SSY281 Model Predictive Control 2023
PSS 3 - Optimization

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Lecture Refresh

Cf PSS notes.

Exercise 1

Norm-1 objective as an LP

Show that the norm-1 optimization problem:

$$\min_x \|Ax\|_1 \tag{1}$$

can be equivalently written as a linear program with the form:

$$\begin{aligned} \min_z \quad & c^\top z && \text{(2a)} \\ \text{s.t.} \quad & Fz \leq g. && \text{(2b)} \end{aligned}$$

Exercise 2

Convex QP and KKT conditions

Consider the objective function $V(x) = x_1^2 - x_1 x_2 + x_2^2 - 3x_1$ and the following QP problem:

$$\begin{aligned} \text{minimize} \quad & V(x) = x_1^2 - x_1 x_2 + x_2^2 - 3x_1 \\ \text{subject to} \quad & x_1 \geq 0 \\ & x_2 \geq 0 \\ & x_1 + x_2 \leq 2 \end{aligned} \tag{P-I}$$

1. Rewrite the optimization problem (P-I) in standard form.
2. Show that the problem (P-I) is convex.

3. Show that the solution obtained graphically from looking at Figure 1 satisfies the KKT optimality conditions (and hence that this solution is optimal indeed).

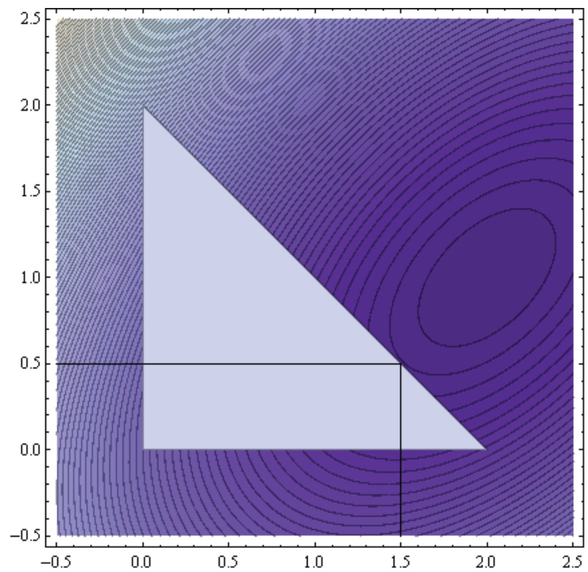

Figure 1: Feasible set overlayed on top of the level curves of the objective function. The level curves decrease in value from the outside to the inside, i.e. the innermost level curve represents the lowest value of the objective function compared to the outer level curves. Here, the constraint $x_1 + x_2 \leq 2$ is tangential to the "optimal" level curve $V(x) = -11/4$.

Exercise 3

Softening the constraints

Consider the following standard QP problem with *hard* linear inequality constraints on the optimization variable x:

$$\begin{aligned} \text{minimize} \quad & V(x) = \frac{1}{2} x^\top Q x + p^\top x \\ \text{subject to} \quad & Ax \leq b \end{aligned} \tag{3}$$

In some practical applications, we can relax the hard constraints by introducing slack variables ϵ (this is called *constraint softening*). ϵ can be seen as the amounts by which constraints are allowed to be violated. These amounts should be kept small when possible, which can be done by adding a penalty on the size of ϵ in the cost function.

Following these guidelines, rewrite the QP (3) with soft constraints only and show that the new optimization problem is also a standard QP problem. You can use a quadratic penalty term for ϵ.