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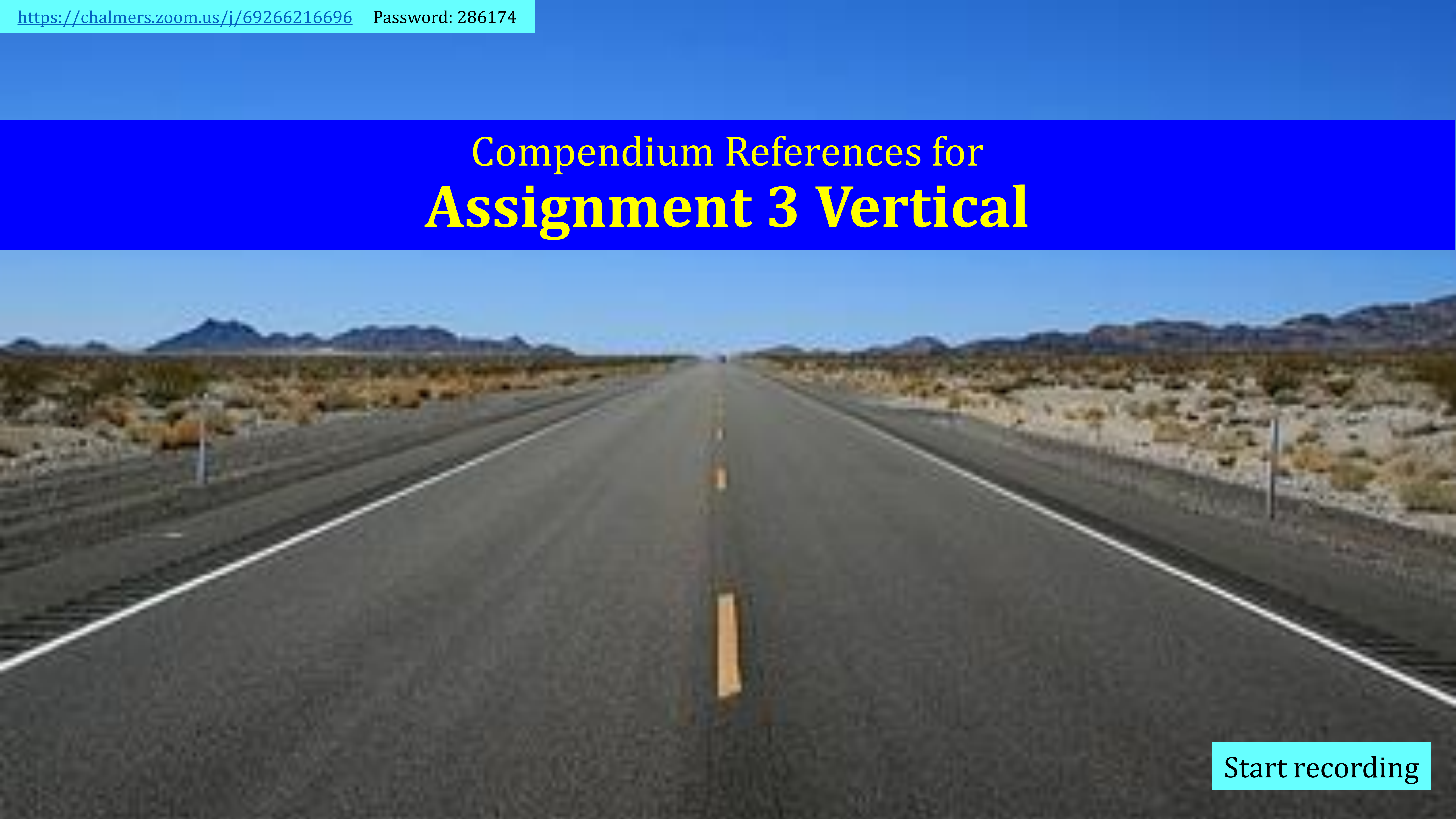
Compendium References for
Assignment 3 Vertical



Start recording

Design tasks

Administration (points, hand-in, etc): See Course Memo.

...and, these will take well care of you:

- Assistants for Design task 1, Longitudinal, 10 p:
  - **Lead:** Sachin.Janardhanan@chalmers.se ...[SJ]
  - Support: Luigi.Romano@chalmers.se ...[LR]
- Assistants for Design task 2, Lateral, 25 p:
  - **Lead**: Yansong.Huang@chalmers.se ...[YH]
  - Support: Sachin.Janardhanan@chalmers.se ...[SJ]
  - For Simulator Lab at CASTER:
    - Admin: Johannes.Jensen@casterchalmers.se ...[JJ]
      - Instructors: Ajit Kumar Madhava Prakash, Nitesh Nikkam, Tarun Kadri Sathiyan, Ruixuan Jiang ...[instr]
- Assistants for Design task 3, Vertical, 15 p:
  - **Lead:** Luigi.Romano@chalmers.se ...[LR]
  - Support: Yansong.Huang@chalmers.se ...[YH]

Luigi will soon do an introduction of the Design task 3,
but Bengt will first point out useful parts in Compendium ...

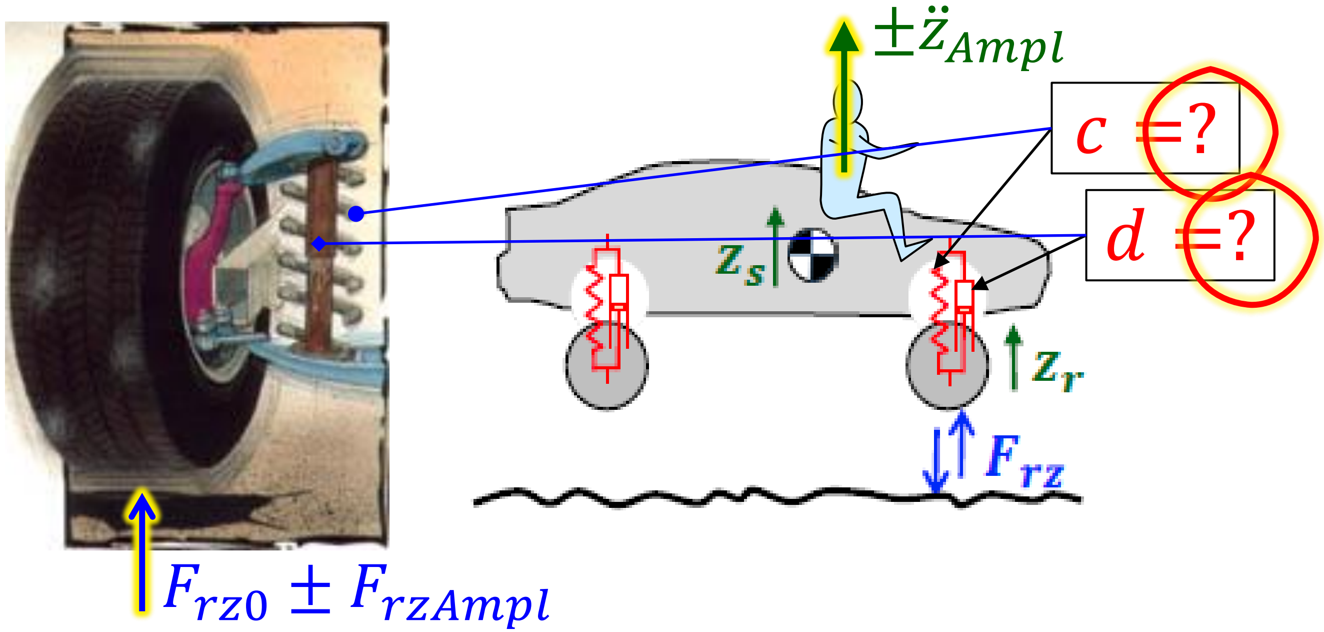
Design Tasks

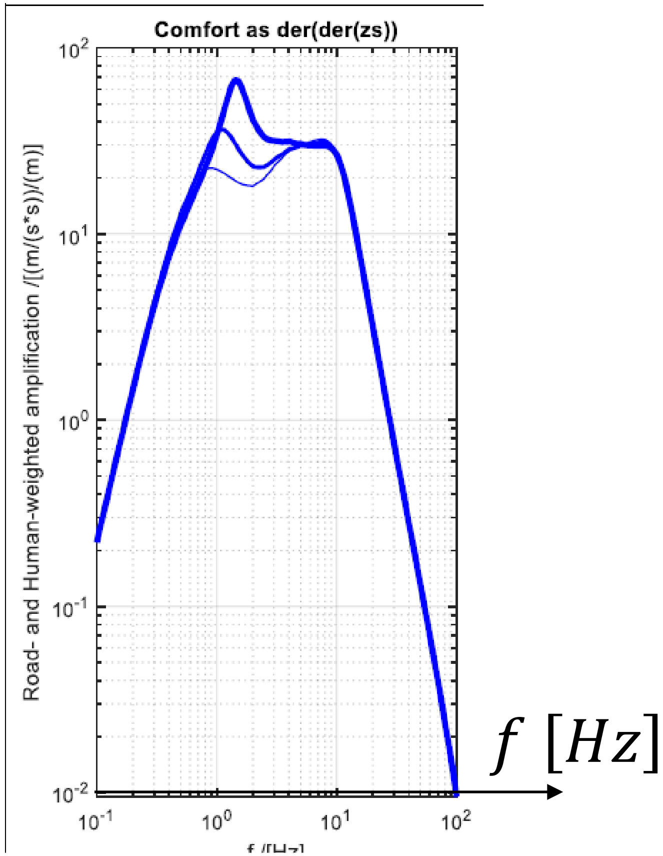
Learning objectives	Reading
Design Task 1: Longitudinal • Functions: Acceleration (uphill, various road friction) • What to engineer: Distribution of propulsion between front and rear axle (FWD/RWD) • Method: Simulation • Tools: Matlab Symbolic toolbox, "Home-coded" time-integration (for conceptual understanding of simulation)	• Figure 2-21, 2.2.3.4.1 Magic Formula Tyre Model, Eq [2.1] • 1.5.4.1.1 General Mathematics Tools • 1.5.1.1.3 Physical Modelling • 1.5.1.1.4 Mathematical Modelling, 1.5.2.1 Free-Body Diagrams • 1.5.1.1.5 Explicit Form Modelling, 1.5.1.1.6 Computation • Figure 3-24, Eq [3.13] • 3.5.2.5 Traction Control, TC *
Design Task 2: Lateral • Functions: Yaw balance in steady state high speed, Step steer response, Brake in curve • What to engineer: Distribution of roll-stiffness and brake force between front and rear axle • Method: Simulation, Driving experience, model integration and log data analysis • Tools: Simulink (for learning one commonly used tool for simulation), Motion platform driving simulator (for driving experience and log data analysis)	• Figure 4-11 • Figure 4-15 • Figure 4-19 • 4.3.6 Steady State Cornering Gains * • Eq [1.1][4.17] • Eq [4.18] • Figure 4-47 • 1.5.1.1.4.5 Affine and Linear form (ABCD form) • Figure 4-38, Eq [4.39] • Eq [2.47]
Design Task 3: Vertical • Functions: Comfort for stationary vibrations, Road grip due to stationary varying vertical tyre force • What to engineer: Wheel suspension stiffness and damping • Method: Frequency analysis • Tools: Matlab (for learning one commonly used tool for matrix computations)	• Figure 5-1, Figure 5-12, [5.44] • Figure 5-3, Eq [5.4], Eq [5.13], 4.4.3.1.1 Solution with Fourier Transform • Eq [5.45] • Figure 5-5, Figure 5-20

"Common thread" between the 3 Design tasks


Learning objectives
Design Task 1: Longitu
Functions: Accelerat
What to engineer: Di
front and rear axle (
ulation
Tools: Mat b Symbo
integration for con
simulation)
Design Task 2: Lateral
Functions: Ya balan
Step steer r ponse
What to en eer: Di
brake force between
Method: Simulation,
integration and log d
Tools: Simulink (for
for simulation), Moti
(for driving experien
Design Task 3: Vertica
Functions: Comfort f
grip due to stationar
What to engineer: W
damping
Method: Frequency analysis
Tools: Matlab (for learning one co ly used tool
for matrix computations)

Chapter	Function	Subsystem (to design/engineer)	Method	Tool
Long	Acceleration	Prop	Simulation	Matlab
Lat	Yaw balance	Susp & Brk	Simulation	Simulink & DrivSim
Vert	Comfort & Road grip	Susp	Freq Analysis	Matlab



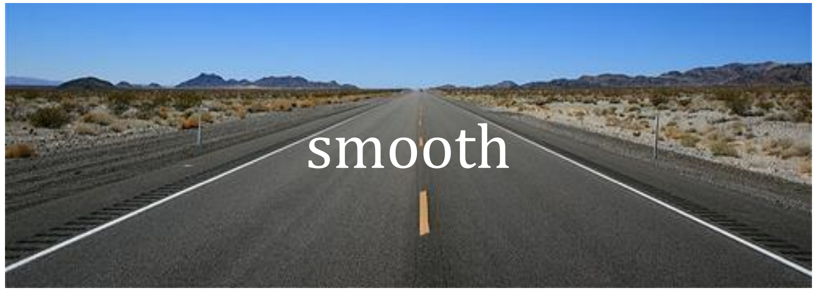


Design Task 3: Vertical

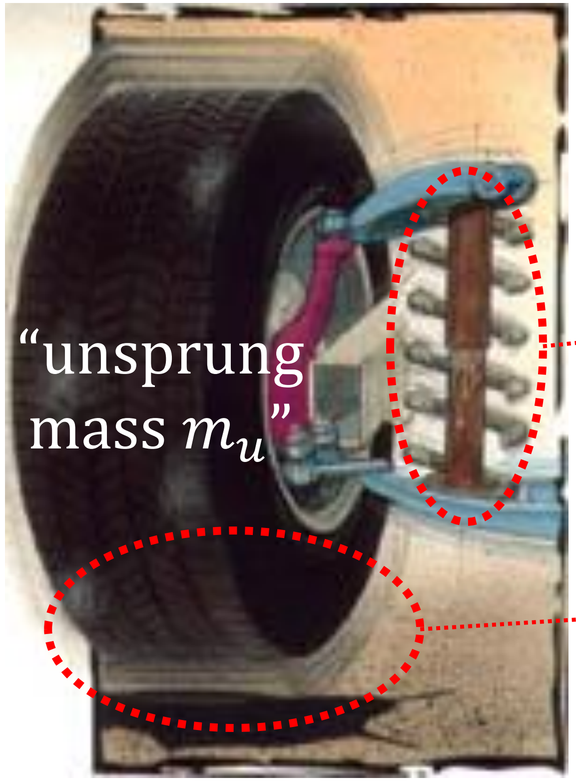
- Functions: Comfort for stationary vibrations, Road grip due to stationary varying vertical tyre force
- What to engineer: Wheel suspension stiffness and damping
- Method: Frequency analysis
- Tools: Matlab (for learning one commonly used tool for matrix computations)

The following slides shows quickly these ”recommended readings”.

- Figure 5-1, Figure 5-12, [5.44]
- Figure 5-3, Eq [5.4], Eq [5.13], 4.4.3.1.1 Solution with Fourier Transform
- Eq [5.45]
- Figure 5-5, Figure 5-20


smooth


rough


“unsprung mass m_u”

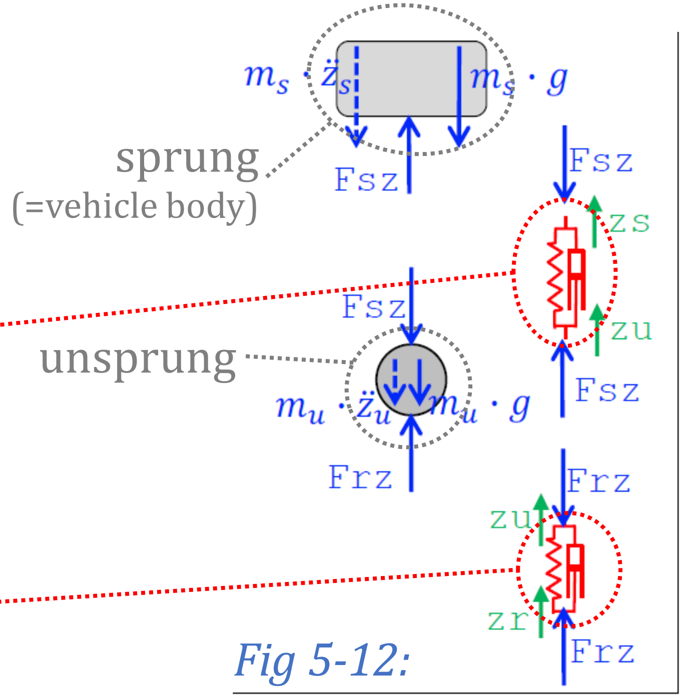

$m_s \cdot \ddot{z}_s$
$m_s \cdot g$
sprung
(=vehicle body)
Fsz
Fsz
zs
zu
Fsz
unsprung
Fsz
$m_u \cdot \ddot{z}_u$
$m_u \cdot g$
Frz
Frz
zu
zr
Frz

Fig 5-12:

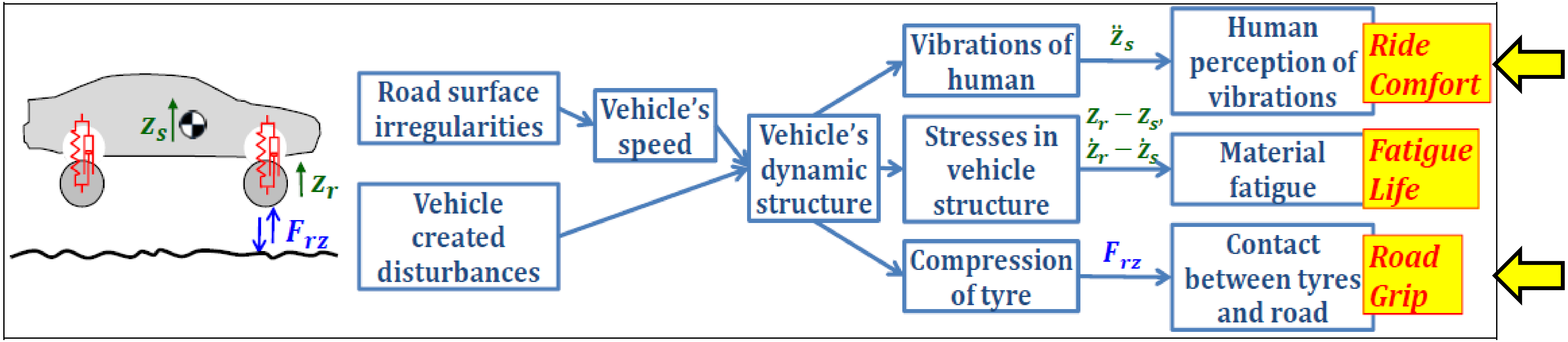


Figure 5-1: Different types of knowledge and functions in the area of vertical vehicle dynamics, organised around the vehicle's dynamic structure.

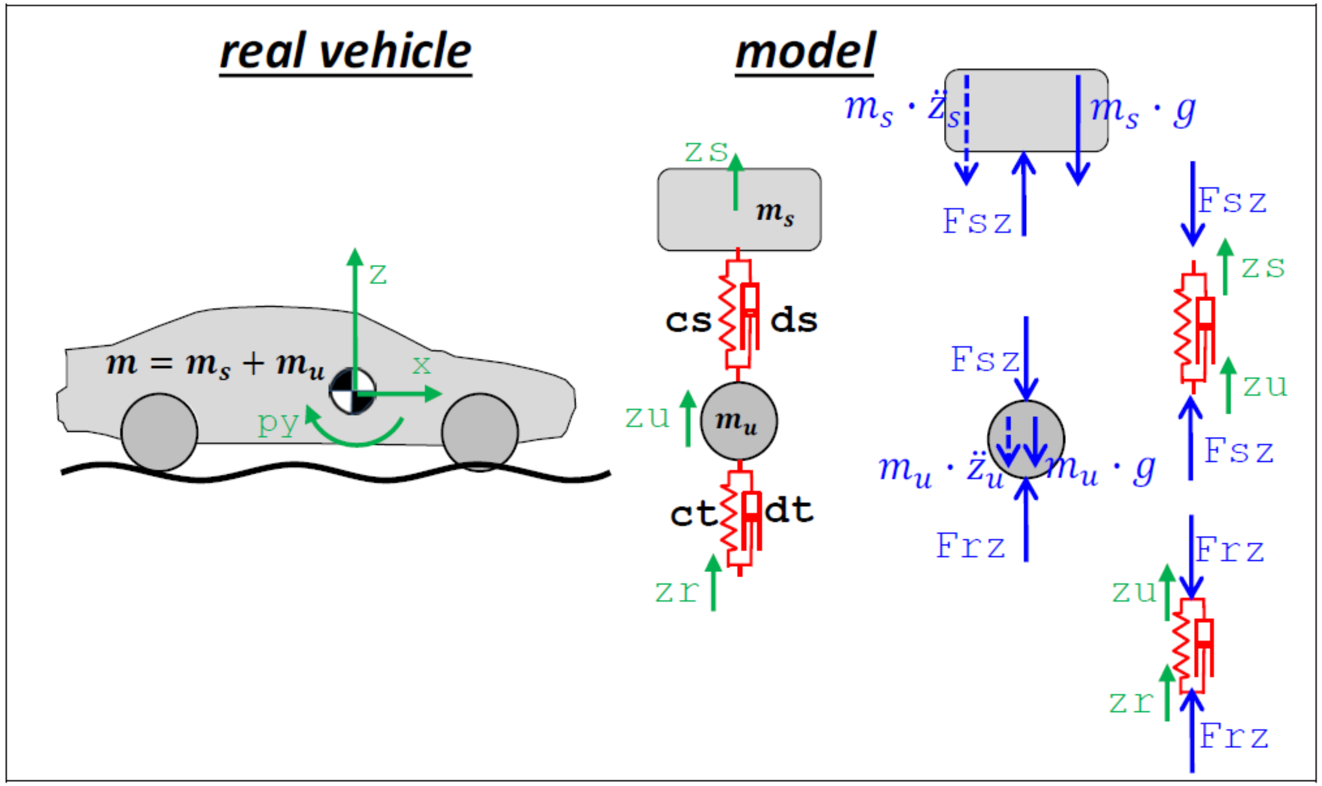


Figure 5-12: One-dimensional model with two dynamic degrees of freedom

These are sometimes difficult to understand. Explained on next slide.

The corresponding mathematical model becomes as follows:

Equilibrium:

$$m_s \cdot \ddot{z}_s = F_{sz} - m_s \cdot g;$$

$$m_u \cdot \ddot{z}_u = F_{rz} - F_{sz} - m_u \cdot g;$$

Constitution (displacements measured from static equilibrium):

$$F_{sz} = \underbrace{c_s \cdot (z_u - z_s) \boxed{+ m_s \cdot g}}_{linear\ spring\ c_s} + d_s \cdot (\dot{z}_u - \dot{z}_s);$$

$$F_{rz} = \underbrace{c_t \cdot (z_r - z_u) + \boxed{(m_s + m_u) \cdot g}}_{linear\ spring\ c_t} + d_t \cdot (\dot{z}_r - \dot{z}_u);$$

Excitation: $z_r = z_r(t);$

[5.44]

Explain with a simpler model

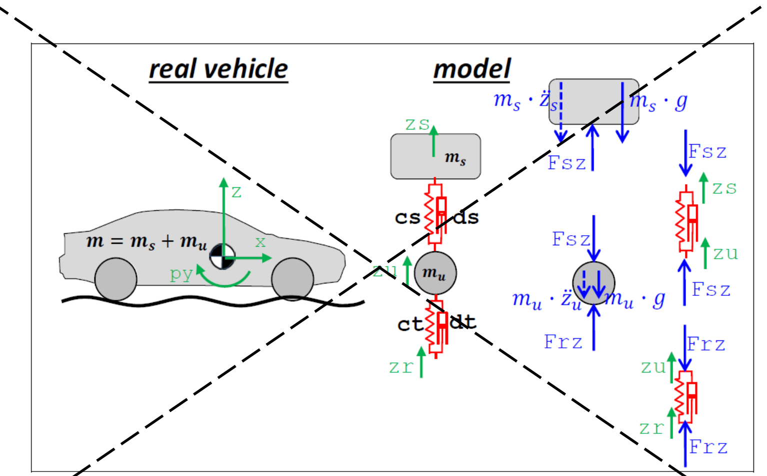


Figure 5-12: One-dimensional model with two dynamic degrees of freedom

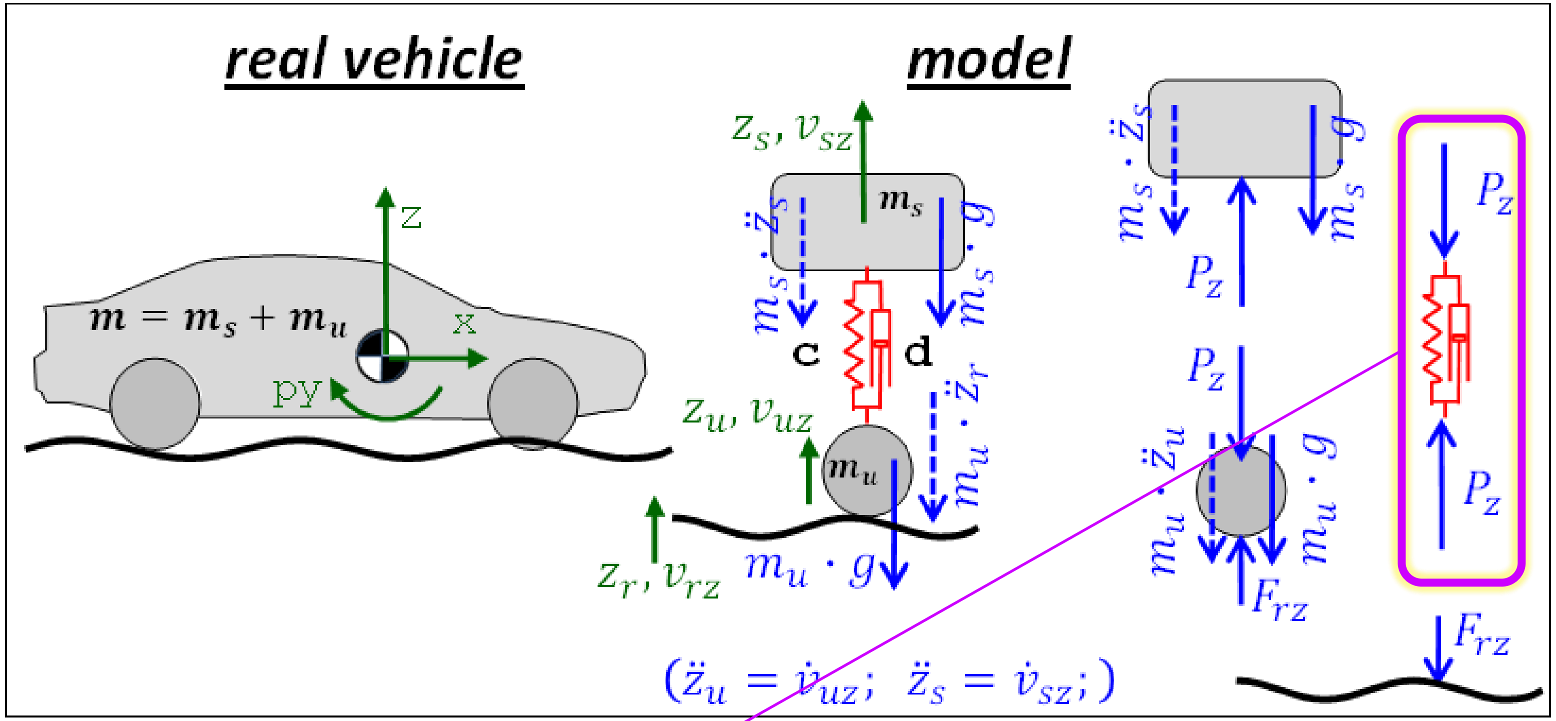


Figure 5-9: One-dimensional model with 1 dynamic degree of freedom

The mathematical model becomes as follows:

Equilibrium sprung mass: $P_z - m_s \cdot \ddot{z}_s - m_s \cdot g = 0;$

Equilibrium unsprung mass: $F_{rz} - P_z - m_u \cdot \ddot{z}_r - m_u \cdot g = 0;$

Constitution: $P_z = \underbrace{c \cdot (z_u - z_s) + P_{zs0}}_{linear\ spring\ c} + d \cdot (\dot{z}_u - \dot{z}_s);$

where z_u and z_s are measured from positions such that $P_{zs0} = m_s \cdot g;$

Compatibility: $z_u = z_r;$

Excitation: $z_r = z_r(t);$ [5.37]

The z_u and z_s can also be measured from other positions. However, the shown way is practical since the constant term in the total system of equations will vanish. The shown way can be physically understood as measuring from the positions for the system in static equilibrium, i.e. when the vehicle is stand-still or driving on a perfectly flat road.

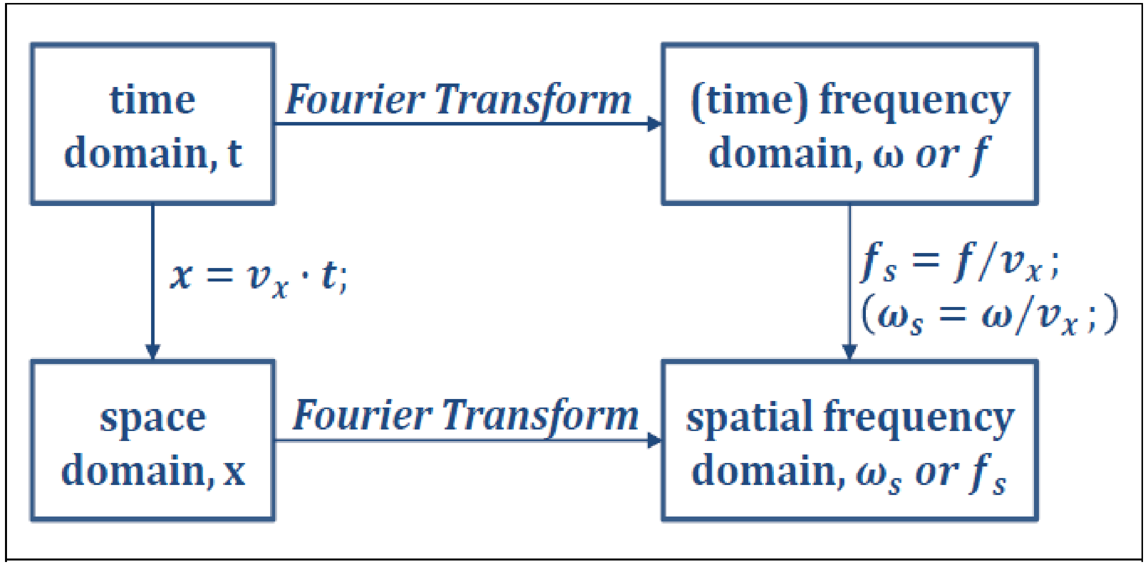


Figure 5-3: Four domains and transformations between them

$$
\begin{aligned}
&Variable: && z && = z(t);\\
&MeanSquare: && MS(z) && = \frac{\int_0^{t_{end}} z^2 \cdot dt}{t_{end}};\\
&RootMeanSquare: && \boxed{RMS(z) = \sqrt{\frac{\int_0^{t_{end}} z^2 \cdot dt}{t_{end}}};}
\end{aligned}
\qquad [5.4]
$$

$$
\begin{aligned}
&\boxed{H_{z_1 \to \dot{z}_2} = j \cdot \omega \cdot H_{z_1 \to z_2};}\\
&H_{z_1 \to \ddot{z}_2} = j \cdot \omega \cdot j \cdot \omega \cdot H_{z_1 \to z_2} = -\omega^2 \cdot H_{z_1 \to z_2};\\
&H_{z_1 \to z_2 - z_3} = H_{z_1 \to z_2} - H_{z_1 \to z_3};
\end{aligned}
\qquad [5.13]
$$

4.4.3.1 Single Frequency Response

We used Fourier transform already in "*Module 4:7 Stationary oscillating steering*". Let's recap!

4.4.3.1.1 Solution with Fourier Transform

Eq [4.51] can be transformed to the frequency domain ($\mathcal{F}$ denotes Fourier transform, i.e. $\mathcal{F}(\xi(t)) = \int_{-\infty}^{\infty} e^{-j\cdot\omega\cdot t}\cdot\xi(t)\cdot dt$):

$$\begin{cases} j\cdot\omega\cdot\boldsymbol{\mathcal{F}}\left(\begin{bmatrix} v_y \\ \omega_z \end{bmatrix}\right) = \boldsymbol{A}\cdot\boldsymbol{\mathcal{F}}\left(\begin{bmatrix} v_y \\ \omega_z \end{bmatrix}\right) + \boldsymbol{B}\cdot\mathcal{F}(\delta_f); \\ \boldsymbol{\mathcal{F}}\left(\begin{bmatrix} \omega_z \\ a_y \end{bmatrix}\right) = \boldsymbol{C}\cdot\boldsymbol{\mathcal{F}}\left(\begin{bmatrix} v_y \\ \omega_z \end{bmatrix}\right) + \boldsymbol{D}\cdot\mathcal{F}(\delta_f); \end{cases}$$

Solving for states and outputs, using $\mathcal{F}(\dot{z}) = j\cdot\omega\cdot\mathcal{F}(z)$;, gives:

$$\begin{cases} \begin{bmatrix} v_y \\ \omega_z \end{bmatrix} = \mathcal{F}^{-1}\left(\boldsymbol{\mathcal{F}}\left(\begin{bmatrix} v_y \\ \omega_z \end{bmatrix}\right)\right) = \mathcal{F}^{-1}\left((j\cdot\omega\cdot\boldsymbol{I}-\boldsymbol{A})^{-1}\cdot\boldsymbol{B}\cdot\mathcal{F}(\delta_f)\right); \\ \begin{bmatrix} \omega_z \\ a_y \end{bmatrix} = \mathcal{F}^{-1}\left(\boldsymbol{\mathcal{F}}\left(\begin{bmatrix} \omega_z \\ a_y \end{bmatrix}\right)\right) = \mathcal{F}^{-1}\left(\boldsymbol{C}\cdot\boldsymbol{\mathcal{F}}\left(\begin{bmatrix} v_y \\ \omega_z \end{bmatrix}\right) + \boldsymbol{D}\cdot\mathcal{F}(\delta_f)\right); \end{cases}$$

Expressed as transfer functions:

$$\boldsymbol{H}_{\delta_f\to\begin{bmatrix} v_y \\ \omega_z \end{bmatrix}} = \frac{1}{\mathcal{F}(\delta_f)}\cdot\boldsymbol{\mathcal{F}}\left(\begin{bmatrix} v_y \\ \omega_z \end{bmatrix}\right) = \frac{(j\cdot\omega\cdot\boldsymbol{I}-\boldsymbol{A})^{-1}\cdot\boldsymbol{B}\cdot\mathcal{F}(\delta_f)}{\mathcal{F}(\delta_f)} = (j\cdot\omega\cdot\boldsymbol{I}-\boldsymbol{A})^{-1}\cdot\boldsymbol{B};$$

$$\boldsymbol{H}_{\delta_f\to\begin{bmatrix} \omega_z \\ a_y \end{bmatrix}} = \frac{1}{\mathcal{F}(\delta_f)}\cdot\boldsymbol{\mathcal{F}}\left(\begin{bmatrix} \omega_z \\ a_y \end{bmatrix}\right) = \boldsymbol{C}\cdot\boldsymbol{H}_{\delta_f\to\begin{bmatrix} v_y \\ \omega_z \end{bmatrix}} + \boldsymbol{D} = \boldsymbol{C}\cdot(j\cdot\omega\cdot\boldsymbol{I}-\boldsymbol{A})^{-1}\cdot\boldsymbol{B}+\boldsymbol{D}; \qquad [4.52]$$

We have derived the transfer functions. The subscript tells that the transfer function is for the vehicle operation with excitation=input= δ_f and response=output= $[v_y \quad \omega_z]^T$ and output= $[v_y \quad a_y]^T$. The transfer function has dimension 2x1 and is complex. It operates as follows:

$$\text{Amplitudes:}\begin{cases} \begin{bmatrix} \hat{v}_y \\ \hat{\omega}_z \end{bmatrix} = \left|H_{\delta_f\to\begin{bmatrix} v_y \\ \omega_z \end{bmatrix}}\right|\cdot\hat{\delta}_f; \\ \begin{bmatrix} \hat{\omega}_z \\ \hat{a}_y \end{bmatrix} = \left|H_{\delta_f\to\begin{bmatrix} \omega_z \\ a_y \end{bmatrix}}\right|\cdot\hat{\delta}_f; \end{cases} \qquad [4.53]$$

The same can be formulated with matrices and Fourier transforms:

$$\begin{bmatrix} m_s & 0 \\ 0 & m_u \end{bmatrix} \cdot \begin{bmatrix} \ddot{z}_s \\ \ddot{z}_u \end{bmatrix} + \begin{bmatrix} d_s & -d_s \\ -d_s & d_s + d_t \end{bmatrix} \cdot \begin{bmatrix} \dot{z}_s \\ \dot{z}_u \end{bmatrix} + \begin{bmatrix} c_s & -c_s \\ -c_s & c_s + c_t \end{bmatrix} \cdot \begin{bmatrix} z_s \\ z_u \end{bmatrix} = \begin{bmatrix} 0 \\ d_t \end{bmatrix} \cdot \dot{z}_r + \begin{bmatrix} 0 \\ c_t \end{bmatrix} \cdot z_r;$$

$$\Rightarrow$$

$$\Rightarrow \boldsymbol{M} \cdot \begin{bmatrix} \ddot{z}_s \\ \ddot{z}_u \end{bmatrix} + \boldsymbol{D} \cdot \begin{bmatrix} \dot{z}_s \\ \dot{z}_u \end{bmatrix} + \boldsymbol{C} \cdot \begin{bmatrix} z_s \\ z_u \end{bmatrix} = \boldsymbol{D_r} \cdot \dot{z}_r + \boldsymbol{C_r} \cdot z_r; \Rightarrow$$

$$\Rightarrow \boldsymbol{M} \cdot \left(-\omega^2 \cdot \begin{bmatrix} \mathcal{F}(z_s) \\ \mathcal{F}(z_u) \end{bmatrix}\right) + \boldsymbol{D} \cdot \left(j \cdot \omega \cdot \begin{bmatrix} \mathcal{F}(z_s) \\ \mathcal{F}(z_u) \end{bmatrix}\right) + \boldsymbol{C} \cdot \begin{bmatrix} \mathcal{F}(z_s) \\ \mathcal{F}(z_u) \end{bmatrix} =$$

$$= \boldsymbol{D_r} \cdot \left(j \cdot \omega \cdot \mathcal{F}(z_r)\right) + \boldsymbol{C_r} \cdot \mathcal{F}(z_r); \Rightarrow$$

$$\Rightarrow \left(-\omega^2 \cdot \boldsymbol{M} + j \cdot \omega \cdot \boldsymbol{D} + \boldsymbol{C}\right) \cdot \begin{bmatrix} \mathcal{F}(z_s) \\ \mathcal{F}(z_u) \end{bmatrix} = \left(j \cdot \omega \cdot \boldsymbol{D_r} + \boldsymbol{C_r}\right) \cdot \mathcal{F}(z_r);$$

[5.45]

5.4.3.1 Response to a Single Frequency Excitation

We can find the transfer functions via Fourier transform, starting from Eq [~~4.57~~]: [5.45]

$$\begin{bmatrix} H_{z_r \to z_s} \\ H_{z_r \to z_u} \end{bmatrix} = \begin{bmatrix} \mathcal{F}(z_s) \\ \mathcal{F}(z_u) \end{bmatrix} \cdot \frac{1}{\mathcal{F}(z_r)} = \left(-\omega^2 \cdot \boldsymbol{M} + j \cdot \omega \cdot \boldsymbol{D} + \boldsymbol{C}\right)^{-1} \cdot \left(j \cdot \omega \cdot \boldsymbol{D_r} + \boldsymbol{C_r}\right);$$

[5.46]

In Matlab: `H=inv(-w^2*M+1j*w*D+C)*(1j*w*Dr+Cr);`

Roads

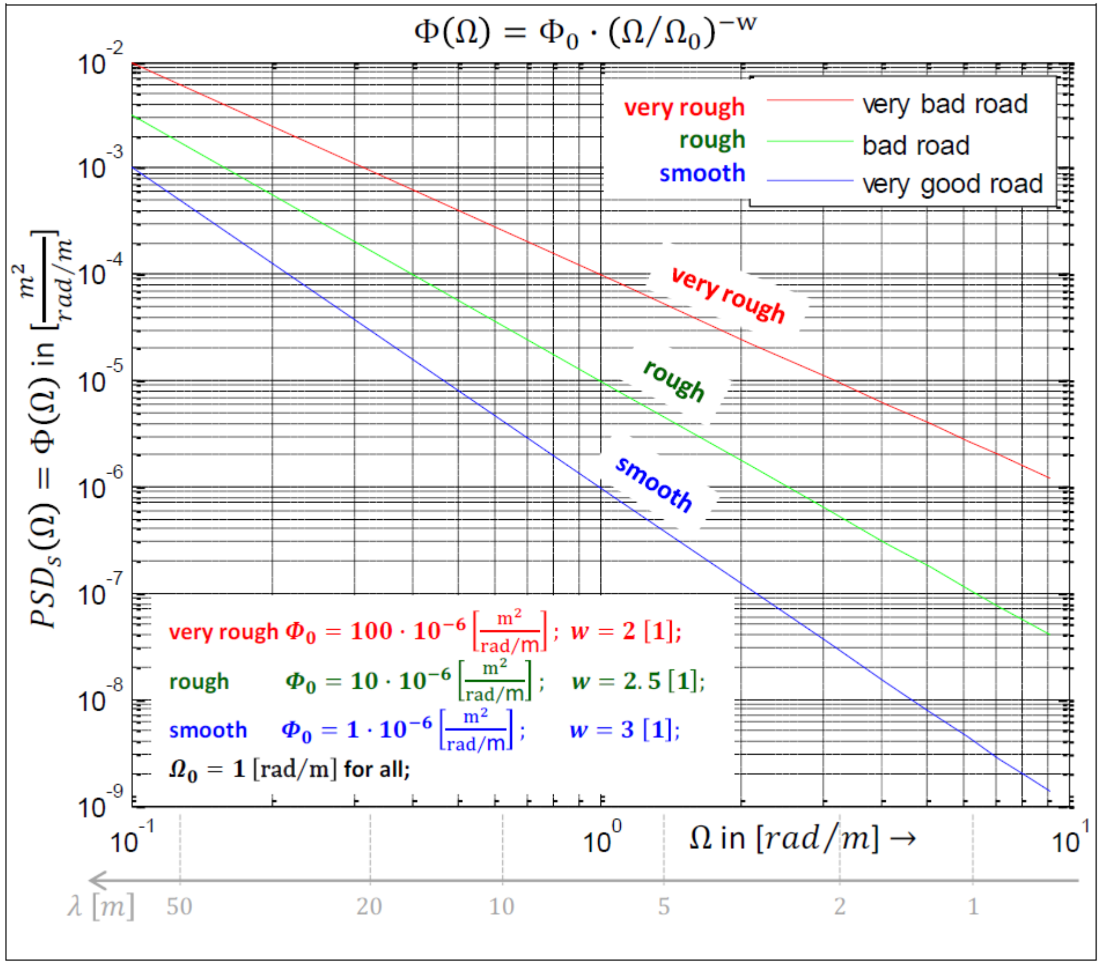


Figure 5-5: PSD spectra for the three typical roads in Figure 5-4.

Humans

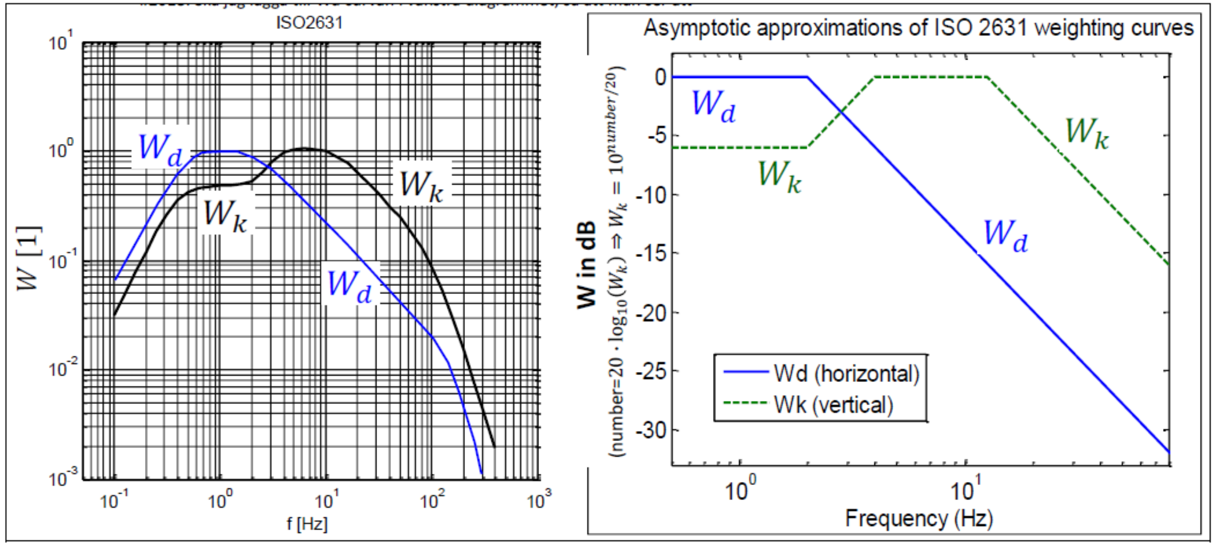
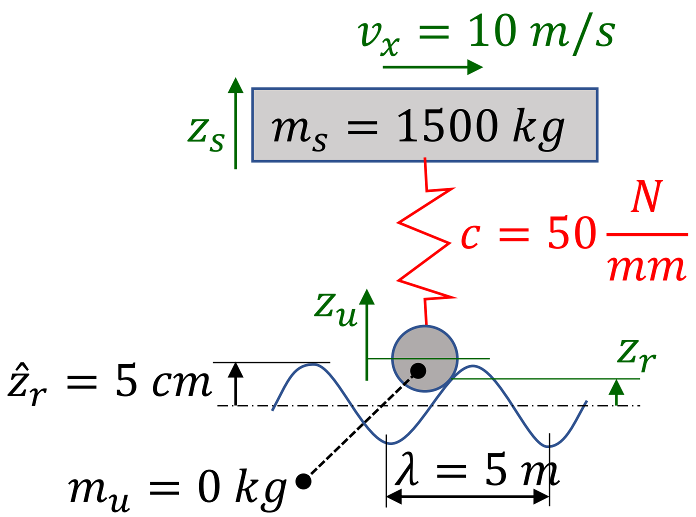


Figure 5-20: Human Sensitivity Filter Function. From (ISO 2631). Right: Asymptotic approximation

"Micro Problem", p1(3)



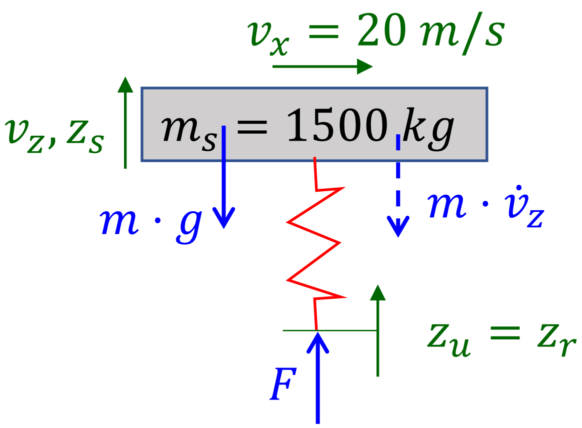
Task: Calculate $RMS(\ddot{z}_s)$

(Because it is a established measure for vertical ride comfort…)

I leave this problem, with solution, as a self-study problem.
Not sure that you need it, but take a look in case you get stuck in DesignTask3.

...continuation of "Micro Problem", p2(3)

Physical model:



Mathematical model:

Equilibrium: $0 = F - m \cdot g - m \cdot \ddot{z}_s$;

Constitution: $F = m \cdot g + c \cdot (z_u - z_s)$;

Compatibility: $z_u = z_r$; $\dot{z}_s = v_z$;

Excitation: $z_r = \hat{z}_r \cdot \sin\left(\frac{2 \cdot \pi}{\lambda} \cdot t\right)$;

Explicit form model:

$z_r \leftarrow \hat{z}_r \cdot \sin\left(\frac{2 \cdot \pi \cdot v_x}{\lambda} \cdot t\right)$;

$z_u \leftarrow z_r$;

$F \leftarrow m \cdot g + c \cdot (z_u - z_s)$;

$\dot{v}_z \leftarrow F/m - g$;

$\dot{z}_s \leftarrow v_z$;

...so, model is complete...

...but we shall not simulate.

Instead frequency analysis.

We manipulate to a single state 2nd order ODE (one single eq): Eliminate F and z_u:

$$m \cdot \ddot{z}_s + c \cdot z_s = c \cdot \hat{z}_r \cdot \sin\left(\frac{2 \cdot \pi \cdot v_x}{\lambda} \cdot t\right); \quad [1]$$

And we keep the excitation as general (z_r instead of $\hat{z}_r \cdot \sin\left(\frac{2 \cdot \pi \cdot v_x}{\lambda} \cdot t\right)$):

$$m \cdot \ddot{z}_s + c \cdot z_s = c \cdot z_r; \quad [2]$$

…continuation of “Micro Problem”, p3(3)

Computation

We look for $RMS(\ddot{z}_s) = \frac{|\hat{\ddot{z}}_s|}{\sqrt{2}}$.

From linear systems, we know/understand that the response is harmonic with same frequency as excitation:
$z_s = \hat{z}_s \cdot \sin(\omega \cdot t + \varphi)$, so that $|\hat{\ddot{z}}_s| = \omega^2 \cdot |\hat{z}_s|$.

So, we look for $|\hat{z}_s|$.
(We could find $\hat{z}_s$ with an ansatz $z_s = \hat{z}_s \cdot \sin(\omega \cdot t + \varphi)$ and insert in differential equation [1]. But we will use Fourier transform instead, because it can handle larger systems easier.)
Fourier transform the differential equation [2]:

$$\mathcal{F}(m \cdot \ddot{z}_s + c \cdot z_s) = \mathcal{F}(c \cdot z_r);$$
$$m \cdot \mathcal{F}(\ddot{z}_s) + c \cdot \mathcal{F}(z_s) = c \cdot \mathcal{F}(z_r);$$
$$m \cdot (j \cdot \omega)^2 \cdot \mathcal{F}(z_s) + c \cdot \mathcal{F}(z_s) = c \cdot \mathcal{F}(z_r);$$
$$(-m \cdot \omega^2 + c) \cdot \mathcal{F}(z_s) = c \cdot \mathcal{F}(z_r);$$
$$\frac{\mathcal{F}(z_s)}{\mathcal{F}(z_r)} = \frac{c}{c - m \cdot \omega^2} = H(\omega) = H_{z_r \to z_s};$$

Fourier transform the differential equation [2]:

$$|\hat{z}_s| = |H_{z_r \to z_s} \cdot \hat{z}_r| = \left|\frac{c}{c - m \cdot \omega^2} \cdot \hat{z}_r\right| = \left|\frac{c}{c - m \cdot \left(\frac{2 \cdot \pi \cdot v_x}{\lambda}\right)^2} \cdot \hat{z}_r\right| = \left|\frac{50000}{50000 - 1500 \cdot \left(\frac{2 \cdot \pi \cdot 10}{\lambda}\right)^2} \cdot 0.05\right| = |-0.0134| = 0.0134[m];$$

Now, calculate RMS: $RMS(\ddot{z}_s) = \frac{|\hat{\ddot{z}}_s|}{\sqrt{2}} = \frac{\omega^2 \cdot |\hat{z}_s|}{\sqrt{2}} = \frac{\left(\frac{2 \cdot \pi \cdot 10}{5}\right)^2 \cdot 0.0134}{\sqrt{2}} = 1.4938\ [m/s^2]$.

Reflection: Rather uncomfortable, although the vehicle speed is low (10 m/s=36 km/h). This is because suspension lacks dampers (!) and excitation frequency ($= f = v_x/\lambda = 10/5 = 2\,Hz$) is close to eigenfrequency ($f_e = \sqrt{c/m} \cdot\ 2 \cdot \pi \approx 1\ Hz$).