LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 3.15

a. The differential equation in the problem text can be written:

$$\dddot{y} + \ddot{y} - 4\dot{y} - 4y = \dot{u} - 2u$$

After Laplace tranformation this equation is :

$$s^3 y + s^2 y - 4sy - 4y = su - 2u$$
$$\Rightarrow G(s) = \frac{y(s)}{u(s)} = \frac{s-2}{s^3 + s^2 - 4s - 4}$$

The controller canonical form is:

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & 4 & -1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \qquad (1)$$
$$\mathbf{y} = [-2 \quad 1 \quad 0]\mathbf{x}$$

b. The eigenvalues may be found from: $\det(\lambda \mathbf{I} - \mathbf{A}) = 0 \Rightarrow \lambda = \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$

One eigenvalue in the right half plane: the system is unstable.

$\mathbf{M}_s = [\mathbf{B} \quad \mathbf{AB} \quad \mathbf{A}^2\mathbf{B}]$, $\quad det(\mathbf{M}_s) \neq 0$ because the controller canonical form is always controllable.

$$\mathbf{M}_o = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 1 \\ 4 & 4 & -3 \end{bmatrix}, \quad det(\mathbf{M}_o) = 0$$

$\Rightarrow$ The system is not observable and therefore it is not minimal either.

Problem 3.15 (continued)

c. The transfer function is:

$$G(s) = \frac{s-2}{(s-2)(s+1)(s+2)} = \frac{1}{s^2+3s+2}$$

and the the controller canonical form is:

$$\dot{\mathbf{z}} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}\mathbf{z} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u, \qquad \mathbf{y} = \begin{bmatrix} 1 & 0 \end{bmatrix}\mathbf{z} \qquad (2)$$

There is no cancellation of poles/zeros in the second order transfer function, so one can be sure that the second order state space model is controllable and observable and therefore also minimal.

d. (1) is controllable and therefore also stabilizable. (2) has two stable eigenvalues

$$\lambda = \begin{bmatrix} -1 \\ -2 \end{bmatrix}.$$

The unstable eigenvalue $\lambda = 2$ in (1) must therefore belong to the non-observable sub-space.

$\Rightarrow$ (1) is not detectable.

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