Ginzburg Landau Theory

1. Given $F[\psi, A]$, minimizing it wrt ψ.

$$F[\psi, A] \cong \int d^3r \left[\alpha |\psi|^2 + \frac{\beta}{2} |\psi|^4 + \frac{B^2}{8\pi} + \frac{1}{2m^*} \left| \left(\frac{\hbar}{i}\nabla - \frac{q}{c} A \right) \psi \right|^2 \right]$$

$$= \int d^3r \left[\alpha |\psi|^2 + \frac{\beta}{2} |\psi|^4 + \frac{B^2}{8\pi} + \frac{1}{2m^*} \left[\left(\frac{\hbar}{i}\nabla - \frac{q}{c} A \right) \psi \right]^* \left[\left(\frac{\hbar}{i}\nabla - \frac{q}{c} A \right) \psi \right] \right]$$

Simplifying last term,

$$\psi^* \left[\frac{\hbar}{i}\nabla - \frac{q}{c} A \right]^* \left[\frac{\hbar}{i}\nabla - \frac{q}{c} A \right] \psi$$

$$= \psi^* \left[\frac{\hbar}{i}\nabla - \frac{q}{c} A \right]^2 \psi$$

$$F[\psi, A] = \int d^3r \left[\alpha |\psi|^2 + \frac{\beta}{2} |\psi|^4 + \frac{B^2}{8\pi} + \frac{1}{2m^*} \psi^* \left(\frac{\hbar}{i}\nabla - \frac{q}{c}A \right)^2 \psi \right]$$

$$\frac{\delta F}{\delta \psi^*} = \int d^3r \left[\alpha\psi + \beta |\psi|^2 \psi + \frac{1}{2m^*} \left(\frac{\hbar}{i}\nabla - \frac{q}{c}A \right)^2 \psi \right]$$

$$= 0$$

$$\Rightarrow \boxed{\frac{1}{2m^*} \left(\frac{\hbar}{i}\nabla - \frac{q}{c}A \right)^2 \psi + \left(\alpha + \beta |\psi|^2 \right) \psi = 0}$$

G.L Eq^n (1)

{occuring at eqb^m}

describe variation of order parameter ψ with space close to the surface of S.C in presence of A.

$\underbrace{\text{S.C}}_{\text{Superconductor}}$

② Given $F[\psi, A]$, now minimizing it wrt A.

$$F[\psi, A] \simeq \int d^3r \left[\alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m^*}\psi^* \left(\frac{\hbar}{i}\nabla - \frac{q}{c}A\right)^2 \psi + \frac{(\nabla\times A)^2}{8\pi} \right]$$

$$F[\psi, A+a] \simeq \int d^3r \left[\alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m^*}\psi^* \left[\left(\frac{\hbar}{i}\nabla - \frac{q}{c}A\right) + \left(-\frac{q}{c}a\right)\right]^2 \psi + \frac{(\nabla\times A + \nabla\times a)^2}{8\pi} \right]$$

$$= F[\psi, A] + \int d^3r \left[\frac{1}{4\pi}(\nabla\times A)\cdot(\nabla\times a) + \frac{1}{2m^*}\left[\left(\frac{\hbar}{i}\nabla - \frac{q}{c}A\right)\psi\right]^* \left(-\frac{q}{c}a\psi\right) + \frac{1}{2m^*}\left(-\frac{q}{c}a\psi\right)^* \left[\left(\frac{\hbar}{i}\nabla - \frac{q}{c}A\right)\psi\right] \right]$$

$$= F[\psi, A] +$$

$$\int d^3r \left[-\frac{q a}{2m^* c} \left[\psi^* \left(\frac{\hbar}{i} \nabla \psi\right) + \psi \left(\frac{\hbar}{i} \nabla \psi\right)^* \right] + \frac{q^2}{m^* c^2} A a \, |\psi|^2 + \frac{1}{4\pi} (\nabla \times B) . a \right]$$

$$\frac{\delta F}{\delta A} = \frac{F[\psi, A+a] - F[\psi, A]}{a}$$

G.L Eq^n

(2)

$$\Rightarrow \boxed{ -\frac{i q \hbar}{2m^* c} \left[\psi (\nabla \psi^*) - \psi^* (\nabla \psi) \right] + \frac{q^2 A}{m^* c^2} |\psi|^2 + \frac{1}{4\pi} (\nabla \times B) = 0 }$$

$$\nabla \times B = \frac{4\pi}{c} j \longrightarrow \text{Ampere's law}$$

$$\boxed{\begin{aligned} \mathbf{j} &= \frac{c}{4\pi}\nabla\times B \quad (1) \\ \mathbf{j} &= \frac{iq\hbar}{2m^*c}\left[\psi(\nabla\psi^*) - \psi^*(\nabla\psi)\right] \\ (2) \quad & - \frac{q^2A}{m^*c}|\psi|^2 \end{aligned}}$$

from (2) j is obtaining due to order parameter ψ, & this j will generate from B (B_{induced}) (1)

for uniform $\psi(r)$,

$$j_s = -\frac{q^2|\psi|^2}{m^*c}A$$

$$j_s = -\frac{e^2 n_s}{mc}A$$

{ that's why A is present in G.L eq^n }

{ reproduce London eq^n }

$$\frac{q^2|\psi|^2}{m^*} = \frac{e^2 n_s}{m}$$

$$(-2e)^2 \qquad m^* = 2m \equiv 2m_e$$

$$n_s = \frac{\cancel{m}}{\cancel{e^2}} \frac{4\cancel{e^2}|\psi|^2}{2\cancel{m}}$$

$$n_s = 2|\psi|^2$$

$$\lambda = \sqrt{\frac{mc^2}{4\pi e^2 n_s}} = \sqrt{\frac{mc^2}{8\pi e^2 |\psi|^2}}$$

$$\lambda = \sqrt{\frac{mc^2}{8\pi e^2 \left(-\frac{\alpha}{\beta}\right)}}$$

$$\left\{\begin{aligned} \alpha &= \alpha'(T-T_c) \\ \beta &= \beta \end{aligned}\right.$$

$$\lambda, \xi \propto \frac{1}{\sqrt{T_c - T}}$$

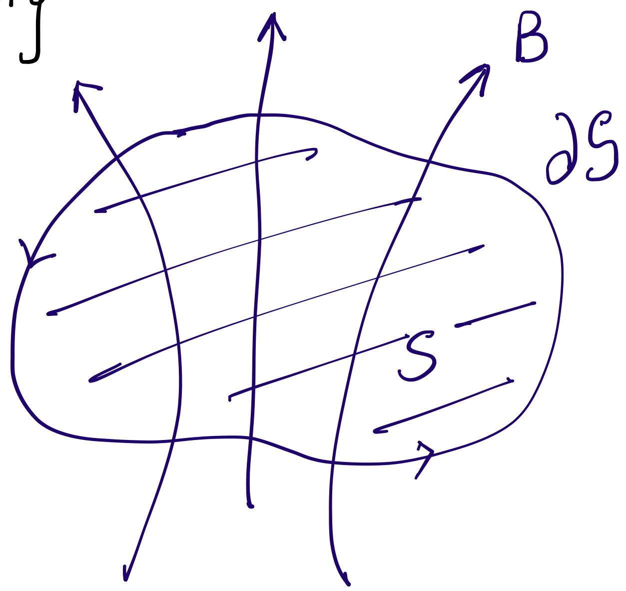
$\lambda \rightarrow$ penetration of B

$\xi \rightarrow$ variation of coarse grained S.C w.f

$$\underbrace{\kappa}_{\text{dimensionless}} = \underbrace{\frac{\lambda(T)}{\xi(T)}}_{\text{ratio}} \quad \left\{ \text{roughly temp. independent} \right\}$$

(G.L parameter)

③ Deriving flux quantization using GL theory



$$\Phi = \int_S da \cdot B = \int_S da \cdot (\nabla \times A)$$

$$= \oint_{\partial S} ds \cdot A$$

$$j = \frac{iq\hbar}{2m^*}\left([\nabla\psi^*]\psi - \psi^*\nabla\psi\right) - \frac{q^2}{m^* c}|\psi|^2 A$$

$$A = -\frac{m^* c}{q^2|\psi|^2} j + \frac{i\hbar c}{2q|\psi|^2}\left([\nabla\psi^*]\psi - \psi^*\nabla\psi\right)$$

$$\Phi = \oint_{\partial S} ds \cdot \left\{ -\frac{m^* c}{\underbrace{q^2}_{4e^2}\underbrace{|\psi|^2}_{n_s/2}}\, j + \frac{i\hbar c}{2q|\psi|^2}\left([\nabla\psi^*]\psi - \psi^*\nabla\psi\right)\right\}$$

$$= \oint_{\partial S} ds \left\{ -\frac{mc}{e^2 n_s}\, j + i\,\frac{\hbar c}{2q}\,(-2i\nabla\phi)\right\}$$

$$\left\{\begin{array}{l} \psi = |\psi| e^{i\phi} \\ \psi^* = |\psi| e^{-i\phi} \\ \psi(\nabla\psi^*) = (-i\nabla\phi)\psi^*\psi \quad \ldots (1) \\ \psi^*(\nabla\psi) = (i\nabla\phi)\psi^*\psi \quad \ldots (2) \\ (1) - (2) \\ \Rightarrow \; -2i(\nabla\phi)|\psi|^2 \\ j = -n_s e v_s \end{array}\right.$$

$$= \frac{mc}{e}\oint_{\partial S} ds \cdot v_s + \frac{\hbar c}{2e}\oint_{\partial S} ds \cdot \underbrace{\nabla\phi}_{\text{change of phase}}$$

$$\Phi - \frac{mc}{e} \oint_{\partial S} ds \cdot v_s$$

$$= \frac{\hbar c}{2e} (2\pi n)$$

$$= \frac{hc}{2e} n \; .$$

$$= n \Phi_0$$

$$\Phi' = n \Phi_0 = \Phi - \frac{mc}{e} \oint_{\partial S} ds \cdot v_s$$

$\downarrow$

fluxoid

$$\Phi' = \Phi \longrightarrow \oint_{\partial S} ds \cdot v_s = 0$$

$$v_s = 0 \Longrightarrow \underline{\underline{j_s = 0}}$$