# Ginzburg Landau theory for 2D Superconductors using PQED formalism

$$F = \int d^2r \left\{ a|\psi|^2 + \frac{b}{2}|\psi|^4 + \frac{1}{2m^*}\left|\left(\frac{\hbar}{i}\nabla_r + \frac{2e}{c}A\right)\psi\right|^2 \right\}$$

$$+ \frac{1}{8\pi}\int d^2r\, d^2r' \left(\nabla_r \times A(r)\right) \frac{1}{\sqrt{-\nabla^2_{r-r'}}} \left(\nabla_{r'} \times A(r')\right)$$

$$= \int d^2r \left\{ a|\psi|^2 + \frac{b}{2}|\psi|^4 + \underbrace{\frac{1}{2m^*}\left(\left(\frac{\hbar}{i}\nabla_r + \frac{2e}{c}A(r)\right)\psi\right)^*\left(\left(\frac{\hbar}{i}\nabla_r + \frac{2e}{c}A(r)\right)\psi\right)} \right\}$$

$$+ \frac{1}{8\pi}\int d^2r\, d^2r' \left(\nabla_r \times A(r)\right) \frac{1}{\sqrt{-\nabla^2_{r-r'}}} \left(\nabla_{r'} \times A(r')\right)$$

$$\int d^2r \quad \psi^* \left(-\frac{\hbar}{i}\nabla_r + \frac{2e}{c}A(r)\right)\left(\frac{\hbar}{i}\nabla_r + \frac{2e}{c}A(r)\right)\psi$$

$$= \int d^2r \quad \psi^* \left(\frac{\hbar}{i}\nabla_r + \frac{2e}{c}A(r)\right)^2 \psi$$

$$\frac{\delta F}{\delta \psi^*(r)} = a\psi + \frac{b}{2}|\psi|^2\psi + \frac{1}{2m^*}\left(\frac{\hbar}{i}\nabla_r + \frac{2e}{c}A(r)\right)^2\psi$$

$$= 0$$

$$\Rightarrow \boxed{\left(a + \frac{b}{2}|\psi|^2\right)\psi + \frac{1}{2m^*}\left(\frac{\hbar}{i}\nabla_r + \frac{2e}{c}A(r)\right)^2\psi = 0}$$

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$$\frac{\delta F}{\delta A(x)} = \int d^2r \quad \psi^* \frac{\delta}{\delta A(x)} \Big[ -\frac{\hbar^2}{2m^*} \nabla_r^2 \, \psi(r)$$

$$+ \frac{4e^2}{2m^* c^2} A^2(r)\, \psi(r) + \frac{\hbar}{i} \frac{2e}{c} \frac{1}{2m^*} \underbrace{\nabla_r \left(A(r)\, \psi(r)\right)}$$

$$+ \frac{\hbar}{i} \frac{2e}{c} \frac{A(r)}{2m^*} \nabla_r \, \psi(r) \Big]$$

$$+ \int d^2r \, d^2r' \quad \frac{\delta}{\delta A(x)} \left[ \nabla_r \times A(r) \frac{1}{\sqrt{\nabla^2_{r-r'}}} \nabla_{r'} \times A(r') \right]$$

$$\psi(r) \left(\nabla_r A(r)\right) + A(r) \left(\nabla_r \psi(r)\right)$$

$$= \int d^2r \quad \psi^* \Big[ \frac{4e^2}{m^* c^2} A(r)\, \delta(r-x) \; \psi(r)$$

$$+ \frac{\hbar e}{i m^* c} \delta(r-x) \; \nabla_r \psi(r)$$

$$+ \frac{\hbar e}{i m^* c} \delta(r-x) \; \nabla_r \psi(r) \Big]$$

$$+\int d^2r\, d^2r' \left\{ \frac{\delta}{\delta A(x)} \left[\nabla_r \times A(r)\right] \frac{1}{\sqrt{\nabla^2_{r-r'}}} \left[\nabla_{r'} \times A(r')\right] \right.$$

$$\left. + \left[\nabla_r \times A(r)\right] \frac{\delta}{\delta A(x)} \left[\frac{1}{\sqrt{\nabla^2_{r-r'}}} \left(\nabla_{r'} \times A(r')\right)\right] \right\}$$

$$= \frac{4e^2 A(x) |\Psi|^2}{m^* c^2} - \frac{i 2\hbar e}{m^* c} \nabla_x \Psi(x)$$

$$+ \int d^2r\, d^2r' \left\{ \left(\frac{\delta}{\delta A(x)} \left[\nabla_r \times A(r)\right]\right) \frac{1}{\sqrt{\nabla^2_{r-r'}}} \left[\nabla_{r'} \times A(r')\right] \right.$$

$$\left. + \left[\nabla_r \times A(r)\right] \frac{\delta}{\delta A(x)} \left[\frac{1}{\sqrt{\nabla^2_{r-r'}}} \left(\nabla_{r'} \times A(r')\right)\right] \right\}$$

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$$\frac{1}{\sqrt{\nabla^2_{r-r'}}} \rightarrow G(r-r')$$

$$\left[\nabla_r \times A(r)\right]_i = \epsilon_{ijk}\, \partial_{r_j} A_k$$

$$\frac{\delta \left[\nabla_r \times A(r)\right]_i}{\delta A_m(x)} = \epsilon_{ijk}\, \partial_{r_j}\, \delta_{km}\, \delta(r-x)$$

$$= \epsilon_{ijm}\, \partial_{r_j}\, \delta(r-x)$$

$$= \nabla_r \times [\mathbb{1}\, \delta(r-x)]$$

$$① \int d^2r\, d^2r'\; \epsilon_{ijm} \left(\partial_{r_j} \delta(r-x)\right) G(r-r')\, B_i(r')$$

$$= \int d^2r'\, \epsilon_{ijm} \left[ \int d^2r \left(\partial_{r_j} \delta(r-x)\right) G(r-r') \right] B_i(r')$$

$$= \int d^2r'\; \epsilon_{ijm} \left[ - \int d^2r \left(\partial_{r_j} G(r-r')\right) \delta(r-x) \right] B_i(r')$$

$$= - \epsilon_{ijm} \int d^2r'\; \partial_{x_j}\, G(x-r')\, B_i(r')$$

$$= - \epsilon_{ijm}\, \partial_{x_j} \int d^2r'\; G(x-r')\, B_i(r')$$

$$\frac{1}{\sqrt{\nabla^2_{r-r'}}} \left[\nabla_{r'} \times A(r')\right]_i = G(r-r')\, \epsilon_{ijk}\, \partial_{r'_j} A_k(r')$$

$$\frac{\delta}{\delta A_m(x)} \frac{1}{\sqrt{\nabla^2_{r-r'}}} \left[\nabla_{r'} \times A(r')\right]_i =$$

$$G(r-r')\, \epsilon_{ijk}\, \partial_{r'_j} \sim$$

$$\delta_{km}\,\delta(r'-x)$$

$$= G(r-r')\,\epsilon_{ijm}\,\partial_{r'_j}\,\delta(r'-x)$$

$$② \int d^2r\, d^2r' \; B_{0_i}(r)\, G(r-r')\, \epsilon_{ijm}\, \partial_{r'_j}\, \delta(r'-x)$$

$$= \int d^2r\; B_{0_i}(r)\,\epsilon_{ijm}\left[\int d^2r' \; G(r-r')\,\partial_{r'_j}\,\delta(r'-x)\right]$$

$$= \epsilon_{ijm} \int d^2r\; B_{0_i}(r)\left[-\int d^2r'\,\delta(r'-x)\,\partial_{r'_j}\,G(r-r')\right]$$

$$= -\epsilon_{ijm}\int d^2r\; B_{0_i}(r)\,\partial_{x_j}\,G(r-x)$$

$$= -\epsilon_{ijm}\,\partial_{x_j}\int d^2r\; B_{0_i}(r)\,G(r-x)$$

$$\underbrace{\int d^2r \; G(x-r)\; B_i(r)}_{\text{from } 1 \; (r \leftrightarrow r')} = \underbrace{\int d^2r \; B_i(r)\; G(r-x)}_{\text{from } 2}$$

$$G(x-r) = G(r-x)$$

$$1 + 2 \implies -2\,\epsilon_{ijm}\,\partial_{x_j} \int d^2r \; G(r-x)\, B_i(r)$$

$$= 2\,\epsilon_{mji}\,\partial_{x_j} \int d^2r \; G(r-x)\, B_i(r)$$

$$= 2\; \nabla_x \times \int d^2r \; \frac{1}{\sqrt{\nabla^2_{r-x}}} \; \nabla_r \times A(r)$$

$$= 2\; \nabla_x \times \underbrace{\left[ \int d^2r \; G(r-x)\; \epsilon_{ijm}\, \partial_{r_j} A_m \right]}$$

$$\downarrow$$

$$\epsilon_{ijm} \int d^2r \; G(r-x)\, \partial_{r_j} A_m$$

$$= - \epsilon_{ijm} \int d^2 r \; A_m \partial_{r_j} G(r-x)$$

$$\boxed{\frac{4e^2}{m^* c^2} A(x) \, |\Psi(x)|^2 - \frac{i 2 \hbar e}{m^* c} \nabla_x \Psi(x) + 2 \nabla_x \times \left[ \int d^2 r \frac{1}{\sqrt{\nabla^2_{r-x}}} \nabla_r \times A(r) \right] = 0}$$

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