LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 6.2

a. In order that *f(x)* is a proper p.d.f. it is necessary that

$$\int_{-\infty}^{\infty} f(\chi)d\chi = 1 = \int_{-\infty}^{\infty} \frac{a}{\chi^2+1}\,d\chi$$

$$= a\ \text{atan}(\chi)\Big|_{-\infty}^{\infty}$$

$$= a\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right] = a\pi$$

$$\Rightarrow a = \frac{1}{\pi}$$

b. The distribution function corresponding to the p.d.f. is given by

$$F(x) = \int_{-\infty}^{x} f(\chi)d\chi = \frac{1}{\pi} = \int_{-\infty}^{x} \frac{1}{1+\chi^2}\,d\chi$$

$$= \frac{1}{\pi}[\text{atan}\ \chi]\Big|_{-\infty}^{x} = \frac{1}{\pi}[\text{atan}\ x - \text{atan}\ (-\infty)]$$

$$= \frac{1}{\pi}\left[\text{atan}\ x + \frac{\pi}{2}\right] = \frac{1}{\pi}\text{atan}\ x + \frac{1}{2}$$

c. It is given that the range of X is:

$$\frac{1}{4} \le X \le 1$$

Thus the probability which must be found is:

$$Pr\left\{\frac{1}{4} \le X \le 1\right\} = \int_{\frac{1}{2}}^{1} f(\chi)d\chi = \frac{1}{\pi}\left[\text{atan}(1) - \text{atan}\left(\frac{1}{2}\right)\right] = \frac{1}{\pi}\left(\frac{\pi}{4} - 0.148\pi\right) = 0.102$$

d. It is given that

$$\frac{1}{4} \le X^2 \le 1 \Rightarrow \frac{1}{2} \le X \le 1 \quad \text{or} \quad -1 \le X \le -\frac{1}{2}$$

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$$\Rightarrow Pr\left\{\frac{1}{4} \le X^2 \le 1\right\} = \int_{-1}^{-\frac{1}{2}} f(\chi)d\chi + \int_{\frac{1}{2}}^{1} f(\chi)d\chi$$

so that

$$Pr\left\{\frac{1}{4} \le X^2 \le 1\right\} = \frac{2}{\pi}\int_{\frac{1}{2}}^{1} \frac{1}{1+\chi^2}d\chi = \frac{2}{\pi}\left[\operatorname{atan}(1) - \operatorname{atan}\left(\frac{1}{2}\right)\right]$$

$$= \frac{2}{\pi}\left(\frac{\pi}{4} - 0.148\pi\right) = 0.204$$

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