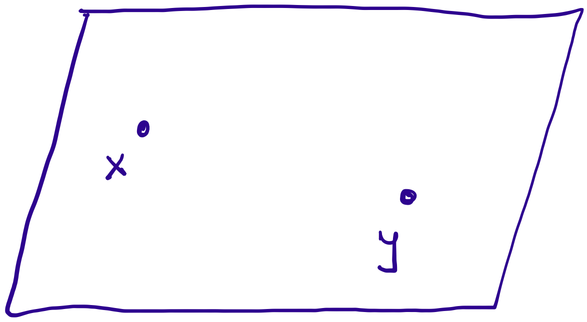
Coulomb Potential

$$j^{\mu}(r,t) = \begin{cases} \delta(r-x) + \delta(r-y) & \mu = 0 \\ 0 & \mu = 1,2 \end{cases}$$



Static charges at x & y.

$$E = \frac{e}{2} \int d^2 r \; j^{\mu}(r) A_{\mu}(r)$$

$$= \frac{e}{2} \int d^2 r \; j^{\mu}(r) \left[\int dt' d^2 r' \, e \, G_{\mu\nu}(r-r') \, j^{\nu}(r') \right]$$

{ Similar to what is done in ϕ^4 theory }

$$= \frac{e^2}{2} \int d^2 r \, d^2 r' \, dt'$$

$$\left\{ j^{\mu}(r) \left[\int \frac{d^2 k}{(2\pi)^2} \frac{d\omega}{2\pi} \, e^{i[k\cdot(r-r') - \omega(t-t')]} G_{\mu\nu}(\omega, k) \right] j^{\nu}(r') \right\}$$

When we integrate over t', we get $\delta(\omega)$ & then when we integrate over ω, we get $G_{\mu\nu}(\omega=0, k)$.

$$= \frac{e^2}{2} \int d^2r \, d^2r' \left\{ j^\mu(r) \left[\int \frac{d^2k}{(2\pi)^2} e^{ik\cdot(r-r')} G_{\mu\nu}(0,k) \right] j^\nu(r') \right\}$$

not QED_{3+1} propagator but PQED

$$= \frac{e^2}{2} \int d^2r \, d^2r' \left\{ \left[\delta(r-x) + \delta(r-y) \right] \left[\int \frac{d^2k}{(2\pi)^2} e^{ik\cdot(r-r')} G_{00}(0,k) \right] \left[\delta(r'-x) + \delta(r'-y) \right] \right\}$$

Only 0^{th} component of j is non-zero, i.e charge $\neq 0$. Current is zero.

$$G^{\mu\nu}(\omega, k) = \frac{P^{\mu\nu}}{2\left[-\omega^2 + k^2\right]^{1/2}}$$

$$P^{\mu\nu} = \delta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}$$

need to derive this using similar way we derive for QED

$$G^{00}(0, k) = \frac{P^{00}}{2(-\omega^2+k^2)^{1/2}} = \frac{1 - \omega^2/k^2}{2\left(k^2-\omega^2\right)^{1/2}}$$

$$= \frac{(k^2 - \omega^2)^{1/2}}{2k^2}$$

$$E = \frac{e^2}{2} \int d^2r\, d^2r' \left\{ \left[\delta(r-x) + \delta(r-y)\right] \left[\int \frac{d^2k}{(2\pi)^2} e^{ik\cdot(r-r')} \frac{(k^2-\omega^2)^{1/2}}{2k^2}\right] \left[\delta(r'-x) + \delta(r'-y)\right] \right\}$$

$$= \frac{e^2}{2} \int d^2r \, d^2r' \; \delta(r-x) \left[\int \frac{d^2k}{(2\pi)^2} e^{ik\cdot(r-r')} \frac{(k^2-\omega^2)^{1/2}}{2k^2} \right] \delta(r'-y)$$

$$+$$

$$\frac{e^2}{2} \int d^2r \, d^2r' \; \delta(r-y) \left[\int \frac{d^2k}{(2\pi)^2} e^{ik\cdot(r-r')} \frac{(k^2-\omega^2)^{1/2}}{2k^2} \right] \delta(r'-x)$$

$$\left\{ \text{neglecting self interaction terms} \right\}$$

$$= \frac{e^2}{2} \int \frac{d^2k}{(2\pi)^2} e^{ik\cdot(x-y)} \frac{(k^2-\omega^2)^{1/2}}{2k^2}$$

$$+$$

$$\frac{e^2}{2} \int \frac{d^2k}{(2\pi)^2} e^{-ik\cdot(x-y)} \frac{(k^2-\omega^2)^{1/2}}{2k^2}$$

$$= \frac{e^2}{2} \int \frac{d^2k}{(2\pi)^2} \left[e^{ik\cdot(x-y)} + e^{-ik\cdot(x-y)} \right] \frac{1}{2\sqrt{k^2}}$$

$$\left\{ k \longrightarrow -k \right\}$$

$$= \frac{e^2}{2} \int \frac{d^2k}{(2\pi)^2} \frac{e^{ik\cdot(x-y)}}{\sqrt{k^2}}$$

$$= \frac{e^2}{4\pi |x-y|}$$

$\left\{ \text{Same for } QED_4 \right\}$