Flight Dynamics & Control

Tutorial Sheet 2

This tutorial sheet is meant to help you remember concepts forst covered in Introduction to Aerospace and consolidate your understanding of Flight Dynamics by asking you to carry out a full stability analysis for an existing aircraft. Flight mechanics is not so much about remembering endless formulae (I never can) but instead being able to use your understanding of key concepts to derive any needed expression from first principles. Therefore try this tutorial sheet without looking at the notes.

The Piaggio P.180 Avanti, seen in figure 1, is a twin turboprop executive transport that first flew in the 1980s and features a rather unique three lifting surface design. The lift generated by the fuselage and the inclusion of a lifting canard lead to a more highly loaded wing and a more efficient aerodynamic design. They also make the stability analysis a little bit harder! Key design characteristics for the aircraft can be found in tables 1 and 2. The aircraft is controlled in pitch using an elevator mounted on the horizontal tail. The canard features a set of plain, non-extending flaps, used to maintain trim when the main wing's single slotted, extending flaps are extended and which are not used to control the aircraft.

Static Stability

1. Estimate the whole aircraft lift curve slope and zero-lift angle of attack and drag polar. You may assume elevator deflections to have a minimal effect on lift and drag.
2. From first principles, derive an expression for estimating the aircraft's stick-fixed static margin. You may ignore the effect of the propulsive system on the aircraft's static stability. What is the aftmost allowable position for the CG? Calculate the value of x_{CG} such that the aircraft flies with a static margin of 7.5%.
3. For the CG position identified above and the operating conditions in 2, calculate the elevator deflection necessary to trim the aircraft. Do not forget the effects of thrust.
4. Calculate the canard flap deflection (δ_{fC}) required to keep the aircraft in trim following a deflection of the wing flaps to the landing configuration, $\delta_{fW} = 30°$.
5. The foremost CG position is typically determined based on trimability considerations during landing. Assuming both the canard and wing flaps are deflected as found in Q4, what is the foremost alowable CG position for the Avanti? The maximum landing weight for this aircraft is 4,965 kg and $C_{L_{max}} = 2.3$.

Dynamic Stability

6. Using the aerodynamic data provided and the CG position from Q2, estimate the U, W, q and $\dot{W}$ derivatives for the Avanti in the equilibrium condition specified in table 2. Try to derive the formulae for each of the 12 derivatives from first principles.

7. Use MATLAB to characterise the dynamic response of the aircraft. Compare the results obtained from analysing the full system to those obtained using the approximate methods in chapter 10 of your notes. Make sure you are able to derive these yourself.

8. To analyse the stick-free dynamic response of the aircraft to a perturbation, the elevator deflection δ_E, must be added as an additional degree of freedom to our system of longitudinal equations of motion. Starting from the longitudinal equations of motion

$$\begin{aligned} m\dot{U} &= X^a - mg\sin\Theta \\ m(\dot{W} - Uq) &= Z^a + mg\cos\Theta \\ I_{yy}\dot{q} &= M^a \end{aligned}$$

and the elevator hinge moment equation

$$I_E\ddot{\delta}_E = H_E = \frac{1}{2}\rho V_\infty^2 S_E c_E \left(b_0 + b_H\alpha_H + b_E\delta_E + b_T\delta_T\right),$$

use small-perturbation theory to show that the state-space representation of the aircraft's longitudinal motion starting from an equilibrium, level flight condition, is

$$\begin{bmatrix} m & -X_{\dot{W}} & 0 & 0 & 0 & 0 \\ 0 & m - Z_{\dot{W}} & 0 & 0 & 0 & 0 \\ 0 & -M_{\dot{W}} & I_{yy} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -H_{\dot{W}} & 0 & 0 & I_E & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{U} \\ \dot{W} \\ \dot{q} \\ \dot{\Theta} \\ \ddot{\delta}_E \\ \dot{\delta}_E \end{bmatrix} = \begin{bmatrix} X_U & X_W & X_q & -mg & 0 & X_{\delta_E} \\ Z_U & Z_W & Z_q + mU_e & 0 & 0 & Z_{\delta_E} \\ M_U & M_W & M_q & 0 & 0 & M_{\delta_E} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ H_U & H_W & H_q & 0 & 0 & H_{\delta_E} \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} U \\ W \\ q \\ \Theta \\ \dot{\delta}_E \\ \delta_E \end{bmatrix}.$$

You may assume that the effect of the elevator's rate of rotation and rotational acceleration on the aircraft's aerodynamics is negligible.

9. Derive expressions for the hinge moment derivatives (H_U, H_W, H_q, $H_{\dot{W}}$, H_{δ_E}) and the elevator control derivatives (X_{δ_E}, Z_{δ_E}, M_{δ_E}). Use the data provided to evaluate these derivatives at the equilibrium condition specified in table 2.

10. Use MATLAB to analyse the dynamic response of the aircraft to perturbations in its states.

    (a) How does the stick-free response differ from the stick fixed one?

    (b) How would the addition of a torsion spring, with a torsional stiffness of 10^4 Nm/rad, to the elevator assembly affect the aircraft's responce?

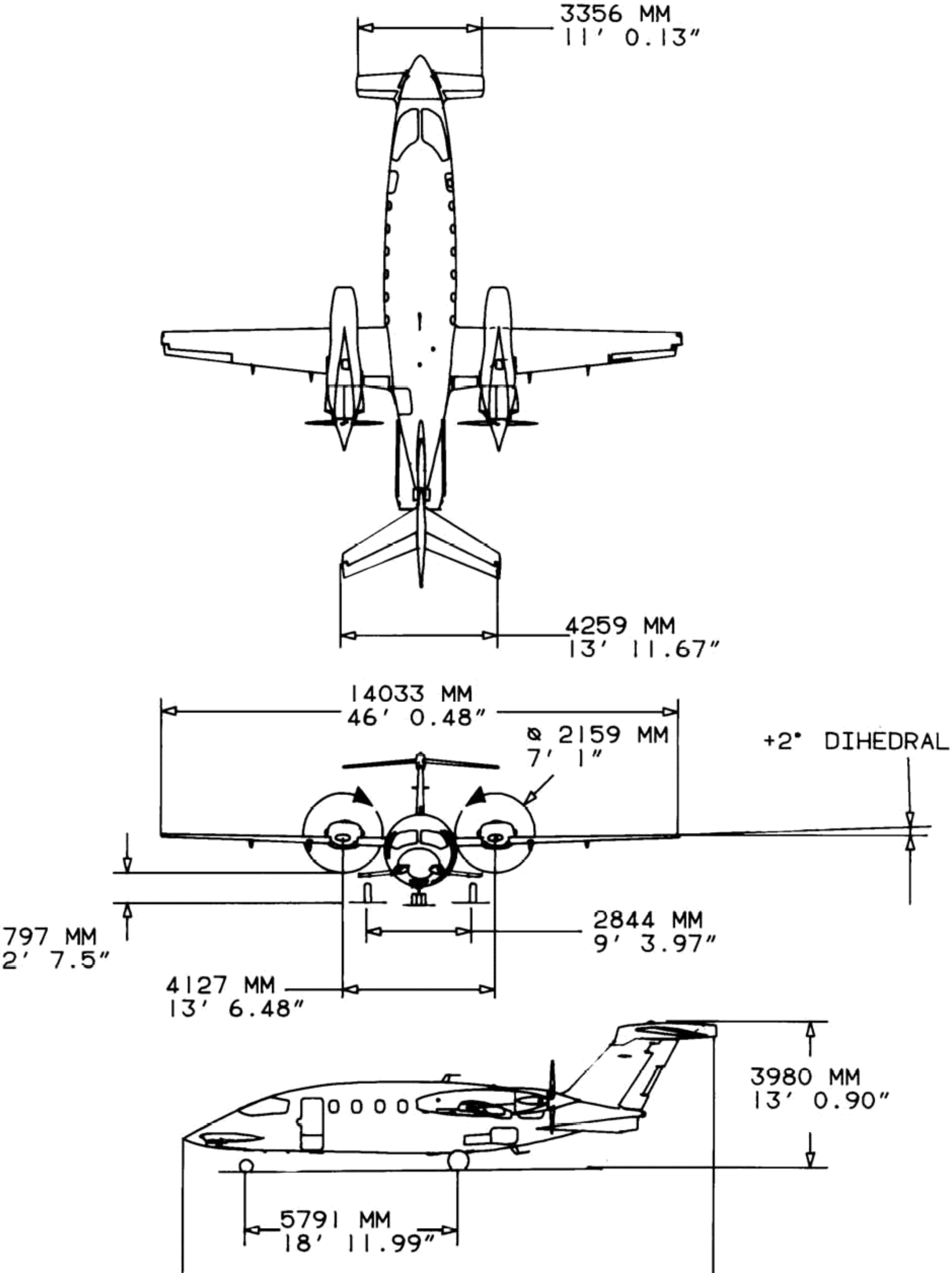

Figure 1: 3-view drawing of the Piaggio P.180 Avanti

Table 1: Lifting Surface Geometric & Aerodynamic Specifications

	Wing			Canard			Horizontal Tail			
Planform area	S_{ref}	=	16.00	S_C	=	2.19	S_H	=	3.83	m^2
Mean Aero Chord	$\bar{c}$	=	1.18	$\bar{c}_C$	=	0.65	$\bar{c}_H$	=	0.92	m
Longitudinal AC position	x_W	=	7.59	x_C	=	0.62	x_H	=	13.00	m
Vertical AC position	z_W	=	0.87	z_C	=	0.07	z_H	=	2.99	m
Aspect ratio	$Æ_W$	=	12.31	$Æ_C$	=	5.14	$Æ_H$	=	4.74	
Quarter Chord Sweep	Λ_W	=	-0.50	Λ_C	=	0.00	Λ_H	=	30.00	deg
Lift-curve slope	a_W	=	8.17	a_C	=	3.40	a_H	=	4.23	1/rad
Zero-lift AoA	α_{0W}	=	-2.40	α_{0C}	=	-1.40	α_{0H}	=	2.50	deg
Setting angle	i_W	=	1.60	i_C	=	0.00	i_H	=	0.00	deg
Zero-lift C_M	$C_{M_{0W}}$	=	-0.46	$C_{M_{0C}}$	=	-0.0429	$C_{M_{0H}}$	=	0.04	
Up/Downwash gradient		-		$d\varepsilon_U/d\alpha$	=	0.15	$d\varepsilon/d\alpha$	=	0.32	
Induced Drag k factor	k_W	=	1.1	k_C	=	2.21	k_H	=	2.5	

Table 2: Additional Aerodynamic Characteristics

Fuselage Effects				
Fuselage Pitching Moment Coef	$(dC_M/d\alpha)_F$	=	0.76	1/rad
Drag				
Aircraft Zero-lift Drag Coef	C_{D_0}	=	0.022	
Wave drag divergence Mach No	M_{DD}	=	0.65	
Canard Zero Lift Drag Coef	C_{D_0C}	=	0.00685	
Tailplane Zero-lift Drag Coef	C_{D_0H}	=	0.00951	
Wing Flaps (Approach Configuration - $\delta_{fW} = 30°$)				
Flapped area ratio	$(S_f/S)_W$	=	0.55	
Flap extension ratio	$(c'/c)_W$	=	1.25	
Flap zero-lift increment	ΔC_{L_0W}	=	0.932	
Flap zero-lift drag increment	ΔC_{D_0}	=	0.017	
Canard Flaps				
Flap lift increment	$\Delta C_{L_C}/\delta_{fC}$	=	1.463	1/rad
Elevator				
Elevator to tailplane chord ratio	c_E/c_H	=	0.4	
Max Elevator deflections	$(\delta_E)_{max}$	=	$\pm 20°$	
Elevator lift curve slope	a_E	=	3.08	1/rad
Zero-lift Hinge Moment Coef.	b_0	=	0	1/rad
Tailplane Hinge Moment slope	b_H	=	-0.502	1/rad
Elevator Hinge Moment slope	b_E	=	-0.879	1/rad
Trim-Tab Hing Moment slope	b_T	=	0	1/rad
Propulsion				
Thrust-line position (above baseline)	z_T	=	0.97	m
Inertial Propetries at mid-cruise				
Aircraft Mass	m	=	4,836	kg
Vertical CoG (above baseline)	z_{CG}	=	0.85	m
Pitching 2^{nd} moment of Inertia	I_{yy}	=	7.529×10^4	kg m^2
Elevator polar moment of Inertia	I_E	=	0.261	kg m^2
Operating Conditions				
Altitude	h	=	30,000	ft
Mach	M_∞	=	0.62	