MOCK TEST OF MATHEMATICS FOR UNIVERSITY ADMISSION

Paper 1

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1.

Consider the points (x, y) on the circle with centre $(1, 0)$ and radius 1. What is the largest value $x + y$ can take?

A 2

B $\frac{3}{2}$

C $\frac{7}{4}$

D $1 + \sqrt{2}$

E $2\sqrt{2}$

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2.

A (non-zero) geometric sequence with ratio μ and first term a is such that its sum to infinity is equal to:

(i) 2 times the sum to infinity of the (non-zero) geometric sequence with ratio $\frac{1}{2}\mu$ and first term b

(ii) 3 times the sum to infinitely of the (non-zero) geometric sequence with ratio $\frac{1}{3}\mu$ and first term b

A there is only one possible value of μ but infinitely many for a, b

B there are infinitely many possible values of μ but only one for each a and b

C there are infinitely many possible values for $\mu,\ a,\ b$

D there is only one triple $\mu,\ a,\ b$ which works

E there are two possible values for μ, a and b

F there are no values of $\mu,\ a,\ b$ which satisfy the condition

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3.

Let T be a right-angled triangle with hypothenuse 1 with α as one of its acute angles. The value of $\tan\alpha$ determines:

A the length of one additional side only

B the length of all sides only

C the area of T only

D the other acute angle only

E the side lengths and the area of T only

F the side lengths and the other acute angle only

G the area of T and the other acute angle only

H all properties of T

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4.

The function r satisfies $r(0) + r(2) = \frac{1}{3}$ and $r\left(\frac{4}{3}\right) = 0$, as well as the equation:

$$\frac{\mathrm{d}^2 r}{\mathrm{d}x^2} = \frac{1}{(1-x)^4} + \frac{1}{(1-x)^3}$$

This defines r as:

A $\dfrac{2}{3(1-x)^2} - \dfrac{x}{2(1-x)^2}$

B $\dfrac{4+3x}{6(1-x)^2}$

C $\dfrac{4-3x-(1+x)^2}{6(1-x)^2}$

D $\dfrac{4}{3(1-x)^2} - \dfrac{x}{(1-x)^2}$

E $1 + \dfrac{4-3x}{6(1-x)^2}$

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5.

$p_1(x),\ p_2(x),\ p_3(x)\ p_4(x)$ are 4 quadratics such that the polynomial

$$p_1(x)p_2(x)p_3(x)p_4(x)$$

has 4 roots. Write Δ_k as the discriminant of p_k. Then:

A $\Delta_1\Delta_2\Delta_3\Delta_4 = 0$

B $\Delta_1\Delta_2\Delta_3\Delta_4 \leqslant 0$

C $\Delta_1\Delta_2\Delta_3\Delta_4 \geqslant 0$

D $\Delta_1\Delta_2\Delta_3\Delta_4 > 0$

E $\Delta_1\Delta_2\Delta_3\Delta_4 < 0$

F there is not enough information to determine the sign of $\Delta_1\Delta_2\Delta_3\Delta_4$.

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6.

The distance between the curves $(x-1)^2+(y+1)^2=7$ and $y=x+7$ is:

A $\sqrt{7}+\frac{1}{8}\sqrt{2}$

B $\frac{5}{2}\sqrt{2}-\sqrt{7}$

C $\frac{9}{\sqrt{2}}-\sqrt{7}$

D $\frac{18}{\sqrt{2}}-4\sqrt{7}$

E $4\sqrt{2}-\sqrt{7}$

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7.

Find the value of

$$\int_0^1 \frac{x+1}{\sqrt[3]{x}}\,\mathrm{d}x + \int_{-1}^0 \frac{x+2}{\sqrt[3]{x+1}}\,\mathrm{d}x$$

A $\frac{6}{5}$

B $\frac{21}{10}$

C $\frac{7}{3}$

D $\frac{19}{6}$

E $\frac{21}{5}$

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8.

How many solutions does the equation $\sin(\cos x) = 0$ have in $[0, 2\pi]$?

A 0

B 1

C 2

D 3

E 4

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9.

Let k be a positive integer, and suppose we differentiate the function

$$(1+x)(1+x^2)(1+x^3)\cdots(1+x^k)$$

121 times, at which point the result reaches 0. What is k?

A 5

B 5!

C 115

D 15

E 11

F 16

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10.

Let f be a function defined over the real numbers. The function is reflected through the bissector of $[1,3]$, parallel to the y-axis. The resulting function can be written as:

A $f\left(2-x\right)$

B $f\left(-2-x\right)$

C $f\left(x-2\right)$

D $f(4-x)$

E $f(x+4)$

F $f\left(x-4\right)$

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11.

The minimum of the quadratic $p(x)$ is m and the maximum of the quadratic $q(x)$ is also m. These are achieved at the same value of x.

The number of roots of the polynomial $p(x) + q(x)$ is:

A either 0 or 2

B at least 1

C at most 1

D either 0, 1, or 2

E either 0, 2, or infinitely many

F 0, 1, 2 or infinitely many

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12.

Find the full set of values (x, y) such that the following simultaneous equations hold:

$$\begin{cases} 2^x + 4^y = 1 \\ 2^y + 4^x = 1 \end{cases}$$

A $x = y = \log_2(\sqrt{5} - 1) - 1$

B $x = -y = \log_2(\sqrt{5} - 1) - 1$ or $y = -x = \log_2(\sqrt{5} - 1) - 1$

C $x = y = \log_2(1 + \sqrt{5}) - 1$

D $x = -y = \log_2(1 + \sqrt{5}) - 1$ or $y = -x = \log_2(1 + \sqrt{5}) - 1$

E there are no solutions

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13.

For how many values of ϑ does $\sin\vartheta + \cos\vartheta = \frac{3}{2}$ if $0 \leqslant \vartheta < 2\pi$?

A 0

B 1

C 2

D 4

E 8

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14.

Consider the polynomial:

$$x^3 + qx^2 + q^2x + q^3$$

Let A be the (unique) intersection between the curve and the x-axis, B the intersection between the tangent to the curve at A and y-axis, and C the intersection between the normal to the curve at A and the y-axis.

For which value(s) of $q \neq 0$ is the triangle ABC isosceles with apex at A?

A ± 1

B $\pm \frac{1}{\sqrt{2}}$

C $\pm \frac{1}{2}$

D $\pm \sqrt{2}$

E $\pm 2(\sqrt{2} - 1)$

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15.

We pick 4 apples from a basket containing 2 red apples and a certain number $\geqslant 2$ of green apples. It is given that the probability that we pick both red apples is twice the probability that we do not pick either red apple.

How many green apples must be in the basket?

A 2

B 3

C 4

D 5

E 6

F 7

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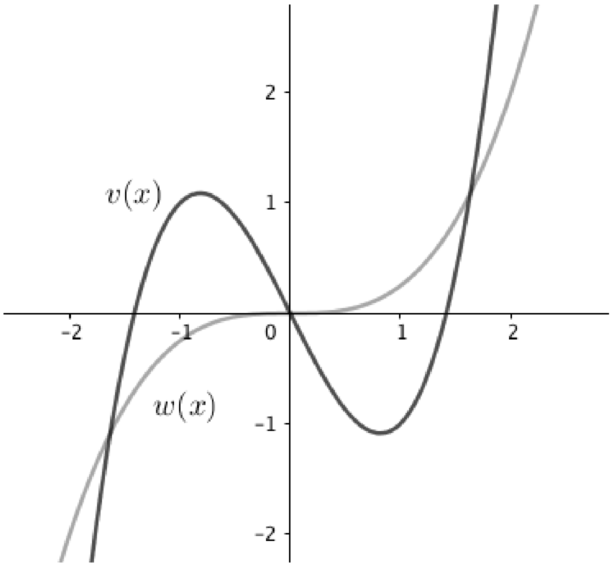
16.

The functions v and w intersect at 0, a and $-a$ with $a > 0$ and are depicted in the graph below:



Consider the following list of integrals:

(1) $\displaystyle\int_{-a}^{a} \left|v(x) - w(x)\right| \mathrm{d}x$

(2) $\displaystyle\int_{0}^{a} 2\big(w(x) - v(x)\big) \,\mathrm{d}x$

(3) $\displaystyle\int_{0}^{a} w(x) - w(-x) + v(-x) - v(x) \,\mathrm{d}x$

Which of these represent(s) the area of the region between the curves in the interval $(-a, a)$?

A (1) only

B (2) only

C (3) only

D (1) and (2) only

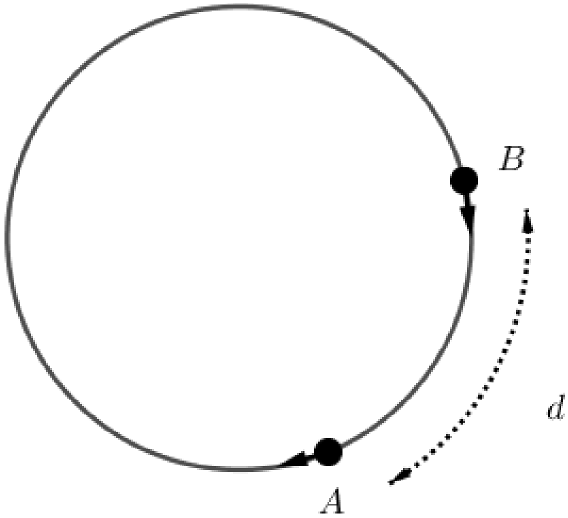
E (2) and (3) only

F (1) and (3) only

G (1), (2), and (3)

17.

Below is a circle with radius 1 and two points A and B which travel at constant velocity clockwise around the circle. It takes t_A seconds for A to complete a full revolution, and it takes B t_B seconds, where $t_A < t_B$.



If the shortest distance between the points is d as shown on the diagram, what time will it take for the points to meet next?

A $d \cdot \dfrac{t_A t_B}{t_B - t_A}$

B $\left(1 - \dfrac{2\pi}{d}\right) \dfrac{t_A t_B}{t_B - t_A}$

C $(t_A^{-1} - t_B^{-1})$

D $(2\pi - d)\left(\dfrac{1}{t_A} - \dfrac{1}{t_B}\right)$

E $\dfrac{t_B - t_A}{2\pi - d}$

F $\dfrac{1}{d}\left(\dfrac{1}{t_A} - \dfrac{1}{t_B}\right)$

18.

$o_1,\ o_2,\ o_3\ldots$ is a (non-zero) arithmetic sequence such that, for each positive integer n, the integral of $x-n$ over $(0,o_n)$ is equal to $\frac{1}{2}o_n$ i.e.

$$\int_0^{o_n} x-n\,\mathrm{d}x = \frac{o_n}{2} \quad \text{for all } n \geqslant 1$$

The value of o_{100} is:

A $50+\frac{1}{2}$

B 51

C 100

D $100+\frac{1}{2}$

E 101

F 199

G 201

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19.

n lines are drawn in the plane, such that no two are parallel and no three meet at the same point. This separates the plane into a number of regions r. Another line is then drawn, neither parallel to another line nor passing through an existing intersection.

The number of regions is now:

A $n + r$

B $\binom{n}{2} + r$

C $n + r + 1$

D $n - 1 + r$

E $\binom{n-1}{2} + r$

F dependant on where the line is drawn

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20.

Define the function:

$$f(x) = x + \frac{1}{x}, \qquad x \neq 0$$

and write $f_k(x)$ for the k^{th} iteration of f, for instance $f_3(x) = f(f(f(x)))$.

What is the value of $f'_{2019}(1)$?

A 0

B 1

C 2019

D $\frac{2020}{2019}$

E $\frac{2019}{2020} + \frac{2020}{2019}$

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