

Unit IV: Markowitz Portfolio Theory

Risk and Return

Unit IV: Markowitz portfolio theory - Risk and return for one security, two security and portfolio. Efficient frontier, Investor utility

Introduction:

Investment decisions are backed by various motives. Some people make investment to acquire control and enjoy prestige associated with it, some to display their wealth and some just for the sake of putting their excess money in some place or the other. But most of the people invest with an aim to get certain benefits in future. These future benefits are the returns you get on the investment. **Return is the driving force behind investment.** It represents rewards for making an investment.

In the case of a fixed income security like a debenture, the returns you get are in the form of periodic interest payment of principal at the end of the maturity.

Similarly in the case of an equity shares, the returns are in the form of dividends and the price appreciation of the shares.

Concept of risk and return:

The investors not only like return but also dislike risk. So in order to discuss the portfolio selection within the context of the risk and return, we require i) the understanding of what the risk and return are; ii) what creates them; and iii) how can they be measured.

Measurement of return: The return may be defined as i) realised return that is the return which was earned or could have been earned; and ii) expected return that is the return the investor anticipate to earn over some future period.

The expected return is the predicted return which may occur or may not occur. Measuring the realised return allows an investor to assess how the future expected returns may be.

For an investor, the return from the investment is the expected cash inflows in terms of dividends, interest, bonus and capital gains. Total return is measured by taking the yield plus the price change. This can be put in the formula as under:

$$k = \frac{D_1 + P_1 - P_0}{P_0}$$

Where, k = Expected return from the investment.

P_0 = Market price at time 0. (Beginning)

P_1 = Market price at time 1, (End) and

D_1 = Cash dividend for the period 1

Example: ABC Ltd is evaluating the rate of return on two of its assets I and II. The asset I was purchased a year ago for Rs. 5,00,000 and since then it has generated cash inflow of the Rs.12,000. Presently it can be sold for a price of Rs. 5,50,000.

Asset II was purchased few years ago and its market price in the beginning and at the end of the year was Rs.2,40,000 and Rs.2,36,000 respectively. The asset II has generated cash inflow of Rs.34,000 during the year. Find out the rate of return on these assets.

Risk:

Risk in investment means that the future returns from that investment are unpredictable. The concept of risk may be defined as the possibilities that the actual return may not be same as expected. In other words, risk may be defined as a chance of variation or chance of loss. **Investment having greater chances of the variations or chances of loss is considered as more risky than those with the lesser chances of variations.**

Following different types of risks are involved in a financial decision.

- Capital Risk:** The risk of incurring a capital loss due to downward changes in the market price of a security is defined as the capital risk of that security. Investment in equity shares have this type of risk running with them.
- Income Risk:** There is a risk of variation in return available from the security. Dividends paid by a company on equity shares may vary from one year to another. However, this risk is almost nil in case of bonds and debentures and preference shares.
- Default Risk:** There may be a default in payment of interest or repayment of principal amount by the company and chances of this default is called the default risk. This risk may also include the risk of losing the principal amount of shares and debentures in case of winding up of the company.

Measurement of Risk and return:

1. Return of Single Asset:

Expected Return

$$E(X) = \sum [X \times P(x)]$$

Where,

$E(X)$ = Expected return of security X

X = return of security X

$P(X)$ = Probability of return of security X

Risk of Single Asset:

Standard Deviation:

$$\sigma_x = \sqrt{\text{Variance}}$$

$$= \sqrt{\sum [X - E(x)]^2 \times P}$$

Where,

σ_x = Standard Deviation of security X

$\sqrt{\quad}$ = Square root

X = return of security X

E(X) = Expected return of security X

P = Probability

2. Return of Portfolio (Two Assets):

$$E(R_p) = W_x \times E(X) + W_y \times E(Y)$$

Where,

E(R_p) = Expected return of portfolio

W_x = Proportion of funds invested in security X

W_y = Proportion of funds invested in security Y

E(X) = Expected return of security X

E(Y) = Expected return of security Y

W_x + W_y = 1.

Risk of Portfolio (Two Assets):

$$\sigma_p = \sqrt{W_x^2 \sigma_x^2 + W_y^2 \sigma_y^2 + 2W_x W_y \gamma_{xy} \sigma_x \sigma_y}$$

Where,

σ_p = Standard deviation of portfolio consisting securities X and Y

W_x, W_y = Proportion of funds invested in Security X and Security Y

σ_x , σ_y = Standard deviation of returns of Security X and Security Y

γ_{xy} = Correlation coefficient between returns of Security X and Security Y

The Correlation Coefficient (γ_{xy}) can be calculated as follows:

$$\gamma_{xy} = \frac{(\text{Cov}_{xy})}{\sigma_x \sigma_y}$$

The Covariance of Security X and Security Y (Cov_{xy}) can be presented as follows:

$$(\text{Cov}_{xy}) = \sigma_x \sigma_y \rho_{xy} \text{ or}$$

$$(\text{Cov}_{xy}) = p \times [X - E(x)] \times [Y - E(y)]$$

3. Expected Rate of Return on Investment: (CAPITAL ASSET PRICING MODEL):

CAPM shows the risk and return relationship of an investment in the formula given below:

$$E(R_s) = I_{RF} + \beta (R_m - I_{RF})$$

Where,

E(R_s) = Expected rate of return on any individual security (or portfolio of securities)

I_{RF} = Risk free rate of return

R_m = Expected rate of return on the market portfolio.

(R_m - I_{RF}) = Risk premium

β = Market sensitivity index of individual security (or portfolio of securities)

4. Beta Coefficient:

$$\beta = \frac{(\text{Cov}_{xp})}{\text{Var}_m}$$

$$= \frac{\sigma_x \sigma_m \gamma_{xm}}{\sigma_m^2}$$

$$= \frac{\sigma_x \gamma_{xm}}{\sigma_m}$$

Where,

β = Beta of individual security

(Cov_{xp}) = Covariance of returns of individual security with market portfolio

Var_m = Variance of returns of market portfolio (σ_m^2)

γ_{xm} = Correlation coefficient between the returns of individual security and the market portfolio

σ_x = Standard deviation of returns of individual security

σ_m = Standard deviation of returns of market portfolio

5. Risk-Return Trade off:

$$R_m - r_x = \frac{R_m - I_{RF}}{\sigma_m}$$

Where,

R_m = Market rate of return

I_{RF} = Risk free return

r_x = Rate of return on individual investment

σ_m = Standard deviation of returns of market portfolio

QUESTIONS

Q. No. 1: The following information is given:

Risk-free rate of return = 3%;

Expected rate of return on market portfolio = 16%;

β of a security = 0.7

(i) Find out the expected rate of return of the security.

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(ii) If another security has an expected return of 24%, what must be its beta?

Q. No. 2: The risk free interest rate is 8% and expected return on market portfolio is 15%. Calculate the expected return of the following security.

Security	β
P	0.5
Q	2
R	1.6
S	1

Q. No. 3: The risk free rate is 4% and market risk premium is 8.6% and beta of security is 1.3. What is the expected return of security under CAPM? What would be the expected return if the beta would be doubled?

Q. No. 4: Find out the expected return on security from the following:

Risk free rate is 9%; return on market portfolio is 18%; If security has beta of i) 1.5; ii) 1; and iii) 2.5

Q. No. 5: Calculate the expected return on portfolio A1 with the following data:

- (i) Risk free rate of return = 5%
- (ii) Expected return on market portfolio = 12%
- (iii) Market sensitivity index. = 0.75

Q. No. 6: Calculate the expected rate of return for security Z from the following information IRF = 10%; RM = 18%; $\beta = 1.35$.

Q. No. 7: a) Calculate the market sensitivity index and the expected return on the investment from the following data:

Standard deviation of an asset = 2.5%
 Market standards deviation = 2.0%
 Risk-free rate of return = 13.0%
 Expected return on market portfolio = 15.0%
 Correlation coefficient of portfolio with market = 0.8
b) What will be the expected return on the portfolio if portfolio beta is 0.5 and the risk-free return is 10%?

Q. No. 8: The following data relate to two securities A and B.

	A	B
Expected Return	22%	17%

Beta Factor (β) 1.5 0.7

Assume IRF = 10% and RM = 3%.

Find out whether the securities A and B are correctly priced? Also show the graphical presentation of the above situation.

Q. No. 9: An investor is seeking the price to pay a security, whose standards deviation is 3.00%. The correlation coefficient for the security with the market is 0.8 and the market standards deviation is 2.2%. The return from Government Securities is 5.2% and from the market portfolio is 9.8%. The investor knows that, by calculating the required return, he can then determine the price to pay for the security. What is the required return on the security?

Q. No. 10: The following information is available in respect of security X and Y.

Security	prob.	Expected Return
X	1.8	22.00%
V	1.6	20.40%

If risk free rate is 7%, are these securities correctly priced? What would the risk free rate has to be if (they are correctly priced?

Q. No. 11: The following information is available in respect of the return from security X under different economic conditions:

State	Return	Probability
Good	20%	0.1
Average	16%	0.4
Bad	10%	0.3
Poor	3%	0.2

Find out the expected return of the security and the risk associated with that.

Q. No. 12: Calculate the beta factor of the following investments. If the acceptance of the investment is worth while based upon its level of risk. The free rate may be taken at 6%.

Probability	Return on Market	Return on Investment
1/3	9%	6%
1/3	12%	30%
1/3	18%	18%

Q. No. 13: You are presented with the following information concerning the returns on the shares of C

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Ltd and on the market portfolio, according to the various conditions of the economy.

Condition	Probability	Return C Ltd	Return market
1	0.2	15%	10%
2	0.4	14%	16%
3	0.4	26%	24%

The current risk-free interest rate is 9%.

Required:

- Calculate the coefficient of correlation between the returns on C Ltd. and the market portfolio.
- Calculate the total risk (i.e. standards deviation) of C Ltd. and discuss why this is not the most appropriate measure of risk to be used in making investment decisions.
- Calculate the beta factor, for C Ltd and briefly discuss its significance. Is C Ltd efficiently priced according to the CAPM and the information given above?

Q. No. 14: The following are the different state of economy, the probability of occurrence of that state and the expected rate of return from security A and B in these different states.

Economy Condition	Probability	Return on S Ltd	Return on R Ltd
Recession	0.20	-15%	20%
Normal	0.45	20%	30%
Boom	0.35	50%	40%

Find out the expected returns and the standard deviations for these two securities. Suppose, an investor has Rs. 20,000 to invest. He invests Rs. 15,000 in security A and balance in security B, what will be the expected return and the standards deviation of the portfolio?

Q. No. 15: From the following information calculate expected rate of return, variance and standard deviation of security A and B.

Economic Condition	Probability	Return A	Return B
Good	0.5	40%	0%
Bad	0.5	0%	40%

Q. No. 16: The shares of Hypothetical Ltd has following anticipated association with return and probability:

Return (%)	Probability
-20	0.05
-10	0.10

10	0.20
15	0.25
20	0.20
25	0.15
30	0.05

You are required to calculate expected return on security its variance and standard deviation.

Q. No. 17: From the following information calculate expected return for the securities and portfolio.

State of Economy	Probability	Return X (%)	Return Y (%)
A	0.1	-8	14
B	0.2	10	-4
C	0.4	8	6
D	0.2	5	15
E	0.1	-4	20

The investor decides to invest 50% of his funds in security X and balance 50% in security Y.

Q. No. 18: " Mr. X is presently concerned with the investment of Rs, 1,00,000. He has two securities S1 and S2 for this purpose. The relevant information in respect of these two securities is as follows:

Particulars	S1	S2
Expected return	12%	20%
σ of return	10%	10%

Coefficient of correlation γ between S1 and S2 = 0.15
He has decided to consider only five portfolios of S1 and S2 as follows:

- All funds invested in S1
- 50% of funds in each of S1 and S2.
- 75% of funds in S1 and 25% in S2
- 25% of funds in S1 and 75% in S2
- All funds Invested in S2.

Find out:

- Expected return under different portfolios.
- Risk factor associated with these portfolios.
- Which portfolio is best for him from the point of risk, and
- Which portfolio is best for him from the point of view of return?

Q. No. 19: Following information is available in respect of two securities X and Y. Find out the expected return and variance of the portfolio consisting of 40% of X and 60% of Y.

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Security	Exp. Return	σ
X	15%	50
Y	20%	30
γ_{xy}	-0.45	

Q. No. 20: Following information is available in respect of market index and share prices of ABC Ltd:

Year	Index	Share Price (Rs.)
1998	218	10.90
1999	230	12.00
2000	248	13.30
2001	250	14.00
2002	282	18.00
2003	297	19.90
2004	288	18.10
2005	290	19.80
2006	320	22.50
2007	356	25.50
2008	371	28.00

Find out β of the shares of ABC Ltd. (Ignore dividends.)