# SSB-30806: Systems biology from genes to ecosystems

(Lecture by Prof. Christian Fleck[1], Period 2; 2014/15)

## Exercise sheet

### General remarks

- For exercise 3) you need MatLab functions which are supplied via the Blackboard. Download those files and put them in your current MatLab folder (or change your MatLab folder to where you saved them).
- If you need any MatLab function (other than the provided ones) and are not sure how to use it, use the `help` function to find out. The syntax is `help` *functionname*.

### 1) The Gierer-Meinhardt activator-inhibitor system

The Gierer-Meinhardt model is one of the prototypes of a reaction-diffusion systems. It is an activator-inhibitor mechanism represented by the following PDEs:

$$\frac{\partial A}{\partial t} = k_1 - k_2 A + k_3 \frac{A^2}{B} + D_A \nabla^2 A \tag{1a}$$

$$\frac{\partial B}{\partial t} = k_4 A^2 - k_5 B + D_B \nabla^2 B \tag{1b}$$

The system can be nondimensionalised which leads to the dimensionless system:

$$\partial u_t = \gamma \left( a - bu + c\frac{u^2}{v} \right) + \nabla^2 u \tag{2a}$$

$$\partial v_t = \gamma \left( u^2 - v \right) + d\nabla^2 v \tag{2b}$$

The boundary and initial conditions are given by:

$$(\mathbf{n} \cdot \nabla) \begin{pmatrix} u \\ v \end{pmatrix} = 0, \quad \mathbf{r} \text{ on } \partial B; \qquad u(\mathbf{r}, 0),\ v(\mathbf{r}, 0) \text{ given,} \tag{3}$$

where $\partial B$ is the closed boundary of the reaction diffusion domain $B$ and $\mathbf{n}$ is the unit outward normal to $\partial B$.

1.1) Determine the spatially homogeneous steady state $(u_0, v_0)$ of the dimensionless system (Eqn. (2)) and linearise the system around this steady state.

1.2) Assume the solutions $\mathbf{w}$ to be in the form

$$\mathbf{w} \propto e^{\lambda t}$$

[1] `Christian.Fleck@wur.nl`

and determine criteria for the stability of the homogeneous steady state.

1.3) Determine the eigenfunctions (also called modes) $\mathbf{W}_k$ that satisfy the boundary conditions and the wavenumbers $k^2$ on a one-dimensional domain $B = [0, p]$. Calculate the eigenvalues $\lambda(k^2)$ as a function of the wavenumbers.

## 2) Turing space and spatial instability

2.1) Plot the dispersion relation ($\lambda(k^2)$) in a range of $0 \leq k^2 \leq 1.5$ (in steps of 0.01) for the parameter set

$$a = 0.1 \qquad b = 1 \qquad c = 5 \qquad \gamma = 1 \qquad d = 20$$

2.2) Vary $d$ from 5 to 10 in steps of 1. Plot the dispersion relation for each value. What is (roughly) the critical value for $d$ where the bifurcation occurs? What does change at the bifurcation?

## 3) Numerical integration of the two-dimensional Gierer-Meinhardt model

3.1) Implement the dimensionless Gierer-Meinhardt system (Eqn. (2)) in the file `gierer_meinhardt.m`. In the file you only need to add the kinetics to the equations, the diffusion term is already included.
**Important:** Use for all calculations elementwise operations (e.g. `.*` instead of `*`).

3.2) The file `integrate_gm.m` contains a function to run a numerical integration of the Gierer-Meinhardt model on a two dimensional domain and plot its solution. It is called with the following command:

```
[t,y] = integrate_gm(a,b,c,gamma,d,size_x,size_y);
```

where `a`, `b`, `gamma` and `d` are the parameters of the model and `size_x` and `size_y` is the size of the domain in $x$ and $y$ direction, respectively. The system is perturbed with random noise. Run the simulation on a $20 \times 30$ grid with the parameters from 2.1. The function produces two plots, one in which the initial conditions and the final pattern is plotted, and one where you see the temporal development of the solution (it's like a movie, once it shows up, don't click anywhere else, just wait until it's done).

3.3) Change the parameter $d$ to a value of 5. Run the simulation as in 3.2 and compare the results.