SSY281 Model Predictive Control 2023
PSS 2 - State Estimation and Setpoint Tracking

Rémi Lacombe

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Course reminder

Cf PSS notes.

Exercise 1

Unconstrained Tracking Problem

1. For an unconstrained system, show that the following condition is sufficient for feasibility of the target problem for any z_{sp}.

$$\text{rank}\begin{bmatrix} I-A & -B \\ HC & 0 \end{bmatrix} = n + n_c \tag{1}$$

   where n is the number of states, n_c is the number of controlled outputs, and p is the total number of measured outputs. The size of the matrix in (1) is $(n+n_c) \times (n+m)$.
   Find a counter-example to prove that condition (1) is sufficient only, and not necessary.

2. Show that the condition (1) implies that the number of controlled variables without offset is less than or equal to the number of manipulated variables and the number of measurements, i.e. $n_c \leq m$ and $n_c \leq p$.

3. Show that (1) implies the rows of H are independent.

4. Does (1) imply that the rows of C are independent? If so, prove it, if not provide a counter-example.

5. By choosing H, how can one satisfy (1) if one has installed redundant sensors so several rows of C are identical?

Exercise 2

Redundant sensors
A constant variable x is measured by two different sensors with different accuracy. The system is described by

$$\begin{aligned} x(k+1) &= x(k) \\ y(k) &= Cx(k) + e(k) \end{aligned}$$

with

$$C = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

and $e(k)$ is a zero-mean white-noise vector with covariance matrix

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}$$

1. Estimate x as

   $$\hat{x}(k) = a_1 y_1(k) + a_2 y_2(k)$$

   and determine the constants a_1 and a_2 so that the mean value of the estimation error is zero and so that the variance of the estimation error is as small as possible. Compare the variance with that obtained when using only one of the sensors.

2. Compare your solution above with that obtained with the stationary Kalman filter.