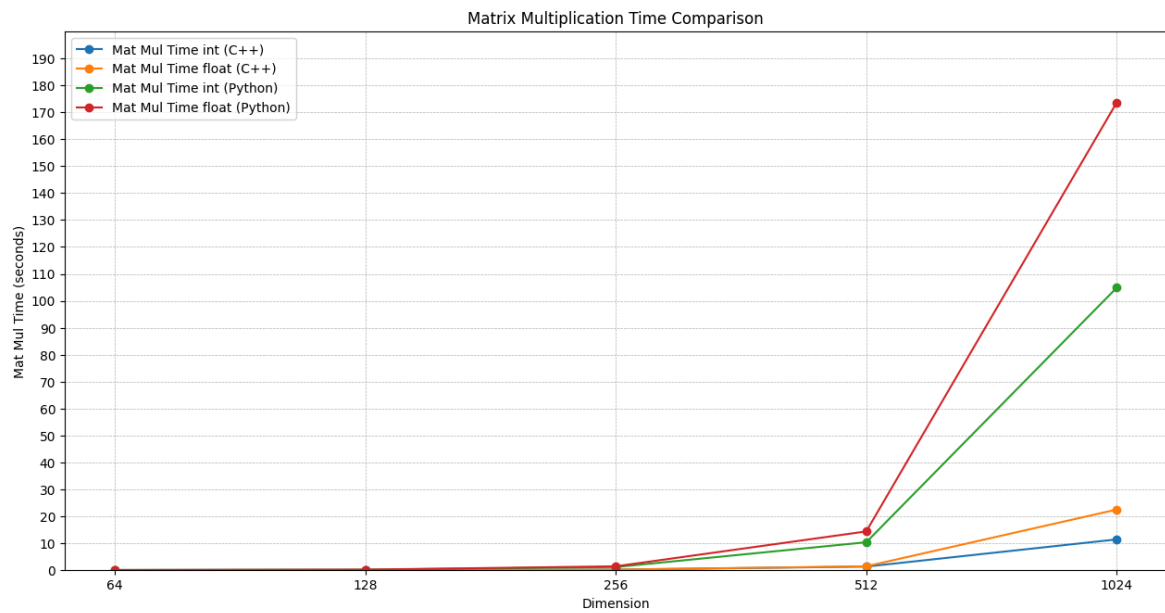
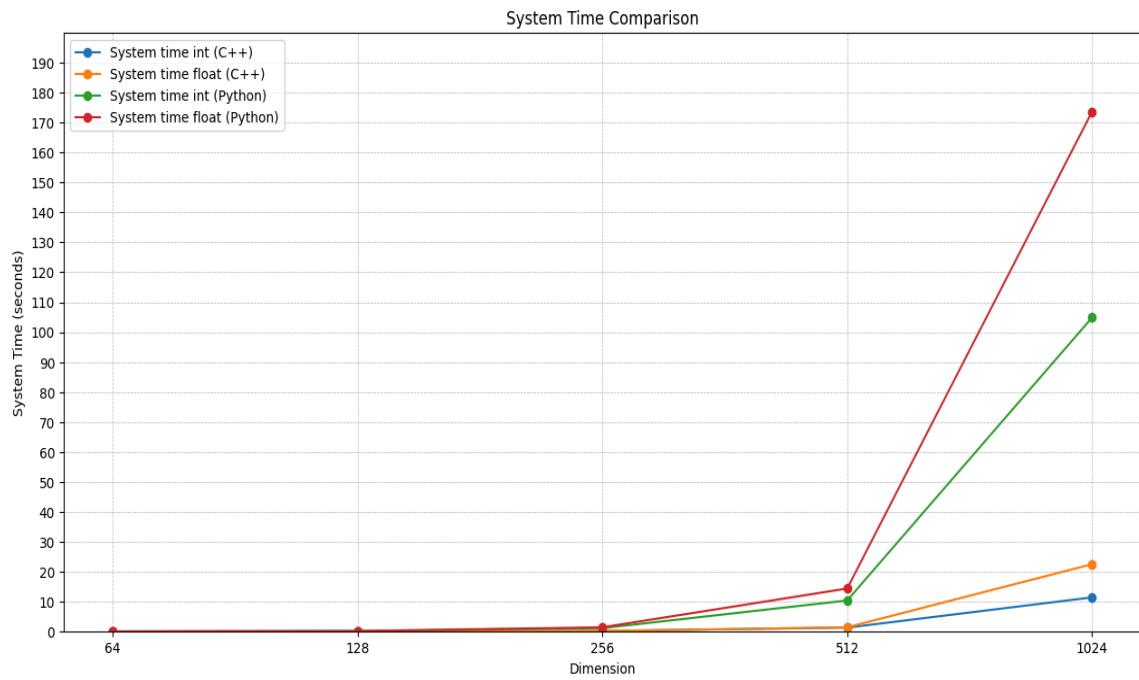
For CPP:

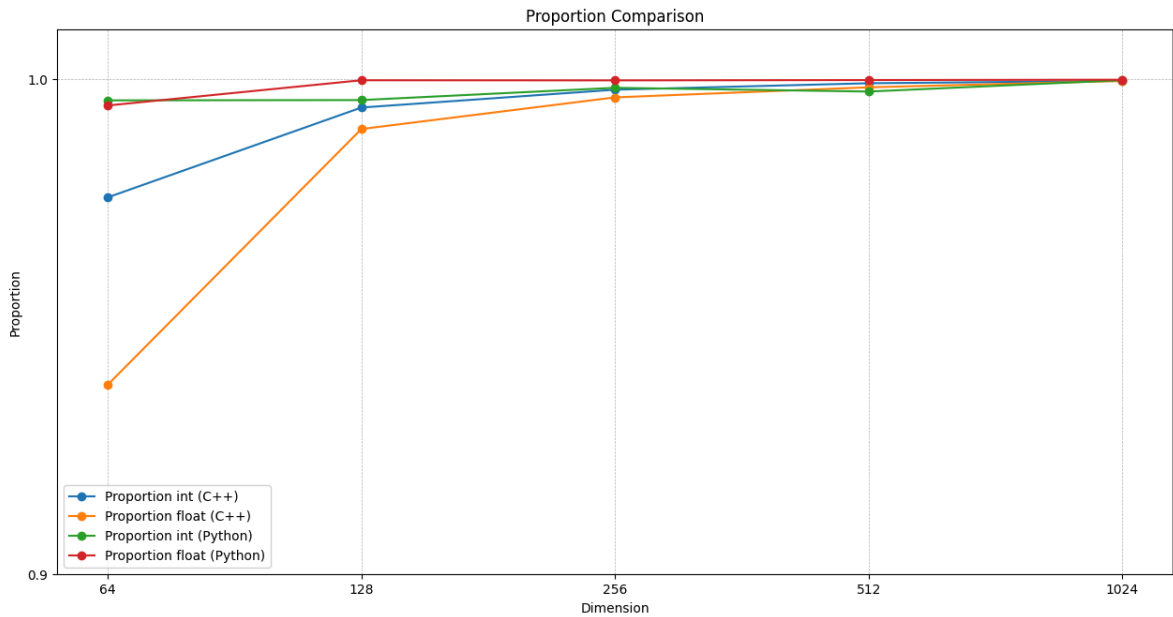
Dimension	Type	System Time (s)	Multiplication Time (s)	Proportion
64	Integer	0.002548	0.0024871	0.9760989
64	Floating-Point	0.0027406	0.0025714	0.9382617
128	Integer	0.0197397	0.019626	0.99424
128	Floating-Point	0.0192835	0.0190888	0.9899033
256	Integer	0.1674563	0.1670953	0.9978442
256	Floating-Point	0.1617419	0.1611423	0.9962929
512	Integer	1.2892412	1.2881428	0.999148
512	Floating-Point	1.3998034	1.3974659	0.9983301
1024	Integer	11.3875216	11.3832001	0.9996205
1024	Floating-Point	22.4779611	22.4692746	0.9996136

For Python:

Dimension	Type	System Time (s)	Multiplication Time (s)	Proportion
64	Integer	0.010989428	0.010942102	0.9956935
64	Double	0.02064085	0.02053053	0.9946553
128	Integer	0.13636899	0.135789633	0.9957515
128	Double	0.162477493	0.162437431	0.9997534
256	Integer	1.125142097	1.123148216	0.9982279
256	Double	1.392737627	1.392357934	0.9997274
512	Integer	10.35852647	10.33232647	0.9974707
512	Double	14.3518312	14.3488369	0.9997914
1024	Integer	104.8778133	104.8537605	0.9997707
1024	Double	173.6508904	173.6268759	0.9998617

c) Graphical representation





In terms of performance, C++ generally exhibits faster system and multiplication times compared to Python, largely due to its compiled nature and more efficient memory management. Conversely, Python tends to have slower execution times, particularly with larger matrices, because of its interpreted nature and additional overhead. In both languages, a significant proportion of the total system time is devoted to matrix multiplication, underscoring the dominant role this operation plays in the overall execution time.