LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 4.6

a. The state vector for the hydraulic servo is:

$$\mathbf{x} = \begin{bmatrix} x \\ \dot{x} \\ p_1 - p_2 \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \\ \Delta p \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

The equations (3.413) and (3.414) are:

$$q_1 = A_c\dot{x} + \frac{V}{\beta}\dot{p}_1 + C_l(p_1 - p_2)$$
$$q_2 = A_c\dot{x} + \frac{V}{\beta}\dot{p}_1 + C_l(p_1 - p_2)$$

Adding these equations and introducing the new variable: $q_1 + q_2 = 2ku$:

$$ku = A_c\dot{x} + \frac{V}{2\beta}\dot{\Delta p} + C_l\Delta p$$

Equation (3.416) becomes

$$M\ddot{x} = f + A_c\Delta p - C_f\dot{x}$$

and one has the state equations:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{1}{M}(f + A_c x_3 - C_f x_2)$$
$$\dot{x}_3 = \frac{2\beta}{V}(ku - A_c x_2 - C_l x_3)$$

or:

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{C_f}{M} & \frac{A_c}{M} \\ 0 & -\frac{2\beta A_c}{V} & -\frac{2BC_l}{V} \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ \frac{2\beta k}{V} \end{bmatrix}u + \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \end{bmatrix}f$$

LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 4.6

The output equation is: $y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \mathbf{x}$

Now inserting the numerical value using the units from example 3.26:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -0.2 & 300 \\ 0 & -700 & -4.667 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 93.33 \end{bmatrix}, \quad \mathbf{B}_v = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

The eigenvalues for **A** are:

$$\lambda_{\mathbf{A}} = \begin{cases} 0 \\ -2.434 \pm j458.3 \qquad \text{(see for example page 193)} \end{cases}$$

b. Using now Matlab and Ackermann's formula, the eigenvalues for the closed loop system are:

$$>> \ \text{evr} = [-20 \quad -12 \pm j*12 \quad -12 - j*12]$$

Using

$$>> \ \text{K} = \text{acker(A,B,evr);}$$

one finds:

$$\mathbf{K} = [0.2057 \quad -7.473 \quad 0.4193]$$

c. The closed loop system is:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u + \mathbf{B}_v v, \qquad y = \mathbf{C}x$$
$$u = -\mathbf{K}\mathbf{x} + r$$

LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 4.6

or:

$$\begin{cases} \dot{\mathbf{x}} = (\mathbf{A} - \mathbf{BK})\mathbf{x} + \mathbf{B}r + \mathbf{B}_v v = \mathbf{A_K}x + \mathbf{B}r + \mathbf{B}_v\mathbf{V} \\ y = \mathbf{C}x \end{cases}$$

Stationary state for $r = 0$ is:

$$\dot{\mathbf{x}} = 0 \Rightarrow \mathbf{A_K}\mathbf{x}_0 + \mathbf{B}_v v_0 = 0$$
$$\Rightarrow \mathbf{x}_0 = -\mathbf{A}_\mathbf{K}^{-1}\mathbf{B}_v v_0$$

for $v_0 = 50$ (note that the force unit is 10 Newton) we find

$$x_0 = \begin{bmatrix} 0.7604 \\ 0 \\ -0.3333 \end{bmatrix}$$

and thus

$$y_0 = Cx_0 = 0.7604 \text{ cm}$$

d. The augmented state vector is: $\mathbf{x}_a = \begin{bmatrix} x \\ x_i \end{bmatrix}$

$$\mathbf{A}_1 = \begin{bmatrix} \mathbf{A} & 0 \\ -\mathbf{C} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.2 & 300 & 0 \\ 0 & -700 & -4.667 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$$

LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 4.6

$$\mathbf{B}_1 = \begin{bmatrix} \mathbf{B} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 93.33 \\ 0 \end{bmatrix}, \quad \mathbf{B}_{v1} = \begin{bmatrix} \mathbf{B}_v \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} C & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

again using use MATLAB and Ackermann’s formula:

```
>> A1 = [A  zeros(3, 1);–C  0];
>> B1 = [B;0];   Bv1 = [Bv;0];   C1 = [C  0];
>> evri = [evr  -16];
>> K1 = acker(A1,B1,evri);
```

One obtains:

$$\mathrm{K1} = [0.6446 \quad -7.448 \quad 0.5907 \quad -3.292]$$

and therefore

$$\mathbf{K} = [0.6446 \quad -7.448 \quad 0.5907] \quad \text{and} \quad K_i = -3.292$$

e. The overall closed loop system is then:

$$\dot{\mathbf{x}}_a = \mathbf{A}_{\mathbf{K}1}\mathbf{x}_a + \mathbf{B}_r r + \mathbf{B}_{v1} v \quad \text{where} \quad \mathbf{A}_{\mathbf{K}1} = \mathbf{A}_{\mathbf{K}1} = \mathbf{A}_1 - \mathbf{B}_1\mathbf{K}_1$$

Stationary state for $r = 0$:

$$\mathbf{A}_{K1}\mathbf{x}_{a0} + \mathbf{B}_{v1}v_0 = 0 \Rightarrow \mathbf{x}_{a0} = -\mathbf{A}_{\mathbf{K}1}^{-1}\mathbf{B}_{v1}v_0$$

For $v_0 = 50$ one finds:

LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 4.6

$$\mathbf{x}_{a0} = \begin{bmatrix} 0 \\ 0 \\ -0.3333 \\ -0.06489 \end{bmatrix} \quad \text{and} \quad y_0 = 0$$

f. In MATLAB one can generate a time vector:

$$t = 0{:}0.01{:}1;$$

Unit step response for the system without integration:

$$>> \ [y, x] = \text{step(AK,B,C,0,1,t);}$$

and with integration

$$>> \ [\text{y1, x1}] = \text{step(AK1,Br,C1,0,1,t);}$$

Note: here $\mathbf{B}_r = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T$.

The responses may be plotted with the command (see plots on following page).

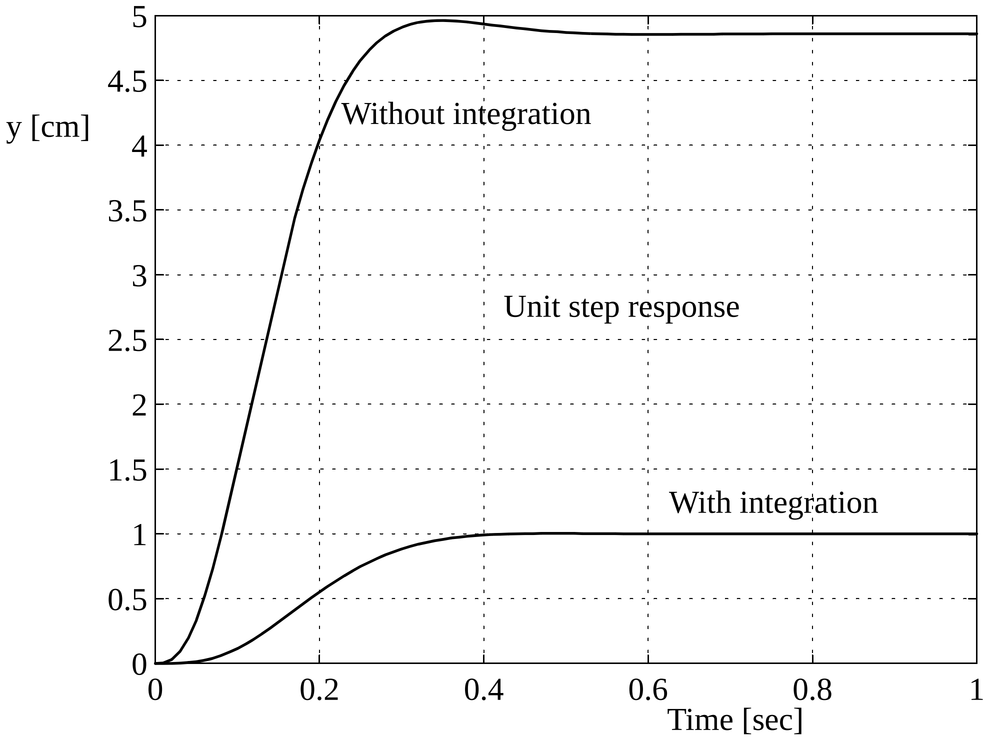
```
>>  plot(t,y,t,y1),  grid on
```

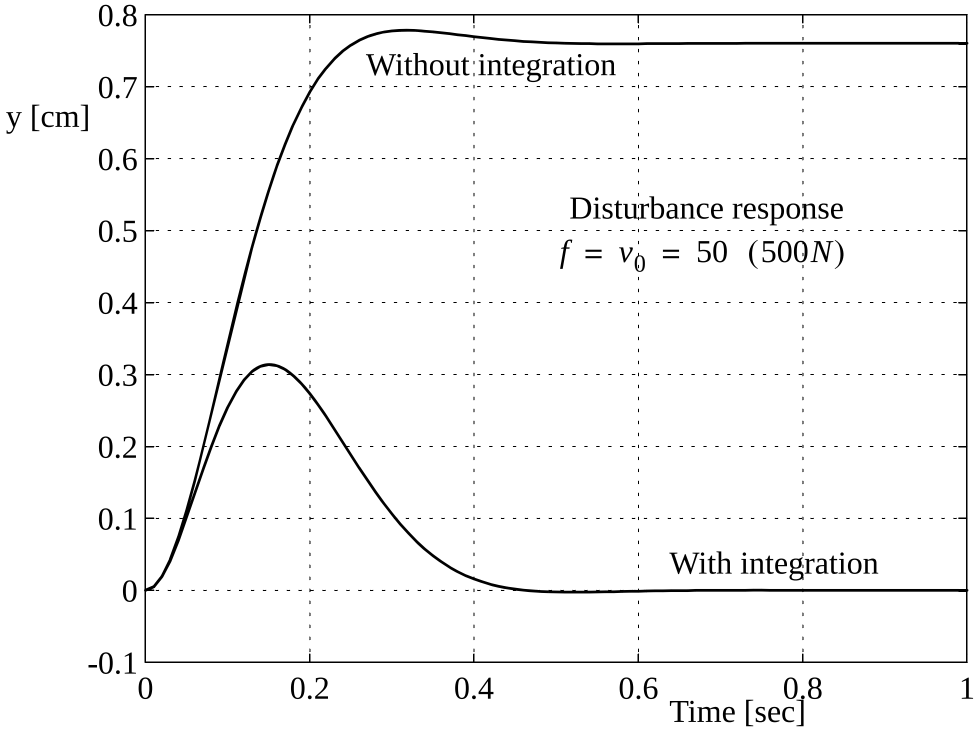
The responses for a 500N force disturbance $(v_0 = 50)$, can be calculated by the commands:

```
>>  [yf,xf] = step(AK,BV,C,0,1,t);
>>  [yf1, xf1] = step(AK1,BV1,C1,0,1,t);
>>  plot(t,50*yf,t,50*yf1),grid on
```

The plots of the responses are on the last page.

Problem 4.6


y [cm]
Without integration
Unit step response
With integration
Time [sec]


y [cm]
Without integration
Disturbance response
$f = v_0 = 50 \ (500N)$
With integration
Time [sec]

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