Sol. 4.1 Andrew ID: sporsh g

Lateral Dynamics

$$s_1 = \begin{bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{bmatrix} \qquad u = \begin{bmatrix} \delta \\ F \end{bmatrix}$$

$$\dot{s}_1 = \begin{bmatrix} \dot{y} \\ \ddot{y} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} \dot{y} \\ -\dot{\psi}\dot{x} + \frac{2C_\alpha}{m}\cos\delta \cdot \delta - \frac{2C_\alpha \cos\delta\, \dot{y}}{m\dot{x}} - \frac{2C_\alpha l_f \dot{\psi}}{m\dot{x}}\cos\delta - \frac{2C_\alpha \dot{y}}{m\dot{x}} + \frac{2C_\alpha l_r \dot{\psi}}{m\dot{x}} \\ \dot{\psi} \\ \frac{2 l_f C_\alpha \delta}{I_z} - \frac{2 l_f C_\alpha}{I_z \dot{x}}\dot{y} - \frac{2 l_f^2 C_\alpha \dot{\psi}}{I_z \dot{x}} + \frac{2 l_r C_\alpha \dot{y}}{I_z \dot{x}} - \frac{2 l_r^2 C_\alpha \dot{\psi}}{I_z \dot{x}} \end{bmatrix}$$

$$= f(s_1, u)$$

Equilibrium Point: $\dot{y} = 0$

$\dot{\psi} = 0$

$$\frac{2 l_f C_\alpha \delta}{I_z} = 0 \Rightarrow \delta = 0 \quad \text{— (1)}$$

$$\frac{2C_\alpha \delta \cos\delta}{m} = 0 \Rightarrow \delta = 0, \pi/2 \quad -(2)$$

From (1) & (2) $\rightarrow \delta = 0 \Rightarrow \bar{u}$

Linearize about the equilibrium point using Taylor Expansion:

$$\delta \dot{s}_1 = \underset{0}{f(\bar{s}_1, \bar{u})} + \left.\frac{\partial f(s_1,u)}{\partial s_1}\right|_{\substack{s_1=\bar{s}_1 \\ u=\bar{u}}} \delta s_1 + \left.\frac{\partial f(s_1,u)}{\partial u}\right|_{\substack{s_1=\bar{s}_1 \\ u=\bar{u}}} \delta u$$

$$\left.\frac{\partial f}{\partial s_1}\right|_{\substack{s_1=\bar{s}_1 \\ u=\bar{u}}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-2C_\alpha}{m\dot{x}}(\cos\delta+1) & 0 & -\dot{x} + \frac{2C_\alpha}{m\dot{x}}(-l_f\cos\delta + l_r) \\ 0 & 0 & 1 & 0 \\ 0 & \frac{2C_\alpha}{I_z\dot{x}}(l_r - l_f) & 0 & \frac{-2C_\alpha}{I_z\dot{x}}(l_f^2 + l_r^2) \end{bmatrix}$$

$$\Rightarrow A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \dfrac{-4C_\alpha}{m\dot{x}} & 0 & -\dot{x} + \dfrac{2C_\alpha}{m\dot{x}}(l_r - l_f) \\ 0 & 0 & 1 & 0 \\ 0 & \dfrac{2C_\alpha(l_r - l_f)}{I_z\dot{x}} & 0 & \dfrac{-2C_\alpha}{I_z\dot{x}}\left(l_f^2 + l_r^2\right) \end{bmatrix}$$

$$\left.\frac{\partial f}{\partial u}\right|_{\substack{s_1 = \bar{s}_1 \\ u = \bar{u}}} = \begin{bmatrix} 0 & 0 \\ \dfrac{2C_\alpha}{m} & 0 \\ 0 & 0 \\ \dfrac{2l_fC_\alpha}{I_z} & 0 \end{bmatrix} = B_1$$

$$\dot{s}_1 = A_1 s_1 + B_1 u$$

Now, for Longitudinal Dynamics

$$S_2 = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \qquad u = \begin{bmatrix} \delta \\ F \end{bmatrix}$$

$$\dot{s}_2 = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \underbrace{\begin{bmatrix} \dot{x} \\ \frac{1}{m}(F - fmg) \end{bmatrix}}_{\Rightarrow f(s_2, u)} + \begin{bmatrix} 0 \\ \dot{\varphi} y \end{bmatrix}$$

Finding Equilibrium points $\bar{S}_2$, $\bar{u}$

$$\dot{x} = 0 \Rightarrow \bar{S}_2$$

$$m\dot{\varphi} y = fmg - F$$

$$\Rightarrow F = (fg - \dot{\varphi} y)m \Rightarrow \bar{u}$$

Linearize using Taylor Expansion:

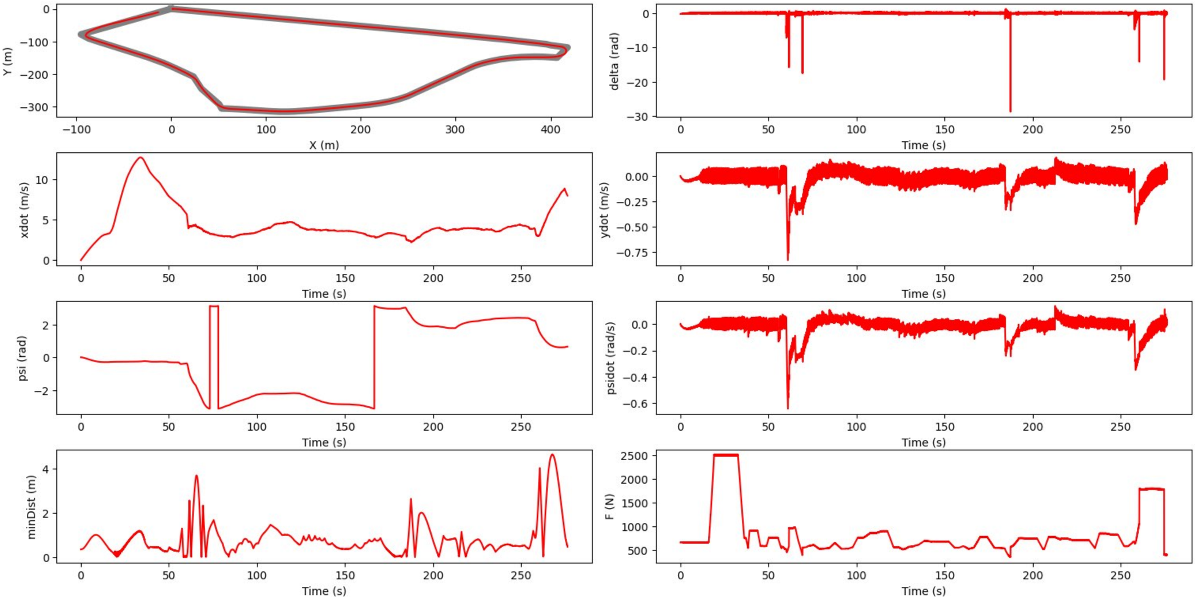
$$\delta \dot{s}_2 = f(\bar{s}_2, \bar{u}) + \left.\frac{\partial f(s_2, u)}{\partial s_2}\right|_{\substack{s_2 = \bar{s}_2 \\ u = \bar{u}}} \delta s_2 + \left.\frac{\partial f(s_2, u)}{\partial u}\right|_{\substack{s_2 = \bar{s}_2 \\ u = \bar{u}}}$$

$$\left.\frac{\partial f(s_2, u_2)}{\partial s_2}\right|_{\substack{s_2 = \bar{s}_2 \\ u = \bar{u}}} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = A_2$$

$$\left.\frac{\partial f(s_2, u)}{\partial u}\right|_{\substack{s_2 = \bar{s}_2 \\ u = \bar{u}}} = \begin{bmatrix} 0 & 0 \\ 0 & 1/m \end{bmatrix} = B_2$$

$$\dot{s}_2 = A_2 s_2 + B_2 u$$

$$\dot{s}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} s_2 + \begin{bmatrix} 0 & 0 \\ 0 & 1/m \end{bmatrix} u + \begin{bmatrix} 0 \\ \dot{\psi}\,\dot{y} \end{bmatrix} \hookrightarrow \text{Disturbance}$$



Y (m)
X (m)
delta (rad)
Time (s)
xdot (m/s)
Time (s)
ydot (m/s)
Time (s)
psi (rad)
Time (s)
psidot (rad/s)
Time (s)
minDist (m)
Time (s)
F (N)
Time (s)