Assigment I - Analytical Solution

Artemii Bykov

$$\begin{cases} y' = -x + \frac{y(2x+1)}{x} \\ y(1) = 3 \\ x \in [1; 18.2] \end{cases}$$

Rewrite equation to standart form Linear Nonhomogenous O.D.E of 1st order:

$$y' - y(2 + \frac{1}{x}) = -x$$

Solution of equation:

$$y = uy_1$$

Write and solve **complementory equation**:

$$(y_1)' - y_1(2 + \frac{1}{x}) = 0$$

$$\frac{d(y_1)}{y_1} = (2 + \frac{1}{x})dx$$

$$\ln(|y_1|) = 2x + \ln(|x|)$$

$$y_1 = xe^{2x}$$

Let's find u:

$$u' = \frac{-x}{xe^{2x}} = -e^{-2x}$$

Integrate both parts:

$$\int u' = \int -e^{-2x}dx$$

$$u = \frac{e^{-2x}}{2} + c_1$$

$$y = \frac{x}{2} + c_1xe^{2x}$$

Now let's solve IVP: $y(1) = 3$

$$3 = \frac{1}{2} + c_1e^2$$

$$c_1 = \frac{8}{3}e^2$$

Answer: $y = \frac{x}{2} + \frac{8}{3}xe^{2x-2}$