LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 2.10

a. The heat conduction power flows are:

$$\begin{aligned} q_c &= k_c(T_s - T_a) \\ q_g &= k_g(T_s - T_g) \end{aligned}$$

The radiation power flow is:

$$q_r = k_r(T_s^4 - T_a^4)$$

From the conservation of energy:

$$\begin{aligned} C_s\dot{T}_s &= q - q_g - q_c - q_r \\ C_g\dot{T}_g &= q_g \end{aligned}$$

where C_s and C_g are the heat capacities.

The states of the system are: T_s , $T_g \Rightarrow \mathbf{x} = \begin{bmatrix} T_s \\ T_g \end{bmatrix}$.

With the input, *u*, the output, $T_g = y$ and the disturbance, $T_a = v$.

The nonlinear state equation for the production oven is:

$$\begin{bmatrix} \dot{T}_s \\ \dot{T}_g \end{bmatrix} = \begin{bmatrix} \frac{1}{C_s}(ku - k_g(T_s - T_g) - k_c(T_s - T_a) - k_r(T_s^4 - T_a^4)) \\ \frac{1}{C_g}k_g(T_s - T_g) \end{bmatrix}$$

b. To find the stationary state one has to define: $\dot{T}_s = 0$ and $\dot{T}_g = 0$. This implies that

$$\begin{cases} ku_0 - k_g(T_{so} - T_{go}) - k_c(T_{so} - T_{ao}) - k_r(T_{so}^4 - T_{ao}^4) = 0 \\ T_{so} = T_{go} \end{cases}$$

LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 2.10

or

$$ku_o - k_c(x_{10} - v_0) - k_r(x_{10}^4 - v_0^4) = 0$$

c. To find the linearized model of the system the following incremental variables are defined:

$$T_s = x_1 = x_{10} + \Delta x_1 \qquad u = u_0 + \Delta u$$
$$T_g = x_2 = x_{20} + \Delta x_2 \qquad T_a = v = v_0 + \Delta v$$

$$\dot{x}_1 = \frac{1}{C_s}(ku - (k_g + k_c)x_1 + k_g x_2 + k_c v - k_r x_1^4 + k_r v^4) = f_1$$
$$\dot{x}_2 = \frac{1}{C_g} k_g (x_1 - x_2) = f_2$$

Now the Jacobians of the state equation can be calculated:

$$\mathbf{A} = \begin{bmatrix} \dfrac{\partial f_1}{\partial x_1} & \dfrac{\partial f_1}{\partial x_2} \\ \dfrac{\partial f_2}{\partial x_1} & \dfrac{\partial f_2}{\partial x_2} \end{bmatrix}_0 = \begin{bmatrix} -\dfrac{k_g + k_c}{C_s} - \dfrac{4k_r}{C_s}x_{10}^3 & \dfrac{k_g}{C_s} \\ \dfrac{k_g}{C_g} & -\dfrac{k_g}{C_g} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \dfrac{\partial f_1}{\partial u} \\ \dfrac{\partial f_2}{\partial u} \end{bmatrix}_0 = \begin{bmatrix} \dfrac{k}{C_s} \\ 0 \end{bmatrix}, \quad \mathbf{B}_v = \begin{bmatrix} \dfrac{\partial f_1}{\partial v} \\ \dfrac{\partial f_2}{\partial v} \end{bmatrix}_0 = \begin{bmatrix} \dfrac{k_c}{C_s} + \dfrac{4k_r}{C_s}v_0^3 \\ 0 \end{bmatrix}$$

$$\mathbf{C} = [0 \quad 1]$$

❐