LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 3.11

a. The system is given by the state equation:

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u \qquad , \qquad \mathbf{y} = [a \quad -1]\mathbf{x}$$

The eigenvalues of the system are determined from the equation:

$$\det(\lambda\mathbf{I} - \mathbf{A}) = \det\begin{bmatrix} \lambda & -1 \\ -1 & \lambda \end{bmatrix} = \lambda^2 - 1 = 0 \quad \text{for}$$

$$\lambda = \begin{cases} 1 \\ -1 \end{cases}$$

One eigenvalue in the right half plane: the system is thus unstable

b. The transfer function of the system can be found from:

$$\mathbf{G}(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$$

$$(s\mathbf{I} - \mathbf{A})^{-1} = \begin{bmatrix} s & -1 \\ -1 & s \end{bmatrix} = \frac{1}{s^2 - 1}\begin{bmatrix} s & 1 \\ 1 & s \end{bmatrix}$$

$$\mathbf{G}(s) = [a \ \ -1]\frac{1}{s^2 - 1}\begin{bmatrix} s & 1 \\ 1 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= [a \ \ -1]\begin{bmatrix} \dfrac{1}{s^2 - 1} \\ \dfrac{s}{s^2 - 1} \end{bmatrix} = \frac{a}{s^2 - 1} - \frac{s}{s^2 - 1} = \frac{a - s}{(s + 1)(s - 1)}$$

For a = 1 one has:

$$G(s) = -\frac{1}{s + 1} \qquad \text{one pole} \ \ s = -1$$

$$\Rightarrow \text{BIBO-stable}$$

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