LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 3.9

1. The first matrix is:

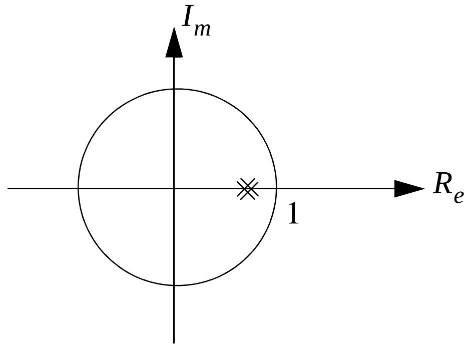
$$\mathbf{F} = \begin{bmatrix} \frac{1}{2} & \frac{1}{8} \\ -\frac{1}{2} & 1 \end{bmatrix}$$

The eigenvalues are determined from

$$\det(\lambda \mathbf{I} - \mathbf{F}) = 0$$

$$\Rightarrow \det \begin{bmatrix} \lambda - \frac{1}{2} & -\frac{1}{8} \\ \frac{1}{2} & \lambda - 1 \end{bmatrix} = \left(\lambda - \frac{1}{2}\right)(\lambda - 1) + \frac{1}{16} = \lambda^2 + \frac{3}{2}\lambda + \frac{9}{16}$$

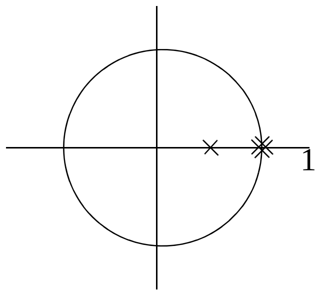
$$= 0 \quad \text{for} \quad \lambda = \begin{cases} \frac{3}{4} \\ \frac{3}{4} \end{cases}$$



Both of these poles are within the unit circle: this $\Rightarrow$ the system is symptotically stable.

2. Matlab produces the following result for the matrix:

$$\lambda = \begin{cases} 1 \\ 1 \\ 0,5 \end{cases}$$

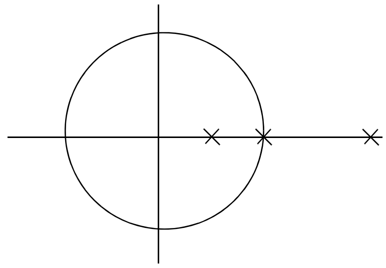


Problem 3.9 (continued)

Double eigenvalue on the unit circle: this $\Rightarrow$ the system is unstable .

3. Matlab gives the follow pole placement:

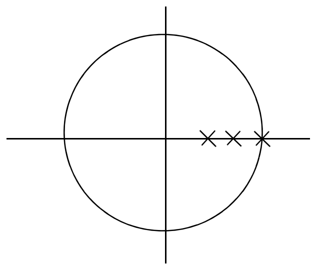
$$\lambda = \begin{cases} 0,5 \\ 1 \\ 2 \end{cases}$$



One eigenvalue outside the unit circle: this $\Rightarrow$ The system is unstable .

4. Matlab gives the follow pole placement:

$$\lambda = \begin{cases} 0,5 \\ 0,75 \\ 1 \end{cases}$$



One eigenvalue is on the unit circle and the rest within the circle:

this $\Rightarrow$ the system is Lyapunov stable .

❐