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```
clear
close all
```

Problem Definition

```
A = 4*10^(-4);
E = 200e9;
L = 0.6;
AE = A*E;
rho = 7850;
nel = 3;
nn = 4;
xh = linspace(0,L,nn);
LM = [1 2 3;
      2 3 4];
idf = [2 3];
idd = [1 2];
```

FEM

Initialize global arrays

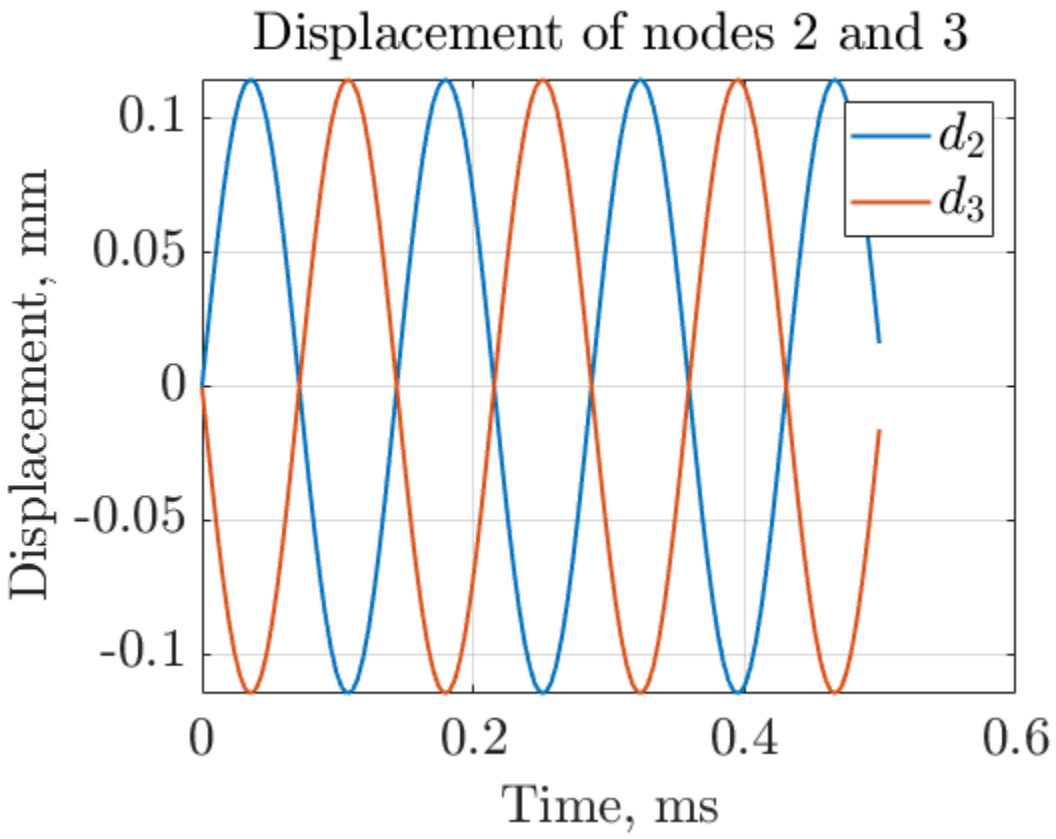
```
M =  zeros(nn,nn);
K = zeros(nn,nn);

% Quadrature points and weights
wgp = [1 1]';
xgp = [-1/sqrt(3) 1/sqrt(3)]';
ngp = length(xgp);

% Shape functions and their derivatives
N = [0.5*(1-xgp),0.5*(1+xgp)];
dN = [-0.5 0.5].*ones(length(xgp),1);

% Assembly
for el = 1:nel
    % element connetivity
    lm = LM(:,el);
    for gp = 1:ngp
        % element jacobian
```

```
            j = dN(gp,:)*xh(lm)';
            % local stiffness matrix
            ke = AE*wgp(gp)*dN(gp,:)'*dN(gp,:)./j;
            % add to global stiffness
            K(lm,lm) = K(lm,lm)+ke;
            % local mass matrix
            me = A*rho*wgp(gp)*N(gp,:)'*N(gp,:).*j;
            % add to global mass
            M(lm,lm) = M(lm,lm)+me;
        end
    end

    % Lumped mass matrix by row-sum
    Mdiag = sum(M,2);

    % Free nodes
    Mfdiag = Mdiag(idf);
    Kf = K(idf,idf);

    % Initial conditions (at idf nodes)
    dn = zeros(2,1);
    vn = [5;-5];
    an = zeros(2,1);

    c = sqrt(E/rho);    % wave speed
    h = L/nel;          % min element length
    dt = 0.1*h/c;       % time step
    tend = 0.5*10^(-3); % final time

    % history array
    ds= zeros(2,ceil(tend/dt));
    vs= zeros(2,ceil(tend/dt));
    vs(:,1) = vn;

    %time step counter
    k = 1;

    % Newmark's method, gamma = 0.5, beta = 0
    for t = 0:dt:tend-dt
        dn = dn + dt*vn +  dt^2/2*an;
        aold = an;
        an = -(Kf*dn)./Mfdiag;
        vn = vn + dt/2*(an + aold);
        ds(:,k+1) = dn;
        vs(:,k+1) = vn;
        k = k+1;
    end
```

Plot Displacements

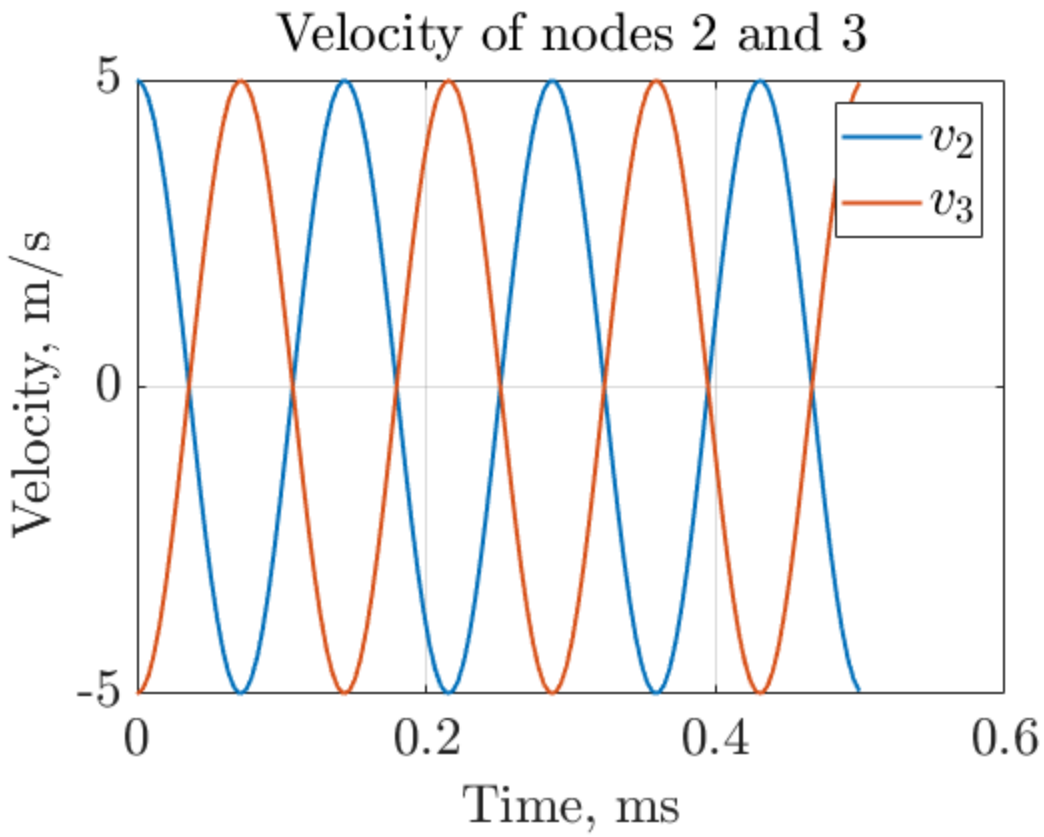
```
    t = 0:dt:tend;
    fig = figure;
    plot(t.*1000,ds(1,:).*1000,'LineWidth',1.5);
```

```
hold on
plot(t.*1000,ds(2,:).*1000,'LineWidth',1.5);
ax = gca;
ax.FontSize = 20;
set(gca,'TickLabelInterpreter','L')
title('Displacement of nodes 2 and 3','FontSize',20,'Interpreter','l')
xlabel('Time, ms','FontSize',20,'Interpreter','l')
ylabel('Displacement, mm','FontSize',20,'Interpreter','l')
legend({'$d_2$','$d_3$'},'FontSize',20,'Interpreter','l')
grid on
```



Plot Velocities

```
t = 0:dt:tend;
fig = figure;
plot(t.*1000,vs(1,:),'LineWidth',1.5);
hold on
plot(t.*1000,vs(2,:),'LineWidth',1.5);
ax = gca;
ax.FontSize = 20;
set(gca,'TickLabelInterpreter','L')
title('Velocity of nodes 2 and 3','FontSize',20,'Interpreter','l')
xlabel('Time, ms','FontSize',20,'Interpreter','l')
ylabel('Velocity, m/s','FontSize',20,'Interpreter','l')
legend({'$v_2$','$v_3$'},'FontSize',20,'Interpreter','l')
grid on
```



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