ØRSTED • DTU AUTOMATION

Linear Systems Control

Solutions to problems

Problem 3.10

$$x_{kf1} = \begin{bmatrix} 0 & 1 \\ -1 & \frac{5}{2} \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_k \qquad y_k = [-2 \;\; 1] x_k$$

Eigenvalues:

$$\begin{bmatrix} z & -1 \\ 1 & z - \frac{5}{2} \end{bmatrix} = z^2 - \frac{5}{2}z + 1 = 0 \Rightarrow z = \begin{cases} 2 \\ \frac{1}{2} \end{cases}$$

Natural modes:

$$m_i = \begin{cases} 2^k \\ \frac{1}{2}k \end{cases}$$

The system has an eigenvalue outside the unit circle and it is not asymptically internally stable.

Transfer furnction

$$\begin{aligned} H(z) &= C(zI - A)^{-1}B \\ &= [-2 \;\; 1] \begin{bmatrix} z & -1 \\ 1 & z - \frac{5}{2} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{z^2 - \frac{5}{2}z + 1} [-2 \;\; 1] \begin{bmatrix} z - \frac{5}{2} & 1 \\ -1 & z \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ &= \frac{1}{z - \frac{1}{2}} \end{aligned}$$

Only one pole (within the unit circle).

$\Rightarrow$ The system is BIBO-stable (externally stable)

The unstable pole/eigenvalue $z = 2$ is cancelled in the transfer function.

NOTE: BIBO-stability is not the same as asymptotic internal stability.