ДЗ 4

Осипенко 595

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101

$$\iiint_T (x^2+y^2+z^2)dxdydz \quad 0<x<a, 0<y<b, 0<z<c$$

$$\int_0^a (x^2+y^2+z^2)dx = \frac{x^3}{3} + x(y^2+z^2)\Big|_0^a = \frac{a^3}{3} + a(y^2+z^2)$$

$$\int_0^b (\frac{a^3}{3} + a(y^2+z^2))dy = \frac{ay^3}{3} + y\left(\frac{a^3}{3} + az^2\right)\Big|_0^b = \frac{ab^3}{3} + b(\frac{a^3}{3} + az^2)$$

$$\int_0^c (\frac{ab^3}{3} + b(\frac{a^3}{3} + az^2))dz = \underline{\frac{abc^3}{3} + \frac{ab^3c}{3} + \frac{a^3bc}{3}}$$

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$$\iiint_T xyzdxdydz \quad x^2+y^2+z^2=1, x=0, y=0, z=0$$

$$\int_0^{\sqrt{1-y^2-x^2}} xyzdz = \frac{yx(1-y^2-x^2)}{2}$$

$$\int_0^{\sqrt{1-x^2-z^2}} \frac{yx(1-y^2-x^2)}{2}dy = -\frac{x(1-2z^2)^2}{8} + (1-2z^2)(-\frac{x^3}{4} + \frac{x}{4})$$

$$\int_0^{\sqrt{1-y^2-z^2}} \left(-\frac{x(1-2z^2)^2}{8} + (1-2z^2)(-\frac{x^3}{4} + \frac{x}{4})\right)dx =$$

$$\underline{(\frac{1}{16} - \frac{z^4}{4})(-y^2-z^2+1) + (\frac{z^2}{8} - \frac{1}{16})(-y^2-z^2+1)}$$

надеюсь правильно

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$$\iiint_T x dx dy dz \quad x = 0, y = 0, z = 0, y = 3, x + z = 2$$

$$\int_0^{2-z} dx \int_0^3 dy \int_0^{2-x} x dz = \int_0^{2-z} dx \int_0^3 (2x - x^2) dy =$$

$$\int_0^{2-z} (6x - 3x^2) dx = \underline{3(2-z)^2 - (2-z)^3 = (2-z)^2(1+z)}$$