Digital Signal Processing
Assignment 1

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Contents

1 Software Installation

Run the following commands

```
sudo apt−get update
sudo apt−get install libffi−dev libsndfile1
    python3−scipy python3−numpy python3−
    matplotlib
sudo pip install cffi pysoundfile
```

2 Digital Filter

2.1 Download the sound file from

```
wget https://raw.githubusercontent.com/
    gadepall/
EE1310/master/filter/codes/Sound_Noise.
    wav
```

http://tlc.iith.ac.in/img/sound/Sound_Noise.wav in the link given below.

2.2 You will find a spectrogram at https://academo.org/demos/spectrum-analyzer. Upload the sound file that you downloaded in the spectrogram and play. Observe the spectrogram. What do you find?

Solution: There are a lot of yellow lines between 440 Hz to 5.1 KHz. These represent the synthesizer key tones. Also, the key strokes are audible along with background noise. By observing spectrogram, it clearly shows that tonal frequency is under 4kHz. And above 4kHz only noise is present.

2.3 Write the python code for removal of out of band noise and execute the code.

Solution:

```
import soundfile as sf
from scipy import signal
#read .wav file
input_signal,fs = sf.read('Sound_Noise.wav')
#sampling frequency of Input signal
sampl_freq=fs
#order of the filter
order=4
#cutoff frquency 4kHz
cutoff_freq=4000.0
#digital frequency
Wn=2*cutoff_freq/sampl_freq
# b and a are numerator and denominator
# polynomials respectively
b, a = signal.butter(order,Wn, 'low')
#filter the input signal with butterworth fi
output_signal = signal.filtfilt(b, a,input_s
#output signal = signal.lfilter(b, a, input_
#write the output signal into .wav file
sf.write('Sound_With_ReducedNoise.wav',outpu
```

2.4 The output of the python script in Problem 2.3 is the audio file Sound_With_ReducedNoise.wav. Play the file in the spectrogram in Problem 2.2. What do you observe?

Solution: The key strokes as well as background noise is subdued in the audio. Also, the signal is blank for frequencies above 5.1 kHz.

3 Difference Equation

3.1 Let

$$x(n) = \left\{ \underset{\uparrow}{1}, 2, 3, 4, 2, 1 \right\} \tag{3.1}$$

Sketch $x(n)$.

3.2 Let

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2), \quad y(n) = 0, n < 0 \tag{3.2}$$

Sketch $y(n)$.

Solution: The following codes yields Fig. 3.2.

```
wget https://github.com/Sparsh−gupta26/
    EE3900/codes/xnyn.py

wget https://github.com/Sparsh−gupta26/
    EE3900/codes/xnyn.c
```

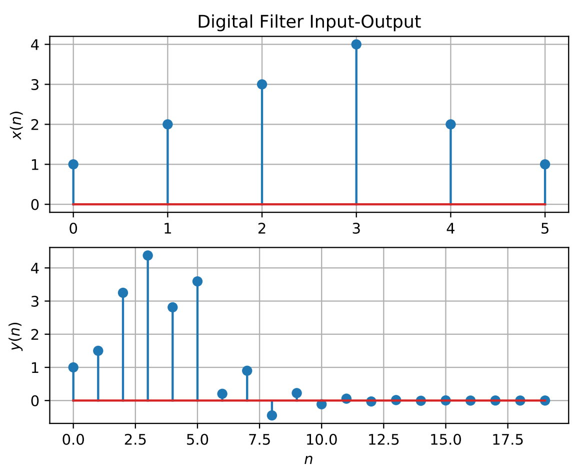

Fig. 3.2

4 Z-transform

4.1 The Z-transform of $x(n)$ is defined as

$$X(z) = \mathcal{Z}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{4.1}$$

Show that

$$\mathcal{Z}\{x(n-1)\} = z^{-1}X(z) \tag{4.2}$$

and find

$$\mathcal{Z}\{x(n-k)\} \tag{4.3}$$

Solution: From (4.1),

$$\mathcal{Z}\{x(n-1)\} = \sum_{n=-\infty}^{\infty} x(n-1)z^{-n} \tag{4.4}$$

$$= \sum_{n=-\infty}^{\infty} x(n)z^{-n-1} = z^{-1}\sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{4.5}$$

resulting in (4.2). Similarly, it can be shown that

$$\mathcal{Z}\{x(n-k)\} = z^{-k}X(z) \tag{4.6}$$

4.2 Obtain $X(z)$ for $x(n)$ defined in problem (3.1).

Solution: From (3.1)

$$X(z) = \mathcal{Z}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

Since, our $x(n)$ is of valid size with valid indices varying from 1 to 6. Therefore,

$$\mathcal{Z}\{x(n)\} = \sum_{n=1}^{6} x(n)z^{-n} \tag{4.7}$$

$$\mathcal{Z}\{x(n)\} = x(1)z^{-1} + x(2)z^{-2} + x(3)z^{-3} + x(4)z^{-4} + x(5)z^{-5} + x(6)z^{-6} \tag{4.8}$$

Which from (3.1) becomes,

$$\mathcal{Z}\{x(n)\} = 1 \cdot z^{-1} + 2 \cdot z^{-2} + 3 \cdot z^{-3} + 4 \cdot z^{-4} + 2 \cdot z^{-5} + 1 \cdot z^{-6} \tag{4.9}$$

4.3 Find

$$H(z) = \frac{Y(z)}{X(z)} \tag{4.10}$$

from (3.2) assuming that the Z-transform is a linear operation.

Solution: Applying (4.6) in (3.2),

$$Y(z) + \frac{1}{2}z^{-1}Y(z) = X(z) + z^{-2}X(z) \tag{4.11}$$

$$\implies \frac{Y(z)}{X(z)} = \frac{1 + z^{-2}}{1 + \frac{1}{2}z^{-1}} \tag{4.12}$$

4.4 Find the Z transform of

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases} \tag{4.13}$$

and show that the Z-transform of

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{4.14}$$

is

$$U(z) = \frac{1}{1 - z^{-1}}, \quad |z| > 1 \tag{4.15}$$

Solution: It is easy to show that

$$\delta(n) \overset{\mathcal{Z}}{\rightleftharpoons} 1 \tag{4.16}$$

and from (4.14),

$$U(z) = \sum_{n=0}^{\infty} z^{-n} \tag{4.17}$$

$$= \frac{1}{1 - z^{-1}}, \quad |z| > 1 \tag{4.18}$$

using the formula for the sum of an infinite geometric progression.

4.5 Show that

$$a^n u(n) \overset{\mathcal{Z}}{\rightleftharpoons} \frac{1}{1 - az^{-1}} \quad |z| > |a| \tag{4.19}$$

Solution: let $x(n) = a^n u(n)$,

$$\mathcal{Z}\{x(n)\} = \mathcal{Z}\{a^n u(n)\} = \sum_{n=0}^{\infty} a^n z^{-n} \tag{4.20}$$

Using the formula for the sum of an infinite geometric progression, we get,

$$\mathcal{Z}\{a^n u(n)\} = \frac{1}{1 - az^{-1}} \quad |z| > |a| \tag{4.21}$$

4.6 Let

$$H\left(e^{J\omega}\right) = H\left(z = e^{J\omega}\right). \tag{4.22}$$

Plot $|H\left(e^{J\omega}\right)|$. Is it periodic ? If so, find the period. $H(e^{J\omega})$ is known as the *Discrete Time Fourier Transform* (DTFT) of $x(n)$.
Solution: The following code plots Fig. 4.6. And, further the it is periodic with a period of ~ 6.378.
Note - for a function to be periodic $f(t) = f(t+T)$ and here T be our period.

```
wget https://github.com/Sparsh−gupta26/
    EE3900/codes/dtft.py
```

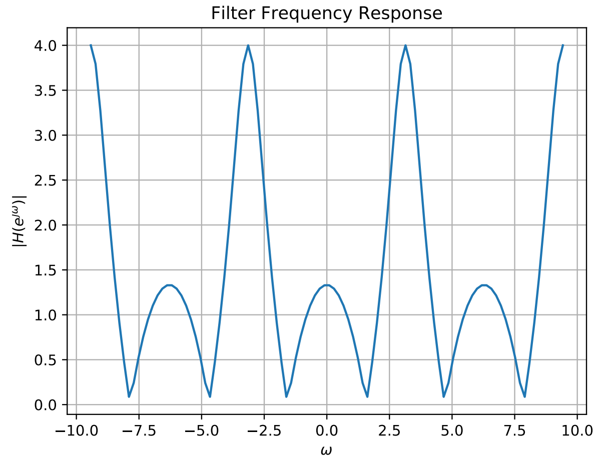

Fig. 4.6: $|H\left(e^{J\omega}\right)|$

4.7 Express $h(n)$ in terms of $H(e^{J\omega})$.
Solution: $h(n)$ can be expressed as follows:

$$h(n) = \frac{1}{2\pi} \int_0^{2\pi} H\left(e^{\jmath\omega}\right) e^{\jmath n\omega} d\omega \tag{4.23}$$

We know that,

$$H\left(e^{\jmath\omega}\right) = \sum_{n=0}^{\infty} h(n) z^{-\jmath n\omega} \tag{4.24}$$

Substitute the above acquired expression in (4.23)

$$h(n) = \frac{1}{2\pi} \int_0^{2\pi} \sum_{k=0}^{\infty} h(k) z^{-\jmath k\omega} e^{\jmath n\omega} d\omega \tag{4.25}$$

$$h(n) = \frac{1}{2\pi} \sum_{k=0}^{\infty} h(k) \int_0^{2\pi} z^{\jmath(n-k)\omega} d\omega \tag{4.26}$$

The above integral is 1 when $k = n$ and 0 otherwise.

$$\implies h(n) = h(n)$$

Hence we can say (4.23) is a correct relation between $h(n)$ and $H(e^{\jmath\omega})$

5 Impulse Response

5.1 Find an expression for $h(n)$ using $H(z)$, given that

$$h(n) \overset{\mathcal{Z}}{\rightleftharpoons} H(z) \tag{5.1}$$

and there is a one to one relationship between $h(n)$ and $H(z)$. $h(n)$ is known as the *impulse response* of the system defined by (3.2).

Solution: From (4.12),

$$H(z) = \frac{1}{1+\frac{1}{2}z^{-1}} + \frac{z^{-2}}{1+\frac{1}{2}z^{-1}} \tag{5.2}$$

$$\implies h(n) = \left(-\frac{1}{2}\right)^{n} u(n) + \left(-\frac{1}{2}\right)^{n-2} u(n-2) \tag{5.3}$$

using (4.19) and (4.6).

5.2 Sketch $h(n)$. Is it bounded? Convergent?
Solution: The following code plots Fig. 5.2.

```
wget https://raw.githubusercontent.com/
    gadepall/EE1310/master/filter/codes/hn.py
```

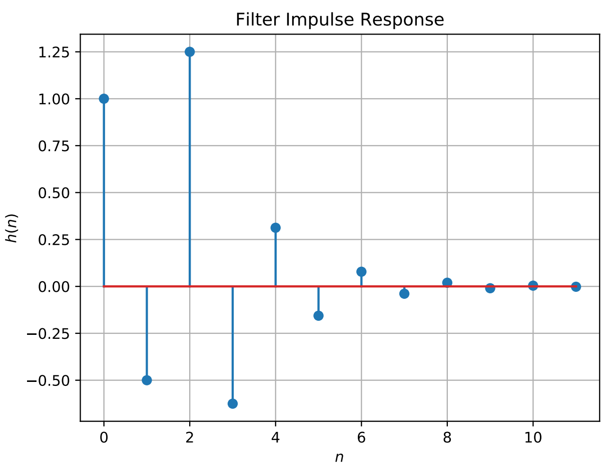

Fig. 5.2: $h(n)$ as the inverse of $H(z)$

5.3 The system with $h(n)$ is defined to be stable if

$$\sum_{n=-\infty}^{\infty} h(n) < \infty \tag{5.4}$$

Is the system defined by (3.2) stable for the impulse response in (5.1)?

5.4 Compute and sketch $h(n)$ using

$$h(n) + \frac{1}{2}h(n-1) = \delta(n) + \delta(n-2), \tag{5.5}$$

This is the definition of $h(n)$.
Solution: The following code plots Fig. 5.4. Note that this is the same as Fig. 5.2.

```
wget https://raw.githubusercontent.com/
    gadepall/EE1310/master/filter/codes/hndef
    .py
```

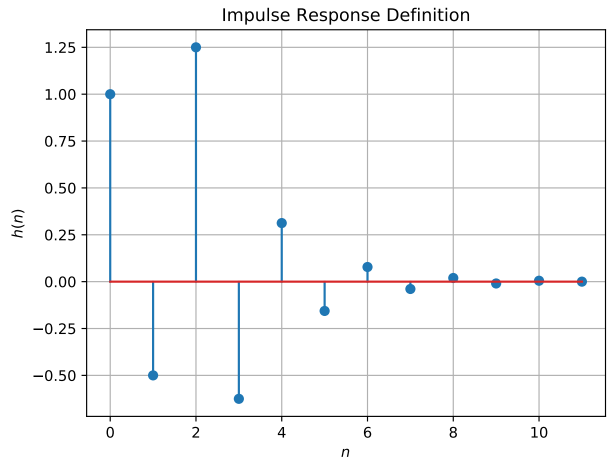

Fig. 5.4: $h(n)$ from the definition

5.5 Compute

$$y(n) = x(n) * h(n) = \sum_{n=-\infty}^{\infty} x(k)h(n-k) \tag{5.6}$$

Comment. The operation in (5.6) is known as *convolution*.
Solution: The following code plots Fig. 5.5. Note that this is the same as $y(n)$ in Fig. 3.2.

```
wget https://raw.githubusercontent.com/
    gadepall/EE1310/master/filter/codes/
    ynconv.py
```

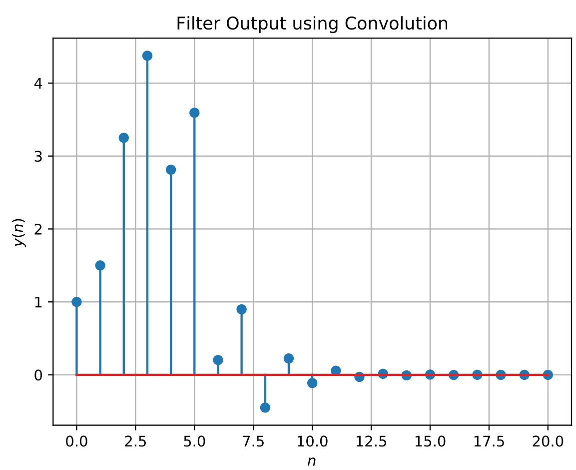

Fig. 5.5: $y(n)$ from the definition of convolution

5.6 Show that

$$y(n) = \sum_{n=-\infty}^{\infty} x(n-k)h(k) \tag{5.7}$$

6 DFT and FFT

6.1 Compute

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n) e^{-J2\pi kn/N}, \quad k = 0, 1, \dots, N-1 \tag{6.1}$$

and $H(k)$ using $h(n)$.

6.2 Compute

$$Y(k) = X(k)H(k) \tag{6.2}$$

6.3 Compute

$$y(n) = \frac{1}{N} \sum_{k=0}^{N-1} Y(k) \cdot e^{J2\pi kn/N}, \quad n = 0, 1, \dots, N-1 \tag{6.3}$$

Solution: The following code plots Fig. 5.5. Note that this is the same as $y(n)$ in Fig. 3.2.

```
wget https://raw.githubusercontent.com/
    gadepall/EE1310/master/filter/codes/yndft.
    py
```

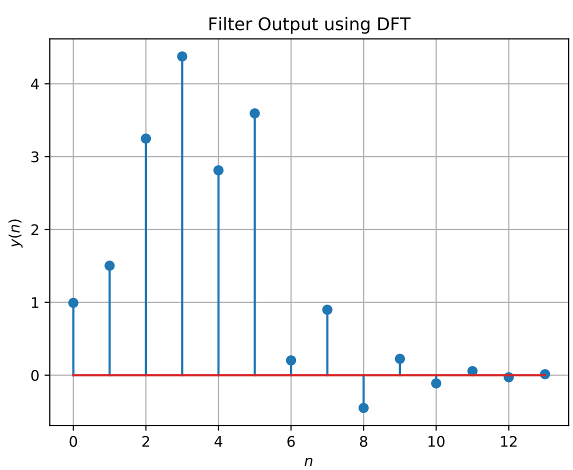

Fig. 6.3: $y(n)$ from the DFT

6.4 Repeat the previous exercise by computing $X(k), H(k)$ and $y(n)$ through FFT and IFFT.

6.5 Wherever possible, express all the above equations as matrix equations.

7 Exercises

Answer the following questions by looking at the python code in Problem 2.3.

7.1 The command

```
output_signal = signal.lfilter(b, a,
    input_signal)
```

in Problem 2.3 is executed through the following difference equation

$$\sum_{m=0}^{M} a(m)\, y(n-m) = \sum_{k=0}^{N} b(k)\, x(n-k) \tag{7.1}$$

where the input signal is $x(n)$ and the output signal is $y(n)$ with initial values all 0. Replace **signal.filtfilt** with your own routine and verify.

7.2 Repeat all the exercises in the previous sections for the above a and b.

7.3 What is the sampling frequency of the input signal?

Solution: Sampling frequency(fs)=44.1kHZ.

7.4 What is type, order and cutoff-frequency of the above butterworth filter

Solution: The given butterworth filter is low pass with order=2 and cutoff-frequency=4kHz.

7.5 Modifying the code with different input parameters and to get the best possible output.