

ARL (A Reversible Programming Language)

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1 Introduction

Reversible computation dates back quite some time but had its first milestone in 1961 when Landauer's principle was proposed[1]. Then in 1973 Bennett[2] formalized a model for Reversible Turing Machines (RTM). Any RTM can only compute injective functions as any input must map to exactly one unique output. Otherwise, reversibility would be ambiguous. Despite this constriction, compared to a classical Turing Machine, there has been some significant research on the topic and its use cases. A lot of the work has been done concerning heat dissipation of reversible vs irreversible languages. But reversibility of programs have also shown uses in the fields of quantum computing, cryptography, and checkpointing of simulations[3]. Already in 1982 the time-reversible language Janus was presented, and later formalized by Yokoyama and Glück[4]. Janus might be the most prevalent reversible programming language, however, the rise of reversible functional languages have been noticeable, probably because of the injectivity restriction. Multiple functional languages have been formalized over the years. The most significant of these might be RFUN, presented Yokoyama, Axelsen, and Glück[4]. RFUN uses a heap manager built on the principle of linearity. Mogensen then presented RCFUN[5] which uses another approach for the heap manager, namely simple sharing using reference counting. Mogensen then later presented RIL, which is an intermediate language, where the heap manager uses maximal sharing[6]. The reasoning for a maximally shared heap manager is to abstract away or eliminate memory management completely from the high-level language, opposed to the linear and reference counting models. For the linear approach, one can only use a variable once and must use that variable exactly once. In reference counting, this constriction is eliminated, as we may use a variable multiple times, however, reference counting limits the way we can construct patterns. RIL with its maximal sharing system resolves this. The constriction for what languages can use RIL is however limited to purely functional languages as RIL uses cons-hashing to improve lookups in the heap. And since RIL is an intermediate language and is tedious and error-prone to write by hand, RIL poses a great choice for a heap manager for a reversible function language. This report concerns itself with a formal description and implementation of such language.

2 The Functional Reversible Language ARL

The Functional Reversible language ARL (A Reversible Language) is an implementation of the simple language presented by Mogensen[6]. ARL implements the core

concepts of the language, meaning ARL in its current state is a simple type-free language, with Pairs as its only construct, in the ML-style family. The syntax for ARL has undergone a few modifications from the original syntax to make the language more manageable and easier to work with as a programmer, adhering to the same philosophy constituting the heap management in RIL. An example showing the syntax of ARL can be seen in fig ?? which inverts a tree. One thing to note from this example is that flip is invertible meaning it satisfies $a \circ a^\dagger \equiv a^\dagger \circ a$ where a is a function. That is, it doesn't matter whether we call the function or uncalls it, it will have the same semantics. Thus the semantics of the supplied program is equivalent to the identity of input as we in the main function calls flip 2, thus invert the tree 2 times. This property however does not have to hold for all functions. They do, however, have to satisfy $a \circ a^\dagger \circ a = a$. We use the dagger from category theory, as this carries nicely over in the syntax of ARL, and has the same mathematical meaning as the functions being injective in the context. We programming one therefore might have to keep track of when a function is partially or fully invertible.

```

fun flip (l,r) = let fl = flip l in
                  let fr = flip r in
                  (fr, lr)
| x          = x

fun main =
  !flip
  flip

```

<i>Program</i>	::=	<i>Main Function</i> ⁺
<i>Main</i>	::=	fun main = <i>FunctionCall</i> ⁺
<i>FunctionCall</i>	::=	!fname fname
<i>Function</i>	::=	fun fname <i>Rules</i>
<i>Rules</i>	::=	<i>Pattern</i> = <i>Def</i> [*] <i>Pattern</i>
		<i>Rules</i> <i>Rules</i>
<i>Pattern</i>	::=	vname
		constant
		vname <> <i>Pattern</i>
		(<i>Pattern</i> :: <i>Pattern</i>)
		(<i>Pattern</i> , <i>Pattern</i>)
		vname as (<i>Pattern</i> , <i>Pattern</i>)
<i>Def</i>	::=	let <i>Pattern</i> = fname <i>Pattern</i> in
		let <i>Pattern</i> = !fname <i>Pattern</i> in
		let <i>Pattern</i> = loop fname <i>Pattern</i> in
		let <i>Pattern</i> = !loop fname <i>Pattern</i> in

Figure 1: Syntax of ARL

As shown in Figure~1 and from the flip function in Figure??, there are some changes to the original grammar presented by Mogensen. These changes have been

carefully selected to make the syntax cleaner and somewhat easier, and more relatable to programmers who are not comfortable with reversible programming languages. These are as follows

- **Introduction of a main**

the introduction of a Main function, is most likely the biggest adjustment. The need for a Main function stems from the fact that the original presentation of the language provides no interface for IO. Since the heap manager in RIL is maximally shared and uses a con-hashing algorithm, ARL cannot allow any updating of variables and no way of doing side-effects, there is no obvious way to do IO. The main function will contextually serve as an entry point and be a pipeline over the function-calls invoked inside Main. That is the input from stdin will be the argument for the first function, and since the compilation ensures that inputs and outputs will be placed in the same register or rather variable in the case of RIL, the input for the next function call will be the result of the previous one. This might not be the optimal solution and might thus change as ARL evolves and the needs change. But for now, this simple interface will work. Furthermore, there is no current way for the program to output anything, this has to be decided in the compilation. One solution could be to provide the compiler with a flag stating whether or not the user wants any form of output.

- **Two ways of constructing a pair**

It might seem unimportant to have multiple ways to construct pairs, and at first hand, it is, as they have the same semantical meaning. However, the decision to do so is to give programmers an easier time. A pure mathematical function can only take a single argument, where this argument might be an argument-vector. This is unsurprisingly also the case for ARL. The difference between ARL and other programming languages is that ARL only takes a single argument where most other popular languages take arbitrary many, either by currying as in Haskell or vector-like in C-style languages. However, these are pure abstractions. And in the same manner, we can abstract away any single argument restriction in ARL. With the distinction between cons as `(::)` representing the cons from ML-style languages and cons as `(.)` representing a dotted pair from LISP. One can interpret any `(::)` as a list with a head and tail and `(.)` as vector/tuple abstractly giving a C-like parameter list. Letting `(::)` having a higher level of precedence than `(.)`, following `(x::xs,y)` will construct

```
Pair (Pair (Var 'x') (Var 'xs')) (Var 'y') .
```

One can see this as a dotted pair with `car` being a list with a head `x` and a tail `xs`, and a `cdr` of any construct `y`. This abstraction might make it easier for the programmer despite them being equivalent. Furthermore, it is also allowed to introduce arbitrary many cons operators as this will get folded the same as the example above.

- **More readable let declarations**

The let declarations have likewise been modified in the same philosophy as the rest of the modifications, to make it more approachable by using familiar or close to familiar syntax to ML. Thus instead of having function call on the RHS of the assignment and function uncall on the LHS, we consistently delimit calls and uncalls to the RHS, denoting a difference with a prefix `!`, since this is the symbol closest resembling a dagger. the same concept holds for loops.

- **Change of != to <>**

This is simply a minor syntactical change, changing the denotation of != to <> as the inequality operator. This has been reasoned to having a more relatable ML-style syntax.

3 Parsing

The compiler for ARL has been written in Haskell using Megaparsec as the parsing library. This was chosen over lexer/parser tools such as Alex/Happy, because of familiarity and because ARL as a language is quite small, thus making it pretty easy to implement. Megaparsec was chosen over other parsing libraries such as Parsec for 2 main reasons. First ARL is an indentation sensitive language, chosen to have quite strict rules, which we will see later on. Second Megaparsec makes position handling extremely easy giving the exact position of when parsing failed without having to bundle the AST with positions.

3.1 AST

The implementation of the abstract syntax tree is almost true to the Grammar presented in 1. There are however three minor changes. Instead of rules looking like

```
data Rules = P Pattern [Def] Pattern
           | R Rules Rules
```

it simply will be a product type of the constructor Rule, and then the Func sum type will take a list of rules, as such:

```
data Func = Func ID [Rules]
```

```
data Rule = Rule { args :: Pattern, body :: [Def], output :: Pattern }
```

This change is mainly reasoned by being easier to parse and evaluate. the meaning should not change. For the same reason we introduce another pattern namely a NilNil, essentially this is a constant value, however, NilNil as a legal value in RIL depends on the build procedure that will create it. We, therefore, want it to have its own constructor as this simply makes implementation easier. Lastly, we earlier described the usefulness of having two ways of creating pairs, in the AST, we however only have one constructor for these as we can use some build-in functionality of megaparsec to enforce precedence without rewriting our grammar.

3.2 Parsers

3.2.1 Basics.

Comments are based on f#. line-comments is the same as in // and block-comments is (* *). Identifiers can be any string starting with a lower character followed by any alphanumerical character, a dash, or an underscore.¹

¹any code described the following subsections can be found in appendix ?? or in the file Parser.hs

3.2.2 Functions

As described ARL has been chosen to have some strict indentation rules. This is forced to make the code readable. We must thus enforce the specific rules in the parser. Firstly we ensure that a function is always declared in column 0. This makes for a fine structure but might need to be changed in the future if we allow for nested function declarations. we will then consume the unnecessary garbage. A function will then either be a Main function or a pure function, if we encounter a main we will then parse the function calls. Here we enforce another indentation rule. A function call, must reside directly under the function name of main (which is “main”), like in the example code in Figure-??. In the parsing we do not enforce a single main function, instead, we handle this in the pre-processing. If we however encounter a non-main function (from here just function) we will parse its rules. Like in the main function we ensure that a rule (other than the first, which must be on the same line) resides under the function name. that is the guard | must be placed here. Other than this indentation handling, the parser is simply a sequence of parsers and combinators.

3.2.3 Rules

The parsing for the Rule sum type is in itself quite simple as most of the indentation is handled in the function parser. Although the rule parser also will have to do some indentation enforcement, it will pass on its indentation level for the let-declarations parser, to make certain that let definitions is deeper indented than the rule, along with forcing let declarations to be lined up with the resulting pattern. Again this is simply used to establish a structure for the body of a particular function pattern, also called a rule.

3.2.4 Let declarations

Unsurprisingly the let declaration follows a similar structure as the other parsers. overall we can reduce a let declaration to either of two, it is a function call/uncall or it is a loop. These are very similar in structure so we will only go over the simple case for function calls. again we ensure the indentation is correct, throwing a parse error otherwise. we then use the same strategy as we did for function calls in main to distinguish between a call and an uncall using the observing function. depending on whether the symbol ! is present before the function identifier, we get a Left or a Right value which we then convert to the appropriate type. This function has a lot of duplicate code, as the loop/unloop construct is very similar. This could potentially be eliminated.

3.2.5 Patterns

Patterns are the most atomic part of the grammar, as its only non-terminal symbol is that of Pattern. It is thus also the easiest to parse. We construct a parser for each terminal and combine these using the parser combinators. We can see that whenever we encounter a [[]] we have a NilNil constructor. for integer constants we simply wrap the constant value in the Const constructor, we, however, omit to change the value to its internal representation in RIL which would be $2n+1$. The reason for this is that we want to distinguish between the syntactical and semantical meaning of the program. It is further noticeable that we also wrap nil as a Const with a value of 2. A variable is simply the identifier wrapped in our Var constructor. A not equal pattern is likewise simply the identifier and a recursive pattern call. The same holds for the as constructor,

however, the second part of an `as` can only be a pair. For `Pairs` we can see?? it makes use of the `MakeExprParser` which specifies associativity and precedence for the two ways of constructing pairs. Lastly, we also want to allow to wrap any `Pattern` in parenthesis.

4 Semantics

4.1 RIL

Before we explain the semantics of ARL, we will shortly go over RIL. At its core, RIL is a set of blocks consisting of an Entry, a Body, and an Exit. Both entries and exits are one of 3 constructs, either a conditional entry/jump, an unconditional entry/jump, or a subroutine entry/exit. These work fairly similar to regular jumps and labels, known from other languages. The biggest difference is the conditional entry, which is not present in those languages. It is the inverse of the conditional jump and is used when run in reverse. It is worth noting, that since RIL has such a basic structure it is a parameterless language, meaning subroutines will use specific variables for their computation. The body of a block consists of statements or subroutine calls. Therefore any control-flow will be in its own block. However, when looking at RIL code this might not be immediately obvious. As RIL is reversible, the statements in a body are quite limited to the form $L_1 \text{ cross } = R_1 \text{ dot } R_2$, or $L_1 \text{ <-> } L_2$. where L is a variable or a memory location and R is either an L or an integer constant in the range $-1^{31}-2^{31}-1$, with some restrictions to which L value R can take. The reason for this that loss of information cannot be as the statement would be non-invertible. The swap likewise swaps the value of the two L 's, ensuring no loss of information. the following table shows how each RIL instruction inverts, which will be useful as a reference for evaluating patterns in reverse.

4.1.1 Value representation

RIL furthermore has a different value representation than ARL. RIL is as mentioned an intermediate language, with a syntax of very simple instructions. It thus uses specific patterns of machine words for different values.

- 0 represents the absence of a value.
- ARL's pairs are in RIL represented as a pointer to a 3-word block memory, where the first word is the reference count, the second and third word is the first and second part of the pair respectively. the RIL pointer is always represented as a multiple of 4. An instance of this is `nilnil ([[]])`, which simply is a pair of two empty lists, represented by 2, and is constructed by an initialize procedure looking as such:

insert code

- Integer constants n in ARL will be translated to $2n + 1$ in RIL since constants in RIL has to be an odd number. This ensures that constants and pairs don't get mixed up.
- The last type of word in RIL is even numbers, whose value is not a multiple of 4. In its current state, only one symbol (`~[]~/nil`) is present, which is represented as the value 2.

4.1.2 Subroutines

RIL also has 3 subroutines, which are used by the heap manager to manage the reference counts of nodes along with ensuring maximal sharing. These are used for some of the more complicated patterns in ARL. `copy` - is used to copy values, which is what allows us to use the same variable multiple times. `fields` - is used in the “as” pattern. `cons` - is used for pairs.

4.1.3 Copy

the `copy` subroutine uses the variables `copyP` and `copyQ`. `Copy` assumes `copyP` to be bigger than 0 and `copyQ` to be equal to 0. This makes sense, since 0 is the absence of a value, and thus `copyP` cannot be 0 as there would be no value to copy and it must be a positive integer as it is an index in memory. `CopyQ` likewise needs to be 0 as would not be a true copy of `copyP` if it weren't. If `copyP` is a pointer the reference count is increased and `copyQ` is set to the same value as `copyP` using `copyQ += copyP`. Called in Reverse `copy` assumes the two variables to be equivalent, as this is the only way to “destroy” a variable without loss of information. This happens by subtracting `copyP` from `copyQ`. thus `copyQ` will be 0. again if `copyP` is a pointer the reference counter is decreased.

4.1.4 Fields

Fields have 3 variables, `fieldsA`, `fieldsD`, and `fieldsP`. We have previously described how an as pattern is an identifier and a pair. the identifier will be the pointer to this pair and will be located in the `fieldsP` variable. The other two variables must be 0, to ensure correctness. It will then set the `fieldsA` and `fieldsD` to the second and third word of the pair respectively, which corresponds to the car and cdr of the pair. In reverse, the `fieldsA` and `fieldsD` will be cleared.

4.1.5 Cons

The `cons` subroutine is quite a bit more complex than the other two. This is because it also has to allocate and deallocate nodes and it is implemented using hashing to make lookup more efficient. We will not go over the specifics, but only the general functionality. `Cons` take two arguments `consA` and `consD`, which must be values (not 0). These values will be cleared, or possibly more intuitive they will be placed as second and third word of the pair, if the pair doesn't already exist on the heap, otherwise the reference count will be increased, while `consA` and `consD` are cleared. The pointer to the pair (`consA`, `consD`) will be in `consP`. Called in reverse a pair is deconstructed, deallocating the pointer if the reference count reaches 0 and increasing the reference count for the `consA` and `consD` fields.

4.2 Functions and Rules

Figure~2 show how we evaluate functions and rules, where $R[r_i]$ is the translation of the Rules and f_i and f'_i represent entry points and exit points respectively. Essentially a function will have its entry point, and jump immediately to the entry of the first rule. It will evaluate each rule sequentially until one is evaluated correctly, that is, its exit point has been reached it will then terminate the function/subroutine (this is a simplification). If no rules are matched the function will assert a false statement, thus exiting with a

$$\begin{array}{ll}
F\llbracket f \ r_1 | \dots | r_n \rrbracket = & R\llbracket p_i = d_i^1 \dots d_i^n o_i \rrbracket = \\
\begin{array}{l}
\text{begin } f \\
\text{skip} \\
--> f_1 \\
f'_1 <-- \\
\text{skip} \\
\text{end } f \\
R\llbracket r_1 \rrbracket \\
\vdots \\
R\llbracket r_n \rrbracket \\
f_{n+1} <-- \\
\text{assert } A \neq A \\
--> f'_{n+1}
\end{array} & \begin{array}{l}
f_i <-- \\
P\llbracket p_i \rrbracket A \\
A \neq 0 \ --> f_i + 1 \\
D\llbracket d_i^1 \rrbracket \\
\vdots \\
D\llbracket d_i^n \rrbracket \\
\frac{f'_{i+1} <-- \ A \neq 0;}{P\llbracket o_i \rrbracket A} \\
--> f'_i
\end{array}
\end{array}$$

Figure 2: Semantics of functions and rules

failure, which essentially means a function cannot be called on any construct only those matching the rules.

Rules are introduced by their entry point f_i . From here p_i , which is the parameter pattern of the rule will be evaluated. Essentially what we do when we evaluate a pattern in the forward direction we try to move it out of A , which is the variable chosen for input and output as RIL as stated is parameterless. If A is correctly distributed to p_i , the value of A will be 0 and we can thus ignore the conditional jump. and proceed to evaluate the body of the rule. Is A however not equal to 0, it means that the pattern was not correctly matched and thus we want to make the jump, which leads us to the next rule. If the jump is not taken the body can safely be evaluated. We can then see there is an exit point for f'_{i+1} . The reason for this is we have to evaluate the result of each previous rule to make sure the output is disjoint, meaning the function is injective. It is also worth noting that when evaluating the result o_i , we evaluate it inversely. This can be seen as a construction of A based on o_i , whereas the $P\llbracket p_i \rrbracket A$ could be the deconstruction of A into p_i . Lastly, we will take an unconditional jump to right before the result in the previous rule, to do the disjoint checking as described.

4.3 Patterns

4.3.1 Variables

There are two different ways a variable needs to be compiled. The most basic rule $x \leftrightarrow v$, with x being the variable, will be valid whenever x first occurs in a pattern. Called in reverse this is simply the same instruction. For every occurrence of x that is not the first occurrence, we will need to use the copy subroutine, described earlier. When evaluating a variable that has occurred previously we first need to check whether or not x and v are identical. This is a prerequisite for the copy subroutine to work as it results in an assertion failure in the copy subroutine otherwise. we then switch the values into the variables that are used in the routine. we switch v into copyQ as this is the value that will be consumed. x will be switched into copyP as this is the value that will be saved. When evaluated in reverse, we check that v is 0 as this again would result in an assertion failure, we move x into copyP and makes a copy into copyQ and

$P\llbracket x \rrbracket v =$	$\overline{P\llbracket x \rrbracket} v =$
$x \leftrightarrow v$	$x \leftrightarrow v$
when x is first occurrence	when x is first occurrence
$P\llbracket x \rrbracket v =$	$\overline{P\llbracket x \rrbracket} v =$
$v \neq x \rightarrow l_1;$	$v \neq 0 \rightarrow l_1;$
$v \leftrightarrow \text{copyQ};$	$x \leftrightarrow \text{copyP};$
$x \leftrightarrow \text{copyP};$	$\text{call copy};$
$\text{uncall copy};$	$x \leftrightarrow \text{copyP};$
$x \leftrightarrow \text{copyP};$	$v \leftrightarrow \text{copyQ};$
$l_1 \leftarrow v \neq 0;$	$l_1 \leftarrow v \neq x;$

Figure 3: Semantics of variables

move it back to x and v.

4.3.2 Constants

$P\llbracket k \rrbracket v =$	$\overline{P\llbracket k \rrbracket} v =$
$v \neq k \rightarrow l_1;$	$v \neq 0 \rightarrow l_1;$
$v -= k;$	$v += k;$
$l_1 \leftarrow v \neq 0;$	$l_1 \leftarrow v \neq x;$

Figure 4: Semantics of constants

Constants are quite simple. firstly the constant need to be equivalent to v for the pattern to match. Once again this we want to extract the constant k from v, getting v to equal 0 if the pattern matches. This can only be the case when they are equivalent. In the case they are, we simply subtract k from v, and since k is a constant and will never change we cannot and there is no need to do anything to k. In reverse we do the opposite we check if v is 0 if it is we can set it to the value of k.

4.3.3 Pairs

When translating a pair to RIL, we first start by checking whether or not v is a pointer to a pair. This can be done by checking $v \ \& \ 3$, as pointers always will have 11 in their 2 least significant bits. If v simply is not a pair, we can skip the entire unfolding of v, jumping straight to the bottom. is v however a pair, we move v into consP, as we need to deconstruct by uncalling cons. the car and cdr will then be in consA and consD. we however have to move these to two newly created variables t_1 and t_2 . this might seems unnecessary at first but whenever we have nested patterns, not moving consA and consD out to new variables will make the program fail as these will not be 0 in the uncall to cons in the nested pair. when moved accordingly, we can then evaluate p_1 under t_1 . After this evaluation we need to check if v was correctly cleared. If t_1 is 0 we can move on to evaluate p_2 under t_2 . Is it the case that t_1 is not 0 we jump to entry l_3 and reconstruct v_1 . Once again this is to ensure we don't lose any information while evaluating a pattern we will then proceed to reconstruct v by doing the inverse

$P\llbracket(p_1, p_2)\rrbracket v =$	$\overline{P\llbracket(p_1, p_2)\rrbracket v} =$
$v \ \& \ 3 \ \rightarrow \ l_1;$	$v \ \& \ 3 \ \rightarrow \ l_1;$
$v \ \leftrightarrow \ \text{consP};$	$v \ == \ 0 \ \rightarrow \ l_3;$
$\text{uncall cons};$	$v \ \leftrightarrow \ \text{consP};$
$t_1 \ \leftrightarrow \ \text{consA};$	$\text{uncall cons};$
$t_2 \ \leftrightarrow \ \text{consD};$	$t_1 \ \leftrightarrow \ \text{consA};$
$P\llbracket p_1 \rrbracket t_1;$	$t_2 \ \leftrightarrow \ \text{consD};$
$t_1 \ != \ 0 \ \rightarrow \ l_2;$	$P\llbracket p_1 \rrbracket t_1;$
$P\llbracket p_2 \rrbracket t_2;$	$t_1 \ != \ 0 \ \rightarrow \ l_2;$
$t_2 \ == \ 0 \ \rightarrow \ l_3;$	$\overline{l_3 \ \leftarrow \ t_2 \ == \ 0};$
$\overline{l_2 \ \leftarrow \ t_1 \ != \ 0};$	$\overline{P\llbracket p_2 \rrbracket t_2};$
$\overline{P\llbracket p_1 \rrbracket t_1};$	$\overline{l_2 \ \leftarrow \ t_1 \ != \ 0};$
$t_1 \ \leftrightarrow \ \text{consA};$	$\overline{P\llbracket p_1 \rrbracket t_1};$
$t_2 \ \leftrightarrow \ \text{consD};$	$t_1 \ \leftrightarrow \ \text{consA};$
$\text{call cons};$	$t_2 \ \leftrightarrow \ \text{consD};$
$v \ \leftrightarrow \ \text{consP};$	$\text{call cons};$
$\overline{l_3 \ \leftarrow \ v \ == \ 0};$	$v \ \leftrightarrow \ \text{consP};$
$\overline{l_1 \ \leftarrow \ v \ \& \ 3};$	$\overline{l_1 \ \leftarrow \ v \ \& \ 3};$

Figure 5: Semantics of pairs

sequence of operations as when we deconstructed the pair. Do we on the other hand evaluate p_2 correctly we can jump to entry l_4 . When evaluated inversely we start by checking whether v is a pointer, skipping the entire thing if it isn't. we then check whether v is 0. if it is we jump to entry l_4 , and proceed to construct v by evaluating t_2 and t_1 inversely, calling `cons` and moving into v . If v is not 0 we have to deconstruct it even further, by uncalling `cons` and make evaluate t_1 . Overall the procedure will deconstruct a pair in forward direction and create a pair in the inverse direction.

4.3.4 As pattern

An `as` pattern is almost identical to the `pair`, the only difference is that we want to keep the integrity of x , which is done by using the `fields` sub-routine. Just like with a `pair`, we check if v is in fact a `pair`. we will then move v into `fieldsP`, calling `fields` and then distributing the pointer to x , `fieldsA` to t_1 and `fieldD` to t_2 . Again t_1 and t_2 needs to be unique newly created variables, such that we encounter any trouble with nested patterns. The rests of the evaluation of an `as` pattern is the same as for `pairs`, since the only difference between an `as` pattern and a `pair` pattern is that we in the `as` pattern want to keep a reference to the `pair`. In reverse the same principles also holds.

4.3.5 Not equal (<>)

For a `not equal` pattern, we first need to assume x is 0 otherwise our two updates, first to x then to v , would compromise the integrity of v . For instance in the case of `flip` the rule $| \ x = x$ could be written as $| \ x \ \<\> \ (l, r) = x \ \<\> \ (fr, fl)$. In such a case v would not be a pointer ($v \ \& \ 3$), thus we skip the entire evaluation of p . we would then subtract, v from x , do nothing once again, and then subtract a value larger than v from v , which is nonsensical. Therefore x must be 0 before the evaluation. As

$P\llbracket xas(p_1, p_2) \rrbracket v =$	$\overline{P\llbracket xas(p_1, p_2) \rrbracket v} =$
$v \ \& \ 3 \ \rightarrow l_1;$	$v \ \& \ 3 \ \rightarrow l_1;$
$v \ \leftrightarrow \text{fieldsP};$	$v \ == \ 0 \ \rightarrow l_3;$
$\text{call fields};$	$v \ \leftrightarrow \text{fieldsP};$
$x \ \leftrightarrow \text{fieldsP};$	$\text{call fields};$
$t_1 \ \leftrightarrow \text{fieldsA};$	$x \ \leftrightarrow \text{fieldsP};$
$t_2 \ \leftrightarrow \text{fieldsD};$	$t_1 \ \leftrightarrow \text{fieldsA};$
$P\llbracket p_1 \rrbracket t_1;$	$t_2 \ \leftrightarrow \text{fieldsD};$
$t_1 \ != \ 0 \ \rightarrow l_2;$	$P\llbracket p_1 \rrbracket t_1;$
$P\llbracket p_2 \rrbracket t_2;$	$t_1 \ != \ 0 \ \rightarrow l_2;$
$t_2 \ == \ 0 \ \rightarrow l_3;$	$l_3 \ \leftarrow t_2 \ == \ 0;$
$l_2 \ \leftarrow t_1 \ != \ 0;$	$\overline{P\llbracket p_2 \rrbracket t_2};$
$\overline{P\llbracket p_1 \rrbracket consA};$	$l_2 \ \leftarrow t_1 \ != \ 0;$
$x \ \leftrightarrow \text{fieldsP};$	$\overline{P\llbracket p_1 \rrbracket t_1};$
$t_1 \ \leftrightarrow \text{fieldsA};$	$x \ \leftrightarrow \text{fieldsP};$
$t_2 \ \leftrightarrow \text{fieldsD};$	$t_1 \ \leftrightarrow \text{fieldsA};$
$\text{uncall fields};$	$t_2 \ \leftrightarrow \text{fieldsD};$
$v \ \leftrightarrow \text{fieldsP};$	$\text{uncall fields};$
$l_3 \ \leftarrow v \ == \ 0;$	$v \ \leftrightarrow \text{fieldsP};$
$l_1 \ \leftarrow v \ \& \ 3;$	$l_1 \ \leftarrow v \ \& \ 3;$

Figure 6: Semantics of As pattern

$P\llbracket x \neq p \rrbracket v =$	$\overline{P\llbracket x \neq p \rrbracket v} =$
$\text{assert } x \ == \ 0;$	$v \ += \ x$
$P\llbracket p \rrbracket v$	$P\llbracket p \rrbracket v$
$x \ += \ v;$	$x \ -= \ v;$
$\overline{P\llbracket p \rrbracket v}$	$\overline{P\llbracket p \rrbracket v}$
$v \ -= \ x$	$\text{assert } x \ == \ 0;$

Figure 7: Semantics of <> pattern

explained, after the assertion we want to deconstruct p under v , then update x with $x \ += \ v$, setting x to v . here v should have its original value as it should skip moving v into p , else x would be equal to p . we then reconstruct p under v and subtract the value of x from v . In its core this is a simple swap, however, if p matches v , v should be 0 and no update to x is happening.

4.4 Let definitions

4.4.1 function calls

A call consists of 4 parts. First, we want to evaluate p_2 under A in inverse. We want to construct A from p_2 . this should prepare A to be the input for f . the call to f then happens, and the result is always placed in A . we then evaluate p_1 under A , moving the value from A into p_1 . lastly, we need to assert that A is 0. this assertion is important,

$$\begin{array}{lcl}
D[\text{let } p_1 = \text{call } f p_2 \text{ in}] = & & D[\text{let } p_1 = \text{loop } f p_2 \text{ in}] = \\
\overline{P[p_2]A} & & l_1 \leftarrow A \neq 0; \\
\text{call } f; & & \overline{P[p_2]A} \\
\overline{P[p_2]A} & & P[p_1]A \\
\text{assert } A == 0; & & A == 0 \rightarrow l_2; \\
& & \text{uncall } f \\
& & \rightarrow l_1 \\
& & l_2 \leftarrow \\
& & \text{assert } x == 0;
\end{array}$$

Figure 8: Semantics of let definitions

as it ensures us that the result of f is in fact a matching pattern to p_1 . For instance, if f returns 7, we cannot assign 7 to a pair, thus such a construct should fail.

4.4.2 Loops

Loops are useful in situations where tail-recursive functions are needed. but since these are not allowed we can write these as our loop construct. The loop will keep calling f until p_1 is matched. we first have an entry l_1 . This is where the loop starts. we then construct A from p_2 . Then right after we deconstruct A into p_1 . if A is 0 it means p_1 was matched correctly and we do not call the function f as we jump to entry l_2 . if A is not 0 p_1 is not matched and we call the function f . We then jump back to l_1 , repeating the procedure until p_1 is matched.* Evaluation

5 Compiler implementation

The implementation of the evaluation functions for ARL is built on a stack of monad-transformers. The reason for choosing such a solution is that monads are a well-integrated part of Haskell and it makes it a lot easier to implement the recursive calls to the different functions as we can use `do` notation to lift our functions into the monad. Furthermore, we both have an environment we want to pass on to the different eval-functions and some states to make it a lot easier to ensure that entries and exits are unique and that variables are correct etc. And probably most importantly, the stack allows for easy extensibility as we can easily add new monad transformers to our stack. The stack looks as follows:

```

type Eval a = ReaderT Env (StateT RilState Identity) a

runEval env st ev = runIdentity $ runStateT (runReaderT ev env) st

```

As can be seen from Figure-??, the stack is fairly simple. The `eval` type takes an arbitrary type `a`, we only use `String` as this allows us to write the RIL code directly to a file. our string is then wrapped in an identity monad, this in itself is useless, but works well with other monads. This again is better for extensibility as, we can always substitute for another monad such as `IO`, which cannot be stacked as a transformer. The identity monad is then wrapped in a state transformer, where the state itself is of the type `RilState`, which is a product type we will go over later in this section. And

lastly, we wrap `readerT` around the `State`. In the future, it could be useful to add the error monad to the stack to handle failures, which we currently don't do, or the writer monad to add some kind of logging.

5.1 Why reader?

The reader monad is extremely useful in our case as we have an environment we want to pass around to the different function, and it makes it easier to manage if this is not passed around as parameters but is kept isolated in the environment which can then be locally set to the specific function calls. From section~?? it might be clear that we often use `A` as the variable, we evaluate under, however in some cases this change, for instance when evaluating a pair where we need to evaluate t_1 and t_2 . Therefore we might want to keep track of this. This at first seems like a state but since it never changes inside of any function we can define it in the environment. The second part of the environment is a map. We use this to keep track of which variables are alive in the program. These should be stored on the stack before a function call. this is fairly simple to do since the control flow of ARL is extremely simple. One solution might be to search the AST from the bottom up, however since the control-flow is as simple as it is, we extract all variable IDs from a Rule into a list of ID lists. we then check if a variable in a list is in any of the following lists. If this is the case the variable must be alive. we can then zip these results with the unique identifier for a let declaration, constructing our map. Thus the Environment looks as follows:

```
type Env = (String, M.Map String [ID])

baseEnv = ("A", M.empty)
```

5.2 RilState

As mentioned, there is some state in RIL that we want to keep track of to make everything easier to grasp. The `RilState` can be seen in figure~??, where one can notice that there is quite a lot of fields for the product type. Firstly there is `RuleNo`, this simply is a counter on rules, which `rLabel` is simply the string version of `ruleNo`, so we don't have to call `show` whenever we need the rule number. This might be a bit excessive. `fNameS` will be set at the beginning of the `evalFun`, and is used together with the unique identifiers for patterns and let declarations to ensure that label names do not occur multiple times. We can exploit this since we know, any function name needs to be unique and every rule needs to be unique. `LabNo` and `label` are the same duality as `ruleNo` and `rLabel` and will number jumps and entries inside the rules. Once again to enforce no duplication of labels. `pVars` is the last field of the state. `pVars` is used to check if a variable has previously occurred in a pattern. Now that we have already gotten over how we use the reader monad, the reader might seem like a good solution for this. It would be if it weren't for how pairs are evaluated. As described in section?? we need to rebuild t_1 if it is not correctly matched, which is opposes some problems. Therefore an easier solution is to add a variable to `pVars` when it is first encountered, otherwise generating duplicate code, and then resetting this map back to empty right before we check $\overline{P[p_1]t_1}$.

```
data RilState = RilState { ruleNo :: Int
                          , rLabel :: String
                          , fNameS :: ID
```

```

, labNo :: Int
, label :: String
, pVars :: M.Map ID Int
}

```

5.3 Generating RIL code

Just like in the parser, we have an `eval` function for each non-terminal in the AST. We use the `do` notation to generate the state etc. we need for a specific function, and then we want to wrap the string inside the monad. we construct the strings, by creating a list of strings, where each string is a RIL instruction, which then gets intercalated, with newlines to preserve structure in the RIL file. To make the code easier to read we abstract away the operations. functions with names `v(EQ|NEQ)(0|x)(E|J)`, will be conditional jumps and entries, where `v` is equal or not to 0 or `x`. Plus and sub is the updates `(+=)` `(-=)` respectively. Swap is `(<->)`. Furthermore, we have defined swap functions for each of the variables used in the 3 subroutines described in section~?? as these are used quite often, e.g. `consP x` swaps `x` with `consP`.

6 Results

6.0.1 TODO describe tests

6.0.2 TODO describe how well the project has come along

When it comes to the actual ARL compiler it is still in its early stages. First and foremost they are no optimizations implemented. One such optimization could be dead code removal, which would make the actual RIL file less cluttered. Furthermore, there is very little error-handling implemented in the ARL compiler itself. As described in section~?? MegaParsec does fine error handling on its own and we let any syntax error be handled by the library. We then check that functions are not defined multiple times, but this is where the error-checking stops. The reason for this is the compiler does not do a whole lot of static checks. However, the need for these checks is also very limited, when keeping in mind there are no types that need to be unified, type-checked, etc. One thing that is not

7 How to use - code structure

7.0.1 TODO describe the code structure and how to run the program.

In its current state the compiler is still a bit tedious to use, since no good interface have been implemented. The ARL compiler will simply generate a RIL file, which has to be compiled by the RIL compiler, which then in turn needs to be compiled using a C compiler.

8 Conclusion

8.0.1 TODO

References

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```
funP :: Parser (Either Main Func)
funP = L.nonIndented scn $ L.lineFold scn p
  where
    p sc'      = do rword "fun"
                  ind <- L.indentLevel
                  id <- identifier
                  case id of
                    "main" -> Left <$> mainP
                    _      -> Right <$> rest id ind
    rest id ind = do r <- ruleP ind;
                    rs <- many $ try rules
                    mapM_ (\(_,x) -> when (x /= mkPos 5)
                      (L.incorrectIndent EQ (mkPos 5) x)) rs
                    return $ Func id $ r:map fst rs
    rules      = do scn
                  ind <- L.indentLevel; symbol "|"
                  r <- ruleP ind
                  return (r,ind)
    mainP      = do symbol "="; some $ try funC

defP :: Pos -> Parser Def
defP ind = try call <|> try loop <?> "Let def"
  where
```

```

call    = do L.indentGuard scn EQ ind;
          rword "let"
          lhs <- patternP
          symbol "="
          uncall <- observing $ symbol "!"
          fname <- identifier
          rhs <- patternP
          rword "in"
          scn
          case uncall of
            Left _ -> return $ Call lhs fname rhs
            Right _ -> return $ Uncall lhs fname rhs

patternP :: Parser Pattern
patternP = try as <|> try neq <|> try nilnil <|> var <|> const'
          <|> try pair <|> parLE <?> "Pattern"
where
  nilnil = rword "[[]]" >> return NilNil
  const' = (integer <|> nils) <&> Const
  nils   = rword "[]" >> return 1
  var    = identifier <&> Var
  neq    = do ident <- identifier; rword "<>"; Neq ident <$> patternP
  as     = do ident <- identifier; rword "as"; As ident <$> pair
  pair   = parens pairP
  pairP  = makeExprParser patternP
          [
            [InfixR $ Pair <$ symbol ":@"],
            [InfixR $ Pair <$ symbol ","]
          ]
  parLE  = parens patternP

```