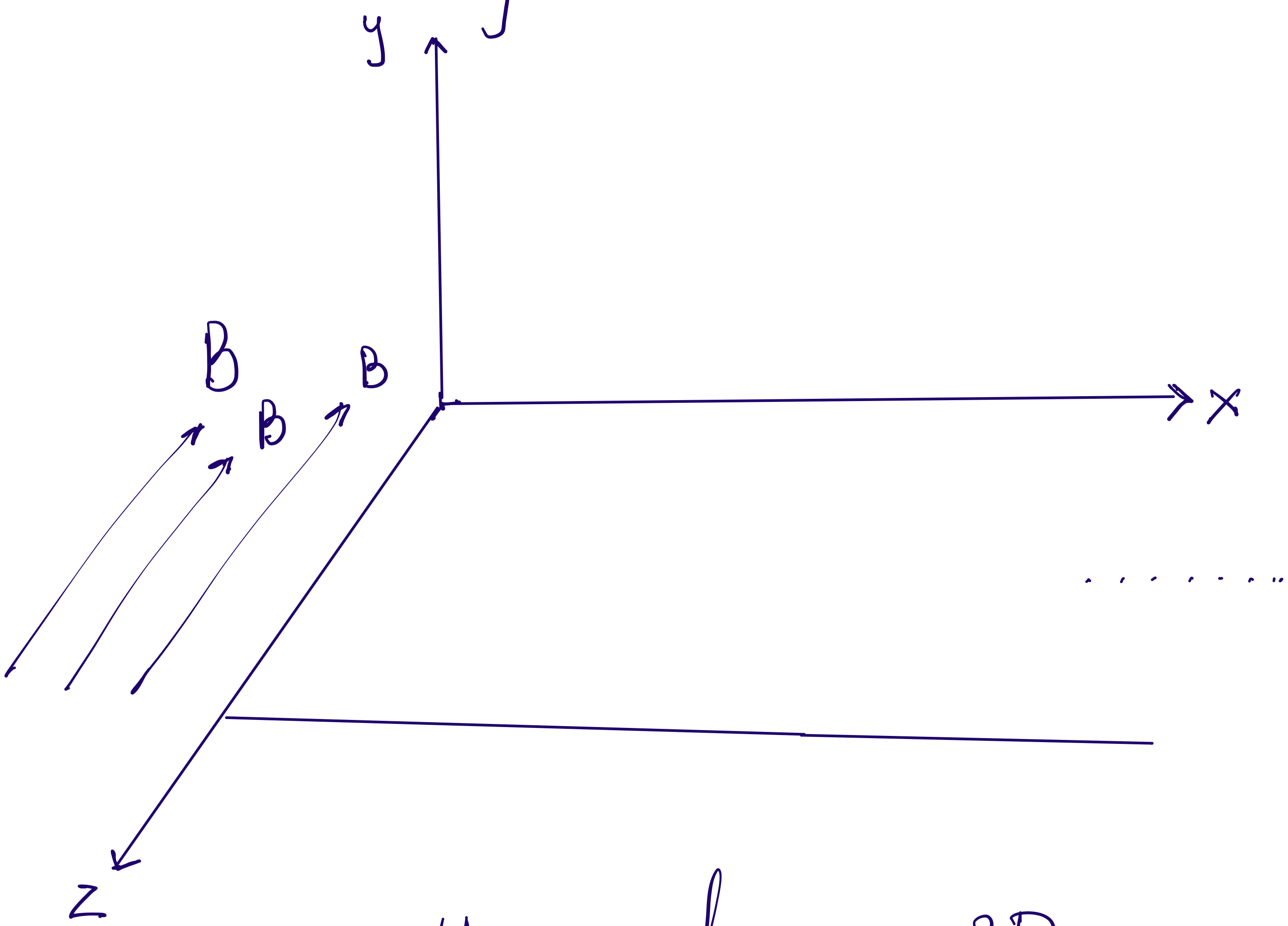
Superconductivity



We consider a 2D semi-infinite superconductor in the XZ plane & apply magnetic field in the z direction. We need to show Meissner effect by showing how B decays with x.

$$\boxed{\nabla \times j_s = -\frac{e^2 n_s}{mc} B}$$

Second London Eq^n

$$\nabla \times B = \frac{4\pi}{c} j_s + \frac{4\pi}{c} j_n$$

$$\nabla \times \nabla \times B = -\frac{4\pi e^2 n_s}{mc^2} B + \frac{4\pi}{c} \sigma_n \nabla \times E$$

$$= -\frac{4\pi e^2 n_s}{mc^2} B - \frac{4\pi}{c} \sigma_n \cancelto{0}{\frac{\partial B}{\partial t}}$$

$\left\{ \text{interested in stationary state} \right\}$

$$-\nabla(\nabla \cdot B) + \nabla^2 B = \frac{4\pi e^2 n_s}{mc^2} B$$

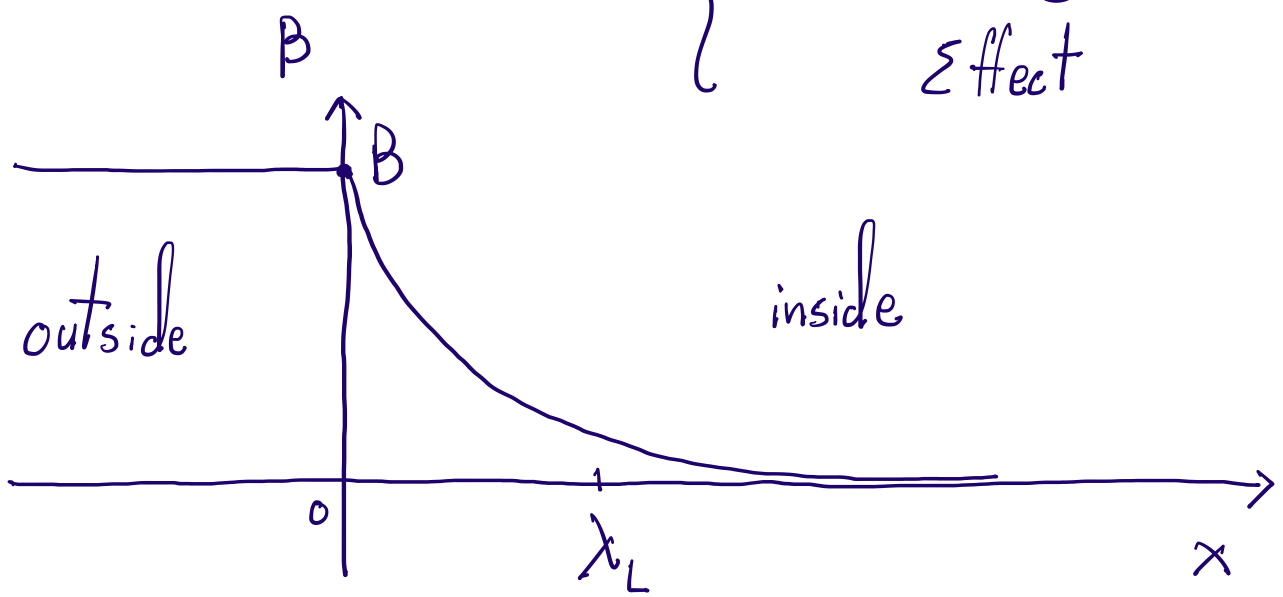
$$\underbrace{\lambda_L}_{\text{London penetration depth}} = \sqrt{\frac{mc^2}{4\pi e^2 n_s}}$$

$$\nabla^2 B = \frac{1}{\lambda_L^2} B$$

$$B(x) = B\hat{y}\, e^{-x/\lambda_L} \quad \text{for } x \geq 0$$

In the bulk, $B \longrightarrow 0$

{Meissner Ochsenfeld Effect}



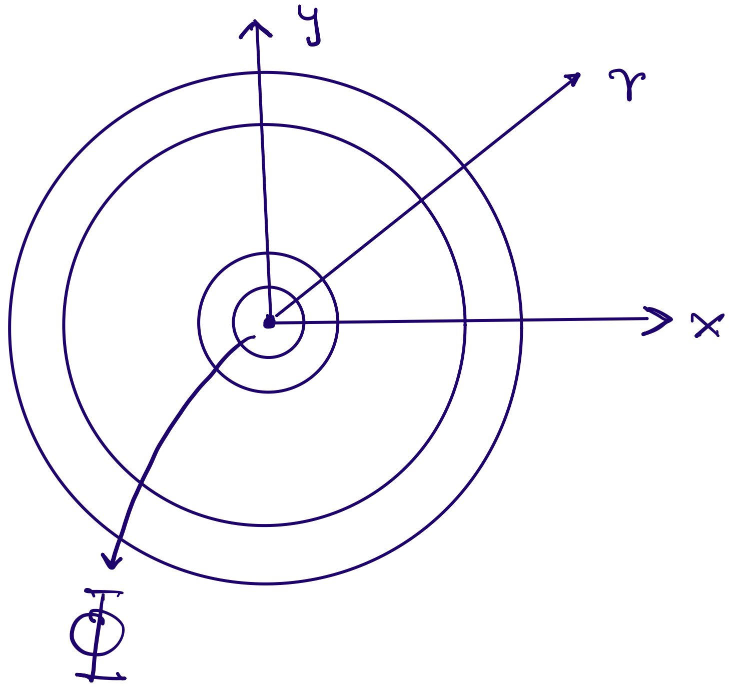
$$\begin{cases} \nabla \times \vec{j}_s = -\dfrac{c}{4\pi\lambda_L^2} B \\ \nabla \cdot \vec{j}_s = 0 \end{cases}$$

$$\vec{j}_s(x) = -\frac{c}{4\pi\lambda_L} B\hat{z}\, e^{-x/\lambda_L}$$

Flux Quantization

Now considering 2 concentric superconducting cylindrical shells that are thick compared to λ_L.

$$\Phi = \int d^2r \; B_\perp$$



$\Phi \longrightarrow$ penetrates inner hole & thin surface layer with order of λ_L.

outside inner cylinder, $B = 0$

$$\oint dl \cdot A = \iint d^2r \; B = \Phi$$

$$A_\varphi = \frac{\Phi}{2\pi r}$$

$$A = \nabla \frac{\Phi}{2\pi} \varphi$$

$\left\{ \text{only non-zero element based on London Gauge} \right\}$

$$A \in \left[0,\ \frac{\Phi}{2\pi r}\,\hat{\varphi}\right]$$

$$A \longrightarrow A + \nabla\chi \qquad \left\{\chi = \frac{\Phi}{2\pi}\varphi\right\}$$

$$\psi_s \longrightarrow \exp\left(-\frac{ie}{\hbar c}\sum_j \chi(r_j)\right)\psi_s$$

continuous but multi valued

$$\psi_s^{\Phi} = \exp\left(-\frac{ie}{\hbar c}\sum_j \frac{\Phi\,\varphi_j}{2\pi}\right)\psi_s^0$$

$$= \exp\left(-\frac{ie\Phi}{hc}\sum_j \varphi_j\right)\psi_s^0$$

ψ_s^0: w.f at $\Phi = 0$ & $A = 0$

$$\varphi_j \longrightarrow \varphi_j + 2\pi$$

exp. factor should not change for w.f ψ_s to be single valued & continuous

$$\frac{e\Phi}{hc} \in Z$$

$$\Rightarrow \boxed{\Phi = \frac{nhc}{e}}$$

$n \in Z$

$$\Phi_0 = \frac{hc}{2e}$$

$\left\{ \begin{array}{l} \text{as } 2\ e^{\ominus} \text{ could form} \\ \text{Boson that could BEC} \end{array} \right\}$