

THE UK UNIVERSITY INTEGRATION BEE

2021/22



ROUND ONE

Friday, 15 October 2021



UNIVERSITY OF
CAMBRIDGE

Sponsored by



Jane Street

1. $\int \sqrt{x^3 \sqrt{x^4 \sqrt{x^5 \sqrt{x \cdots}}}} dx$
2. $\int_0^{2\pi} \cos^{420}(x) dx$
3. $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos \theta}}{(\sqrt{\cos \theta} + \sqrt{\sin \theta})^5} d\theta$
4. $\int_0^1 \frac{\sin(\ln x)}{\ln x} dx$
5. $\int_0^4 \frac{\ln(x)}{\sqrt{4x - x^2}} dx$
6. $\int_0^\infty \left(\frac{\ln x}{1+x} \right)^2 dx$
7. $\int_0^\infty \frac{\ln(x)}{x^2 + 2x + 2} dx$
8. $\int_{-\infty}^\infty \frac{dx}{(x^3 + \frac{1}{x^3})^2}$
9. $\int \frac{x-1}{(x+1)\sqrt{x^3+x^2+x}} dx$
10. $\int_0^1 \frac{x}{\sin(x)} dx$
11. $\int_1^\infty \left(\frac{\ln x}{x} \right)^{2011} dx$
12. $\int_0^\infty \frac{\sin x}{x^n} dx \quad (0 < n < 2)$
13. $\int_0^{\frac{\pi}{2}} \frac{\cos x}{(1 + \sqrt{\sin(2x)})^2} dx$
14. $\int_0^{\frac{\pi}{2}} \ln(7997 \sin^2 \theta + 7945 \cos^2 \theta) d\theta$
15. $\int \frac{dx}{\csc x + 1}$
16. $\int_0^{\frac{\pi}{2}} \frac{\{\tan(x)\}}{\tan(x)} dx$, where $\{x\}$ is the fractional part of x
17. $\int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{2 + \sin x} dx$
18. $\int \sqrt{\frac{1}{x} - 1} dx$
19. $\int_0^{2\pi} e^{3\cos \theta} \cos(3 \sin \theta) d\theta$

20. $\int \frac{dx}{1 + 2x^2 - 2\sqrt{x^2 + x^4}}$
21. $\int_0^a \frac{x dx}{\cos(x) \cos(a - x)}$
22. $\int_0^1 \frac{\arctan x}{1 + x} dx$
23. $\int_0^{\frac{\pi}{2}} \frac{\ln(\cos x)}{\sin x} dx$
24. $\int \frac{\tan x}{\sqrt{\sec^3 x + 1}} dx$
25. $\int_0^\infty \frac{\arctan(x)}{x(\log(x)^2 + 1)} dx$
26. $\int_1^e \frac{x - \ln x + 1}{x(x + 1)^2 + x \ln^2 x} dx$
27. $\int_0^\infty \frac{\cos(ax)}{x^2 + b^2} dx$
28. $\int (\sec x)^{(1+\sec x)} (\tan x + \sin x) dx$
29. $\int_0^\infty \lfloor x \rfloor e^{-x} dx$
30. $\int_0^1 \frac{\arctan(x^2)}{1 + x^2} dx$
31. $\int_0^1 \frac{dx}{1 + \lfloor \frac{1}{x} \rfloor}$
32. $\int_0^\infty \frac{\cos(ax) - \cos(bx)}{x^2} dx$
33. $\int_0^{2\pi} e^{\cos x} \cos(\sin x) \cos(5x) dx$
34. $\int_{-\infty}^\infty \frac{dx}{\cosh^2(x)}$
35. $\int_0^1 W(x) dx$, where W is the Lambert W function, the solution to $W(x)e^{W(x)} = x$
36. $\int_0^1 \frac{\arctan^2 x}{x} dx$
37. $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cot x^{\tan x \cot x} - \tan x^{\cot x \tan x} dx$
38. $\int_3^5 \ln \Gamma(x) dx$
39. $\int \frac{dx}{\sin^4(x) + \cos^4(x)}$
40. $\int_0^\infty \frac{\cos(\ln x)}{(1 + x)^2} dx$