Pseudo Quantum Electrodynamics (PQED) formalism for Electromagnetic Screening in 2D Superconductors

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Outline

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Motivation

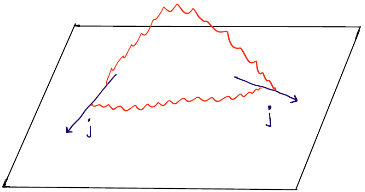

Figure: Matter current j (with blue arrow) confined in a plane but electromagnetic interaction (red colour) having both in and out of plane interactions

- **Problem**:
  - Screening in 2D superconductors is computationally expensive.
  - Many approaches neglect screening or use approximations that fail in 3D environments.
- **Goal**: Develop **tractable** 3D screening framework.
- **Approach**: Use PQED for:
  - Simplified macroscopic theories (e.g., Ginzburg-Landau)
  - Advanced microscopic superconductivity models

Electrodynamics in (3+1)D

$$\mathscr{L}_{QED_{3+1}}[A_\mu, j^\mu_{3+1}] = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - ej^\mu_{3+1}A_\mu + \mathscr{L}_M + \mathscr{L}_{GF} \qquad (1)$$

- Performing the quadratic functional integral over the electromagnetic field

$$Z[j^\mu_{3+1}] = Z_0 \int DA_\mu \exp\left\{-\int d^4z_E \left[\frac{1}{4}F^2 + ej^\mu_{3+1}A_\mu - \frac{\xi}{2}A_\mu\partial^\mu\partial^\nu A_\nu\right]\right\}$$

$$= \exp\left\{\frac{e^2}{2}\int d^4z_E d^4z'_E \;\, j^\mu_{3+1}(z)G_{\mu\nu}(z-z')j^\nu_{3+1}(z')\right\}$$

$$= \exp\left\{\frac{e^2}{2}\int d^4z_E d^4z'_E \;\, j^\mu_{3+1}(z)\left[\frac{1}{\Box}\right]j^\nu_{3+1}(z')\right\} \qquad (2)$$

$$= \exp\left\{-S_{eff}[j^\mu_{3+1}]\right\} \qquad (3)$$

Electrodynamics of matter confined to a plane

- $S_{eff} \rightarrow$ Electromagnetic interaction
- Assuming now that the matter is constrained on a plane.

$$j^{\mu}_{3+1} = \begin{cases} j^{\mu}(x^0, \mathrm{r})\,\delta(z) & \mu = 0,1,2 \\ 0 & \mu = 3 \end{cases} \tag{4}$$

- Now, integrating over k_3,

$$S_{\rm eff}[j^{\mu}] = \frac{e^2}{2}\int d^2r\, d^2r'\, d\tau\, d\tau' \;\; j^{\mu}(r,\tau)\Big[\int \tfrac{d\omega\, d^2k}{(2\pi)^3}\, \tfrac{e^{i[k\cdot(r-r')-\omega(\tau-\tau')]}}{(\omega^2+k^2)^{1/2}}\Big] j^{\mu}(r',\tau') \tag{5}$$

- **Imp**: The above effective interaction can be obtained from a theory in a (2+1)D space-time: The Pseudo Quantum Electrodynamics [1]

$$\mathscr{L}_{PQED} = -\frac{1}{4}F_{\mu\nu}\left[\frac{2}{\sqrt{\Box}}\right]F_{\mu\nu} - ej^{\mu}A_{\mu} + \mathscr{L}_{GF} \tag{6}$$

[1] E.C. Marino, *Quantum Field Theory Approach to Condensed Matter Physics*

Superconductivity

- Superconductivity:
  - **zero electrical resistivity** below a critical temperature T_c.
  - **Meissner effect**: Expulsion of magnetic field

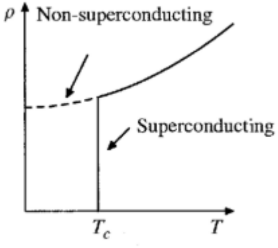

Figure: Resistivity as function of temperature
Source: Superconductivity, Superfluids and Condensates by James Annett

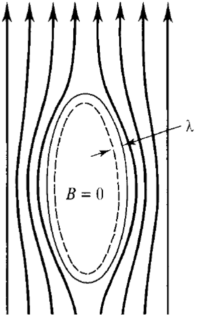

Figure: Meissner Effect
Source: Introduction to Superconductivity by M Tinkham

Ginzburg Landau (GL) Theory

- Macroscopic description of superconductivity: Can be modeled using **GL theory**.
- GL theory for Superconductors:

$$F[\Psi, A] = \int d^3r \left[\alpha|\Psi|^2 + \frac{\beta}{2}|\Psi|^4 + \frac{1}{2m^*}\Psi^* \left(\frac{\hbar}{i}\nabla + \frac{2e}{c}A \right)^2 \Psi + \frac{B^2}{8\pi} \right] \tag{7}$$

- GL theory for 2D Superconductors using PQED formalism:

$$F = \int d^2r \left[\alpha|\Psi|^2 + \frac{\beta}{2}|\Psi|^4 + \frac{1}{2m^*}\Psi^* \left(\frac{\hbar}{i}\nabla + \frac{2e}{c}A \right)^2 \Psi \right]$$
$$+ \frac{1}{8\pi} \int d^2r\, d^2r'\, B(r) \frac{1}{\sqrt{-\nabla^2_{r-r'}}} B(r') \tag{8}$$

Ginzburg Landau (GL) Theory

- Minimizing F (for both cases) with respect to Ψ, we obtain

$$\frac{\delta F}{\delta \Psi^*(r)} = 0$$

$$\implies \left(\alpha + \frac{\beta}{2}|\Psi|^2\right)\Psi + \frac{1}{2m^*}\left(\frac{\hbar}{i}\nabla_r + \frac{2e}{c}A(r)\right)^2 \Psi(r) = 0 \qquad (9)$$

- Minimize F (with non-local term) with respect to A, we obtain,

$$\frac{\delta F}{\delta A(x)} = 0$$

$$\implies \frac{4e^2}{m^* c^2} A(x)|\Psi|^2 - \frac{i2\hbar e}{m^* c}\nabla_x \Psi(x) + 2\nabla_x \times \left[\int d^2 r \frac{1}{\sqrt{-\nabla^2_{r-x}}} B(r)\right] = 0 \qquad (10)$$

Meissner Effect

- Consider a semi-infinite superconductor in the X-Z plane filling the half space $x > 0, z > 0$.
- Magnetic field $B\hat{z}$ is applied parallel to the surface.
- Using **London Equations**:

$$B(x) = B\hat{z}e^{-x/\lambda_L} \qquad x \geq 0 \tag{11}$$

- **Using PQED**: The effective Free Energy F_{eff} obtained after integrating out A_T and A_L components is:

$$F_{eff}[\theta, J] = -\frac{1}{2}\int \frac{d^2k}{(2\pi)^2}\left[\frac{J_T(-k)J_T(k)}{\rho_s + \frac{|k|}{4\pi}} + \frac{J_L(-k)J_L(k)}{\rho_s} + \frac{i\hbar c}{e}k\theta(k)J_L(-k)\right]$$

$$\frac{\delta F_{eff}}{\delta\theta} = 0 \implies J_L(k) = 0$$

Note: Here $\Psi(r) = \sqrt{n_s}e^{i\theta(r)}$ and $\rho_s = \frac{4e^2 n_s}{m^* c^2}$

Meissner Effect

- Effective Photon Propagator:

$$G_T(k) = \rho_s + \frac{|k|}{4\pi}$$

  - Denominator changes from $|k|$ (bare PQED) to $|k| + M$ (Higgs PQED).
  - reflects partial photon mass generation.

$$\implies B(x) = \int \frac{dk}{2\pi} e^{ikx} B(k) = \int dk e^{ikx} \frac{2k}{|k| + M} J_T(k), \;\; M = 4\pi\rho_s$$

- At large distances, [2]

$$B(x) \approx \frac{1}{M^2 x^3}$$

[2] E.C. Marino, *Quantum Field Theory Approach to Condensed Matter Physics*

Conclusion and Outlook

- **Conclusion:** The electromagnetic screening in 2D superconductors is modified by PQED, leading to a different asymptotic behavior in the long-distance versus short-distance limit.
- **Outlook:** The updated electrodynamics equations can be incorporated into macroscopic or microscopic theories to study the screening and magnetic response in thin superconducting systems.