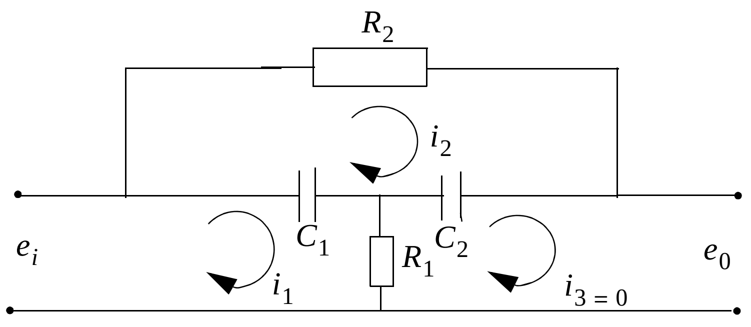
LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 2.5

a. The first filter (a.) is:



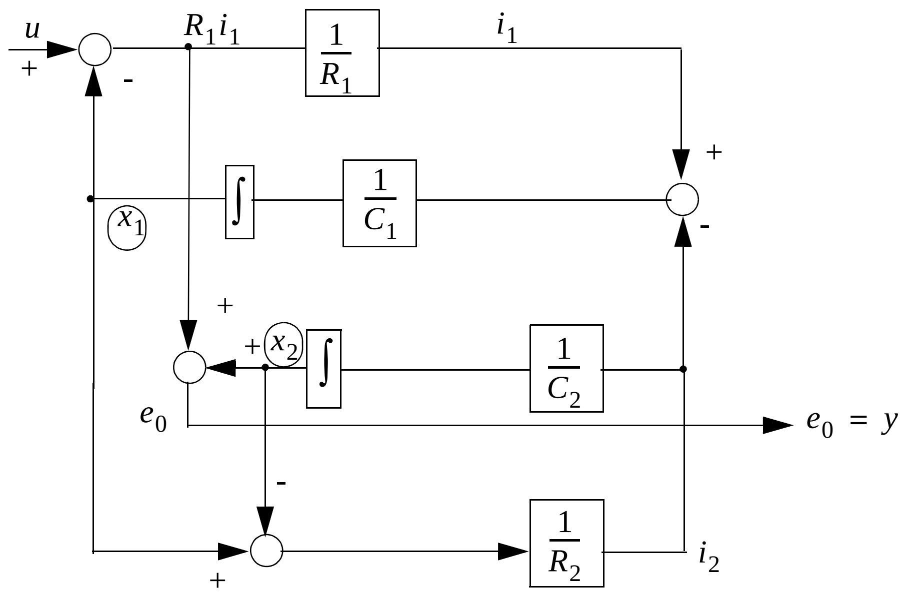
With the mask currents as drawn and using Ohm's law, one obtains:

$$e_i = \frac{1}{C_1}\int(i_1 - i_2)dt + R_1 i_1$$

$$0 = \frac{1}{C_1}\int(i_2 - i_1)dt + R_2 i_2 + \frac{1}{C_2}\int i_2 dt$$

$$-e_0 = -R_1 i_1 - \frac{1}{C_2}\int i_2 dt$$

b. These equations have the block diagram below:



Problem 2.5 (continued)

c. The state equations for bridged T filter (a.) are:

$$\dot{x}_1 = \frac{1}{C_1}\left(\frac{1}{R_1}(u - x_1) - \frac{1}{R_2}(x_1 - x_2)\right)$$

$$\dot{x}_2 = \frac{1}{C_2}\frac{1}{R_2}(x_1 - x_2)$$

$$y = e_0 = x_2 - x_1 + u$$

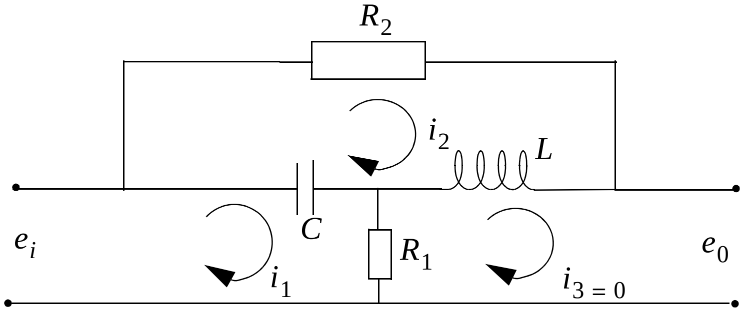
$$\dot{x}_1 = \frac{1}{C_1R_1}u - \frac{1}{C_1R_1}x_1 - \frac{1}{C_1R_2}x_1 + \frac{1}{C_1R_2}x_2$$

$$\dot{x}_2 = \frac{1}{C_2R_2}x_1 - \frac{1}{C_2R_2}x_2$$

$$\dot{\mathbf{x}} = \begin{bmatrix} -\dfrac{R_1 + R_2}{C_1R_1R_2} & \dfrac{1}{C_1R_2} \\ \dfrac{1}{C_2R_2} & \dfrac{1}{C_2R_2} \end{bmatrix} + \begin{bmatrix} \dfrac{1}{C_1R_1} \\ 0 \end{bmatrix} u$$

$$\mathbf{y} = [-1 \quad 1]\mathbf{x} + u$$

The second filter (b.) is:



This filter is decribed by the mask equations:

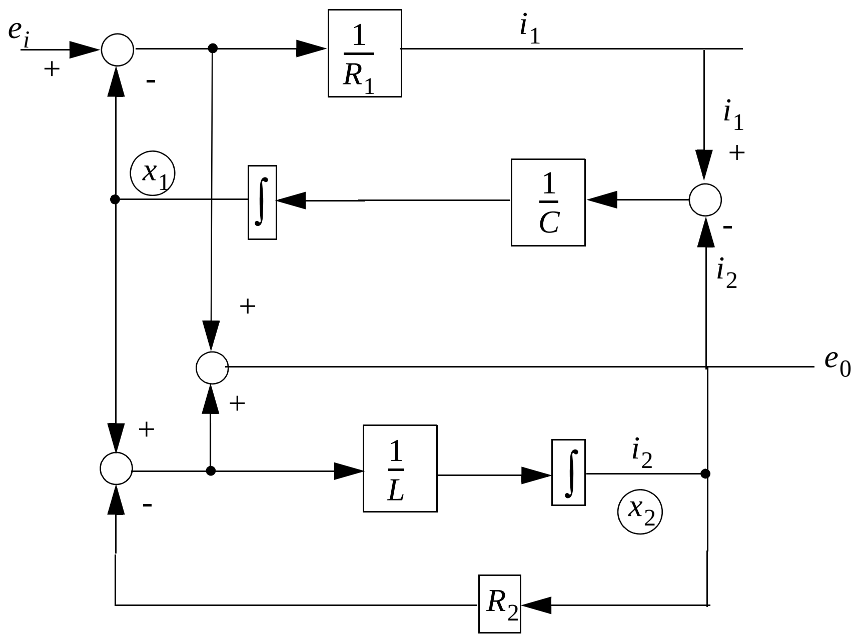
LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 2.5 (continued)

$$
\begin{aligned}
e_i &= \frac{1}{C}\int (i_1 - i_2)dt + R_1 i_1 \\
0 &= \frac{1}{C}\int (i_2 - i_1)dt + R_2 i_2 + L\frac{di_2}{dt} \\
-e_0 &= -R_1 i_1 - L\frac{di_2}{dt}
\end{aligned}
$$

With the block diagram:



This block diagram can be translated into the state equations:

$$
\begin{aligned}
\dot{x}_1 &= \frac{1}{C}\left(\frac{1}{R_1}(e_i - x_1) - x_2\right) = \frac{1}{CR_1}e_i - \frac{1}{CR_1}x_1 - \frac{1}{C}x_2 \\
\dot{x}_2 &= \frac{1}{L}(-R_2 x_2 + x_1) = -\frac{R_2}{L}x_2 + \frac{1}{L}x_1 \\
y &= e_i - x_1 + x_1 - R_2 x_2
\end{aligned}
$$

Problem 2.5 (continued)

$$\dot{\mathbf{x}} = \begin{bmatrix} -\dfrac{1}{CR_1} & -\dfrac{1}{C} \\ \dfrac{1}{L} & -\dfrac{R_2}{L} \end{bmatrix} \mathbf{x} + \begin{bmatrix} \dfrac{1}{CR_1} \\ 0 \end{bmatrix} e_i$$

$$\mathbf{y} = [0 \quad -R_2]\mathbf{x} + e_i$$

❐