

1 Basic Mathematics

1. If

$$\begin{aligned}1^2x_1 + 2^2x_2 + 3^2x_3 + 4^2x_4 + \dots + 9^2x_9 &= 1 \\2^2x_1 + 3^2x_2 + 4^2x_3 + 5^2x_4 + \dots + 10^2x_9 &= 11 \\1^2x_1 + 3^2x_2 + 5^2x_3 + 7^2x_4 + \dots + 17^2x_9 &= 111\end{aligned}$$

then find:

- a.) $3^2x_1 + 4^2x_2 + 5^2x_3 + 6^2x_4 + \dots + 11^2x_9$
b.) $3^2x_1 + 5^2x_2 + 7^2x_3 + 9^2x_4 + \dots + 19^2x_9$

Solution:

$$\sum_{r=1}^9 (r^2x_r) = 1 \quad (1.1)$$

$$\sum_{r=1}^9 ((r+1)^2x_r) = 11 \quad (1.2)$$

$$\sum_{r=1}^9 ((2r-1)^2x_r) = 111 \quad (1.3)$$

a.) let

$$\sum_{r=1}^9 ((r+2)^2x_r) = \sum_{r=1}^9 ((Xr^2 + Y(r+1)^2 + Z(2r-1)^2)x_r) \quad (1.4)$$

$$\sum_{r=1}^9 ((r^2 + 4 + 4r)x_r) = \sum_{r=1}^9 ((r^2(X + Y + 4Z) + r(2Y - 4Z) + (Y + Z))x_r) \quad (1.5)$$

so,

$$X + Y + Z = 1 \quad (1.6)$$

$$2Y - 4Z = 4 \quad (1.7)$$

$$Y + Z = 4 \quad (1.8)$$

so,

$$X = -5 \quad (1.9)$$

$$Y = \frac{10}{3} \quad (1.10)$$

$$Z = \frac{2}{3} \quad (1.11)$$

so by (1.4), (1.9), (1.10), and (1.11),

$$\sum_{r=1}^9 ((r+2)^2x_r) = \sum_{r=1}^9 (((-5)r^2 + \left(\frac{10}{3}\right)(r+1)^2 + \left(\frac{2}{3}\right)(2r-1)^2)x_r) \quad (1.12)$$

$$\sum_{r=1}^9 ((r+2)^2 x_r) = -5 \sum_{r=1}^9 (r^2 x_r) + \frac{10}{3} \sum_{r=1}^9 ((r+1)^2 x_r) + \frac{2}{3} \sum_{r=1}^9 ((2r-1)^2 x_r) \quad (1.13)$$

so by (1.1), (1.2), and (1.3),

$$\sum_{r=1}^9 ((r+2)^2 x_r) = (-5)(1) + \left(\frac{10}{3}\right)(11) + \left(\frac{2}{3}\right)(111) \quad (1.14)$$

$$\sum_{r=1}^9 ((r+2)^2 x_r) = \frac{317}{3} \quad (1.15)$$