LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 3.6

a. The eigenvalues of the matrix can be found as:

$$\begin{vmatrix} \lambda + 4 & 3 \\ -1 & \lambda \end{vmatrix} = \lambda(\lambda + 4) + 3 = 0 \quad \text{for} \quad \lambda = \begin{cases} -1 \\ -3 \end{cases}$$

with the corresponding eigenvectors:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \qquad \mathbf{v}_2 = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

b. The diagonal transformed system can be found using the modal matrix:

$$\mathbf{M} = \begin{bmatrix} 1 & -3 \\ -1 & 1 \end{bmatrix} \Rightarrow \mathbf{M}^{-1} = \begin{bmatrix} -\frac{1}{2} & -\frac{3}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

$$\mathbf{A}_t = \mathbf{M}^{-1}\mathbf{A}\mathbf{M} = \begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix} = \Lambda$$

$$\mathbf{B}_t = \mathbf{M}^{-1}\mathbf{B} = \begin{bmatrix} -\frac{3}{2} \\ -\frac{3}{2} \end{bmatrix}, \qquad \mathbf{C}_t = \mathbf{C}\mathbf{M} = \{-1 \quad 1\}$$

c. The transformed state transition matrix is:

$$\Phi_t(t) = \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-3t} \end{bmatrix} = e^{\Lambda t}$$

d. To come back to the state transiton matrix of the original system use the expression:

$$e^{\mathbf{A}t} = \mathbf{M}e^{\Lambda t}\mathbf{M}^{-1} \qquad \text{(equation 3.133)}$$

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$$e^{\mathbf{A}t} = \begin{bmatrix} 1 & -3 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-3t} \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & -\frac{3}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{2}e^{-t} + \frac{3}{2}e^{-3t} & -\frac{3}{2}e^{-t} + \frac{3}{2}e^{-3t} \\ \frac{1}{2}e^{-t} - \frac{1}{2}e^{-3t} & \frac{3}{2}e^{-t} - \frac{1}{2}e^{-3t} \end{bmatrix}$$

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