LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 5.11

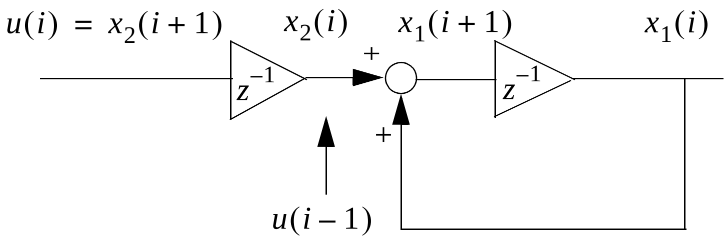
a. The state equation of the stock resupply problem can be found by letting $x(i)$ = stock supply on day i and $x_2(i)$ = delayed order on day *i*. This means that

$$x_1(i) = x(i), \qquad x_2(i) = u(i-1)$$
$$x_1(i+1) = x_1(i) + x_2(i)$$
$$\Rightarrow \; x_2(i+1) = u(i)$$

$$\Rightarrow \begin{pmatrix} x_1(i+1) \\ x_2(i+1) \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1(i) \\ x_2(i) \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(i)$$

$$y(i) = (1 \quad 0)\begin{pmatrix} x_1(i) \\ x_2(i) \end{pmatrix}$$

Block diagram:



b. What is desired is a regulator for which can achieve a steady state output given a step in put in at most 2 days. The system is of second order. This implies that a deadbeat regulator is to be designed.

To design such a regulator let

$$|\lambda\mathbf{I} - \mathbf{F} + \mathbf{GK}| = \lambda^2$$

this implies that

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$$\Rightarrow \quad |\lambda\mathbf{I} - \mathbf{F} + \mathbf{GK}| = \left|\begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} - \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}(k_1 k_2)\right|$$

$$= (\lambda - 1)(\lambda + k_2) + k_1$$

$$= \lambda^2 + (k_2 - 1)\lambda - k_2 + k_1$$

$$= \lambda^2$$

$$\Rightarrow \quad k_2 - 1 = 0, \quad -k_2 + k_1 = 0 \quad \Rightarrow \quad k_1 = k_2 = 1$$

c From the specification what is required is a steady state LQR regulator.The Riccato equation which must be solved is

$$\mathbf{P}_d = \mathbf{R}_{1d} + \mathbf{F}^T\mathbf{P}_d[\mathbf{I} + \mathbf{G}\mathbf{R}_2^{-1}\mathbf{G}\mathbf{P}]^{-1}\mathbf{F}$$

where

$$\mathbf{K} = (k_1 \quad k_2), \qquad \mathbf{P} = \begin{pmatrix} p_{11} & p_{12} \\ p_{11} & p_{22} \end{pmatrix}$$

$$R_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad R_2 = \rho$$

One finds that

$$\mathbf{K}_d = [\mathbf{R}_{2d} + \mathbf{G}^T\mathbf{P}_d\mathbf{G}]^{-1}\mathbf{G}\mathbf{P}_d\mathbf{F} = \frac{1}{\rho + p_{22}}(\, p_{12} \quad p_{22}\,)$$

and

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$$p_{11} = p_{11} + 1 - \frac{\rho_{12}}{\rho + p_{22}} p_{12}$$

$$p_{12} = p_{11} + 1 - \frac{\rho_{12}}{\rho + p_{22}} p_{12}$$

$$p_{22} = p_{11} + 1 - \frac{\rho_{12}}{\rho + p_{22}} p_{12}$$

$$\Rightarrow p_{11} = p_{12} = p_{22} = p$$

$$p^2 - p - \rho = 0 \quad \Rightarrow \quad p = \frac{1 \pm \sqrt{1 + 4\rho}}{2}$$

$$\Rightarrow \quad k_1 = k_2 = \frac{1 + \sqrt{1 + 4\rho}}{2\rho + 1 + \sqrt{1 + 4\rho}}$$

d. Note:

$$\rho \gg 1 \Rightarrow k = \frac{1}{1 + \sqrt{\rho}}$$

$$\rho \ll 1 \quad \Rightarrow \quad k = 1 - \rho \quad \Rightarrow \quad \rho \to 0$$

gives a deadbeat regulator.

With this type of regulator the input level will be very high as the state power level is not weighted (punished) at all.

❐