LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 5.8

a. The index which must be minimized implies that what is required is a steady state LQR regulator. This means that a Riccati equation must be solved which is

$$-\dot{\mathbf{P}} = \mathbf{0} = \mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} + \mathbf{R}_1 - \mathbf{P}\mathbf{B}\mathbf{R}_2^{-1}\mathbf{B}^T\mathbf{P}$$

$$0 = -\alpha p - p\alpha + 1 - p\ \beta\ \frac{1}{\rho}\ \beta\ p$$

$$= -2\alpha p + 1 - \frac{\beta^2}{\rho}\ p^2$$

$$\Rightarrow \left(\frac{\beta^2}{\rho}\ p^2 + 2\alpha p - 1\right) = 0$$

$$p = \frac{-\alpha \pm \sqrt{\alpha^2 + \frac{\beta^2}{\rho}}}{\frac{\beta^2}{\rho}} = -\frac{\alpha\rho}{\beta^2} + \frac{\rho}{\beta^2}\sqrt{\alpha^2 + \frac{\beta^2}{\rho}}$$

$$k = -\mathbf{R}_2^{-1}\mathbf{B}^T\mathbf{P} = -\frac{1}{\rho}\ \beta\ p = -\frac{\alpha}{\beta} + \sqrt{\frac{\alpha^2}{\beta^2} + \frac{1}{\rho}}$$

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