SPARSH AGARWAL
9075905142

HW5

CS 576

A2) (a) $x = \{-1, 0, 1, 10, 11, 19\}$

(a) $\mu_1 = 0, \mu_2 = 17$

$c_0 = \mu_1$ as $(0-(-1))^2 < (17-(-1))^2$

$c_1 = \mu_1$

$c_2 = \mu_1$

$c_3 = \mu_2$ as $(10-0)^2 > (10-17)^2$

$c_4 = \mu_2$ as $(11-0)^2 > (11-17)^2$

$c_5 = \mu_2$

$$\mu_1 = \frac{1+0-1}{3} = 0$$

$$\mu_2 = \frac{10+11+19}{3} = \frac{40}{3} = 13.33$$

$ED\ \mu_1 = -1 + 0 + 1 = 0$

$ED\ \mu_2 = -(17-10) + (11-17) + (19-17)$

$= -7 - 6 + 2$

$= -11$

second iteration

$c_0 = f_1$

$c_1 = f_1$

$c_2 = f_1$

$c_3 = f_2$

$c_4 = f_2$

$c_5 = f_2$

since the elements clustering did not change.
The values & assignments will remain same
We have achieved convergence

(b) $f_1 = 7, f_2 = 17$

$c_0 = f_1$

$c_1 = f_1$

$c_2 = f_1$

$c_3 = f_1$

$c_4 = f_1$

$c_5 = f_2$

$f_1 = \frac{21}{5} = 4.2$

$f_2 = 19$

$\Sigma D_{f_1} = (-1-7) + (0-7) + (1-7) + (10-7) + (11-7)$
$= -8 - 7 - 6 + 3 + 4 = -14$

$\Sigma D_{f_2} = 19 - 17 = 2$

$c_0 = f_1$

$c_1 = f_1$

$c_2 = f_1$

$c_3 = f_1$

$c_4 = f_1$

$c_5 = f_2$

since the elements clustering did not change
we have achieved convergence

(c) No, K-means clustering would not give optimal ~~s~~ solution as the solution depends on the initial assumption of clusters, which can produce wrong solution. By penalizing farther objects exponentially & rewarding nearer objects exponentially, we can get better results. Eg- something using a ~~Ga~~ Gaussian function

A3) (a) $f(S_1|M_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{S_1 - M_1}{\sigma}\right)^2}$ $\underbrace{\frac{1}{\sqrt{2\pi}}\left(e^{-\frac{1}{2}}\right)}_{A}\left(\frac{3-1}{\sigma}\right)^2 =$ 0.053990967

$f(S_2|M_1) = \frac{A^{\left(\frac{2-1}{\sigma}\right)^2}}{\sqrt{2\pi}} =$ ~~0.053990967~~ 0.24197072

$f(S_3|M_1) = \frac{A^{\left(\frac{-2-1}{\sigma}\right)^2}}{\sqrt{2\pi}} =$ 0.00443185

$f(S_4|M_1) = \frac{A^{\left(\frac{-3-1}{\sigma}\right)^2}}{\sqrt{2\pi}} =$ 0.00013383

$f(S_5|M_1) = \frac{A^{\left(\frac{2-1}{\sigma}\right)^2}}{\sqrt{2\pi}} =$ 0.24197072

$f(S_1|M_2) = \frac{A^{(3-2)^2}}{\sqrt{2\pi}} =$ 0.24197072

$f(S_2|M_2) = \frac{A^{(2-2)^2}}{\sqrt{2\pi}} =$ 0.39894228

$f(S_3|M_2) = \frac{A^{(-2-2)^2}}{\sqrt{2\pi}} =$ 0.00013383

$f(S_4|M_2) = \frac{A^{(-3-2)^2}}{\sqrt{2\pi}} =$ ~~0.1.4867~~ 0.0000014867

$f(S_5|M_2) = \frac{A^{(2-2)^2}}{\sqrt{2\pi}} =$ 0.39894224

$$\gamma_{11} = \frac{\frac{1}{2} f(S_1 | M_1)}{\frac{1}{2} f(S_1|M_1) + \frac{1}{2} f(S_1|M_2)} = 0.18242552$$

$$\gamma_{12} = 1 - \gamma_{11} = 0.81757448$$

$$\gamma_{21} = \frac{\frac{1}{2} f(S_2|M_1)}{\frac{1}{2} f(S_2|M_1) + \frac{1}{2} f(S_2|M_2)} = 0.37754$$

$$\gamma_{22} = 1 - \gamma_{21} = 0.62245933$$

$$\gamma_{31} = \frac{\frac{1}{2} f(S_3|M_1)}{\frac{1}{2} f(S_3|M_1) + \frac{1}{2} f(S_3|M_2)} = 0.970687$$

$$\gamma_{32} = 1 - \gamma_{31} = 0.02931223$$

$$\gamma_{41} = \frac{\frac{1}{2} f(S_4|M_1)}{\frac{1}{2} f(S_4|M_1) + \frac{1}{2} f(S_4|M_2)} = 0.98901$$

$$\gamma_{42} = 1 - \gamma_{41} = 0.01098687$$

$$\gamma_{51} = \frac{\frac{1}{2} f(S_5|M_1)}{\frac{1}{2} f(S_5|M_1) + \frac{1}{2} f(S_5|M_2)} = 0.3775$$

$$\gamma_{52} = 1 - \gamma_{51} = 0.62245933$$

(b)

$$\mu_1 = \frac{\sum_i S_i \times \gamma_{i1}}{\sum \gamma_{i1}} = \cancel{0.125932} \; -0.9840725$$

$$\mu_2 = \frac{\sum_i S_i \times \gamma_{i2}}{\sum_i \gamma_{i2}} = 2.30692104$$

$$P_1 = \frac{\sum_i \gamma_{i1}}{n=5} = 0.57944155$$

$$P_2 = \frac{\sum_i \gamma_{i2}}{n=5} = 0.42055845$$