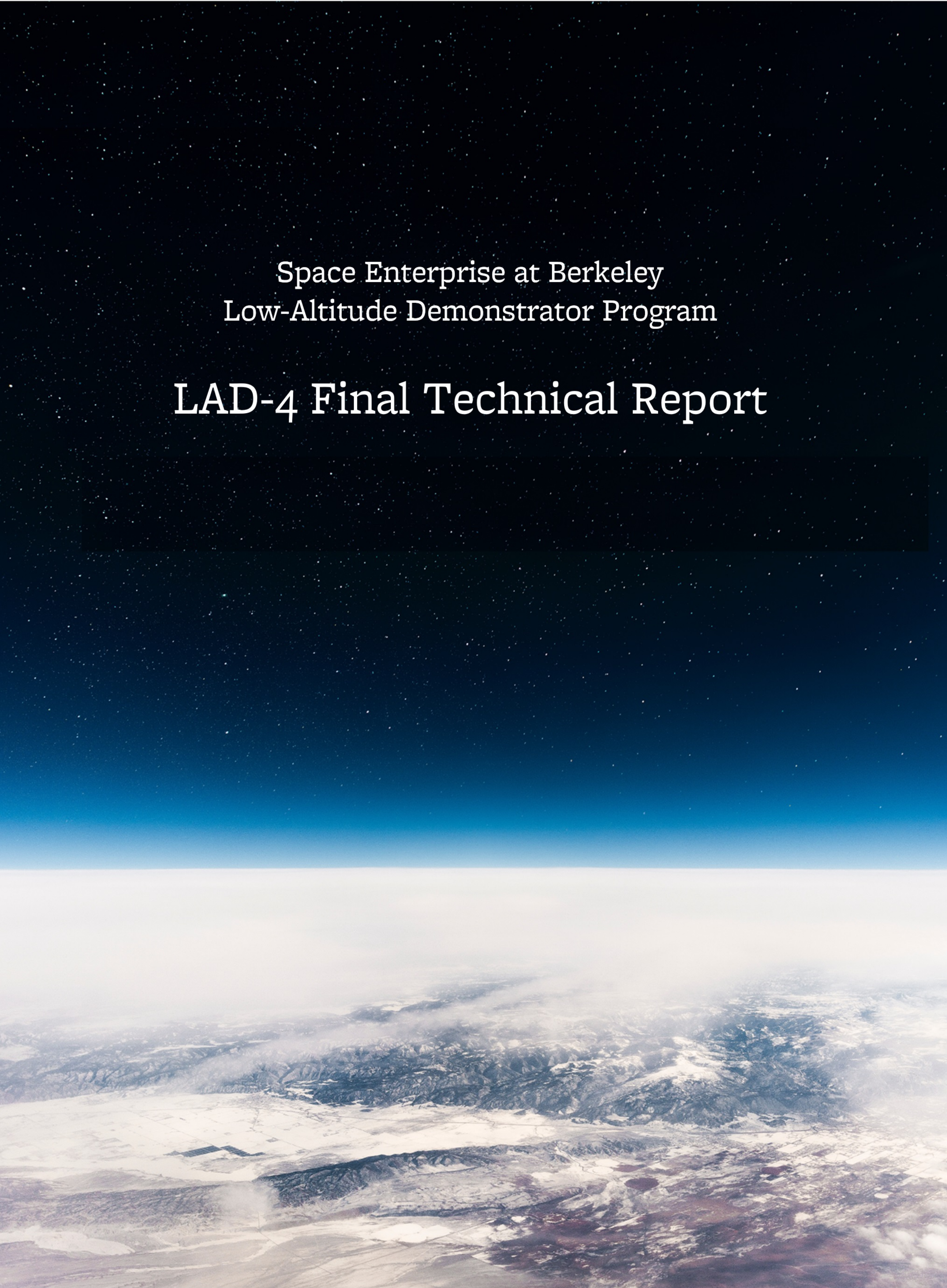


Space Enterprise at Berkeley
Low-Altitude Demonstrator Program

# LAD-4 Final Technical Report

# Contents

## 1 Introduction

Space Enterprise at Berkeley (SEB) is a fully student-run organization founded in 2016 with the goal of developing rocket technologies and achieving spaceflight. Since then, the team has extensively developed its *Low-Altitude Demonstrator* (LAD) program in order to flight-test critical infrastructure and fine-tune its design and in-house manufacturing processes. Each LAD flight in the team's history has served the purpose of validating important components and designs to be used on future rockets, and an accelerated design cycle allows for rapid empirical testing. In 2019, the team debuted its EUREKA space-shot program; the propulsion system of EUREKA-1, the team's first liquid-fuel rocket, is currently under development. To flight-test and validate the avionics and recovery system that will fly aboard EUREKA-1 and successive vehicles, LAD-4 was built within a seven-week span in the spring semester of 2020. Beyond its primary use as a test vehicle, LAD-4 was an educational project that allowed newer members to develop skills in the basics of rocket design, experience composite material manufacturing, and taste the sweetness of flight. LAD-4 was also designed to break university records for altitude and speed, and to be the first supersonic vehicle built by a student team at the University of California, Berkeley.

## 2 Abstract

LAD-4 "*Updog*", is Space Enterprise at Berkeley's third fully-completed flight vehicle in the Low Altitude Demonstrator (LAD) program. As the fourth LAD airframe to be constructed overall, considerations regarding cost savings, time savings, and material efficiency were paramount. As a test vehicle, the critical aspect of LAD-4's airframe design was reusability and modularity; the rocket was designed to be flown up to ten times as necessary. This led to the natural progression of LAD-4's design to allow for the ability to rapidly replace damaged parts, to house different motor configurations in future flights, and to integrate active aerodynamic control modules for further test projects. The rocket has a nominal, minimal outside diameter of 6.5 inches, a height of 90 inches, and is powered by an Aerotech M1340W-PS solid motor. On March 7, 2020, at the Friends of Amateur Rocketry site in Cantil, California, LAD-4 flew on its maiden flight to Mach 1.2 and an apogee of 11 193 feet above ground level, successfully establishing new UC Berkeley records for altitude and speed.

## 3 Systems Architecture Overview

With the goals of reusability and modularity in mind, LAD-4 was designed to use as many student researched and developed (SRAD) components as possible, with the exception of the use of a commercial/off-the-shelf (COTS) solid motor, which was determined to be the most economical and time-efficient strategy with respect to project development time. The rocket's aerostructure, recovery, and avionics systems were fully designed and fabricated by the team, using COTS parts only where necessary and within the framework of a student-developed system (e.g. a COTS barometric altimeter was used for apogee detection). These decisions were made in accordance with the mission goal of validating these components for future EUREKA flights.

The following table lists important top-level parameters for the rocket.

| **Parameter** | **Value** | **Unit** |
|---|---|---|
| Length | 90 | inches |
| Nominal outer diameter | 6.5 | inches |
| Dry mass | 13 | kilograms |
| Theoretical (projected) apogee | 13142 | feet |
| Theoretical (projected) maximum speed | 1.07<br>356.6 | Mach<br>meters per second |
| Motor impulse, total (manufacturer specification) | 7673 | newton-seconds |

Table 1: LAD-4 Top-Level Parameters

The figures on the following page illustrate the internal configuration of LAD-4. The top figure is a diagram from the OpenRocket simulator program and the bottom figure is a CAD model of the rocket and its systems with each component located precisely. The rocket was designed using a combination of commercial simulation software (e.g. OpenRocket) and a custom numerical calculator (section 3.2.2) in order to establish important parameters such as stability, target mass, and predicted apogee. A CAD model of the rocket ensured that components would fit together during integration as designed and provided a basis for machining procedures (e.g. laser cutting).

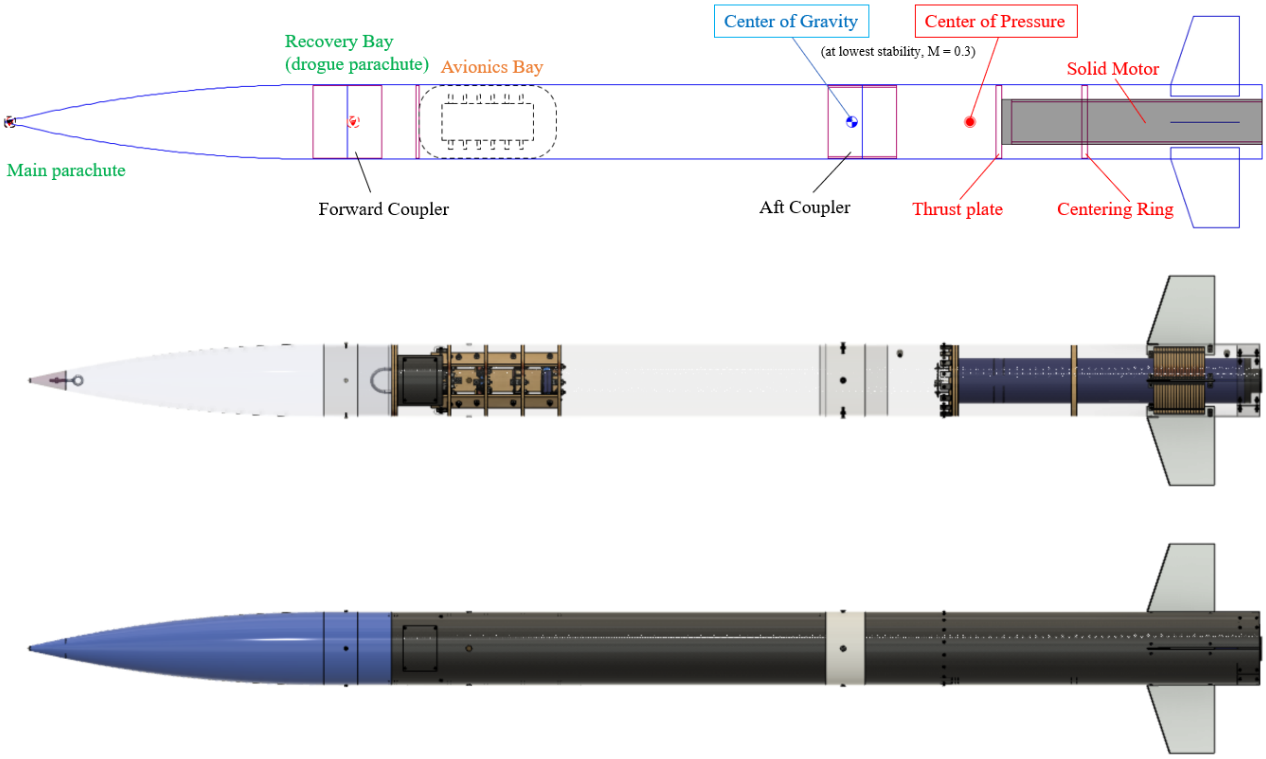



Figure 1: System Overview

### 3.1 Airframe

The LAD-4 airframe consists of a nosecone and two main body tubes, which house four main sections. The *recovery bay* is situated in the aft portion of the nosecone and the forward portion of the forward tube, sealed by the forward pressure bulkhead. Aft of the bulkhead is the *avionics bay*, which consists of the apogee detection and charge deployment system, a custom data-logging system, and a radio transmitter module. In the aft airframe tube, the thrust plate sits forward of the motor housing, which retains and centers the *motor assembly*. The *fin can section*, which contains the architecture to secure and stabilize the fins, is located at the aft end of the rocket.

#### 3.1.1 Nosecone

LAD-4 features a fiberglass nosecone manufactured in-house by team members. Fiberglass was chosen to maintain radio transparency, with the avionics bay in close proximity.

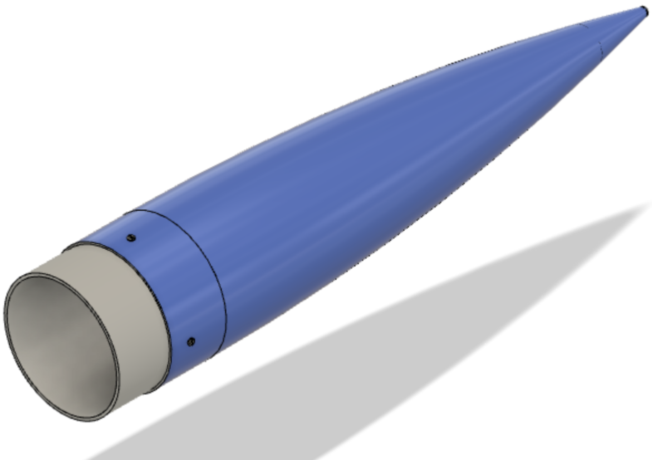


Figure 2: Nosecone with forward coupler

**Design** The nosecone of the rocket was a tangent ogive shape with a total length of 30 inches, which includes a 3-inch straight section built in for additional recovery equipment space. As the principal criterion for nosecone design was aerodynamic, studies of drag at various speeds were done with multiple ogive designs, as well as conical and power-law designs. It was determined that the tangent-ogive shape was most appropriate for the transsonic range of LAD-4.

**Manufacturing** The nosecone was made using two identical negative molds, each the shape of half of the nosecone. The two molds were produced out of medium-density fiberboard (MDF) on a Shopbot CNC mill.

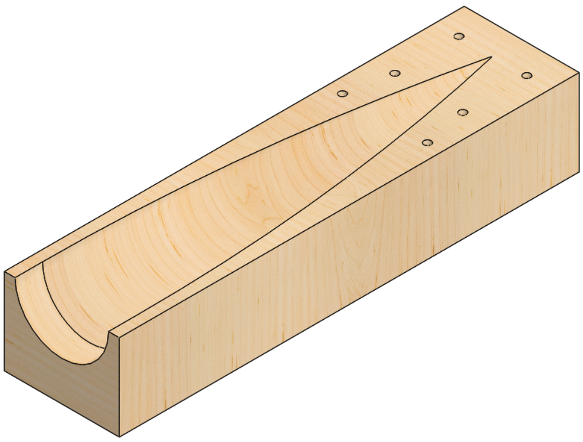


Figure 3: Nose Cone Mold Half

The molds were prepared with two coats of paste wax ("turtle wax") and two coats of polishing wax. After the wax dried according to the manufacturer's instructions, the nosecone was laid up with seven layers of E-glass fiberglass and TAP Plastics epoxy resin. The layers were thoroughly saturated and applied in quick succession. Fiberglass "templates" for each layer were cut out in the shape of a triangle prior to the layup process to save material. The two halves were joined together using wooden dowels as alignment pins in the 6 holes visible in Figure 3. After the halves were aligned, fiberglass patches were applied along the interior seam between each mold half, followed by a slurry of epoxy resin and chopped fiberglass fibers. The nosecone was then allowed to cure according to the epoxy manufacturer's directions for a minimum of 24 hours.

After the epoxy cured, the nosecone was removed from the mold. After removal, a number of small surface imperfections were observed on the cone. Most notably, there was a small ridge along the line of the seam between the mold halves. The ridge and other surface defects were sanded, filled in using Bondo body filler, and finished with successive rounds of sanding and filling until a smooth, uniform surface finish without ridges, dents, or other substantial imperfections was achieved. The inside of the nosecone was cleaned using a wire brush attached to a power drill, and the uneven bottom edge of the nose cone was finished on a tile saw to achieve a true bottom mating surface against the upper airframe tube.

**Eye-Bolt** The tip of the nosecone was filled to a height of 4 inches with a fiberglass-resin slurry, and an eye-bolt was submerged into the wet slurry and suspended in place such that when the resin fully cured, the "stem" of the eye-bolt was entombed in hardened slurry and the "eye" of the eye-bolt was fully exposed. The "eye" served as a hard point for the attachment of the nose cone to the parachute and recovery gear.

#### 3.1.2 Body Tubes

The two fiberglass/carbon-fiber body tubes of the rocket were also manufactured by team members using continuous fiber sheets. The rocket features a monocoque load-carrying aerostructure, which does not require the use of longerons to provide stiffness or strength.

**Design** Because of the high aspect ratio and low thickness of the airframe, the principal failure modality of the body tubes to be considered is buckling. The state of loading on the airframe consists of two major sources of compressive stress: aerodynamic drag and an internal force from the motor, which is passed through to the airframe via a thrust plate. Thus the maximum total stress occurs at the end of the motor burn, where the dynamic pressure is at a maximum (the point of "max $q$"). The dynamic pressure over time, calculated using motor parameters and the manufacturer's thrust curve, is shown in the following figure.

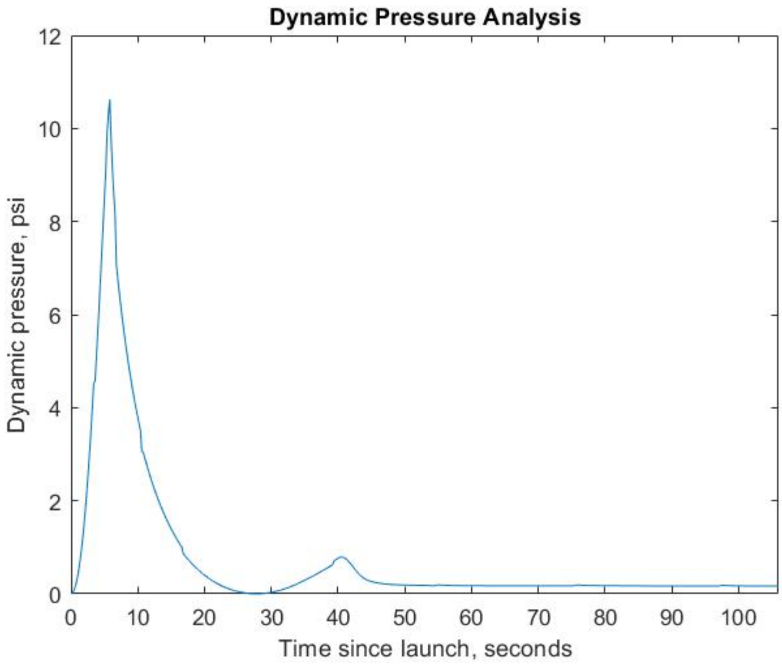



Figure 4: Dynamic pressure on the rocket as a function of time. The peak dynamic pressure occurs at the end of motor burn, around 5.7 seconds into the flight.

LAD-4's airframe is primarily composed of bidirectionally-woven fiberglass with the exterior layer substituted for carbon fiber, chosen for its high specific strength and stiffness. The decision to use fiberglass as the principal material was due to its lower cost compared to other materials used in fiber-reinforced polymer composites. An analysis of the critical buckling load on an airframe tube was conducted to determine the amount of material needed for the airframe in order to withstand the maximum dynamic pressure with a minimum safety factor of 2.0. Though a degree of variability is introduced in the manual production of airframe tubes, the properties of the laminate were determined using the laminated plate theory described by R.F. Gibson (*Principles of Composite Material Mechanics*, fourth edition), and the necessary thickness was calculated using Euler's formula for beam buckling (the calculation is given in Appendix C). In all, four layers of fiberglass and one layer of carbon fiber were used. The main body tubes consisted of two sections laid up independently: one 48-inch section and one 38-inch section. The nominal inside diameter of the tubes was 6.3", and the nominal outside diameter of the tubes was 6.5". As a final step in the design process, the rocket was mocked up in a simulation program and evaluated for its stability. The results are plotted below.

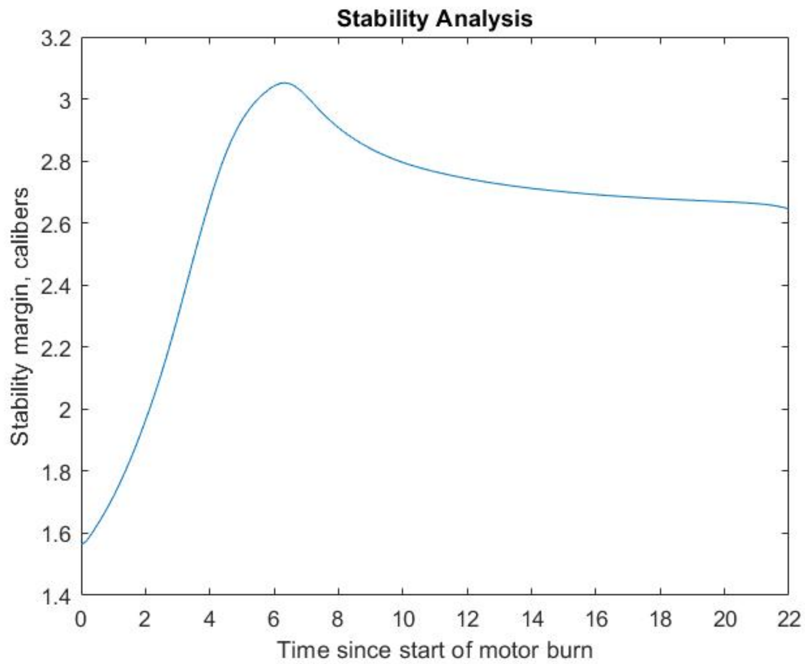



Figure 5: Stability analysis obtained from a simulation program (OpenRocket) and plotted.

In the duration of flight before apogee is reached and the drogue parachute is deployed, the stability was determined to be within a safe envelope (greater than 1.5 calibers and less than 6 calibers).

**Manufacturing** The body tubes were laid up by hand using a wet layup process. A cardboard mailing tube with a 6" outside diameter was used as a central mandrel, onto which a layer of mylar was wrapped tightly to aid the airframe removal post-cure. The mandrel was wetted with epoxy, and the woven fiberglass cloth, pre-cut to size by layer, was wrapped onto the mandrel and subsequently wetted out. This process was repeated layer by layer for the four fiberglass layers and one carbon fiber layer, rotating the seam by approximately 60 degrees each time to prevent ridges. After the last (carbon fiber) layer was wetted out, peel-ply film was wrapped tightly around the layup to create a consistent surface finish by soaking up excess epoxy. The tube was then left to cure for a minimum of 24 hours, as given in the epoxy manufacturer's guidelines.

After the epoxy was fully cured, the body tube exteriors were repeatedly sanded down and re-wetted with epoxy until the surface finish was smooth. Increasingly finer-grit sandpaper was used, up to 3000 grit sandpaper for the final pass. The ends of the airframe tubes were then removed using the tile saw, such that each tube had two clean, perpendicular ends. This resulted in a slightly shorter overall length of each tube; the final airframe tubes had lengths of 45 inches and 35 inches.

**Static port holes** During the final assembly process, four static port holes of diameter 0.25" were drilled in the airframe near the Stratologger barometric altimeter module to normalize the internal pressure with the atmospheric pressure, enabling the barometric altimeter to function properly.

**Access panel** An access panel was created in the forward body tube for access to the Avionics bay after integration. The access panel was a 1.5" by 2.5" cutout in the airframe, secured via rivnuts, with a backing plate installed within the airframe. The backing plate was made using a scrap section of 6"-diameter carbon fiber tube, and permanently affixed to the airframe with epoxy. The access panel could then be removed and reattached to the backing plate freely.

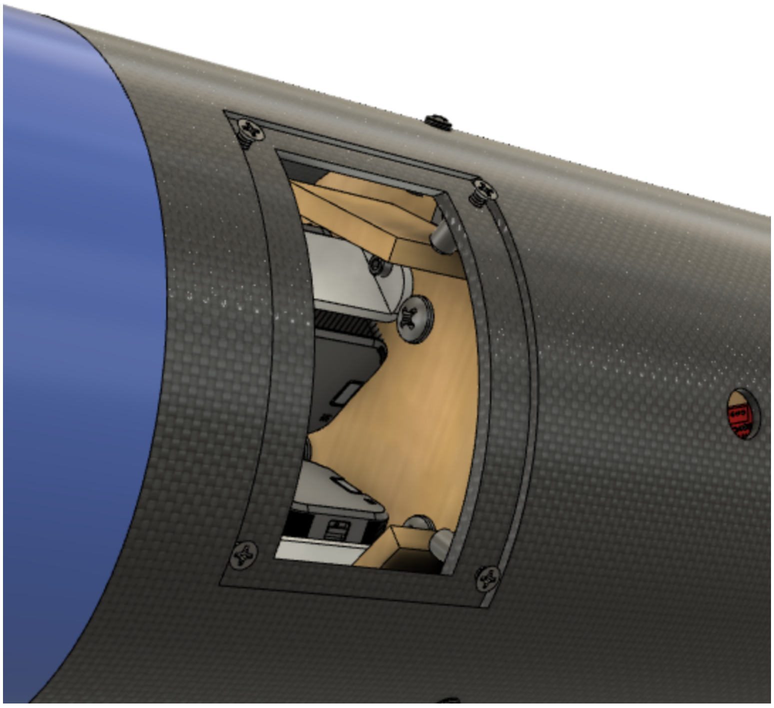


Figure 6: Access Panel detail. Aft of the access panel, a camera hole is visible.

**Fin slots** Fin slots, extending 9" from the aft end of the airframe, were created using a router and a custom laser-cut plywood jig. The rectangular jig consisted of a box with a missing side for the excess length of tube to protrude through as well as a removable top with a 0.13" by 9" slot for the router bit to stick through. The router was pushed along a groove on the removable top parallel to the slot and cut the tube beneath. The jig also included a square frame press fitted on the protruding end of the tube to ensure that the slot centerlines were 90 degrees apart.

#### 3.1.3 Coupler Sections

There are two coupler sections used to connect the three manufactured portions of the airframe. Both coupler sections had a phenolic-like core, made of 6"-diameter Blue Tube (Always Ready Rocketry, Inc., Mt. Vernon, WA) cut to length. The Blue Tube was inserted into 6"-inner-diameter mailing tubes and epoxied into place, forming a bilayer structure. The outside of the mailing tubes were then sanded down until they were able to be inserted the correct distance into the body tubes. The *forward coupler* was epoxied onto the aft portion of the nosecone and is used to bridge the nosecone and the forward body tube. The forward coupler is attached to the forward body tube using three nylon bolts, designed to shear at apogee and eject the nosecone. The *aft coupler* attaches the forward body tube to the aft body tube and is bolted into both composite skins.

#### 3.1.4 Fins

LAD-4 is equipped with four metal fins, made from 6061 aluminum sheets of 1/8" thickness.

**Design** Aluminum was chosen as the fin material due to its high specific stiffness and for its ability to be machined consistently. The team experimented with all-carbon fiber fins but found that variance in the layup process resulted in nonuniform fins. The aluminum fins, manufactured using a laser cutter, have the profile shown below, with a 6" span, a 2" sweep distance, and a 6" root chord. The bottom 1" of the fin, below the slot, is completely internal and provides contact surface area for the fin can assembly. The slot in the forward section of the fin slides into a corresponding slot made in the airframe, and locks the fin into place, preventing rotational motion. Additionally, the leading edge of each fin is bevelled to a 45 degree angle to reduce the induced drag.

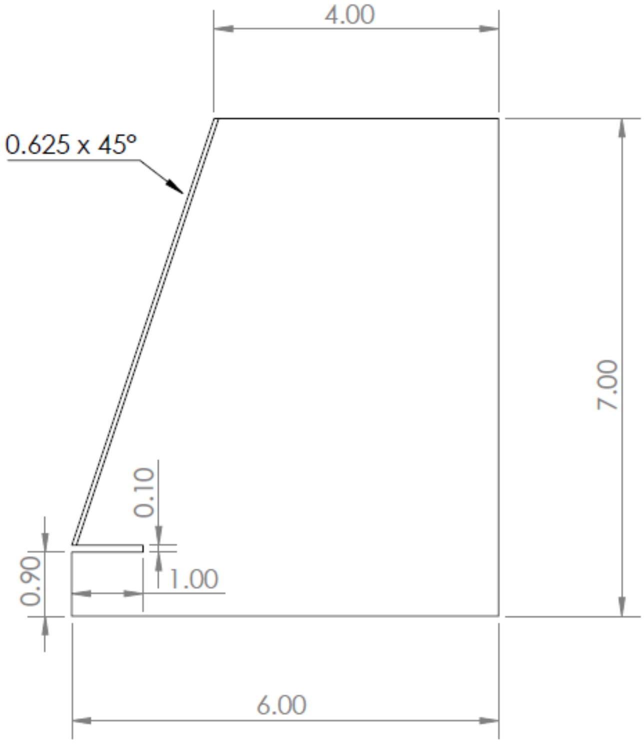



Figure 7: Fin design geometry. All dimensions are in inches; fin thickness is uniformly 0.125".

A principal concern in fin design is aeroelastic flutter, a phenomenon that occurs above a critical *flutter velocity* whereby any transverse oscillations on the fin are amplified rather than damped, leading to the catastrophic failure of the fin and extreme instability in subsequent flight. The flutter velocity is dependent largely on the fin geometry and material. Using the fin geometry and material given, the flutter speed was calculated to be in excess of Mach 2, much higher than the predicted maximum velocity in flight. The calculation of flutter speed is given in Appendix C; the designed fins were determined to be safe against flutter with a factor of safety of 1.97.

**Manufacturing** The fin profile was drawn using AutoCAD and the DXF file was sent to a Fablight laser cutter for manufacturing. A laser cutter was used to ensure consistent dimensions across all fins and to minimize the kerf distance. The bevel was done freehand using a belt sander. After the final assembly of the rocket, the fins were cleaned and polished using a 6000 grit whetstone to achieve a smooth finish.

#### 3.1.5 Fin Can

In accordance with the project goal of flying the LAD-4 airframe several times, the fin retention system is designed to be completely modular. Consequently, the fin can section allows for the simple replacement of a single damaged fin (e.g. from a rough landing) between flights. This means that the fins are not permanently bonded to any portion of the airframe. In previous LAD rockets, permanent fin fillets were created using a carbon fiber slurry and used to bond the fins to the airframe, followed by a tip-to-tip layup. Though the fillet-layup method is effective in adding the needed strength and stiffness to the fins, it effectively renders the aft body tube and fins as one complete unit. In LAD-4, a new, completely modular *fin can* was designed and manufactured to allow for the same structural stability with the added benefit of replaceability.

**Design** The design goals of the fin can section were: (a) to retain each fin and provide structural stability against translational motion in any dimension, (b) prevent a bending moment about the fin root that could lead to dangerous flutter conditions, and (c) allow for the easy removal and replacement of a single damaged fin and avoid any sort of permanent bonding. To strengthen the fin-fuselage attachment, a through-wall mounting system is used: a set of wooden centering rings, attached around the motor housing, mate with the fins, themselves attached to the motor housing via L-brackets. Together with the centering rings, a slot cut into the leading edge of each fin which mates with a slot cut into the airframe ensures alignment is exact and repeatable, while retaining the fin against circumferential and longitudinal motion. The L-bracket system provides a pre-load against a bending moment, damping oscillations in the fin. The entire assembly can be assembled independent of the rest of the airframe, slid in via the airframe slots, and secured by tightening bolts into the L-brackets and rivet nuts on the airframe.

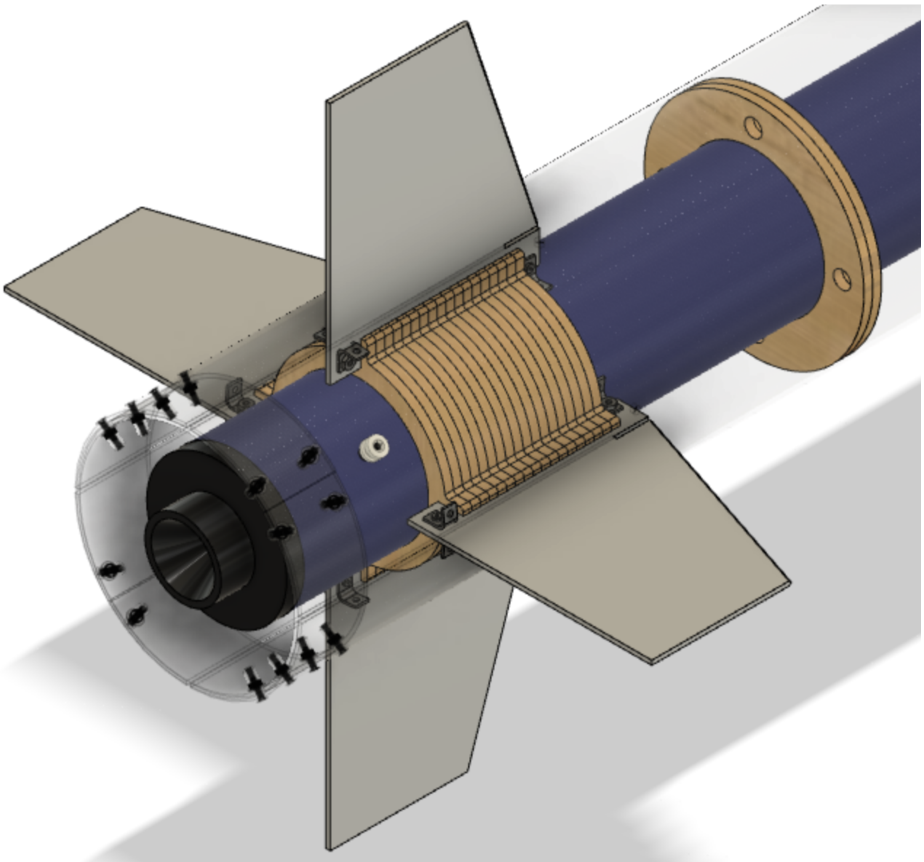


Figure 8: Detail of Fin Can and Fin Attachment System

**Manufacturing** The eighteen wooden rings were manufactured by laser cutting out ⅛" plywood. The wooden rings were then attached to the motor housing one at a time with the first ring being 6½" from the bottom of the housing. A single layer of epoxy was applied in between each ring and the motor housing. Once all the rings were attached, two layers of carbon fiber were applied to the outside of the wooden ring assembly, across all of the rings. The L-brackets were attached first to each fin in a designated position, so that they could be tightened upon integration of the fin can section.

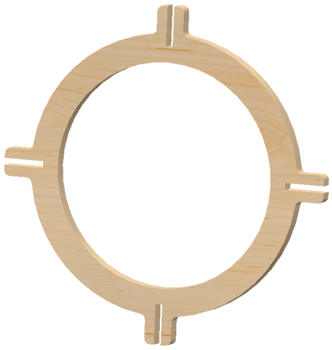


Figure 9: One of eighteen wooden fin alignment and retention rings.

**Integration** First, the fin can assembly was created by inserting all 4 fins into the grooves formed by the wooden rings. Then, the entire assembly is attached to the airframe by aligning the fins to the slots made in the aft body tube and sliding the fins such that the slots in the fins interlock with the slots in the aft body tube. Once the fin can was seated, the L brackets on the fin can were secured to the rivnuts in the airframe. Beneath the fins, small composite 'band-aids' were mounted to the inside of the aft body tube at the aftmost end to stabilize sections between the fin slots. The 'band-aids' were small, rectangular portions cut out of a previously-constructed 6"-diameter airframe tube, so that they were pre-formed into the correct radius.

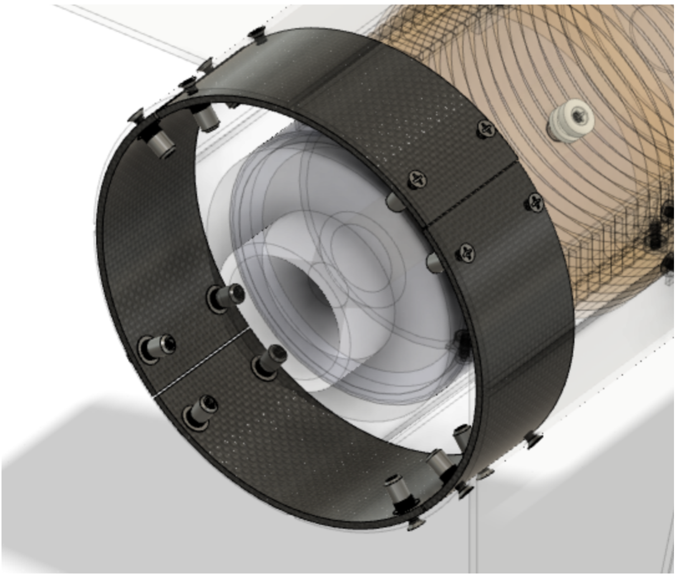


Figure 10: Fin Slot "Band-Aids"

#### 3.1.6 Bulkheads

**Thrust plate** The thrust plate, responsible for transferring loads from the engine through to the airframe, was created from two ¼" plywood discs glued together with four layers of fiberglass and one layer of carbon fiber laid up on both sides for a total of 10 composite layers. The thrust plate was attached to the airframe at a position that allowed contact with the motor assembly.

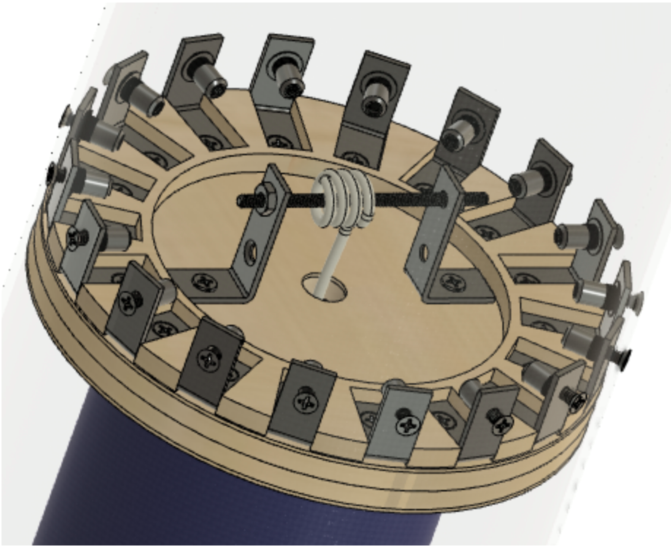


Figure 11: Thrust plate detail, showing the motor retention system (center). The eighteen bolts around the circumference of the thrust plate are screwed through the airframe, securing the thrust plate in place. In the center, a length of paracord secures the motor to the thrust plate, preventing it from falling out of the rocket.

**Motor retention** For the purposes of retaining the motor and motor housing within the airframe after it had fired, a length of paracord was tied to an eye-bolt threaded to the top of the motor, passed through a small hole drilled in the thrust plate, and tied to a threaded rod set off from the thrust plate.

**Attachment to airframe** For the purposes of thrust transfer to the airframe, the thrust plate was attached to the airframe using a set of eighteen L-brackets, which were bolted into the thrust plate itself. For alignment purposes, in the other hole of each L-bracket, a threaded insert (rivnut) was installed. This allowed a bolt to be threaded in from the outside of the airframe in a secure manner. A laser-cut wood piece was attached to the top of the thrust plate to align the L-brackets.

**Pressure plate** The pressure plate served as a bulkhead between the deployment charge/recovery compartment and the avionics bay. It is necessary for the purposes of (a) protecting the avionics bay and the rest of the rocket from the heat and forces of the explosive charge, and (b) to facilitate the buildup of pressure within the recovery compartment that would fracture the shear pins and eject the nosecone. It also served as the forward stop for the avionics bay. The pressure plate had a core made of ¼”-thick wood and was laid up with four layers of fiberglass on each side. No carbon fiber was used to lay up the pressure plate to ensure that radio transmission was possible from the avionics bay to the exterior of the rocket. A small hole was drilled in the pressure plate for the passage of wires between the avionics bay and the charges.

Additionally, a U-bolt was attached to the forward side of the pressure plate. This U-bolt served as the hard point on the airframe for the drogue parachute. The pressure plate was constructed on its own and epoxied into the airframe using a fiberglass-resin slurry that created a fillet around the edge of the pressure plate.

## 3.2 Propulsion

Inasmuch as the primary project goal of LAD-4 is to provide a robust vehicle for iterative validation flights of avionics and recovery system components, the propulsion system was designed to use a COTS solid motor. The decision to use a commercially available motor drastically reduces development cost and time, and provides a reliable propulsion source for each flight, reducing the number of variables in the system.

### 3.2.1 Propulsion System Overview

The propulsion system on the maiden flight of LAD-4 uses an Aerotech M1340W-PS solid motor, which consists of three individual ammonium-perchlorate composite propellant (APCP) grains. The mass of the propellant is 4.036 kg, and the motor provides an average thrust of 1346.14 newtons during a 5.7-second burn time. The motor was assembled two days before launch, and integrated into the rocket during final assembly. The included electronic motor starter was used to ignite the engine on the launchpad. The M1340W-PS motor results in a thrust-to-weight ratio of 7.62 on launch, and results in a vehicle speed of 27.89 m/s (91.52 ft/s) off the launch rail, sufficient to withstand significant wind forces.

The following figure, obtained from John Coker for ThrustCurve.org and certified by Tripoli Rocketry Association, Inc., plots the thrust of the motor over time.

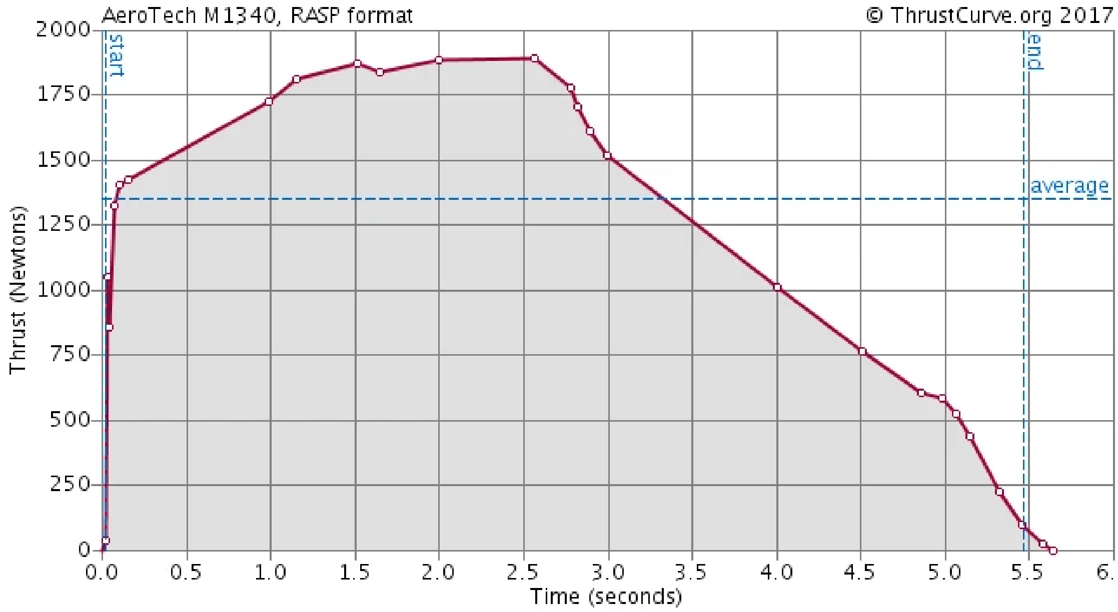



Figure 12: Thrust curve for the Aerotech M1340W-PS solid motor.

Though an Aerotech M-class motor was used on the first flight, the rocket is capable of integrating with *any* 98mm-diameter motor system. This provides the ability to tailor the rocket's performance (speed and altitude) to the specific goals of the testing performed on that flight.

### 3.2.2 Trajectory Planning

A comprehensive numerical "flight simulator", or trajectory planner, was created to accurately estimate critical flight parameters during manufacturing and testing, using a set of predetermined parameters (independent variables). It is based on a simple premise: model the flight of a rocket only in one dimension using basic kinematic equations. By using a small time-step (0.2s), these kinematic equations can achieve sufficient granularity to create close approximations of the rocket's flight capabilities and final apogee.

On a more detailed level, the simulator required thrust, drag, gravity, and mass data. From these basic requirements, atmospheric density data as a function of altitude, detailed drag properties of the rocket over a wide speed range, and a reliable model relating thrust, altitude, and time were necessary. Once these basic features were in place, more interesting calculations such as dynamic pressure, acceleration ($G$) load, and aerodynamic stability became possible. The calculation of these parameters heavily informed the design of the airframe and recovery systems, allowing weight and cost to be minimized where possible. For example, once a motor with a known thrust curve is chosen, the expected apogee of the rocket and the maximum loading condition on the airframe can be determined.

The one-dimensionality of the model results in a high degree of transparency when evaluating model results and allows for "bounding" of the rocket's flight parameters. Furthermore, meaningful sensitivity analysis on parameters such as mass, drag, airframe size, burn time/thrust, etc., is possible, something that would be substantially more complex and arguably less useful in a six-degree-of-freedom simulation that was too far "into the weeds". The one-degree-of-freedom platform allowed for solid conceptual design work, and a post-flight analysis showed that, importantly, it demonstrated close agreement with empirical flight data. This provides evidence that building a numerical flight simulator enables the team to avoid having to design the entire rocket in detail before getting accurate results that could then be used to guide more refined design and analysis operations.

### 3.3 Recovery

The LAD-4 dual-deployment recovery system consists of commercially purchased parachutes and a Tender Descender parachute release device, stored between the forward body tube and nosecone. Recovery begins at apogee; immediately after the Stratologger module detects a decrease in altitude corresponding with apogee, it issues the command to fire the 10-gram ejection charge, shearing the nylon bolts and separating the nosecone. This allows the drogue parachute to be ejected and inflated, pulling out the main parachute. (The Tender Descender system prevents deployment and inflation of the main parachute at this stage by constraining both ends within its carabiners.) With the drogue parachute inflated, the rocket is able to assume a stable, nose-up attitude for the descent, preventing a dangerous ballistic trajectory. The drogue parachute was sized such that this attitude stabilization was possible, while excessive drift due to wind conditions is avoided during the majority of descent.

The main parachute is held in place without being deployed until an altitude of 700 feet, at which point the Stratologger module issues a second command to fire the charge in the Tender Descender, ejecting its carabiners, which deploys the main parachute and allows it to inflate. The main parachute slows the rocket's speed for landing, minimizing or preventing any damage sustained due to impact with terrain, and allowing the rocket to be recoverable in a re-flyable state.

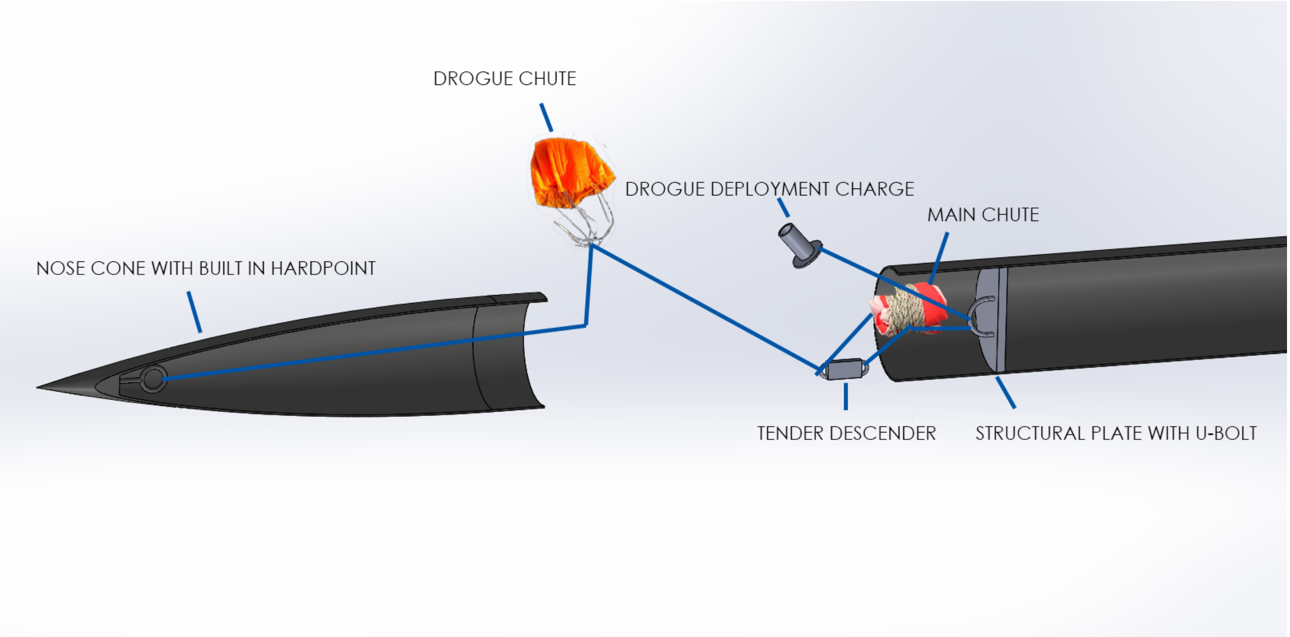



Figure 13: Exploded view of the Recovery system

### 3.3.1 Ejection Charge

The ejection charge was held in a machined metal container attached to a wooden board. It rested above the main chute when packed into the rocket. Before flight, the container was filled with 10 grams of black powder and an e-match was wired to the avionics bay. The charge was attached to the forward body section with a nylon cord.

The size of the ejection charge (10 grams of black powder) is determined mathematically from first principles and tested rigorously and repeatedly on the ground with a 100% success rate in the flight configuration. The black powder must provide a sufficient buildup of pressure to fracture three #6-32 nylon bolts in shear. Calculating the failure loading of each bolt using published shear strength values, it was determined that a total of 228 pounds-force was required to separate all three bolts. The required pressure difference between the inside of the recovery bay and the atmosphere was then calculated, using the surface area given by the inner diameter of the recovery bay (6 inches). This total pressure difference (8 psi) represented the sum of the pressure caused by the black powder detonation and the change in pressure due to the altitude difference of approximately 11 000 feet, as a lower bound. The latter is approximately 5.8 psi, meaning that the black powder charge must be sufficient to generate at least 2.2 psi of pressure in the recovery bay. As verified by a black powder charge calculator online, using 10 grams of black powder is sufficient to ensure shearing of all shear pins on the ground (8 psi), which guarantees the shear threshold will be crossed even in the event of an air leak within the recovery bay.

### 3.3.2 Tender Descender

The Tender Descender is a commercially designed product designed for dual deployment in amateur rocketry. It consists of two carabiners held in place by a pin; this assembly is retained until a small charge (3 grams of black powder) releases the pin, freeing the carabiners. By attaching the top end of the main parachute to one carabiner and the bottom end of the parachute to the other carabiner (and the rocket body), deployment of the main parachute is achieved by sending a command to detonate the charge within the Tender Descender, separating the halves. Before flight, the tender descender was filled with black powder and an e-match was set in place, wired to the avionics bay. It is directly connected to the Stratologger module, which is programmed to issue the command to fire the charge at 700 feet AGL.

### 3.3.3 Parachutes

Two parachutes were packed for a dual deployment recovery. Both parachutes are COTS parts manufactured by Fruity Chutes. The parachute sizes and models were chosen on the basis of terminal velocity (i.e. rate of descent at equilibrium). The drogue parachute allows the rocket to adopt a safe, nose-up attitude and maintain this orientation until the main parachute is deployed, while preventing excessive transverse drifting due to wind conditions near the launch area. The main parachute slows the rocket down to a safe speed for impact with terrain upon landing so that the rocket and its internal components remain undamaged. The drogue parachute was 15" in diameter, and the main parachute was 60" in diameter. Both parachutes had a drag coefficient $C_d$ of 2.20, from the manufacturer's specifications. Using the manufacturer's listed drag coefficients and dimensions for each parachute, this configuration results in a projected terminal velocity of 31.41 m/s (103.05 ft/s) under the drogue chute and 7.85 m/s (25.75 ft/s) under the main chute.

The drogue chute was packed into the nose cone and attached to the top of the main chute via a length of Kevlar shock cord and the Tender Descender, so that the drogue deployment event would allow the main parachute to be released but not inflated. It was also attached to an eye bolt at the tip of the nosecone, which provided a hard point against the shock loading during deployment. The base of the main parachute was attached to a U-bolt on the pressure bulkhead, serving as another hard point, and the top was secured to the Tender Descender. Both parachutes were wrapped using fire-resistant Nomex blankets to protect them from the black powder charge during the separation event.

### 3.4 Avionics

The avionics system aboard LAD-4 consisted of a custom electronics system operating in parallel with a commercial deployment system centered around a PerfectFlite StratologgerCF, which is a barometric altimeter and flight data recorder. This redundancy was implemented in order to test the custom avionics platform in true flight scenarios without having the safety of the rocket depend on untested hardware. The custom platform consisted of a telemetry module and a data logging module. Both of these were built centered around Teensy 4.0 arduino microcontrollers, and were designed to function independently of each other and of the commercial deployment system. Inside the rocket, the avionics modules were arranged on a removable avionics bay, which was designed to be inserted into the forward airframe tube during final assembly.

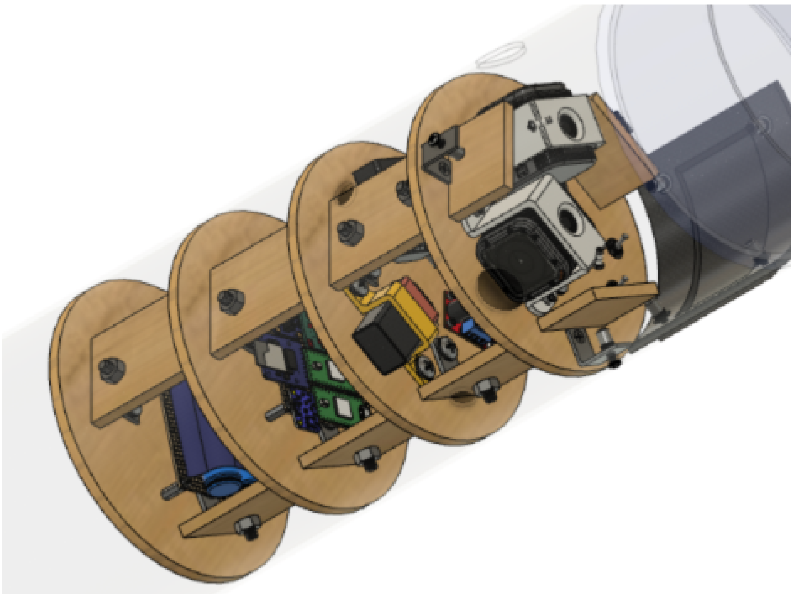


Figure 14: Overview of the Avionics Bay

### 3.4.1 Custom Avionics

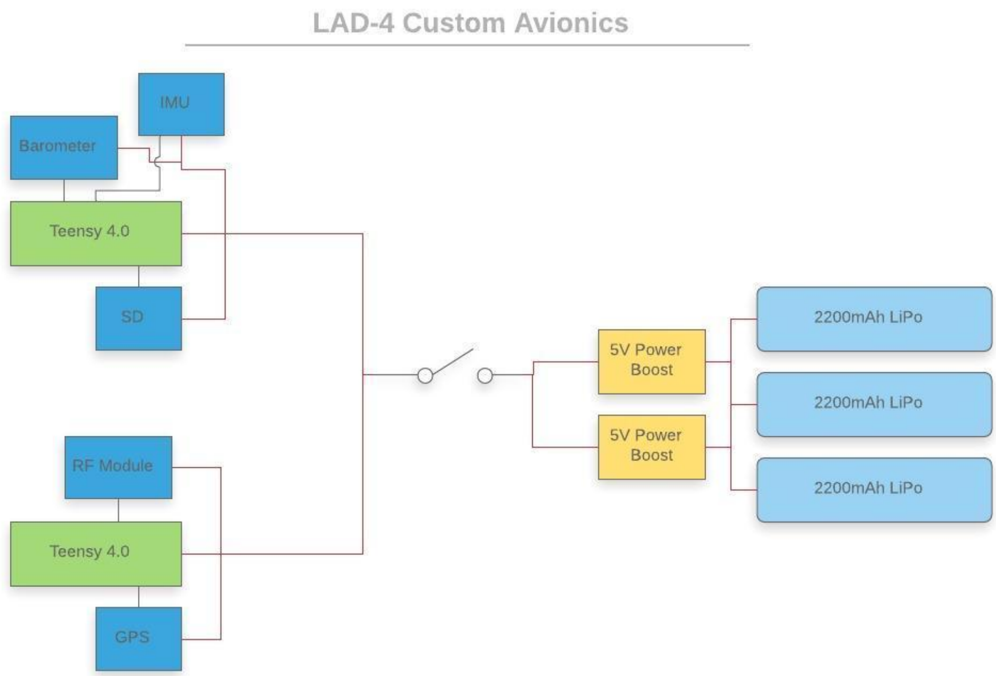



Figure 15: Diagram of the SRAD custom avionics onboard LAD-4

**2200mAh 18650 LiPo** The 2200mAh LiPo batteries were selected for their low cost, high capacity, and small size. Two of these batteries would be able to supply enough voltage for the duration of the flight, but three were used for redundancy.

**5V Power Boost 1000** The 5V Power Boost component was necessary to take the 3.7V output of the LiPo batteries and convert it into the 5V necessary for the microcontrollers and sensors. Two power boosts were used in parallel in order to supply enough current to run all the electronics.

**Teensy 4.0** The Teensy 4.0 was the microcontroller board that was used as the main controller of LAD-4's custom avionics system. The Teensy 4.0 was selected due to its high performance to size/cost ratio, compared to other similarly sized Arduino boards; notably, its extremely small footprint allowed more flexibility for arrangement within the Avionics Bay. There were two independent Teensy 4.0 controllers on board, one to read and record data from the IMU and Barometer during flight, and the other to read GPS data and transmit it to the Ground Station via the RF Module. This separate design allowed data collection and writing from the Barometer and IMU to occur at a higher rate, as well as parallel development of the radio communication and data collection modules.

**BNO055 IMU** The BNO055 is an inertial measurement unit (IMU) that contains an accelerometer, magnetometer, and gyroscope and utilizes an onboard processor to fuse those values into accurate orientation data. This IMU was selected for its precise collection and recording of acceleration data along three axes during flight. This acceleration data was used for post-flight analysis, and in the future will be used for an SRAD apogee detection algorithm.

**BMP388 Barometric Pressure and Altimeter** The BMP388 is a sensor that utilizes its onboard barometer and the change in pressure to determine the altitude of the rocket. This barometer was selected because of its precision of $\pm$ 0.5m. This altitude data was then utilized post-flight to calculate the velocity of the rocket.

**SD Cards** The onboard SD cards were used to store IMU and altitude data during the flight. The SD Cards used had a rating of UHS Speed Class 3 (U3), meaning they were capable of higher data write rates. The data logging functionality of the flight code was adjusted to optimize write-time and increase sampling frequency, polling around 65 Hz. The SD write was the primary data bottleneck, given that the sensors were capable of polling at much higher frequencies.

**GTPA013 GPS** This GPS sensor is accurate to 3 meters in three dimensions and was primarily intended to be used to recover the rocket post-flight. The GPS was meant to communicate with a ground station at the launch pad; however, due to an anomaly before launch, the GPS module was non-functional on this flight. Future LAD-4 flights will validate GPS communications between the GTPA013 and the ground station.

#### 3.4.2 Commercial Avionics

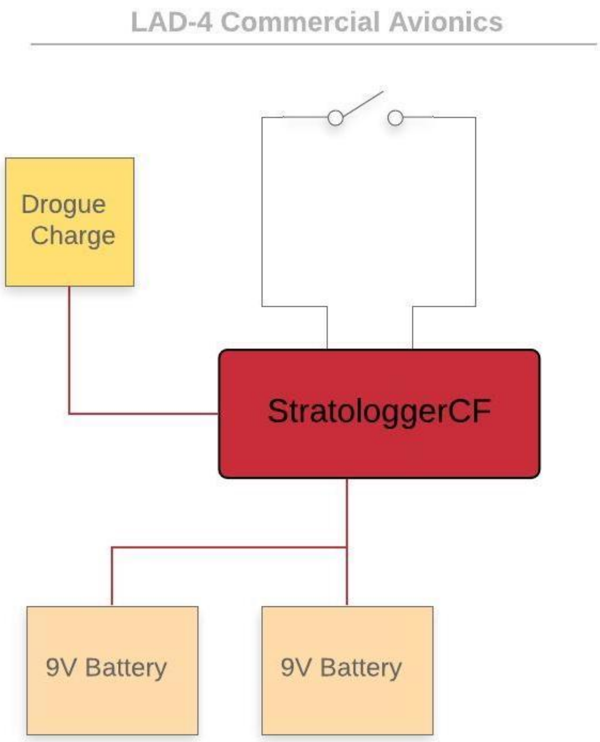



Figure 16: Diagram of the commercial avionics onboard LAD-4

**StratologgerCF** The StratologgerCF is a commercial system that contains an altimeter, accelerometer, and on board processor. It was used as the primary flight computer and altimeter aboard LAD-4. The Stratologger was chosen on the basis that it is able to perform dual deployment, based on an internal, independent apogee detection method and a pre-programmed main chute deployment altitude. The Stratologger was a product that could be trusted to successfully recover the rocket, as well as use as a reference for the custom acceleration- and altitude-data collection module.

**On-Board Cameras** There were two GoPro Hero Session 5 cameras on board, both with UHS Speed Class 3 (U3) SD cards, meaning they were capable of higher data write rates. This allowed footage from two different angles at 4K to be captured during flight. The cameras revealed valuable insight into the mechanics of the parachute deployment, especially during the drogue deployment event at apogee.

#### 3.4.3 Avionics Bay

The avionics bay was a four-level structure that encompassed the entire cross section of the rocket and housed all of the avionics for LAD-4. It was designed to be modular, so that levels could be easily added/removed to fit the changing electronics design. The different levels allowed a clear separation of components between our custom avionics, commercial avionics, and camera systems. The compact structure could be easily put in and taken out of the rocket, allowing for more rapid development.

The avionics bay was designed using Adobe Illustrator to fit the footprint of each component, locate attachment points, and facilitate laser cutting. Laser cut plywood was used for both the base of each level and the three vertical supports. Laser cutting the levels enabled the avionics bay to be constructed to maximize space while still fitting within the constrained space of the internal airframe. Using plywood allowed components to be firmly bolted to the plywood, and the levels themselves to be firmly bolted to the supports using bolts and L-brackets. On the top level, three additional L-brackets were fitted with rivnuts so that the entire avionics bay could be secured through the airframe, via holes that were pre-drilled through the composite material. Previous iterations of a 3D-printed avionics bay structure using PLA, a common thermoplastic used in FDM printing, proved to be too brittle and thus dangerous in the event of a hard landing.

**Avionics Bay Design** The four levels of the avionics bay each housed one component of the avionics system. Counting from the aft end forwards, towards the recovery bay, the levels were as follows:

- Level 1: **Power Level.** The lowest level of the avionics bay housed the power delivery system for the custom avionics system (i.e. the LiPo batteries and Power Boost power supply). Two Power Boosts were used in parallel, one going to the control boards on the underside of level 2, and the other going to the RF module.
  The RF module was attached to the bottom of Level 1. To maximize the cross-product of the two orientation vectors, the two antennae were pointed downwards with orientations maximally separated. The separation of the antennae was contstrained by the airframe diameter.
- Level 2: **Custom Avionics.** This level housed all of the microcontrollers and sensors that were a part of the commercial avionics system.
- Level 3: **Commercial Avionics.** This level housed the StratologgerCF flight computer and the batteries for both the Stratologger and the parachute charges.

- Level 4: **Access Level.** The highest level of the avionics bay housed the wiring for both the drogue and main deployment charges, which was threaded through the pressure bulkhead. The two switches to arm the Stratologger and to power on the custom avionics module were located on this level, easily accessible through the access panel made in the airframe (see the corresponding paragraph in section 3.1.2). Finally, the two GoPro cameras were mounted to this level, aligned with camera holes drilled through the airframe.

# 4 Mission Concept of Operations Overview

## 4.1 Flight Sequence

The flight of LAD-4 consists of four phases: *powered ascent* after liftoff using the solid motor, *unpowered ascent* from the end of motor burn to apogee, *descent under drogue chute* to an altitude of 700 feet above ground level, and *descent under main chute* until landing. Numbered sequentially, the four phases are shown below on a plot of the rocket's altitude above ground level as a function of time.

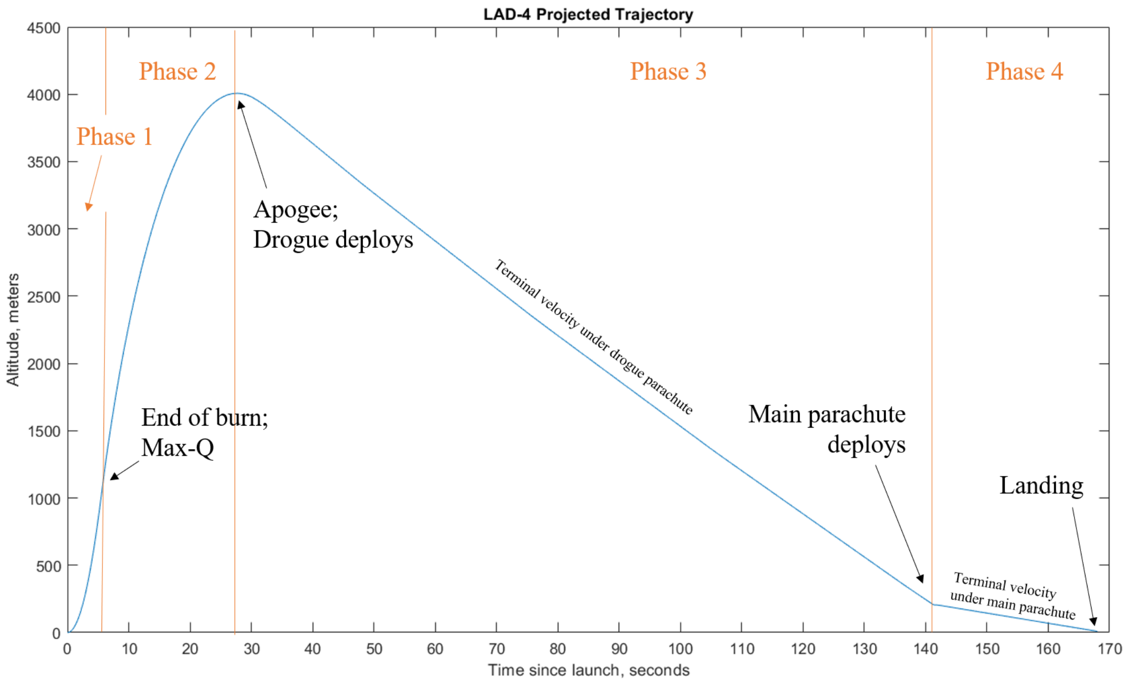



Figure 17: Flight sequence overview with the four phases of flight demarcated.

The graphs on the following page plot altitude above ground level, velocity, and acceleration as a function of flight time for a complete picture of the rocket's trajectory.

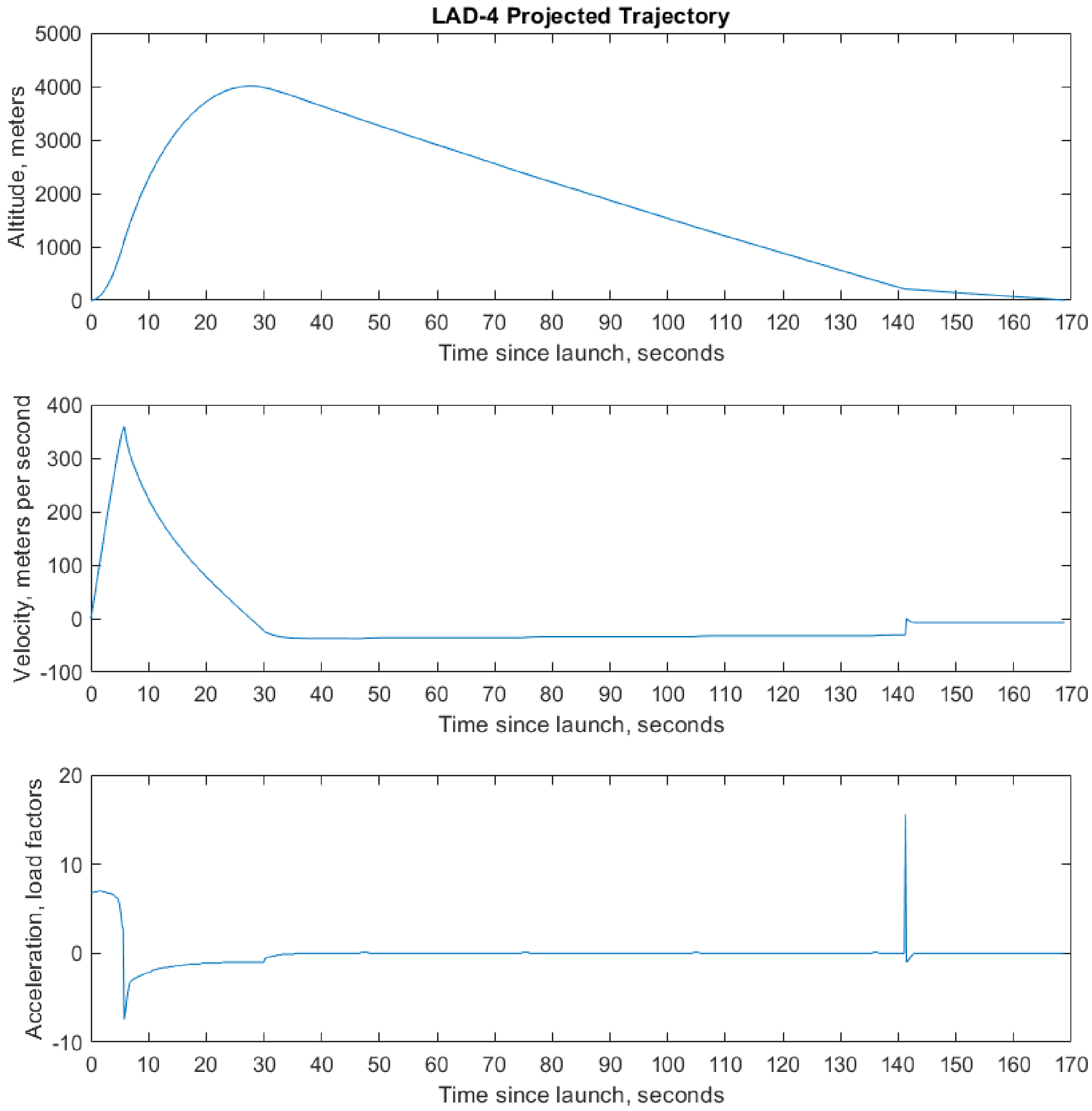



Figure 18: Graphs of predicted kinematic parameters as a function of time over the rocket's flight.

**Phase 1: Powered Ascent** When the igniter is powered, combustion of the solid propellant grain commences, and the flight sequence begins. Seconds later, the rocket exits the 20-foot (6.09 m) launch rail with a speed of 27.89 m/s (91.52 ft/s). The Aerotech M1340W-PS motor is designed to burn for 5.7 seconds, with a total impulse of 7673 newton-seconds applied over this time. At the end of the 5.7 second burn, the rocket's velocity is estimated to be 1161.18 ft/s (353.93 m/s), a maximum for the flight. At this point in the trajectory, the rocket is estimated to be at an altitude of 3491.24 feet above ground level. During this time, the onboard avionics system records altitude data using a barometric altimeter.

**Phase 2: Unpowered Ascent (Coasting)** After the motor finishes its burn, LAD-4 continues on a ballistic trajectory for another 22.1 seconds, which is 27.8 seconds after launch. No longer powered, it loses speed as it approaches its calculated apogee of 13141.66 feet (4005.58 meters) above ground level. Loads on the airframe decrease, and due to the ejection of the propellant mass, the rocket's center of mass shifts forward. The avionics system continues to record altitude data during this phase of the flight. Apogee is reached, and the unpowered ascent phase of flight ends a few seconds later, when the barometer records an increase in pressure associated with a decrease in altitude over time. At this point, the pyrotechnic charge deploys, shearing the nylon bolts of the forward coupler.

**Phase 3: Descent Under Drogue Chute** As a result of the charge deployment and the shearing of the nylon bolts, the nosecone, tethered to the rocket by a shock cord, separates. The drogue parachute deploys and inflates, allowing the rocket to remain in a stable, vertical attitude for the majority of its descent. The rocket reaches its terminal velocity under the small drogue chute about 5 seconds after drogue deployment. During this phase of flight, the avionics system continues to record altitude data.

**Phase 4: Descent Under Main Chute** At an altitude of 700 feet above ground level, the avionics system sends a signal to fire the charge in the Tender Descender system. The Tender Descender separates, releasing the main parachute, which inflates outside of the rocket, decreasing its speed for landing. The drogue parachute, attached to the nosecone and to the main airframe by shock cord, deflates. The entire rocket continues to lose altitude, slowing down and approaching the terminal velocity under the main parachute. This phase of flight continues until the rocket lands. The vehicle is then able to be located and recovered.

### 4.2 Launch Readiness Procedures

Prior to flight, a document outlining the launch day procedures was constructed. The launch day procedures involved final assembly of the rocket and verification of communication with and arming of the onboard electronic system, written as a checklist of steps necessary to prepare the rocket for launch. The checklist is performed as a call-and-response between at least two *independent* sets of people during launch preparation.

The checklist is included as an appendix to this document.

## 5 Maiden Flight Report

LAD-4 launched from the Friends of Amateur Rocketry (FAR) site in the Mojave Desert near Cantil, CA on the afternoon of Saturday, March 7, 2020. The prevailing weather condition at the time of launch was strong winds, with wind speeds at 20 miles per hour to the northeast and gusts of 24 miles per hour; the recorded temperature was 67.5 degrees Fahrenheit (19.7 degrees Celsius).

The command to launch was given around 13:40 local time under partly cloudy skies. Powered ascent was nominal; the motor burned for 5.40 seconds (cf. nominal burn of 5.70 seconds). As observed on the ground and from in-flight footage, the rocket began to undergo pitch-roll locking ("coning") shortly after launch at a frequency of approximately 0.25 Hz. It is hypothesized that this is a result of slight asymmetry in the nose cone and small misalignments of the fins. At the end of burn, the rocket attained a velocity of 411.6 m/s (Mach 1.2), and reached an altitude of 2997 feet (913 m) above ground level. Strong winds at the time of launch caused the rocket to pitch due North into the wind slightly during ascent, reducing the apogee by approximately 15% from the nominal altitude predicted by the trajectory simulation. The rocket's flight was otherwise nominal during the powered ascent phase. Following motor burnout, the rocket continued into unpowered ascent and, 26.00 seconds after launch, reached its apogee of 11 193 feet (3411.6 m) above ground level.

At 26.05 seconds after launch, a signal to detonate the 10g drogue parachute charge was given by the Stratologger module. The charge was set off and the nylon bolts sheared as designed. The drogue parachute was ejected from the airframe and inflated. During the ejection event, the main parachute was inadvertently partially ejected from the airframe along with the drogue. The main parachute thankfully did not inflate due to the Tender Descender carabiners constraining it. During this process, the main parachute also began to tangle with itself due to the violence of the ejection event and surrounding atmospheric conditions.

The rocket continued a nominal drogue descent for another 117.10 seconds until reaching an altitude of 700 feet above ground level (AGL). During this period, the partially ejected main parachute's cords tangled further due to spinning of the airframe during descent. At the time the rocket reached 700 feet AGL, recorded at 143.15 seconds after launch, the Stratologger module issued a signal to fire the smaller (3g) charge inside the Tender Descender. The charge fired, and the Tender Descender performed nominally, releasing the carabiners that retained the top and bottom attachment points of the main parachute. Due to the tangling of the parachute cords, however, the main parachute only partially inflated at this stage, and descent continued at an essentially unchanged rate under the drogue parachute, substantially exceeding design limitations on landing velocity. The rocket touched down in this condition in the desert brush some 8.10 seconds later at a point 1.09 miles (1746.2 meters) east of the launch site, with a total flight time of 151.25 seconds.

The rocket airframe was located and recovered approximately one hour after landing. The airframe was intact, the nosecone and parachute system having landed a few feet away attached via the length of paracord. Due to the impact with terrain at the higher-than-expected velocity under the drogue parachute, the aft-most section of the airframe (approximately 4-6 inches of the slotted section) sustained moderate damage (cracks in the composite matrix) caused by shearing against the aluminum fins. The rest of the rocket was recovered in a stable, flyable condition. The parameters of flight recorded by the onboard computers were later corroborated using footage from in-flight cameras aboard the rocket.

Figure 19: Liftoff of LAD-4

# Appendix A

## System Weights, Measures, and Performance Data

**Table A1: Predicted Flight Data and Analysis**

| | **Measurement** | **Comments** |
|---|---|---|
| Launch Rail Identity | Newman Button-Rail | Provided at launch site |
| Rail Length, feet | 20 | |
| Liftoff Thrust-to-Weight Ratio | 7.62 | |
| Launch Rail Departure Velocity (feet per second) | 91.52 | |
| Minimum Static Margin during Motor Burn | 1.58 | see Figure 5 |
| Maximum Acceleration (*G*) | 6.9 | see Figure 18 |
| Maximum Velocity (feet per second) | 1161.18 | see Figure 18 |
| Predicted Apogee (feet AGL) | 13141.66 | see Figure 18 |

**Table A2: Vehicle Parameters**

| | **Measurement** | **Comments** |
|---|---|---|
| Length (inches) | 90 | |
| Diameter (inches) | 6.5 | Nominal OD |
| Vehicle Dry Weight (pounds) | 28.66 | |
| Liftoff Weight (pounds) | 37.55 | |
| Number of Stages | 1 | |
| Propulsion Type | Solid | |
| Propulsion System Identity | Aerotech M1340W-PS (98mm DMS) | |
| Total Impulse (Newton-seconds) | 7673 | |

**Table A3: Recovery Information**

***First Stage Recovery***

| | **Identity** | **Comments** |
|---|---|---|
| Type | Parachute | Dual-deployment, see section 3.3 ff. |
| Primary Initiation Sensor | Barometer | COTS |
| Secondary Initiation Sensor | Barometer | SRAD |
| Deployment Energy Source | Crimson Powder Charge | |

## Appendix B

## **Pre-Launch Checklist** (excerpted from Launch Procedures document)

**The launch checklist should be performed as a challenge-and-response. Make a check or other mark in the final column after verifying a flight-ready configuration. Do not mark if ALL configuration elements are not met.**

| **Component** | **Flight-ready configuration** | **Y/N 1** | **Y/N 2** |
|---|---|---|---|
| | ***Aft Body Tube*** | | |
| Fins (4) | Secured fully in airframe slots<br>Bolts tightened | | |
| Motor | Installed into airframe<br>Retention cord tied to rod on thrust plate | | |
| Thrust plate bolts | Tightened fully into rivet nuts | | |
| Aft coupler-aft body tube bolts | Tightened fully (from inside) | | |
| Launch rail buttons (2) | Installed and tightened | | |
| | ***Avionics Bay - Flight Computer*** | | |
| Custom avionics system | Radio transmission confirmed | | |
| Stratologger | Connected to 9V battery terminals | | |
| Custom avionics power line | Connected to power switch (outside rocket) | | |
| Go-Pros | Turned on | | |
| Go-Pro SD card | Mounted in Go-Pro | | |
| Custom avionics SD card | Mounted in custom avionics system | | |
| Entire avionics bay | Inserted into rocket airframe<br>Wires threaded through pressure bulkhead<br>Bolts tightened fully into rivet nuts | | |

| *Forward Body Tube - Nosecone* | | | |
|---|---|---|---|
| Pressure vent holes (4) | Present and clear of blockage | | |
| Main parachute, Main parachute Nomex blanket | Folded and wrapped according to defined procedure<br>Bottom line attached to Main Body Tube carabiner<br>Top line attached to Tender Descender end B | | |
| Tender Descender (arbitrary carabiners A and B) | Crimson powder inserted<br>E-match inserted<br>Snapped together with two carabiners in place<br>Internal cord present<br>Connected to U-bolt via string<br>End A connected to main body tube carabiner<br>End B connected to Main parachute *top* and Kevlar line to nosecone<br>Wires passed through to avionics bay | | |
| Drogue Deployment Charge | Crimson powder (10g) inserted into metal tube<br>E-match (with connector) inserted<br>Duct tape applied, seven layers on top and one layer around<br>Wires passed through to avionics bay<br>Charge centering ring affixed to rocket airframe | | |
| U-bolt | Securely affixed to pressure bulkhead<br>Connected to Tender Descender end A<br>Connected to Forward Body Tube Carabiner | | |
| Forward Body Tube Carabiner | Connected to U-bolt<br>Connected to bottom end of Main parachute<br>Connected to Tender Descender end A | | |
| Long Kevlar cord | Shock taping applied<br>Connected to drogue parachute<br>Connected to Tender Descender end B | | |
| Drogue parachute, Drogue parachute Nomex blanket | Folded and wrapped according to defined procedure<br>Connected to the long Kevlar cord towards the forward body tube<br>Connected to the Kevlar cord towards the eye-bolt | | |
| Nosecone | Contains eye-bolt, securely fixed into place<br>Kevlar cord on eye-bolt securely connected to Drogue parachute | | |

| | | | |
|---|---|---|---|
| Shear pins (3) | Installed<br>Hot glue applied | | |
| ***Final Visual Inspection*** | | | |
| Nosecone | Present, airworthy | | |
| Nosecone-forward body tube coupling: shear pins | Installed | | |
| Forward body tube | Present, airworthy, all bolts tightened<br>Pressure and camera holes present | | |
| Aft coupling: bolts | All bolts tightened | | |
| Aft body tube | Present, airworthy, all bolts tightened | | |
| Fins | Present, airworthy, firmly secured | | |
| Motor | Present, centered, and secured to rocket | | |
| ***Launchpad*** | | | |
| Custom avionics system | Powered ON | | |
| Radio connection | Established and confirmed | | |
| Flight cameras | Powered ON<br>Centered in holes | | |
| Stratologger | Switch fixed ON (*audible signal*) | | |
| Access panel | Secured<br>All bolts tightened | | |
| Igniter | Installed | | |
| **GO for launch** | | | |

# Appendix C
# Worked Calculations

## Airframe Thickness

### Assumptions

- The airframe will fail by buckling, due to the compressive forces of drag and thrust from the engine, which are assumed to act axially.
- The airframe is a homogeneous structure having an annular cross-section, with constant inner and outer radii. The outer diameter is, by design, equal to 6.5 inches.
- The airframe will buckle when the total compressive axial loading $P$ exceeds the Euler critical load,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

#### Variable Definitions

Here, $P$ represents the total axial loading acting in compression on the airframe, assumed to consist of the drag force and the thrust force. The magnitude of the force due to aerodynamic drag is given by $P_{drag} = \frac{1}{2} C_D A \rho v^2$, where $C_D$ is the coefficient of drag, assumed to be near 1; $A$ is the cross-sectional area of the airframe; $\rho$ is the density of air; and $v$ is velocity, taken to be the speed at the end of motor burn, which is a maximum. The maximum of the magnitude of the thrust force is read off from the manufacturer's thrust curve.

$E$ is the elastic modulus of the airframe material. It represents the "stiffness", or resistance to elastic deformation, of the fiber-epoxy composite. Because the loading is assumed to be uniaxial, and the fibers are oriented at 0°and 90°, the *rule of mixtures* can be used to calculate the elastic modulus of the material in the longitudinal (loading) direction. The rule of mixtures says that the elastic modulus of the composite material is a *volume-fraction weighted average* of the elastic moduli of the constituent materials.

$I$ is the second moment of area (or *moment of inertia*) for the airframe's cross-sectional area, which is a geometric property independent of the material. For an annulus, the moment of inertia is computed by $I = \frac{\pi}{4}\left(r_2^4 - r_1^4\right)$, where $r_2$ is the outer radius and $r_1$ is the inner radius.

$K$ is the *effective length coefficient* for the "end configuration", which changes with the degrees of freedom that each end has. In this situation, we assume $K = 1$. $L$ is the length of the column, which is the length of the airframe section in this case.

## Calculations

#### Total Axial Loading

The total axial loading consists of a force from drag and a force from the motor thrust. The magnitude of the drag force is $P_{drag} = \frac{1}{2} C_D A \rho v^2$. Here,

the cross-sectional area $A = \frac{\pi d_2^2}{4}$, using the outer diameter $d_2$;
the density of air $\rho = 1.2\mathrm{kg/m^3}$; and
the velocity at max-$q$ is $v = 171.5$ m/s.

Hence the total drag force is $P_{drag} = 337.130$ N.
The peak thrust according to the motor manufacturer is $P_{thrust} = 1634$ N, giving a total axial load of $\boxed{P = 1971.130 \text{ N}}$.

### Airframe Properties

The volume fraction of fiber is assumed to be $V_f = 60\%$, which is close to the ideal value for composite fabrics laid up by hand. From the manufacturer, the fiber elastic modulus is $E_f = 5.52$ GPa, and the matrix elastic modulus is 30 KPa. Within the composite, the fabric is assumed to bear almost all of the applied load, and correspondingly the longitudinal elastic modulus is a *fiber-controlled* property. Using the rule of mixtures to compute the composite's longitudinal elastic modulus $E_1$,

$$E_1 = \sum (E_i V_i) = E_f V_f + E_m(1 - V_f) = \boxed{3.31 \text{ GPa}}$$

The composite layer thickness is taken to be 0.01 inches per layer. The column length is taken to be the length of the entire airframe (80 inches), as the aft coupler is sufficiently long (greater than one caliber) and strongly bonded to both airframe sections. Finally, the effective length factor is taken to be unity.

### Required Moment of Inertia

A factor of safety $X_b$ against buckling equal to 4.0 is used in the calculation. Since every variable in the critical load equation is known except the moment of inertia $I$, it is possible to solve for $I$:

$$X_b P = \frac{\pi^2 EI}{(KL)^2} \implies I = \frac{X_b P (KL)^2}{\pi^2 E}$$

Substituting numerical values, $\boxed{I = 1.42537 \times 10^{-6} \text{ m}^4}$.

### Required Thickness

Finally, with a known outer radius, the inner radius (and subsequently, the thickness) can be calculated. Using the annular cross-sectional geometry,

$$\begin{aligned}
I &= \frac{\pi}{4}\left(\left(\frac{d_2}{2}\right)^4 - \left(\frac{d_1}{2}\right)^4\right) \\
1.42537 \times 10^{-6} \text{ m}^4 &= \frac{\pi}{4}\left(\left(\frac{d_2}{2}\right)^4 - \left(\frac{d_1}{2}\right)^4\right) \\
1.42537 \times 10^{-6} \text{ m}^4 &= \frac{\pi}{4}\left((0.078 \text{ m})^4 - (r_1)^4\right) \\
r_1 &= 0.07705 \text{ m}
\end{aligned}$$

The thickness is then $t = r_2 - r_1 = \boxed{0.974178 \text{ mm}}$.
Dividing by the layer thickness of 0.254 mm per layer, it is seen that a minimum of $\boxed{4 \text{ layers}}$ of composite are needed to prevent buckling.

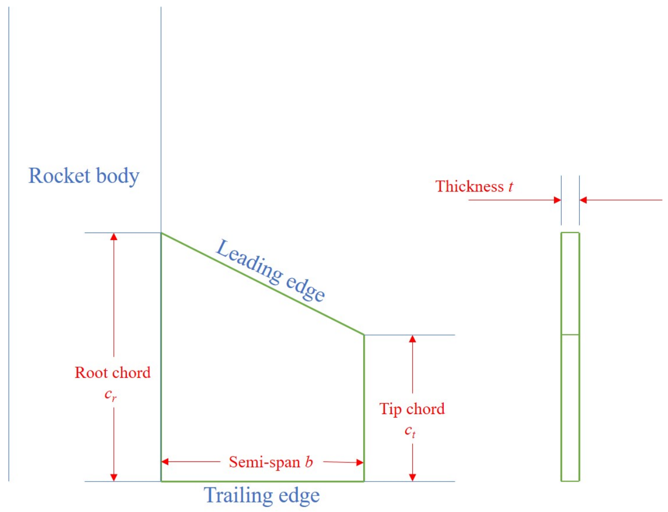
# Fin Flutter Speed

## Assumptions

- The fin has the geometry shown in the figure below with a uniform thickness of 0.125 inches.
- The air temperature (in degrees Fahrenheit) decreases linearly with altitude, and the local speed of sound is solely a function of air temperature.
- Flutter is possible when the rocket body velocity reaches or exceeds the *flutter boundary speed*, given by

$$v_f = a\sqrt{\frac{2G(AR+2)\left(\frac{t}{c_r}\right)^3}{1.337(AR)^3P(\lambda-1)}}$$

This equation is adapted from the article "How to Calculate Fin Flutter Speed" by Zachary Howard, published on page 3 of Issue 291 (July 19, 2011) of the *Peak of Flight* magazine. The constants in this equation assume the use of imperial units: feet, feet per second, degrees Fahrenheit, and pounds per square inch.




### Variable Definitions

The geometry of the fin is defined using the parameters shown in the above figure. For the fins on LAD-4, the root chord length $c_r$ is 6 inches, the semi-span $b$ is 6 inches, and the tip chord length $c_t$ is 4 inches. Thus, the fin has surface area $A = \frac{1}{2}(c_r + c_t)(b) = 30$ in$^2$, aspect ratio $AR = \frac{b^2}{A} = 1.2$, and taper ratio $\lambda = \frac{c_t}{c_r} = 0.667$. The thickness is uniform throughout the fin.

The shear modulus $G$ of the fin material, 6061-T6 Aluminum, is 3 770 000 psi.

Using the trajectory planner (section 3.2.2), the altitude at maximum speed was numerically calculated to be 3140.124 feet. The air pressure, which decreases with height, is calculated to be 13.12 psi at this altitude. The temperature, assumed to be 59°on the ground, is approximately 47.8°at this altitude. Finally, the local speed of sound at the given conditions is a function of absolute temperature alone and is given by $a(T) = \sqrt{1.4 \times 1716.59(T + 460^\circ)}$, with $T$ in degrees Fahrenheit. Using the calculated temperature, $a = 1104.72$ feet per second in this case.

## Calculation

With all parameters on the right-hand side of the flutter boundary speed equation now known, the computation is straightforward; substitution yields $\boxed{v_f = 2295.9 \text{ feet per second}}$, or Mach 2.041. This speed is well over the calculated maximum speed of 1161.18 feet per second (section 4.1), giving a safety factor of 1.97 against flutter.