
Recommender Systems

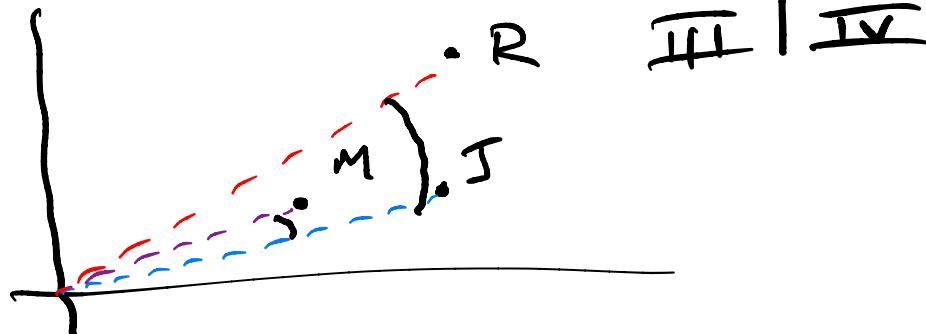
Movie Ratings

Interstellar

Boss Baby

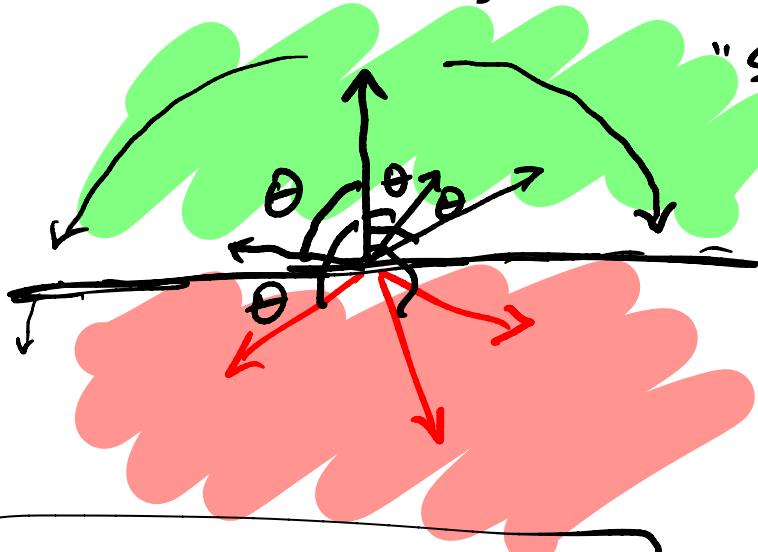
Boss
Baby

James	Matt	Ryland
[8] 2	[5] 2	[8] 6



Interstellar

Cosine Similarity



$$\cos \theta = \frac{a \cdot b}{\|a\| \cdot \|b\|}$$

$$\cos \theta > 0$$

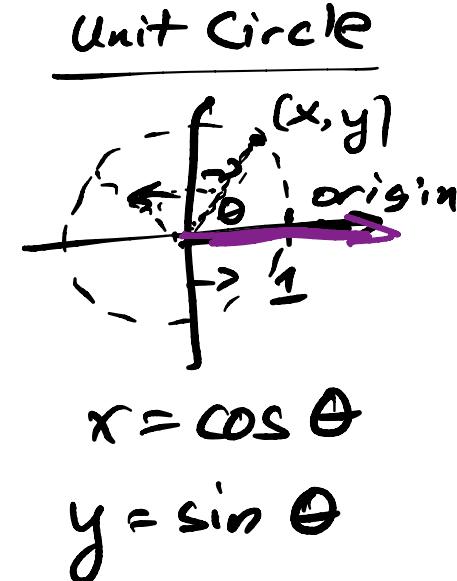
less similar

$$-1 \leq \cos \theta \leq 1$$

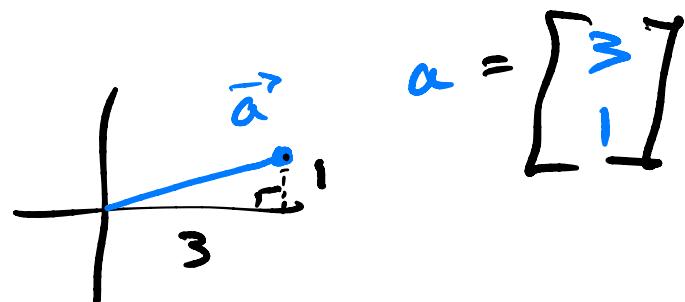
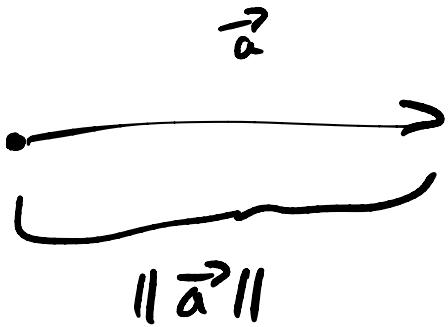
more similar

"same/similar direction"
 $\cos > 0$

$\cos = 0$
"opposite direction"



Vector Magnitude



$$\|\vec{a}\|^2 = 1^2 + 3^2$$

$$\|\vec{a}\| = \sqrt{1^2 + 3^2}$$

$$a = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} \quad \|a\| = \sqrt{1^2 + 2^2 + 3^2 + 4^2}$$

Vector Normalization (Unit Vector)

$$a = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad \begin{aligned} \|a\| &= \sqrt{3^2 + 1^2} \\ &= \sqrt{10} \end{aligned}$$
$$\hat{a} = \begin{bmatrix} 3/\sqrt{10} \\ 1/\sqrt{10} \end{bmatrix}$$

↓
5

- \hat{a} "unit"
- length 1 ✓
 - same direction as a ✓

$$\|\hat{a}\| = \left\| \frac{a}{\|a\|} \right\|$$
$$= \frac{1}{\|a\|} \|a\|$$
$$= 1$$
$$\boxed{\hat{a} = \frac{a}{\|a\|}}$$

scalar positive

Dot Product

↑
same
shape

$$a = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$
$$b = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

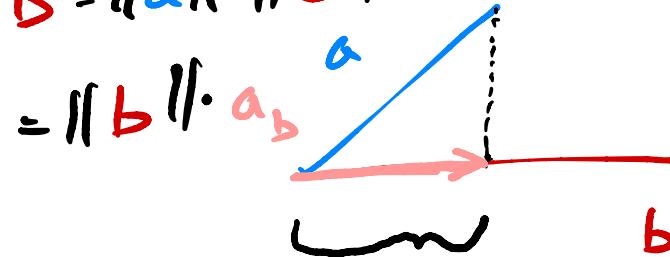
$$a \cdot b = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \frac{3 \cdot (-1) + 1 \cdot (2)}{-3 + 2}$$

$$(\text{vector}) \cdot (\text{vector}) = \underline{\text{scalar}} = 1$$

Dot Product Intuition

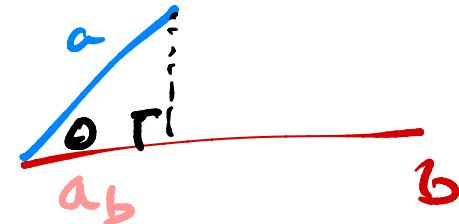


$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \cdot \|\mathbf{b}\| \cdot \cos \theta$$



projection
of \mathbf{a}
onto \mathbf{b}

magnitude
of



$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cos \theta = \frac{ab}{\|\mathbf{a}\|}$$

$$ab = \|\mathbf{a}\| \cos \theta$$

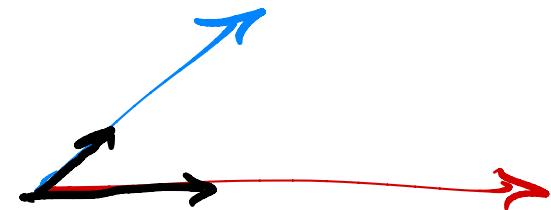
Why do we normalize?

$$a \cdot b = \|a\| \cdot \|b\| \cdot \cos \theta$$

$$\cos \theta = \frac{a \cdot b}{\|a\| \cdot \|b\|}$$

$$= \frac{a}{\|a\|} \cdot \frac{b}{\|b\|}$$

$$= \hat{a} \cdot \hat{b}$$



Content-Based Recommenders

Based on **product features**

Title	Year	Genre	Director	MPAA Rating
Elf	2003	<u>Christmas/Comedy</u>	Jon Favreau	<u>PG</u>
Die Hard	1988	<u>Christmas/Action</u>	John McTiernan	R
How to Train Your Dragon	2010	Animation/ <u>Action</u>	Dean DeBlois Chris Sanders	<u>PG</u>

PG Christmas
PG Action

Collaborative Recommenders: User-Based

Recommendations from **users** with similar **ratings**

	Movie 1	Movie 2	Movie 3
User 1	5	1	?
User 2	4	1	4
User 3	1	5	1

Rows
Col
Val

Users
Movies
RatingS

Collaborative Recommenders: User-Based

Recommendations from **users** with similar **purchases**

	Movie 1	Movie 2	Movie 3
User 1	1	0	NOT WATCHED
User 2	1	0	1
User 3	0	1	0

Rows Users

cols Movies

values did / didn't buy / # units purchased

Collaborative Recommenders: Item-Based

Recommendations from **products** with similar **ratings/purchases**

	User 1	User 2	User 3
Movie 1	4	5	2
Movie 2	2	1	5
Movie 3	5	4	2

"Because you liked Movie 1, here's something else you might like!" <link Movie 3>

Rows Movies
Cols Users
Values ratings