LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 4.8

a. The question here is whether or not it is possible to make a full order observer for the system in Problem 4.3.

$$\mathbf{M}_0 = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ -6 & 6 \end{bmatrix}, \ det(\mathbf{M}_0) = 18$$

$$\Rightarrow rank(\mathbf{M}_0) = 2 \Rightarrow \text{the system is observable}$$

The answer is that: yes it is possible.

b. To make and observer for the system use the method from section 3.9.2:

$$P_{Ch,\mathbf{A}}(\lambda) = \det[\lambda\mathbf{I} - \mathbf{A}] = \begin{bmatrix} \lambda & -1 \\ 2 & \lambda - 1 \end{bmatrix} = \lambda^2 - 2\lambda + 2$$

$$\Rightarrow a_1 = -2, \qquad a_0 = 2$$

$$q_1^T = \mathbf{C} = \begin{bmatrix} 0 & 3 \end{bmatrix}$$

$$q_2^T = \mathbf{CA} + q_{n-1}\mathbf{C} = [-6 \quad 6] - 2[0 \quad 3] = [-6 \quad 0]$$

$$\mathbf{Q} = \begin{bmatrix} q_2^T \\ q_1^T \end{bmatrix} = \begin{bmatrix} -6 & 0 \\ 0 & 3 \end{bmatrix}, \quad \mathbf{Q}^{-1} = \begin{bmatrix} -\frac{1}{6} & 0 \\ 0 & \frac{1}{3} \end{bmatrix}$$

$$\mathbf{A}_t = \mathbf{A}_{0c} = \mathbf{QAQ}^{-1} = \begin{bmatrix} 0 & -2 \\ 1 & 2 \end{bmatrix}$$

$$\mathbf{C}_t = \mathbf{C}_{0c} = \mathbf{CQ}^{-1} = [0 \quad 1] \qquad \text{since } \mathbf{L}_t = \begin{bmatrix} l_{1t} \\ l_{2t} \end{bmatrix}$$

$$\mathbf{A}_{Lt} = \mathbf{A}_{0c} - \mathbf{L}_t\mathbf{C}_{0c} = \begin{bmatrix} 0 & -2 - l_{1t} \\ 1 & 2 - l_{2t} \end{bmatrix}$$

Desired eigenvalues for observer:

$$\lambda_{\mathbf{A_L}} = -4 \pm j4$$

$$P_{Ch,\mathbf{A_L}} = (\lambda + 4 + j4)(\lambda + 4 - j4) = \lambda^2 + 8\lambda + 32$$

Problem 4.8

$$\mathbf{A}_{Lt} = \begin{bmatrix} 0 & -32 \\ 1 & -8 \end{bmatrix} \text{(observer canonical form)}$$

$$\mathbf{A}_{Lt} = \begin{bmatrix} 0 & -32 \\ 1 & -8 \end{bmatrix} = \begin{bmatrix} 0 & -2-l_{1t} \\ 0 & 2-l_{2t} \end{bmatrix}$$

$$\Rightarrow \begin{cases} l_{1t} = 30 \\ l_{2t} = 10 \end{cases}$$

For the original system one has that:

$$\begin{aligned} \mathbf{A_L} &= \mathbf{A} - \mathbf{LC} = \mathbf{Q}^{-1}\mathbf{A}_{0c}\mathbf{Q} - \mathbf{LC}_{0c}\mathbf{Q} = \mathbf{Q}^{-1}\mathbf{A}_{0c}\mathbf{Q} - \mathbf{Q}^{-1}\mathbf{QLC}_{0c}\mathbf{Q} \\ &= \mathbf{Q}^{-1}[\mathbf{A}_{0c} - \mathbf{QLC}_{0c}]\mathbf{Q} = \mathbf{Q}^{-1}\mathbf{AL}_t\mathbf{Q} \end{aligned}$$

$$\Rightarrow \mathbf{L}_t = \mathbf{QL} \quad \Rightarrow \quad \mathbf{L} = \mathbf{Q}^{-1}\mathbf{L}_t = \begin{bmatrix} -5 \\ 3.333 \end{bmatrix}$$

c. The design is now to be repeated using Ackermann's formula:

$$P_{Ch,\,\mathbf{A_L}}(\mathbf{A}) = \mathbf{A}^2 + 8\mathbf{A} + 32\mathbf{I} = \begin{bmatrix} 30 & 10 \\ -20 & 50 \end{bmatrix}$$

$$\mathbf{L} = P_{Ch,\,\mathbf{A_L}}(\mathbf{A})\mathbf{M}_0^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -5 \\ 3.333 \end{bmatrix}$$

d. A block diagram is now to be drawn of the observer and state controller of Problem 4.3.

The observer equation is:

$$\begin{aligned} \dot{\hat{\mathbf{x}}} &= \mathbf{A}\hat{\mathbf{x}} + \mathbf{B}u + \mathbf{L}(y - \hat{y}) \\ &= \begin{bmatrix} 0 & 1 \\ -2 & 2 \end{bmatrix}\hat{\mathbf{x}} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u + \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}(y - \hat{y}) \end{aligned}$$

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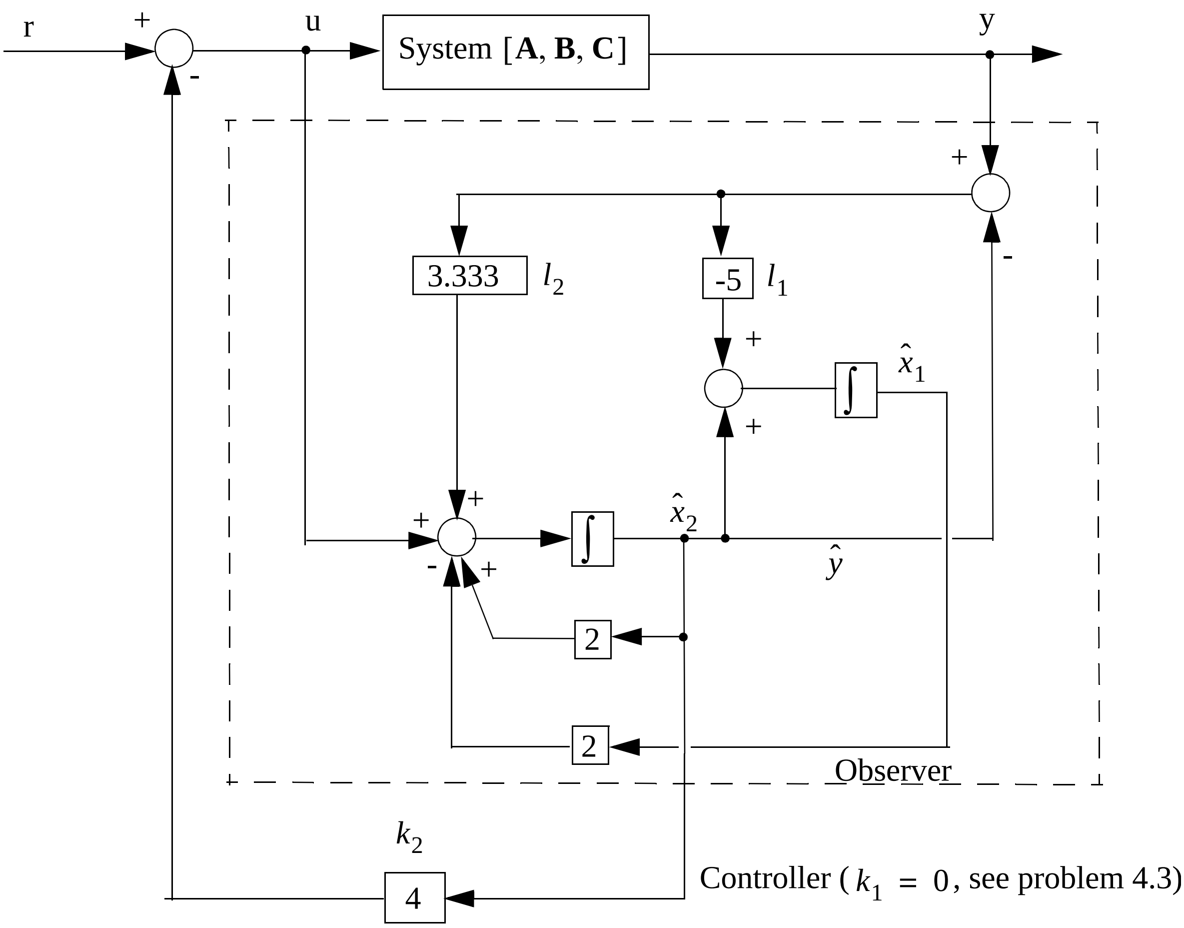
$$\Rightarrow \begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + l_1(y - \hat{y}) \\ \dot{\hat{x}}_2 = -2\hat{x}_1 + 2\hat{x}_2 + u + l_2(y - \hat{y}) \end{cases}$$

The controller is:

$$u = -K\hat{x} + r = -k_1\hat{x}_1 - k_2\hat{x}_2 + r$$



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