

Linear Control Design II - Group Work Problem Module 14

Description

Consider the airplane model (MIMO model).

```
% System matrices:  
A = [-0.0558 -0.9968 0.0802 0.0415;  
      0.598  -0.115  -0.0318 0;  
      -3.050  0.388  -0.4650 0;  
      0       0.0805  1       0]
```

```
A = 4x4  
    -0.0558    -0.9968     0.0802     0.0415  
     0.5980    -0.1150    -0.0318         0  
    -3.0500     0.3880    -0.4650         0  
         0      0.0805     1.0000         0
```

```
B = [0.00729 1e-5; -0.475 0.123; 0.153 1.063; 0 0]
```

```
B = 4x2  
     0.0073     0.0000  
    -0.4750     0.1230  
     0.1530     1.0630  
         0         0
```

```
C = [0 1 0 0; 0 0 0 1]
```

```
C = 2x4  
     0     1     0     0  
     0     0     0     1
```

```
D = [0;0];
```

Problem 1

Is the system observable? Is it detectable?

```
% Fill in your answer here:  
Mo = obsv(A,C);  
rank(Mo)
```

```
ans = 4
```

Problem 2

In module 15 a multivariable state feedback controller for the aircraft was designed to assign the following closed-loop continuous eigenvalues

$$\lambda = \begin{cases} -0.5 \\ -0.6 \\ -0.5 \pm 0.5j \end{cases}$$

Design a discrete time observer for the system and use the sampling period $T_s = 0.1s$. Find the observer gain matrix L that assigns the eigenvalues of the estimation error dynamics to be 4-5 times faster than those of the closed-loop system (in continuous time). Use the Matlab function `place()` and remember that an observer can be designed by solving the dual problem of controller design.

Check the eigenvalues of the estimation error.

% Fill in your answer here:

```
Ts = 0.1;  
[F,G] = c2d(A,B,Ts)
```

```
F = 4x4  
    0.9902    -0.0985     0.0082     0.0041  
    0.0597     0.9855    -0.0028     0.0001  
   -0.2956     0.0525     0.9533    -0.0006  
   -0.0147     0.0104     0.0977     1.0000  
  
G = 4x2  
    0.0031    -0.0002  
   -0.0472     0.0121  
    0.0137     0.1041  
    0.0005     0.0053
```

% observer design

```
lambda = [-0.5 -0.6 -0.5+0.5i -0.5-0.5i];  
P_obsv_d = exp(5*lambda*Ts);  
L = place(F',C',P_obsv_d)'
```

```
L = 4x2  
    0.1476    -0.4695  
    0.3055    -0.1293  
   -0.3282     1.3050  
   -0.0252     0.5947
```

% verify the result

```
log(eig(F-L*C))/Ts
```

```
ans = 4x1 complex  
   -2.5000 + 2.5000i  
   -2.5000 - 2.5000i  
   -2.5000 + 0.0000i  
   -3.0000 + 0.0000i
```

Problem 3

Add the observer to the open loop Simulink model of the aircraft and carry out a simulation with the function on Figure 1 at both inputs, but one at a time. Plot the states, the state estimates and the estimation error.

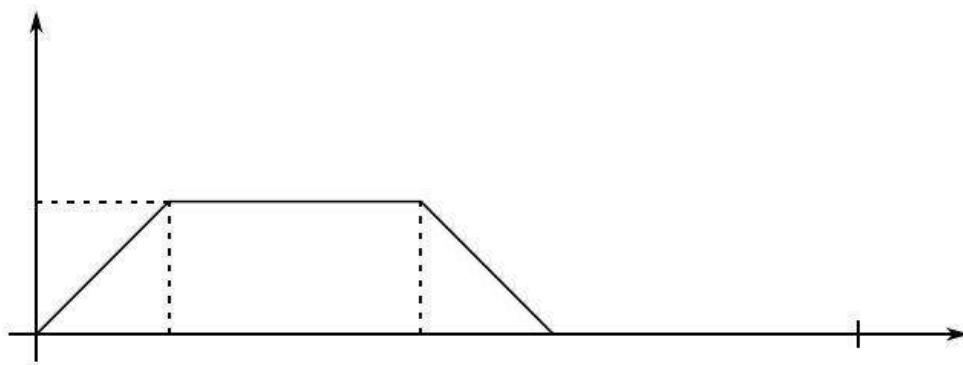


Figure 1 : Input signal for the closed loop system.

% Fill in your answer here:

```
t = 0:0.1:150;  
t1 = 0:0.1:5;  
r1 = 0.0175*t1/5;  
r3 = -0.0175*t1/5+0.0175;  
ra = [zeros(1,800) r1 0.0175*ones(1,99) r3 zeros(1,500)];
```

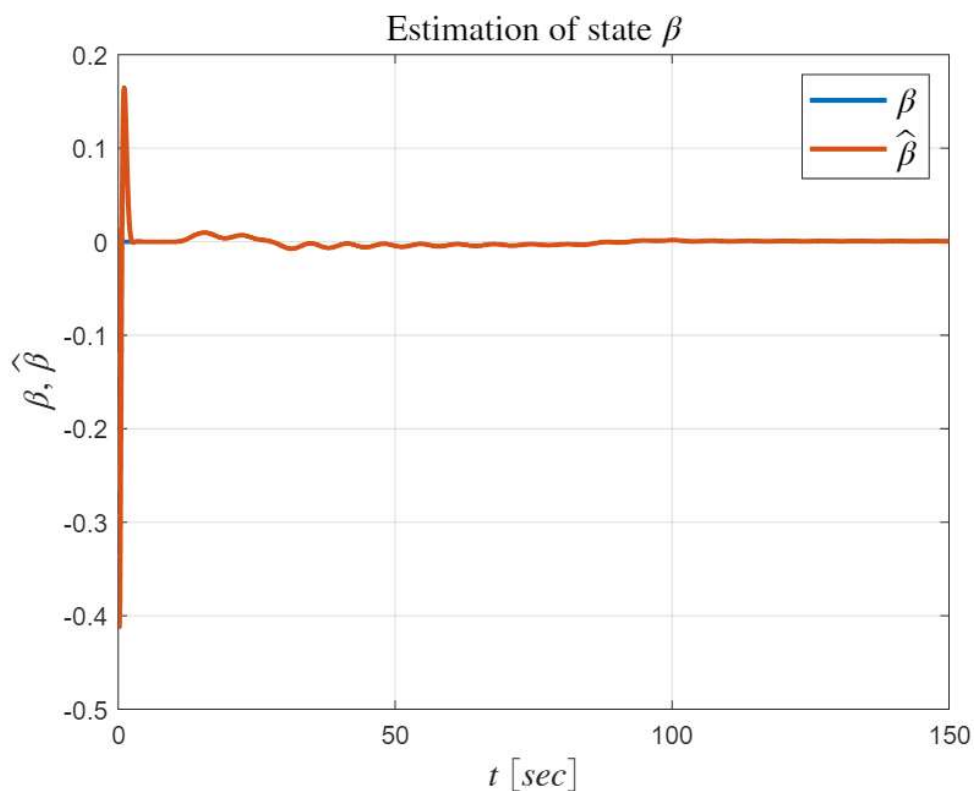
```

r = [zeros(1,100) r1 0.0175*ones(1,99) r3 zeros(1,800) zeros(1,400)];

inp = [t' r' ra'];
x0 = [0;0;0;0];
x_esht_0 = [0.015;-0.03;-0.08;-0.7];
sim('MyobsvOpenLoopSimul', 150, [], inp);
%sim('obsvOpenLoopSimul');
%% Plots
t = states.time;

figure
plot(t,states.signals.values(:,[1,5]), 'LineWidth', 2)
xlabel('$t \; [sec]$', 'Interpreter', 'latex', 'FontSize', 14);
ylabel('$\beta, \; \hat{\beta}$', 'Interpreter', 'latex', 'FontSize', 14);
legend({'$\beta$', '$\hat{\beta}$'}, 'Interpreter', 'latex', 'FontSize', 14);
title('Estimation of state $\beta$', 'Interpreter', 'latex', 'FontSize', 14);
grid

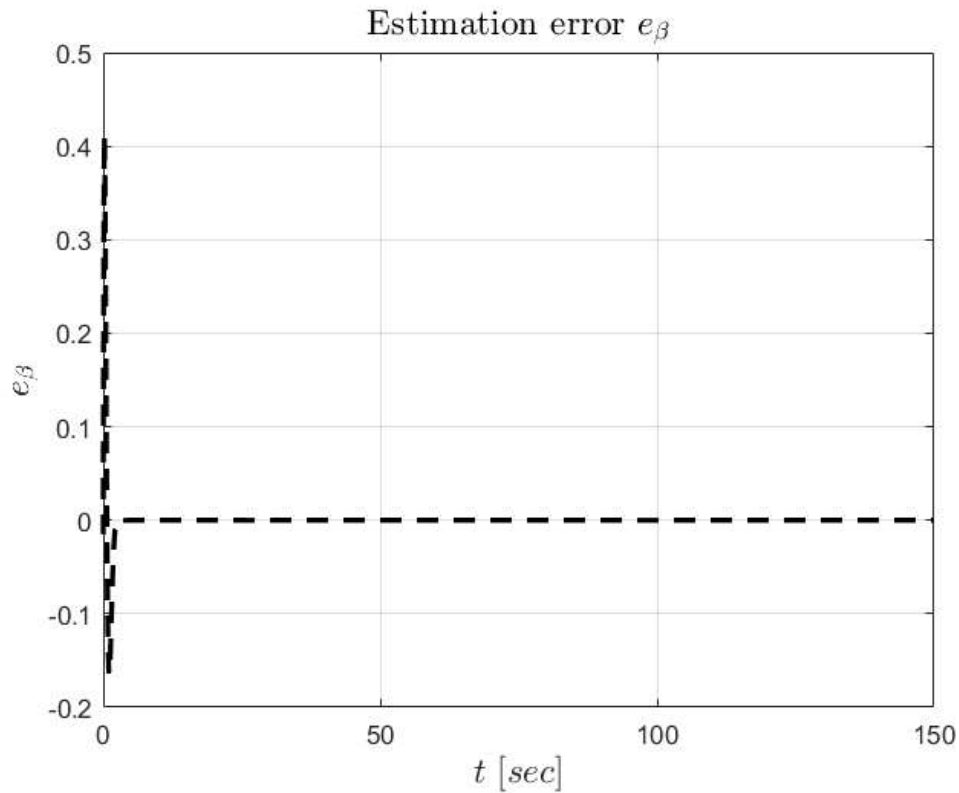
```



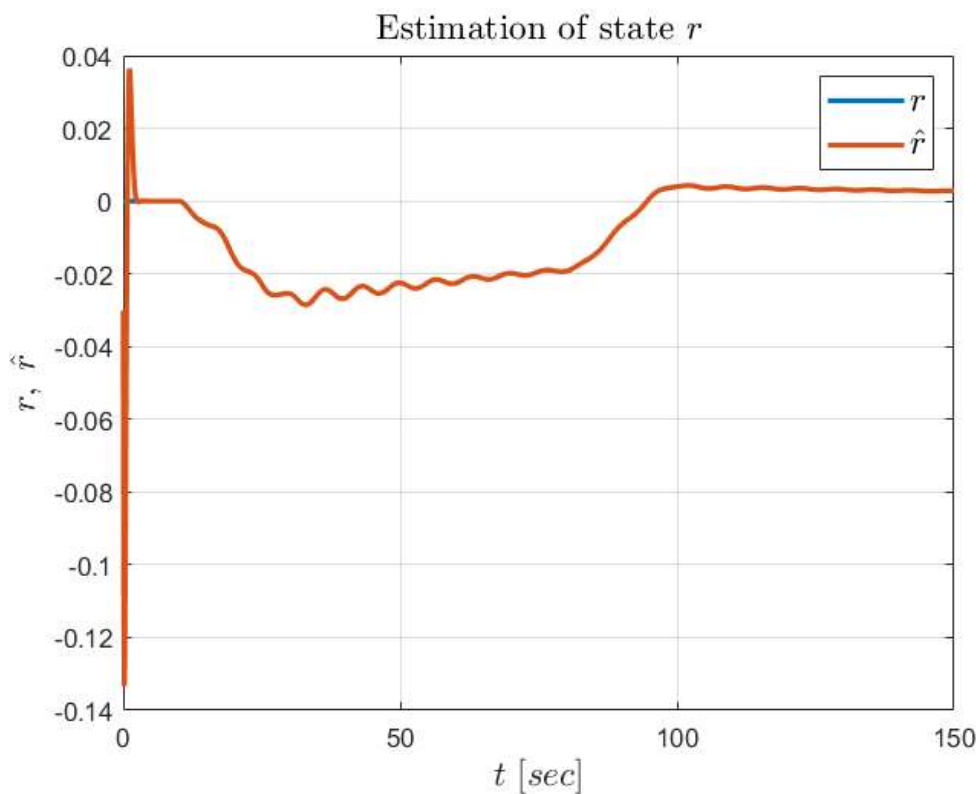
```

figure
plot(t,error.signals.values(:,1), '--k', 'LineWidth', 2);
xlabel('$t \; [sec]$', 'Interpreter', 'latex', 'FontSize', 14);
ylabel('$e_{\beta}$', 'Interpreter', 'latex', 'FontSize', 14);
title('Estimation error $e_{\beta}$', 'Interpreter', 'latex', 'FontSize', 14);
grid

```



```
figure
plot(t,states.signals.values(:,[2,6]), 'LineWidth', 2);
xlabel('$t \; [sec]$', 'Interpreter', 'latex', 'FontSize', 14);
ylabel('$r, \; \hat{r}$', 'Interpreter', 'latex', 'FontSize', 14);
legend({'$r$', '$\hat{r}$'}, 'Interpreter', 'latex', 'FontSize', 14);
title('Estimation of state $r$', 'Interpreter', 'latex', 'FontSize', 14);
grid
```

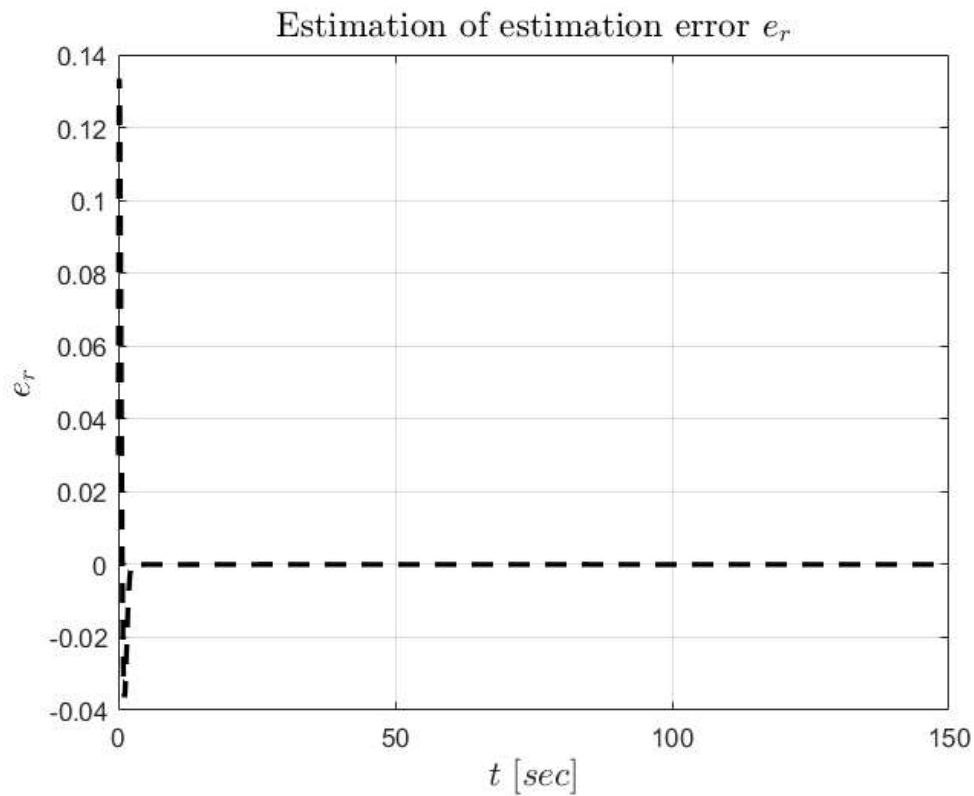


```
figure
plot(t,error.signals.values(:,2), '--k', 'LineWidth', 2);
xlabel('$t \; [sec]$', 'Interpreter', 'latex', 'FontSize', 14);
ylabel('$e_r$', 'Interpreter', 'latex', 'FontSize', 14);
```

```

title('Estimation of estimation error  $e_r$ ', 'Interpreter', 'latex', 'FontSize', 14);
grid

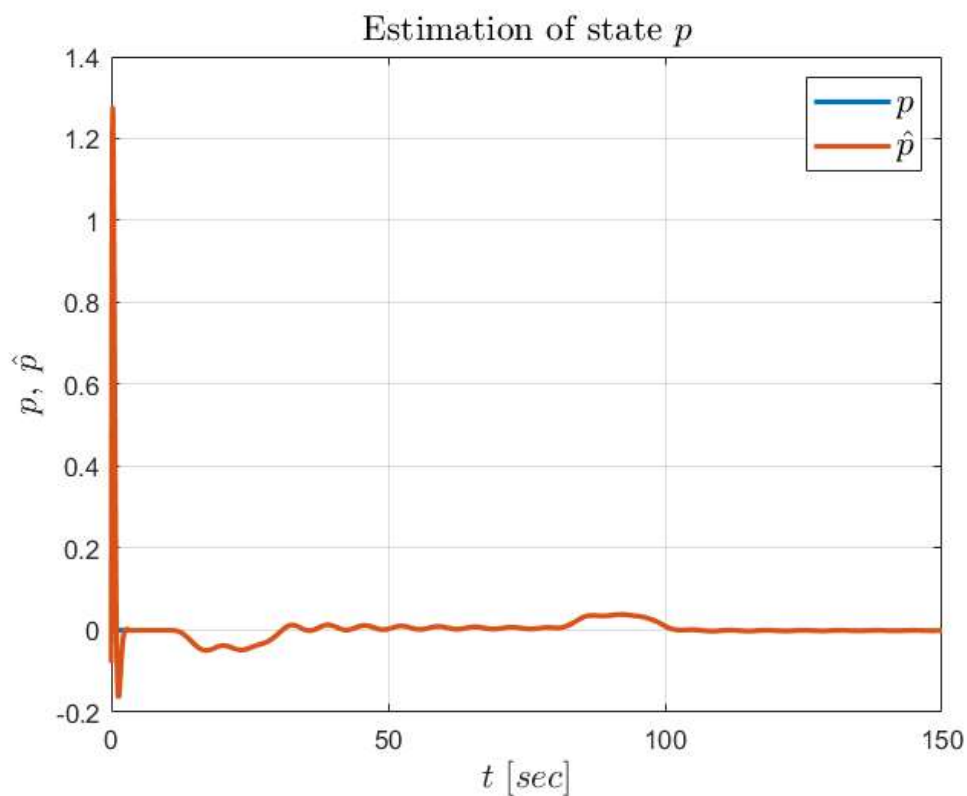
```



```

figure
plot(t,states.signals.values(:,[3,7]), 'LineWidth', 2);
xlabel('$t \; [sec]$', 'Interpreter', 'latex', 'FontSize', 14);
ylabel('$p, \; \hat{p}$', 'Interpreter', 'latex', 'FontSize', 14);
legend({'$p$', '$\hat{p}$'}, 'Interpreter', 'latex', 'FontSize', 14);
title('Estimation of state  $p$ ', 'Interpreter', 'latex', 'FontSize', 14);
grid

```



```

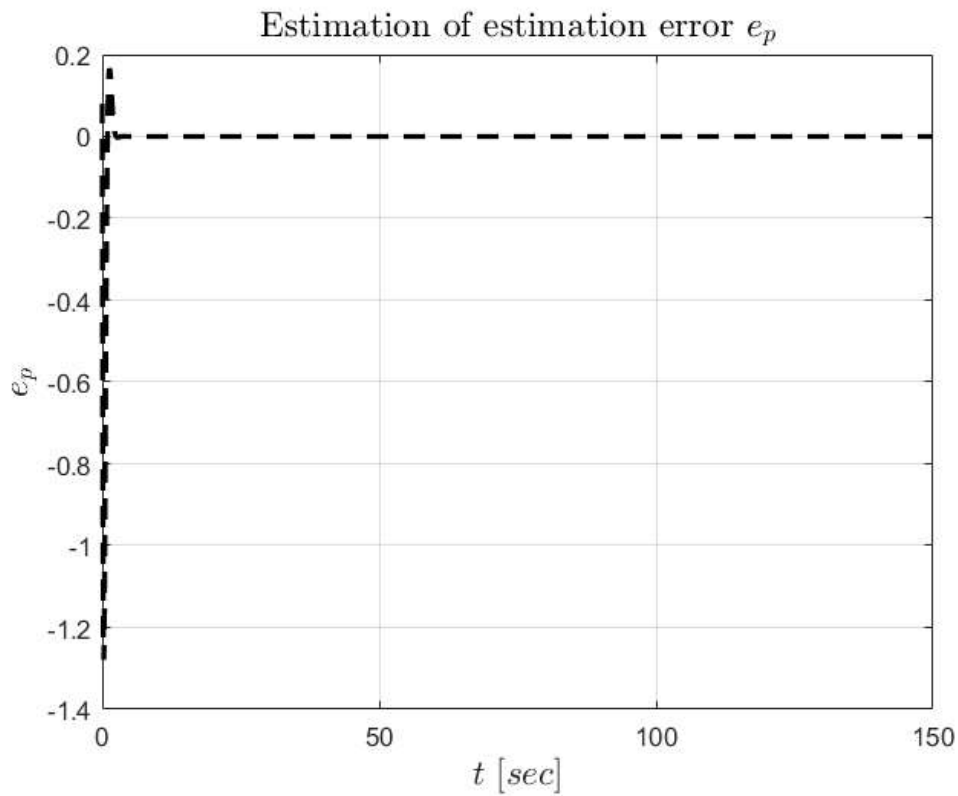
figure
plot(t,error.signals.values(:,3), '--k', 'LineWidth', 2);

```

```

xlabel('$t \; [sec]$', 'Interpreter', 'latex', 'FontSize', 14);
ylabel('$e_p$', 'Interpreter', 'latex', 'FontSize', 14);
title('Estimation of estimation error $e_p$', 'Interpreter', 'latex', 'FontSize', 14);
grid

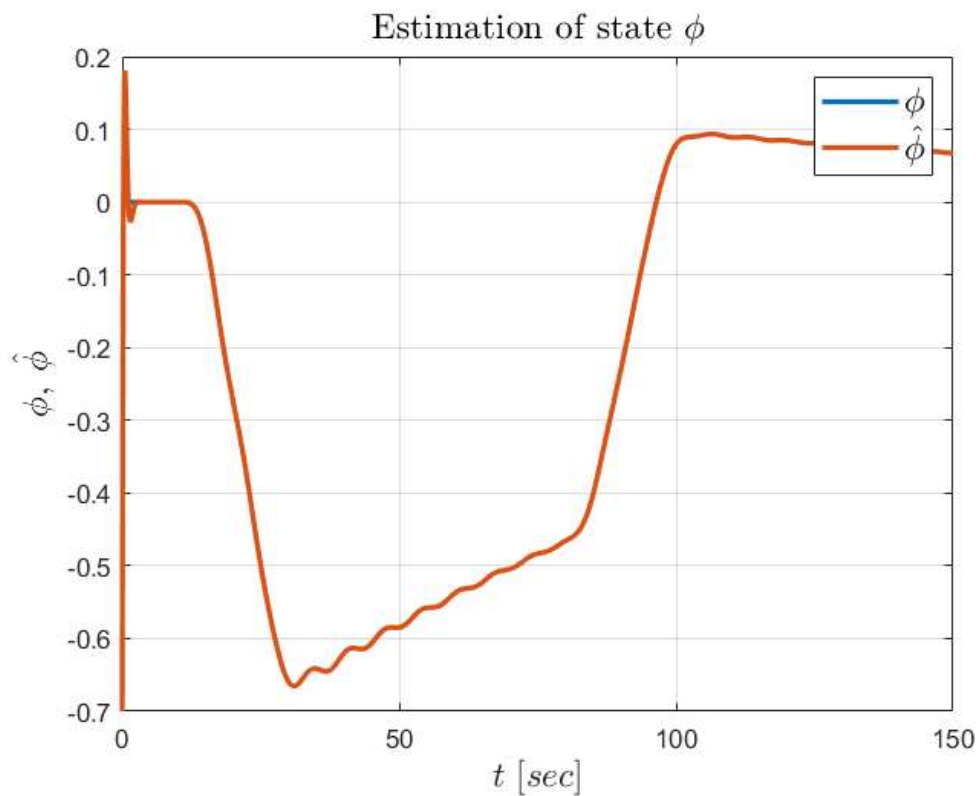
```



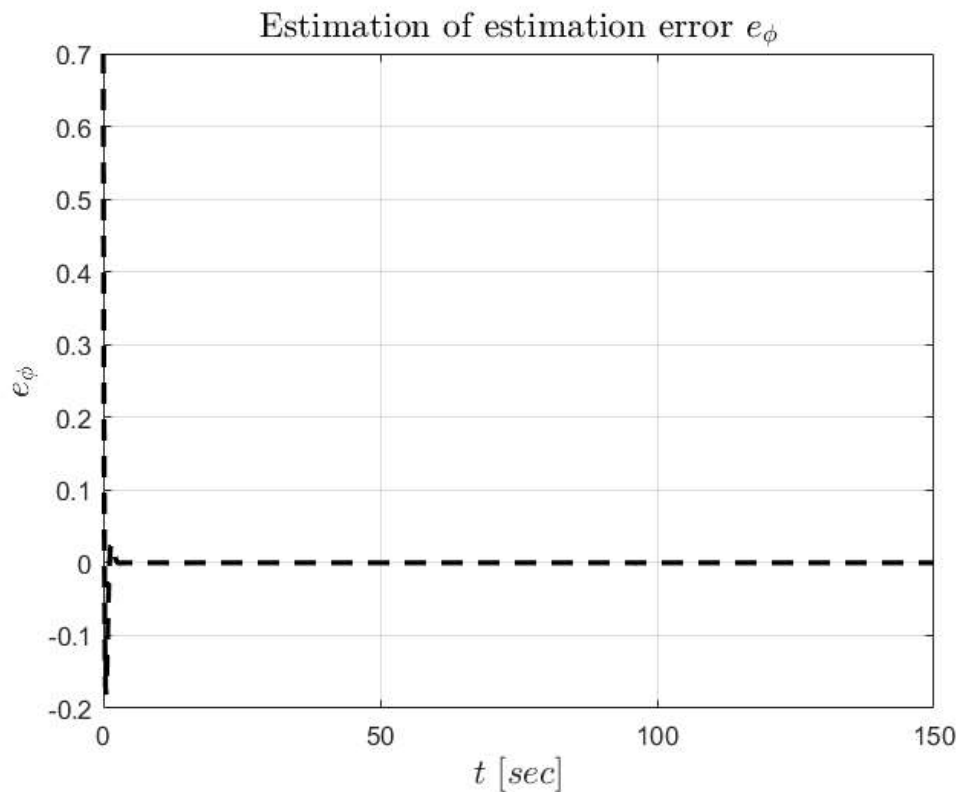
```

figure
plot(t,states.signals.values(:,[4,8]), 'LineWidth', 2);
xlabel('$t \; [sec]$', 'Interpreter', 'latex', 'FontSize', 14);
ylabel('$\phi, \; \hat{\phi}$', 'Interpreter', 'latex', 'FontSize', 14);
legend({'$\phi$', '$\hat{\phi}$'}, 'Interpreter', 'latex', 'FontSize', 14);
title('Estimation of state $\phi$', 'Interpreter', 'latex', 'FontSize', 14);
grid

```



```
figure
plot(t,error.signals.values(:,4), '--k', 'LineWidth', 2);
xlabel('$t \text{ [sec]}$', 'Interpreter', 'latex', 'FontSize', 14);
ylabel('$e_{\phi}$', 'Interpreter', 'latex', 'FontSize', 14);
title('Estimation of estimation error $e_{\phi}$', 'Interpreter', 'latex', 'FontSize', 14);
grid
```



Problem 4

Add the discrete controller designed in the module 15 to the Simulink model and instead of using the true states of the system, use their estimates provided by the observer. Carry out simulations and plot the states, the state estimates, the estimation error, the outputs and the control signals. Compare the performance of the closed-loop systems with and without the observer.

```
% Fill in your answer here:
% input reference
t=0:.1:150;
t1=0:.1:5;
r1=.0175*t1/5;
r3=.0175*(4-(t1+15)/5);
ra=[zeros(800,1)' r1 .0175*ones(99,1)' r3 zeros(1,500)];
r=[zeros(1,100) r1 .0175*ones(99,1)' r3 zeros(800,1)' zeros(1,400)];
inp = [t' r' ra'];

% feedback control gain
ev = [-0.5 -0.6 -0.5+0.5i -0.5-0.5i];
evd = exp(ev*Ts);
Kd = place(F,G,evd);

% initial states of system and obsv
x0 = [0;0;0;0];
x_esht_0 = [0.015;-0.03;-0.08;-0.7]*2;

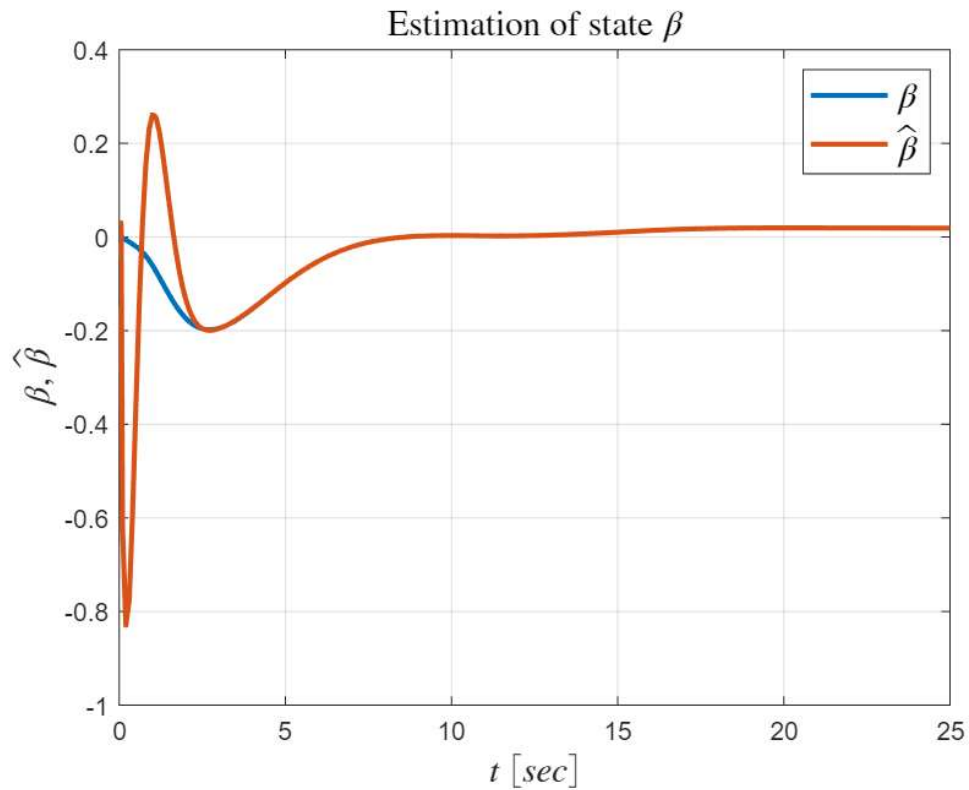
sim('MyobsvCloseLoopSimul.slx',25,[],inp)

%% Plots
```

```

t = states.time;
figure;
plot(t,states.signals.values(:,[1,5]), 'LineWidth', 2);
xlabel('$t \; [sec]$', 'Interpreter', 'latex', 'FontSize', 14);
ylabel('$\beta, \; \hat{\beta}$', 'Interpreter', 'latex', 'FontSize', 14);
legend({'$\beta$', '$\hat{\beta}$'}, 'Interpreter', 'latex', 'FontSize', 14);
title('Estimation of state $\beta$', 'Interpreter', 'latex', 'FontSize', 14);
grid;

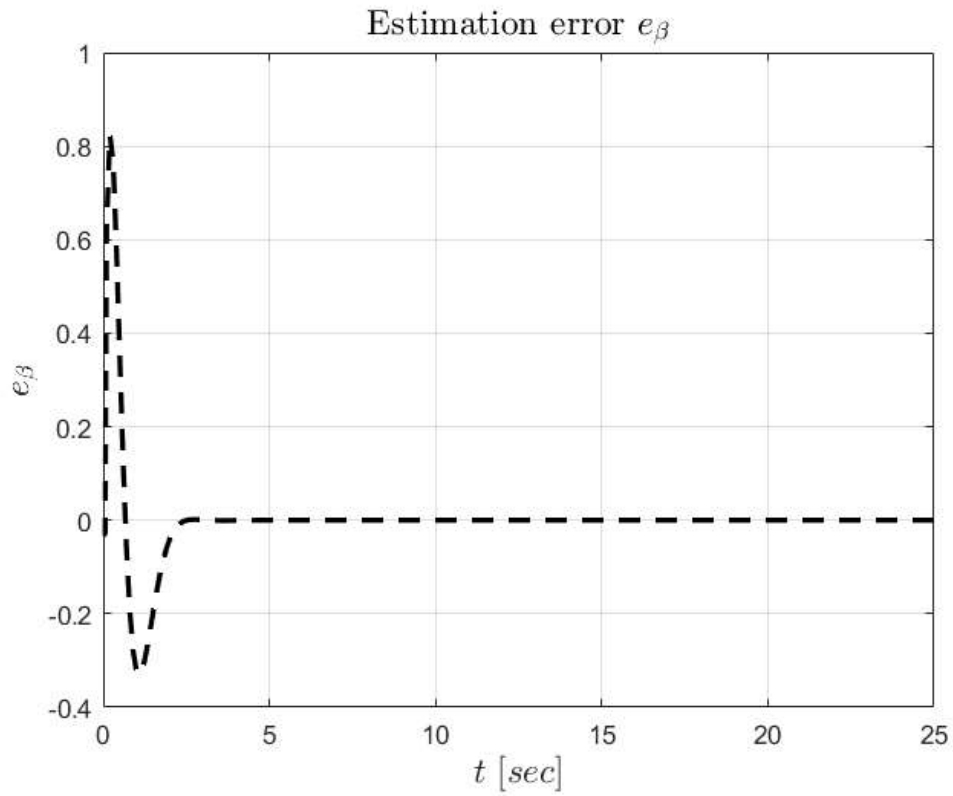
```



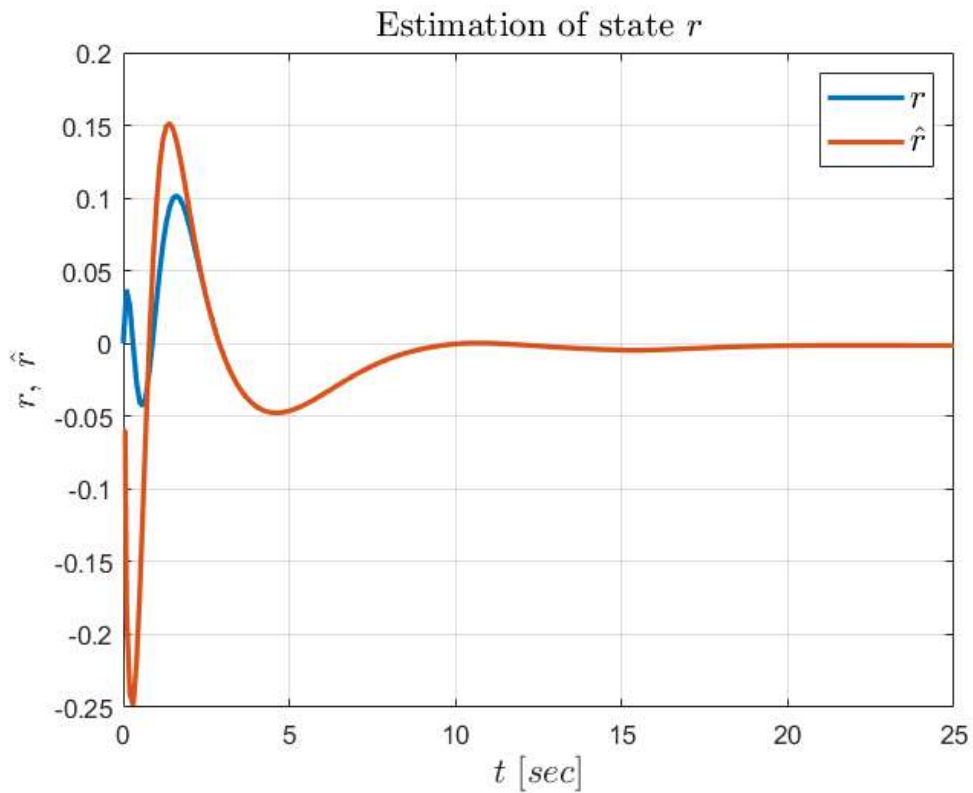
```

figure
plot(t,error.signals.values(:,1), '--k', 'LineWidth', 2);
xlabel('$t \; [sec]$', 'Interpreter', 'latex', 'FontSize', 14);
ylabel('$e_{\beta}$', 'Interpreter', 'latex', 'FontSize', 14);
title('Estimation error $e_{\beta}$', 'Interpreter', 'latex', 'FontSize', 14);
grid;

```

```
figure
plot(t,states.signals.values(:,[2,6]), 'LineWidth', 2);
xlabel('$t \; [sec]$', 'Interpreter', 'latex', 'FontSize', 14);
ylabel('$r, \; \hat{r}$', 'Interpreter', 'latex', 'FontSize', 14);
legend({'$r$', '$\hat{r}$'}, 'Interpreter', 'latex', 'FontSize', 14);
title('Estimation of state $r$', 'Interpreter', 'latex', 'FontSize', 14);
grid;
```

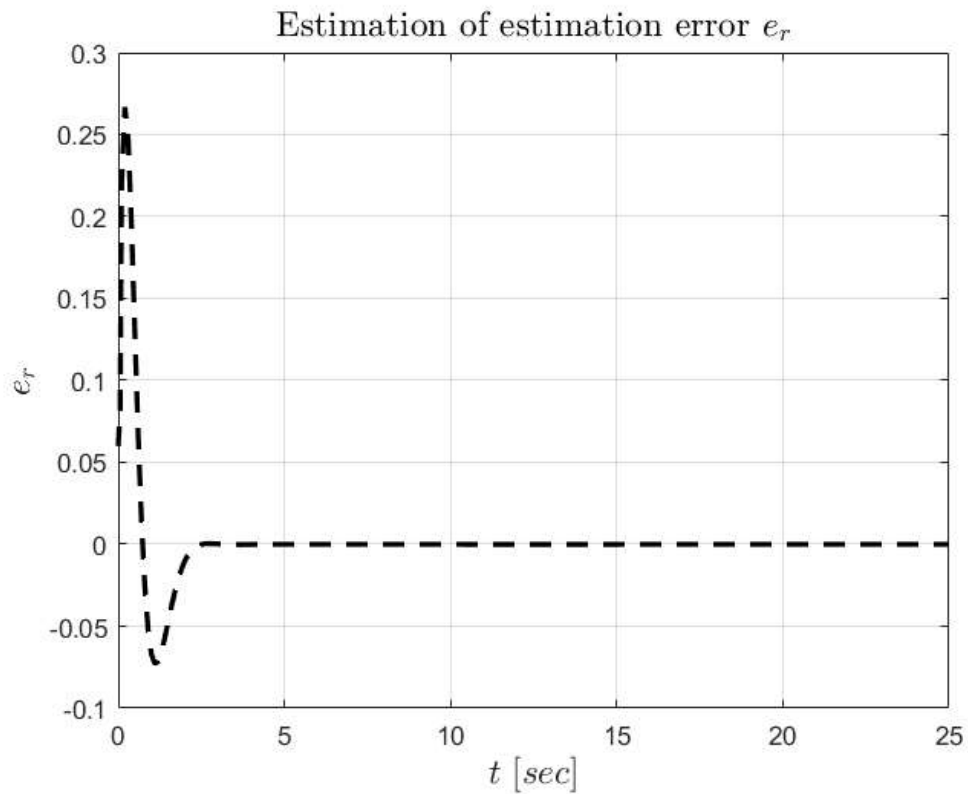


```
figure
plot(t,error.signals.values(:,2), '--k', 'LineWidth', 2);
xlabel('$t \; [sec]$', 'Interpreter', 'latex', 'FontSize', 14);
ylabel('$e_r$', 'Interpreter', 'latex', 'FontSize', 14);
```

```

title('Estimation of estimation error  $e_r$ ', 'Interpreter', 'latex', 'FontSize', 14);
grid;

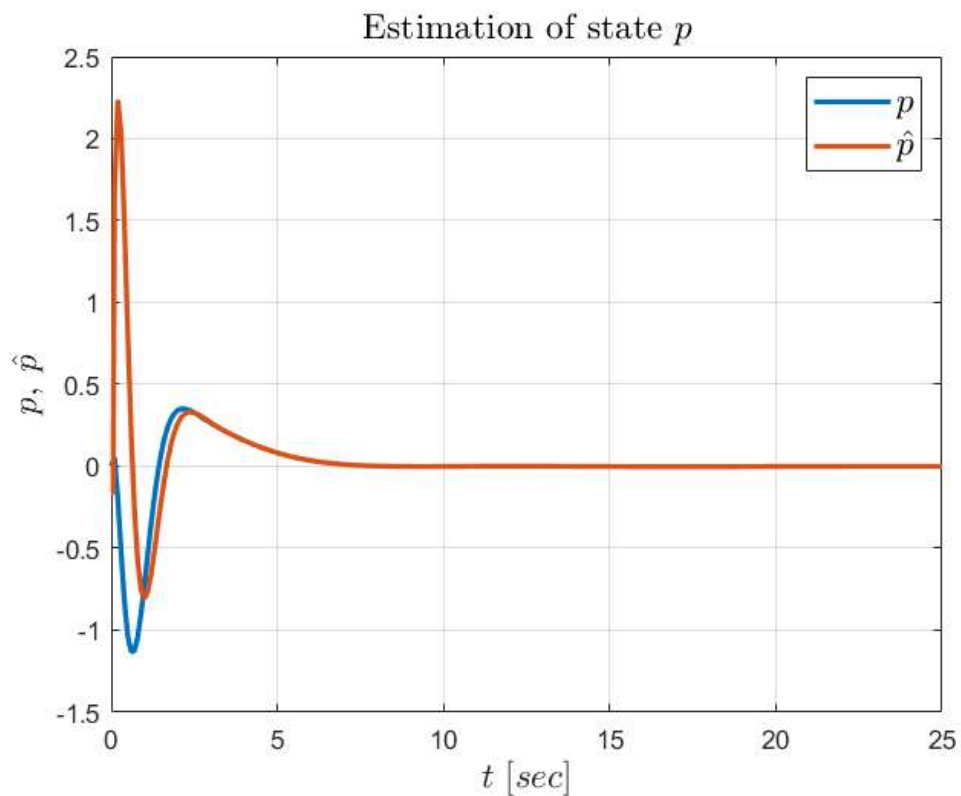
```



```

figure
plot(t,states.signals.values(:,[3,7]), 'LineWidth', 2);
xlabel('$t \; [sec]$', 'Interpreter', 'latex', 'FontSize', 14);
ylabel('$p, \; \hat{p}$', 'Interpreter', 'latex', 'FontSize', 14);
legend({'$p$', '$\hat{p}$'}, 'Interpreter', 'latex', 'FontSize', 14);
title('Estimation of state $p$', 'Interpreter', 'latex', 'FontSize', 14);
grid;

```



```

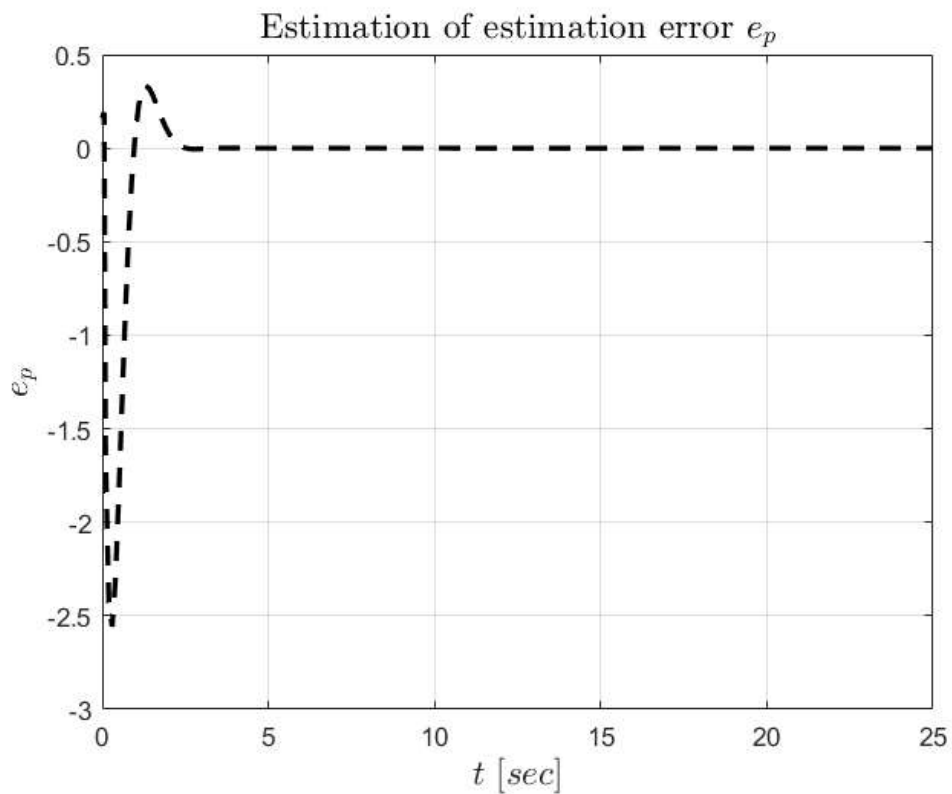
figure
plot(t,error.signals.values(:,3), '--k', 'LineWidth', 2);

```

```

xlabel('$t \; [sec]$', 'Interpreter', 'latex', 'FontSize', 14);
ylabel('$e_p$', 'Interpreter', 'latex', 'FontSize', 14);
title('Estimation of estimation error $e_p$', 'Interpreter', 'latex', 'FontSize', 14);
grid;

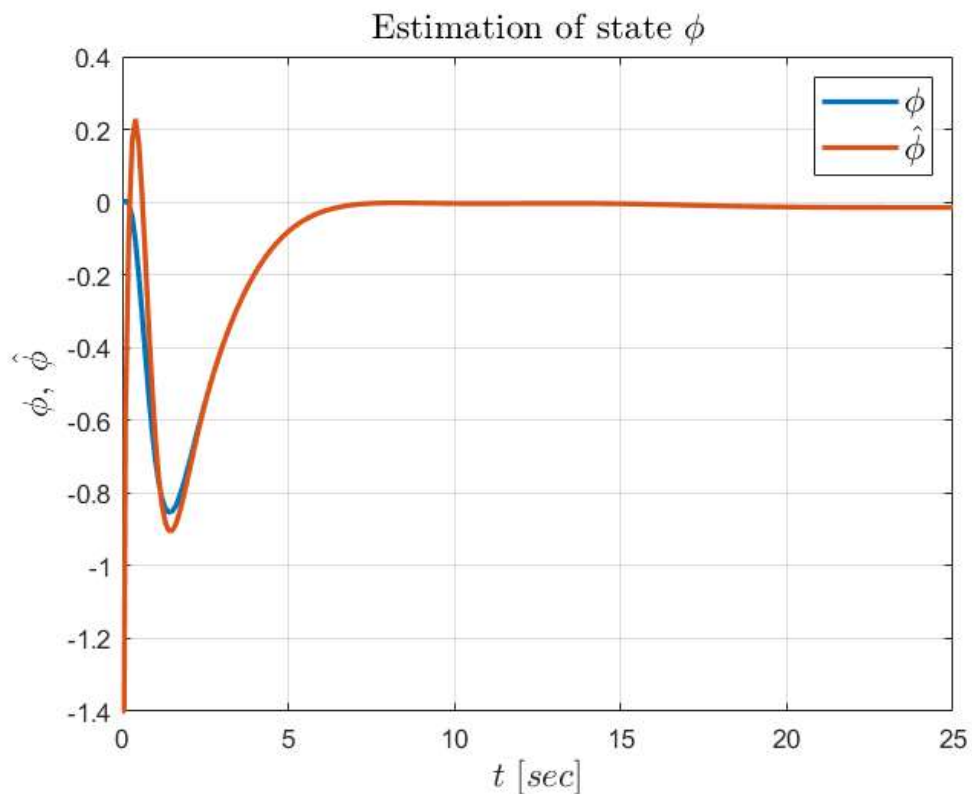
```



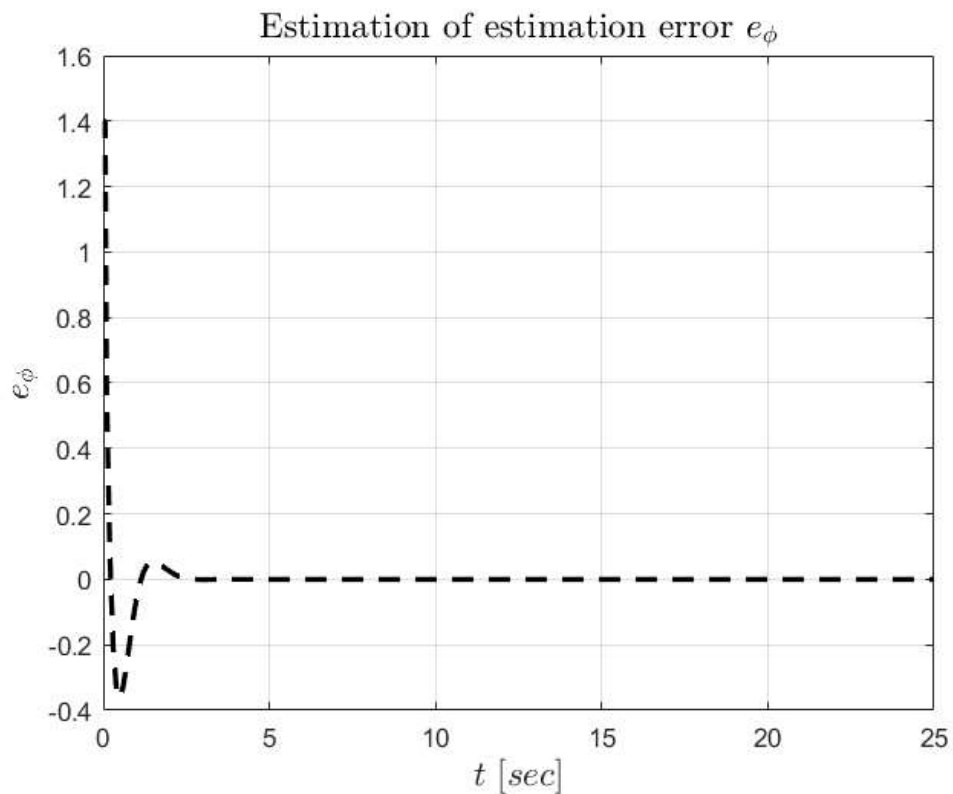
```

figure
plot(t,states.signals.values(:,[4,8]), 'LineWidth', 2);
xlabel('$t \; [sec]$', 'Interpreter', 'latex', 'FontSize', 14);
ylabel('$\phi, \; \hat{\phi}$', 'Interpreter', 'latex', 'FontSize', 14);
legend({'$\phi$', '$\hat{\phi}$'}, 'Interpreter', 'latex', 'FontSize', 14);
title('Estimation of state $\phi$', 'Interpreter', 'latex', 'FontSize', 14);
grid;

```



```
figure
plot(t,error.signals.values(:,4), '--k', 'LineWidth', 2);
xlabel('$t \backslash; [sec]$', 'Interpreter', 'latex', 'FontSize', 14);
ylabel('$e_{\phi}$', 'Interpreter', 'latex', 'FontSize', 14);
title('Estimation of estimation error $e_{\phi}$', 'Interpreter', 'latex', 'FontSize', 14);
grid;
```



Problem 5

Do the estimations obtained from the observer depend on how good the controller is? Can we estimate any of the states if the system is very fast or unstable?

% Fill in your answer here:

Problem 6

Repeat Problem 2-4 using the controller with integral action.

% Fill in your answer here:

```
% augment the system with integrators
Fi = [F zeros(4,2); -C eye(2,2)]
```

```
Fi = 6x6
    0.9902    -0.0985    0.0082    0.0041         0         0
    0.0597     0.9855   -0.0028    0.0001         0         0
   -0.2956     0.0525    0.9533   -0.0006         0         0
   -0.0147     0.0104    0.0977    1.0000         0         0
         0    -1.0000         0         0    1.0000         0
         0         0         0   -1.0000         0    1.0000
```

```
Gi = [G; zeros(2,2)]
```

```
Gi = 6x2
```

```

0.0031    -0.0002
-0.0472    0.0121
0.0137    0.1041
0.0005    0.0053
     0         0
     0         0

```

```
Ci = [C zeros(2,2)]
```

```

Ci = 2x6
     0     1     0     0     0     0
     0     0     0     1     0     0

```

```
% design the controllers
```

```

ev1 = [-0.5 -0.6 -0.5+1i -0.5-1i -0.7 -0.7];
ev1d = exp(ev1*Ts)

```

```

ev1d = 1x6 complex
    0.9512 + 0.0000i    0.9418 + 0.0000i    0.9465 + 0.0950i    0.9465 - 0.0950i    0.9324 + 0.0000i    0.

```

```
K1 = place(Fi,Gi,ev1d)
```

```

K1 = 2x6
    11.5504    0.7679    1.1256    0.3606   -1.4904    0.0041
    27.0476    6.8870    1.7904   -0.3779   -3.4459    0.0583

```

```
Kd = K1(:,1:4)
```

```

Kd = 2x4
    11.5504    0.7679    1.1256    0.3606
    27.0476    6.8870    1.7904   -0.3779

```

```
Ki = -K1(:,5:6)
```

```

Ki = 2x2
    1.4904   -0.0041
    3.4459   -0.0583

```

```
% eigenvalue solution
```

```
cl_eval = 1/Ts*log(eig(Fi-Gi*K1))
```

```

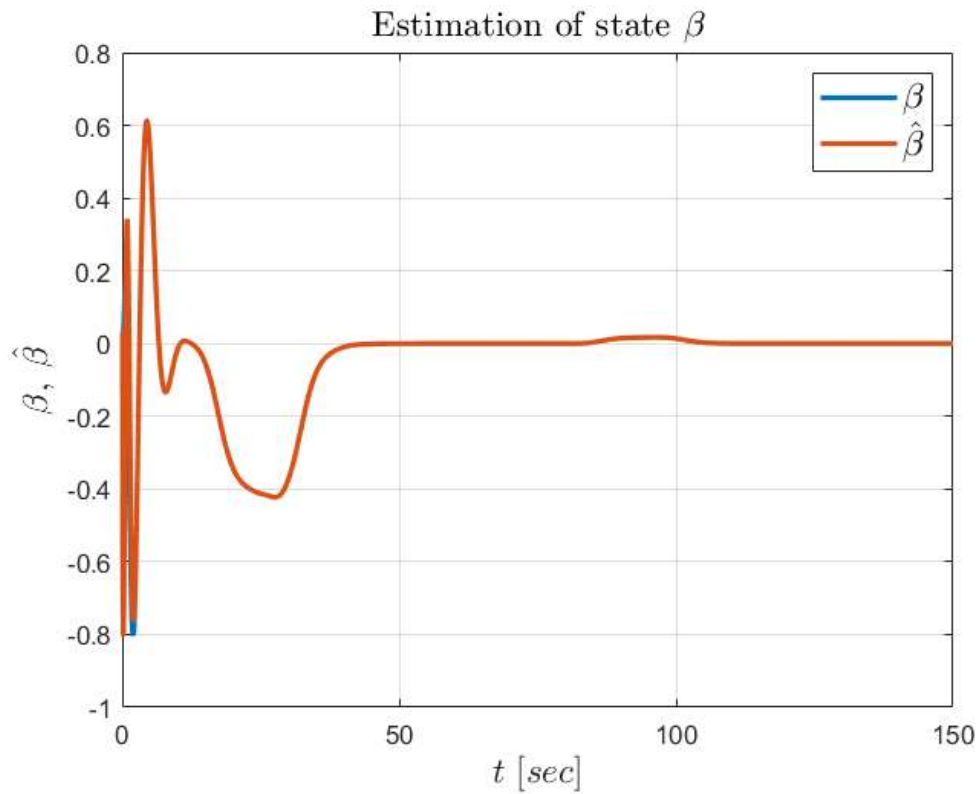
cl_eval = 6x1 complex
   -0.5000 + 1.0000i
   -0.5000 - 1.0000i
   -0.5000 + 0.0000i
   -0.6000 + 0.0000i
   -0.7000 + 0.0000i
   -0.7000 + 0.0000i

```

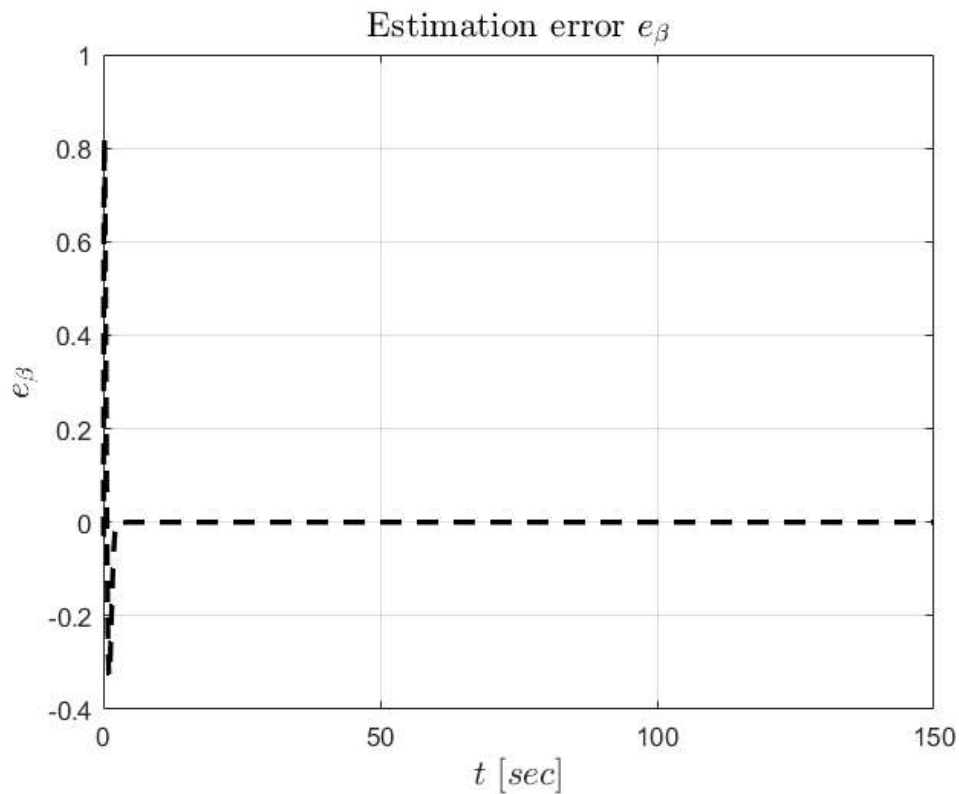
```

sim('MyobsvIntegralSimul.slx',150,[],inp);
%sim('obsvIntegralSimul.slx');
%% Plots
t = states.time;
figure;
plot(t,states.signals.values(:,[1,5]), 'LineWidth', 2);
xlabel('$t \; [sec]$', 'Interpreter', 'latex', 'FontSize', 14);
ylabel('$\beta, \; \hat{\beta}$', 'Interpreter', 'latex', 'FontSize', 14);
legend({'$\beta$', '$\hat{\beta}$'}, 'Interpreter', 'latex', 'FontSize', 14);
title('Estimation of state $\beta$', 'Interpreter', 'latex', 'FontSize', 14);
grid;

```

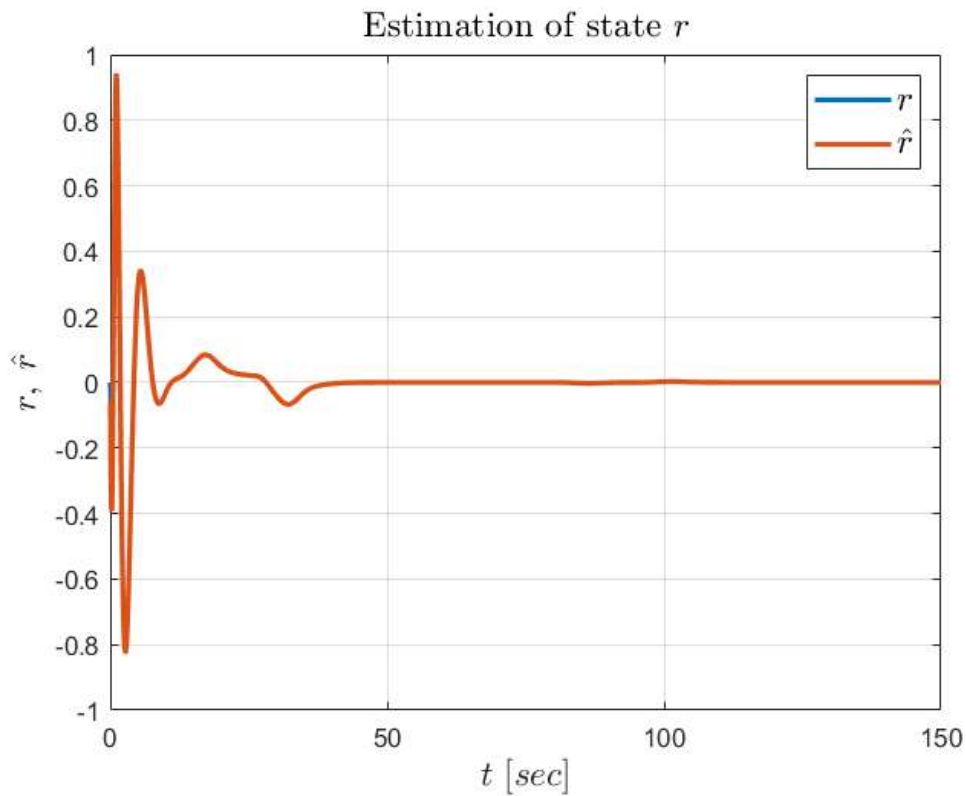


```
figure
plot(t,error.signals.values(:,1), '--k', 'LineWidth', 2);
xlabel('$t \; [sec]$', 'Interpreter', 'latex', 'FontSize', 14);
ylabel('$e_{\beta}$', 'Interpreter', 'latex', 'FontSize', 14);
title('Estimation error $e_{\beta}$', 'Interpreter', 'latex', 'FontSize', 14);
grid;
```

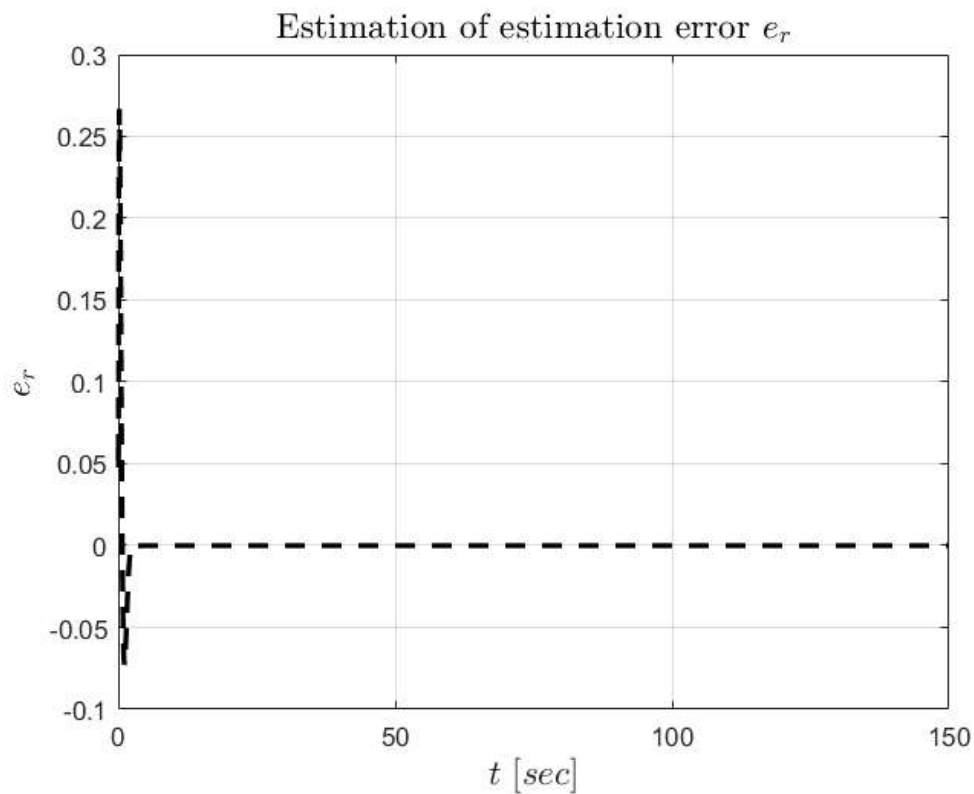


```
figure
plot(t,states.signals.values(:,[2,6]), 'LineWidth', 2);
xlabel('$t \; [sec]$', 'Interpreter', 'latex', 'FontSize', 14);
ylabel('$r, \; \hat{r}$', 'Interpreter', 'latex', 'FontSize', 14);
legend({'$r$', '$\hat{r}$'}, 'Interpreter', 'latex', 'FontSize', 14);
```

```
title('Estimation of state  $r$ ', 'Interpreter', 'latex', 'FontSize', 14);
grid;
```



```
figure
plot(t,error.signals.values(:,2), '--k', 'LineWidth', 2);
xlabel('$t \; [sec]$', 'Interpreter', 'latex', 'FontSize', 14);
ylabel('$e_r$', 'Interpreter', 'latex', 'FontSize', 14);
title('Estimation of estimation error  $e_r$ ', 'Interpreter', 'latex', 'FontSize', 14);
grid;
```

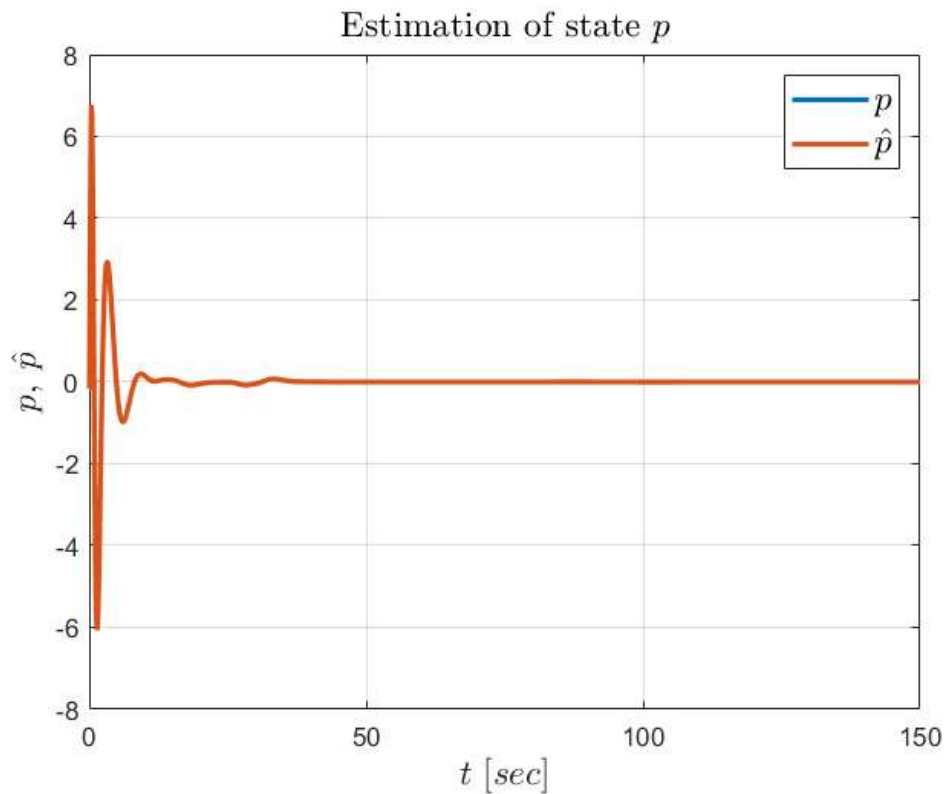


```
figure
plot(t,states.signals.values(:,[3,7]), 'LineWidth', 2);
xlabel('$t \; [sec]$', 'Interpreter', 'latex', 'FontSize', 14);
```

```

ylabel('$p, \hat{p}$', 'Interpreter', 'latex', 'FontSize', 14);
legend({'$p$', '$\hat{p}$'}, 'Interpreter', 'latex', 'FontSize', 14);
title('Estimation of state $p$', 'Interpreter', 'latex', 'FontSize', 14);
grid;

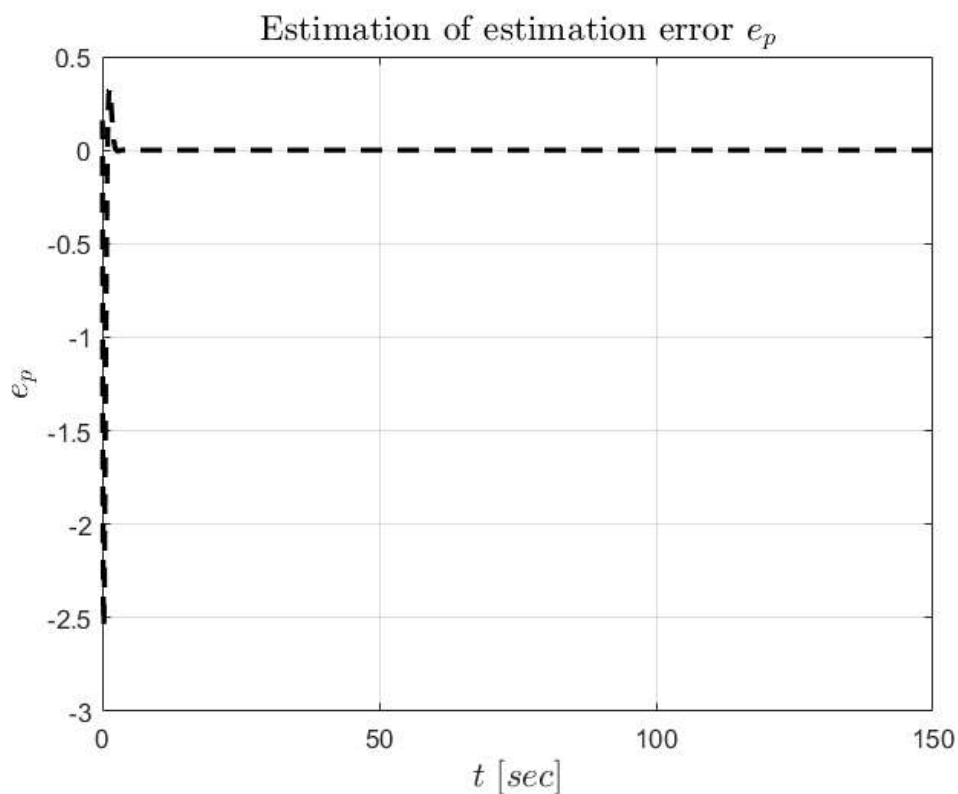
```



```

figure
plot(t,error.signals.values(:,3), '--k', 'LineWidth', 2);
xlabel('$t \text{ [sec]}$', 'Interpreter', 'latex', 'FontSize', 14);
ylabel('$e_p$', 'Interpreter', 'latex', 'FontSize', 14);
title('Estimation of estimation error $e_p$', 'Interpreter', 'latex', 'FontSize', 14);
grid;

```

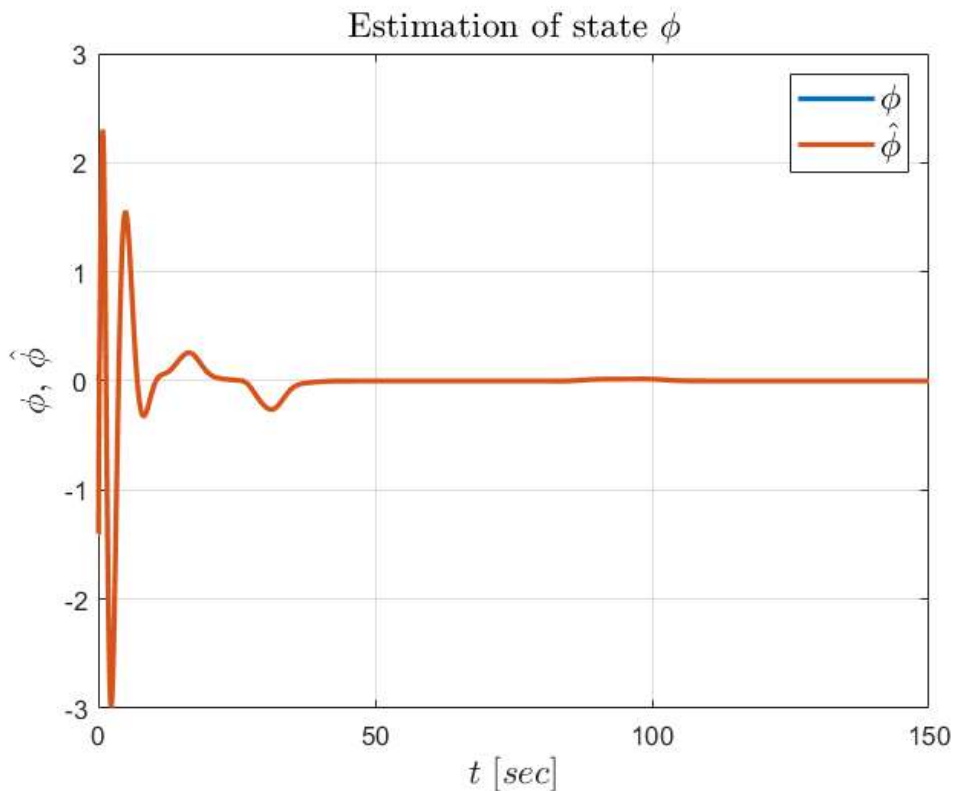


figure


```

plot(t,states.signals.values(:,[4,8]), 'LineWidth', 2);
xlabel('$t \; [sec]$', 'Interpreter', 'latex', 'FontSize', 14);
ylabel('$\phi, \; \hat{\phi}$', 'Interpreter', 'latex', 'FontSize', 14);
legend({'$\phi$', '$\hat{\phi}$'}, 'Interpreter', 'latex', 'FontSize', 14);
title('Estimation of state $\phi$', 'Interpreter', 'latex', 'FontSize', 14);
grid;

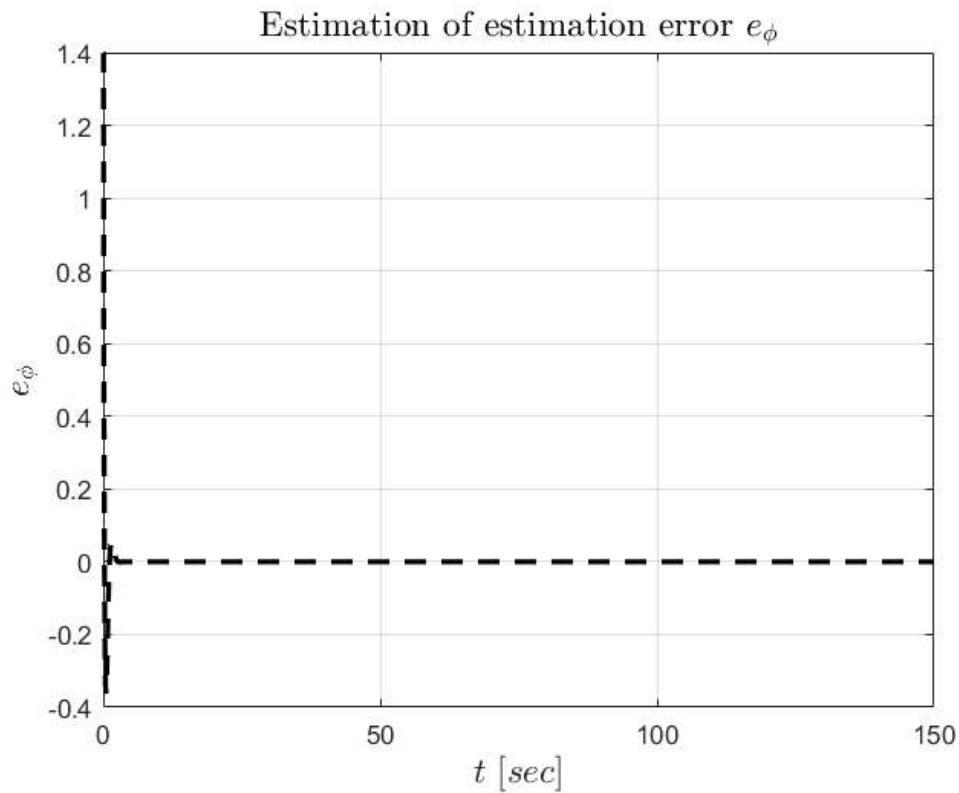
```



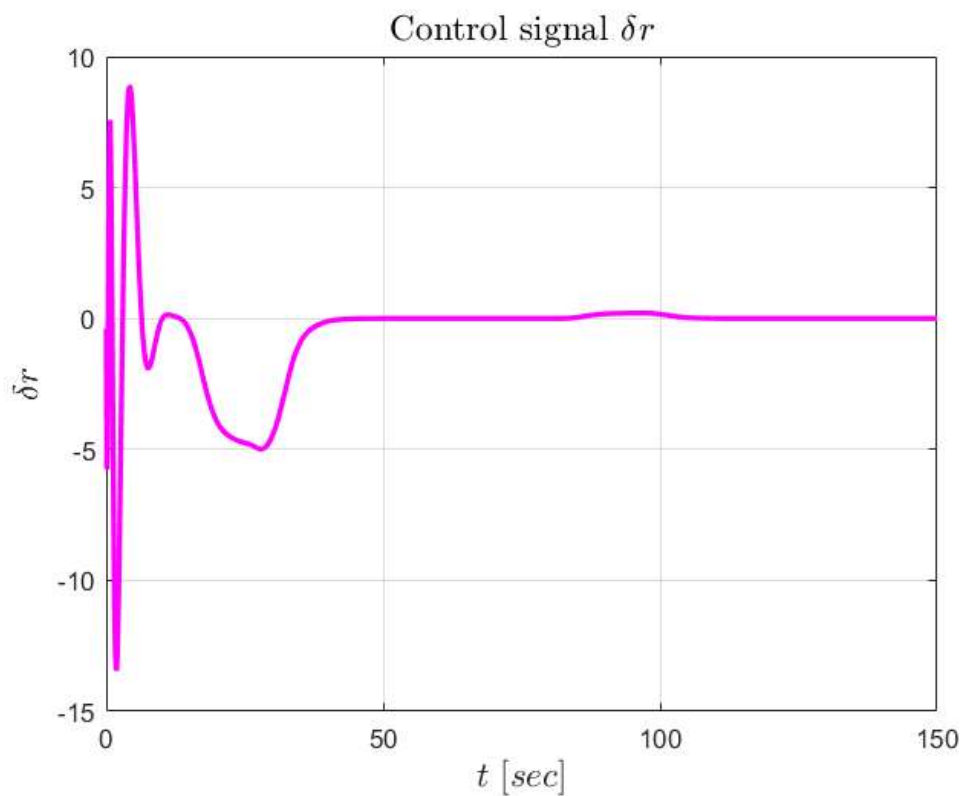
```

figure
plot(t,error.signals.values(:,4), '--k', 'LineWidth', 2);
xlabel('$t \; [sec]$', 'Interpreter', 'latex', 'FontSize', 14);
ylabel('$e_{\phi}$', 'Interpreter', 'latex', 'FontSize', 14);
title('Estimation of estimation error $e_{\phi}$', 'Interpreter', 'latex', 'FontSize', 14);
grid;

```



```
t = control.time;
figure
plot(t,control.signals.values(:,1), 'm', 'LineWidth', 2);
xlabel('$t \; [sec]$', 'Interpreter', 'latex', 'FontSize', 14);
ylabel('$\delta r$', 'Interpreter', 'latex', 'FontSize', 14);
title('Control signal $\delta r$', 'Interpreter', 'latex', 'FontSize', 14);
grid;
```



```
figure
plot(t,control.signals.values(:,2), 'm', 'LineWidth', 2);
xlabel('$t \; [sec]$', 'Interpreter', 'latex', 'FontSize', 14);
```

```
ylabel('$\delta a$', 'Interpreter', 'latex', 'FontSize', 14);  
title('Control signal $\delta a$', 'Interpreter', 'latex', 'FontSize', 14);  
grid;
```

