TMUA/MAT Graphs of Functions

Syllabus

Sketch common functions; transformations of graphs; stationary points / increasing / decreasing functions; intersection with coordinate axes / number of roots; graphs and simultaneous equations.

1. Sketch each of the following functions and find the range for the given domains:

a) $f(x) = x^2 - 8x + 13 \qquad x \in \mathbb{R} \quad x > 0$

b) $f(x) = \sqrt{x-2} \qquad x \in \mathbb{R} \quad 6 < x < 18$

c) $f(x) = \dfrac{2}{x+3} \qquad x \in \mathbb{R} \quad x \geq 1$

d) $f(x) = \dfrac{1}{x-1} + 2 \qquad x \in \mathbb{R} \quad x > 2$

e) $f(x) = 15 - (x-2)^2 \qquad x \in \mathbb{R} \quad 0 \leq x \leq 4$

f) $f(x) = 8 - x^3 \qquad x \in \mathbb{R} \quad 0 \leq x \leq 2$

g) $f(x) = 2 - e^x \qquad x \in \mathbb{R} \quad x \leq 0$

h) $f(x) = 3 - e^{x+1} \qquad x \in \mathbb{R} \quad x \geq -1$

i) $f(x) = 3 - ln\, x \qquad x \in \mathbb{R} \quad 0 < x < 1$

2. Sketch each of the following graphs, stating any values of x for which the function is not defined

a) $y = |3x - 6|$

b) $y = |x^2 - 3x - 4|$

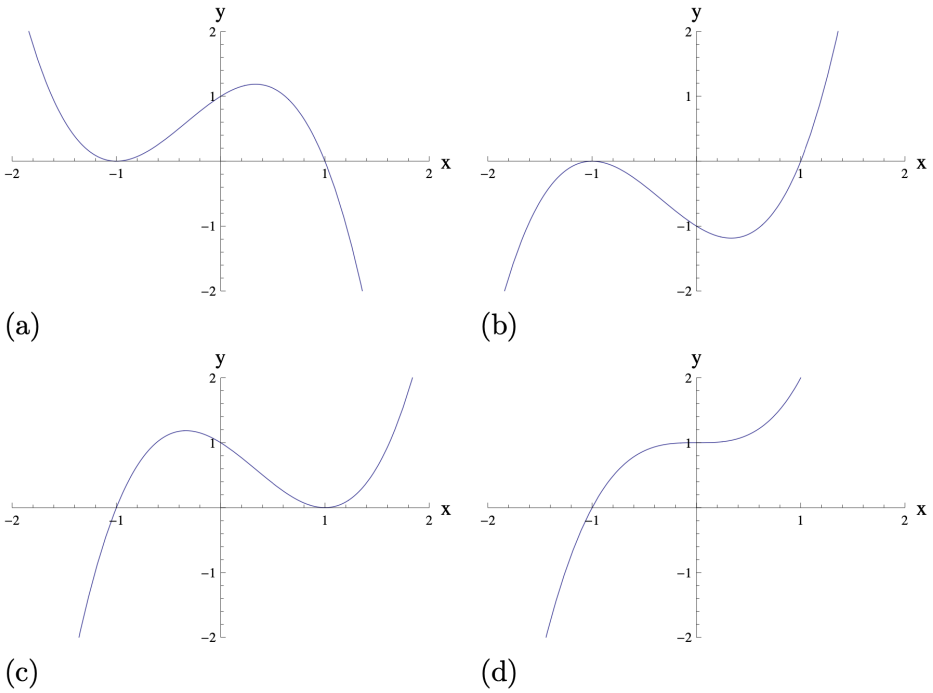
c) $y = \dfrac{1}{1 + x^2}$

d) $y = \dfrac{x^2 + 1}{x - 1}$

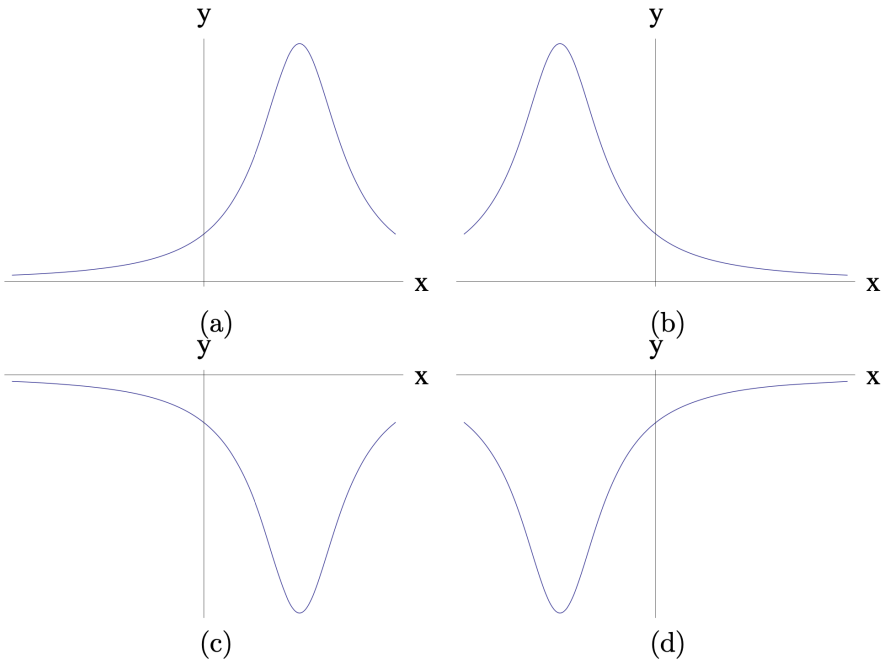
e) $y^2 = x^3$

f) $y = \sqrt[3]{x^3 - x}$

3a) A sketch of the graph $y = x^3 - x^2 - x + 1$ appears on which of the following axes?



b) Which of the following graphs is a sketch of $y = \dfrac{1}{6x - x^2 - 10}$



4. Find the composite function $fg(x)$ and sketch this function.
State any values of x for which the function $fg(x)$ is not valid.

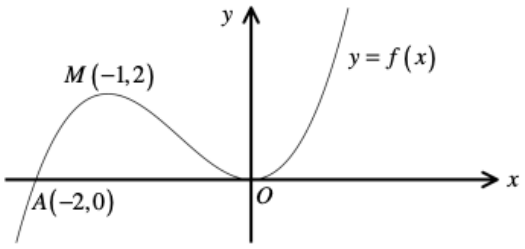
a) $f(x) = x^2 - 4$ $g(x) = 2x - 2$

b) $f(x) = 2x^2 - 3$ $g(x) = \sqrt{x + 4}$

c) $f(x) = 2e^{\frac{1}{2}x}$ $g(x) = ln(4x)$

d) $f(x) = sin\, x$ $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ $g(x) = x - \frac{\pi}{2}$ $x \geq 0$

5. The figure shows the graph of the curve with equation $y = f(x)$



Sketch the graphs of the following functions and include the new coordinates of points A and M.

a) $y = 2f(x) + 1$

b) $y = f(x - 3)$

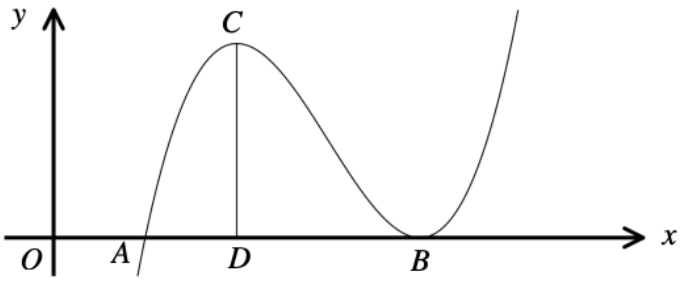
c) $y = f(-\frac{1}{2}x)$

d) $y = f(|x|)$

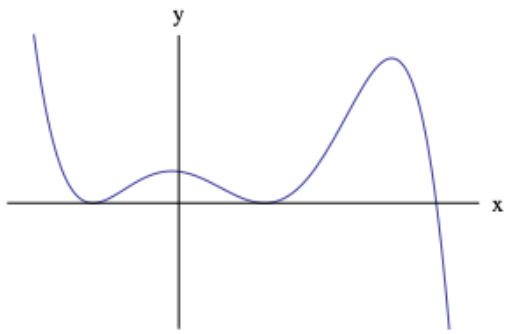
e) $y = -4f(x + 1)$

f) $y = f(2x + 4)$

6. The figure shows a cubic curve whose coefficient of x^3 is 1. The curve crosses the x-axis at A $(a,0)$ and touches the x-axis at B $(b,0)$ where a and b are positive constants such that $a < b$. The point C is a local maximum of the curve. Find the coordinate of D in terms of a and b.



7. Which one of the following equations could possibly be the graph below:



I $\quad y = (3-x)^2(3+x)^2(1-x)$

II $\quad y = -x^2(x-9)(x^2-3)$

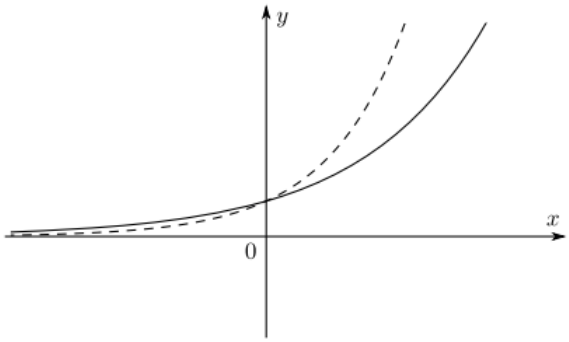
III $\quad y = (x-6)(x-2)^2(x+2)^2$

IV $\quad y = (x^2-1)^2(3-x)$

8. The graphs of two functions are shown.

$y = a^x$ is shown with a solid line where a is a positive real number.

$y = f(x)$ is shown with a dashed line



Which of the following could be true?

I $\quad f(x) = b^x \quad$ for some $b > a$

II $\quad f(x) = b^x \quad$ for some $b < a$

III $\quad f(x) = a^{kx} \quad$ for some $k > 1$

IV $\quad f(x) = a^{kx} \quad$ for some $k < 1$