LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 4.10

From example 2.9 one has the state vector:

$$\mathbf{z} = \begin{bmatrix} H_1 \\ H_2 \\ T_1 \\ T_2 \end{bmatrix}$$

A transformation matrix **Q** is selected such that:

$$\mathbf{x} = \mathbf{Q}\mathbf{z} = 0\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} H_1 \\ H_2 \\ T_1 \\ T_2 \end{bmatrix} = \left[\begin{array}{c} H_2 \\ T_2 \\ \hdashline H_1 \\ H_2 \end{array}\right] = \left[\begin{array}{c} x_1 \\ \hdashline x_2 \end{array}\right]$$

The matrices of the transformed system will then be:

$$\mathbf{A}_t = \mathbf{Q}\mathbf{A}\mathbf{Q}^{-1} = \left[\begin{array}{cc:cc} -0.0667 & 0 & 0.0499 & 0 \\ 0 & -0.0335 & 0 & 0.0335 \\ \hdashline 0.0499 & 0 & -0.0499 & 0 \\ 0 & 0 & 0 & -0.0251 \end{array}\right] = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix}$$

$$\mathbf{B}_t = \mathbf{Q}\mathbf{B} = \left[\begin{array}{cc} 0 & 0 \\ 0 & 0 \\ \hdashline 0.0051 & 0.0051 \\ 0.0377 & -0.0377 \end{array}\right] = \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \end{bmatrix}$$

$$\mathbf{C}_t = \mathbf{C}\mathbf{Q}^{-1} = \left[\begin{array}{cc:cc} 2 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \end{array}\right] = \begin{bmatrix} \mathbf{C}_1 & \mathbf{C}_2 \end{bmatrix}$$

The matrix L is found by application of Matlab's place-function:

Problem 4.10

```
>> L = PLACE(A22', (C1*A12)', [-0, 4 + 0, 5J  -0, 4 - 0, 5J]
```

and then M , N and P can be found with the statements:

```
>> M =  A22-L*C1*A12
>> N =  B2-L*C1*B1
>> P =  (A21-L*C1*A11)*inv(C1) + M*L
```

An m-file which do all the calculations is given on the next page.

The output from running this file is shown on pages 4-6.

The SIMULINK-diagram of the entire system is also shown (on page 7), and on the last page can be seen plots showing simulation results using this SIMULINK-model. Note that the estimates for the states x_3 and x_4 are initiated with "wrong" values. The estimation error vanishes within the first 12-15 sec.

LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 4.10

m-file

```
% Calculates data for the SIMULINK-model opg410.mdl
%
%
a=[-.0499 .0499 0 0
.0499 -.0667 0 0
0 0 -.0251 0
0 0 .0335 -.0335]
b=[.00510 .00510
   0   0
   .0377 -.0377
   0 0]
c=[0 2 0 0
   0 0 0 .1]
q=[0 1 0 0
   0 0 0 1
   1 0 0 0
   0 0 1 0]
at=q*a*inv(q)
bt=q*b
ct=c*inv(q)
a22=at(3:4,3:4)
a12=at(1:2,3:4)
a11=at(1:2,1:2)
a21=at(3:4,1:2)
c1=ct(:,1:2)
b1=bt(1:2,:)
b2=bt(3:4,:)
eobs=[-.4+j*.5 -.4-j*.5]
l=place(a22',(c1*a12)',eobs)'
m=a22-l*c1*a12
n=b2-l*c1*b1
p=(a21-l*c1*a11)*inv(c1)+m*l
```

Problem 4.10

Data generated by the m-file

```
a =
-0.0499    0.0499         0         0
   0.0499   -0.0667         0         0
        0         0   -0.0251         0
        0         0    0.0335   -0.0335

b =
0.0051    0.0051
        0         0
   0.0377   -0.0377
        0         0

c =
0    2.0000         0         0
        0         0         0    0.1000

q =
0     1     0     0
     0     0     0     1
     1     0     0     0
     0     0     1     0

at =
-0.0667         0    0.0499         0
        0   -0.0335         0    0.0335
   0.0499         0   -0.0499         0
        0         0         0   -0.0251

bt =
0         0
        0         0
   0.0051    0.0051
   0.0377   -0.0377

ct =
2.0000         0         0         0
        0    0.1000         0         0
```

LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 4.10

```
a22 =
-0.0499         0
      0   -0.0251

a12 =
0.0499         0
      0    0.0335

a11 =
-0.0667         0
      0   -0.0335

a21 =
0.0499         0
      0         0

c1 =
2.0000         0
      0    0.1000

b1 =
0     0
    0     0

b2 =
0.0051    0.0051
   0.0377   -0.0377

eobs =
-0.4000 + 0.5000i  -0.4000 - 0.5000i
place: ndigits= 15

l =
3.5080    5.0100
 -149.2537  111.9104

m =
-0.4000   -0.0168
  14.8955   -0.4000
```

Problem 4.10

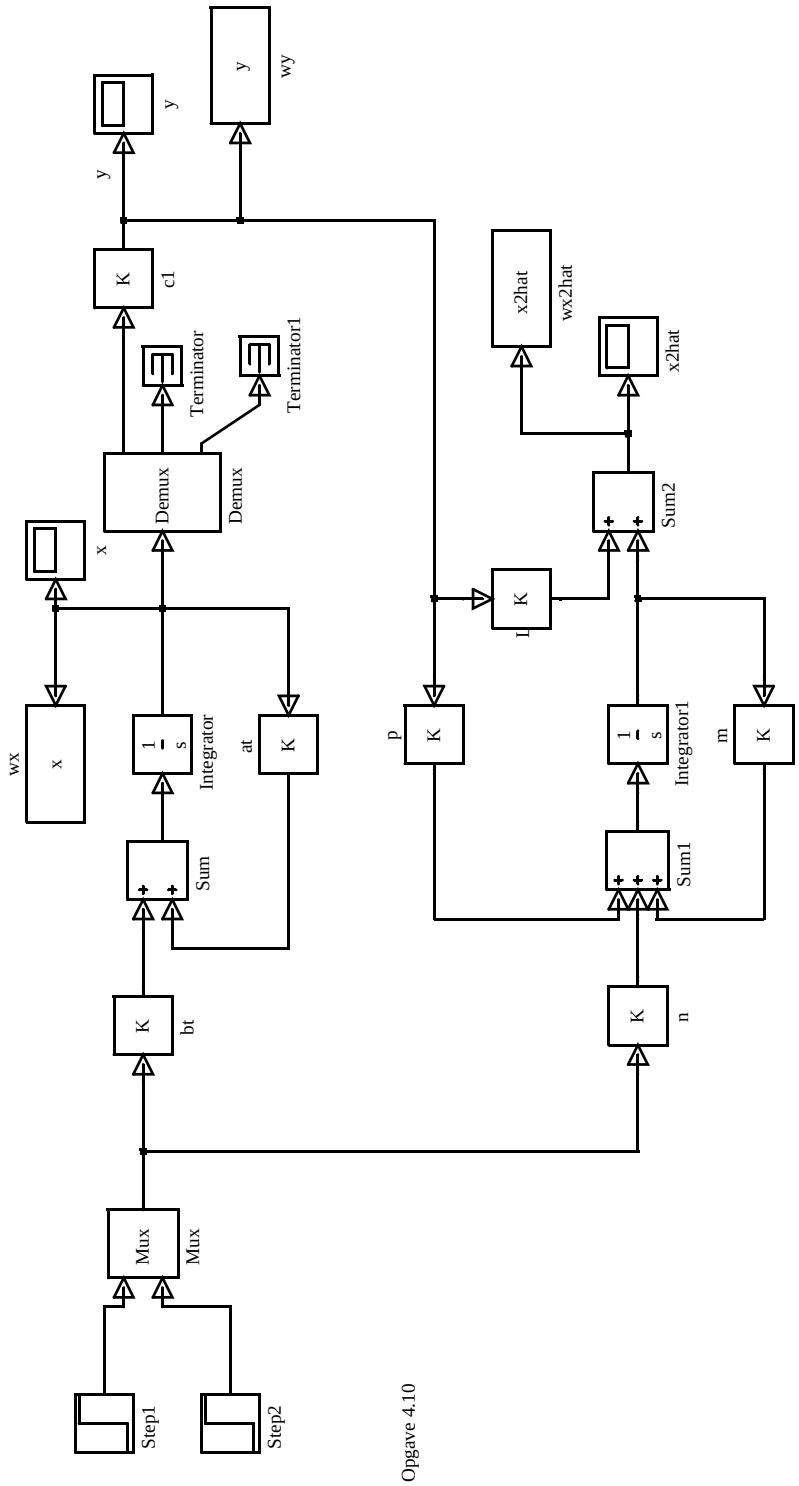
```
n =
0.0051    0.0051
   0.0377   -0.0377

p =
1.3607   -3.7144
  102.0000   33.6117
```

LINEAR SYSTEMS CONTROL

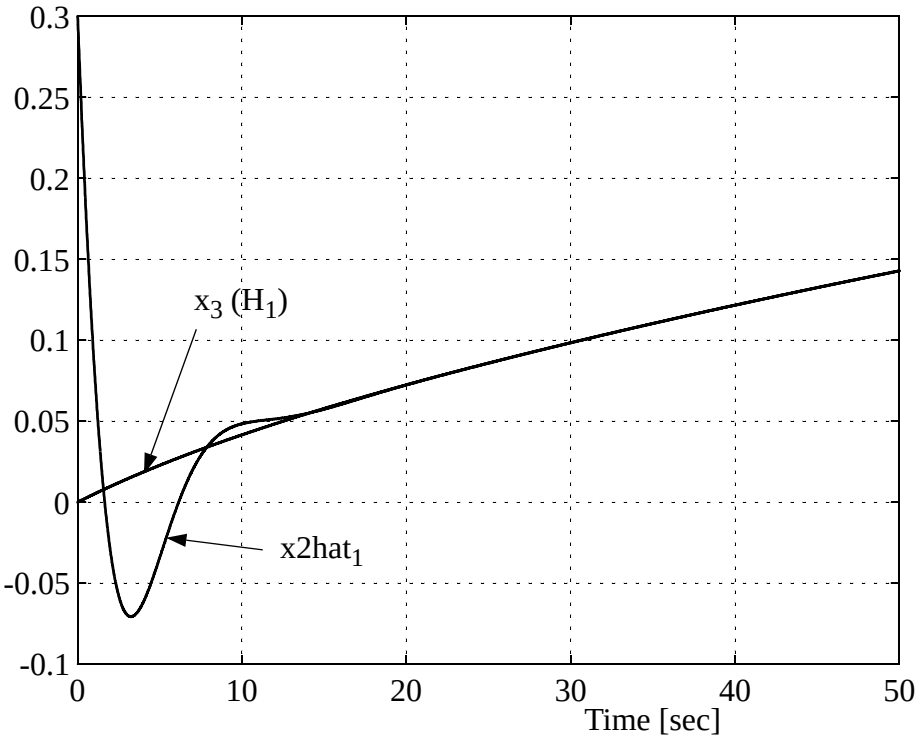
Solutions to Problems

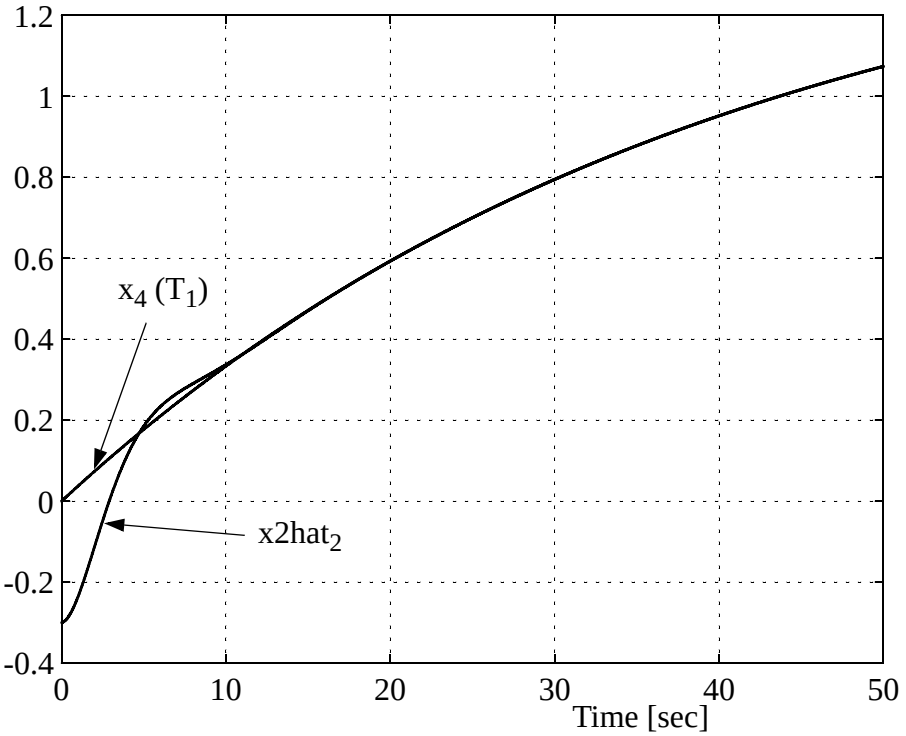
Problem 4.10

SIMULINK diagram



Problem 4.10


0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
0
10
20
30
40
50
Time [sec]
x3 (H1)
x2hat1


1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
10
20
30
40
50
Time [sec]
x4 (T1)
x2hat2

❒