Advent of Code 2022 - Day 11

were going to have some very large numbers and need to track their divisibility by some set of numbers. Luckily, those numbers are prime.

$$S = \{2, 7, 17, 13, 3, 5, 19, 11\}$$

There are three operations, $n * a$, n^2 and $n + a$

The First two are easy for divisibility

let $n = p_1 p_2 p_3 \ldots$, where p_n is a prime factor (aka Fundamental theorem of arithmetic)

<u>$n * a$</u> : will just add a new set of prime factors to the value.

$n * a = (p_1 p_2 p_3 \ldots)_n (p_1 p_2 p_3 \ldots)_a$ and so divisibility will gain those factors. of a.

As the number n gets very large, we can just discard factors not $\in S$ or repeated factors.

<u>n^2</u> : Simply repeats all current factors, and will not change divisibility by any prime.

$$n^2 = p_1 p_1 p_2 p_2 p_3 p_3 \ldots$$

<u>$n + a$</u> : Heres the tough one... addition very much can change the entire set of factors

ex) $28 = 7 \cdot 7 \cdot 2$, $28 + 1 = 29 \rightarrow$ prime

however there may be a maximum value of n, above which addition acts the same as in lower registers

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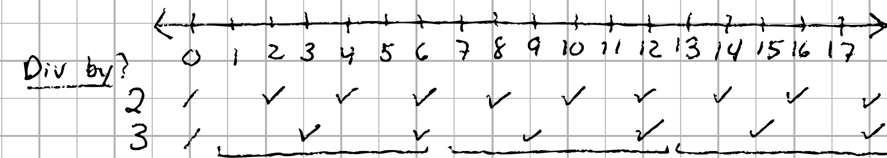
- lets just look at 2 & 3 for now $S' = \{2, 3\}$



the divisibility repeats every $\prod_i S_i$ values
let $\prod_i S_i = d$ (product of members of set S)

and so under addition, divisibility repeats such that

$$\text{Factors}_{\in S}(n+a) = \text{Factors}_{\in S}((n+a) \% d)$$

For example, $n = 10$, $a = 5$

$n = 10 = (2)(5)$ $\quad$ $n+a = 15 = (3)(5)$

$15 \% 6 = (3)$

both show divisibility by the same value of S'

- okay but now we need to see how the operations chain together, because addition needs at least one "cycle" of factors to add into the next one.

- Can we just modulo by d after every operation?
- I'm hoping we can ignore the squaring altogether since that makes huge numbers

If the multiplication is small enough to stay within a window / cycle, then it will affect addition.

ex) $S = \{2, 3, 5\}$ $\quad d = 30$

$12 = (2)(2)(3) * (2) = 24$ $\quad$ $24 \% 30 = 24$
$12 = (2)(2)(3) * (3) = 36$ $\quad$ $36 \% 30 = 6$

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Now for powers of two, do they affect the next addition?

$S = \{2, 3\}$ $d = 6$

$n = 2 = (2)$ $n^2 = (2)(2) = 4$ yea... $4 + 3 \% d \neq 2 + 3 \% d$

$n = 4 = (2)(2)$ $n^2 = (2)(2)(2)(2) = 16$ $16 \% 6 = 4$

$(4 + n) \% d = (16 + n) \% d$

$4 \% d + \cancel{n \% d} = 16 \% d + \cancel{n \% d}$

only if the squaring takes it to a new window

when is $n^2 > d$? $\rightarrow$ $n > \sqrt{d}$

so for $d = 6$, $n > 2.446$

$n = 3$ $n^2 = 9 = (3)(3)$

$3 \% d + \cancel{n \% d} = 9 \% d + \cancel{n \% d}$

try $S = \{2, 3, 5, 7\}$ $d = 210$, $n > 14.49$

$n = 14$, $n^2 = 196$ X

$n = 15$, $n^2 = 225$

$15 \% d = 225 \% d$ ✓

So if $n > \sqrt{d}$, don't bother squaring! Yay.

So Final algorithm to preserve divisibility by S.

$n + a$:

do $(n + a) \% d$

$n * a$: $(n * a) \% d$

n^2:

do n^2 if $n < \sqrt{d}$, otherwise skip

*okay theres still some problems...

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- lets check the three-case for addition

2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
3		3	6	9		12	15		18	21		24		27	30
5			5		10		15			20		25			30

which does yes, repeat every $2 \cdot 3 \cdot 5 = 30$

- addition is the only operation that "destroys" factors of n.

if $n \% S_1 = 0$, then n is divisible by S_1

$$(n + \alpha S_1) \% S_1 = n \% S_1 + \cancel{\alpha S_1 \% S_1}^{0}$$

likewise for any $S_n \in \{S_1, S_2 \dots S_n\}$, with $d = \prod S_n$
since $S_n \% d = 0$ by definition.

* this is more formal than my earlier conclusion so its probably not this causing the problem.

- on a whim I removed my power rule and it fixed everything. why?

- to preserve addition, $n^1 \% d = n^2 \% d$

my earlier conclusion was wrong: it actually only works if n's factors all $\in S_n$ <u>and</u> $n > \sqrt{d}$

not a condition worth checking

ex) works: $d = 30, \sqrt{6} = 5.4$, $n = 6, 10, 15$
doesnt: $n = 14, 18, 16 \dots$