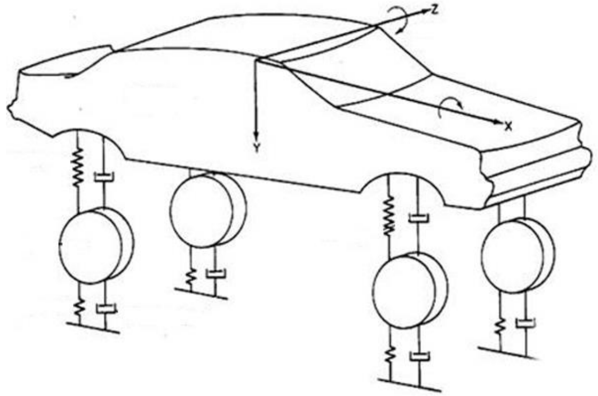
Assignment 3

Vertical dynamics

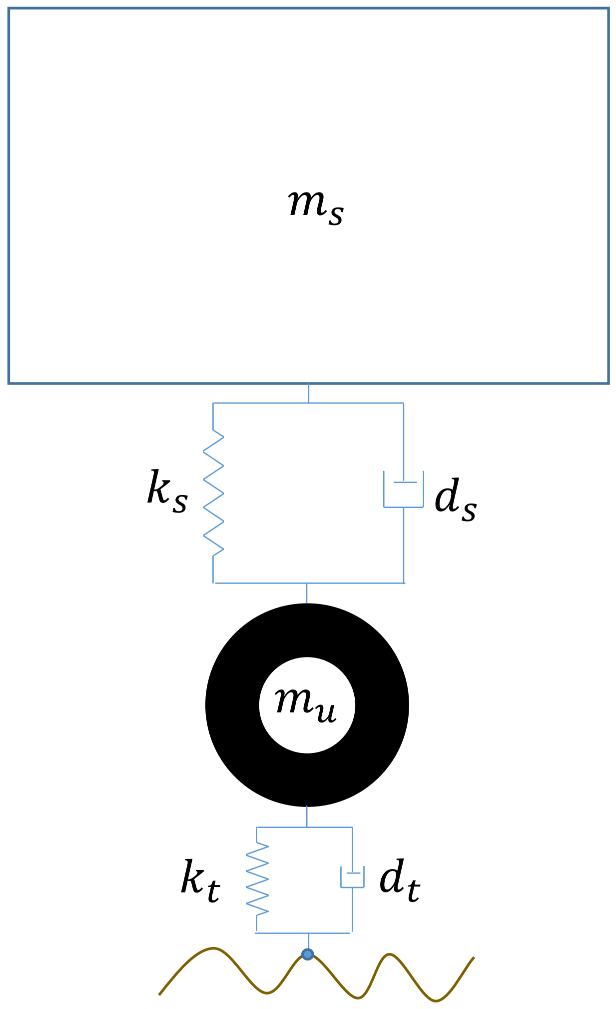


Assignment objectives

- Understand the influence of spring stiffness and damping.
- Understand how the road roughness affects road grip and ride comfort.
- Propose a wheel suspension stiffness and a damping for a certain transportation task.

Task 1

Quarter car model (6 p)



Task 1.1: Derive equations

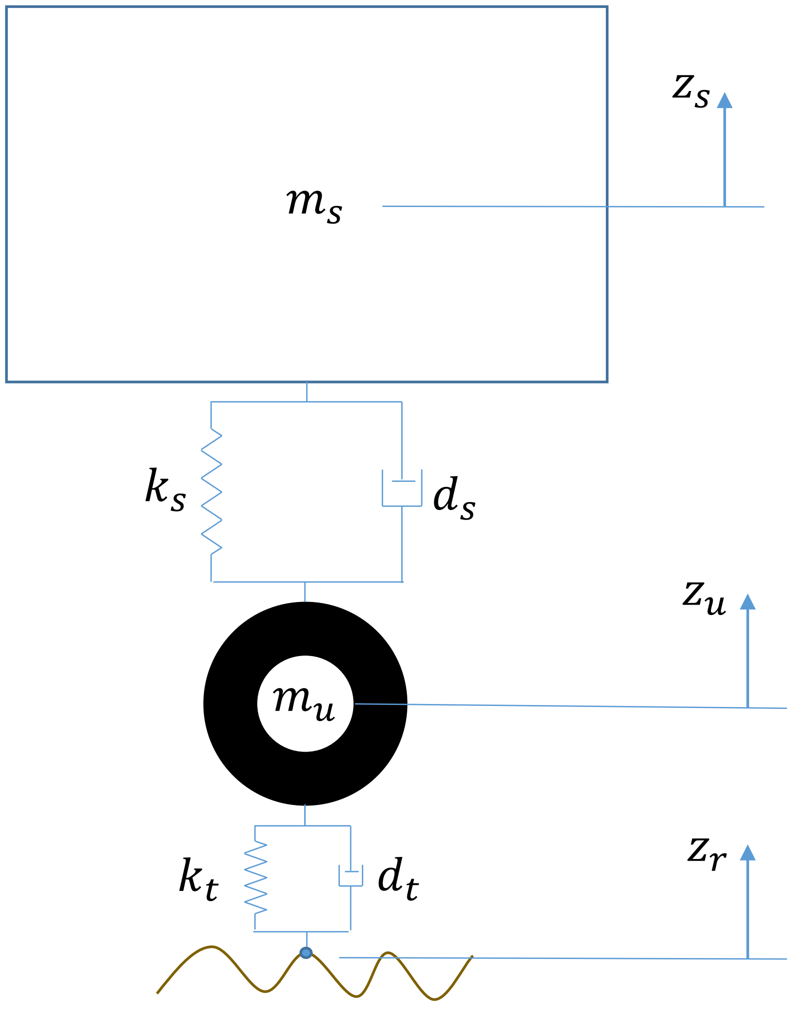
- Draw free body diagram
- Derive equations of motion (see compendium)
- Derive the state-space form of the dynamical system (linear time invariant system).

$$\dot{\boldsymbol{x}} = \boldsymbol{A} \cdot \boldsymbol{x} + \boldsymbol{B} \cdot \boldsymbol{u}$$

where,

State vector of dimension 4

Input: $\boldsymbol{u} = z_r$



Task 1.2: Transfer function

- Given the state-space equations:

$$\dot{\boldsymbol{x}} = \boldsymbol{A} \cdot \boldsymbol{x} + \boldsymbol{B} \cdot \boldsymbol{u}$$
$$\boldsymbol{y} = \boldsymbol{C} \cdot \boldsymbol{x} + \boldsymbol{D} \cdot \boldsymbol{u}$$

where $\boldsymbol{y}$ is an output, show that the transfer funcion $\boldsymbol{H}(\omega)$ can be calculated as

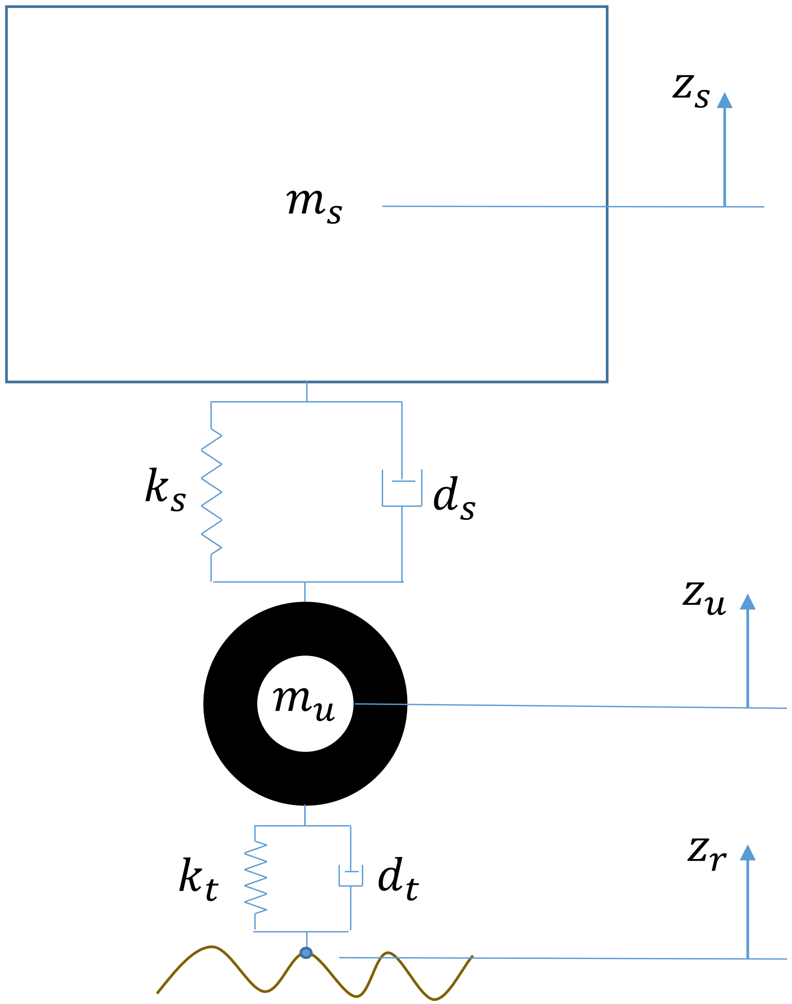
$$\boldsymbol{H}(\omega) = \boldsymbol{C} \cdot (j\omega \cdot \boldsymbol{I}_n - \boldsymbol{A})^{-1} \cdot \boldsymbol{B} + \boldsymbol{D}$$

by assuming harmonic state, input and output:

$$\boldsymbol{x} = \boldsymbol{X} \cdot e^{j\omega t}, \ \boldsymbol{u} = \boldsymbol{U} \cdot e^{j\omega t}, \quad \boldsymbol{y} = \boldsymbol{Y} \cdot e^{j\omega t}$$

transfer funcion $\boldsymbol{H}(\omega)$ is defined by

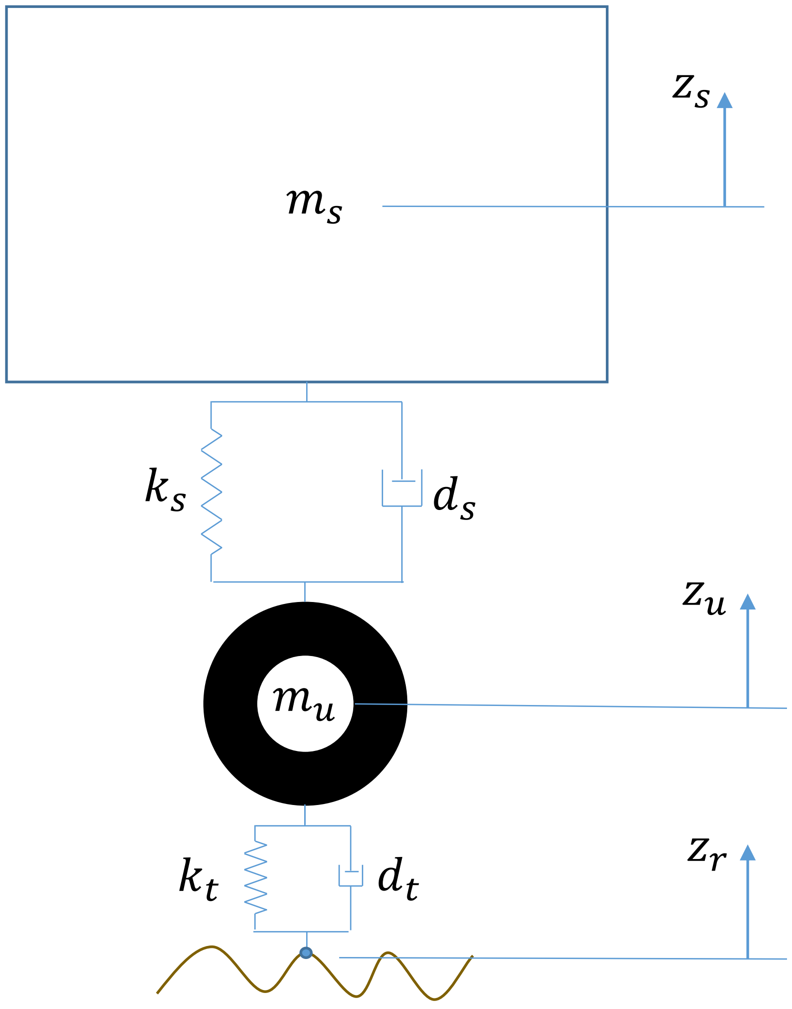
$$\boldsymbol{Y} = \boldsymbol{H}(\omega) \cdot \boldsymbol{U}$$



Task 1.3: Plot

- Define a single input single output (SISO) system using Task 1.2 by finding corresponding B, C and D matrices, where the input is road displacement (z_r) for three different outputs:

1. Ride comfort, $\ddot{z}_s$
2. Suspension travel, $(z_u - z_s)$
3. Road grip, ΔF_{RZ}

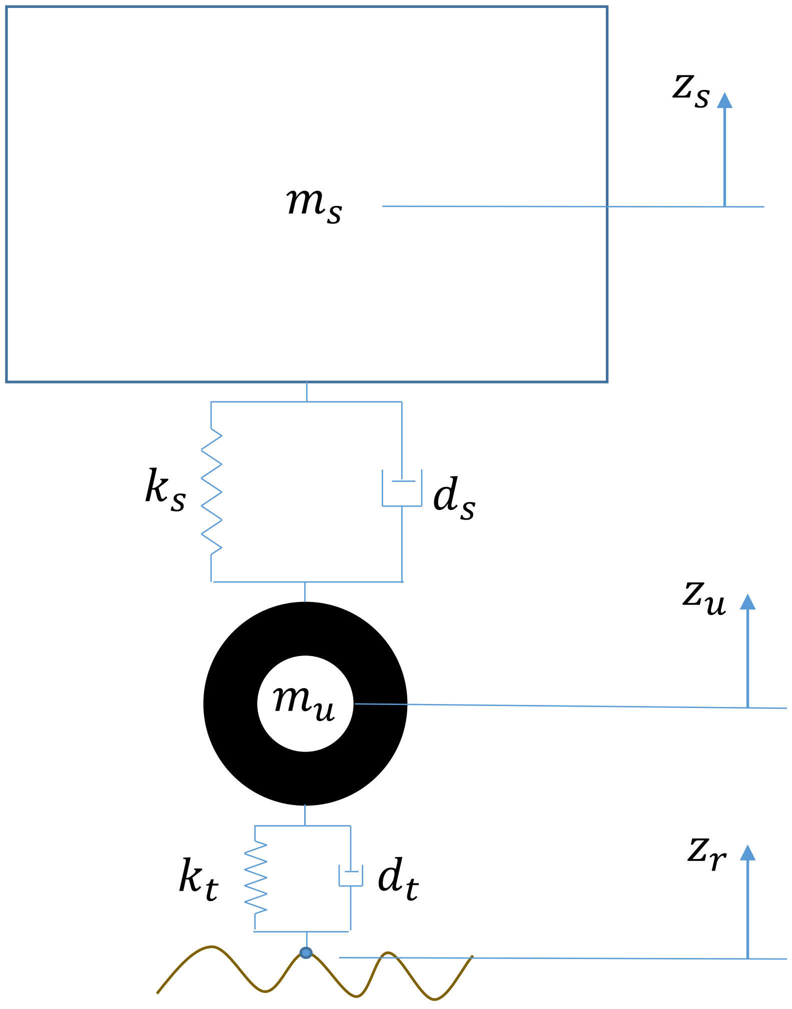


Task 1.3: Plot

- Then, find the corresponding transfer functions:

1. Ride comfort $H(\omega)_{z_r \to \ddot{z}_s}$
2. Suspension travel $H(\omega)_{z_r \to (z_u - z_s)}$
3. Road grip $H(\omega)_{z_r \to \Delta F_{RZ}}$

- Plot magnitude of these for $\omega \in [0.1, 50]\ [Hz]$

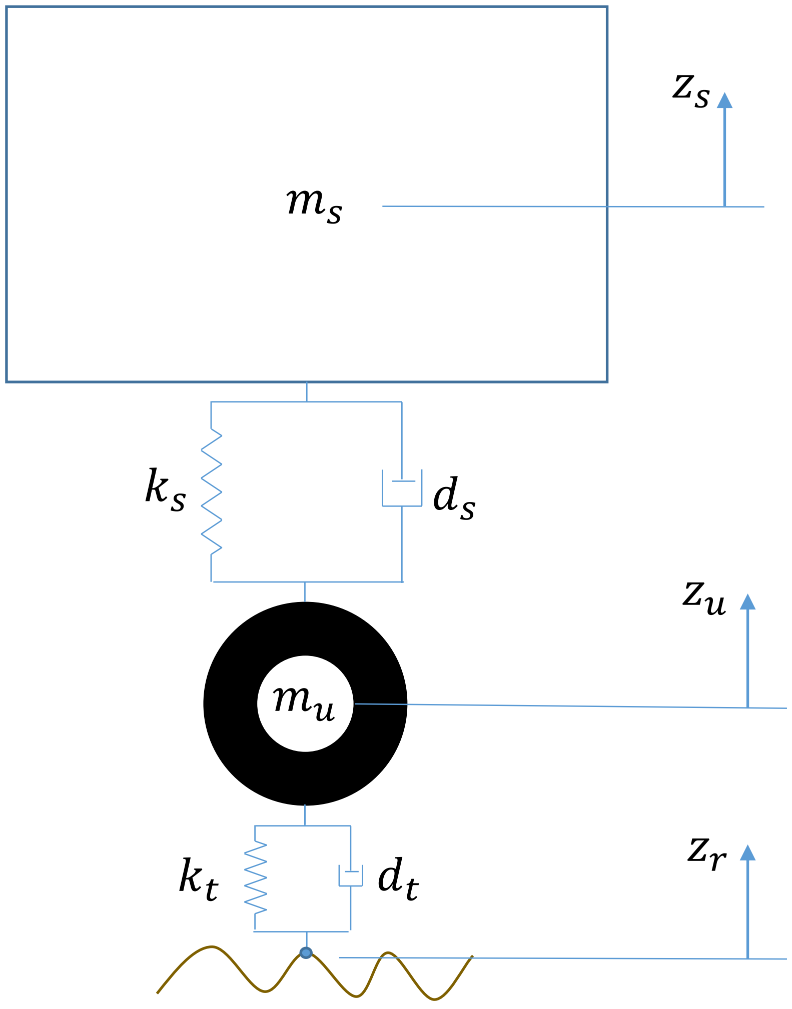


Task 1.4: Natural frequencies

- Calculate the natural frequencies and compare with the graphs.

$$\omega_{Bounce} = \sqrt{\frac{1/\left(\frac{1}{c_s} + \frac{1}{c_t}\right)}{m_s}}$$

$$\omega_{WheelHop} = \sqrt{\frac{c_s + c_t}{m_u}}$$



Suspension stiffness and damping (5 p)

- Study ride comfort and road grip
  - Varying suspension properties: stiffness and damping

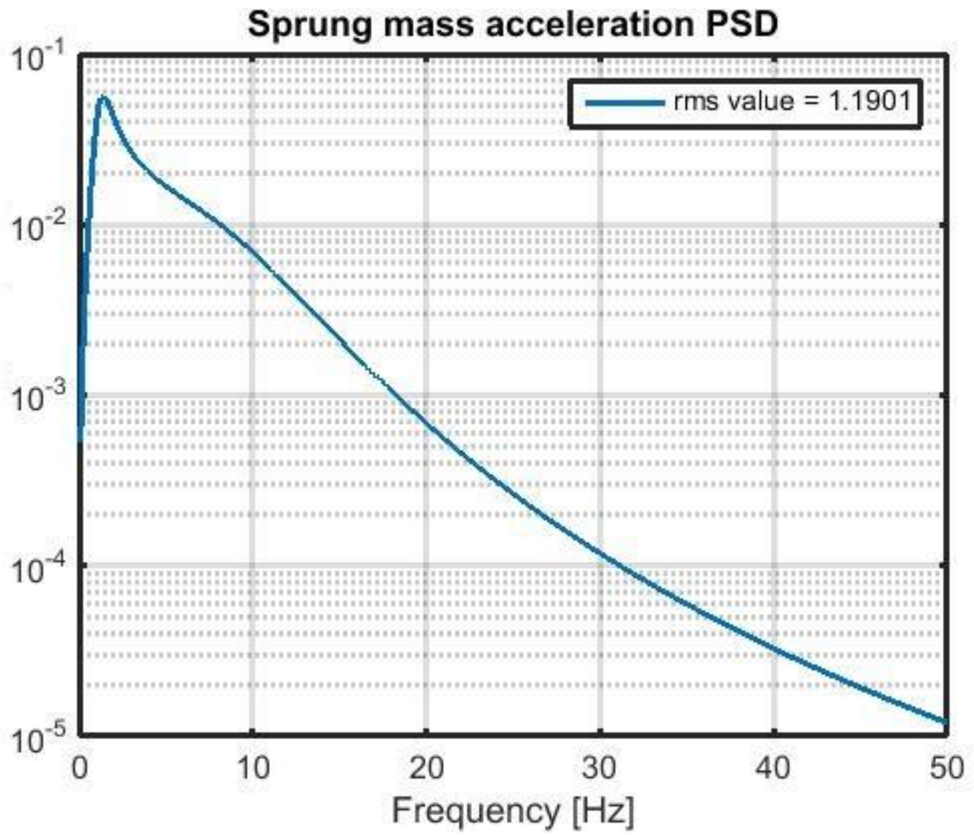
- Road profile is given

- Assume speed is constant

- Lots of help in the compendium and
in skeleton code.

Task 2.1: Response spectra

- Plot the PSD (power spectral density) of ride comfort and road grip
- Calculate RMS-values



Task 2.2: Balance ride comfort and road grip

- Vary stiffness and damping, compute RMS-values
- Try to identify an optimal setting for each stiffness and damping value
- Plot these and draw conslusion

Ride comfort (4 p)

- A messenger company has hired you as a consultant
- One standard transport mission has particular poor ride comfort
  - EC Directive 2002/44 impose vibration exposure limits
- You are asked to analyze and find a solution

Task 3.1: Calculate exposure values

- Use model from task one and results from task two
- For a speed of 110 km/h, compute ride comfort
- Filter values using ISO standard (handout script will help you)
- Average over time (8 h driving)

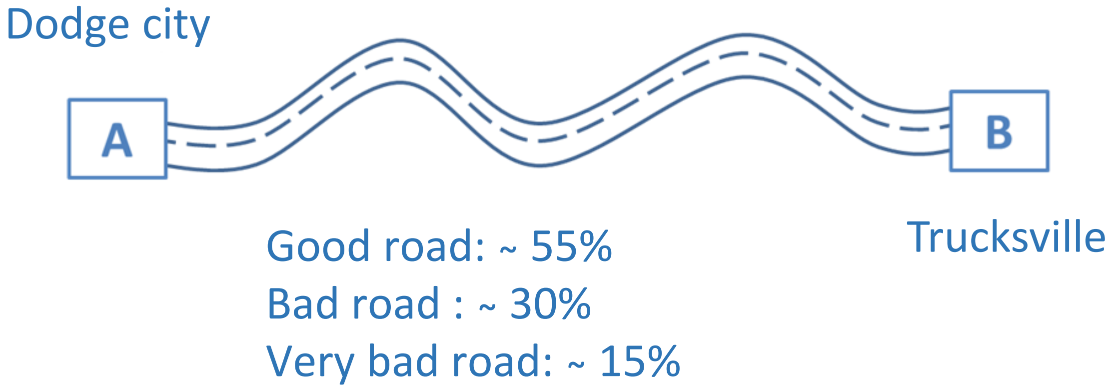
Dodge city

Good road: ~ 55%
Bad road : ~ 30%
Very bad road: ~ 15%

Trucksville

Task 3.2: Modify the strategy

- Suggest individual speeds for the different parts
- Exposure limits must be respected!
- How many deliveries can be made in 8 h?



Deadlines and support

- Assistants

Fredrik Bruzelius
fredrik.bruzelius@chalmers.se

Sachin Janardhanan
sachin.janardhanan@chalmers.se

- Maximum 15 points
  - Pass ≥ 6 points, task 1 and 2
- Submit one .pdf-file and one zip file (Matlab codes) in Canvas
- Hard deadline 2022-01-15, 23.59

Thank you!
and
Good luck!