LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 6.1

Experiment with a coin: the coin is flipped two times.

In order to assign numbers to the sample space, assign: T (Tails) = 0 , H (Heads) = 1.

Sample	HH	HT	TH	TT
x	2	1	1	0

a. Construction of the probability density function:

$$Pr(HH) = \frac{1}{4}, \quad Pr(HT) = Pr(TH) = \frac{1}{4}, \quad Pr(TT) = \frac{1}{4}$$

$$Pr(X = 0) = Pr(TT) = \frac{1}{4}$$

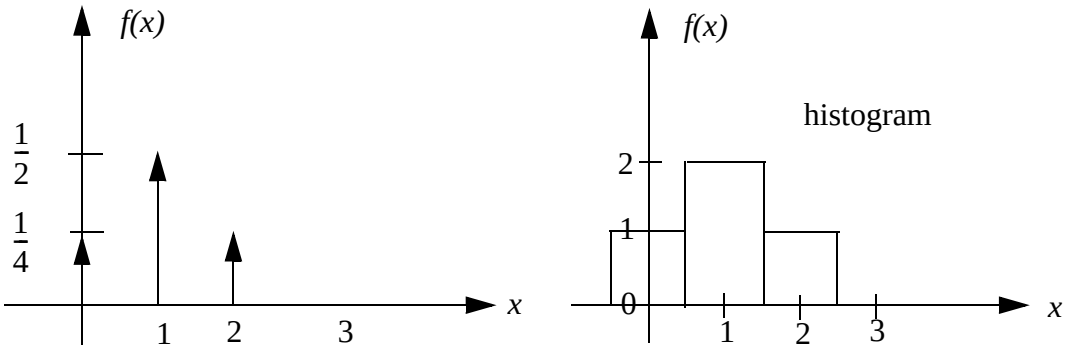
$$Pr(X = 1) = Pr(HT \ v \ TH) = Pr(HT) + Pr(TH)$$

$$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$Pr(X = 2) = Pr(HH) = \frac{1}{4}$$

The probability density function table is sketched on the figures below:

x	0	1	2
$f(x)$	1/4	1/2	1/4



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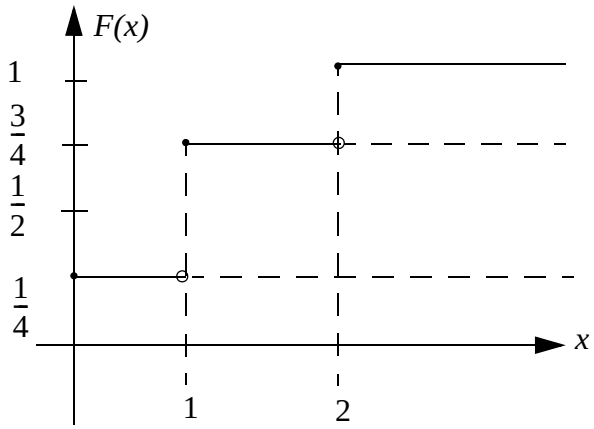
b. Probability distribution function is sketched on the figure below:

$$F(x) = \begin{cases} 0, & -\infty < x < 0 \\ Pr(X = 0)U(x) = \frac{1}{4}, & 0 \le x < 1 \\ Pr(X = 0)U(x) + Pr(X = 1)U(X-1) & \\ \quad = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}, & 1 \le x < 2 \\ Pr(X = 0)U(x) + Pr(X = 1)U(X-1) & \\ + Pr(X = 2)U(X-2) = \frac{1}{4} + \frac{1}{2} + \frac{1}{4} = 1, & \\ & 2 \le x < \infty \end{cases}$$



c. The probability of finding X during the experiment:

$$E\{X\} = x_1 Pr(X = x_1) + \ldots x_n Pr(X = x_n)$$

$$= \sum_{j=1}^{n} x_j \, Pr(X = x_1)$$

$$= 0 \cdot \frac{1}{4} + 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4} = 1$$

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d. The probability of finding X^2:

$$E\{X^2\} = x_1^2 Pr(x = x_1) + \ldots + x_n^2 Pr(x = x_0)$$
$$= 0 \cdot \frac{1}{4} + 1^2 \cdot \frac{1}{2} + 2^2 \ \frac{1}{4} = 1 \cdot \frac{1}{2}$$

e. Given the probability of finding X and X^2, the standard deviation is easily found:

$$\sigma_x^2 = \{E(X-m)^2\} = E\{X^2\} - [E\{X\}]^2$$
$$= \left(\frac{3}{2}\right)^2 - 1^2 = \frac{5}{4}$$
$$\sigma_x = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$$

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