LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 3.12

a. The state equations for the target system are:

$$\dot{\mathbf{x}} = \begin{bmatrix} -3 & 4 \\ -2 & 3 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\mathbf{u}, \qquad \mathbf{y} = \begin{bmatrix} -1 & 2 \\ 1 & -2 \end{bmatrix}\mathbf{x}$$

To investigate the controllability of the system the rank of the controllability matrix is tested.

$$\mathbf{M}_c = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad \text{rank}(\mathbf{M}_c) = 1$$

$\Rightarrow$ the system is not controllable

To investigate the observability of the system the rank of the observability matrix is tested.

$$\mathbf{M}_0 = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 1 & -2 \\ -1 & 2 \\ 1 & -2 \end{bmatrix} \quad \text{rank}(\mathbf{M}_0) = 1$$

$\Rightarrow$ the system is not observable

b. Now the input matrix of the system is changed:

$$\mathbf{B} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}, \qquad \mathbf{C} = \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix}$$

$$\mathbf{M}_c = \begin{bmatrix} 1 & 1 & 5 & 1 \\ 2 & 1 & 4 & 1 \end{bmatrix} \quad \text{rank}(\mathbf{M}_c) = 2$$

$\Rightarrow$ the system is controllable

$$\mathbf{M}_o = \begin{bmatrix} 1 & 2 \\ 1 & -2 \\ -7 & 10 \\ 1 & -2 \end{bmatrix} \quad \text{rank}(\mathbf{M}_o) = 2 \quad \Rightarrow \text{the system is observable}$$

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