Predicting team performance through analysis of a CDPF model

Lukas Campbell

November 19, 2023

Analysis

In this project I will compile and analyze a data set from the NHL[1], using the statistics for the Toronto Maple Leafs NHL team from the 2000-01 season to the 2022-23 season to create my Cobb-Douglas Production function. I will use the goals scored and the games played as the independent variables in my Cobb-Douglas Function, and the function will create the surface that predicts the wins for a season. I will then compare the predictions with the actual wins for the season. My data spans the wins, the goals scored, and the games played, of 32 teams; the NHL teams for the 2022-2023 season.

Using the statistics on the NHL Website[1], I end up with:

Productivity Variable: Number of Wins in the Season

Metric 1: Number of games played in the season

Metric 2: Number of goals scored in the season

The next page contains the data table with the compiled data for the Toronto Maple Leafs.

Table 1: NHL Statistics for the Toronto Maples leafs from the 2000-2001 to 2022-23 seasons

Season	Games Played	Goals Scored	Wins
2022/2023	82	278	50
2021/2022	82	312	54
2020/2021	56	186	35
2019/2020	70	237	36
2018/2019	82	286	46
2017/2018	82	270	49
2016/2017	82	250	40
2015/2016	82	192	29
2014/2015	82	206	30
2013/2014	82	222	38
2012/2013	48	145	26
2011/2012	82	227	35
2010/2011	82	213	37
2009/2010	82	210	30
2008/2009	82	244	34
2007/2008	82	228	36
2006/2007	82	254	40
2005/2006	82	254	41
2003/2004	82	242	45
2002/2003	82	236	44
2001/2002	82	249	43
2000/2001	82	232	37

2. Showing the alternate expression of the CDPF:

$P(L, K) = bL^{\alpha}K^{1-\alpha} \Rightarrow y = \ln b + \alpha x$

$P(L, K) = bL^{\alpha}K^{1-\alpha}$

$\Rightarrow \ln P(L, K) = \ln bL^{\alpha}K^{1-\alpha} \Rightarrow \ln P(L, K) = \ln B + \ln L^{\alpha} + \ln K^{1-\alpha}$

$\Rightarrow \ln P = \ln b + \alpha \ln L + \ (1 - \alpha) \ln K$

$\Rightarrow \ln P = \ln b + \alpha \ln L + \ln K - \alpha \ln K$

$\Rightarrow \ln P = \ln b + \ln K + \alpha(\ln L - \ln K)$

$\Rightarrow \ln P - \ln K = \ln B + \ln K + \alpha(\ln L - \ln K)$

$\Rightarrow \ln \frac{P}{K} = \ln B + \alpha \ln \frac{L}{K}$

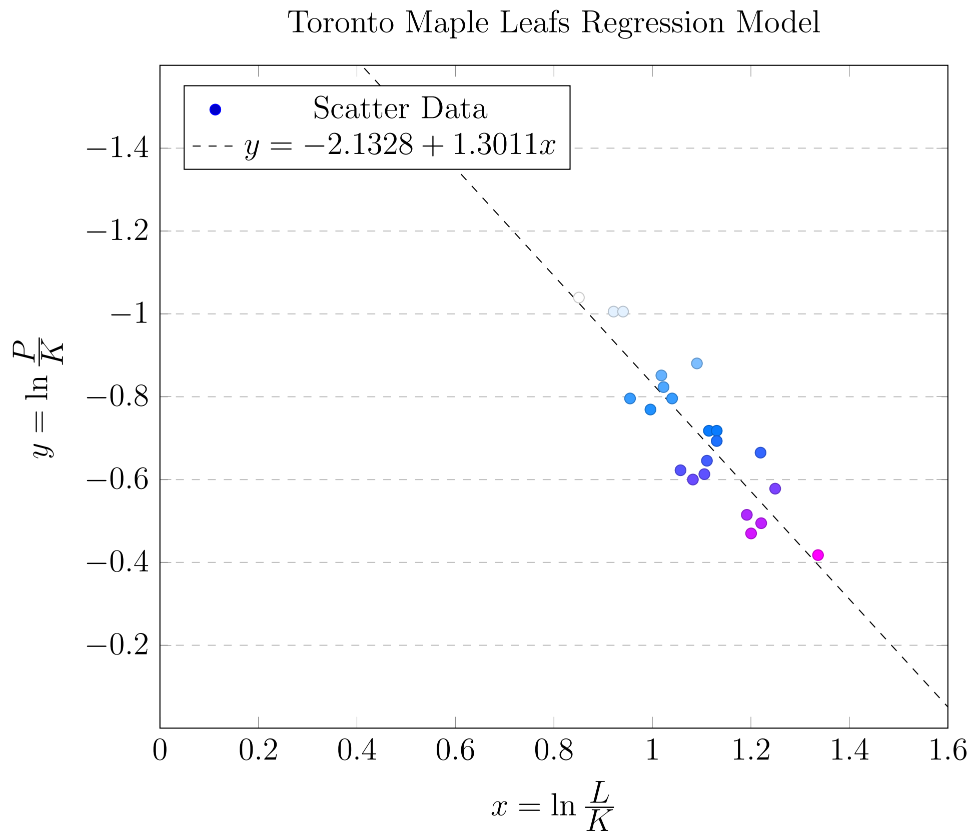
$x = \ln \frac{L}{K}$, $y = \ln \frac{P}{K}$

$\ln \frac{P}{K} = \ln b + \alpha \ln \frac{L}{K} \Rightarrow y = \ln B + \alpha x$

In this case, my P is games won, my L is the goals scored, and my K is games played, per season respectively.

3. Finding my CPDF values for my dataset

Using the data from Table 1, equating $x = \ln \frac{L}{K}$ and $y = \ln \frac{P}{K}$, and compiling that data, we get the following graph:



The regression model for my data follows the equation $y = -2.1328 + 1.3011x$, which when comparing to the equation $y = \ln b + \alpha x$, it is clear that $\ln b = -2.1328$ and $\alpha = 1.3011$. As $\ln b = -2.1328 \Rightarrow b = \exp(-2.1328)$

4. Creating my CDPF for my dataset

Using the values calculated above:

The CPDF takes the form $P(L, K) = bL^{\alpha}K^{1-\alpha}$

Where:

$P = \textit{Number of games won per season}$

$L = \textit{Number of goals scored per season}$

$K = \textit{Number of games played per season}$

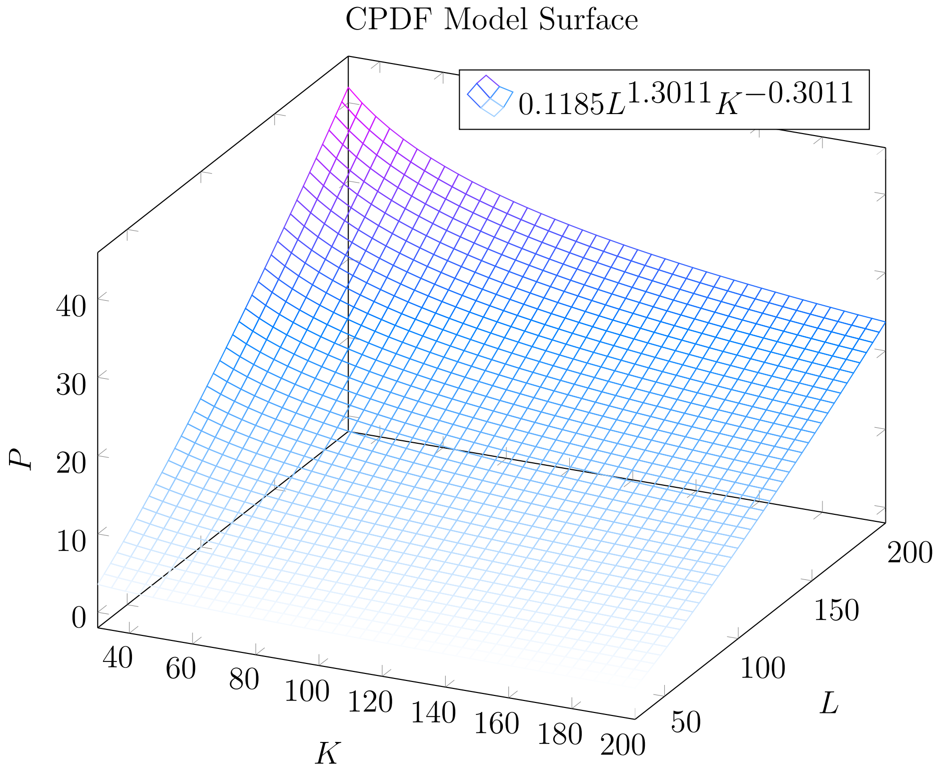
$\alpha = 1.3011$ *&* $b = \exp(-2.1328)$

$\therefore$

$P(L, K) = \exp(-2.1328)L^{1.3011}K^{1-1.3011}$

$\Rightarrow P(L, K) = 0.1185L^{1.3011}K^{-0.3011}$

4. *cont.* Representing my CPDF Model Graphically



5. Evaluating the accuracy of my CPDF

Using Excel to perform the computations:

Table 2: Comparing Games Won to Model Prediction

Season	W (P)	Predicted Wins
2022/2023	50	47.58
2021/2022	54	55.29
2020/2021	35	31.64
2019/2020	36	40.55
2018/2019	46	49.37
2017/2018	49	45.81
2016/2017	40	41.44
2015/2016	29	29.39
2014/2015	30	32.21
2013/2014	38	35.51
2012/2013	26	23.97
2011/2012	35	36.55
2010/2011	37	33.65
2009/2010	30	33.03
2008/2009	34	40.15
2007/2008	36	36.76
2006/2007	40	42.31
2005/2006	41	42.31
2003/2004	45	39.72
2002/2003	44	38.45
2001/2002	43	41.23
2000/2001	37	37.6

As with any data model, there are outliers. However, most predictions are within just a few wins of their real-life counterparts. This model can quite effectively predict a small ballpark in which the wins the Toronto Maple Leafs will fall, based on their goals scored and games played, over a season. For some data points this model is almost perfectly accurate, meaning with an extra 20 data points, this model can very accurately predict the team's performance based on those parameters.

6. What do the partial derivatives of P and ∇P represent

The partial derivatives of P, depending on if they are in terms of L or K, would represent the rate of increase in wins per season at some given L and K if L was increasing and K was constant, or vice versa. This would allow a team manager to see the rate at which their seasons success is increasing given some level of increase in games played and a constant number of goals scored, or some level of increase in goals scored with a constant number of games played. The gradient vector represents the direction with the highest rate of change, meaning depending on the values given the teams current games played and goals scored a team manager would be able to figure out what to prioritize during training, and to change things accordingly to maximize the teams success.

7. Maximizing seasonal wins using Lagrange Multipliers

Let n represent the cost of producing goals scored *(L)* and let m represent the cost of producing games played *(K)*

Given a fixed budget p, and the rates for the unit cost of goals scored and games played, we get:

$p = nL + mK$

Calculating the values of n and m using Lagrangian Multipliers:

Let $P(L,K) = bL^{\alpha}K^{1-\alpha}$ with boundary condition $p = nL + mK$

$\nabla P(L,K) =< \frac{\partial P}{\partial L}, \frac{\partial P}{\partial K} >=< \alpha b L^{\alpha-1}K^{1-\alpha}, -(1-\alpha)bL^{\alpha}K^{-\alpha} >$

$\nabla p(L,K) =< \frac{\partial p}{\partial L}, \frac{\partial p}{\partial K} >=< n, m >$

$\nabla P(L,K) = \lambda \nabla p(L,K)$

$\alpha b L^{\alpha-1}K^{1-\alpha} = \lambda n \Rightarrow \frac{\alpha b L^{\alpha-1}K^{1-\alpha}}{n} = \lambda$

$-(1-\alpha)bL^{\alpha}K^{-\alpha} = \lambda m \Rightarrow \frac{-(1-\alpha)bL^{\alpha}K^{-\alpha}}{m} = \lambda$

$\Rightarrow \frac{\alpha b L^{\alpha-1}K^{1-\alpha}}{n} = \frac{-(1-\alpha)bL^{\alpha}K^{-\alpha}}{m}$

$\Rightarrow \frac{\alpha L^{-1}K^{1-\alpha}}{n} = \frac{-(1-\alpha)K^{-\alpha}}{m} \Rightarrow \frac{\alpha K}{nL} = \frac{\alpha-1}{m} \Rightarrow K = \frac{n(\alpha-1)L}{m\alpha}$

We then know:

$p = nL + mK \Rightarrow p = nL + \frac{n(\alpha-1)L}{\alpha} \Rightarrow L(\frac{2n\alpha-n}{\alpha}) = p$

$\Rightarrow L = \frac{\alpha p}{2n\alpha-n}$

As $K = \frac{n(\alpha-1)L}{m\alpha}$, then $K = \frac{(\alpha-1)n\frac{p\alpha}{2n\alpha-n}}{m\alpha} = \frac{(\alpha-1)\frac{p\alpha}{2\alpha-1}}{m\alpha} \Rightarrow \frac{(\alpha-1)p}{(2\alpha-1)m}$

Therefore, $K = \frac{(\alpha-1)p}{(2\alpha-1)m}, L = \frac{\alpha p}{2n\alpha-n}$

8. Use of Lagrange Multiplier results in allocating budget

The results of the previous step would indicate the values of goals scored and games played which would maximize the teams success in a season. This means that in order to allocate the budget most effectively, a team manager would use the values found previously for the L and K variables. It would allow a manager to prioritize processes which result in increases or decreases in either value and to maximize the teams success effectively.

Conclusion

This project describes different aspects of how a team can maximize their success in training. From what is seen in the CPDF function, increasing goals scored has a larger impact on seasonal wins than the games played, however both do positively effect the teams overall wins. By using Lagrange Multipliers, a team manager would be able to prioritize which aspect of training leads to the greatest success and act accordingly. This project has laid out how each variable affects the seasonal wins, and which direction the team manager should take the team in order for maximum success.

Bibliography

1. “NHL Statistics for the Toronto Maple Leafs.”
NHL.Com, https://www.nhl.com/stats/teams