

✓ Relational Algebra

Tables

$\{x, y, +, -, *, /\}$

$U, \cap, \cup, -$

Query languages

tools to obtain from a DBMS

Set Theory

✓ Relation Algebra

Tuple Relational Calc

Logic

SQL

program

add & modify delete

Procedural vs Declarative languages:

✓ Relational Algebra
✓ C/C++/Java
→ Clear instructions
on how to generate
outputs given
inputs

✓ SQL

→ What we want & NOT how to obtain it

Basic Relational Operators:

Projection: Π

one-reln
 $\Pi_{\text{cid, cname}}(\text{Cust})$

cid	cname
i ₁	n ₁
i ₂	n ₁
i ₃	n ₂

cid	cname	czip
i ₁	n ₁	z ₁
i ₂	n ₁	z ₂
i ₃	n ₂	z ₃

Cust

Unary
operators

$\Pi_{\text{cname}}(\text{Cust}) =$

name another-reln

n ₁
n ₁
n ₂

Relation is a set of tuples

cname
n ₁
n ₂

Selection:

Unary

$$\underbrace{\bigvee_{cname=n_1}}_{\text{Condition}} (\underline{\underline{Cust}}) = \begin{array}{|c|c|c|} \hline cid & cname & czip \\ \hline i_1 & n_1 & z_1 \\ \hline i_2 & n_1 & z_2 \\ \hline \end{array}$$

propositional logic statement
 $\textcircled{10} (\vee, \wedge, \neg) \checkmark \textcircled{10}$

$$\underline{\underline{\bigvee_{(cname=n_1) \wedge (czip=z_1)}}} (\underline{\underline{Cust}}) = \begin{array}{|c|c|c|} \hline cid & cname & czip \\ \hline i_1 & n_1 & z_1 \\ \hline \end{array}$$

$$\underline{\underline{\prod cid}} \left(\underline{\underline{\bigvee_{cname=n_1}}} (\underline{\underline{Cust}}) \right) = \begin{array}{|c|} \hline cid \\ \hline i_1 \\ \hline i_2 \\ \hline \end{array} \checkmark$$

cid	Cname	Czip	
i ₁	n ₁	z ₁	✓
i ₂	n ₁	z ₂	✓
i ₃	n ₂	z ₃	✗

Cust

$$\underline{\underline{(2+3) \times 4}} \checkmark$$

Rename:

✓ P cname / custName (Cust)

=

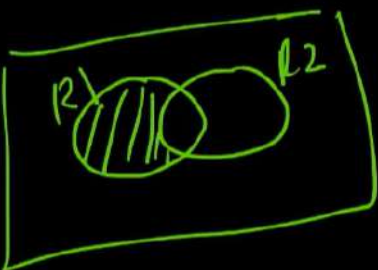
cid	<u>CustName</u>	czip
i ₁	n ₁	z ₁
i ₂	n ₁	z ₂
i ₃	<u>(10)</u> n ₂	z ₃

(10)

cid	Cname	czip
i ₁	n ₁	z ₁
i ₂	n ₁	z ₂
i ₃	n ₂	z ₃

Cust

Set-operations: $\underbrace{U, \cap, -}$

$$R_1 \cup R_2 = \{ (1, a, d), (2, b, \beta), (3, c, \gamma), (4, d, \delta), (2, c, \gamma) \}$$


$$R_1 \cap R_2 = \{ (1, a, d) \}$$

$$R_1 - R_2 = R_1 - (R_1 \cap R_2) = \{ (2, b, \beta), (3, c, \gamma) \}$$

$$\checkmark U = C_1 \times C_2 \times C_3$$

C1	C2	C3
1	a	d
2	b	β
3	c	γ

⑩
 R_1

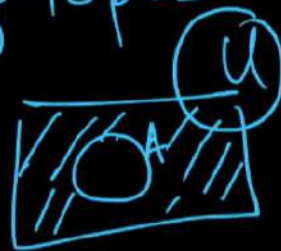
$$R_1^c = U - R_1$$

C1	C2	C3
1	a	d
4	d	δ
2	c	γ

R_2

Relation is a set of tuples

$\{ R_1^c \Rightarrow \text{don't often use} \}$



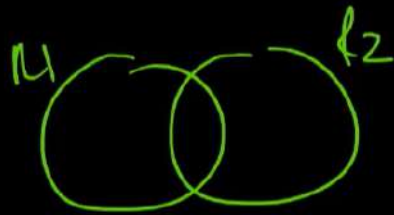
Compatibility of Relations:

✓ $R_1(A, C_2, C_3)$, $R_2(A, C_2)$ → NOT Compatible

$R_1(A, C_2, C_3)$ (10) , $R_2(A, C_2, C_4)$ (10) → NOT Compatible

$R_1(A, C_2, C_3)$, $\left\{ \begin{array}{l} R_2(A, C_2, C_5) \\ C_5/C_3 \quad \text{BAN} \end{array} \right\}$ → Compatible
{ C3 & C5 have same domain }

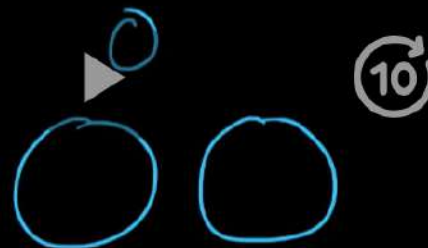
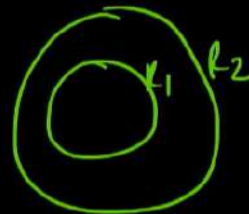
① $\overset{m}{R_1} \cup \overset{n}{R_2}$



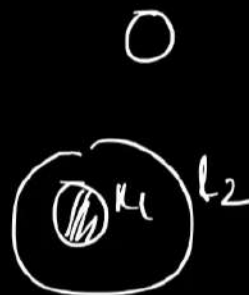
② $R_1 \cap R_2$

⑩

$\overset{\text{min}}{\text{max}(m, n)}$



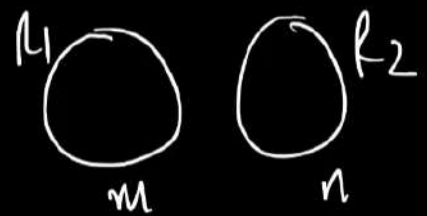
⑩



$\overset{\text{max}}{m+n}$

$\overset{\text{min}(m, n)}{\text{min}(m, n)}$

m



③ $R_1 - R_2 \neq R_2 - R_1$

Cross-product / Cartesian-product:

$$\begin{array}{|c|} \hline A, B \\ \hline R \subseteq \underline{A \times B} \\ \hline \end{array}$$

{ Relation is a set of tuples ✓

$R_1 \times R_2$:

	c1	c2	c3	c3	c4
1	1	a	x	α	β
1	1	a	x	1	α
2	2	b	y	α	β
2	2	b	y	1	α

c1	c2	c3
1	a	x
2	b	y

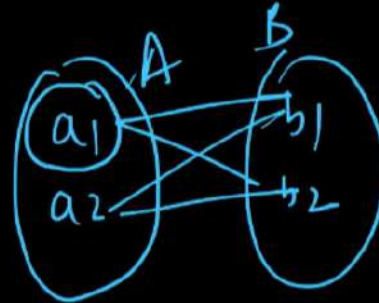
R_1 (m)

c3	c4
α	β
1	α

R_2 (n)

$R_1 \times R_2$: $m \times n$ tuples

$$A \times B = \{(a_1, b_1), (a_1, b_2), (a_2, b_1), (a_2, b_2)\}$$



$$R_1(\underbrace{x_1, x_2, x_3, \dots, x_m}_{\substack{\downarrow \\ m\text{-attrib}}}) \times R_2(\underbrace{s_1, s_2, s_3, \dots, s_n}_{\substack{\downarrow \\ m\text{-attrib}}})$$

$$= \left\{ \underbrace{\left(\underbrace{x_1, x_2, \textcircled{10}, \dots, x_m}_{\substack{\downarrow \\ m\text{-attrib}}} \rightarrow \underbrace{s_1, s_2, \dots, s_n}_{\substack{\downarrow \\ m\text{-attrib}}} \right)}_{m+n \text{ tuple}} \mid \begin{array}{l} (x_1, x_2, \dots, x_m) \in \underline{R_1} \\ \text{and} \\ (s_1, s_2, \dots, s_n) \in \underline{R_2} \end{array} \right\}$$

$R \times R$ ✓

$R_1 \times R_2 \times R_3$ ✓

$R \times R \times R$



40:42 / 41:20



Autoplay



✓ Natural Join (Derived Operators) SQL

$R \bowtie S$ =

C1	C2	C3	C4
1	a	α	1
(10)			

✓

practical

$$\left\{ R \bowtie S = \prod_{C1, C2, C3, C4} \left(\bigwedge_{\substack{R.C2 = S.C2 \\ R.C3 = S.C3}} \right) \right\}$$

C1	C2	C3	C2	C3	C4
1	a	α	a	α	1
2	b	β	a	γ	2

(10) R

S

R(C1, C2, C3)

S(C2, C3, C4)

step by step.

✓
R.C2

R.C1 R.C2 R.C3 S.C2 S.C3 S.C4

Conditional Join: Θ -joins $R \bowtie_{\Theta} S$

$$R \bowtie_c S = \sigma_c(\underline{R \times S})$$

$$R \bowtie_c S = \sigma_{R.C1 < S.C4}(\underline{R \times S})$$

\leq $>$ $<$ $>$
 \neq $=$

c1	c2	c3		c2	c3	c4
①	a	α	\times	a	d	①
②	b	β	\checkmark	a	γ	2

R S

⑩

R. c1	R. c2	R. c3	S. c2	S. c3	S. c4
1	a	α	a	γ	2

\checkmark



15:30 / 40:15



Autoplay



Left-outer join:

$$R \bowtie S = \underbrace{R \bowtie S}_U \quad \checkmark$$

all ¹⁰ tuples from the left relation that failed the join with NULL values appended

c1	c2	c3		c2	c3	c4
1	a	α	\checkmark	a	d	1
2	b	β	\times	a	γ	2

R S

c1	c2	c3	c4
1	a	α	1
2	b	β	NULL

Left-outer join:

$$R \underset{\checkmark}{\bowtie} \underset{\checkmark}{S} = \underset{U}{\overset{\checkmark}{(R \bowtie S)}}$$

all ¹⁰ tuples from the left relation that failed the join with NULL values appended

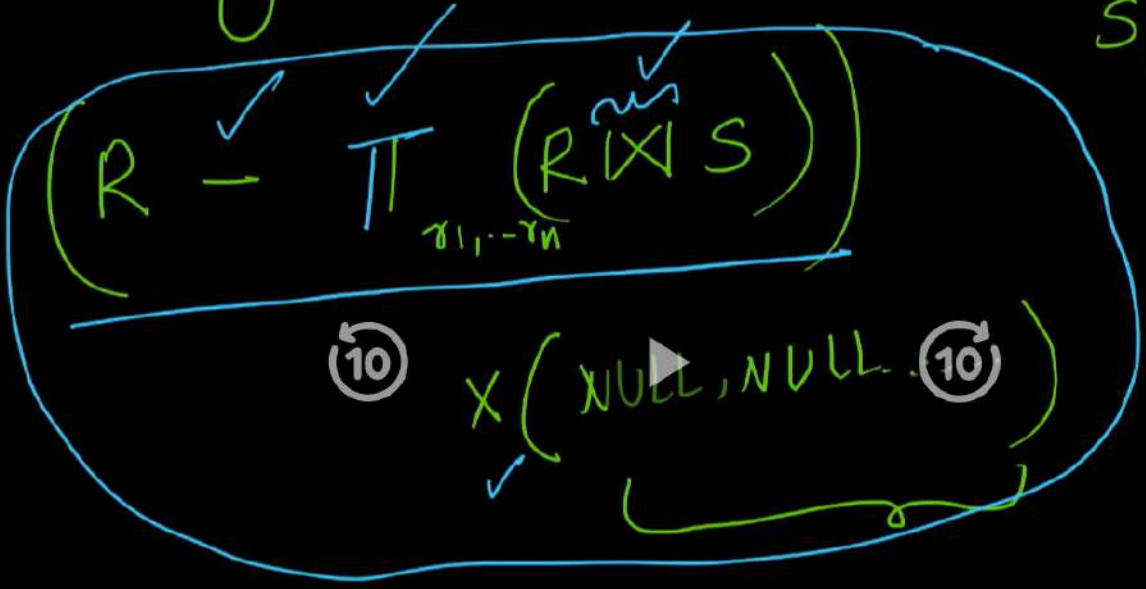
c1	c2	c3		c2	c3	c4
1	a	α	✓	a	d	1
2	b	β	✗	a	γ	2

R

c1	c2	c3	c4
1	a	α	1
2	b	β	NULL

$R \bowtie S = R \bowtie S$ ✓

U



$R(\gamma_1, \gamma_2, \dots, \gamma_n)$

$S(s_1, s_2, \dots, s_m)$

$c_1 \ c_2 \ c_3$
 $1, a, d$

$c_1 \ c_2 \ c_3$
 $2, b, \beta$ $X(NULL)$

$2, b, \beta, NULL$

wikipedia

Right outer join:

$$R \times S = R \times S \cup$$

(NULL, NULL, ...)

$$X \left(S - \pi_{S_1, S_2, \dots, S_m}(R \times S) \right)$$

wikipedia

c1	c2	c3	
1	a	α	
2	b	β	

R

10

c1	c2	c3	c4
1	a	α	1
NULL	a	γ	2

R X S

Full outer join

$$R \text{ X } S = R \text{ X } S \checkmark$$

$$U$$

$$\textcircled{10} \text{ X } S \blacktriangleright \checkmark$$

C1	C2	C3		C2	C3	C4	
✓ 1	a	α		a	d	1	✓
X 2	b	β		a	γ	2	X

R S

⑩

✓

C1	C2	C3	C4
1	a	α	1
2	b	β	NULL
NULL	a	γ	2

Division

$$R \div S$$

$$R/S$$

$$\begin{cases} 2 \div 3 = 0.66 \\ 2 = 0.66 \times 3 \end{cases}$$

cid	cat
1	C1
1	C2
2	C1
3	C2
4	C1
4	C2

cat
C1
C2

T cid

1
4

TXS

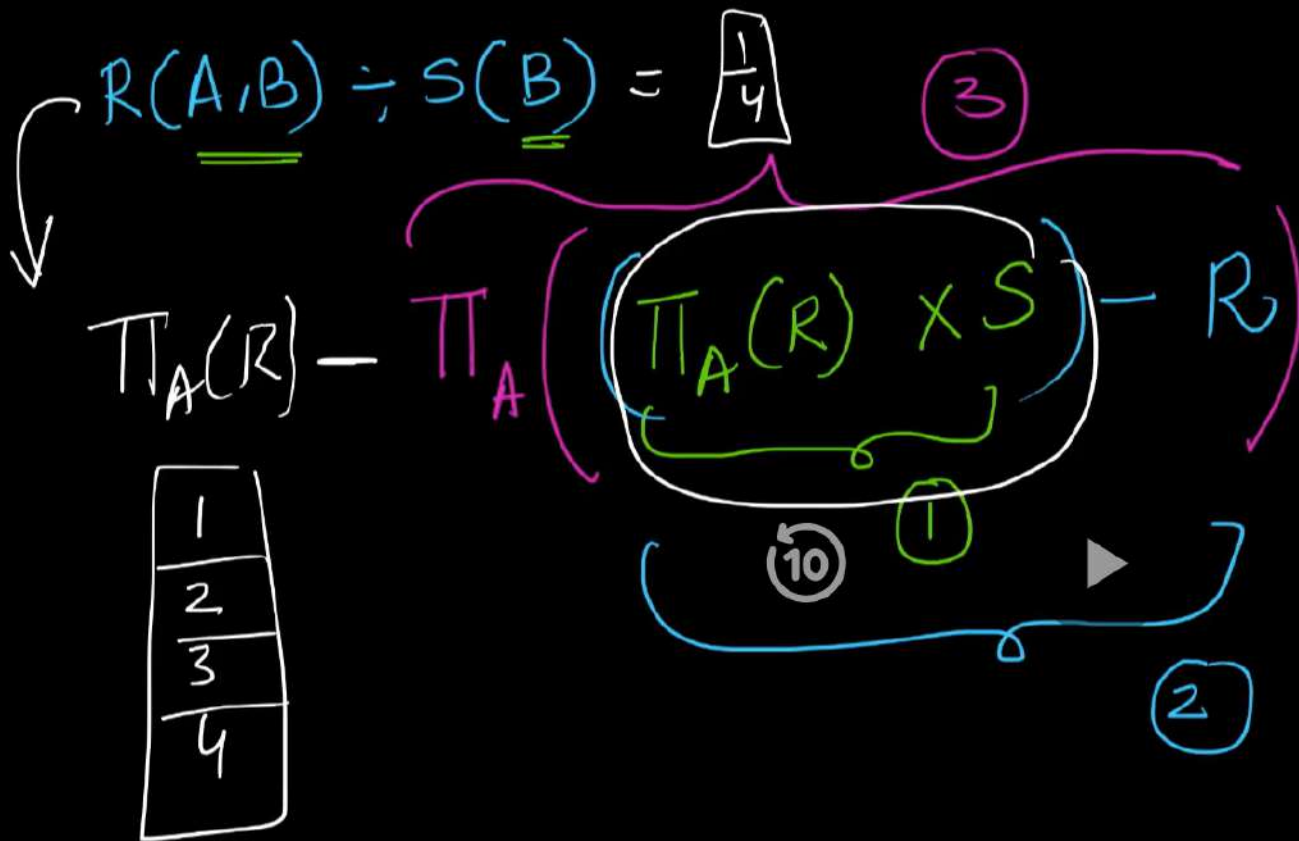
1, C1
1, C2
4, C1
4, C2

R

1
4

(Q) Customers
whom
purchased
from
every
category

$$\begin{cases} R \div S = T \\ R \supseteq T \times S \end{cases}$$



A	B
1	a
1	b
2	a
3	b
4	a
4	b

R

B
a
b

S

(1):

1	a
1	b
2	a
2	b
3	a
3	b
4	a
4	b

(2):

2	b
3	a

(3)

2
3

Completeness: you can represent any relational operators
using $\{ \cup, -, \times, \Pi, \sigma \}$

⑩



⑩

Complete set
of operators
in RA



39:11 / 40:15



Autoplay



Consider the following relation P(X, Y, Z), Q(X, Y, T) and R(Y, V):
How many tuples will be returned by the following relational algebra query?

$$\pi_x(\sigma_{(P.Y=R.Y \wedge R.V=V_2)}(P \times R)) - \pi_x(\sigma_{(Q.Y=R.Y \wedge Q.T>2)}(Q \times R))$$

$$\boxed{X_2} - \boxed{X_1}$$

$$A - B = A - (A \cap B)$$

$$\boxed{X_2} \checkmark$$

P		
X	Y	Z
X1	Y1	Z1
X1	Y1	Z2
X2	Y2	Z2
X2	Y4	Z4

Q		
X	Y	T
X2	Y1	2
X1	Y2	5
X1	Y1	6
X3	Y3	1

R	
Y	V
Y1	V1
Y3	V2
Y2	V3
Y2	V2

1

Consider the relations $r(A, B)$ and $s(B, C)$, where $s.B$ is a primary key and $r.B$ is a foreign key referencing $s.B$.

Consider the query

✓ $Q: r \bowtie (\sigma_{B < 5}(s))$

$$\rightarrow \begin{array}{|c|c|c|} \hline \alpha & 4 & a \\ \hline \end{array}$$

Let LOJ denote the natural left outer-join operation. Assume that r and s contain no null values.

Which one of the following is **NOT** equivalent to Q ?

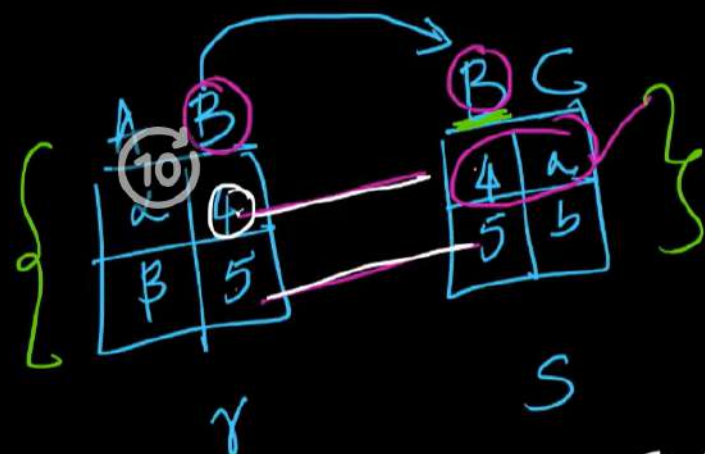
1. $\sigma_{B < 5}(r \bowtie s)$ ✓

2. $\sigma_{B < 5}(r \text{ LOJ } s)$

✓ 3. $r \text{ LOJ } (\sigma_{B < 5}(s))$

4. $\sigma_{B < 5}(r) \text{ LOJ } s$

$\rightarrow (\alpha, 4, a)$ (10)



$$\begin{array}{|c|c|} \hline \alpha & 4 \\ \hline \beta & 5 \\ \hline \end{array} \bowtie \begin{array}{|c|c|} \hline 4 & a \\ \hline 5 & \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline \alpha & 4 & a \\ \hline \beta & 5 & \text{NULL} \\ \hline \end{array}$$

$\alpha, 4, a$

$r \bowtie s$

Consider a database that has the relation schema $CR(\text{StudentName}, \text{CourseName})$.
An instance of the schema CR is as given below.

The following query is made on the database.

$T1 \leftarrow \pi_{\text{CourseName}}(\sigma_{\text{StudentName}='SA'}(CR))$

$T2 \leftarrow CR \div T1$

The number of rows in $T2$ is

4

10

CourseName
CA
CB
CC

StudentName
SA
SC
SD
SF

CR	
StudentName	CourseName
SA	CA
SA	CB
SA	CC
SB	CB
SB	CC
SC	CA
SC	CB
SC	CC
SD	CA
SD	CB
SD	CC
SD	CD
SE	CD
SE	CA
SE	CB
SF	CA
SF	CB
SF	CC



16:26 / 22:22



Autoplay



Consider two relations $R_1(A,B)$ with tuples $(1,5)$, $(3,7)$ and $R_2(A,C) = (1,7)$, $(4,9)$. Assume that $R(A,B,C)$ is the full natural outer join of R_1 and R_2 . Consider the following tuples of the form (A,B,C) : $a = (1,5,\text{null})$, $b = (1,\text{null},7)$, $c = (3,\text{null},9)$, $d = (4,7,\text{null})$, $e = (1,5,7)$, $f = (3,7,\text{null})$, $g = (4,\text{null},9)$. Which one of the following statements is correct?

Which one of the following statements is correct?

1. R contains a, b, e, f, g but not c, d .
2. R contains all a, b, c, d, e, f, g .
3. R contains e, f, g but not a, b .
4. R contains e but not f, g .

10



10

R_1

A	B
1	5
3	7

R_2

A	C
1	7
4	9

$R:$

A	B	C
1	5	7
3	7	NULL
4	NULL	9

$$R = R_1 \bowtie R_2$$



19:30 / 22:22



Autoplay



Suppose (A, B) and (C, D) are two relation schemas. Let r_1 and r_2 be the corresponding relation instances. B is a foreign key that refers to C in r_2 . If data in r_1 and r_2 satisfy referential integrity constraints, which of the following is ALWAYS TRUE?

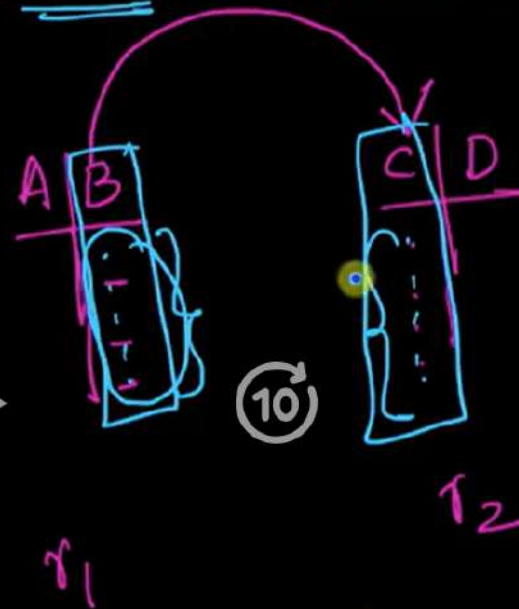
✓ (A) $\Pi_B(r_1) - \Pi_C(r_2) = \emptyset$ ✓

(B) $\Pi_C(r_2) - \Pi_B(r_1) = \emptyset$

(C) $\Pi_B(r_1) = \Pi_C(r_2)$

(D) $\Pi_B(r_1) - \Pi_C(r_2) \neq \emptyset$

10



$B - C = \emptyset$



$C \supseteq B$
 $B \subseteq C$



22:03 / 22:22



Autoplay

