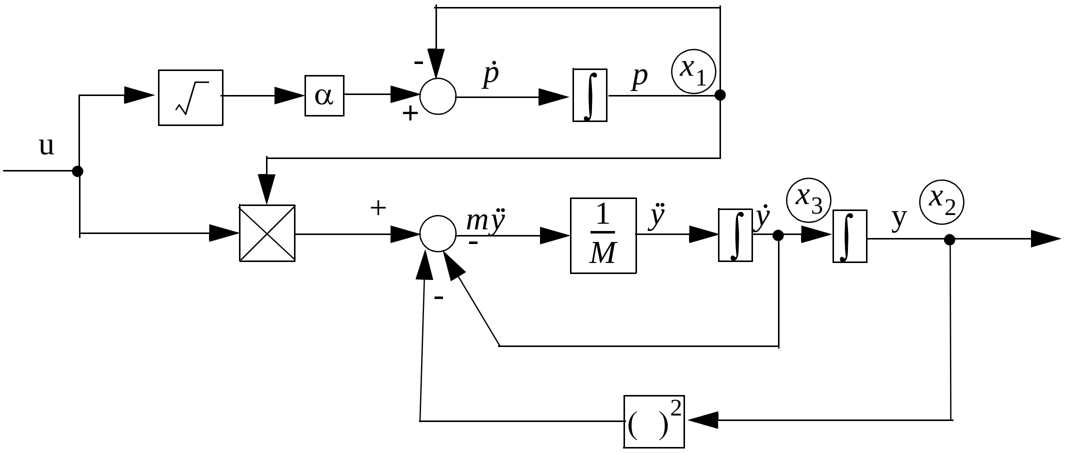
LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 2.8

a. A block diagram for the system can be drawn from the given equations.



b. The state equations for the nonlinear system can be derived as follows by choosing the state vector: $\mathbf{x} = [p \quad \dot{y} \quad y]^T = [x_1 \quad x_2 \quad x_3]^T$

$$\begin{aligned}
\dot{x}_1 &= -x_1 + \alpha\sqrt{u} \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= \frac{1}{m}(-x_2^2 - x_3 + x_1 u) \\
y &= x_2
\end{aligned}$$

The stationary stated can be found by setting $\dot{\mathbf{x}} = 0$:

$$\left.\begin{aligned}
0 &= -x_{10} + \alpha\sqrt{u_0} \\
0 &= x_{30} \\
0 &= -x_{20}^2 - x_{30} + x_{10}u_0
\end{aligned}\right\} \Rightarrow \left\{\begin{aligned}
x_{30} &= 0 \\
x_{10} &= a\sqrt{u_0} \\
x_{20} &= \pm\sqrt{\alpha u_0^{\frac{3}{2}}}
\end{aligned}\right.$$

c. Now defining:

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$$\begin{aligned} x_1 &= x_{10} + \Delta x_1 & \qquad u &= u_0 + \Delta u \\ x_2 &= x_{20} + \Delta x_2 & \qquad y &= y_0 + \Delta y \\ x_3 &= x_{30} + \Delta x_3 \end{aligned}$$

the linearized state space model is:

$$\Delta\dot{\mathbf{x}} = \mathbf{A}\Delta\mathbf{x} + \mathbf{B}\Delta\mathbf{u}, \quad \Delta\mathbf{y} = [0 \quad 1 \quad 0]\Delta\mathbf{x}$$

where:

$$\mathbf{A} = \begin{bmatrix} \frac{\partial \dot{x}_1}{\partial x_1} & \frac{\partial \dot{x}_1}{\partial x_2} & \frac{\partial \dot{x}_1}{\partial x_3} \\ \frac{\partial \dot{x}_2}{\partial x_1} & \frac{\partial \dot{x}_2}{\partial x_2} & \frac{\partial \dot{x}_2}{\partial x_3} \\ \frac{\partial \dot{x}_3}{\partial x_1} & \frac{\partial \dot{x}_3}{\partial x_2} & \frac{\partial \dot{x}_3}{\partial x_3} \end{bmatrix}_0, \quad \mathbf{B} = \begin{bmatrix} \frac{\partial \dot{x}_1}{\partial u} \\ \frac{\partial \dot{x}_2}{\partial u} \\ \frac{\partial \dot{x}_3}{\partial u} \end{bmatrix}_0$$

Choosing $x_{20} = \sqrt{\alpha u_0^{\frac{3}{2}}}$ one obtains:

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ \frac{u_0}{m} & -\frac{2}{m}\sqrt{\alpha u_0^{\frac{3}{2}}} & -\frac{1}{m} \end{bmatrix}, B = \begin{bmatrix} \frac{\alpha}{2\sqrt{u_0}} \\ 0 \\ \frac{\alpha}{m}\sqrt{u_0} \end{bmatrix}$$

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