IDC402 Term Paper

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Jerk circuit with thresholding

The original images, videos, and source codes are attached to this document under the same directory.

Jerk circuits can be modeled by nonlinear third-order ordinary differential equations. Some variants can be simple to implement in electronic circuits. Here I have derived the circuit from what is described by Kengne et al. (International Journal of Bifurcation and Chaos - 2016) [1], and combined it with thresholding (Murali & Sinha - Physical Review E - 2003) [2] and simplified it to use only 4 opamps.

1. Circuit

1.1. Description

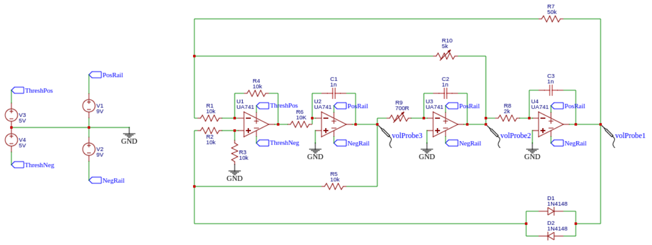

Figure 1: Schematic circuit diagram

The circuit consists of four operational amplifiers, labeled from U1 to U4 [Figure 1]. The U1 serves as a unit gain differential amplifier, followed by U2, U3, and U4 which work as successive integrators. The U1 has variable supply voltage (via V3 and V4), fed by a digital-to-analog converter via a low pass filter, thereby effectively clipping the output to the input voltage range. This serves the purpose of thresholding. The nonlinearity in the circuit is provided by the two zener diodes (D1 and D2), and, when applicable, the thresholding system.

1.2. SPICE simulations

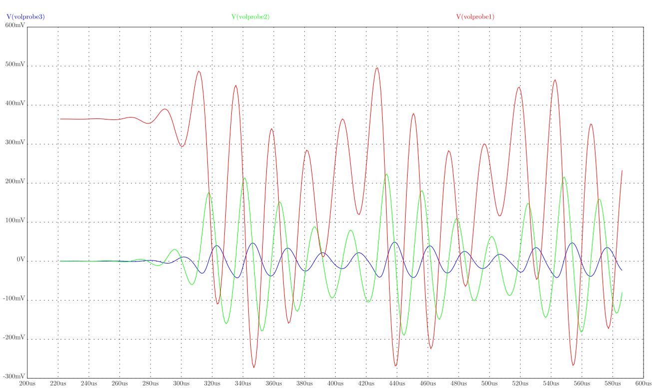

Figure 2: Simulation of the circuit in LTspice

The circuit is simulated in LTspice. Self-excited oscillations are observed, where a brief transient period is followed by a stable period 4 cycle.

2. Numerical simulations

2.1. Modelling the system

The circuit can be modeled by the following system of nonlinear ordinary differential equations -

$$dx = y$$

$$dy = \sigma z$$

$$dz = \begin{cases} dz^* & \text{if } dz^* \leq \tau \\ \tau & \text{if } dz^* > \tau \end{cases}$$

where $dz^* = x - \gamma y - z - \eta \sinh(\rho x)$ and τ is the thresholding parameter which is a linear function of the positive threshold voltage [ThreshPos in Figure 1]. The constants η and ρ depend on the intrinsic characteristics of the diodes. For the IN4148 diodes used in the circuit, the values of η and ρ are 2.6820×10^{-4} and 4.0485 respectively (at room temperature).

In all of the following numerical simulations, unless otherwise specified, γ is fixed to be 2, and σ is fixed to be 9.

2.2. Thresholding

The non-thresholded circuit shows chaotic behaviour, but as the threshold is slowly decreased, the chaotic orbit becomes stable periodic orbits before finally the stable limit cycle ceases to exist. [Figure 3].

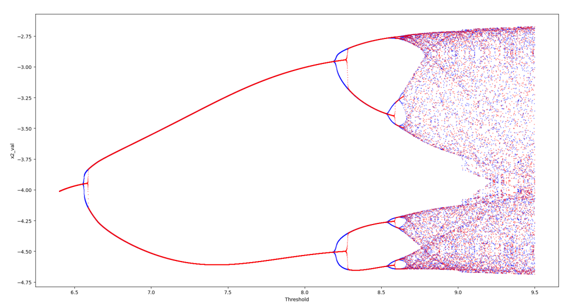

Figure 3: Bifurcation diagram (plotting y for local maxima of z) when the τ is swept slowly from 9.5 to 6.4 (blue) and back to 9.5 (red). Hysteresis can be observed near the bifurcation points.

A video of the evolution of the attractor for various values of τ is attached. In contrast, thresholding both the positive and negative supply pins of the opamp to the same voltage (V3 = V4 in Figure 1) can induce bifurcations from a period 1 orbit for $\sigma = 14$.

2.3. Multiple coexisting attractors

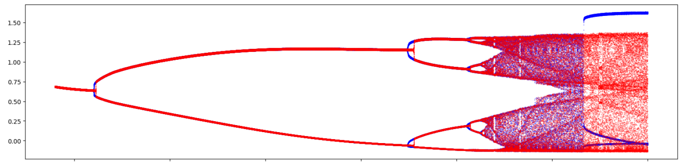

Figure 4: A similar simulation like that of Figure 3 is carried out with a wider range of values of τ ($6 \leq \tau \leq 15$). The coexistence of multiple attractors is visible [period 3 limit cycle (blue) and the chaotic attractor (red)].

For the following Figure 5, Figure 6, and Figure 7, value of x is plotted in the x axis and value of y in the y axis. All figures are on the $z = 0$ plane.

2.4. Poincare section

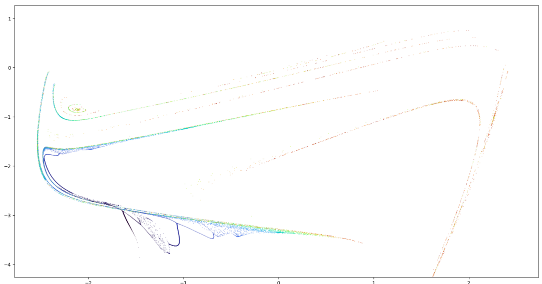

Figure 5: The evolution of the Poincare section of the system as τ is swept from 20 to 2 for $\sigma = 5$. More blue points denote lesser values of τ.

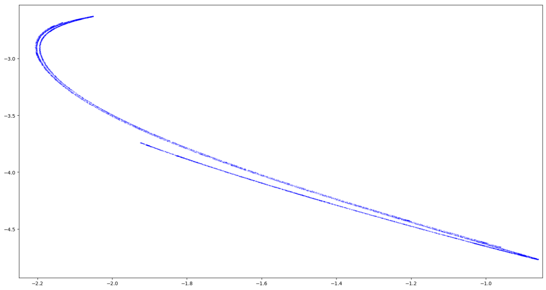

Figure 6: The Poincare map for $\sigma = 9$ and $T = 25$.

2.5. Attractor basin

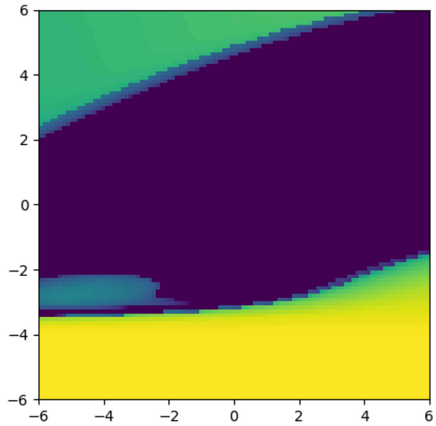

Figure 7: The attractor basin for $z = 0$. The image is colored according to how fast the trajectory diverges. Purple points converge to stable attractors and do not diverge.

3. Experimental study

The circuit is made in a breadboard using the specified components as shown in the schematic. It is yet to be tested.

Bibliography

[1] J. Kengne, Z. T. Njitacke, A. Nguomkam Negou, M. Fouodji Tsostop, and H. B. Fotsin, "Coexistence of Multiple Attractors and Crisis Route to Chaos in a Novel Chaotic Jerk Circuit", *International Journal of Bifurcation and Chaos*, no. 5, p. 1650081, May 2016, doi: 10.1142/S0218127416500814.

[2] K. Murali and S. Sinha, "Experimental realization of chaos control by thresholding", *Physical Review E*, no. 1, p. 16210, Jul. 2003, doi: 10.1103/PhysRevE.68.016210.