LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 2.3

a. The stationary state is defined by the expression: $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}_0, \mathbf{y}_0) = \mathbf{0}$

Every vector: $\mathbf{x}_0 = \{0 \;\; 0 \;\; p_0 \;\; 0\}^T$ is a solution if $u_0 = 0$ is selected.

For the given numerical values given in the problem text the non-linear state equation are:

$$\dot{\mathbf{x}} = \begin{bmatrix} x_2 \\ \dfrac{45\sin x_1 - 0.75\sin x_1 \cos x_1 \cdot x_2^2 - 1.5\cos x_1 \cdot u}{3 - 0.75\cos^2 x_1} \\ x_4 \\ \dfrac{0.5\sin x_1 \cdot x_2^2 - 7.5\sin x_1 \cos x_1 + u}{3 - 0.75\cos^2 x_1} \end{bmatrix}$$

The Jacobian of the state equation above with the parameter values can be calculated from:

$$\mathbf{A} = \left\{ \frac{\partial f_i}{\partial x_j} \right\}_{x_1 = 0,\; x_2 = 0},$$

with the following results:

$$a = (3 - 0.75\cos^2 x_1)(45\cos x_1 - 0.75\, x_2^2(\cos^2 x_1 - \sin^2 x_1) + 1.5\sin x_1 \cdot u)$$

$$b = (45\sin x_1 - 0.75\sin x_1 \cos x_1 \cdot x_2^2 - 1.5\cos x_1 \cdot u)(2 \cdot 0.75\cos x_1 \sin x_1)$$

$$\frac{\partial f_2}{\partial x_1} = \frac{a - b}{(3 - 0.75\cos^2 x_1)^2}$$

$$\frac{\partial f_2}{\partial x_2} = \frac{-2 \cdot 0.75\sin x_1 \cos x_1 \cdot x_2}{3 - 0.75\cos^2 x_1}$$

Problem 2.3 (continued)

$$c = (3 - 0.75\cos^2 x_1)(0.5\, x_2^2 \cos x_1 + 7.5\, (\sin^2 x_1 - \cos^2 x_1))$$

$$d = (0.5\, \sin x_1 \cdot x_2^2 - 7.5\, \sin x_1 \cos x_1 + u)(2 \cdot 0.75 \cos x_1 \sin x_1)$$

$$\frac{\partial f_4}{\partial x_1} = \frac{c - d}{(3 - 0.75\, \cos^2 x_1)^2}$$

$$\frac{\partial f_4}{\partial x_2} = \frac{2 \cdot 0,5 \sin x_1 \cdot x_2}{3 - 0.75 \cos^2 x_1}$$

After insertion of numerical values for x_0 one obtains:

$$\left.\frac{\partial f_2}{\partial x_1}\right|_0 = \frac{2.5\ 45}{(2.25)^2} = 20$$

$$\left.\frac{\partial f_2}{\partial x_2}\right|_0 = 0$$

$$\left.\frac{\partial f_4}{\partial x_1}\right|_0 = \frac{2.25\ (-7.5)}{(2.25)^2} = -3.3$$

$$\left.\frac{\partial f_4}{\partial x_2}\right|_0 = 0$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 20 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -3.33 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{B} = \left\{\frac{\partial f_i}{\partial u}\right\}_0 = \left[\begin{array}{c} 0 \\ \dfrac{-1.5\, \cos x_1}{3 - 0.75\, \cos^2 x_1} \\ 0 \\ \dfrac{1}{3 - 0.75\, \cos^2 x_1} \end{array}\right]_{x_1 = 0,\, x_2 = 0} = \begin{bmatrix} 0 \\ -0.667 \\ 0 \\ 0.444 \end{bmatrix}$$

Problem 2.3 (continued)

b. The same result can be obtained from problem 2.1.

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