SPARSH AGARWAL
9075 905 142
CS 576

HW 4

Problem 2

ⓐ

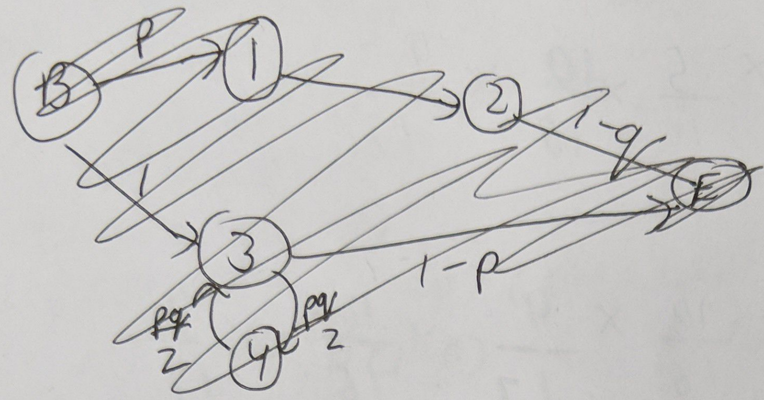
$$L(1) = 0$$
$$L(2) = p(1-p)$$
$$L(3) = ((1-p)+p)(p)(1-p)$$
$$= p(1-p)$$
$$L(4) = (1-p)^2 p(1-p) + (1-p)(p)(p)(1-p) + p(p)^2(1-p)$$
$$= \cancel{(1-p)\,p\,[(1-p)^2 + p\,p^2]}$$
$$(1-p)^3 p \left[1 + \frac{p}{(1-p)} + \frac{p^2}{(1-p)^2}\right]$$

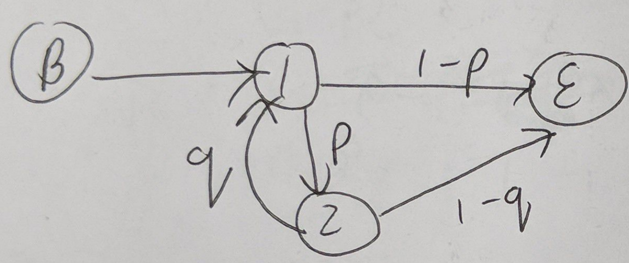
$$L(5) = (1-p)^3 p(1-p) + (1-p)^2 p\,p(1-p) + (1-p)p^3(1-p) +$$
$$p^4(1-p)$$
$$= (1-p)^4 p \left[1 + \left(\frac{p}{1-p}\right) + \frac{p^2}{(1-p)^2} + \frac{p^3}{(1-p)^3}\right]$$
$$= (1-p)^4 p \left[\frac{\left(\frac{p}{1-p}\right)^4 - 1}{\frac{p}{1-p} - 1}\right]$$
$$= (1-p)^4 p \left[\frac{p^4 - (1-p)^4}{(1-p)^3 (2p-1)}\right]$$
$$= (1-p)\,p \left[\frac{p^4 - (1-p)^4}{(2p-1)}\right] = (1-p)p\,(p^2 + (1-p)^2)$$

$$\therefore \text{ for } L(l) = \left.\begin{array}{ll} (1-p)^{l-1}\, p \left[\dfrac{\left(\frac{p}{1-p}\right)^{l-1}-1}{\frac{p}{1-p}-1}\right] & \forall l \geq 2 \\ 0 \ \forall l < 2 & \end{array}\right\} = L(l)$$

$\cancel{L(l) = 0 \text{ for } l<2}$

(b)





$$P(l) = \begin{cases} 0 & \text{if } l = 0 \\ (1-p)(pq)^{(l-1)/2} & \text{for } l>0 \text{ \& } l \text{ odd} \\ p(1-q)(pq)^{(l-2)/2} & \text{for } l>0 \text{ \& } l \text{ even} \end{cases}$$

Problem 3

(a)

	B	A	C	G	E
B	0	4/6	0	2/6	0
A	0	4/16	10/16	2/16	0
C	0	4/17	5/17	4/17	4/17
G	0	5/11	1/11	3/11	2/11
E	0	0	0	0	0

(5) (i) AGG $\Rightarrow \frac{4}{6} \times \frac{2}{16} \times \frac{3}{11} \times \frac{2}{11} = 0.0041322314$

(ii) GAC $= \frac{2}{6} \times \frac{5}{11} \times \frac{10}{16} \times \frac{4}{17} = 0.02228163992$

(iii) ACAC $= \frac{4}{6} \times \frac{10}{16} \times \frac{4}{17} \times \frac{10}{16} \times \frac{4}{17} = 0.01441753171$

(iv) AA $= \frac{4}{6} \times \frac{4}{16} \times 0 = 0$