LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 5.9

In the exercise text the performance index should be written as:

$$J = \frac{1}{2}\mathbf{x}^T(t_1)\mathbf{x}(t_1) + \int_{t_0}^{t_1} (\mathbf{x}^T(t)\mathbf{x}(t) + r_2 u^2(t))\ dt\ ,$$

where r_2 is a constant. There is an error in the book text.

a. The solution to this exercise is given in the book text in examples 5.3 and 5.5. It is a good idea go throught these examples in detail and to try to integrate the differential equations using Matlab different input weights and various initial conditions.

b. The stationary solution of the three differential equations in example 5.3 can be found by solving the following algebraic equations simultaneously (see example 5.5):

$$0 = r_p - \frac{1}{r_2}p_{12}^2$$

$$0 = p_{11} - \frac{1}{r_2}p_{12}p_{22}$$

$$0 = r_v + 2p_{12} - \frac{1}{r_2}p_{22}^2$$

This can be done by solving the first equation for p_{12} and then using this expression in the third equation to find p_{12}. Then it is easy to use the second equation to find p_{11}. The results are (see example 5.5):

$$p_{12} = \sqrt{r_p r_2}$$

$$p_{22} = \sqrt{2r_2p_{12} + r_v r_2}$$

$$p_{11} = \frac{1}{r_2}p_{12}p_{22}$$

Using the values given in example 5.3 for r_p, r_v and r_2, one finds that $p_{12} = 1.732$, $p_{22} = 2.732$ and $p_{11} = 4.732$. These values are those which can be observed at $t = 0$ in figure 5.9. Remember the three differential equations are solve backwards in time.

The corresponding LQR gain is given by:

LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 5.9

$$\mathbf{K} = \frac{1}{r_2}\mathbf{B}^T\mathbf{P}_\infty = \frac{1}{r_2}\begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} = \frac{1}{r_2}\begin{bmatrix} p_{11} & p_{22} \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{\frac{r_p}{r_2}} & \sqrt{\frac{r_p}{r_2}}\sqrt{2\sqrt{r_p r_2} + r_v} \end{bmatrix}$$

❒