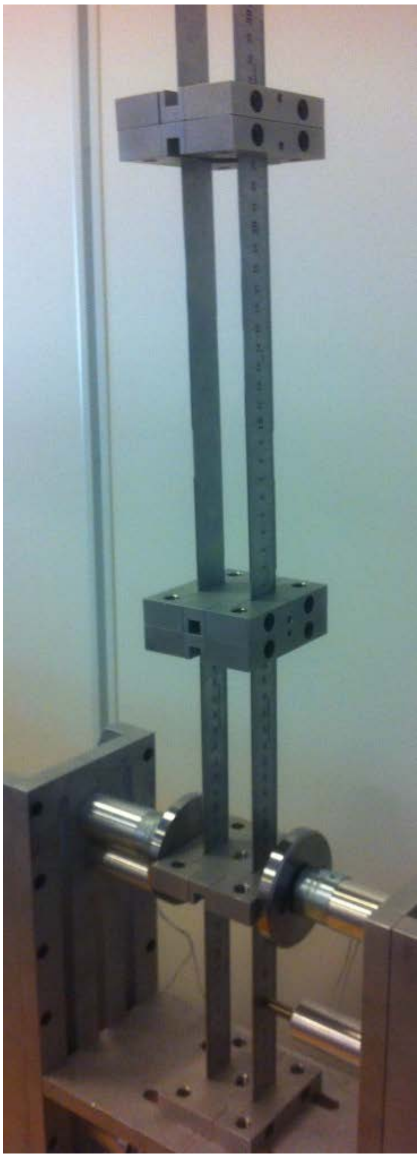
Module 4 – Similarities Transformation & Modal Analysis

In the modules 2 and 3 you have obtained a mathematical model based on principles and laws of Mechanics and Electromagnetism. Such a mathematical model uses 7 state variables which form the state vector **x**(t). With the matrices **A**, **B**, **C**, and **D** we are able to describe the dynamic behavior of the electro-mechanical system presented in the figure below.



$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u, \quad \mathbf{x} \in \mathbb{R}^7,\ u \in \mathbb{R}$$
$$y = \mathbf{C}\mathbf{x} + Du \quad y \in \mathbb{R}$$

$$\mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_1 & x_2 \end{bmatrix}^T$$
$$= \begin{bmatrix} i & q_1 & \dot{q}_1 & q_2 & \dot{q}_2 & q_3 & \dot{q}_3 \end{bmatrix}^T$$

$$\mathbf{A} = \begin{bmatrix} -\frac{R}{L} & 0 & -\frac{K_i}{L} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{K_i}{m_1} & -\frac{k_1+k_2+K_s}{m_1} & 0 & \frac{k_2}{m_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{k_2}{m_2} & 0 & -\frac{k_2+k_3}{m_2} & 0 & \frac{k_3}{m_2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \frac{k_3}{m_3} & 0 & -\frac{k_3}{m_3} & 0 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \frac{1}{L} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \mathbf{C}^T = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},\ D = 0$$

The mathematical models are not unique, which you will show by building an equivalent mathematical model based on the transformation to the modal coordinates **z**(t):

$$\boxed{\mathbf{z} = \mathbf{P}\mathbf{x} \Leftrightarrow \mathbf{x} = \mathbf{P}^{-1}\mathbf{z}}$$

where

$$\dot{\mathbf{z}}(t) = \mathbf{PAP}^{-1}\mathbf{z}(t) + \mathbf{PBu}(t) \qquad \dot{\mathbf{z}}(t) = \mathbf{\Lambda z}(t) + \mathbf{B}_t\mathbf{u}(t)$$

$$\mathbf{y}(t) = \mathbf{CP}^{-1}\mathbf{z}(t) + \mathbf{Du}(t) \qquad \mathbf{y}(t) = \mathbf{C}_t\mathbf{z}(t) + \mathbf{D}_t\mathbf{u}(t)$$

and **P** and its inverse **M**, and **Λ** are the matrices composed of eigenvectors and eigenvalues of the state matrix **A**. Using two different linearization points ($\mathbf{x_0}$ and $\mathbf{i_0}$) and the parameters provided in the modules 2 and 3.

1) Calculate **Λ**.
2) Calculate **M**.
3) Calculate **P**, i.e. the inverse of **M**.
4) Calculated **C**t.
5) Calculate the time constant of the electro-mechanical system.
6) Calculate the damped natural frequencies of the electro-mechanical system
7) Calculate the damping ratios associated to the damped natural frequencies of the system.
8) Explain the physical meaning of the natural modes of the electro-mechanical system and their link to eigenvectors and eigenvalues (modal superposition).