Longitudinal Multidimensional Item Response Modelling in Preschool Children's Mental State Understanding

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Theory of Mind

Definition

Ability to perceive our own mental states as well as from others, such as beliefs, desires and intentions and know that they differ from one person to another.

Main Features

- Developed in the first years of life (4 years old).
- Understand social environment and how to interact in it.
- Different mental state tasks to identify the acquisition of this ability in children.

Let's take a look:

Aims

1. Use Item Response Model in the context of Theory of Mind.
2. Analyse the response patterns of mental state tasks delivered to children through item response models.
3. Identify latent sub-dimensions in the response patterns by applying multidimensional item response theory.
4. Analyse each latent factor under the Bayesian and Longitudinal approach of item analysis.

Data Description

Participants

86 British children (Female = 41, Male = 45) from different preschools and day nurseries located in Northern Lancashire. Age: Between 30 and 33 months when recruited.

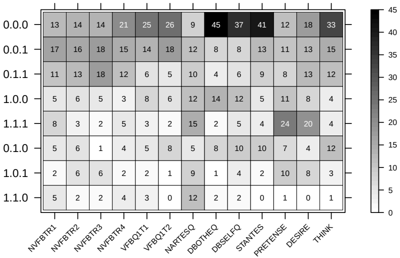
Measures

8 mental state tasks (13 questions three times in intervals of 4 months). A correct response scored ‘1’ and an incorrect response scored ‘0’.

- Standard Location Change
- Deceptive Box
- Pretence, Desire and Think
- Narrative
- Verbal and Non-Verbal (2 and 4 trials repectively)

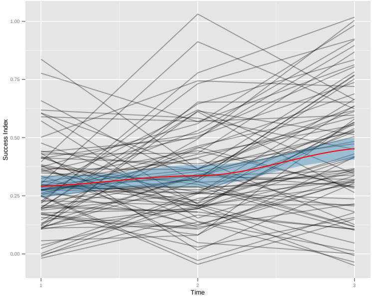
Exploratory Analysis

Response Patterns



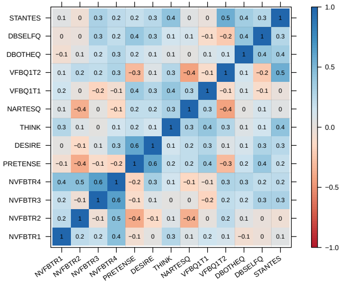
- Only complete observations taken into account.
- Most difficult tasks: Standard Location Change and Deceptive Box.

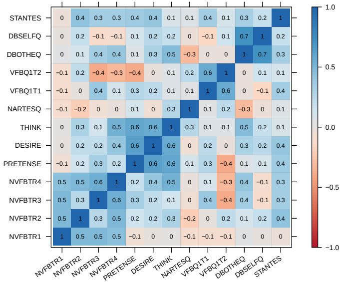
Total Performance Trend

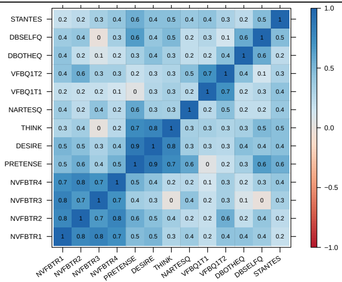


General trend is increasing

Correlation Analysis







- Time 1 and 2 (Above)
  Time 3 (Below)
- Correlation across time.
- Correlation between possible latent factors.

Item Response Theory (IRT)

- Used in assessment and evaluation research.
- A set of latent variable techniques.
- Model the interaction between a subject's ability (i.e., latent trait) and item level stimuli (difficulty, guessing, etc.).
- Main focus is on the pattern responses.
- Highlights how responses can be thought in probabilistic terms.
- Most of the IRT models specify a single latent trait (i.e., unidimensional); however, many psychological constructs are multidimensional in nature.

Two-Parameter Logistic Model (2PL)

The probability of an individual i to answer correctly the item j is:

$$P(x_{ij} = 1 \mid \theta_i, \alpha_j, d_j) = \frac{1}{1 + exp\{-D\alpha_j(\theta_i - d_j)\}}$$

Where, $i = 1, ..., N$ participants, $j = 1, ..., n$ test items, θ_i is the latent ability of subject i, α_j and d_j are the discrimination and difficulty parameters, and D is a scaling constant with a common value of 1.7.

- The higher (lower) the discrimination parameter, the (less) better the item is able to distinguish between low and high ability levels.

Multidimensional Item Response Modeling

The probability of answering a dichotomous item correctly is:

$$\Phi(x_{ij} = 1|\theta_i, \alpha_j, d_j) = \frac{1}{1 + exp[-D(\alpha_j^T \theta_i + d_j)]}$$

Where, $i = 1, ..., N$ participants, $j = 1, ..., n$ test items, m latent factors $\theta_i = (\theta_{i1}, ..., \theta_{im})$ with associated item slopes $\alpha_j = (\alpha_1, ..., \alpha_m)$, d_j is the item intercept and D is a scaling adjustment (usually 1.702).

- The slopes are the multidimensional discrimination parameters (one for each latent factor).
- The intercept is proportional to the item difficulty.

Exploratory and Confirmatory

Exploratory Item Analysis

- The number of dimensions are not assumed known before.
- Estimated by comparing nested models or ...
- By rotating the factor loadings matrix to make a more clear structure (e.g. *'varimax'* to make the factors orthogonal, *'oblimin'*, *'oblimax'*, etc.).

Confirmatory Item Analysis

- More than one factor is present in the data.

Bifactor Method

A single factor is present in all the items, but with additional clusters of local dependencies formed by other independent specific factors (Gibbons and Hedeker, 1992).

The models are based on **Full Information Factor Analysis**

Exploratory Factor Analysis

Factor Loading Matrix after Varimax Rotation

	F1	F2	F3	F4
NVFBTR1	**0.72**	-0.10	-0.12	0.04
NVFBTR2	**0.67**	-0.08	-0.28	-0.23
NVFBTR3	**0.75**	-0.19	-0.07	0.20
NVFBTR4	**0.85**	-0.18	-0.09	-0.12
PRETENSE	0.13	**-0.94**	0.09	-0.05
DESIRE	0.27	**-0.73**	-0.15	-0.14
THINK	0.18	**-0.58**	-0.22	-0.19
NARTESQ	0.02	-0.32	-0.14	0.13
VFBQ1T1	0.05	-0.33	**-0.56**	0.27
VFBQ1T2	0.15	0.05	**-0.97**	-0.07
DBOTHEQ	0.13	-0.31	-0.07	**-0.49**
DBSELFQ	0.00	-0.39	-0.10	**-0.57**
STANTES	0.36	-0.28	-0.24	-0.23

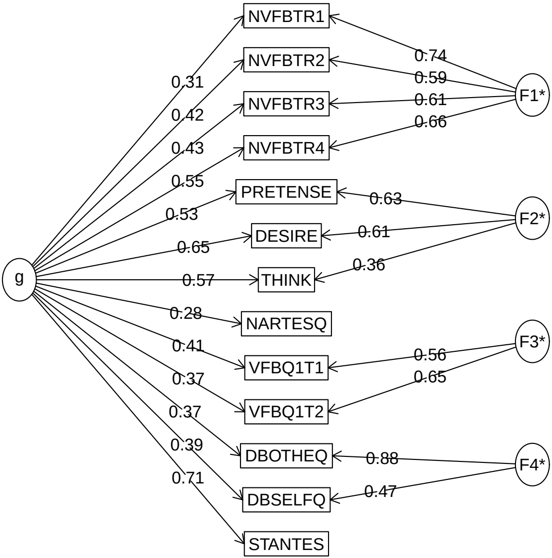
Model	AIC	AICc	SABIC	BIC	logLik	X^2	df	p
2 Factors	3667.51	3681.05	3682.05	3802.53	-1795.76			
3 Factors	3656.66	3680.21	3675.41	3830.75	-1779.33	32.86	11	0.00
4 Factors	3655.07	3690.83	3677.65	3864.70	-1768.54	21.58	10	0.02

Confirmatory Factor Analysis

Factor Loading Matrix - Bifactor Method

	G	F1	F2	F3	F4
NVFBTR1	0.31	0.74			
NVFBTR2	0.42	0.59			
NVFBTR3	0.43	0.61			
NVFBTR4	0.55	0.66			
PRETENSE	0.53		0.63		
DESIRE	0.65		0.61		
THINK	0.57		0.36		
NARTESQ	0.28				
VFBQ1T1	0.41			0.56	
VFBQ1T2	0.37			0.65	
DBOTHEQ	0.37				0.88
DBSELFQ	0.39				0.47
STANTES	0.71				

Bifactor Model Path



Causality Analysis
First Stage: Bayesian Longitudinal Analysis

Data Structure

The responses for subject i to the first dimension Non Verbal False Belief have the form:

$$\boldsymbol{X}_{if_1} = (\boldsymbol{X}_{if_1 1} \quad \boldsymbol{X}_{if_1 2} \quad \boldsymbol{X}_{if_1 3}) = \begin{pmatrix} x_{i11} & x_{i12} & x_{i13} \\ x_{i21} & x_{i22} & x_{i23} \\ x_{i31} & x_{i32} & x_{i33} \\ x_{i41} & x_{i42} & x_{i43} \end{pmatrix}$$

where the number of rows represents the number of items in the first dimension ($n_{f_1} = 4$) and the columns stand for each time point $t = 1, 2, 3$.

Remaining factors: $n_{f_2} = 3, n_{f_3} = 2, n_{f_4} = 2, n_{f_5} = 1, n_{f_6} = 1.$

Let the ability vector in the factor f of T times be:

$$\boldsymbol{\theta}_{i,f,1:T} = (\theta_{if1}, \theta_{if2}, \ldots, \theta_{ifT})'$$

where θ_{ift} represents the ability of the i-th subject in the latent dimension f at time t.

For our data:

$$\boldsymbol{\theta}_{i,f,1:3} = (\theta_{if1}, \theta_{if2}, \theta_{if3})'$$

Take into account for correlation structure

Latent abilities on the same subject will be more correlated than latent abilities among different subjects.

Assumption

Subjects' abilities for each factor f come from a multivariate normal distribution: $\boldsymbol{\theta}_{i,f,1:T} \sim N(\boldsymbol{\mu}_\theta, \boldsymbol{\Sigma}_\theta)$

Autoregressive AR(1)

- Constant variance across time.
- Exponential correlation decrease as the lag between times increases.

$$\boldsymbol{\Sigma}_\theta = \sigma^2 \begin{pmatrix} 1 & \rho & \rho^2 \\ \rho & 1 & \rho \\ \rho^2 & \rho & 1 \end{pmatrix}$$

Unstructured Covariance

- Not specific pattern in the covariance structure $\boldsymbol{\Sigma}_\theta$.
- Cholesky decomposition to get identifiability

$$\boldsymbol{\Sigma}_\theta = \boldsymbol{L}_\theta \boldsymbol{L}'_\theta$$

Random Effects

Model the ability of each individual i in each latent dimension f as a linear combination of random coefficients and time.

$$\theta_{ift} = \gamma_{if}^{(0)} + \gamma_{if}^{(1)} t$$

Prior Distributions

Choice of prior distributions for each *f* latent dimension

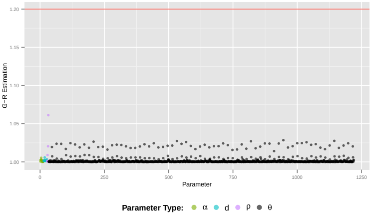
Parameters			AR(1)	Unstructured	Random Effects
Discrimination		α_j	$N(1,1)\ I[\alpha_j > 0]$	$N(1,1)\ I[\alpha_j > 0]$	$N(1,1)\ I[\alpha_j > 0]$
Difficulty		d_j	$N(0,1)$	$N(0,1)$	$N(0,1)$
Latent Ability (θ_i)	$\boldsymbol{\mu}_\theta$	$\mu_{\theta_{i1}}$	0	0	-
		$\mu_{\theta_{i2}}$	$N(0,1)$	$N(0,1)$	-
		$\mu_{\theta_{i3}}$	$N(0,1)$	$N(0,1)$	-
	$\boldsymbol{\Sigma}_\theta$	σ	1	-	-
		ρ	$U(-1,1)$	-	-
		L_{ii}	-	$\text{Gamma}(1,1)$	-
		$L_{ij}\ [i>j]$	-	$N(0,1)$	-
		$\gamma_i^{(0)}$	-	-	$N(0,1)$
	$\gamma_i^{(1)}$	$\mu_{\gamma_i^{(1)}}$	-	-	$N(0,1)$
		$\tau_{\gamma_i^{(1)}}$	-	-	$\text{Gamma}(1,1)$

Features of Modelling

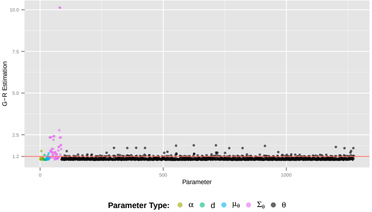
- 3 chains of 10000 iterations with a burn-in phase of 5000 and final results pooled in a single chain.
- Employment of a **BUGS** (**B**ayesian inference **U**sing **G**ibbs **S**ampling) code called from the free software R.

Convergence Diagnostics

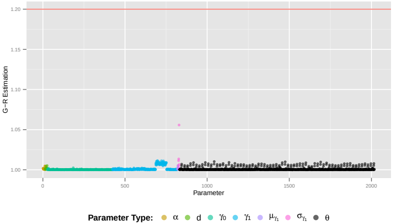
Autoregressive AR(1)



Unstructured Covariance



Random Effects

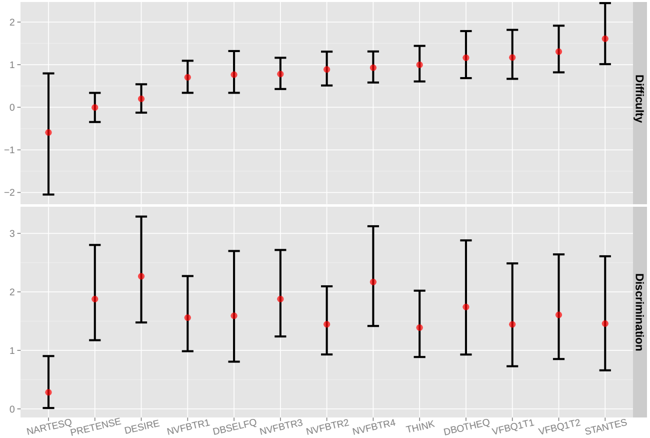


Gelman Rubin Statistic

The decision rule is that if $\hat{R} < 1.2$ it can be said that convergence is attained.

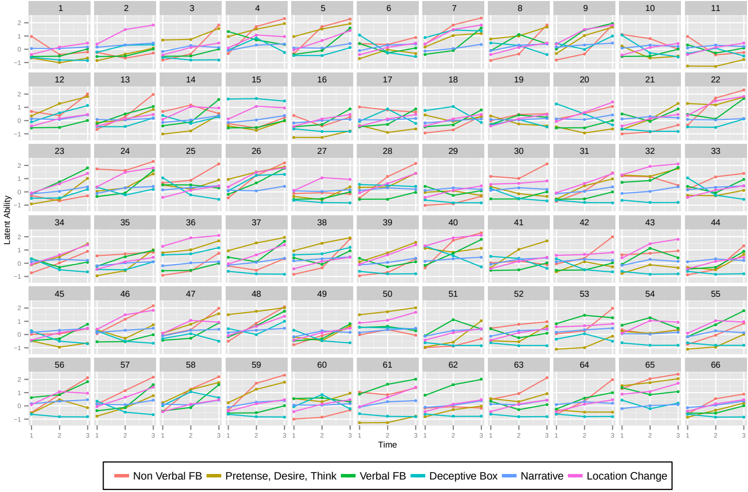
- Not significant for Unstructured Covariance

Estimation Results - AR(1)



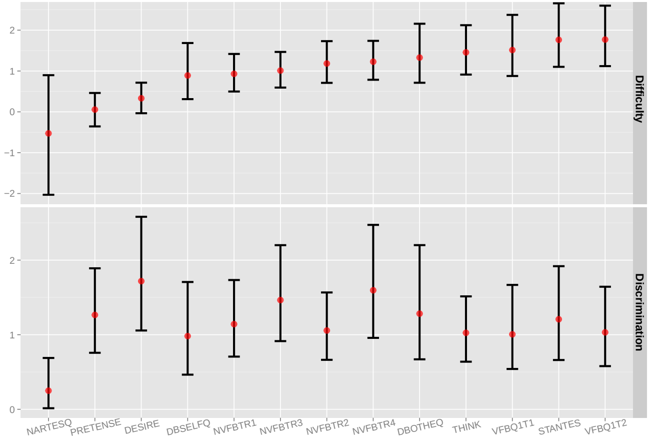
Credibility Interval of Item parameters

Estimation Results - AR(1)



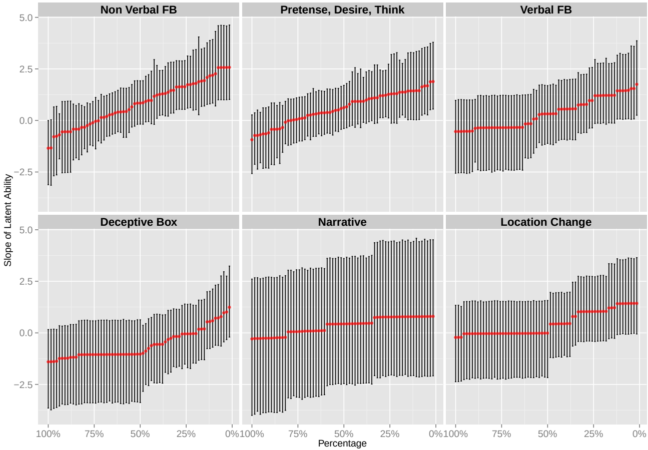
Estimated Latent Ability by subject

Estimation Results - Random Effects



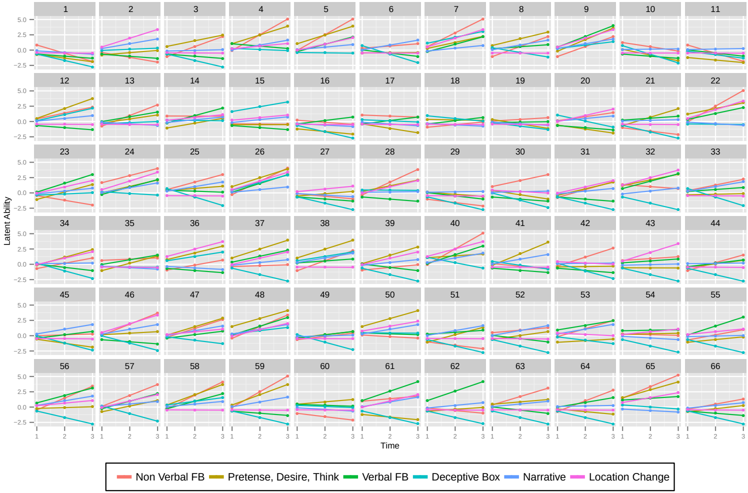
Credibility Interval of Item parameters

Estimation Results - Random Effects



Credibility Intervals of the 6 latent abilities slopes

Estimation Results - Random Effects



Estimated Latent Ability by subject

Model Selection

Deviance Information Criterion (DIC)

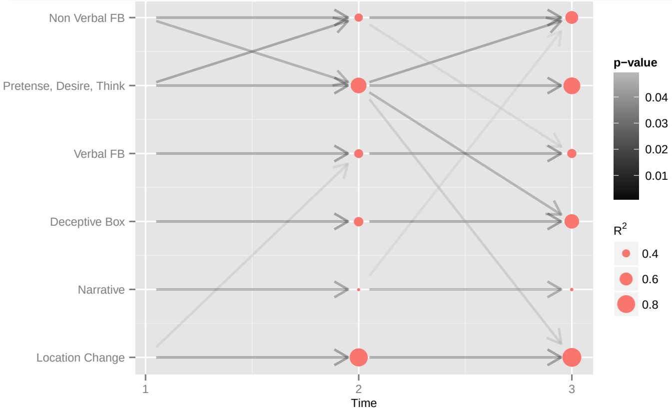
- Generalization of the AIC criterion for the bayesian framework.
- $DIC = \bar{D} + p_D$, where $\bar{D}$ describes the model fitting measured by the posterior expectation of the deviance $\bar{D} = E_{\theta|y}(D(\theta))$, and p_D stands for the complexity of the model measured by the effective number of parameters $p_D = \bar{D} - D(E_{\theta|y}(\theta))$
- Smaller values of DIC suggests a better model.

Summary of DIC criterion

Model	DIC	$Q_{0.025}$	$Q_{0.975}$
AR(1) Covariance Structure	2312.46	2205.88	2418.96
Unstructured Covariance	2242.62	2124.69	2359.80
Random Effects	2337.56	2258.15	2415.93

Second Stage: Ability Regression

Regression of the latent ability factors of $t = 2, 3$ against the latent ability of the previous instant of times.



Path Diagram of Causality - Model AR(1).
The p-values have not been adjusted for multiple comparison.

Conclusions

1. Children before 4 years old successfully passed some mental state tasks, mainly Pretense, Desire and Non Verbal False Belief.
2. Theory of Mind could be reduced to 6 latent abilities through the Bifactor Model.
3. The easy items were Pretense and Desire and the most difficult item was Standard Location Change.
4. Non Verbal False Belief was the ability with more significant improvement across time, but this did not happen for the abilities of Deceptive Box and Standard Location Change.
5. The Narrative task seemed to have a random pattern with children responding correctly or wrong at non specific time.
6. Regarding the causal analysis, Pretense, Desire and Think affects the development of most of the others abilities.

Further Work

- Consider the correlation between latent abilities in the model.
- A guessing parameter for each item should be considered in the model because a lot of guessing could be seen in the exploratory analysis.
- Covariates could be included in the modelling since there is information available of age, sex and institution.
- Multilevel Modelling or Dynamic Latent Trait Models could be used in the future.