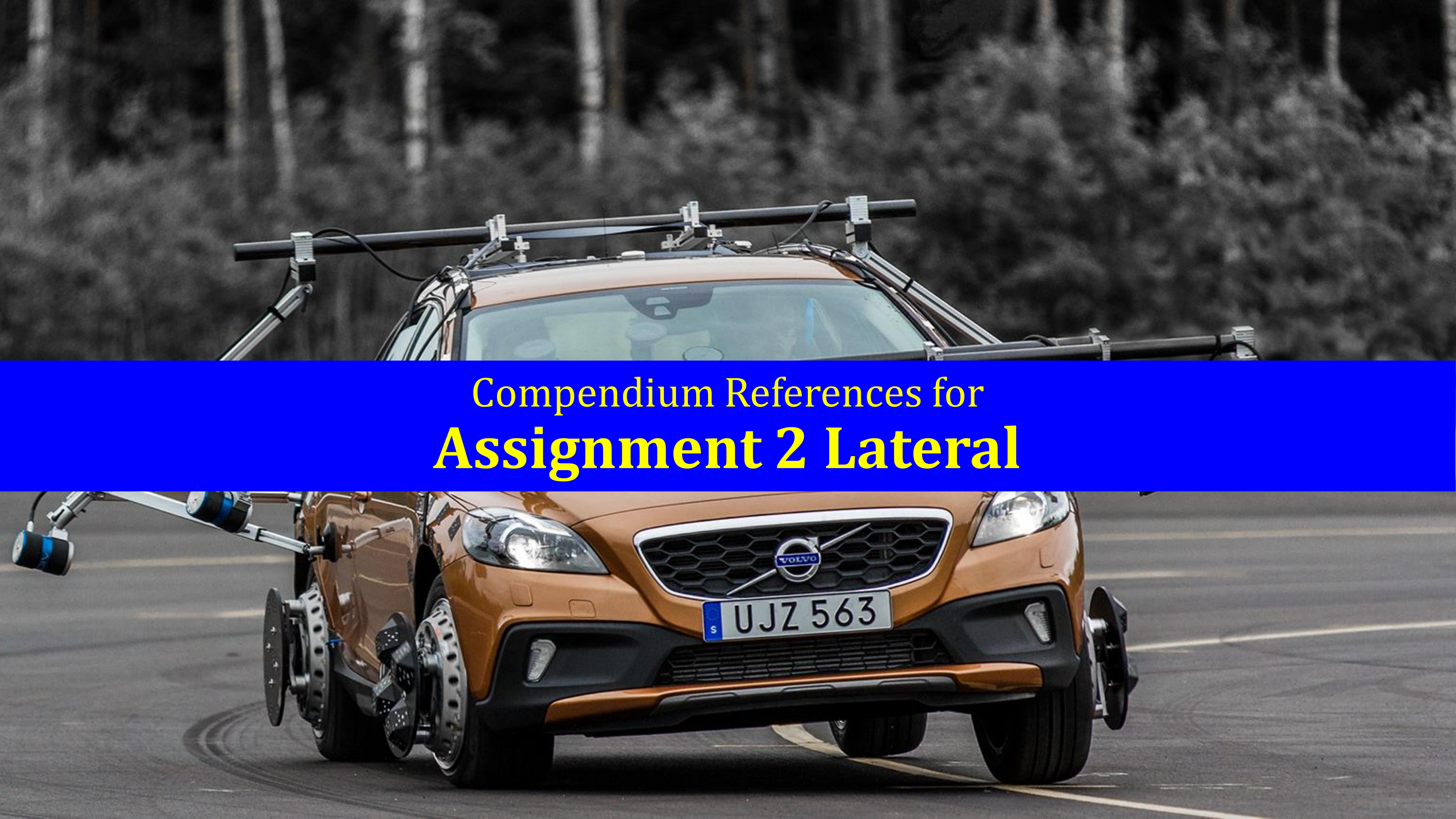
Compendium References for

Assignment 2 Lateral



Design tasks

Administration (points, hand-in, etc): See Course Memo.

...and, these will take well care of you:

- Assistants for Design task 1, Longitudinal, 10 p:
  - **Lead:** Yansong.Huang@chalmers.se +46 73 860 47 19 ...[YH]
  - Support: Fredrik.Bruzelius@chalmers.se, +46 73 1431 365 ...[FB]
- Assistants for Design task 2, Lateral, 25 p:
  - **Lead**: Sachin.Janardhanan@chalmers.se +46 76 553 8053 ...[SJ]
  - Support: Yansong.Huang@chalmers.se +46 73 860 47 19 ...[YH]
  - For Simulator Lab at CASTER:
    - Admin: Sam.Azadi@casterchalmers.se ...[SA]
    - Instructors: 4-6 personer ...[instr]
- Assistants for Design task 3, Vertical, 15 p:
  - **Lead:** Fredrik.Bruzelius@chalmers.se, +46 73 1431 365 ...[FB]
  - Support: Sachin.Janardhanan@chalmers.se +46 76 553 8053 ...[SJ]

Sachin will soon do an introduction of Design task 2,
but Bengt will first point out useful parts in Compendium ...

Design Tasks

Learning objectives	Reading
Design Task 1: Longitudinal • Functions: Acceleration (uphill, various road friction) • What to engineer: Distribution of propulsion between front and rear axle (FWD/RWD) • Method: Simulation • Tools: Matlab Symbolic toolbox, "Home-coded" time-integration (for conceptual understanding of simulation)	• Figure 2-21, 2.2.3.4.1 Magic Formula Tyre Model, Eq [2.1] • 1.5.4.1.1 General Mathematics Tools • 1.5.1.1.3 Physical Modelling • 1.5.1.1.4 Mathematical Modelling, 1.5.2.1 Free-Body Diagrams • 1.5.1.1.5 Explicit Form Modelling, 1.5.1.1.6 Computation • Figure 3-24, Eq [3.13] • 3.5.2.5 Traction Control, TC *
Design Task 2: Lateral • Functions: Yaw balance in steady state high speed, Step steer response, Brake in curve • What to engineer: Distribution of roll-stiffness and brake force between front and rear axle • Method: Simulation, Driving experience, model integration and log data analysis • Tools: Simulink (for learning one commonly used tool for simulation), Motion platform driving simulator (for driving experience and log data analysis)	• Figure 4-11 • Figure 4-15 • Figure 4-19 • 4.3.6 Steady State Cornering Gains * • Eq [1.1][4.17] • Eq [4.18] • Figure 4-47 • 1.5.1.1.4.5 Affine and Linear form (ABCD form) • Figure 4-38, Eq [4.39] • Eq [2.47]
Design Task 3: Vertical • Functions: Comfort for stationary vibrations, Road grip due to stationary varying vertical tyre force • What to engineer: Wheel suspension stiffness and damping • Method: Frequency analysis • Tools: Matlab (for learning one commonly used tool for matrix computations)	• Figure 5-1, Figure 5-12, [5.44] • Figure 5-3, Eq [5.4], Eq [5.13], 4.4.3.1.1 Solution with Fourier Transform • Eq [5.45] • Figure 5-5, Figure 5-20

”Common thread” between the 3 Design tasks

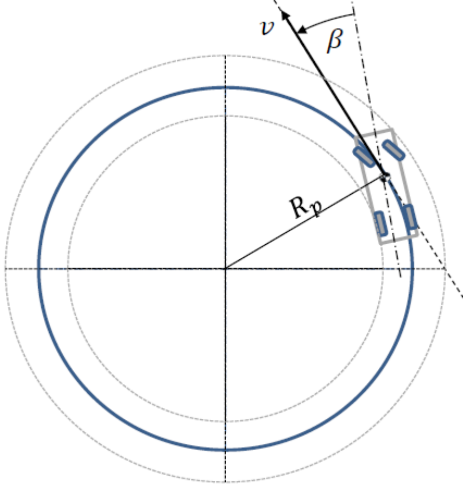

Learning objectives

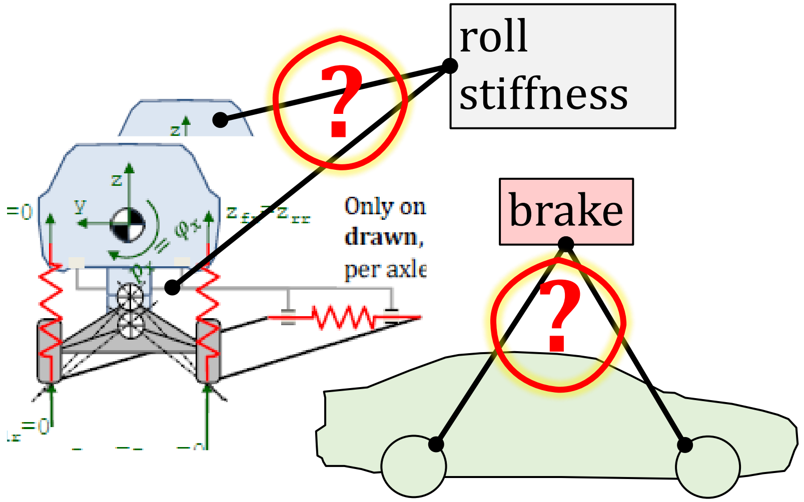
Design Task 1: Longitu
- Functions: Accelerat
- What to engineer: Di
front and rear axle (
- simulation
- Tools: Matl b Symbo
integration for con
simulation)

Design Task 2: Lateral
- Functions: Ya balan
Step steer response
- What to engineer: Di
brake force between
- Method: Simulation,
integration and log d
- Tools: Simulink (for
for simulation), Moti
(for driving experien

Design Task 3: Vertica
- Functions: Comfort f
grip due to stationar
- What to engineer: W
damping
- Method: Frequency analysis
- Tools: Matlab (for learning one commonly used tool
for matrix computations)

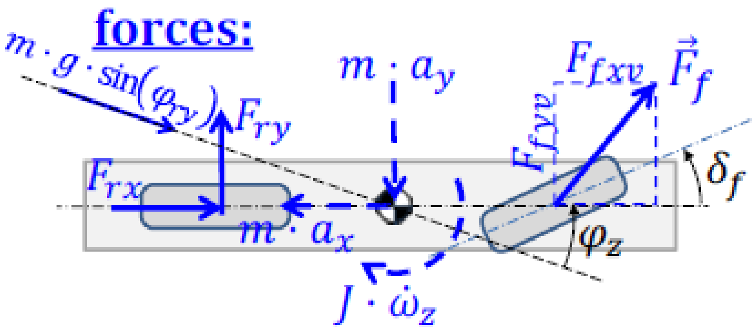
Long	Acceleration	Prop	Simulation	Matlab
Lat	Yaw balance	Susp & Brk	Simulation	Simulink & DrivSim
Vert	Comfort & Road grip	Susp	Freq Analysis	Matlab





Design Task 2: Lateral

- Functions: Yaw balance in steady state high speed, Step steer response, Brake in curve
- What to engineer: Distribution of roll-stiffness and brake force between front and rear axle
- Method: Simulation, Driving experience, model integration and log data analysis
- Tools: Simulink (for learning one commonly used tool for simulation), Motion platform driving simulator (for driving experience and log data analysis)



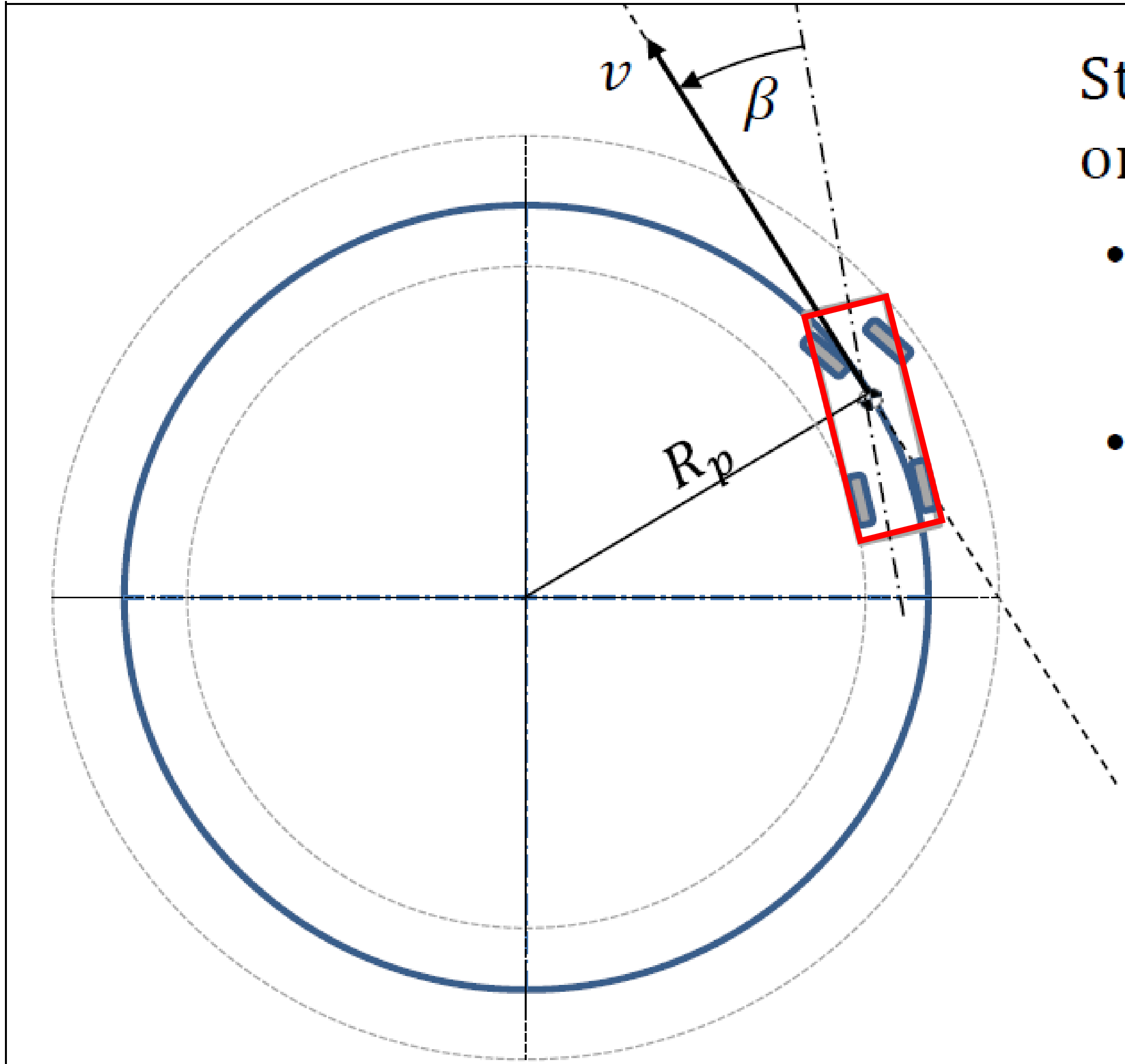
The following slides about these ”recommended readings”.

Quickly now, but more on lectures.

- Figure 4-11
- Figure 4-15
- Figure 4-19
- 4.3.6 Steady State Cornering Gains *
- Eq [1.1][4.17]
- Eq [4.18]
- Figure 4-47
- 1.5.1.1.4.5 Affine and Linear form (ABCD form)
- Figure 4-38, Eq [4.39]
- Eq [2.47]

Note: The figure and equation numbers on the following slides can be different from 2021 year’s compendium. Sorry

Task 1, Steady State Cornering at High Speed



Steady state cornering can be defined by either 3 or 2 quantities, depending on assumptions:

- For a **general vehicle** it is fully defined only by all **3 quantities**, e.g. $[v_x, v_y, \omega_z]$ or $[v, \beta, R_p]$.
- For a **certain vehicle** it is often considered as fully defined by **2 inputs from driver**: accelerator pedal and steering wheel. (Then, one have neglected other inputs from driver, such as brake pedal and parking brake. One have also neglected other possible automatic actuation, such as all wheel drive distribution and ESC-braking one wheel.)

Figure 4-13: Steady state cornering. (β will be negative for larger v_x, i.e. vehicle will point inwards.)

Task 1, Steady state model

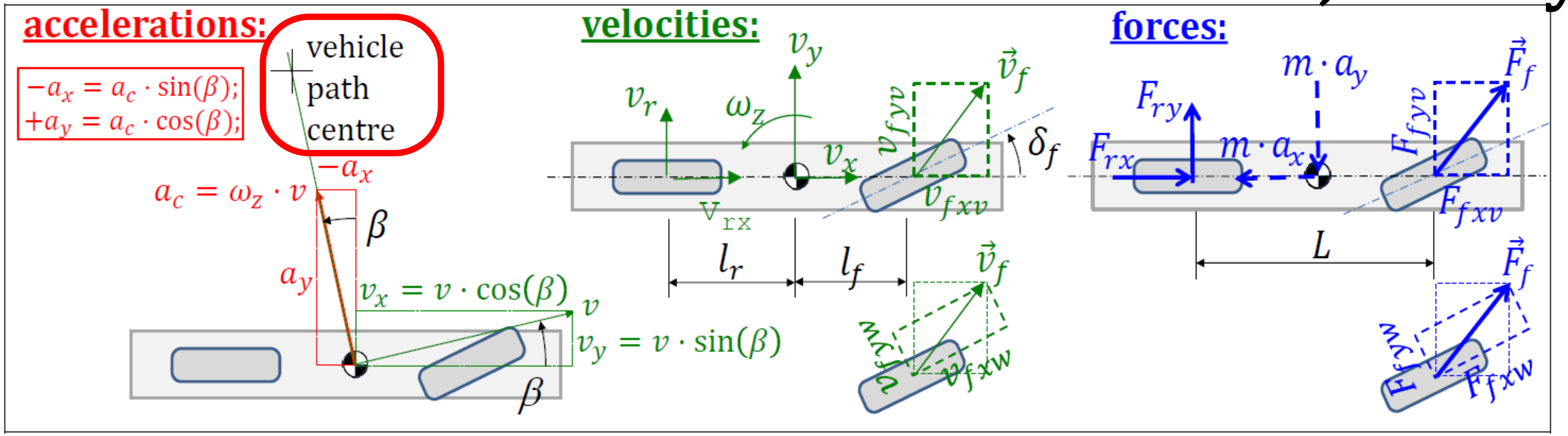
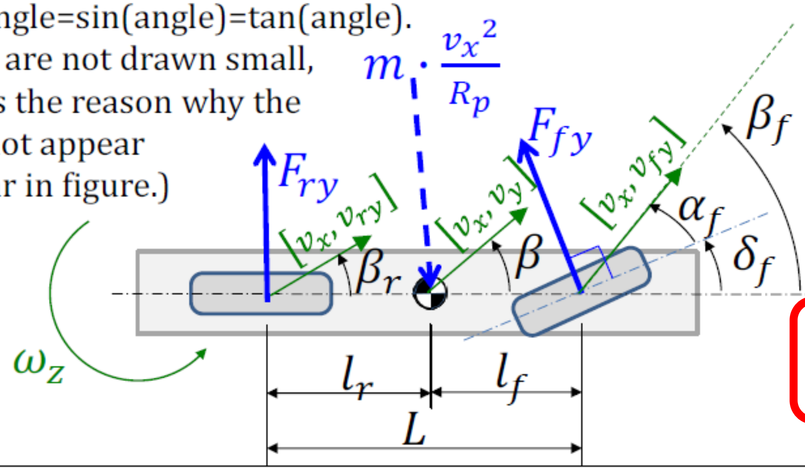


Figure 4-17: One-track model for Steady State Cornering. Dashed forces are "fictive forces".

$C_i = 2 \cdot C_{i,OneWhl,y}$ for $i = f, r$
(Assignment allows that we neglect other compliances than the tyres)

Physical model:

- Path radius >> the vehicle. Then, all forces (and centripetal acceleration) are approximately co-directed.
- Small tyre and vehicle side slip. Then, angle=sin(angle)=tan(angle). (Angles are not drawn small, which is the reason why the forces not appear co-linear in figure.)



Mathematical model:

Equilibrium: $m \cdot \frac{v_x^2}{R} \approx F_{fy} + F_{ry}$; $\quad 0 \approx F_{fy} \cdot l_f - F_{ry} \cdot l_r$;

Constitution: $F_{fy} = -C_f \cdot s_{fy}$; $\quad F_{ry} = -C_r \cdot s_{ry}$;

Compatibility:

$$s_{fy} \approx (v_y + l_f \cdot \omega_z)/v_x - \delta_f;$$
$$s_{ry} \approx (v_y - l_r \cdot \omega_z)/v_x;$$
$$\omega_z \approx v_x / R_p;$$

Eliminate $F_{fy}, F_{ry}, s_{fy}, s_r, \omega_z, v_y$ yields:

$$\boxed{\delta_f \approx \frac{L}{R_p} + K_u \cdot \frac{m \cdot v_x^2}{R_p}; \quad \textit{where } K_u = \frac{C_r \cdot l_r - C_f \cdot l_f}{C_f \cdot C_r \cdot L};}$$

Figure 4-21: Simpler derivation final step in Equation [4.9].

Task 1, Steady state "gains vs speed"

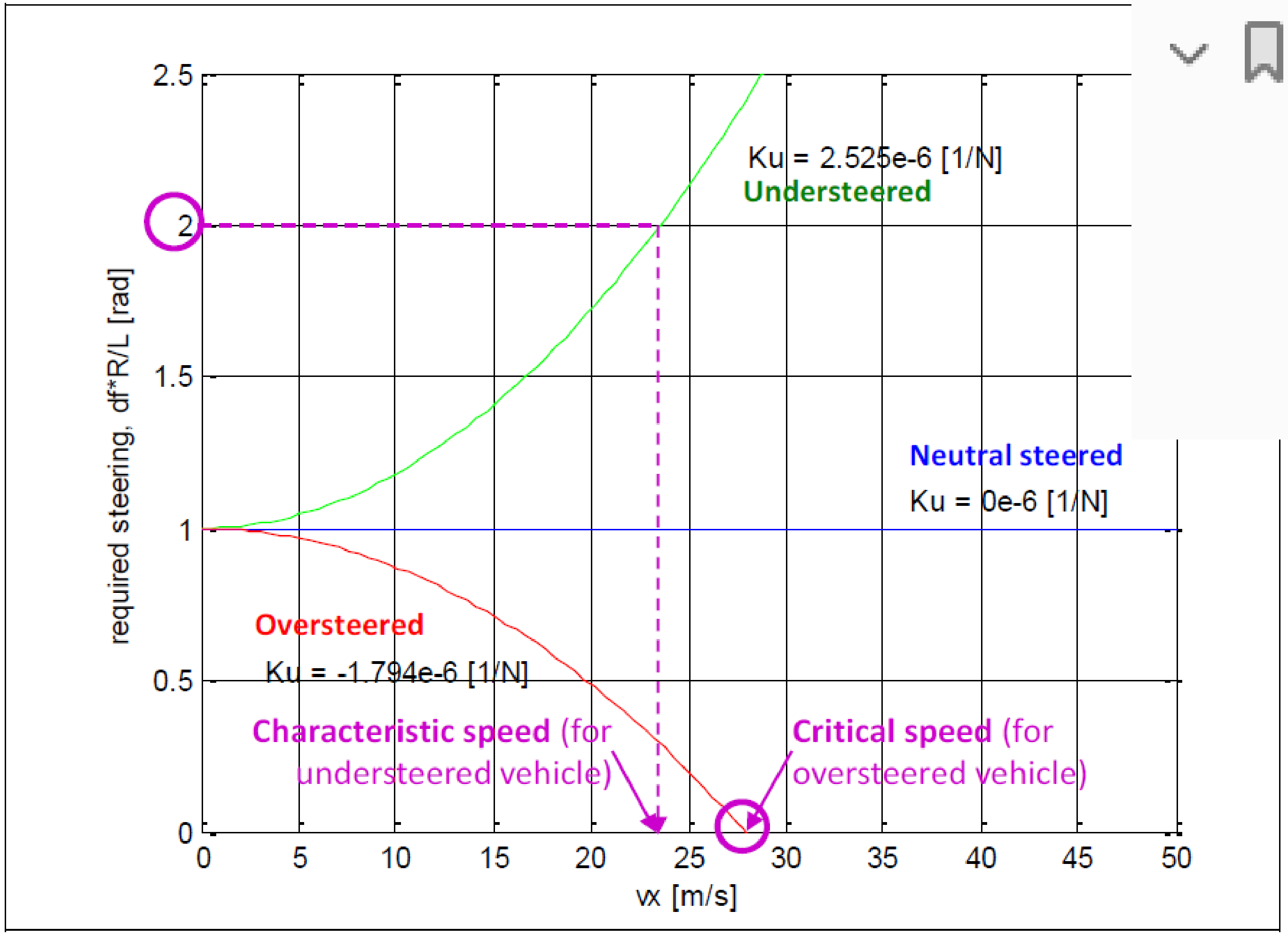
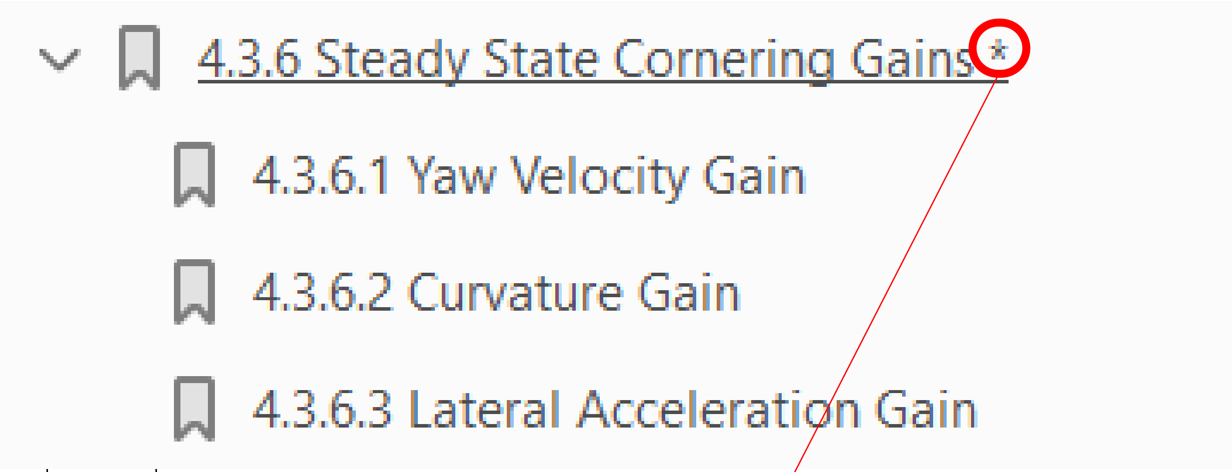


Figure 4-25: Normalized steer angle ($\delta_f \cdot R / L$) for Steady State Cornering



"*" marks "Function"

K_u denotes "understeering gradient"

$$\textbf{Normalized required steering angle} = \frac{\delta_f \cdot R_p}{L} = 1 + K_u \cdot \frac{m \cdot {v_x}^2}{L}; \qquad [4.17]$$

Task 1, Critical/Characteristics Speed

4.3.5 Critical and Characteristic Speed *

*Function definition: **Critical speed** is the speed above which the vehicle becomes unstable in the sense that the yaw velocity grows largely for a small disturbance in, e.g., steer angle.*

*Function definition: **Characteristic speed** is the speed at which the vehicle requires twice as high steer angle for a certain path radius as required at low speed (Figure 4-25). (Alternative definitions: The speed at which the yaw velocity gain reaches maximum (Figure 4-28). The speed at which the lateral acceleration gain per longitudinal speed reaches its highest value. (Figure 4-30).)*

C_f denotes front **axle** (lateral) slip stiffness, which is approximately the sum of both front **tyres**' slip stiffnesses.

$$\delta_f = \frac{L}{R_p} + K_u \cdot \frac{m \cdot v_{x,crit}{}^2}{R_p} = 0 \quad \Rightarrow \quad \boxed{v_{x,crit} = \sqrt{\frac{L}{-K_u \cdot m}}} = \sqrt{\frac{C_f \cdot C_r \cdot L^2}{(C_f \cdot l_f - C_r \cdot l_r) \cdot m}};$$

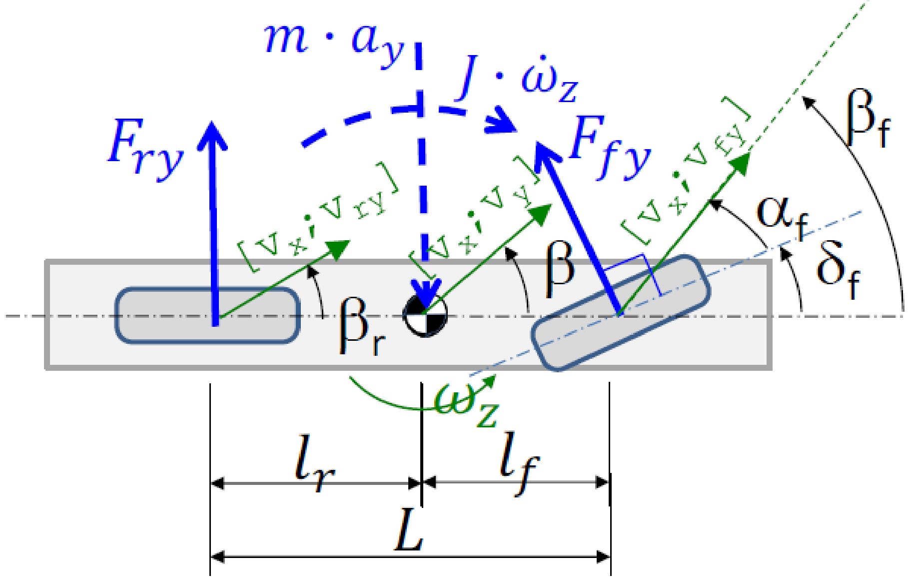
$$\delta_f = \frac{L}{R_p} + K_u \cdot \frac{m \cdot v_{x,char}{}^2}{R_p} = 2 \cdot \frac{L}{R_p} \quad \Rightarrow \quad \boxed{v_{x,char} = \sqrt{\frac{L}{K_u \cdot m}}} = \sqrt{\frac{C_f \cdot C_r \cdot L^2}{(C_r \cdot l_r - C_f \cdot l_f) \cdot m}};$$

[4.18]

Task 2, Transient Model

Physical model:

- Path radius >> the vehicle. Then, all forces (and centripetal acceleration) are approximately co-directed.
- Small tyre and vehicle side slip ⇒ angle=sin(angle)=tan(angle). (Angles are not drawn small, which is why the forces not appear parallel in figure.)



Mathematical model:

Equilibrium:

$$m \cdot (\dot{v}_y + \omega_z \cdot v_x) \approx F_{fy} + F_{ry};$$
$$J \cdot \dot{\omega}_z \approx F_{fy} \cdot l_f - F_{ry} \cdot l_r;$$

Constitution:

$$F_{fy} = -C_f \cdot s_{fy}; \quad F_{ry} = -C_r \cdot s_{ry};$$

Compatibility:

$$\delta_f + s_{fy} \approx \delta_f + \alpha_f = \beta_f \approx \frac{v_{fy}}{v_x} = \frac{v_y + l_f \cdot \omega_z}{v_x};$$
$$s_{ry} \approx \alpha_r = \beta_r \approx \frac{v_{ry}}{v_x} = \frac{v_y - l_r \cdot \omega_z}{v_x};$$

Eliminate $F_{fy}, F_{ry}, \alpha_f, \alpha_r, \beta_f, \beta_r$ **yields:**

$$m \cdot \dot{v}_y + \frac{C_f + C_r}{v_x} \cdot v_y + \left(\frac{C_f \cdot l_f - C_r \cdot l_r}{v_x} + m \cdot v_x\right) \cdot \omega_z \approx C_f \cdot \delta_f;$$
$$J \cdot \dot{\omega}_z + \frac{C_f \cdot l_f - C_r l_r}{v_x} \cdot v_y + \frac{C_f \cdot l_f^2 + C_r \cdot l_r^2}{v_x} \cdot \omega_z \approx C_f \cdot l_f \cdot \delta_f;$$

Figure 4-48: Less general derivation of the Linear One-Track Model, i.e. Eq [4.50].

1.5.1.1.3.2 § Affine and Linear form (ABCD form)

- $\dot{\boldsymbol{x}} = \boldsymbol{A} \cdot \boldsymbol{x} + \boldsymbol{B} \cdot \boldsymbol{u}; \quad \boldsymbol{y} = \boldsymbol{C} \cdot \boldsymbol{x} + \boldsymbol{D} \cdot \boldsymbol{u}$; where $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$ are matrices.

Task 3,
Lateral Load Transfer and Roll Stiffness

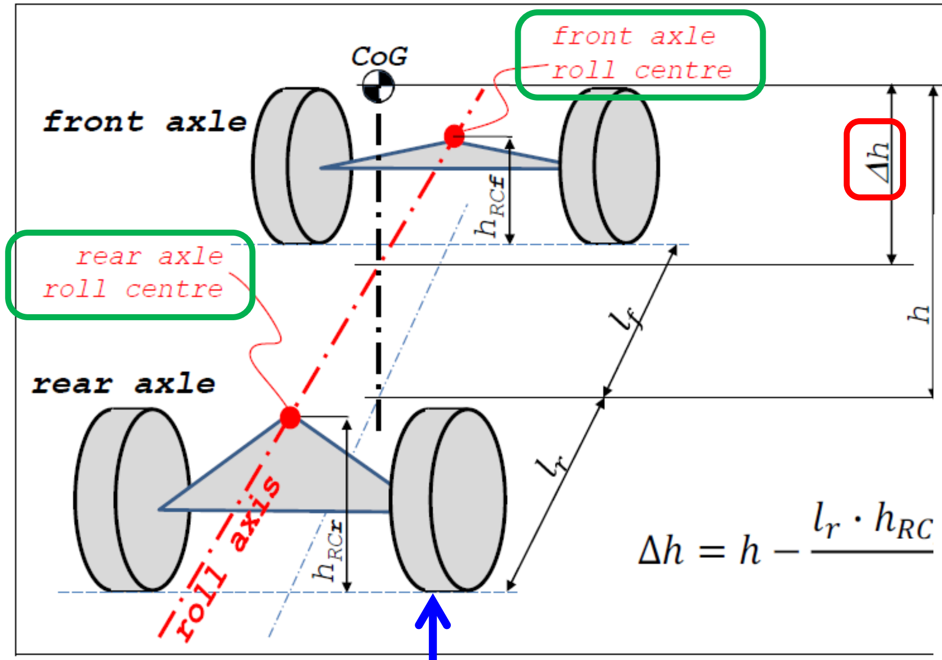


Figure 4-39: Roll axis for a two-axle vehicle. (Note that the picture may
centres and roll axis are above wheel centre, but this is normally

$$F_{flz} = m \cdot \left(\frac{g \cdot l_r}{2 \cdot L} - a_y \cdot \left(\frac{h_{RCf} \cdot l_r}{L \cdot w} + \frac{\Delta h}{w} \cdot \frac{c_{f,roll}}{c_{vehicle,roll}} \right) \right);$$

$$F_{frz} = m \cdot \left(\frac{g \cdot l_r}{2 \cdot L} + a_y \cdot \left(\frac{h_{RCf} \cdot l_r}{L \cdot w} + \frac{\Delta h}{w} \cdot \frac{c_{f,roll}}{c_{vehicle,roll}} \right) \right);$$

$$F_{rlz} = m \cdot \left(\frac{g \cdot l_f}{2 \cdot L} - a_y \cdot \left(\frac{h_{RCr} \cdot l_f}{L \cdot w} + \frac{\Delta h}{w} \cdot \frac{c_{r,roll}}{c_{vehicle,roll}} \right) \right);$$

$$F_{rrz} = m \cdot \left(\frac{g \cdot l_f}{2 \cdot L} + a_y \cdot \left(\frac{h_{RCr} \cdot l_f}{L \cdot w} + \frac{\Delta h}{w} \cdot \frac{c_{r,roll}}{c_{vehicle,roll}} \right) \right);$$

[4.38]

One part of load transfer is “direct” via roll centres

The other part is distributed (“less direct”) via roll stiffness is distributed

Task 4,
Combined Tyre Slip

Lateral Force on front Axle: $F_{fy} = F_{flyv} + F_{fryv}$;

$F_{flyv} = \sin(\delta) \cdot F_{flxw} + \cos(\delta) \cdot F_{flyw} \approx F_{flyw}$;

$F_{flyw} = F_{fly}\left(\mu, F_{flz}, v_{fly}, \omega_{fl}, v_{flx}\right) \approx F_{fly}\left(F_{flz}, s_{fly}, \mu, F_{flx}\right)$;

$$F_y = \sqrt{1 - \left(\frac{F_x}{\mu \cdot F_z}\right)^2} \cdot \boxed{F_y\Big|_{F_x = s_x = 0}};$$

[2.45]

Any lateral slip model, e.g. Magic Formula for lateral slip: $F_y(\mu \cdot F_z, s_y)$;