1.1 a

$$\text{Given, } p = [p_x, p_y]^T$$
$$x = [x, y]^T$$
$$W(x;p) = x + p = \begin{bmatrix} x + p_x \\ y + p_y \end{bmatrix} = \begin{bmatrix} 1 & 0 & x \\ 0 & 1 & y \end{bmatrix} * \begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix}$$
$$\Rightarrow \frac{\partial W(x;p)}{\partial p^T} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

b

Following the lecture slides, after linearizing the objective function by multivariate Tayor expansion,

$$\sum_x [I_{t+1}(W(x;p+\Delta p)) - I_t(x)]^2 = \sum_x [I_{t+1}(W(x;p)) + \bigtriangledown I_{t+1}\frac{\partial W}{\partial p}\Delta p - I_t(x)]^2$$

Least squares problem: Minimize to solve for Δp

$$\min_{\Delta \mathrm{p}} \sum_x [\bigtriangledown I_{t+1}\frac{\partial W}{\partial p}\Delta p - \{I_t(x) - I_{t+1}W(x;p)\}]^2$$

$$\text{Comparing with} \quad arg \min_{\Delta \mathrm{p}} ||A\Delta p - b||_2^2$$

$$A = \bigtriangledown I_{t+1}\frac{\partial W}{\partial p}$$
$$b = I_t(x) - I_{t+1}W(x;p)$$

c

To find $\Delta \mathrm{p}$, A^TA should be a full rank matrix, that is, $det(A^TA) \neq 0$

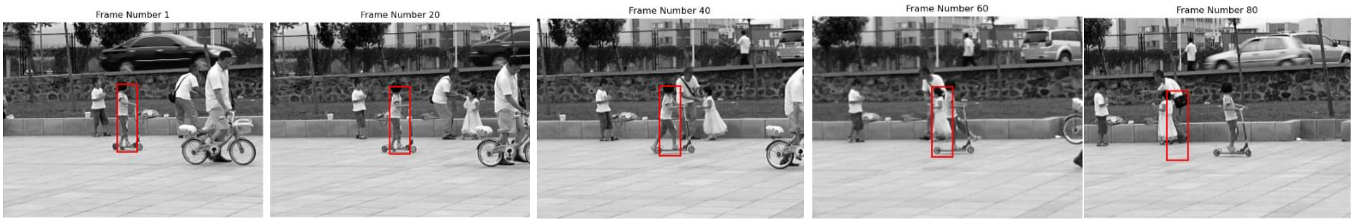
A^TA should be invertible

1.2

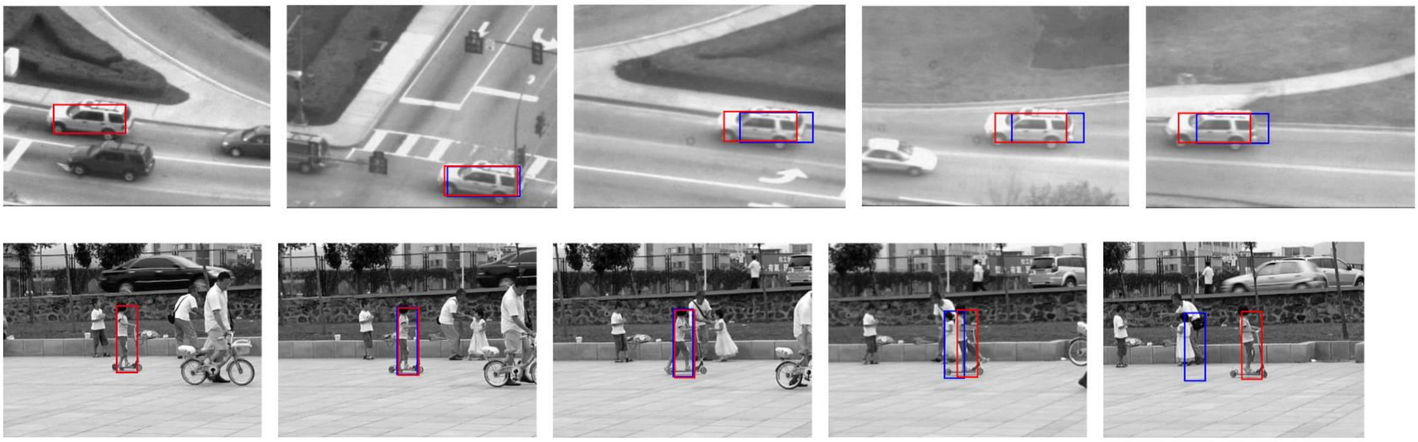
See code

1.3


Frame Number 1
Frame Number 100
Frame Number 200
Frame Number 300
Frame Number 400


Frame Number 1
Frame Number 20
Frame Number 40
Frame Number 60
Frame Number 80

1.4



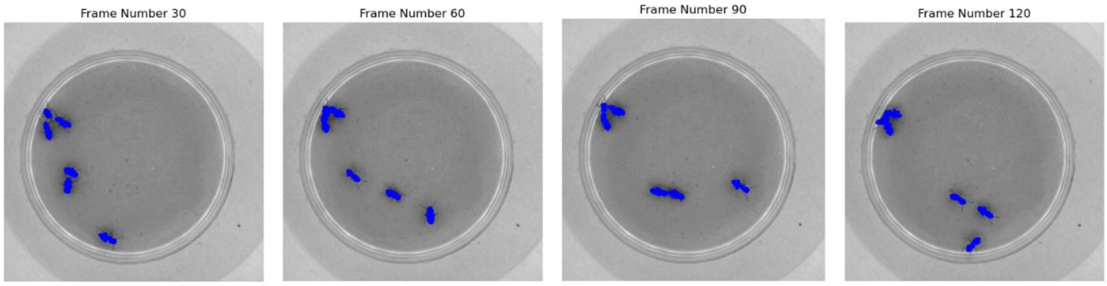
2.1

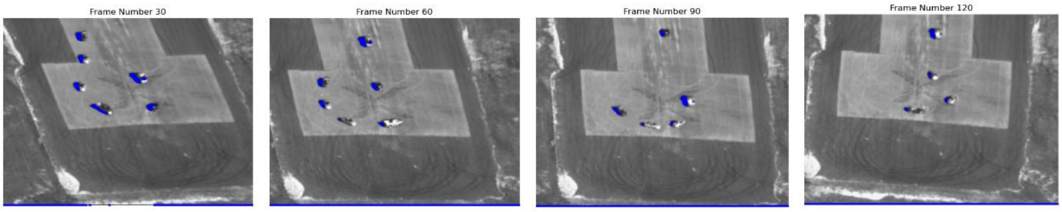
See code

2.2

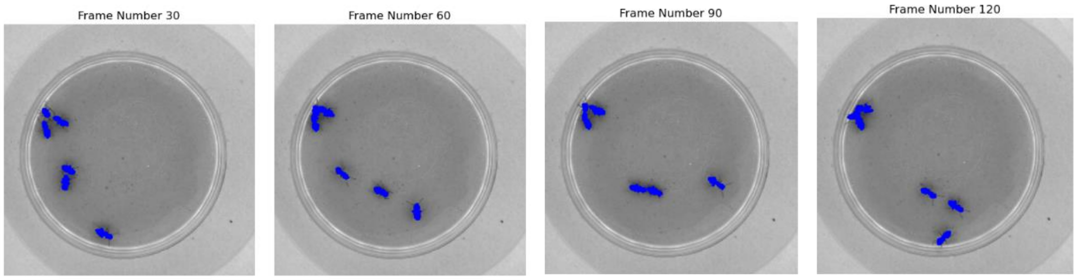
See code

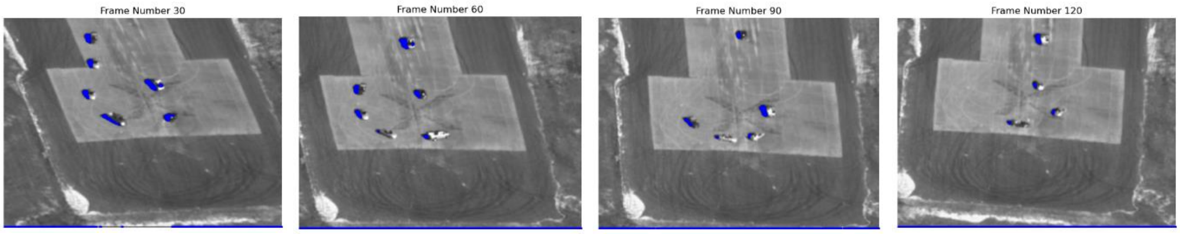
2.3 The moving objects are marked in blue





3.1





The traditional (Additive) approach – function LucasKanadeAffine, is computationally heavier than the inverse composition strategy. The reason is that the Jacobian, the gradient of the template, and the Hessian matrices are all constant, and hence don't need to be recomputed. All these 3 matrices have been pre-computed outside the loop, and only the error image is computed every iteration.

The time required by the program to track using the inverse composition strategy was almost half compared to tracking using the additive strategy.