SSY281 Model Predictive Control 2023
PSS 6 - Stability

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Lecture Refresh

Cf PSS notes.

Exercise 1

Simple Lyapunov function

Consider the autonomous system: $x(k+1) = \begin{bmatrix} 0.5 & 1 \\ -0.1 & 0.2 \end{bmatrix} x(k)$.

Find a Lyapunov function of the form $V(x) = x^\top Sx$ for this system.

Exercise 2

Stability of unconstrained infinite horizon RHC

Consider the infinite horizon RHC minimization problem:

$$V(\boldsymbol{u}, x_0) = \sum_{k=0}^{\infty} x(k)^\top Q x(k) + u(k)^\top R u(k) \tag{1a}$$

$$\text{s.t.} \quad x(k+1) = Ax(k) + Bu(k), \tag{1b}$$

with $Q = I_2,\ R = 1,\ A = \begin{bmatrix} 1 & 1 \\ -1 & 5 \end{bmatrix}$, and $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Show that $V(\boldsymbol{u}, x_0)$ is a Lyapunov function for the closed-loop system.

Exercise 3

Infinite to finite horizon RHC

Suppose that the following infinite horizon ($N = \infty$) LQ problem is to be solved:

$$\begin{aligned} \text{minimize} \quad & V(k) = \sum_{i=0}^{\infty} \left\{ ||z(k+i)||_Q^2 + ||\Delta u(k+i)||_R^2 + ||u(k+i)||_S^2 \right\} \\ \text{subject to} \quad & x(k+1) = 0.9x(k) + 0.5u(k), \\ & z(k) = C_z x(k). \end{aligned} \tag{P-I}$$

1. Show that if A is stable and $Q \succ 0$, then (P-I) can be equivalently formulated as a **finite horizon problem**. We will assume that there is an index M such that: $u(k+i) = 0, \quad \forall i \geq M$.

2. Use this result to write the infinite horizon problem (P-I) as a quadratic program (QP) for the case $M = 2$. You may assume that $Q = 1$, $R = 0$, $S = 0$ and $C_z = 1$, for the sake of simplicity.