LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 4.2

a. Here MATLAB can be used directly.

The controllability matrix is:

$$\mathbf{M}_c = \begin{bmatrix} 4.2 & 0.1 \\ 2 & 0.05 \end{bmatrix}, \quad det(\mathbf{M}_c) = 0.005.$$

With MATLAB's acker-function (Ackermann's formula) one can find $\mathbf{K}$:

```
>> ev = [–sqrt(2) + j* sqrt(2)    –sqrt(2)-j*sqrt(2)]
>> K = acker (A, B, ev)
```

The result is:

$$\mathbf{K} = [-1571.7 \quad 3223.4].$$

The gains are 300-500 times larger than in problem 4.1.

The plots of y(t) and u(t) are shown below.

The strange appearance of $y(t)$ is due to the fact that the system in this case has a zero in the right half plane ($s = 0.0238$) and therefore the sytem is not a minimum-phase. This cannot be changed by the controller. See the remark in the notes on page 230. $u(t)$ is identical with the control signal in problem 4.1 However, if one plots the to components which constitute $u(t)$:

$$u = -\mathbf{K}\mathbf{x} + r = -\begin{bmatrix} k_1 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + r =$$

$$-k_1 x_1 - k_2 x_2 + r = -u_a - u_b + r$$

The result is seen on the last page.

Here it is clear what the large gains mean. Control signals with such large amplitudes may be very difficult to realize.

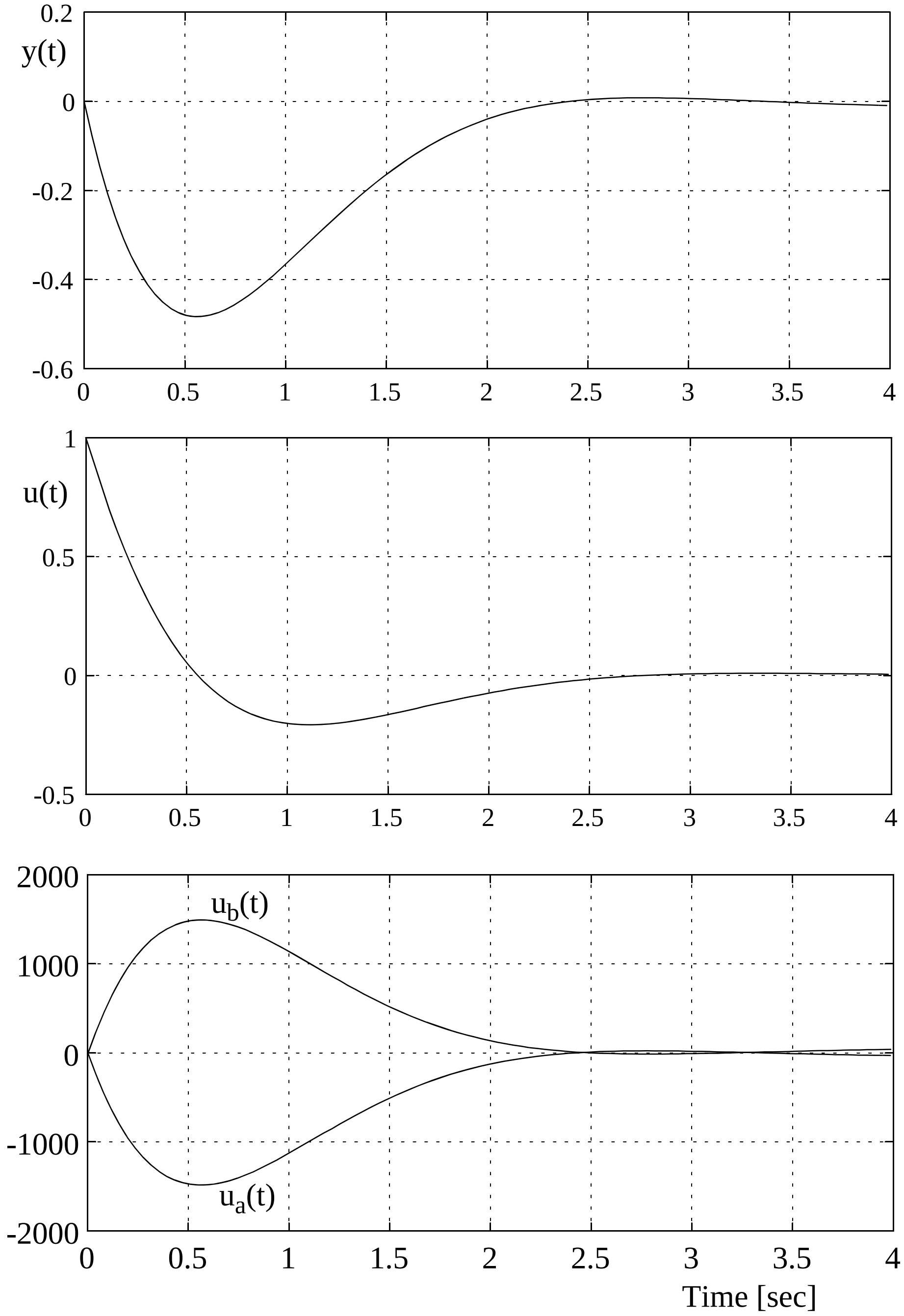
LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 4.2

b. The problem with this system is that it is close to being non-controllable. ($\mathbf{M}_c$ is almost singular).

In order to live up to the demands on the placement of the eigenvalues, the controller must generate control signals of an unrealistic size.


y(t)
0.2
0
-0.2
-0.4
-0.6
0 0.5 1 1.5 2 2.5 3 3.5 4
u(t)
1
0.5
0
-0.5
0 0.5 1 1.5 2 2.5 3 3.5 4
$u_b(t)$
$u_a(t)$
2000
1000
0
-1000
-2000
0 0.5 1 1.5 2 2.5 3 3.5 4
Time [sec]

❐