Sol 1.1

$$\tilde{p_l} = \tilde{p_r} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\text{We know that,} \quad \tilde{p_r}^T F \, \tilde{p_l} = 0$$

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow f_{33} = 0$$

Sol 1.2

$$p_l = [u_l \quad v_l \quad 1]$$
$$p_r = [u_r \quad v_r \quad 1]$$
$$t = \begin{bmatrix} t_1 \\ 0 \\ 0 \end{bmatrix}$$
$$R = I_{3x3}$$
$$\text{Essential Matrix, } E = [t_\times]R$$
$$\Rightarrow E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$\Rightarrow E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix}$$
$$\text{We know that epipolar line, } l_l = p_r^T E$$
$$\Rightarrow [u_r \quad v_r \quad 1] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix}$$
$$\Rightarrow l_l = [0 \quad t_1 \quad -v_r t_1]$$
$$\Rightarrow \text{Epipolar line in the Left Camera is, } y = v_r \qquad \ldots 1$$
$$\text{Similarly, Epipolar line in the Right Camera is, } y = v_l \qquad \ldots 2$$

From Equations, 1 & 2, the two cameras have epipolar lines parallel to the X-axis

Sol 1.3

$$\text{At timestamp t1,}$$
$$p_1 = KR_1P + KT_1$$
$$\Rightarrow P = R_1^{-1}K^{-1}p_1 - R_1^{-1}T_1$$
$$\text{At timestamp t2,}$$
$$p_2 = KR_2P + KT_2$$
$$\Rightarrow p_2 = (KR_2R_1^TK^{-1})p_1 + (-KR_2R_1^TT_1 + KT_2)$$
$$\Rightarrow R_{rel} = KR_2R_1^TK^{-1}$$
$$T_{rel} = -KR_2R_1^TT_1 + KT_2$$
$$E = [T_{rel\,\times}]R_{rel}$$
$$F = (K^{-1})^TEK^{-1}$$
$$\Rightarrow F = (K^{-1})^T[T_{rel\,\times}]R_{rel}K^{-1}$$

Sol 1.4

The transformation between the object and its mirror image is pure translation (planar object).

$$\Rightarrow R_{rel} = I_{3\times 3}$$

$$T_{rel} = [t_x \quad t_y \quad t_z]$$

$$F = (K^{-1})^T E K^{-1}$$

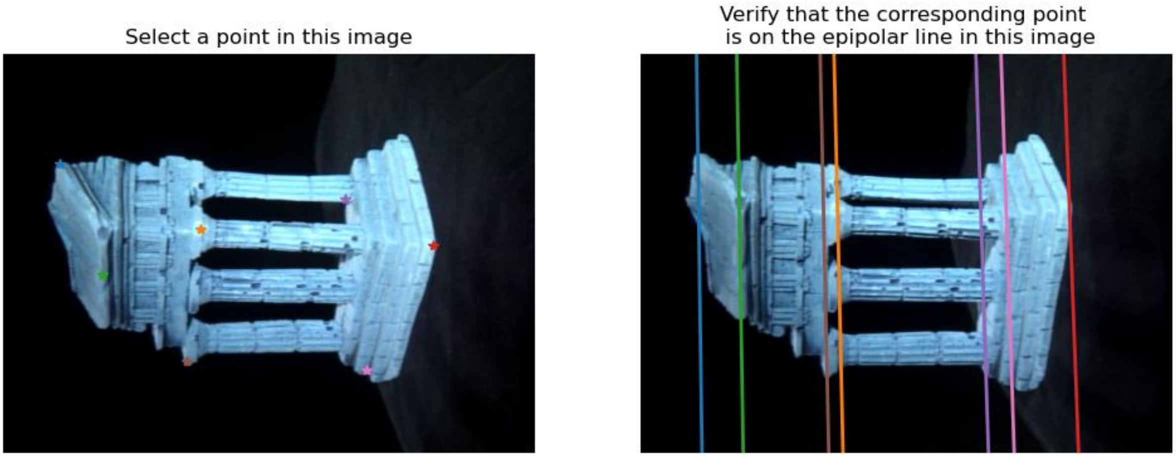
$$F = (K^{-1})^T \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix} K^{-1}$$

$$F^T = (K^{-1})^T \begin{bmatrix} 0 & t_z & -t_y \\ -t_z & 0 & t_x \\ t_y & -t_x & 0 \end{bmatrix} K^{-1}$$

$$\Rightarrow F = -F^T$$

Therefore, Fundamental matrix is skew-symmetric

Sol 2.1



Fundamental Matrix [[9.78833287e-10 -1.32135929e-07 1.12585666e-03]

[-5.73843315e-08 2.96800276e-09 -1.17611996e-05]

[-1.08269003e-03 3.04846703e-05 -4.47032655e-03]]

Sol 3.1

See Code

Sol 3.2

Consider pts1 corresponding to left and pts2 corresponding to right image

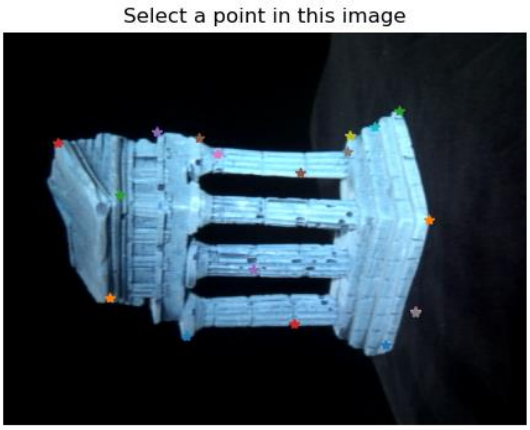
$$A_i = \begin{bmatrix} y_{li} m_{l3}^T - m_{l2}^T \\ m_{l1}^T - x_{li} m_{l3}^T \\ y_{ri} m_{r3}^T - m_{r2}^T \\ m_{r1}^T - x_{ri} m_{r3}^T \end{bmatrix}$$

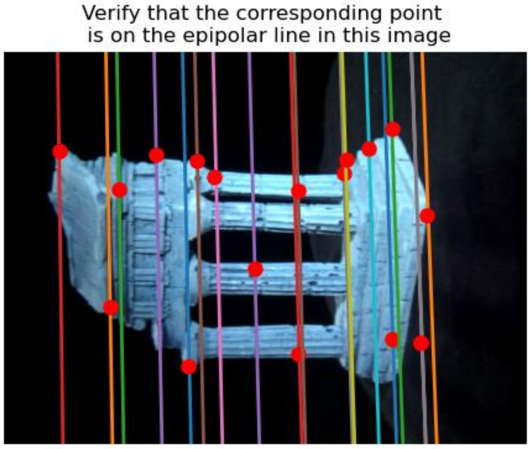
Where m_{li} and m_{ri} are rows in left and right camera projection matrices respectively

Sol 3.3

See code

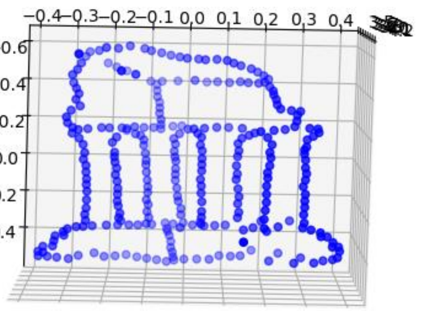
Sol 4.1

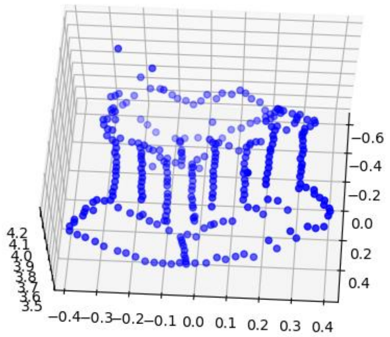


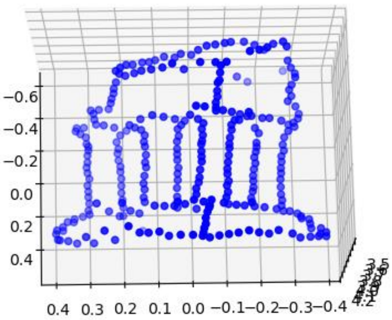


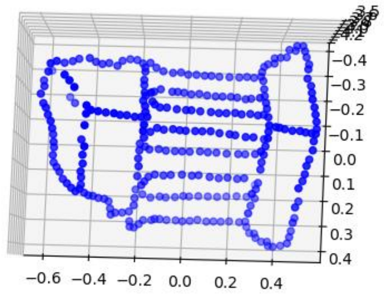
Function *epipolarMatchGUI* in the helper.py has been modified a bit to save the output q4_1.npz

Sol 4.2


−0.4 −0.3 −0.2 −0.1 0.0 0.1 0.2 0.3 0.4
−0.6 −0.4 −0.2 0.0 0.2 0.4

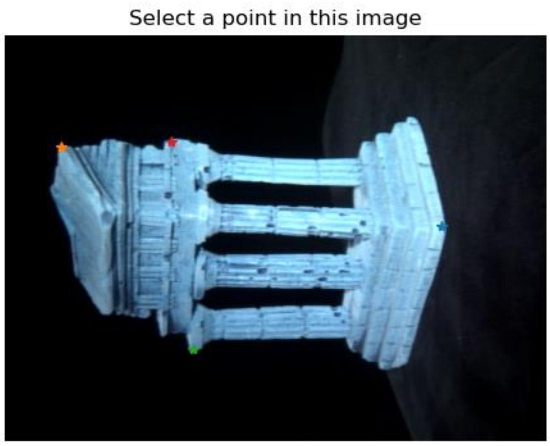

−0.6 −0.4 −0.2 0.0 0.2 0.4
4.2 4.1 4.0 3.9 3.8 3.7 3.6 3.5
−0.4 −0.3 −0.2 −0.1 0.0 0.1 0.2 0.3 0.4

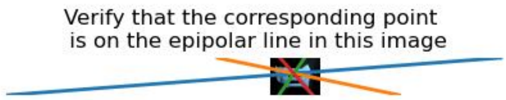

−0.6 −0.4 −0.2 0.0 0.2 0.4
0.4 0.3 0.2 0.1 0.0 −0.1 −0.2 −0.3 −0.4


−0.4 −0.3 −0.2 −0.1 0.0 0.1 0.2 0.3 0.4
−0.6 −0.4 −0.2 0.0 0.2 0.4

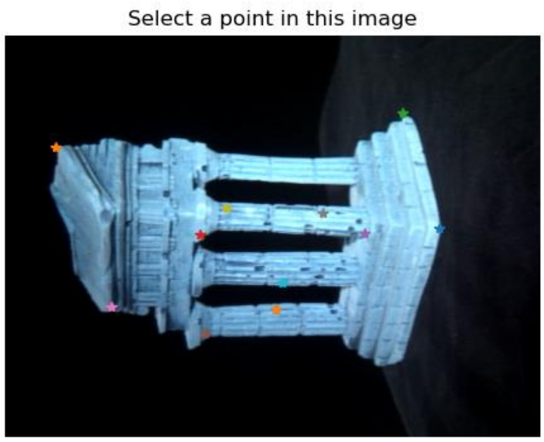
Sol 5.1

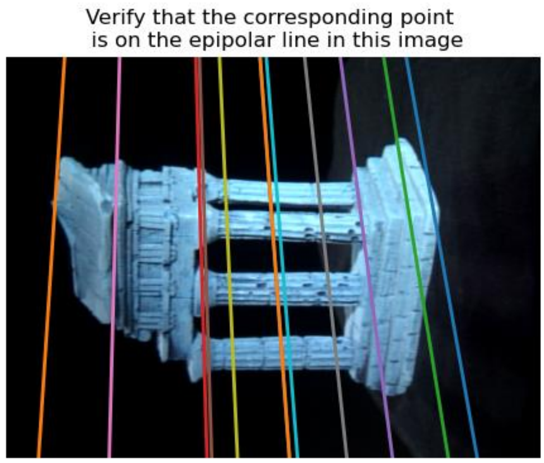
Without Ransac, result from *some_corresp_noisy.npz* :





RANSAC:





Cost function – pr^T * F * pl has been used

Number of inliers: 140

As the number of iterations increases, the accuracy of the fundamental matrix improves.

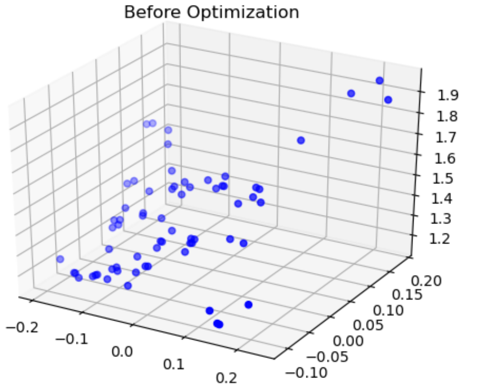
With the increase in the tolerance, more number of points are included, which also improves the accuracy of F, but only up to a limit. After which, the noise gets counted as inliers and the accuracy deteriorates.

Sol 5.2

See Code

Sol 5.3

Initial Reprojection error: 2065.14



Reprojection error after optimization: 59.36