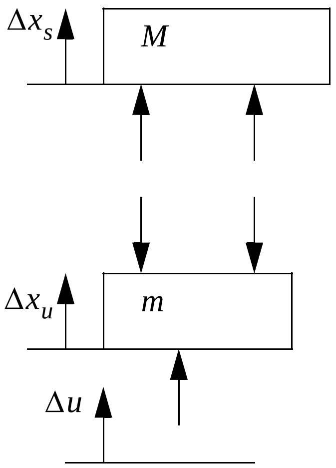
LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 2.4



State convention:
$\Delta u > \Delta x_u > \Delta x_s$
$\Delta\dot{x}_u > \Delta\dot{x}_s$

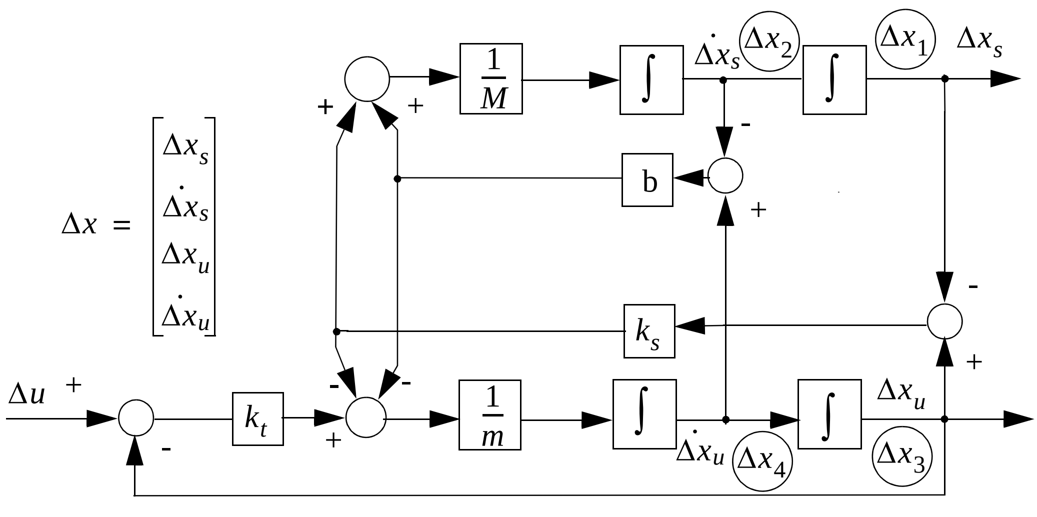
a. Using the sign convention defined on the figure above, Newton's second law applied to M gives:

$$M\Delta\ddot{x}_s = k_s(\Delta x_u - \Delta x_s) + b(\Delta\dot{x}_u - \Delta\dot{x}_s)$$

Newton's second law applied to m gives:

$$m\Delta\ddot{x}_u = k_t(\Delta u - \Delta x_u) - k_s(\Delta x_u - \Delta x_s) - b(\Delta\dot{x}_u - \Delta\dot{x}_s)$$

b. The block diagram of the overall system is then as below:



LINEAR SYSTEMS CONTROL

Solutions to Problems

Problem 2.4

c. The linearized state equations are then:

$$\begin{aligned}
\Delta\dot{x}_1 &= x_2 \\
\Delta\dot{x}_2 &= \frac{1}{M}(k_s(-\Delta x_1 + \Delta x_3) + b(-\Delta x_2 + \Delta x_4)) \\
\Delta\dot{x}_3 &= x_4 \\
\Delta\dot{x}_4 &= \frac{1}{m}(k_t(-\Delta x_3 + \Delta u) - k_s(\Delta x_3 - \Delta x_1) - b(\Delta x_4 - \Delta x_2))
\end{aligned}$$

or in matrix form:

$$\Delta\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_s}{M} & -\frac{b}{M} & \frac{k_s}{M} & \frac{b}{M} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m} & \frac{b}{m} & -\frac{k_t + k_s}{m} & -\frac{b}{m} \end{bmatrix} \Delta\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_t}{m} \end{bmatrix} \Delta u = \mathbf{A}\Delta\mathbf{x} + \mathbf{B}\Delta u$$

with the output equations:

$$\Delta\mathbf{y} = \begin{bmatrix} \Delta x_s \\ \Delta x_u \\ \Delta f_t \\ \Delta\ddot{x}_s \end{bmatrix} = \begin{bmatrix} \Delta x_1 \\ \Delta x_3 \\ k_t(\Delta u - \Delta x_3) \\ -\frac{k_s}{M}\Delta x_1 - \frac{b}{M}\Delta x_2 + \frac{k_s}{M}\Delta x_3 + \frac{b}{M}\Delta x_4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -k_t & 0 \\ -\frac{k_s}{m} & -\frac{b}{M} & \frac{k_s}{M} & \frac{b}{m} \end{bmatrix} \Delta\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ k_t \\ 0 \end{bmatrix} \Delta u = \mathbf{C}\Delta\mathbf{x} + \mathbf{D}\Delta u$$

❒