

# Design Task 1 (10 points)

## Longitudinal Dynamics

### Introduction

In this design task **you shall propose front or rear wheel drive** for a two-axle driven passenger vehicle. The vehicle should have the best possible acceleration performance during different uphill gradients and different road friction conditions.

This design task will give some insights how vehicle and tires can be modelled for evaluation of vehicle longitudinal performance. You will carry out:

- Modelling of road surface conditions (dry, wet, ice) using a longitudinal slip tyre model
- Modelling longitudinal vehicle dynamics using equations of motion (incl. longitudinal load transfer and resistance forces)
- Modelling in steps “Physical model”, “Mathematical model” and “Explicit form model” including how to document those steps.
- Simulation using ordinary differential equation (ODE) integration method
- Evaluation of vehicle acceleration performance from the simulation results

For education purpose the used vehicle model contains several simplifications.

Before you start with a task, please read through the whole description and make sure that you consider all the given information!

### Task 1: Tire Characteristics on different Road Conditions (1p)

#### a) Plot tyre model (0.5p)

Plot the tyre’s normalised traction force (sometimes called utilized friction) for  $0 < s_x = \text{slip} < 1 = 100\%$ ; what is the physical meaning of “longitudinal slip”? Use the tyre model “Magic Tyre”:

$$\frac{F_x}{F_z} = D \cdot \sin\left(C \cdot \arctan\left(B \cdot s_x - E \cdot (B \cdot s_x - \arctan(B \cdot s_x))\right)\right) \quad (\text{Eq. 1})$$

$$\text{With:} \quad K = \frac{3 \cdot \pi}{180} \quad (\text{Eq. 2})$$

$$\text{And:} \quad B = 100 \cdot \frac{\arctan(K)}{C \cdot D} \quad (\text{Eq. 3})$$

The parameters for the different road conditions:

	D	C	E
<b>Dry Asphalt</b>	1.00	1.45	-4.00
<b>Wet Asphalt</b>	0.60	1.35	-0.20
<b>Ice</b>	0.10	1.50	0.80

## b) Plot experimental tyre behaviour (0.5p)

Plot the following friction  $\mu$ -slip experimental data and find the C, D, and E coefficients to fit the data using the Magic Formula. Use the definitions for K and B above. The value for E is between -0.5 and -4. Illustrate the influence of the factor E by plotting  $E=0$  and  $E=-4$  for the data while having C and D constant at the best fit values.

To find the coefficients C, D, and E it is recommended to manipulate manually the values to find the best curve fitment.

Experimental data:

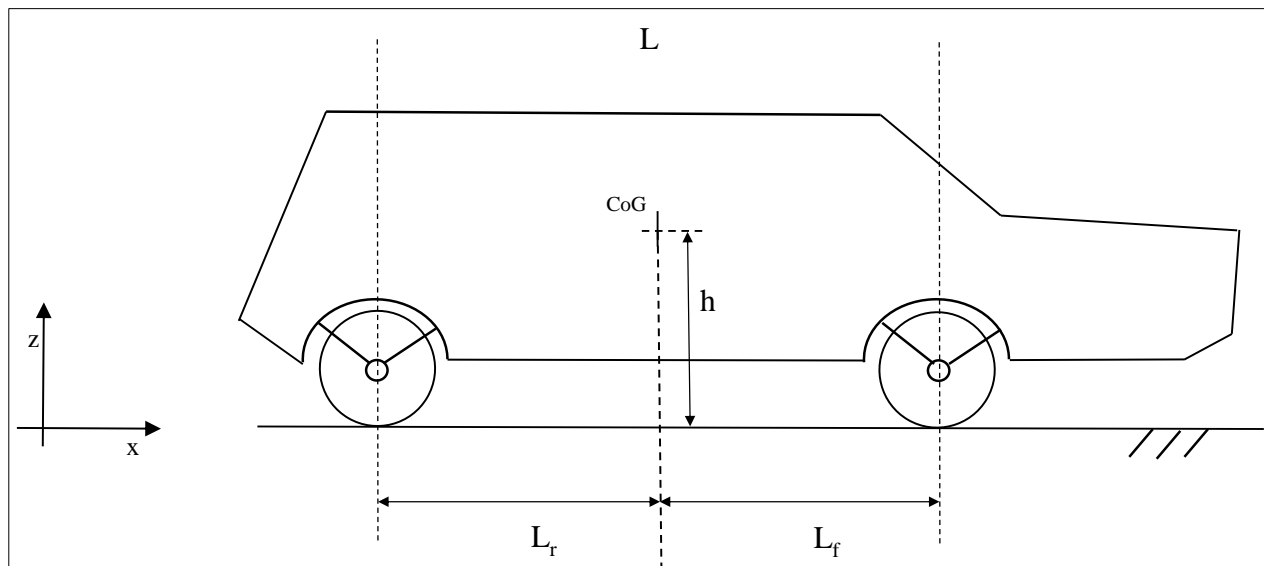
Slip, $s_x$	0.0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45
Normalized longitudinal tyre force, $F_x/F_z$	0.0	0.25	0.53	0.77	0.89	0.95	0.94	0.92	0.90	0.86

Slip, $s_x$	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
Normalized longitudinal tyre force, $F_x/F_z$	0.85	0.83	0.81	0.80	0.79	0.78	0.77	0.76	0.75	0.74

## Task 2: Vehicle Model (5p)

### Task 2a: Physical model (incl. free body diagram) (1p)

Draw a free body diagram of the following vehicle with the mass M, assuming that each axle (front and rear) have **one wheel** (the vehicle is driving forward towards the right).



The free body diagram should separate into the chassis, the wheels and the ground. Assume for the wheels that translational inertia (mass) is negligible compared to the rotational inertia  $J$  and that a rolling resistance is present (the horizontal offset  $e$  of vertical wheel force to hub is very small compared to distance of CoG to axes), and air resistance should be considered. The vehicle has a rigid suspension, thus there is

neither heave nor pitch. The propulsion torques of each axle and the relation between slip and normalized longitudinal tyre force,  $f_x(s_x)$ , for both front and rear tyres should be regarded as given.

**Show** and list all relevant variables and parameters in the model that are necessary to derive the equations of motion and denote them in a nomenclature.

### Task 2b: Mathematical model (1p)

Derive the equations of motion from the free body diagram for the front axle, rear axle, and complete vehicle. Which additional equations need to be considered to get as many equations as unknown variables? *Hint: How can the traction force be calculated?*

The unknown variables you have to describe with the equations are:

- Traction force on front axis,  $F_{fx}$
- Traction force on rear axis,  $F_{rx}$
- Normal force on front axis,  $F_{fz}$
- Normal force on rear axis,  $F_{rz}$
- Front wheel rotational acceleration,  $\dot{\omega}_f$
- Rear wheel rotational acceleration,  $\dot{\omega}_r$
- Vehicle acceleration,  $\dot{v}$

Derive all equations of motion respectively for the front axle, rear axle, and complete vehicle. Ensure that the equations you derive are independent of each other, so the equation system becomes solvable.

### Task 2c: Update models with road gradient (1p)

The vehicle is driving up a slope with the road inclination angle  $\theta$ . Rotate the x,z-coordinate system with the vehicle. Draw a new free-body diagram and update the mathematical model (the equations).

### Task 2d: Solve the equations (1p)

Solve the equations of motion that you got in task 2c to get explicit expressions for  $F_{fx}$ ,  $F_{rx}$ ,  $F_{fz}$ ,  $F_{rz}$ ,  $\dot{\omega}_f$ ,  $\dot{\omega}_r$  and  $\dot{v}$ .

What is the influence of  $F_{air}$  on load ( $F_z$ ) transfer between axles? How can it be eliminated? Show by equations.

You are allowed to use symbolic toolbox of Matlab to solve the system of equations.

### Task 2e: Integrate the equations (1p)

Update the Matlab function "Sub\_wheel\_slip" so that it calculates the slip. The inputs to this function are vehicle speed,  $v$ , tyre angular speed,  $\omega$ , and tyre radius, CONST.R.

## Task 3: Simulation of Longitudinal Dynamics (4p)

This simulation study will give some insights into a vehicle's longitudinal dynamics and limitations of acceleration performance on slopes and low friction road surface. A set of MATLAB scripts are given which provide a template for a two-axle model of a vehicle.

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The program can be started from the file 'Simulation\_main.m'.

*Read through the code so that you understand the different m-files and are able to modify them!*

You are the chief engineer of a racing team that wants to compete in a new series of acceleration performance competitions. For this racing there is the additional requirement that only serial production vehicles in a specific horse power class may be used! The vehicle you have available is a Saab 9-3.

### **Task 3a: Implement and validate model (1p)**

Implement the solution to the equations into the Matlab model, in the file 'Sub\_vehicle\_dynamics.m'. Run simulations and verify that the results are plausible. This means that you present the model output in some scenario and explain whether it makes sense or not. Just checking that the model runs without error messages is not a verification of the results.

### **Task 3b: Select between FWD and RWD (3p)**

Now you are going to prepare the car for the competition. The track is 100 m long and has an uphill slope of 8 degrees. Before the race you are able to configure the vehicle as either front wheel drive or rear wheel drive. However, there is a high risk of rain and in that case the road surface would be wet.

Identify the optimum drive configuration to drive the track in shortest possible time. You will need to consider both wet and dry road surface to be fully prepared. Is the setting the same in both wet and dry? Explain why or why not.