

Reduced Rat Race Couplers

Name: Sparsh Arya

Registration Number:17BEC0656

Slot: F1

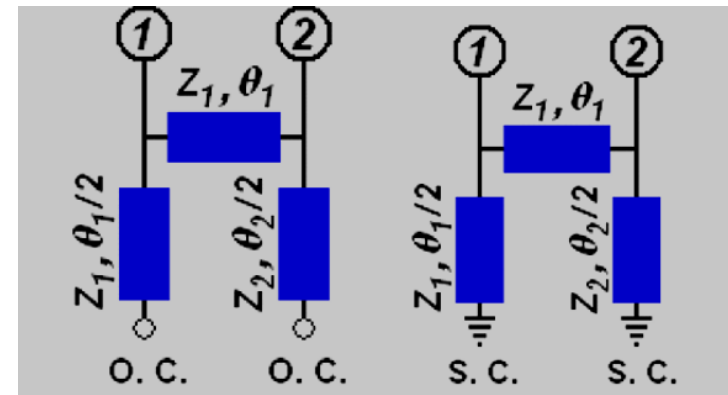
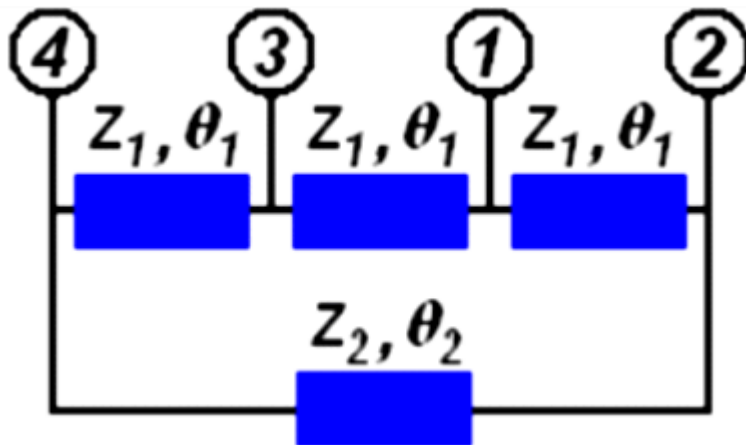
Subject: Microwave Engineering

Abstract and Overview

- The 180 ° rat-race coupler is one of the most fundamental components in microwave frequency range. It is often used in various circuit applications such as mixers, multipliers, amplifiers, beam formers, etc.
- The conventional ring coupler had an electrical length of 1.5λ at the operating frequency and, hence, occupied a large circuit area.
- One efficient approach of miniaturization is to increase effective electrical length by using a slow wave effect. Slow wave effects are realized usually by increasing effective series inductance or shunt capacitance of the transmission line.
- A slotted ground plane is used to increase series inductance, shunt open stubs and artificial transmission lines were used to increase shunt capacitance. Another way of increasing capacitance is to use a spiral compact microstrip resonant cell.
- It is shown that an infinite number of solutions exist for coupler design at a given frequency. For all the solutions, characteristic impedances are less than the conventional $Z_0\sqrt{2}$. In some cases, coupler electrical lengths are less than 1.5λ .
- Due to the lower Z values, the slow wave effect can be used for more effective miniaturization than a $1.5\text{-}\lambda$ coupler under the same fabrication limit of a high impedance line.
- Bandwidth variations of these unconventional couplers are also investigated.

Design

- A signal applied to port 1 will be evenly split into two in-phase components at ports 2 and 3, and port 4 will be isolated.
- If the input is applied to port 4, it will be equally split into two components with a 180° phase difference at ports 2 and 3, and port 1 will be isolated.
- When operated as a combiner, with input signals applied at ports 2 and 3, the sum of the inputs will be formed at port 1, while the difference will be formed at port 4.
- The design is simplified using even and odd mode analysis.



FORMULA

Port 1 Transmission parameters

$$A_e = \cos \theta_1 - \frac{Z_1}{Z_2} \frac{\sin \theta_1}{\cot \frac{\theta_2}{2}}$$

$$B_e = jZ_1 \sin \theta_1$$

$$C_e = j \frac{\cos \theta_1}{Z_1 \cot \frac{\theta_1}{2}} - j \frac{\sin \theta_1}{Z_2 \cot \frac{\theta_1}{2} \cot \frac{\theta_2}{2}} + j \frac{\sin \theta_1}{Z_1} + j \frac{\cos \theta_1}{Z_2 \cot \frac{\theta_2}{2}}$$

$$D_e = \cos \theta_1 - \frac{\sin \theta_1}{\cot \frac{\theta_1}{2}}$$

$$A_o = \cos \theta_1 + \frac{Z_1}{Z_2} \frac{\sin \theta_1}{\tan \frac{\theta_2}{2}}$$

$$B_o = jZ_1 \sin \theta_1$$

$$C_o = -j \frac{\cos \theta_1}{Z_1 \tan \frac{\theta_1}{2}} - j \frac{\sin \theta_1}{Z_2 \tan \frac{\theta_1}{2} \tan \frac{\theta_2}{2}} + j \frac{\sin \theta_1}{Z_1} - j \frac{\cos \theta_1}{Z_2 \tan \frac{\theta_2}{2}}$$

$$D_o = \cos \theta_1 + \frac{\sin \theta_1}{\tan \frac{\theta_1}{2}}$$

Port 4 Transmission parameters

$$A_e = \cos \theta_1 - \frac{\sin \theta_1}{\cot \frac{\theta_1}{2}}$$

$$B_e = jZ_1 \sin \theta_1$$

$$C_e = j \frac{\cos \theta_1}{Z_2 \cot \frac{\theta_2}{2}} - j \frac{\sin \theta_1}{Z_2 \cot \frac{\theta_1}{2} \cot \frac{\theta_2}{2}} + j \frac{\sin \theta_1}{Z_1} + j \frac{\cos \theta_1}{Z_1 \cot \frac{\theta_1}{2}}$$

$$D_e = \cos \theta_1 - \frac{Z_1}{Z_2} \frac{\sin \theta_1}{\cot \frac{\theta_2}{2}}$$

$$A_o = \cos \theta_1 + \frac{\sin \theta_1}{\tan \frac{\theta_1}{2}}$$

$$B_o = jZ_1 \sin \theta_1$$

$$C_o = -j \frac{\cos \theta_1}{Z_2 \tan \frac{\theta_2}{2}} - j \frac{\sin \theta_1}{Z_2 \tan \frac{\theta_1}{2} \tan \frac{\theta_2}{2}} + j \frac{\sin \theta_1}{Z_1} - j \frac{\cos \theta_1}{Z_1 \tan \frac{\theta_1}{2}}$$

$$D_o = \cos \theta_1 + \frac{Z_1}{Z_2} \frac{\sin \theta_1}{\tan \frac{\theta_2}{2}}$$

At operating frequency, matching ($S_{11} = 0$) and isolation ($S_{41} = 0$) conditions for port 1 excitation yield

$$\frac{Z_1}{Z_2} = -\tan \frac{\theta_1}{2} \tan \frac{\theta_2}{2} \quad (5a)$$

$$\sin \theta_1 \left(\frac{Z_1}{Z_0} - \frac{Z_0}{Z_1} \right) - Z_0 \cos \theta_1 \left(\frac{\tan \frac{\theta_1}{2}}{Z_1} + \frac{\tan \frac{\theta_2}{2}}{Z_2} \right) + \frac{Z_0}{Z_2} \frac{\sin \theta_1}{\cot \frac{\theta_1}{2} \cot \frac{\theta_2}{2}} = 0. \quad (5b)$$

Assuming $Z_1 = Z_2 = Z$, we get

$$\theta_2 = 180^\circ + \theta_1$$

$$Z = Z_0 \sqrt{\left\{ 3 - \frac{1}{2} \left(\cot^2 \frac{\theta_1}{2} + \tan^2 \frac{\theta_1}{2} \right) \right\}}.$$

$$S_{31} = 1/X$$

$$S_{21} = 1/X$$

$$S_{34} = 1/X$$

$$S_{24} = -1/X$$

where

$$X = \left[2 \cos \theta_1 + j \frac{1}{2} \sin \theta_1 \left(\frac{Z}{Z_0} + \frac{2Z_0}{Z} \right) - j \frac{Z_0}{Z} \cos \theta_1 \cot \theta_1 \right].$$

CALCULATIONS

Calculations

For the given circuit, we have assumed that $\theta_1 = 90^\circ$
 $\theta_2 = 270^\circ$. Thus, $\theta_2 = 180^\circ + \theta_1$.

Also, value of $Z_1 = Z_2 = 71.9 \approx 2$.
 $Z_0 = 50 \Omega$

Thus,

$$Z = Z_0 \left[\sqrt{3 - \frac{1}{2} \left(\cot^2 \frac{\theta_1}{2} + \tan^2 \frac{\theta_1}{2} \right)} \right]$$

$$= 50 \left[\sqrt{3 - \frac{1}{2} \left(\cot^2 90^\circ + \tan^2 90^\circ \right)} \right]$$

$$= 50 \left[\sqrt{3 - \frac{1}{2} (1+1)} \right]$$

$$= 50 [\sqrt{3-1}] = 50\sqrt{2}.$$

$$S_{31} = 1/x$$

$$S_{21} = 1/x$$

$$S_{34} = 1/x$$

$$S_{24} = -1/x$$

$$X = \left[2 \cos 90^\circ + j \sin 90^\circ \left(\frac{50\sqrt{2}}{50} + \frac{100}{50\sqrt{2}} \right) - j \frac{50}{50\sqrt{2}} \cot 90^\circ \right]$$

$$= \left[j \left(\frac{\sqrt{2} + \sqrt{2}}{2} \right) \right]$$

$$= j\sqrt{2}$$

Thus,

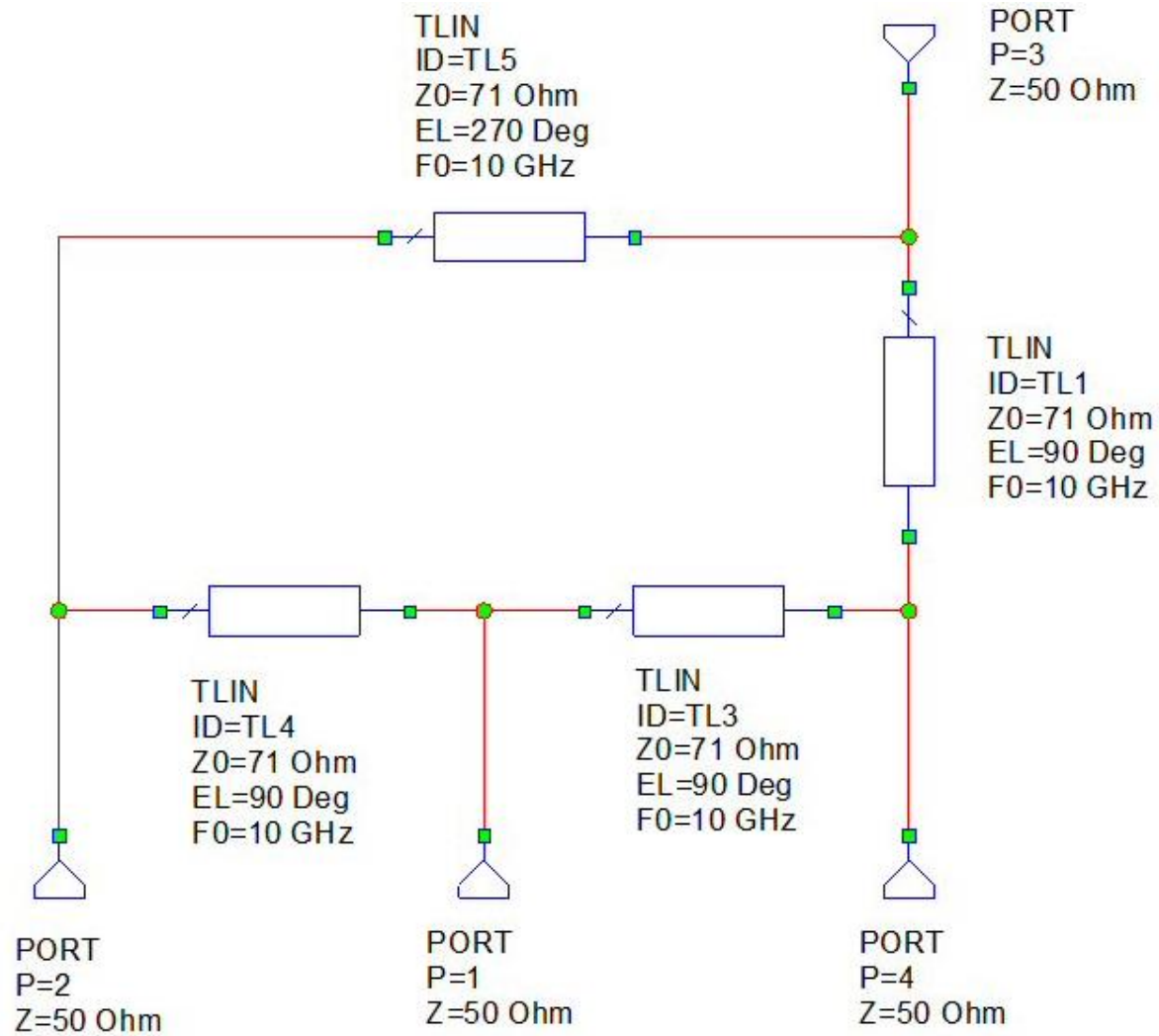
$$S_{31} = -j\sqrt{2}$$

$$S_{21} = -j\sqrt{2}$$

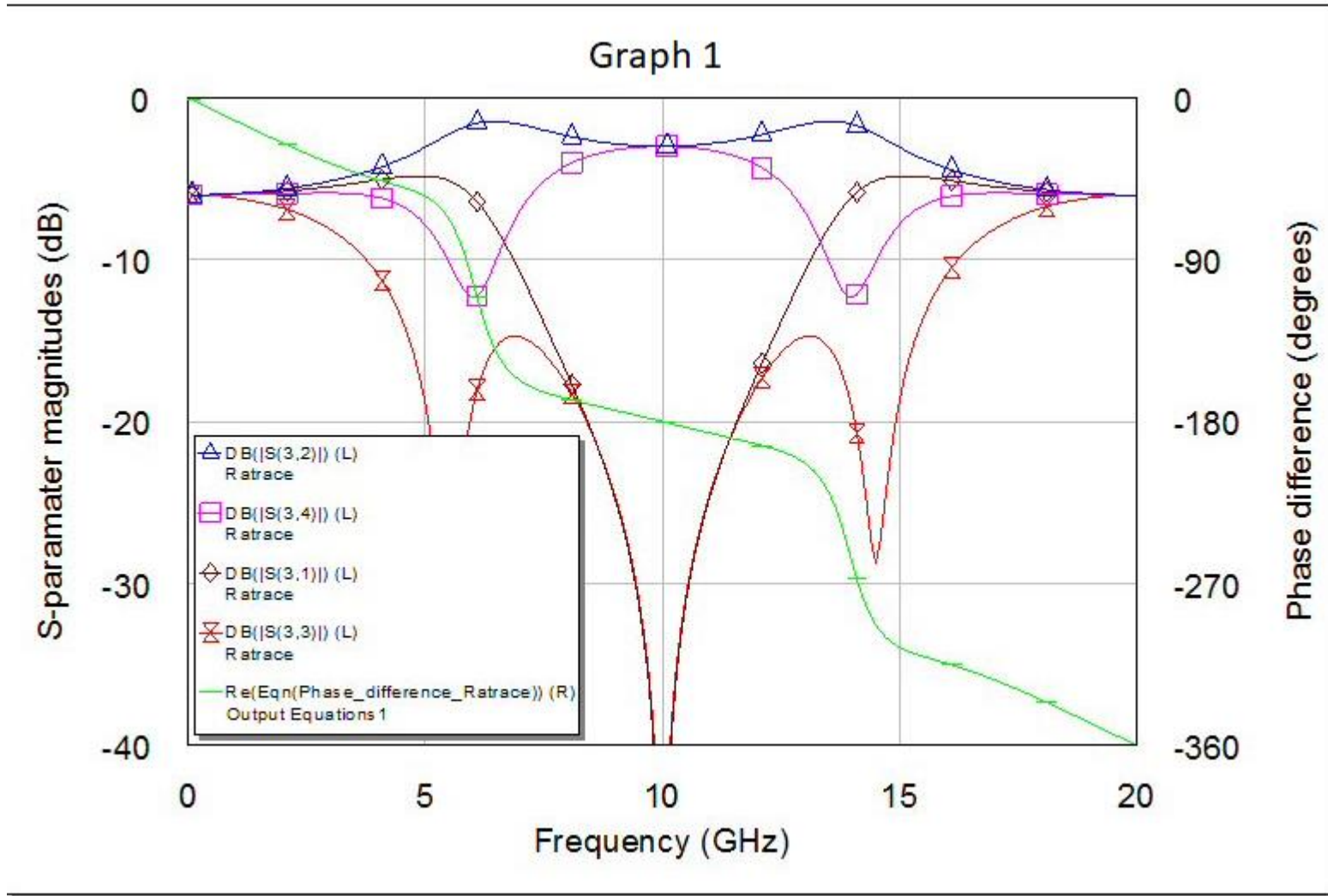
$$S_{34} = -j\sqrt{2}$$

$$S_{24} = j\sqrt{2},$$

Schematic



Graphical analysis for schematic

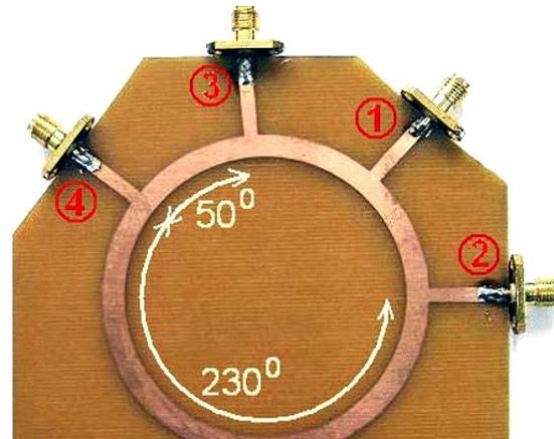


Graphical inference

In the following, coupler bandwidth variations with Z, θ_1, θ_2 are considered. The various coupler upper and lower cutoff frequencies are defined for ± 0.5 -dB amplitude imbalance, $0 \pm 5^\circ$ and $180 \pm 5^\circ$ phase imbalances between the outputs, respectively, for the sum and difference port excitations and for 20-dB matching and isolation. It is observed that amplitude and phase responses are symmetrical about the operating frequency at $\theta_1 = 90^\circ$ and its odd multiples. The coupler band of operation shifts towards higher frequencies if θ_1 decreases below 90° and vice versa. As an example, the transmission line S -parameters for $\theta_1 = 90^\circ$ and 60° are shown in Fig. 4. Due to this asymmetry in the S -parameters, upper (f_2) and lower (f_1) cutoff frequencies are not symmetrical about the operating frequency. In bandwidth variation plots, the fractional bandwidths are defined as $2(f_2 - f_1)/(f_2 + f_1)$.

Conclusion

- Design of Unconventional rat-race couplers has been presented.
- Even and odd mode analysis for the coupler is verified.
- Bandwidth variation in terms of matching, isolation, amplitude and phase imbalance has been presented.
- Amplitude and phase imbalance bandwidth are better than conventional coupler.
- Fabricated coupler has 50.7% more area than conventional coupler.



References

- Microwave Engineering- David M. Pozar
- https://www.tutorialspoint.com/microwave_engineering/microwave_engineering_introduction.htm
- <https://www.microwaves101.com/encyclopedias/waveguide-mathematics>
- [https://en.wikipedia.org › wiki › Microwave_engineering](https://en.wikipedia.org/wiki/Microwave_engineering)