Electrodynamics of a particle confined to a plane

$$j^{\mu}_{3+1} = \begin{cases} j^{\mu}(x^0, r)\ \delta(z) & \mu = 0, 1, 2 \\ 0 & \mu = 3 \end{cases}$$

$$r = (x, y)$$

$$S_{eff}(j^{\mu}_{3+1}) = \frac{e^2}{2} \int d^2r\, d^2r'\, dz\, dz'$$

$$j^{\mu}(r, z) \Big[\underbrace{\int \frac{d\omega\, d^2k\, dk_3}{(2\pi)^4} \frac{e^{i[k\cdot(r-r') - \omega(z-z')]}}{(\omega^2 + k^2 + k_3^2)}}_{\downarrow} \Big] j^{\mu}(r', z')$$

F.T of $\frac{1}{\Box}$

$$= \frac{e^2}{2} \int d^2r\, d^2r'\, dz\, dz'$$

$$j^{\mu}(r, z) \Big[\int \frac{d\omega\, d^2k}{(2\pi)^4} \frac{e^{i[k\cdot(r-r') - \omega(z-z')]}}{[\omega^2 + k^2]^{1/2}} \frac{2\pi}{2} \Big]$$

$$j^{\mu}(r', z')$$

$$\left\{ \int_{-\infty}^{\infty} \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} = \frac{\pi}{a} \right\}$$

$$= \frac{e^2}{2} \int d^2 r \, d^2 r' \, d\tau \, d\tau' \; \frac{j^\mu(r,\tau)}{2} \left[\int \frac{d\omega \, d^2 k}{(2\pi)^3} \frac{e^{i[k\cdot(r-r') - \omega(\tau-\tau')]}}{\sqrt{\omega^2+k^2}} \right] j^\mu(r',\tau')$$

The above effective action can be derived from the (2+1)D Gauge theory

$$\mathcal{L}_{PQED} = -\frac{1}{4} F^{\mu\nu} \left[\frac{2}{\sqrt{\Box}}\right] F_{\mu\nu} - e j^\mu A_\mu + \frac{\xi}{2} A_\mu \frac{\partial^\mu \partial^\nu}{\Box^{1/2}} A_\nu$$