LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 4.1

a. The state equations of the system are:

$$\dot{\mathbf{x}} = \begin{bmatrix} 1 & -2 \\ 0.5 & -1 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 2 \\ 2 \end{bmatrix}\mathbf{u}$$

$$\mathbf{y} = [-1 \quad 1]\mathbf{x}$$

The eigenvalues are:

$$\det(\lambda\mathbf{I} - \mathbf{A}) = \left\| \begin{bmatrix} \lambda - 1 & 2 \\ -0.5 & \lambda + 1 \end{bmatrix} \right\| = 0$$

$$\Rightarrow (\lambda - 1)(\lambda + 1) + 1 = 0 \Rightarrow \lambda^2 = 0 \qquad \lambda = \begin{cases} 0 \\ 0 \end{cases}$$

b. Testing the controllability of the system:

$$\mathbf{M}_c = \begin{bmatrix} \mathbf{B} & \mathbf{AB} \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 2 & -1 \end{bmatrix}, \qquad det(\mathbf{M}_c)$$

$\Rightarrow$ the system is controllable

$$\mathbf{A}_k = \mathbf{A} - \mathbf{BK} = \begin{bmatrix} 1 & -2 \\ 0.5 & -1 \end{bmatrix} - \begin{bmatrix} 2 \\ 2 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - 2k_1 & -2 - 2k_2 \\ 0.5 - 2k_1 & -1 - 2k_2 \end{bmatrix}$$

$$\lambda_{Cl} = -\sqrt{2} \pm j\sqrt{2} \Rightarrow \begin{cases} \omega_n = 2 \\ \zeta = 0.707 \end{cases}$$

$$\det(\lambda\mathbf{I} - \mathbf{A}_K) = 0 \Rightarrow \det \begin{bmatrix} \lambda - 1 + 2k_1 & 2 + 2k_2 \\ -0.5 + 2k_1 & \lambda + 1 + 2k_2 \end{bmatrix} = 0$$

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$$\Rightarrow \lambda^2 + 2(k_1 + k_2)\lambda - 2k_1 - k_2 = 0$$

$$\lambda = -(k_1 + k_2) \pm \sqrt{(k_1 + k_2)^2 + 2k_1 + k_2}$$

or:

$$\lambda_1 = -(k_1 + k_2) + \sqrt{(k_1 + k_2)^2 + 2k_1 + k_2} \qquad (1)$$
$$\lambda_2 = -(k_1 + k_2) - \sqrt{(k_1 + k_2)^2 + 2k_1 + k_2} \qquad (2)$$

(1) +(2) $\Rightarrow k_1 + k_2 = -\frac{1}{2}(\lambda_1 + \lambda_2) = \sqrt{2}$ (3)

$$\Rightarrow k_1 = \sqrt{2} - k_2 \qquad (4)$$

(1)-(2) $\Rightarrow \lambda_1 - \lambda_2 = 2\sqrt{(k_1 + k_2)^2 + 2k_1 + k_2}$

Inserting (3) og (4) one has:

$$j2\sqrt{2} = 2\sqrt{2 + k_2 + 2(\sqrt{2} - k_2)}$$
$$\Rightarrow k_2 = 6.828$$

(4): $k_1 = -5.414$

c. One can use MATLAB with the commands below to make the plots below:

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```
>>[y, x, t] = step(AK,B,C,0);
>>plot(t,y),grid on
>>[u1, x, t] = step(AK,B,-K,0);
>>plot(t, u1+1),grid on }
```

See the plots below.

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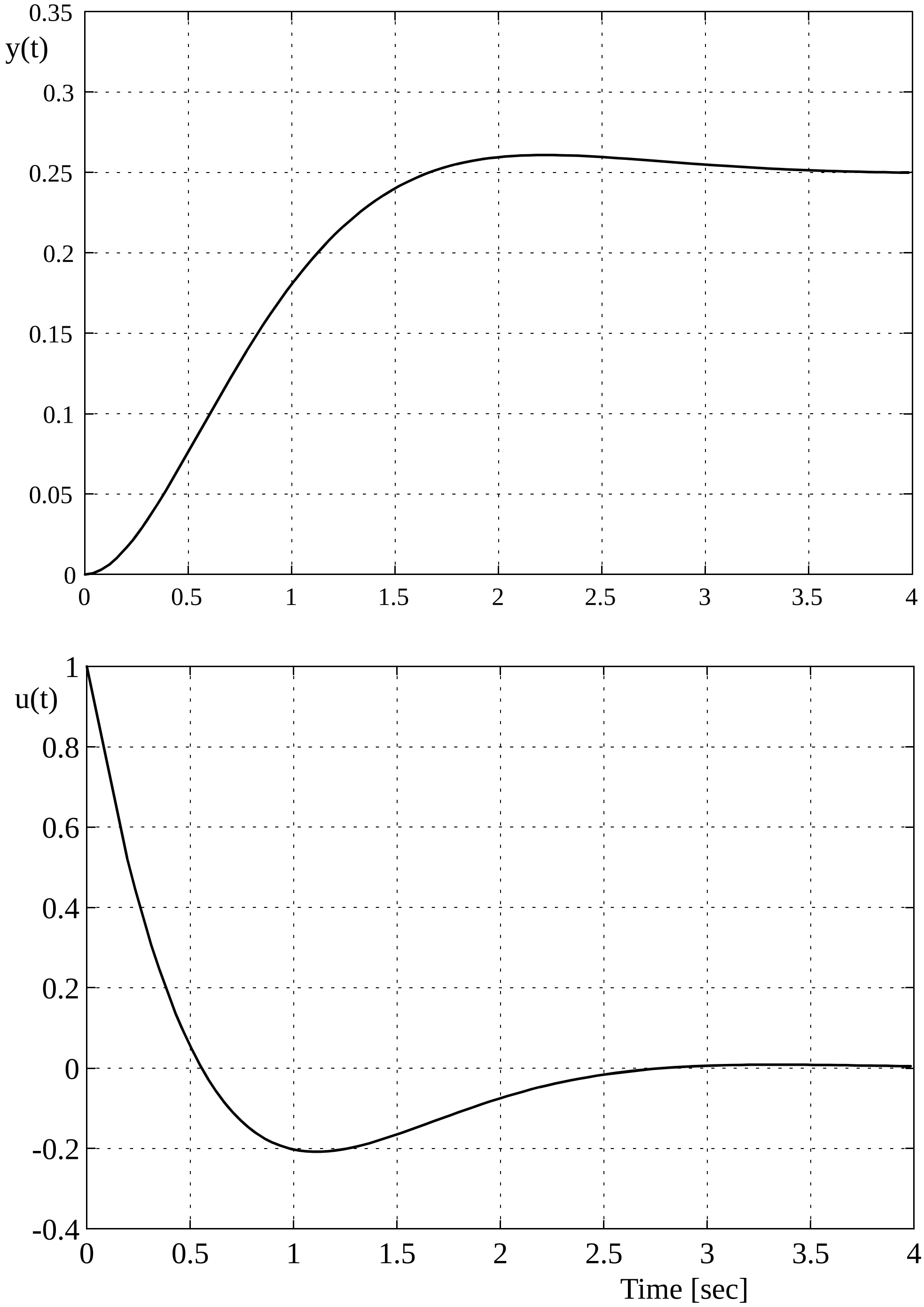
Problem 4.1



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