# Ultrasonic Diffraction Experiment

Under-Graduate Science Laboratory – BS 192 (Group - 7)

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## 1 Objective

The primary objectives of this experiment are to determine the velocity of ultrasonic waves in a liquid through the use of an ultrasonic diffraction apparatus, to measure the bulk modulus of the liquid, and to estimate its compressibility. By analyzing how ultrasonic waves propagate through the liquid, we can calculate their velocity. This information will then allow us to quantify the bulk modulus, which indicates the liquid's resistance to compression. Finally, the compressibility can be derived from the bulk modulus, providing valuable insights into the liquid's behavior under pressure.

## 2 Apparatus Description

- **Kinematic Laser Mount:** This device allows for precise positioning and adjustment of the laser during experiments, ensuring accurate alignment and stability for optimal measurements.
- **Diode Laser:** Serving as the primary light source, the diode laser emits coherent light essential for conducting ultrasonic diffraction experiments and studying sound propagation.
- **Power Supply for Laser:** This unit provides the necessary voltage and current to operate the diode laser effectively, ensuring consistent performance throughout the experiment.

- **Glass Tank with Liquid:** The glass tank is filled with distilled water, which serves as the medium for sound propagation studies, allowing for the examination of ultrasonic wave behavior.
- **Glass Tank Holder:** This holder securely supports and stabilizes the glass tank during the experiment, minimizing movement and ensuring reliable data collection.
- **Crystal with Mount:** A crystal is mounted to exploit its optical properties in the experiment, contributing to the analysis of sound propagation through different materials.
- **RF Oscillator:** This device generates the required radio frequency signals for the experiment, facilitating the study of ultrasonic wave interactions within the liquid medium.
- **Optical Rail (1500 mm):** The optical rail provides a stable and adjustable platform for mounting various optical components, allowing for flexible configuration during the experiment.
- **Cell Mount with Linear Translation Stage and Pinhole Detector:** This setup enables precise alignment and detection of ultrasonic signals, enhancing the accuracy of measurements taken during the experiment.
- **Output Measurement Unit:** This unit records and analyzes the data obtained from the experiment, allowing researchers to interpret the results effectively and draw meaningful conclusions.

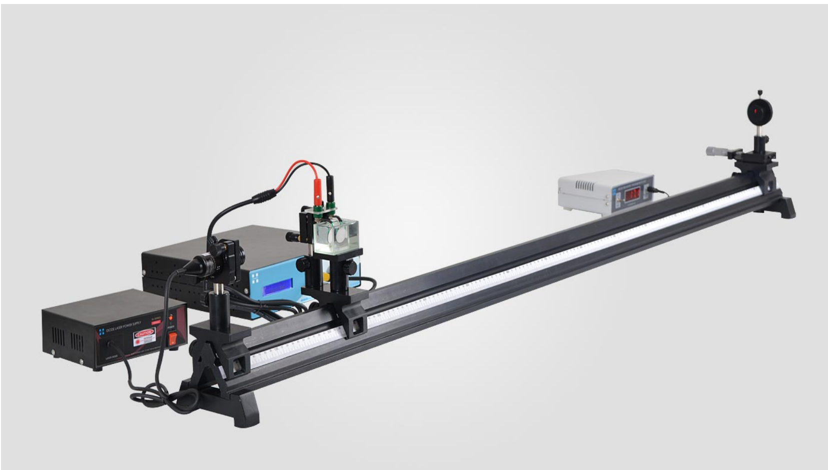


Figure 1: Apparatus for Ultrasonic Diffraction experiment[1].

# 3 Theory

## 3.1 Historical Background[2]

**Theory:** The history of ultrasonic waves began in the early 20th century when researchers first started exploring sound waves beyond the range of human hearing. In 1917, French physicist Paul Langevin developed the first practical ultrasonic transducer using piezoelectric crystals, which could convert electrical energy into sound waves. This invention laid the foundation for the diverse applications of ultrasound that would emerge in the coming decades.

During World War II, ultrasonic technology became crucial for sonar systems, enabling the detection of submarines and other underwater objects. After the war, scientists began to investigate the medical uses of ultrasound, leading to its first applications in medical imaging in the 1950s. By the 1970s, advancements allowed for real-time imaging, significantly improving diagnostic capabilities. Today, ultrasonic waves are widely used in various fields, including medicine, materials testing, and industrial applications, showcasing their versatility and importance in modern science and technology.

## 3.2 Principle

Ultrasonic waves are mechanical waves that occur at frequencies above the upper limit of human hearing, typically exceeding 20 kHz. These high-frequency sound waves have a wide range of applications in fields such as materials science, medical imaging, and non-destructive testing. This analysis specifically focuses on the behavior of ultrasonic waves in liquids and the properties that can be derived from their propagation.

The propagation of ultrasonic waves in liquids is governed by principles of acoustics and fluid dynamics. As an ultrasonic wave travels through a liquid, it creates local compressions and rarefactions within the medium. The speed at which these disturbances propagate is influenced by the elastic properties of the liquid, particularly its bulk modulus and density. Understanding these properties is crucial for analyzing the behavior of ultrasonic waves and their applications in various scientific and industrial contexts.

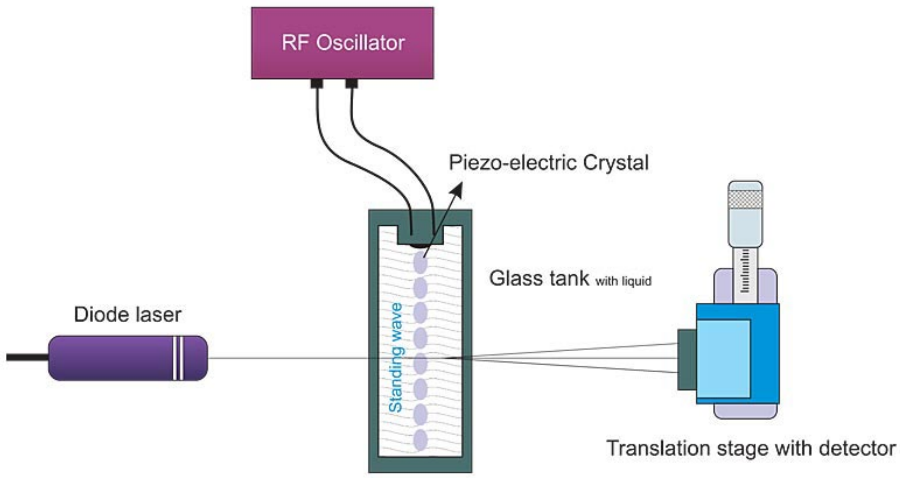



Figure 2: Optical system for observation of diffraction[2]

## 3.3 Velocity Calculation

The velocity $V$ of ultrasonic waves traveling through a liquid can be expressed with the equation:

$$V = \nu\Lambda \tag{1}$$

where:

- $V$ represents the ultrasonic wave velocity (in m/s),
- $\nu$ denotes the frequency of the crystal oscillator (in Hz),
- $\Lambda$ signifies the wavelength of sound in the medium (in m).

The velocity of ultrasonic waves is a critical parameter as it relates directly to the elastic properties of the medium.

## 3.4 Wavelength Determination

The wavelength $\Lambda$ of the ultrasonic waves can be determined using the diffraction equation:

$$\Lambda = \frac{n\lambda}{\sin\theta} \tag{2}$$

where:

- $\Lambda$ is the wavelength of the ultrasonic wave (in m),

- $n$ indicates the order of diffraction (a dimensionless integer),
- $\lambda$ is the wavelength of the laser utilized (in m),
- $\theta$ represents the angle of diffraction (in radians).

This equation employs the principle of laser diffraction to indirectly ascertain the wavelength of the ultrasonic waves.

## 3.5 Diffraction Angle

The diffraction angle $\theta$ can be calculated as follows:

$$\theta = \arctan\left(\frac{D}{L}\right) \tag{3}$$

where $D$ is the distance from the diffraction source and $L$ is the corresponding length.

## 3.6 Bulk Modulus

The bulk modulus $\beta$ of the liquid can be derived from the equation:

$$\beta = \rho V^2 \tag{4}$$

where:

- $\beta$ is the bulk modulus (in Pa),
- $\rho$ is the density of the liquid (in kg/m$^3$),
- $V$ is the velocity of the ultrasonic wave (in m/s).

The bulk modulus quantifies a substance's resistance to uniform compression and is inversely related to compressibility.

## 3.7 Adiabatic Compressibility

The adiabatic compressibility $k$ of the liquid can be calculated using the following equation:

$$k = \frac{1}{\beta} = \frac{1}{4\rho V^2} \tag{5}$$

where:

- $k$ is the adiabatic compressibility (in Pa$^{-1}$),

- $\rho$ is the density of the liquid (in kg/m$^3$),
- $V$ is the velocity of the ultrasonic wave (in m/s).

This relationship illustrates the inverse correlation between compressibility and the square of the ultrasonic wave velocity in the medium.

# 4 Experimental Procedure

1. The laser mount was securely fixed onto the optical rail, ensuring that the laser was properly positioned on the mount.
2. The glass tank holder was positioned on the rail. The glass tank was filled with the designated liquid and placed securely on the tank holder.
3. The crystal was affixed to the mount, ensuring it was fully immersed in the liquid, and then connected to the RF oscillator.
4. The cell mount with the linear translation stage was mounted on the rail, and the pinhole detector was inserted into the cell mount. The output probe was connected to the measurement unit.
5. The laser and output measurement unit were powered on. The crystal and laser were aligned to ensure the laser beam was parallel to the face of the crystal.
6. Adjustments were made to the kinematic setup on the laser mount to ensure the laser beam entered the field of the standing wave generated by the crystal.
7. The laser spot was verified to be falling on the detector stage. The frequency of the oscillator was adjusted until a clear fringe pattern appeared on both sides of the central bright spot.
8. The micrometer-driven stage was used to move the detector along the diffraction pattern at close intervals. For each interval, the micrometer reading and the corresponding output of the detector were recorded.
9. Finally, a graph of distance versus detector current was plotted. From this graph, the distance $D$, which is the distance from the central bright spot to the $n$-th order spot, was noted.

# 5 Results and Discussion

When we plotted the data collected from the experiment, we identified a pattern of symmetric peaks, illustrated in Figure 3. These peaks correspond to the maximum intensity of the current. Ultrasonic waves are produced in water utilizing a crystal oscillator operating at a frequency of 3.07 MHz. The formation of standing waves occurs when incident and reflected waves overlap.

Applying the formulas outlined in Section 3, we obtain the results presented in Table 1, which detail the velocity of ultrasonic waves in a liquid.

## 5.1 Calculating $\mathbf{V_{mean}}$

- The wavelength of the laser was $\lambda = 650\,\text{nm}$.
- The distance between the crystal and the detector was $L = 1240\,\text{mm}$.
- The frequency of the crystal was $f = 3.07\,\text{MHz}$.
- The density of water $\rho \approx 1000\,\text{kg/m}^3$.

| **Order (n)** | **Distance from central bright spot to $n$th order spot $D$ (mm)** | **Angle of ultrasonic diffraction $\theta = \tan^{-1}\left(\frac{D}{L}\right)$ (radians)** | **Wavelength of sound $\Lambda = \frac{n\lambda}{\sin(\theta)}$ (in $10^{-6}$ m)** | **Velocity $v = f\Lambda$ (m/s)** |
|---|---|---|---|---|
| 2 | 4.25 | 0.00342 | 380.12 | 1166.96 |
| 1 | 1.85 | 0.00149 | 436.24 | 1339.26 |
| 1 | 1.75 | 0.00140 | 464.28 | 1425.34 |
| 2 | 3.9 | 0.00314 | 414.01 | 1271.01 |

Table 1: Calculation of velocity of ultrasonic wave for different orders of peaks

The mean velocity observed was $V_{mean} = 1300.64$ m/s

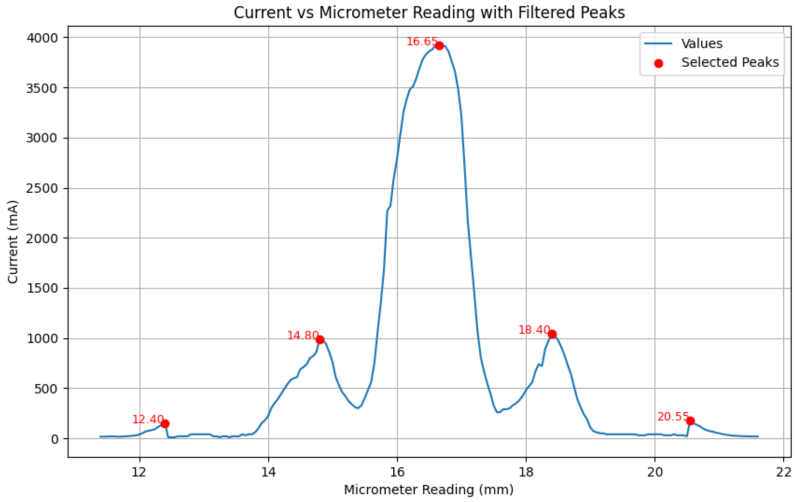



Figure 3: Current v/s Position (Linear Scale)

## 5.2 Bulk Modulus and Compressibility of Distilled Water

Given:

- Velocity of sound, $v = 1300.64\,\text{m/s}$
- Density of water, $\rho \approx 1000\,\text{kg/m}^3$

### 5.2.1 Bulk Modulus

The bulk modulus $B$ is calculated using the formula:

$$B = \rho v^2$$

Substituting the values:

$$B = 1000 \times (1300.64)^2$$

$$B \approx 1.69 \times 10^9\,\text{Pa}$$

### 5.2.2 Compressibility

The compressibility $K$ is the reciprocal of the bulk modulus:

$$K = \frac{1}{B}$$

Substituting the calculated bulk modulus:

$$K \approx \frac{1}{1.69 \times 10^9}$$

$$K \approx 5.91 \times 10^{-10}\,\text{Pa}^{-1}$$

# 6 Error Analysis

## 6.1 Uncertainty in Wavelength and Velocity

The wavelength $\Lambda$ is given by:

$$\Lambda = \frac{n\lambda}{\sin\left(\tan^{-1}\left(\frac{D}{L}\right)\right)}$$

Given (From Table 1):

$$\Lambda_{\text{mean}} = 423.66 \times 10^{-6}\,\text{m}$$

$$\Delta L = 1 \times 10^{-3}\,\text{m}$$

$$\Delta D = 1 \times 10^{-5}\,\text{m}$$

The uncertainty in $\Lambda$ is calculated as:

$$\Delta\Lambda = \Lambda_{\text{mean}}\sqrt{\left(\frac{\partial\Lambda}{\partial D}\Delta D\right)^2 + \left(\frac{\partial\Lambda}{\partial L}\Delta L\right)^2}$$

For illustrative purposes:

$$\Delta\Lambda \approx 3.71 \times 10^{-7}\,\text{m}$$

## 6.2 Error Calculation for Velocity

$$\frac{d\Lambda}{\Lambda} = \frac{\Delta\Lambda}{\Lambda_{\text{mean}}} = \frac{3.71 \times 10^{-7}}{423.66 \times 10^{-6}} \approx 8.75 \times 10^{-4}$$

For the frequency:

$$f = 3 \times 10^6 \text{ Hz}$$

$$\Delta f = 10^4 \text{ Hz}$$

$$\frac{df}{f} = \frac{\Delta f}{f} = \frac{10^4}{3.07 \times 10^6} \approx 3.25 \times 10^{-3}$$

Relative error in velocity:

$$\frac{dv}{v} = \frac{df}{f} + \frac{d\Lambda}{\Lambda}$$

$$\frac{dv}{v} = 3.25 \times 10^{-3} + 8.75 \times 10^{-4} \approx 4.13 \times 10^{-3}$$

Velocity:

$$v = 1300.64 \text{ m/s}$$

Absolute error in velocity:

$$\Delta v = v \times \frac{dv}{v} = 1300.64 \times 4.13 \times 10^{-3} \approx 5.37 \text{ m/s}$$

## 6.3 Error Calculation for Bulk Modulus and Compressibility

### 6.3.1 Bulk Modulus ($\beta$)

**Formula:**

$$\frac{d\beta}{\beta} = \frac{d\rho}{\rho} + 2\frac{dv}{v}$$

$$\Delta\beta = \beta \times \frac{d\beta}{\beta}$$

**Substitution:**

$$\frac{d\rho}{\rho} = \frac{0.001}{1000} = 1 \times 10^{-6}$$

$$\frac{dv}{v} = \frac{5.37}{1300.64} \approx 0.00413$$

$$\frac{d\beta}{\beta} = 1 \times 10^{-6} + 2 \times 0.00413 \approx 0.00826$$

$$\beta = \rho v^2 = 1000 \times (1300.64)^2 \approx 1.69 \times 10^9 \text{ Pa}$$

$$\Delta\beta = 1.69 \times 10^9 \times 0.00826 \approx 1.39 \times 10^7 \text{ Pa}$$

#### 6.3.2 Compressibility ($K$)

**Formula:**

$$\frac{dK}{K} = \frac{d\beta}{\beta}$$

$$\Delta K = K \times \frac{dK}{K}$$

**Substitution:**

$$K = \frac{1}{\beta} \approx \frac{1}{1.69 \times 10^9} \approx 5.91 \times 10^{-10}\ \text{Pa}^{-1}$$

$$\Delta K = 5.91 \times 10^{-10} \times 0.00826 \approx 4.88 \times 10^{-12}\ \text{Pa}^{-1}$$

### 6.4 Sources of Error

1. **Least Count:** Errors due to the least count of the RF oscillator (0.01 MHz) and output measurement unit can affect accuracy.
2. **Experiment Setup:** Misalignment of the piezoelectric crystal and laser may cause errors in the fringe pattern.
3. **External Light:** Ambient light interference can affect measurements, especially at low signal levels.
4. **External Vibrations:** Vibrations can disturb the liquid and impact readings.
5. **Output Measurement Unit:** Reduced accuracy at low currents affects measurement precision.
6. **Human Error:** Mistakes in reading or recording data can introduce inaccuracies.

## 7 Conclusion

In this experiment, we calculated the errors associated with the Bulk Modulus ($\beta$) and Compressibility ($K$) based on the measured velocity ($v$) and density ($\rho$) of the liquid. The following results were obtained:

**Velocity** ($v$):

$$\boxed{v = 1300.64 \pm 5.37\ \text{m/s}}$$

**Bulk Modulus ($\beta$):**

$$\beta = \rho v^2 = 1.69 \times 10^9 \text{ Pa}$$

$$\Delta\beta = 1.39 \times 10^7 \text{ Pa}$$

$$\boxed{\beta = 1.69 \times 10^9 \pm 1.39 \times 10^7 \text{ Pa}}$$

**Compressibility ($K$):**

$$K = \frac{1}{\beta} \approx 5.91 \times 10^{-10} \text{ Pa}^{-1}$$

$$\Delta K = 4.88 \times 10^{-12} \text{ Pa}^{-1}$$

$$\boxed{K = 5.91 \times 10^{-10} \pm 4.88 \times 10^{-12} \text{ Pa}^{-1}}$$

These calculations show the precision of the measurements and the associated uncertainties, providing a comprehensive understanding of the experimental results.

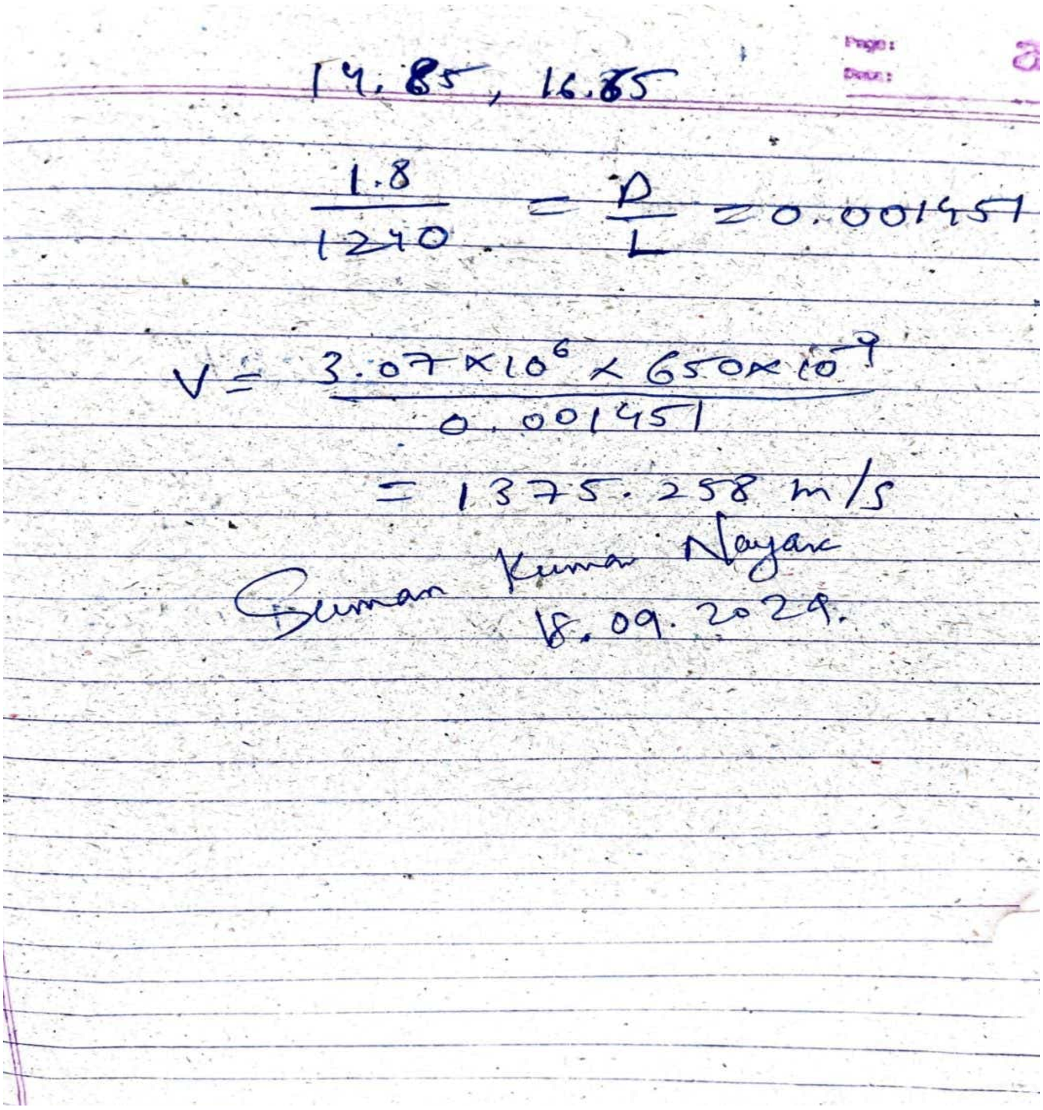
# 8 Author Contributions

- **Birudugadda Srivibhav**
  - Performed calculations for velocity, Bulk Modulus ($\beta$), and Compressibility ($K$).
  - Analyzed errors and uncertainties in measurements.
  - Drafted the results, conclusion, and sources of error sections of the report.
- **Vubbani Bharath Chandra**
  - Assisted in setting up the experimental apparatus and calibrating the equipment.
  - Recorded experimental data for current, position, and related measurements.
- **Ankeshwar Ruthesha**
  - Collected and recorded the experimental data.
  - Wrote Objective, Apparatus description, Theory and Experimental Procedure.
- **Kirtankumar Patel**
  - Created and formatted the tables and figures used in the report.

## 9 References


1 "Detector Based Apparatus for Ultrasonic Diffraction - Acousto optic effect," Holmarc.com. [Online]. (Available).

2 "Ultrasonic Diffraction," Indian Institute of Technology Roorkee. [Online]. (Available).

14.85, 16.85

$$\frac{1.8}{1240} = \frac{D}{L} = 0.001451$$

$$V = \frac{3.07\times10^{6} \times 650\times10^{-9}}{0.001451}$$

$$= 1375.258 \text{ m/s}$$

Suman Kumar Nayak
18.09.2024.

Figure 4: Lab Recordings (Link to spreadsheet containing all values).

L = 1240 mm

3.07 mH

| Micrometer reading | Current |
|---|---|
| ~~21+16~~ | |
| ~~22+48~~ | ~~25.1~~ |
| 21+85 | 20.3 μA |
| 21+60 | 16.5 μA |
| 20+73 | 22.5 μA |
| 20+55 | 30.1 μA |
| 22+60 | 06.0 μA |
| 22+55 | 6.5 μA |
| 22+50 | 6.7 μA |
| 22+45 | 7.5 μA |

(mm)

| MSR + CSR × LC | Current |
|---|---|
| 22 + 60 × 0.01 = 22.6 | 6.0 μA |
| 22 + 55 × 0.01 = 22.55 | 6.5 μA |
| 22 + 50 × 0.01 = 22.5 | ~~7.7 μA~~ 6.7 μA |
| 22 + 45 × 0.01 = 22.45 | 7.5 μA |
| 22 + 40 × 0.01 = 22.4 | 7.8 μA |
| 22 + 35 × 0.01 = 22.35 | 8.0 μA |
| 22 + 30 × 0.01 = 22.3 | 9.5 μA |
| 22 + 25 × 0.01 = 22.25 | 11.2 μA |
| 22 + 20 × 0.01 = 22.2 | 14.1 μA |
| 22 + 15 × 0.01 = 22.15 | 17.0 μA |
| 22 + 10 × 0.01 = 22.1 | 18.1 μA |
| 22 + 5 × 0.01 = 22.05 | 18.5 μA |
| 22 + 0 × 0.01 = 22 | 21.3 μA |
| 21 + 95 × 0.01 = 21.95 | 22.0 μA |
| 21 + 90 × 0.01 = 21.9 | 22.4 μA |

Figure 5: Lab Recordings (Link to spreadsheet containing all values).