LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 2.7

Dividing numerator and denominator through by 4 makes polynomial division easier:

$$H(z) = \frac{0.75\,z^3 + 0.5\,z^2 + 0.25\,z + 1.25}{z^3 + z^2 + 0.5\,z + 2}$$

Now using polynomial division:

$$(0.75\,z^3 + 0.5\,z^2 + 0.25\,z + 1.25) \div (z^3 + z^2 + 0.5\,z + 2) = 0.75$$
$$-0.75\,z^3 - 0.75\,z^2 - 0.375\,z - 1.5$$
$$\text{–––––––––––––––––––––––––––}$$
$$-0.25\,z^2 - 0.125\,z - 0.25$$

$$\Rightarrow H(z) = 0.75 + \frac{-0.25\,z^2 - 1.25\,z - 0.25}{z^3 + z^2 + 0.5\,z + 2}$$

Companion form 1 is then:

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -0.5 & -1 \end{bmatrix}, \quad \mathbf{B}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\mathbf{C}_1 = [-0.25 \quad -1.25 \quad -0.25], \quad \mathbf{D}_1 = [0.75]$$

and companion form 2:

$$\mathbf{A}_2 = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 0 & -0.5 \\ 0 & 1 & -1 \end{bmatrix}, \quad \mathbf{B}_2 = \begin{bmatrix} -0.25 \\ -0.125 \\ -0.25 \end{bmatrix}$$

$$\mathbf{C}_2 = [0 \quad 0 \quad 1], \quad \mathbf{D}_2 = [0.75]$$

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