MMF062 Vehicle motion engineering

Design Task 1

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Abstract

Vehicle motion engineering is used to formulate equations and understanding of a vehicles motion. This report is studying the vehicles behavior in longitudinal axis. A model is used to describe the behaviour of a car during different conditions. The car is tested during dry asphalt, wet asphalt and ice with front wheel drive and rear wheel drive. The performance is measured in the quickest possible time for a 100m dra race at a 8 degree slope.

1 Task1

1.1

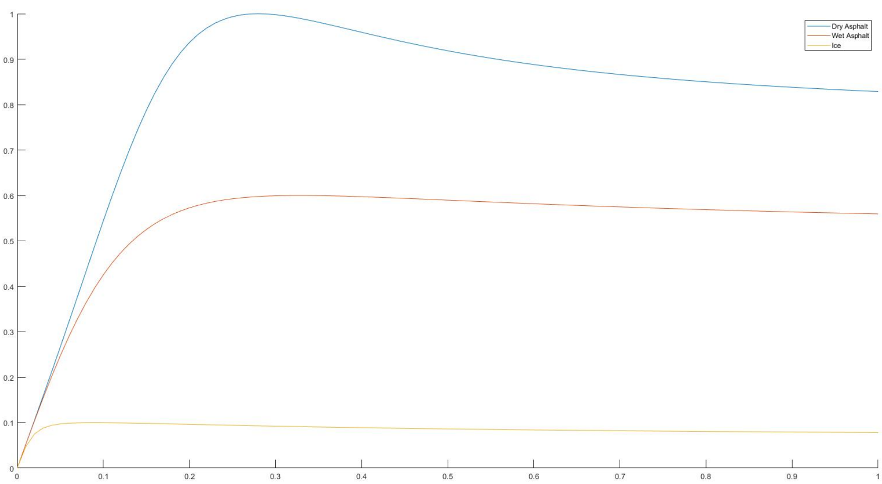

Figure 1: Tyre normalized traction force vs slip for Dry Asphalt, Wet Asphalt and Ice.

1.2 b

To manipulate the data we started by testing one coefficient at a time to find the behaviour of it. The final numbers came to $E = -4$, $D = 0.952$ and $C = 1.5$. By varying The E moves the peak of the curve in x direction. The D coefficient decides the angle after the peak(if $C > 1$). The D variable decides the height of the peak in Y direction.

1.3 1

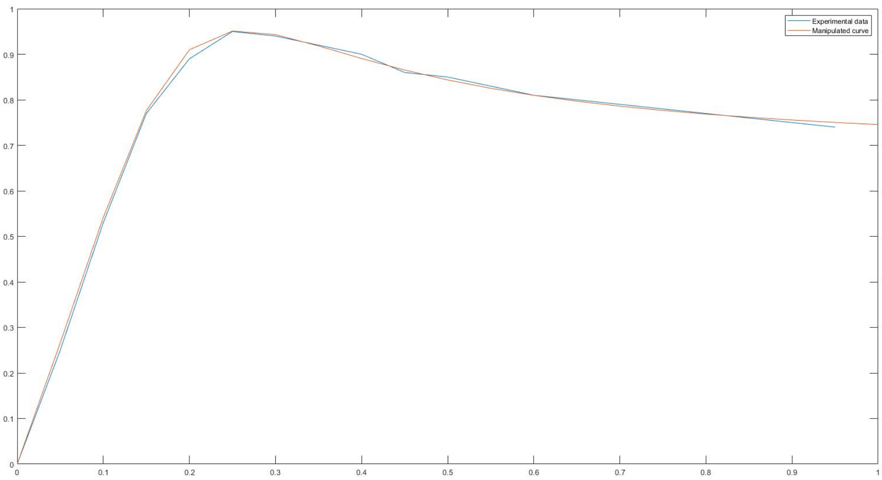

Figure 2: Experimental data vs manipulated data

2 Task2

2.1 a

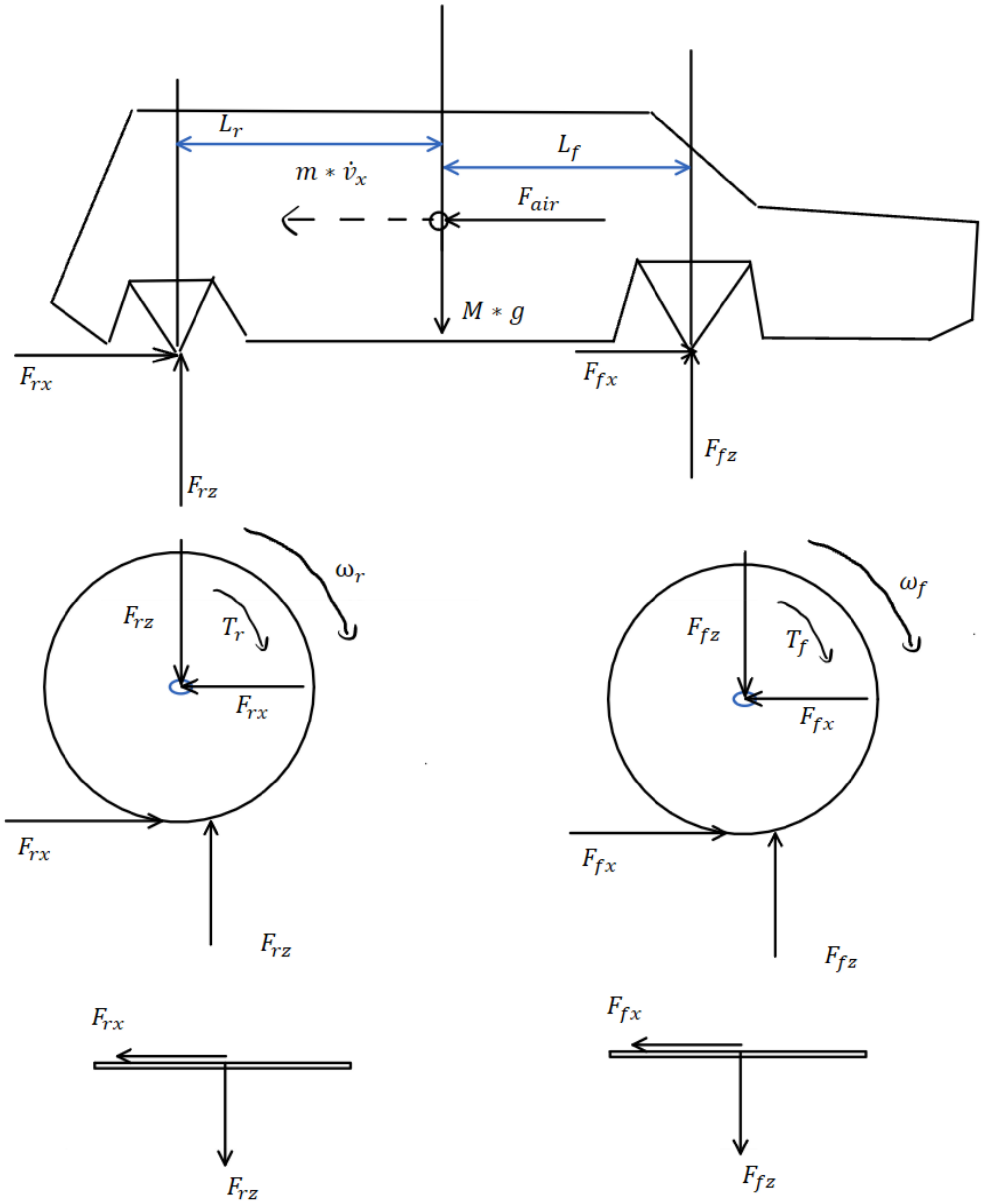

Figure 3: Free body diagram with 0 degree of slope

2.2 b

Derive the equations for the front/rear axles and the complete vehicle. Assuming that the vehicle is driving on a flat road, slope=0.
Equations are as following:
Equilibrium equation for the whole vehicle:

$$0 = F_{\mathrm{fz}} + F_{\mathrm{rz}} - Mg \tag{1}$$

$$M\,\dot{v}_x = F_{\mathrm{fx}} + F_{\mathrm{rx}} - \mathrm{F}_{\mathrm{air}} \tag{2}$$

Moment equilibrium around front contact with groud:

$$0 = F_{\mathrm{rz}}\,(L_f + L_r) - L_f\,M\,g\; - \;h_{air}F_{\mathrm{air}} + hM\,\dot{v}_x \tag{3}$$

Equilibrium for the front axle, around wheel center

$$0 = T_f - J\,\dot{\omega}_f - F_{\mathrm{fx}}\,R - F_{\mathrm{fz}}\,R\,f_r \tag{4}$$

Equilibrium for the rear axle, around wheel center

$$0 = T_r - J\,\dot{\omega}_r - F_{\mathrm{rx}}\,R - F_{\mathrm{rz}}\,R\,f_r \tag{5}$$

Traction force and normal force relation:

$$F_{\mathrm{fx}} = F_{\mathrm{fz}}\,\mu_f \tag{6}$$

$$F_{\mathrm{rx}} = F_{\mathrm{rz}}\,\mu_r \tag{7}$$

f_r is the rolling resistance coefficient. μ_f,μ_r are the coefficient of friction for front and rear wheels respectively, calculated by magic tire formula.

2.3 c

For this task, the vehicle will drive up a slop with the road inclination angle θ. The new three-body diagram and equations are as follows.

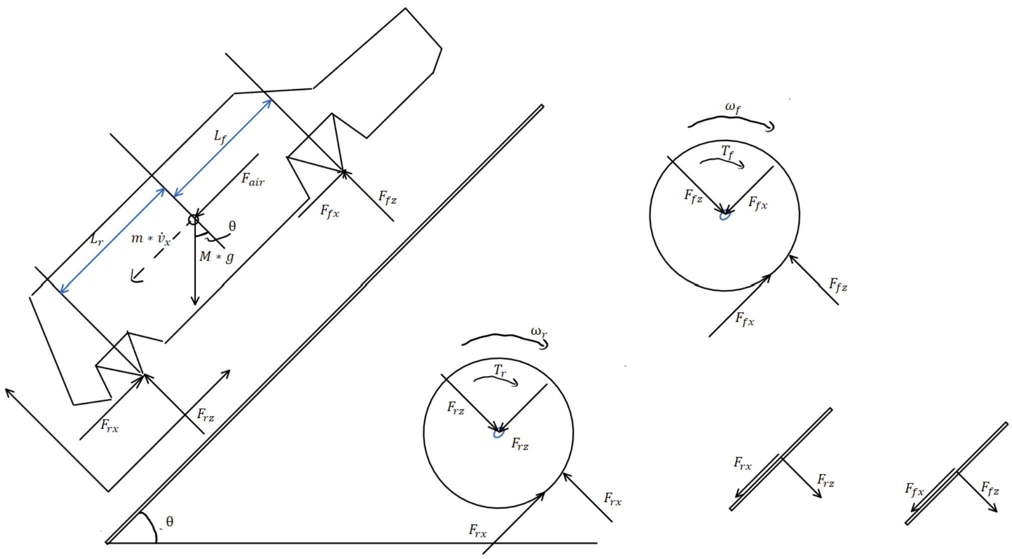

Figure 4: Free body diagram with slope

$$0 = F_{\text{fz}} + F_{\text{rz}} - M\,g\,\cos(\theta) \tag{8}$$

$$M\,\dot{v}_x = F_{\text{fx}} + F_{\text{rx}} - \mathrm{F}_{\text{air}} - M\,g\,\sin(\theta) \tag{9}$$

$$0 = F_{\text{rz}}\,(L_f + L_r) - h\,(M\,\dot{v}_x + M\,g\,\sin(\theta)) - F_{\text{air}}\,h_{\text{air}} - L_f\,M\,g\,\cos(\theta) \tag{10}$$

$$0 = T_f - J\,\dot{\omega}_f - F_{\text{fx}}\,R - F_{\text{fz}}\,R\,f_r \tag{11}$$

$$0 = T_r - J\,\dot{\omega}_r - F_{\text{rx}}\,R - F_{\text{rz}}\,R\,f_r \tag{12}$$

$$F_{\text{fx}} = F_{\text{fz}}\,\mu_f \tag{13}$$

$$F_{\text{rx}} = F_{\text{rz}}\,\mu_r \tag{14}$$

2.4 d

Using MATLAB function *solve* to solve the equations above, we get the results as shown below.

$$
\begin{aligned}
F_{\mathrm{fx}} &= \frac{\mu_f\,(F_{\mathrm{air}}\,h - F_{\mathrm{air}}\,h_{\mathrm{air}} + L_r\,M\,g\,\cos(\theta) - M\,g\,h\,\mu_r\,\cos(\theta))}{L_f + L_r + h\,\mu_f - h\,\mu_r}\\
F_{\mathrm{rx}} &= \frac{\mu_r\,(F_{\mathrm{air}}\,h_{\mathrm{air}} - F_{\mathrm{air}}\,h + L_f\,M\,g\,\cos(\theta) + M\,g\,h\,\mu_f\,\cos(\theta))}{L_f + L_r + h\,\mu_f - h\,\mu_r}\\
F_{\mathrm{fz}} &= \frac{F_{\mathrm{air}}\,h - F_{\mathrm{air}}\,h_{\mathrm{air}} + L_r\,M\,g\,\cos(\theta) - M\,g\,h\,\mu_r\,\cos(\theta)}{L_f + L_r + h\,\mu_f - h\,\mu_r}\\
F_{\mathrm{rz}} &= \frac{F_{\mathrm{air}}\,h_{\mathrm{air}} - F_{\mathrm{air}}\,h + L_f\,M\,g\,\cos(\theta) + M\,g\,h\,\mu_f\,\cos(\theta)}{L_f + L_r + h\,\mu_f - h\,\mu_r}\\
\dot{\omega}_f &= \frac{\begin{array}{c} L_f\,T_f + L_r\,T_f + T_f\,h\,\mu_f - T_f\,h\,\mu_r - F_{\mathrm{air}}\,R\,f_r\,h + F_{\mathrm{air}}\,R\,f_r\,h_{\mathrm{air}} - F_{\mathrm{air}}\,R\,h\,\mu_f + F_{\mathrm{air}}\,R\,h_{\mathrm{air}}\,\mu_f \\ - L_r\,M\,R\,f_r\,g\,\cos(\theta) - L_r\,M\,R\,g\,\mu_f\,\cos(\theta) + M\,R\,f_r\,g\,h\,\mu_r\,\cos(\theta) + M\,R\,g\,h\,\mu_f\,\mu_r\,\cos(\theta)\end{array}}{J\,(L_f + L_r + h\,\mu_f - h\,\mu_r)}\\
\dot{\omega}_r &= -\frac{\begin{array}{c} T_r\,h\,\mu_r - L_r\,T_r - T_r\,h\,\mu_f - L_f\,T_r - F_{\mathrm{air}}\,R\,f_r\,h + F_{\mathrm{air}}\,R\,f_r\,h_{\mathrm{air}} - F_{\mathrm{air}}\,R\,h\,\mu_r + F_{\mathrm{air}}\,R\,h_{\mathrm{air}}\,\mu_r + \\ L_f\,M\,R\,f_r\,g\,\cos(\theta) + L_f\,M\,R\,g\,\mu_r\,\cos(\theta) + M\,R\,f_r\,g\,h\,\mu_f\,\cos(\theta) + M\,R\,g\,h\,\mu_f\,\mu_r\,\cos(\theta)\end{array}}{J\,(L_f + L_r + h\,\mu_f - h\,\mu_r)}\\
\dot{v}_x &= -\frac{\begin{array}{c} F_{\mathrm{air}}\,L_f + F_{\mathrm{air}}\,L_r + F_{\mathrm{air}}\,h_{\mathrm{air}}\,\mu_f - F_{\mathrm{air}}\,h_{\mathrm{air}}\,\mu_r + L_f\,M\,g\,\sin(\theta) + L_r\,M\,g\,\sin(\theta) - \\ L_f\,M\,g\,\mu_r\,\cos(\theta) - L_r\,M\,g\,\mu_f\,\cos(\theta) + M\,g\,h\,\mu_f\,\sin(\theta) - M\,g\,h\,\mu_r\,\sin(\theta)\end{array}}{M\,(L_f + L_r + h\,\mu_f - h\,\mu_r)}
\end{aligned}
\tag{15}
$$

To remove the effect of the F_{air} we need it to have the same height as the CoG which is h. This would cancel the effect of F_{air} on the load transfer. These are the following equations we get after canceling out the F_{air}.

$$
\begin{aligned}
F_{\text{fx}} &= \frac{M\,g\,\mu_f\,\cos(\theta)\,\left(L_r - h\,\mu_r\right)}{L_f + L_r + h\,\mu_f - h\,\mu_r}\\
F_{\text{rx}} &= \frac{M\,g\,\mu_r\,\cos(\theta)\,\left(L_f + h\,\mu_f\right)}{L_f + L_r + h\,\mu_f - h\,\mu_r}\\
F_{\text{fz}} &= \frac{F_{\text{air}}\,h - F_{\text{air}}\,h_{\text{air}} + L_r\,M\,g\,\cos(\theta) - M\,g\,h\,\mu_r\,\cos(\theta)}{L_f + L_r + h\,\mu_f - h\,\mu_r}\\
F_{\text{rz}} &= \frac{F_{\text{air}}\,h_{\text{air}} - F_{\text{air}}\,h + L_f\,M\,g\,\cos(\theta) + M\,g\,h\,\mu_f\,\cos(\theta)}{L_f + L_r + h\,\mu_f - h\,\mu_r}\\
\dot{\omega}_f &= \frac{\begin{array}{c} L_f\,T_f + L_r\,T_f + T_f\,h\,\mu_f - T_f\,h\,\mu_r - L_r\,M\,R\,f_r\,g\,\cos(\theta)\\ - L_r\,M\,R\,g\,\mu_f\,\cos(\theta) + M\,R\,f_r\,g\,h\,\mu_r\,\cos(\theta) + M\,R\,g\,h\,\mu_f\,\mu_r\,\cos(\theta)\end{array}}{J\left(L_f + L_r + h\,\mu_f - h\,\mu_r\right)}\\
\dot{\omega}_r &= -\frac{\begin{array}{c} T_r\,h\,\mu_r - L_r\,T_r - T_r\,h\,\mu_f - L_f\,T_r + L_f\,M\,R\,f_r\,g\,\cos(\theta)\\ + L_f\,M\,R\,g\,\mu_r\,\cos(\theta) + M\,R\,f_r\,g\,h\,\mu_f\,\cos(\theta) + M\,R\,g\,h\,\mu_f\,\mu_r\,\cos(\theta)\end{array}}{J\left(L_f + L_r + h\,\mu_f - h\,\mu_r\right)}\\
\dot{v}_x &= -\frac{\begin{array}{c} F_{\text{air}}\,L_f + F_{\text{air}}\,L_r + F_{\text{air}}\,h\,\mu_f - F_{\text{air}}\,h\,\mu_r + L_f\,M\,g\,\sin(\theta) + L_r\,M\,g\,\sin(\theta)\\ - L_f\,M\,g\,\mu_r\,\cos(\theta) - L_r\,M\,g\,\mu_f\,\cos(\theta) + M\,g\,h\,\mu_f\,\sin(\theta) - M\,g\,h\,\mu_r\,\sin(\theta)\end{array}}{M\left(L_f + L_r + h\,\mu_f - h\,\mu_r\right)}
\end{aligned}
\tag{16}
$$

2.5 e

```
function slip=Sub_wheel_slip(v,w,CONST)

slip= (CONST.R * w - v)/abs(CONST.R * w);

end
```

3 Task3

The following graphs are given at a 8 degree slope on dry asphalt. The acceleration and speed looks normal to a Saab 93. We can compare the F_z plot where we see the force on the rear tyre is increased in the beginning of the run where the acceleration is high constant until 4 seconds into the run where 2nd gear is chosen. Now the acceleration is less due to the car demanding a higher gearing ratio for higher speed and it can be reflected in that the load on the rear tyre is less and closer to the static load distribution.

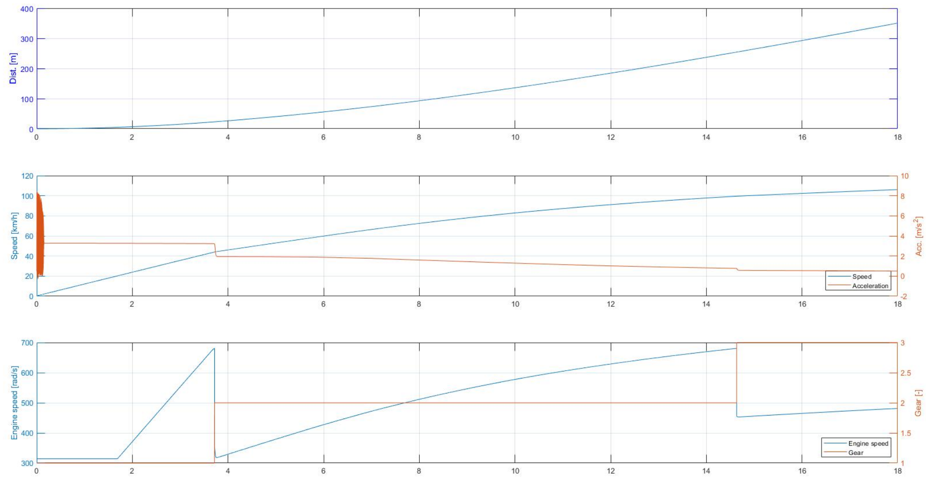

Figure 5: Dry Fwd 8 degrees of road gradient

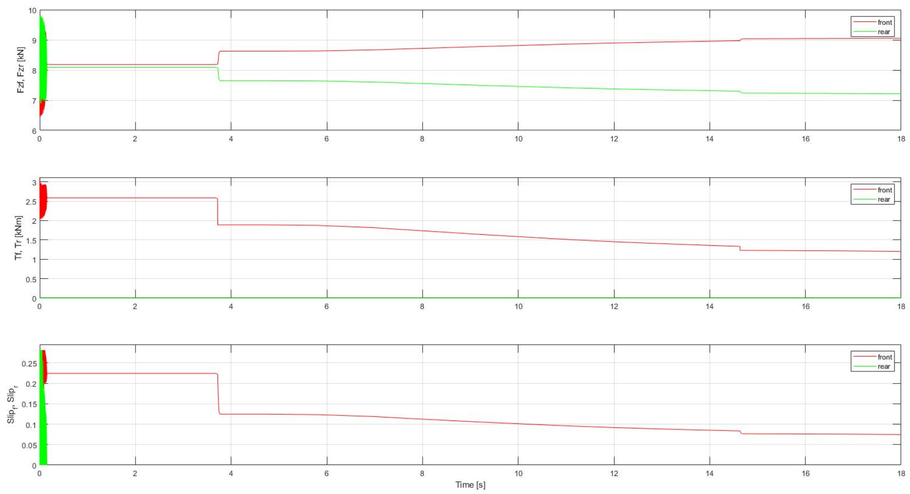

Figure 6: Dry Fwd 8 degrees of road gradient

Another scenario that proves the fidelity of the model is the following. The car is put on a icy road where the friction is low and with a 8 degree slope we can see that the traction force is not enough to keep it going forward and the velocity can be seen going negative due to the slope being too step for the car and it slides backwards.

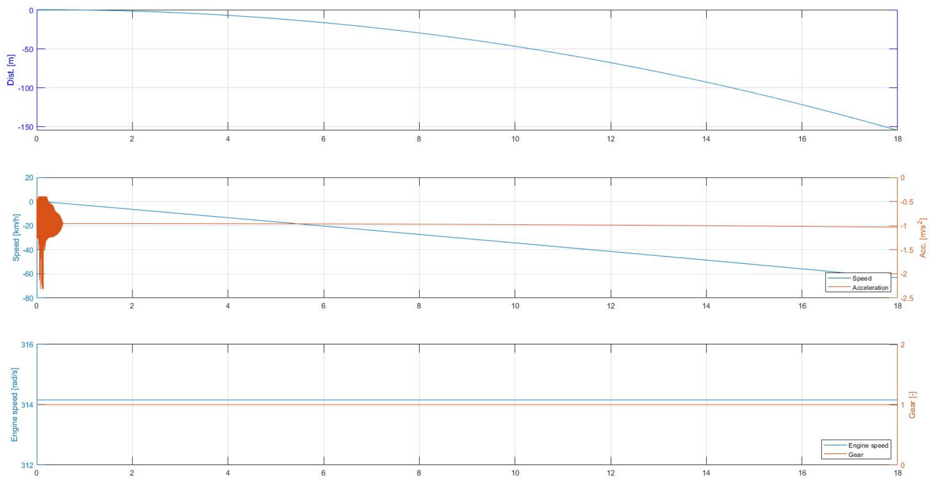

Figure 7: Ice Fwd 8 degrees of road gradient

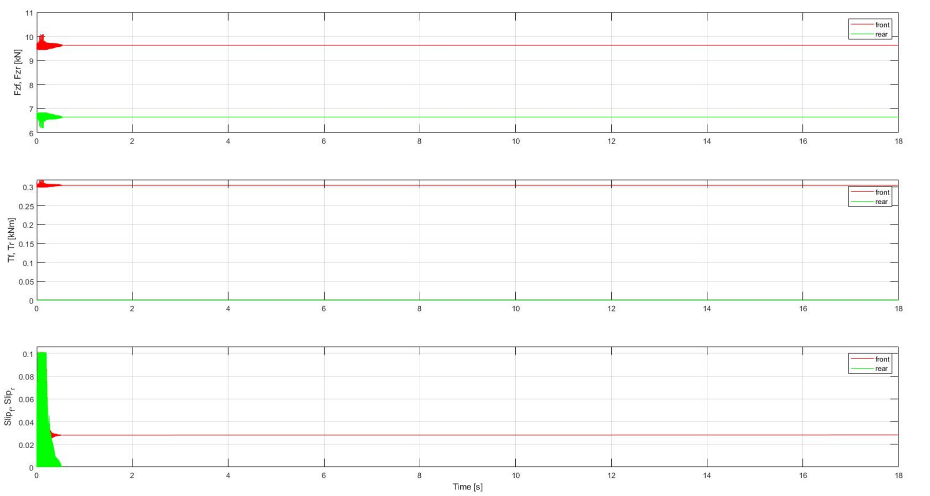

Figure 8: Ice Fwd 8 degrees of road gradient

Comparing this to the following case where the slope is 0 degrees but still on ice. The car is now able to get a positive acceleration and get rolling.

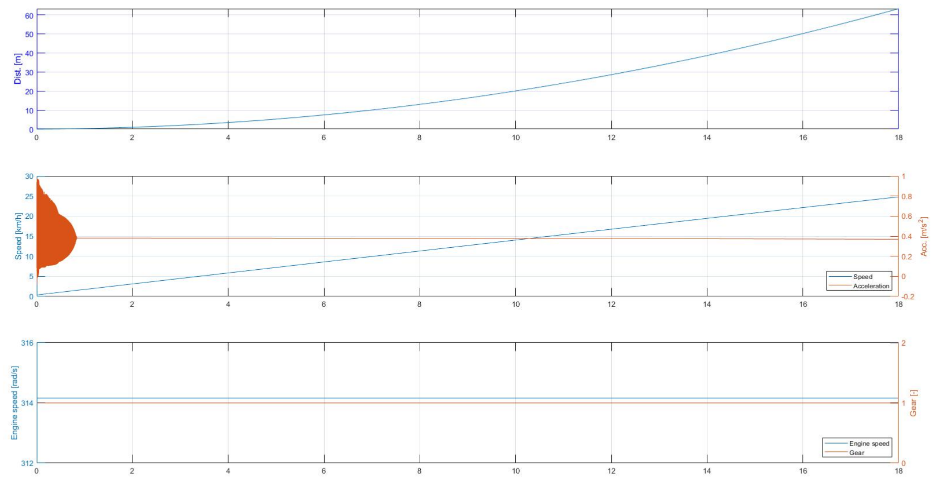

Figure 9: Ice Fwd 0 degrees of road gradient

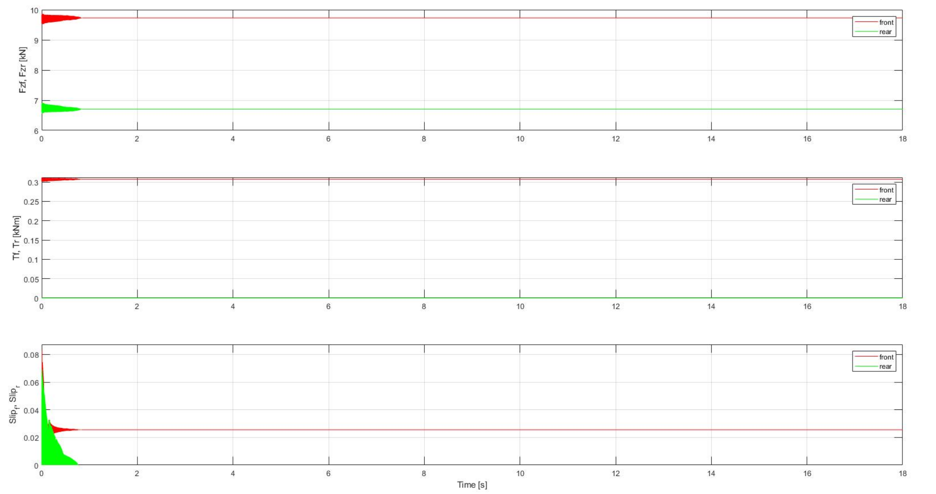

Figure 10: Ice Fwd 0 degrees of road gradient

3.1 b

We got the following times in dry, wet and on ice.

Surface	RWD	FWD
Dry	8.42	8.35
Wet	13.71	11.30
Ice	0	0

This shows that the FWD car is faster in all cases but RWD is very close in the dry. The reason RWD is closer in the dry is due to the acceleration is higher when the friction is higher and with the higher acceleration we are able to produce a bigger load transfer. The load transfer increases the F_z on the rear tyre and makes it possible to output a greater force without exceeding the optimal slip ratio. It follows the graph below where Fz is N.

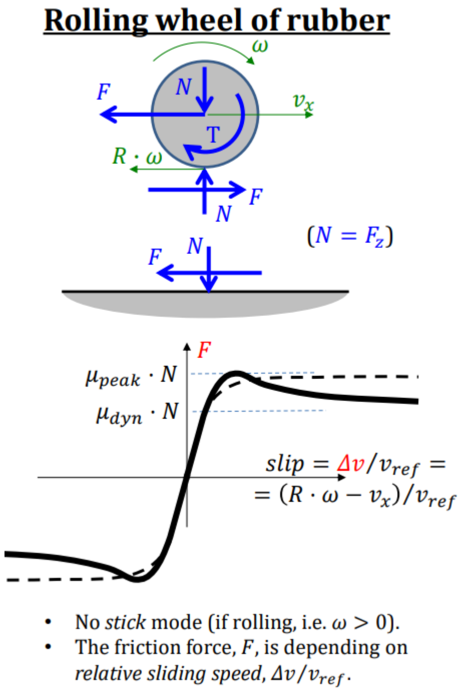

Figure 11: Reference to compendium page 129 of force vs slip ratio curve

This can be proven by moving the CoG from being in the front to being in the middle of the car. We now get the following times in the 100m drag race.

Surface	RWD	FWD
Dry	7.6	9.15
Wet	10.7	13.58
Ice	0	0

It is now clearly seen that the RWD is much faster in the dry due to a higher F_z on the rear during acceleration and it can output more force without slipping, the same is happening for the wet. As an exampe, most RWD cars such as a BMW E46 M3 has a weight distribution of very close to 50/50 due to the differentials, drive axles and more is moved towards the rear of the car compared to a FWD car. This shows that the drive wheel configuration heavily influences the weight distribution. Therefore testing the platform FWD vs RWD but without moving the weight distribution it becomes highly unrealistic.

But in the case that we are able to use RWD or FWD and keeping the original weight distribution then FWD car is faster due to the heavy front and a higher F_z in the front.