LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 7.1

a. A reasonable measurement model is one with additive white noise

$$y = \omega(t) + \omega_2(t)$$

b. A Kalman filter can be constructed for the D.C. motor by appending the state equation with an innovation term using the measurement of the shaft speed

$$\dot{\hat{\omega}}(t) = -\alpha\hat{\omega}(t) + \beta V_a(t) + l(\omega_m - \hat{\omega})$$

where ω_m is the measurement.

In order to calculate the Kalman gain, *l*, the Riccati equation for the system must be solved. This Riccati equation is that for the stationary case.

$$0 = \mathbf{AQ} + \mathbf{QA}^T + \mathbf{V}_1 - \mathbf{QC}^T\mathbf{V}_2^{-1}\mathbf{CQ}$$

Here this equastion becomes

$$0 = -\alpha q - q\alpha + V_1 - q\frac{1}{V_2}q$$

$$\Rightarrow q^2 + 2\alpha V_2 q - V_1 V_2 = 0$$

where the bars indicate that the steady state solution is being found. The solution is:

$$q = \frac{-2\alpha V_2 \pm \sqrt{4\alpha^2 V_2{}^2 + 4V_1V_2}}{2}$$

$$= \alpha V_2\left(\sqrt{1 + \frac{V_1}{\alpha^2 V_2}} - 1\right)$$

The corresponding Kalman gain is

$$l = QC^T V_2^{-1} = \alpha\left(\sqrt{1 + \frac{V_1}{\alpha^2 V_2}} - 1\right)$$

LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 7.1

c. If the signal to noise ratio is poor then $\frac{V_1}{V_2} \ll 1$ and $l \cong 0$. There is no feedback in the observer and thus the observer is a primative model of the system itself. Such a construct is called a feedforward or degenerate observer.

d. If the signal to noise ratio is good then $\frac{V_1}{V_2} \gg 1$ and l will be large. The filter then uses the measurement strongly.

❐