Assignment 3:
Vertical Dynamics (15p)

Introduction

*This assignment aims at **proposing a wheel suspension stiffness and damping** for a passenger vehicle. Also, you shall propose a **strategy for a certain transport task**, so that driver/passenger vibration exposure becomes acceptable.*

This assignment introduces a quarter-car model to investigate vehicle responses to road surface excitation. The engineering problem in mind for task 2 is to select stiffness and damping with respect to ride comfort and road grip. In task 3 the engineering problem is to propose vehicle speed for a transport mission in order to meet legal requirements on ride comfort.

There are five Matlab files that can be downloaded from the course web page that may help the analysis. Four of these files are skeletons in which you are supposed to fill in the missing code. The fifth file named CalculateIsoWeightedRms.m is needed in task 3. It is not mandatory to use any of these files.

For submission, include all Matlab files you have used in a zip file. You should have a separate pdf file for your report. Upload these files in Canvas before the deadline(s). The deadline is Jan 15th, 23:59.

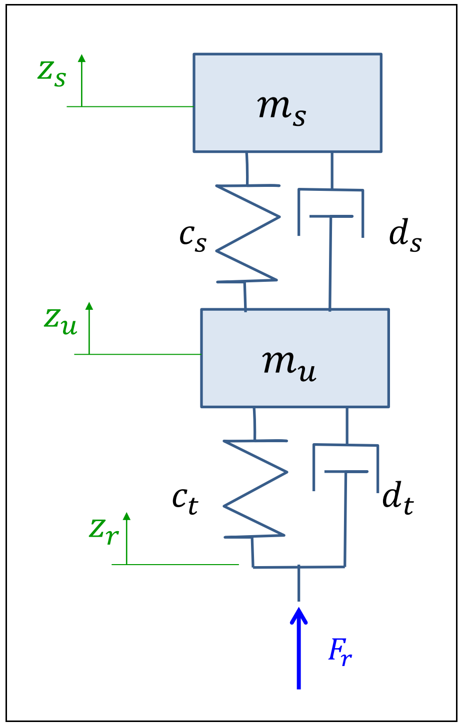
Given Vehicle Data

Quantity	Notation	Value	Unit
Total mass (sprung+unsprung) of the vehicle	m	1860	kg
Unsprung mass (for all 4 wheels)	m_u	185	kg
Wheel base L	L	2.675	m
Distance front axle to CoG	l_f	0.4*L	m
Moment of inertia	I_{yy}	2398	kgm^2
Suspension stiffness, for one wheel on the front		30800	N/m
Suspension stiffness, for one wheel on the rear		29900	N/m
Suspension damping, for one wheel on the front		4500	Ns/m
Suspension damping, for one wheel on the rear		3500	Ns/m
Tire Stiffness		230000	N/m
Tire damping (approximation)		0	N/(m/s)

Task 1: Quarter Car Model Transfer Functions in Matrix Form (6 points)

1.1 Model and derive equations (2 point)

Consider the 2-DOF quarter car model shown below.



Draw free-body diagrams and state the equations of motion in state-space form, i.e.

$\dot{\boldsymbol{x}} = \boldsymbol{A} \cdot \boldsymbol{x} + \boldsymbol{B} \cdot \boldsymbol{u}$ (1)

where $\boldsymbol{x}$s a state vector that contains the displacement of sprung and unsprung masses and their speed (you need 4 states); $\boldsymbol{u} = z_r$ is an input vector which contains road vertical displacement; states and inputs are functions of time. **A** and **B** are constant coefficient matrices. Note that $z_s = 0$, $z_u = 0$, when the system is at rest (static equilibrium). Find **A** and **B** matrices. *Tips: you need to choose your state variables such that you eliminate the derivative of the input. Consider for example*

$$\dot{y} + ay = b_1\dot{u} + b_0 u$$

One choice if the state variable is $x = y - b_1 u$ *which results in the state space*

$$\dot{x} = \dot{y} - b_1\dot{u} = -ay + b_0 u = -a(y - b_1 u + b_1 u) + b_0 u = -ax + (b_0 - ab_1)u$$

1.2 Determine transfer function matrix H(ω) (1 point)

Assume the output of the system, $\boldsymbol{y}$, that we are interested in is given by

$\boldsymbol{y} = \boldsymbol{C} \cdot \boldsymbol{x} + \boldsymbol{D} \cdot \boldsymbol{u}$ (2)

Equations (1) and (2) define a linear time invariant (LTI) system. The system is linear since there is no term containing multiplications of states or inputs. The system is time invariant since the matrices **A**, **B**, **C** and **D** are constant coefficient matrices.

Further, assume that there is a harmonic input to the system given by

$$\boldsymbol{u} = \boldsymbol{U} \cdot e^{j\omega t} \quad (3)$$

Outputs and states can also be assumed harmonic:

$$\boldsymbol{y} = \boldsymbol{Y} \cdot e^{j\omega t} \qquad \boldsymbol{x} = \boldsymbol{X} \cdot e^{j\omega t} \quad (4)$$

where in general the elements of $\boldsymbol{X}$, $\boldsymbol{U}$ and $\boldsymbol{Y}$ are complex. Insert the assumptions into the state-space equations (1) and (2). Show that the transfer function matrix ***H(ω)*** defined by

$$\boldsymbol{Y} = \boldsymbol{H}(\omega) \cdot \boldsymbol{U} \quad (5)$$

can be calculated as

$$\boldsymbol{H}(\omega) = \boldsymbol{C} \cdot (j\omega \cdot \boldsymbol{I}_n - \boldsymbol{A})^{-1} \cdot \boldsymbol{B} + \boldsymbol{D} \quad (6)$$

where, $\boldsymbol{I}_n$ is an identity matrix and *n* is the number of states. You don't need to carry out the actual calculation of the inverse of $(j\omega \cdot \boldsymbol{I}_n - \boldsymbol{A})^{-1}$.
Alternatively, you can derive (6) starting from state-space equations using Laplace transform, while, $s = j\omega$.

1.3 Plot transfer functions H(ω) (2 point)

Find ***B***, ***C*** and ***D*** matrices and corresponding transfer function given by equation (6) for three following single input single output (SISO) systems:

1. The road displacement, z_r, as an input and ride comfort (acceleration of the sprung mass, $\ddot{z}_s$) as an output; notation for the transfer function: $H(\omega)_{z_r \to \ddot{z}_s}$;
2. The road displacement, z_r, as an input and suspension travel (difference in displacement of the unsprung and the sprung mass, $z_u - z_s$) as an output; notation for the transfer function: $H(\omega)_{z_r \to (z_u - z_s)}$;
3. The road displacement, z_r, as an input and tyre force (i.e. road grip defined by difference in displacement of the road and the unsprung mass multiplied with tire stiffness, for the case when tire damping is zero) as an output, notation for the transfer function: $H(\omega)_{z_r \to \Delta F_{rz}}$.

Plot the magnitude of the three transfer functions versus frequencies from 0.1 to 50 Hz. Use dB as the unit for Y axis. Plot the results for both one front and one rear wheel. Write the units of axes.

Hint: You can verify your results using the expressions for the transfer functions stated in the course compendium, equation 5.51.

1.4 Identify natural frequencies (1 point)

Identify the two natural frequencies (for one front and one rear wheel) using the simplified expressions in the compendium in Eq. 5.50 and compare them with the graphs derived in Task 1.3. Explain what you observe with reasoning.

Task 2. Study of the suspension stiffness and damping (5 points)

In this task you will study a vehicle driving with constant speed on a road with a specified vertical profile. You will calculate the road spectra for the given road and use this together with your model to determine the acceleration spectra of the sprung mass and the tyre force. These spectra can be used to calculate the respective rms values. Finally, you will do a study of the suspension spring stiffness and damping using stated rms values as the objective.

Consider <u>one front wheel</u> of a vehicle moving at constant speed. If the longitudinal velocity v_x is constant, the road spectrum can be calculated according to Eq. 5.29 in the course compendium stating:

$G_{z_r}(\omega) = {v_x}^{w-1} \cdot \Phi_0 \cdot \omega^{-w}$, where w=waviness and Φ_0=road severity [m^2/(rad/m)].

In Task 2 you should use the values: w =2.5, Φ_0=10·10^{-6} and v_x=80 km/h

2.1 Plot response spectrum and calculate rms values (2 points)

Plot the response spectra (Power Spectral Density, PSD) and determine the rms values of the:

- Acceleration of the sprung mass; i.e. $G_{\ddot{z}_s}(\omega)$ and $rms_{\ddot{z}_s}$ (Ride comfort),
- Tyre force (Road grip); i.e. $G_{\Delta F_{rz}}(\omega)$ and $rms_{\Delta F_{rz}}$.

Use a bandwidth $\Delta\omega = 2 \cdot \pi \cdot 0.01$ in the approximation of the integral. The response spectra and the rms values can be calculated according to Eqs. 5.30-5.31 in the course compendium stating:

$$G(\omega) = |H(\omega)|^2 \cdot G_{z_r}(\omega)$$

$$rms = \sqrt{\int_0^{\infty} G(\omega)d\omega} \approx \sqrt{\sum_i^N G(\omega_i) \cdot \Delta\omega}$$

2.2 Balance Ride comfort and Tyre force/road grip (3 points)

High levels of the sprung mass acceleration variation mean poor ride comfort. Also, high levels of the tyre force variation mean poor road grip. Study the sensitivity of these measures to changes in the suspension stiffness and damping.

1. For each stiffness value ([0.5, 0.75, 1, 1.25, 1.5, 2]*given stiffness), vary the damping according to [1000, …, 9000 Ns/m]. Calculate the rms values for the sprung mass acceleration and the tyre force

for each setting of stiffness and damping (i.e. in total 6*81 rms values for the sprung mass acceleration).

2. Plot the rms values for the sprung mass acceleration vs. the damping [1000, …, 9000 Ns/m]. One curve per stiffness value. Plot the rms values for the tyre force vs. the damping [1000, …, 9000 Ns/m]. One curve per stiffness value. <u>Identify the optimal damping values for each spring stiffness w. r. t. the acceleration rms and the tyre force rms. Describe the effect of spring stiffness w. r. t. the acceleration rms and the tyre force rms.</u>
3. Plot the identified optimal damping values (w.r.t. sprung mass acceleration and tyre force) from 2. vs. the spring stiffness values ([0.5, 0.75, 1, 1.25, 1.5, 2]*given stiffness). <u>Which of the objectives requires highest damping to be optimal?</u>

Task 3. Ride comfort and the use of ISO-2631 (4 points)

In this task you will consider the management of different road types. You will use the ISO standard (ISO2631) to calculate ISO-filtered vertical whole-body vibration levels and the linked time averaged vibration exposure value. You will also make use of the EC Directive 2002/44 to compare your calculated value with the existing vibration exposure limits.

Note that a Matlab file which calculate an ISO filtered rms value can be found at the course web page (CalculateIsoWeightedRms.m).

Consider following scenario:
A messenger company is based in City A. One of the major customers of the messenger company is located in city B. The road between A and B is 80 km long. The vertical profile of this road can be categorized in three different types:

- Good road (ϕ_0=1E-6, w=3)
- Bad road (ϕ_0=10E-6, w=2.5)
- Very bad road (ϕ_0=100E-6, w=2)

The distribution of these road types is as follows: good road 70% (56km), bad road 17.5% (14 km) and very bad road 12.5% (10 km).

A driver employed by the messenger company works 8 hours per day. He knows that if he drives with a speed of 110 km/h, regardless of road type, he will be able to drive to the customer and back 5 times per day. After the first working day, the driver expresses that the ride comfort has been very poor: the vibration levels were too high. He proposes to his manager that to improve the ride comfort he needs to lower the vehicle speed on some parts of the road. This will not only affect the ride comfort level but also the number of times he can drive to the customer and back per day. The manager of the messenger company contacts you and asks for some suggestions.

- Good road (ϕ_0=1E-6, w=3) 70% of total distance = 56 km
- Bad road (ϕ_0=10E-6, w=2.5) 17.5% of total distance = 14 km
- Very bad road (ϕ_0=100E-6, w=2) 12.5% of total distance = 10 km

3.1 Calculation of daily whole-body exposure values (2 points)

Follow the steps i-iii below and calculate the whole-body vibration exposure value when driving on the specified road in 110 km/h during an eight hour period. Compare the calculated value with the limit specified in Directive 2002/44/EC (partially included in the assignment), consider only the first specified value 1.15 m/s^2.

i. Consider one single front wheel. Calculate sprung mass acceleration response spectra for the different road types. Follow the same procedure as in Task 2.1.
ii. Calculate ISO-filtered sprung mass acceleration rms values (whole-body vibration levels) for the different road types. Use included ISO filter in the matlab file CalculateIsoWeightedRms.m. This function actually calculates a filtered acceleration response spectrum first and then calculate the rms values.
iii. Calculate time averaged vibration exposure value. Use Eq. 5.52 in the course compendium.

3.2 Modification of vehicle velocity (2 points)

Modify the vehicle speed individually on the three different road types so that the vibration exposure limit is not exceeded on each road type. You should do this with the objective that the driver should drive as many times as possible to the customer and back during 8 hours driving while minimizing the averaged vibration exposure. Maximum allowed speed is 110 km/h. Specify your selected speeds. How many times per day can he drive to the customer and back? Round to closest lower integer. (You don't need to make use of any optimization algorithm. Test some different values, maybe in loops, and make conclusions).

Directive 2002/44/EC, p.2:

2. For whole-body vibration:

(a) the daily exposure limit value standardised to an eight-hour reference period shall be 1,15 m/s^2 or, at the choice of the Member State concerned, a vibration dose value of 21 $m/s^{1,75}$;

(b) the daily exposure action value standardised to an eight-hour reference period shall be 0,5 m/s^2 or, at the choice of the Member State concerned, a vibration dose value of 9,1 $m/s^{1,75}$.

Workers' exposure to whole-body vibration shall be assessed or measured on the basis of the provisions of Point 1 of Part B of the Annex.

Full document can be found:
http://eur-lex.europa.eu/legal-content/EN/TXT/?qid=1418071675222&uri=CELEX:32002L0044