LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 2.6

a. For the volume flows one has:

$$\left.\begin{aligned} q_1 &= A_c\dot{x} + \frac{V_1}{\beta}\dot{p}_1 + C_l(p_1 - p_2) \\ q_2 &= A_c\dot{x} - \frac{V_1}{\beta}\dot{p}_2 + C_l(p_1 - p_2) \end{aligned}\right\}$$

$$\Rightarrow \qquad \left.\begin{aligned} \dot{p}_1 &= \frac{\beta}{V_1}(q_1 - A_c\dot{x} - C_l(p_1 - p_2)) \\ \dot{p}_2 &= \frac{\beta}{V_2}(-q_2 + A_c\dot{x} + C_l(p_1 - p_2)) \end{aligned}\right\}.$$

The flow are controlled by the input relation:

$$q_1 = q_2 = ku.$$

Newton's second law applied to M gives:

$$M\ddot{x} = f + A_c(p_1 - p_2) - C_f\ \dot{x}$$

$$\Rightarrow \dot{x} = \frac{1}{M}(f + A_c(p_1 - p_2) - C_f\ \dot{x}).$$

b. The state vector for the system is (see the block diagram below):

$$\mathbf{x} = \begin{bmatrix} x \\ \dot{x} \\ p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} x \\ v \\ p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

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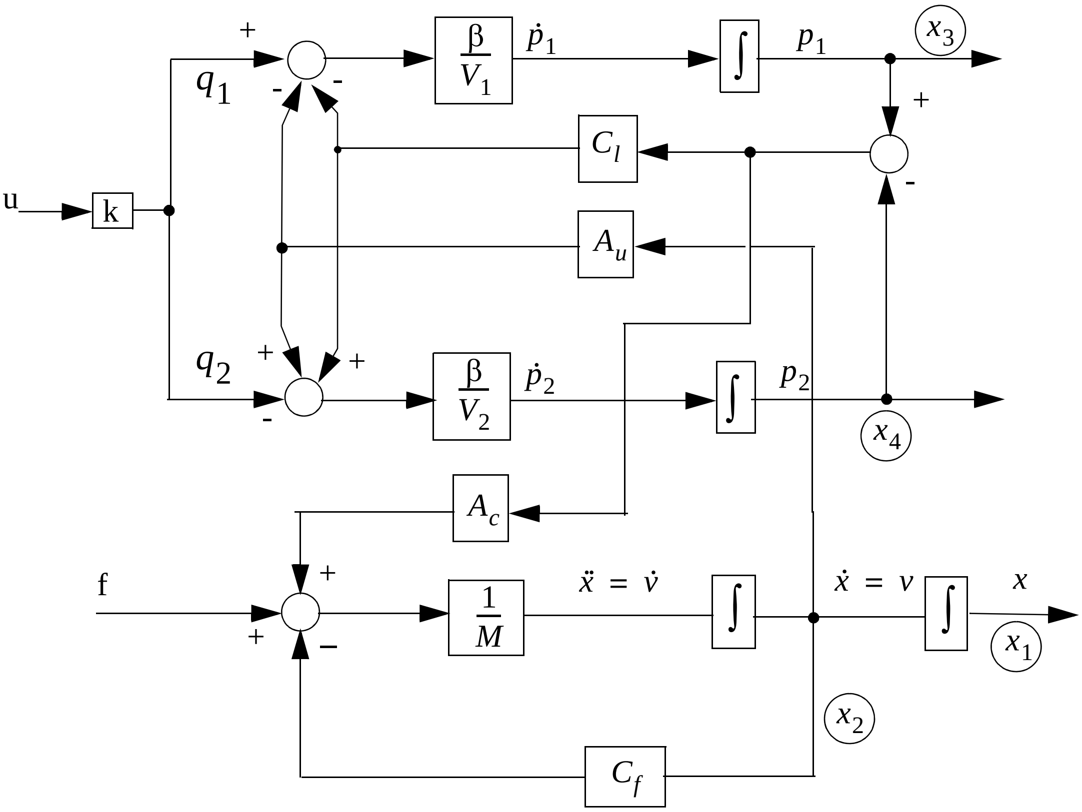
Solutions to Problems

Problem 2.6 (continued)

Blockdiagram:



The state equations for the system can be written in two different ways:

$$
\begin{aligned}
\dot{x} &= v & \qquad & & \dot{x}_1 &= x_2 \\
\dot{v} &= \frac{1}{M}(-C_f v + A_c(p_1 - p_2) + f) & & & \dot{x}_2 &= \frac{1}{M}(-C_f x_2 + A_c(x_3 - x_4) + f) \\
\dot{p}_1 &= \frac{\beta}{V_1}(ku - A_c v - C_l(p_1 - p_2)) & & \text{or} & \dot{x}_3 &= \frac{\beta}{V_1}(ku - A_c x_2 - C_l(x_3 - x_4)) \\
\dot{p}_2 &= \frac{\beta}{V_2}(-ku + A_c v + C_l(p_1 - p_2)) & & & \dot{x}_4 &= \frac{\beta}{V_2}(-ku + A_c x_2 + C_l(x_3 - x_4))
\end{aligned}
$$

This leads to:

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Solutions to Problems

Problem 2.6 (continued)

$$\Rightarrow \mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\dfrac{C_f}{M} & \dfrac{A_c}{M} & -\dfrac{A_c}{M} \\ 0 & -\dfrac{A_c\beta}{V_1} & -\dfrac{C_l\beta}{V_1} & \dfrac{C_l\beta}{V_1} \\ 0 & \dfrac{A_c\beta}{V_2} & \dfrac{C_l\beta}{V_2} & -\dfrac{C_l\beta}{V_2} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ \dfrac{k\beta}{V_1} \\ -\dfrac{k\beta}{V_2} \end{bmatrix}$$

$$\mathbf{B}_v = \begin{bmatrix} 0 \\ \dfrac{1}{M} \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{C} = [1 \;\; 0 \;\; 0 \;\; 0]$$

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