SSY281 Model Predictive Control 2023
PSS 3 - MPT: Invariant and Reachable Sets

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Lecture refresh

• Consider the autonomous system:

$$x(k+1) = Ax(k), \tag{1}$$

with constraints $x(k) \in \mathbb{X}, \forall k$.

The (1-)backward reachable set of $\mathbb{X}$ is:

$$\text{pre}(\mathbb{X}) = \{x \in \mathbb{R}^n | Ax \in \mathbb{X}\}. \tag{2}$$

Likewise, the (1-)forward reachable set of $\mathbb{X}$ is:

$$\text{reach}(\mathbb{X}) = \{Ax | x \in \mathbb{X}\}. \tag{3}$$

• Now, we consider the classical discrete-time LTI system:

$$x(k+1) = Ax(k) + Bu(k), \tag{4}$$

with constraints $x(k) \in \mathbb{X}, u(k) \in \mathbb{U}, \forall k$.

The (1-)backward reachable set of $\mathbb{X}$ is:

$$\text{pre}(\mathbb{X}) = \{x \in \mathbb{R}^n | \exists u \in \mathbb{U} \text{ s.t. } Ax + Bu \in \mathbb{X}\}. \tag{5}$$

Likewise, the (1-)forward reachable set of $\mathbb{X}$ is:

$$\text{reach}(\mathbb{X}) = \{Ax + Bu | x \in \mathbb{X}, u \in \mathbb{U}\}. \tag{6}$$

• A positively invariant set $\mathcal{S}$ is defined as:

$$x(0) \in \mathcal{S} \ => \ x(k) \in \mathcal{S}, \quad \forall k, \tag{7}$$

for an autonomous system (cf. file *Oinf.m*).

• Similarly, a control invariant set $\mathcal{C}$ is defined as:

$$x(k) \in \mathcal{C} \ => \ \exists u(k) \in \mathbb{U} \text{ s.t. } Ax(k) + Bu(k) \in \mathcal{C}, \quad \forall k. \tag{8}$$

Recall that: control invariant terminal constraint set $\mathbb{X}_f \Rightarrow$ recursive feasibility of the RHC.

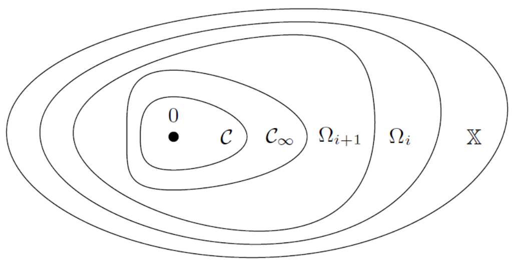

Figure 33: Construction of a maximal control invariant set $\mathcal{C}_\infty$ in $\mathbb{X}$. $\mathcal{C}$ is an arbitrary control invariant set.

The maximum control invariant set in $\mathbb{X}$ is noted $\mathcal{C}_\infty$. It can be computed with the following recursion:

$$\Omega_{i+1} = \text{pre}(\Omega_i) \cap \Omega_i, \quad \Omega_0 = \mathbb{X}. \tag{9}$$

• Let us consider a RHC with stage constraints $\mathbb{X}$, $\mathbb{U}$, and terminal constraints $x(N) \in \mathbb{X}_f$. The feasible set $\mathcal{X}_N$ can be computed as the N-step controllable set $\mathcal{K}_N(\mathbb{X}_f)$ through the recursion:

$$\mathcal{K}_{i+1}(\mathbb{X}_f) = \text{pre}(\mathcal{K}_{i+1}(\mathbb{X}_f)) \cap \mathbb{X}, \quad \mathcal{K}_0(\mathbb{X}_f) = \mathbb{X}_f. \tag{10}$$

Tasks

Question 1: Play around with the file *PreReachAut.m* to familiarize with reachable sets.

Question 2: Play around with the file *PreReach.m* to familiarize with the impact of control inputs on reachable sets.

Question 3: In the file *Cinf.m*, compute $\mathcal{C}_\infty$ using the recursion (9) (i.e. do

NOT use the invariantSet() command of MPT). Investigate how the set sequence changes for different model parameters.

Question 4: In the file *FeasibleSet.m*, find the smallest horizon length N which gives the maximum controllable set (i.e. the largest feasible set $\mathcal{X}_N$). Here, we assume $\mathbb{X}_f = 0$. Use the recursion (10) to compute $\mathcal{X}_N$.