# The design and optimization of a teacup ride

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## Analysis

A teacup ride consists of multiple parameters which can be changed. In the case of this project, those parameters are:

- Radius of teacup
- Rate of rotation of the ride floor
- Rate and direction of rotation of the teacups
- How many people per teacup

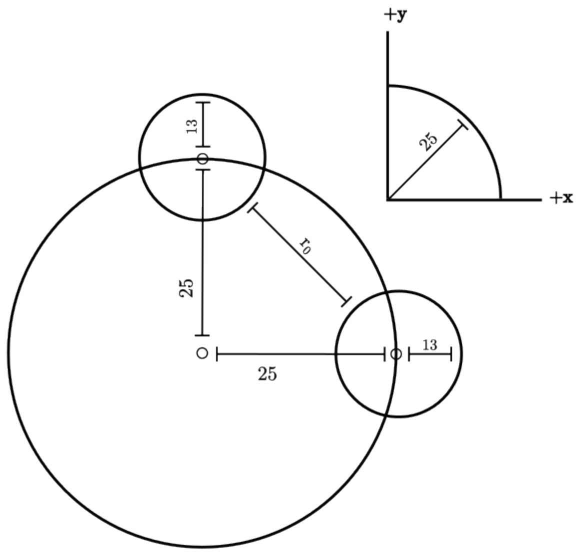
These parameters will be chosen based on a number of criteria, namely:

- Is the teacup large enough that the riders are comfortable and safe during the ride
- Are riders able to comfortably walk between each teacup while boarding
- Riders should not be able to reach out and touch another rider in an adjacent teacup
- The centripetal force the rider experiences cannot be sufficiently large to cause discomfort

(Following the order of the rubric)
Considering this, I have decided on the following:

1. Radii and Distance
   - Radius Teacup: 13 *ft*
   - Radius Person: 4 *ft*
   - Shortest distance between adjacent cups:

Figure 1: Finding dist. ($r_0$)




$r_t$ = Radius of teacup
$P_1$ = Pos. of upper teacup
$P_2$ = Pos. of right teacup
$r_0$ = Shortest Distance

$P_1 = (0, 25)$, $P_2 = (25, 0)$
$\Rightarrow \vec{P_1P_2} = 25\hat{i} - 25\hat{j}$
$\Rightarrow r_0 = \left\| \vec{P_1P_2} \right\| - 2r_t$

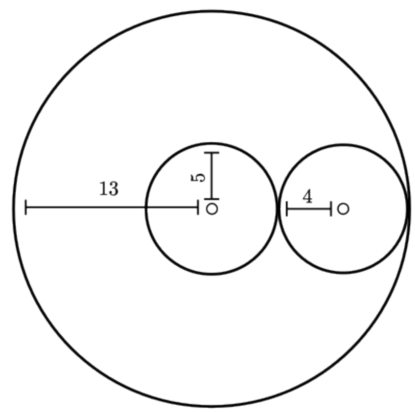
$$\therefore$$

$r_0 = \sqrt{25^2 + (-25)^2} - 2 * 13$
$\Rightarrow r_0 = 25\sqrt{2} - 26$
$r_0 \approx 9.355 ft$

2. Number of Riders

    - Radius of seat: 4 *ft*
    - Radius of teacup: 13 *ft*
    - Inner Radius: 5 *ft*

Figure 2: Finding riders per teacup ($n_r$)




$r_t$ = Radius of teacup
$r_s$ = Radius of seat
$r_i$ = Inner Radius
$A_t$ = Area of teacup
$A_s$ = Area of seat
$A_a$ = Area of seating space
$A_i$ = Inner Area
$n_r$ = Number of riders

$A_t = \pi r_t^2$, $A_s = \pi r_s^2$, $A_i = \pi r_i^2$
$\Rightarrow A_a = A_t - A_i$
$\Rightarrow A_a = \pi r_t^2 - \pi r_i^2$
$\Rightarrow n_r = \left\lfloor \frac{A_a}{A_s} \right\rfloor = \left\lfloor \frac{\pi r_t^2 - \pi r_i^2}{\pi r_s^2} \right\rfloor$

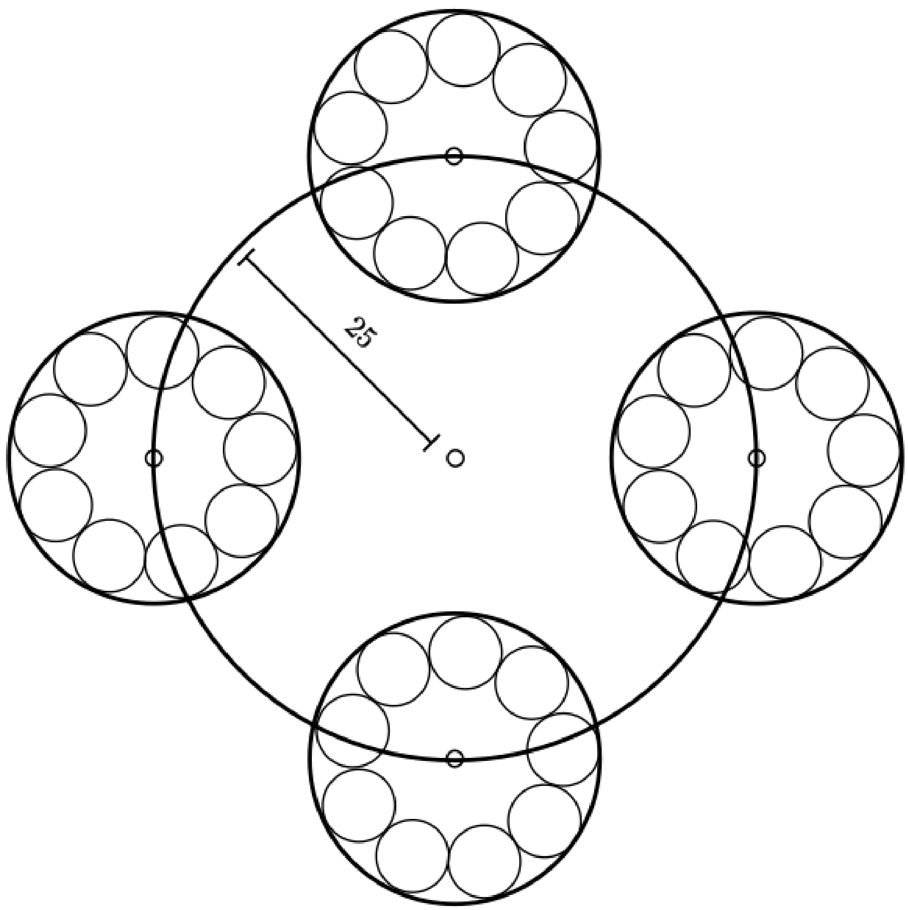
$$\therefore$$

$\Rightarrow n_r = \left\lfloor \frac{13^2\pi - 5^2\pi}{4^2\pi} \right\rfloor = 9$

$n_r = 9$ *riders per teacup*

3. Ride Diagram

Figure 3: Ride Diagram



4. Rate of spin

   *positive rpm indicates counterclockwise dir.*

   - Teacup spin rate: -4 *rpm*
   - Floor spin rate: -4 *rpm*

5. Ride length

   - Length: 120 seconds
   - Reason: 120 seconds is long enough for a rider to enjoy themselves while also ensuring that the rider does not become bored. 120 seconds is also not too long of a wait for those in line, so it's the perfect middle-ground between a ride that is too short for the rider, and a waiting time too long for someone in line

6. Rider path equation

   $r_1$ = Radius of spinning floor, $r_2$ = Radius of teacup
   $\omega_1$ = Angular speed of platform, $\omega_2$ = Angular speed of teacup

   The function describing the path of the rider from the start of the ride is given by:

   $$f(x(t), y(t)) =< r_1 \cos(\omega_1 t) + r_2 \cos(\omega_2 t), r_1 \sin(\omega_1 t) + r_2 \sin(\omega_2 t) >$$
   $$0 \leq t \leq L$$

   $$\omega = \frac{rpm*2\pi}{60} \Rightarrow \omega_1 \;\&\; \omega_2 = \frac{-8\pi}{60} \approx -0.419 \; rad/s$$

   $$r_1 = 25 \; ft, \; r_2 = 13 \; ft$$

   Therefore, the function that describes the path of the rider is:

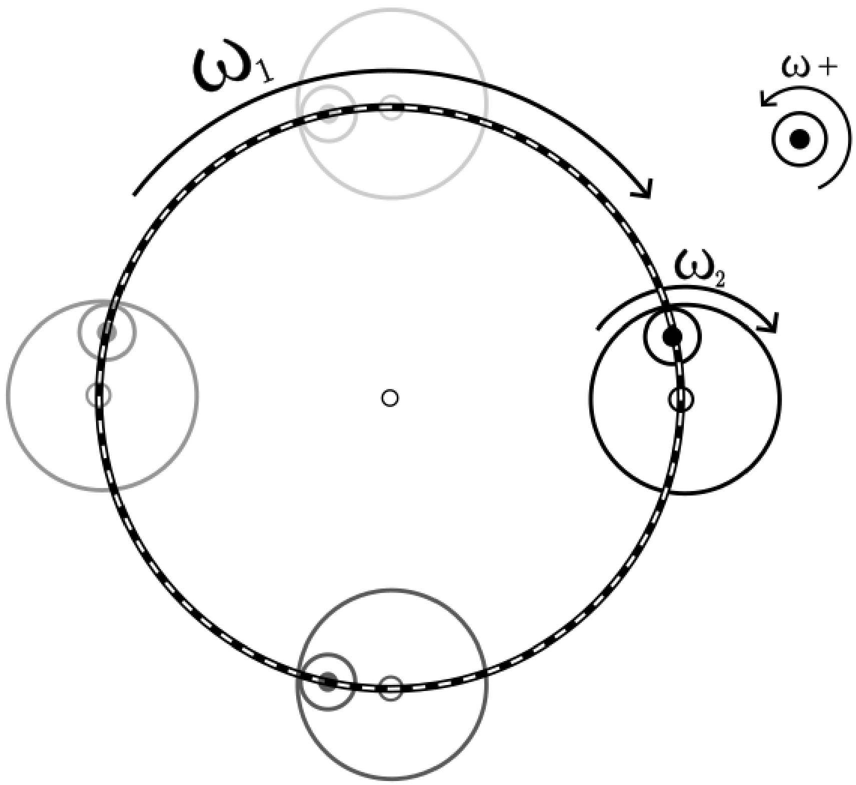
   $$f(x(t), y(t)) =< 25\cos(\frac{-2\pi}{15}t) + 13\cos(\frac{-2\pi}{15}t), 25\sin(\frac{-2\pi}{15}t) + 13\sin(\frac{-2\pi}{15})t >$$

7. Parameter Units

   $$\text{radii}[ft]$$
   $$\omega[rad/s]$$
   $$f(x(t), y(t))[ft,ft]$$
   $$t[s]$$

8. Path of a single rider

Figure 4: Rider Path




(Note: The white dotted line indicates the path traced)

9. Total distance travelled during a ride

    (Assuming an ideal case for this example, but there are infinitely many distances travelled by the rider as the radius of the circle the rider traces is entirely dependent on the riders original position as the ride starts. The same process I will describe can be used for any radius, however to simplify the process I have chosen the ideal case in which the riders position is such that the path traced lies directly on the circumference of the platform)

    As the path followed by the rider in this case is simply a circle, we can calculate the distance travelled in one rotation by finding the circumference of the circle the path forms. To find the period of the rotation, we can simplify the problem by identifying two things. The first being that as the curve the rider follows lies directly on the circumference of the platform, the period of rotation for the rider is the rotational period of the platform. The second being that, as $wr = v$, and $v = \frac{2\pi r}{p}$, we can reorder and substitute, and find that $p = \frac{2\pi}{\omega}$.

    This means that to find the total distance travelled $D$ for a ride with length $L$, platform radius $r_1$, and platform angular velocity of $\omega_1$:

$$D = \frac{\omega_1 L}{2\pi} 2\pi r_1 = \omega_1 r_1 L$$

    Substituting in the values for this ride:

$$D = \left|(-0.419)(25)(120)\right| = 1257ft$$

This process could also be done by parametrizing the path by using cos and sin, and using the arc length formula to calculate the path, however the aformentioned method was more efficient in this case.

10. Computing the magnitude of the riders acceleration

To compute the magnitude of the riders acceleration, we must simply find the magnitude of the acceleration vector $a\vec{(t)}$:

$$f(x(t), y(t)) =< r_1 \cos(\omega_1 t) + r_2 \cos(\omega_2 t), r_1 \sin(\omega_1 t) + r_2 \sin(\omega_2 t) >$$

$$a\vec{(t)} = \frac{d^2}{dt^2}[f(x(t), y(t))]$$

$$a_x(t):$$

$$\frac{d}{dt}[x(t)] = v_x(t) = \frac{d}{dt}[r_1 \cos(\omega_1 t) + r_2 \cos(\omega_2 t)]$$

$$v_x(t) = (-1)(r_1\omega_1 \sin(\omega_1 t) + r_2\omega_2 \sin(\omega_2 t))$$

$$\frac{d}{dt}[v_x(t)] = a_x(t) = \frac{d}{dt}[(-1)(r_1\omega_1 \sin(\omega_1) + r_2\omega_2 \sin(\omega_2 t))]$$

$$a_x(t) = (-1)(r_1\omega_1^2 \cos(\omega_1 t) + r_2\omega_2^2 \cos(\omega_2 t))$$

$$a_y(t):$$

$$\frac{d}{dt}[y(t)] = v_y(t) = \frac{d}{dt}[r_1 \sin(\omega_1 t) + r_2 \sin(\omega_2 t)]$$

$$v_y(t) = (r_1\omega_1 \cos(\omega_1 t) + r_2\omega_2 \cos(\omega_2 t))$$

$$\frac{d}{dt}[v_y(t)] = a_y(t) = \frac{d}{dt}[(r_1\omega_1 \cos(\omega_1) + r_2\omega_2 \cos(\omega_2 t))]$$

$$a_y(t) = (-1)(r_1\omega_1^2 \sin(\omega_1 t) + r_2\omega_2^2 \sin(\omega_2 t))$$

11. (cont.)

$$\vec{a(t)} =< a_x(t), a_y(t) >$$

$$\vec{a(t)} = (-1) < r_1\omega_1^2 \cos(\omega_1 t) + r_2\omega_2^2 \cos(\omega_2 t), r_1\omega_1^2 \sin(\omega_1 t) + r_2\omega_2^2 \sin(\omega_2 t) >$$

$$\|\vec{a}\| = \sqrt{a_x^2 + a_y^2}$$

$$\Rightarrow \|\vec{a}\| = \sqrt{r_1^2\omega_1^4 + r_2^2\omega_2^4 + 2r_1r_2\omega_1^2\omega_2^2 \cos(\omega_1 t - \omega_2 t)}$$

*Note: In this case,* $\omega_1 = \omega_2$

$$\Rightarrow \|\vec{a}\| = \sqrt{r_1^2\omega_1^4 + r_2^2\omega_2^4 + 2r_1r_2\omega_1^2\omega_2^2}$$

$$\Rightarrow \|\vec{a}\| = \sqrt{(r_1\omega_1^2 + r_2\omega_2^2)^2}$$

$$\Rightarrow \|\vec{a}\| = r_1\omega_1^2 + r_2\omega_2^2$$

Substituting in our values:

$$\|\vec{a}\| = 13(-0.419)^2 + 25(-0.419)^2 \approx 6.671 ft/s/s$$

12. Computing G-Forces

Converting feet per second per second to meters per second per second

$\|\vec{a}\|$ *(ft/s/s)*$\frac{\|\vec{a}\|}{3.281}$

Calculating the G-Force: $G = \frac{\|\vec{a}\|}{g}$

Substituting in our values: $\|\vec{a}\| \, (ft/s/s) = \frac{6.671}{3.281} \approx 2.033 m/s/s$

$\Rightarrow G = \frac{2.033}{9.807} \approx 0.207 g\text{'}s$

13. Comparing ride g-forces

The g-forces this ride causes are very low, akin to just being in a car lightly going around a bend. This is sought after as the main riders in a teacup ride will be children, and this ride is soft enough such that it will not cause the children to feel discomfort. Reviewing the guidelines set by the Global Association for the Attractions Industry[1], it is clear that this ride falls well below the limits - a sneeze produces upwards of 2.0 g's, while this ride lies at 0.2 g's - and is a perfect ride for young children, making it more than suitable for the target audience.

# Conclusion

This ride was designed for a younger audience, and the parameters of the ride reflect this. During the project I learnt about how interconnected the design process is, with each aspect relying on the other. It was a little challenging at the start to find the correct values which allowed for adequate space between cups, adequate seating, and adequate g-forces, but that process got easier the more I worked on the project. I was not expecting the distance travelled to be as high as it was, and I found that quite interesting. To recap, the radius of the platform was 25ft, the radius of the teacup was 13ft, and the radius of each seat was 4ft. My each teacup carried 9 people, totalling to 36 riders riding at once. The length of the ride was 120 seconds, and the distance travelled 1257 feet. My teacups and platform spun with the same angular velocity, namely 4 rotations per minute, and in the same direction. The g-forces experienced by riders on this ride is 0.2g's

# Bibliography

1. G-forces. IAAPA. (n.d.). https://www.iaapa.org/amusement-ride-safety/g-forces