

# PSS 1

$$V_N = \sum_{k=0}^{N-1} x(k)^T Q x(k) + u(k)^T R u(k) + x(N)^T P_F x(N)$$

$$Z = \begin{bmatrix} x(1) \\ \vdots \\ x(N) \\ u(0) \\ \vdots \\ u(N-1) \end{bmatrix} = \begin{bmatrix} x \\ u \end{bmatrix}$$

$$V_N = Z^T H Z = \begin{bmatrix} x & u \end{bmatrix} \begin{bmatrix} \bar{Q} & 0 \\ 0 & \bar{R} \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix}$$

$$\bar{Q} = \begin{bmatrix} Q & & \\ & \ddots & \\ & & Q_F \end{bmatrix} \quad \bar{R} = \begin{bmatrix} R & & \\ & \ddots & \\ & & R \end{bmatrix}$$

$$\text{blkdiag}(M_1, M_2) = \begin{bmatrix} M_1 & \\ & M_2 \end{bmatrix}$$

$$\text{KRON}(A, B) = \begin{bmatrix} a_{11}B & a_{12}B & \\ a_{21}B & & \ddots \\ & & & \ddots \\ & & & & a_{nn}B \end{bmatrix}$$

$$\text{KRON}(I, B) = \begin{bmatrix} 1 \times B & 0 \times B & \\ 0 \times B & 1 \times B & \\ & & \ddots & \\ & & & 1 \times B \end{bmatrix}$$

↑  
identity

$$= \begin{bmatrix} B & & \\ & \ddots & \\ & & B \end{bmatrix}$$

$$\begin{aligned} x(1) &= A x(0) + B u(0) \\ \vdots \\ x(k+1) &= A x(k) + B u(k) \end{aligned}$$

$$A_{eq} z = b_{eq}$$

$$\underbrace{\begin{bmatrix} I & 0 & \dots & -B & 0 & \dots & 0 \\ -A & I & 0 & \dots & 0 & -B & 0 \\ & -A & & & & & \\ & & \ddots & & & & \\ & & & -A & I & 0 & -B \end{bmatrix}}_{A_1} \underbrace{\begin{bmatrix} x(1) \\ x(2) \\ \vdots \\ x(N) \\ u(0) \\ \vdots \\ u(N-1) \end{bmatrix}}_{A_2} = \begin{bmatrix} Ax(0) \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} I_m & & & & \\ -A & I_m & & & \\ & -A & \ddots & & \\ & & & -A & I_m \end{bmatrix} \quad A_2 = \begin{bmatrix} -B & & & \\ & -B & & \\ & & \ddots & \\ & & & -B \end{bmatrix}$$

$$A_1 \in \mathbb{R}^{mN \times mN}$$

$$A_2 \in \mathbb{R}^{mN \times mN}$$

$m \rightarrow \dim$  of  $x$   
 $m \rightarrow \dim$  of  $u$

$$|x_2(k)| \leq 0.5 \Leftrightarrow \begin{aligned} x_2(k) &\leq 0.5 \\ x_2(k) &\geq -0.5 \end{aligned}$$

$$\Leftrightarrow \begin{aligned} x_2(k) &\leq 0.5 \\ -x_2(k) &\leq 0.5 \end{aligned}$$

$$A_{im} \succeq \preceq b_{im}$$

$$\underbrace{\begin{bmatrix} 0 & 1 & & \\ 0 & -1 & & \\ & & \ddots & \\ & & & 0 \end{bmatrix}}_F \underbrace{\begin{bmatrix} & & & 0 \\ & & & \\ & & & \\ -1 & & & \\ & -1 & & \\ & & \ddots & \\ & & & -1 \end{bmatrix}}_G \begin{bmatrix} x(1) \\ \vdots \\ x(N) \\ u(0) \\ \vdots \\ u(N-1) \end{bmatrix} \preceq \underbrace{\begin{bmatrix} 0.5 \\ \vdots \\ 0.5 \\ 0.7 \\ \vdots \\ 0.7 \end{bmatrix}}_h$$

$$A_{im} = \begin{bmatrix} F & G \end{bmatrix}$$

$$b_{im} = h$$

$$\Rightarrow \begin{bmatrix} F & G \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} \preceq h$$