1 Basic Mathematics

1. If

$$1^2 x_1 + 2^2 x_2 + 3^2 x_3 + 4^2 x_4 + \dots + 9^2 x_9 = 1$$
$$2^2 x_1 + 3^2 x_2 + 4^2 x_3 + 4^2 x_4 + \dots + 10^2 x_9 = 11$$
$$1^2 x_1 + 3^2 x_2 + 5^2 x_3 + 7^2 x_4 + \dots + 17^2 x_9 = 111$$

then find:

a.) $3^2 x_1 + 4^2 x_2 + 5^2 x_3 + 6^2 x_4 + \dots + 11^2 x_9$

b.) $3^2 x_1 + 5^2 x_2 + 7^2 x_3 + 9^2 x_4 + \dots + 19^2 x_9$

Solution:

$$\sum_{r=1}^{9} (r^2 x_r) = 1 \tag{1.1}$$

$$\sum_{r=1}^{9} ((r+1)^2 x_r) = 11 \tag{1.2}$$

$$\sum_{r=1}^{9} ((2r-1)^2 x_r) = 111 \tag{1.3}$$

a.) let

$$\sum_{r=1}^{9} ((r+2)^2 x_r) = \sum_{r=1}^{9} ((Xr^2 + Y(r+1)^2 + Z(2r-1)^2) x_r) \tag{1.4}$$

$$\sum_{r=1}^{9} ((r^2 + 4 + 4r) x_r) = \sum_{r=1}^{9} ((r^2(X+Y+4Z) + r(2Y-4Z) + (Y+Z)) x_r) \tag{1.5}$$

so,

$$X + Y + Z = 1 \tag{1.6}$$
$$2Y - 4Z = 4 \tag{1.7}$$
$$Y + Z = 4 \tag{1.8}$$

so,

$$X = -5 \tag{1.9}$$
$$Y = \frac{10}{3} \tag{1.10}$$
$$Z = \frac{2}{3} \tag{1.11}$$

so,

$$\sum_{r=1}^{9} ((r+2)^2 x_r) = \sum_{r=1}^{9} (((-5)r^2 + (\frac{10}{3})(r+1)^2 + (\frac{2}{3})(2r-1)^2) x_r) \tag{1.12}$$

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$$\sum_{r=1}^{9} ((r+2)^2 x_r) \;=\; (-5) \sum_{r=1}^{9} (r^2 x_r) \;+\; \frac{10}{3} \sum_{r=1}^{9} ((r+1)^2 x_r) \;+\; \frac{2}{3} \sum_{r=1}^{9} ((2r-1)^2 x_r)$$

$$(1.13)$$

so by (1.1), (1.2), and (1.3),

$$\sum_{r=1}^{9} ((r+2)^2 x_r) = (-5)(1) + \frac{10}{3}(11) + \frac{2}{3}(111) \tag{1.14}$$

$$\sum_{r=1}^{9} ((r+2)^2 x_r) = \frac{317}{3} \tag{1.15}$$