**LINEAR SYSTEMS CONTROL**

**Solutions to Problems**

### Problem 3.14

a. The characteristic polynomial for the control object can be found as:

$$P_{ch,\mathbf{A}} = \det(\lambda\mathbf{I}-\mathbf{A}) = \det\begin{bmatrix} \lambda+2 & 3 & -5 \\ -4 & \lambda-5 & 5 \\ -3 & -4 & \lambda+3 \end{bmatrix} = \lambda^3 - 2\lambda - 4$$

$$\Rightarrow a_0 = -4; \quad a_1 = -2; \quad a_2 = 0$$

b. The system is controllable if $\det(\mathrm{M}_c) \neq 0$ .

$$\mathbf{p}_1 = \mathbf{B} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\mathbf{p}_2 = \mathbf{A}\mathbf{p}_1 + 0 \cdot \mathbf{p}_1 = \mathbf{AB} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$$\mathbf{p}_3 = \mathbf{A}\mathbf{p}_2 - 2\mathbf{p}_1 = \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix} - \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\mathbf{P} = [\mathbf{p}_3 \quad \mathbf{p}_2 \quad \mathbf{p}_1] = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \Rightarrow \mathbf{P}^{-1} = \begin{bmatrix} 1 & 2 & -2 \\ 0 & -1 & 1 \\ -1 & -1 & 2 \end{bmatrix}$$

$$\mathbf{A}_t = \mathbf{A}_{cc} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & 2 & 0 \end{bmatrix}, \qquad \mathbf{B}_t = \mathbf{B}_{cc} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\mathbf{C}_t = \mathbf{C}_{cc} = [-2 \quad 1 \quad 0]$$

The system is not observable as $(\det(\mathrm{M}_o) = 0)$.

If one attempts to set up the transformation matrix Q , one will has:

**Problem 3.14** (continued)

$$\mathbf{q}_1^T = \mathbf{C} = [-2 \quad -5 \quad 5]$$
$$\mathbf{q}_2^T = \mathbf{CA} + 0 \cdot \mathbf{C} = [-1 \quad -1 \quad 0]$$
$$\mathbf{q}_3^T = \mathbf{CA}^2 + 0 \cdot \mathbf{CA} - 2 \cdot \mathbf{C} = [10 \quad 18 \quad 20]$$

$$\mathbf{Q} = \begin{bmatrix} \mathbf{q}_3^T \\ \mathbf{q}_2^T \\ \mathbf{q}_1^T \end{bmatrix} = \begin{bmatrix} 10 & 18 & -20 \\ -1 & 1 & 0 \\ -2 & -5 & 5 \end{bmatrix}$$

Q is singular and therefore the observer canonical form cannot be found by a similarity tranformation. However, the form can be found by duality as mentioned in section 3.9.3.

One finds that:

$$\mathbf{A}_{oc} = \mathbf{A}_{cc}^T = \begin{bmatrix} 0 & 0 & 4 \\ 1 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix}, \qquad \mathbf{B}_{oc} = \mathbf{C}_{cc}^T = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$
$$\mathbf{C}_{oc} = \mathbf{B}_{cc}^T = [0 \quad 0 \quad 1]$$

Note that system $S(\mathbf{A}_{oc},\ \mathbf{B}_{oc},\ \mathbf{C}_{oc})$ is observable but not controllable.

The system $S(\mathbf{A}_{cc},\ \mathbf{B}_{cc},\ \mathbf{C}_{occ})$ is controllable but not observable.

c. The transfer function can be determined directly from the controller canonical form:

$$G(s) = \frac{s-2}{s^3 - 2s - 4} = \frac{s-2}{(s-2)(s+1+j)(s+1-j)}$$
$$= \frac{1}{s^2 + 2s + 2}$$

d. In systems which are not controllable as well as observable it is always possible to cancel zeros/poles in the transfer function.

Note that this is only true for SISO-systems, see for exanple section 3.9.4. ❐