TMUA Multiple Choice Practice - Differentiation

1. The gradient of the curve $y = \dfrac{(4x + \sqrt{x})(x^2 - 3)}{3\sqrt{x}}$ at the point where $x = 1$ is

A $\frac{1}{3}$ [✓ B] 2 C $\frac{10}{3}$ D 4 E $\frac{20}{3}$

$$y = \frac{4x^3 + x^{5/2} - 12x - 3x^{1/2}}{3x^{1/2}} = \frac{4}{3}x^{5/2} + \frac{1}{3}x^2 - 4x^{1/2} - 1$$

$$\frac{dy}{dx} = \frac{10}{3}x^{3/2} + \frac{2}{3}x - 2x^{-1/2}$$

$$= \frac{10}{3}x^{3/2} + \frac{2}{3}x - \frac{2}{\sqrt{x}}$$

At $x = 1$ $\frac{dy}{dx} = \frac{10}{3} + \frac{2}{3} - 2 = 2$

2. A curve C has equation $y = \dfrac{x^2 - 2}{\sqrt{x}}$. Find the gradient of C at the point $(2, \sqrt{2})$.

A $\sqrt{2}$ [✓ B] $\frac{7}{4}\sqrt{2}$ C $\frac{7}{2}\sqrt{2}$ D $4\sqrt{2}$ E $\frac{9}{2}\sqrt{2}$

$$y = x^{3/2} - 2x^{-1/2}$$

$$y' = \frac{3}{2}x^{1/2} + x^{-3/2}$$

At $x = 2$ $y' = \frac{3}{2}\sqrt{2} + \frac{1}{2\sqrt{2}}$

$$= \frac{3}{2}\sqrt{2} + \frac{1}{4}\sqrt{2} = \frac{7}{4}\sqrt{2}$$

3. The curve $y = x^3 + 3\sqrt{5}px^2 + 3px + 13$ has two distinct turning points.

Find the possible values of p.

[✓ A] $p < 0, p > 0.2$

B $p \leq 0, p \geq 0.2$

C $0 < p < 0.2$

D $0 \leq p \leq 0.2$

E $p < 0, p > 1.2$

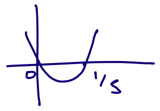
F $p \leq 0, p \geq 1.2$

$$\frac{dy}{dx} = 3x^2 + 6\sqrt{5}px + 3p = 0$$

$\Delta > 0$ $180p^2 - 4(3)(3p) > 0$

$$5p^2 - p > 0$$

$$p(5p - 1) > 0$$



$p < 0$ $p > 0.2$

4. Find the complete set of values of k for which the graph $y = x^3 - 2kx^2 + 4x - k$ has two distinct real stationary points.

A $-3 < k < 3$

B $k < -3$ or $k > 3$

C $-\sqrt{3} < k < \sqrt{3}$

[✓ D] $k < -\sqrt{3}$ or $k > \sqrt{3}$

E all values of k

$\frac{dy}{dx} = 3x^2 - 4kx + 4$

$\Delta > 0$ $\quad 16k^2 - 4(3)(4) > 0$

$k^2 - 3 > 0$

$(k - \sqrt{3})(k + \sqrt{3}) > 0$

$k < -\sqrt{3}$, $k > \sqrt{3}$

5. Given that the cubic equation $f(x) = p^{\frac{2}{3}}x^3 + px^2 + p^{\frac{1}{3}}x + 3$ where p is a positive constant

has exactly one point where $f'(x) = 0$, find the value of p.

A $\frac{1}{4}$ B $\frac{3}{4}$ C 1 [✓ D] 3 E 6

$f'(x) = 3p^{2/3}x^2 + 2px + p^{1/3} = 0$

$\Delta = 0$ $\quad 4p^2 - 4(3p^{2/3})(p^{1/3}) = 0$

$p^2 - 3p = 0$

$p(p-3) = 0$

$p > 0 \quad p = 3$

6. Consider the function given by $f(x) = x^{\frac{1}{5}}(x^2 - 2x + 1)$

The fraction of the interval $0 < x < 2$ for which $f(x)$ is decreasing is

[✓ A] $\frac{5}{11}$ B $\frac{1}{2}$ C $\frac{3}{5}$ D $\frac{5}{6}$ E $\frac{10}{11}$

$f(x) = x^{11/5} - 2x^{6/5} + x^{1/5}$

$f'(x) = \frac{11}{5}x^{6/5} - \frac{12}{5}x^{1/5} + \frac{1}{5}x^{-4/5} < 0$

$x^{-4/5}(11x^2 - 12x + 1) < 0$

↗ >0 for $0 < x < 2$

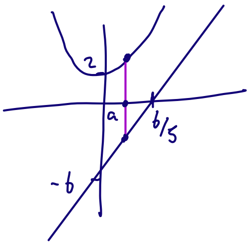
$x^{-4/5}(11x - 1)(x - 1) < 0$

$\frac{1}{11} < x < 1$

Fraction $= \frac{1 - \frac{1}{11}}{2} = \frac{5}{11}$

7. A curve has equation $y = 3x^2 + 2$ and a line has equation $y = 5x - 6$.
What is the shortest distance parallel to the y-axis between the curve and the line?

A $\frac{5}{6}$ B $\frac{6}{5}$ C $\frac{49}{12}$ [✓ D] $\frac{71}{12}$ E 8



$(a, 3a^2+2)$ $(a, 5a-6)$

$D = 3a^2+2-5a+6$

$= 3a^2-5a+8$

$\frac{dD}{da} = 6a - 5 = 0$

$a = \frac{5}{6}$

$D = 3\left(\frac{25}{36}\right) - \frac{25}{6} + 8$

$= \frac{25}{12} - \frac{50}{12} + \frac{96}{12} = \frac{71}{12}$

8. A curve C has equation $y = 2x^3 - 5x^2 + a$ where a is a constant.

The tangent to C at $x = 2$ and the normal to C at $x = 1$ meet on the x-axis.

The value of a is

A $\frac{1}{4}$ B $\frac{2}{3}$ C 4 D 6 [✓ E] $\frac{8}{3}$

$y' = 6x^2 - 10x$

At $x=2$ $y' = 4$ $x=1$ $y' = -4$

$x=2$ $y = 16-20+a = a-4$

$x=1$ $y = a-3$

Tangent $m=4$ $(2, a-4)$

$y-(a-4) = 4(x-2)$

$4-a = 4x-8$

$12-a = 4x$

$40 - 15a = 0$

$a = \frac{40}{15} = \frac{8}{3}$

Normal $m = \frac{1}{4}$ $(1, a-3)$

$y-(a-3) = \frac{1}{4}(x-1)$

$3-a = \frac{1}{4}x - \frac{1}{4}$

$\frac{13}{4} - a = \frac{1}{4}x$

$52 - 16a = 4x$

9. The point P lies on the curve with equation $y = x^2$ so that its distance from the point $Q\,(-5, -1)$ is least. Find the distance PQ.

[✓ A] $2\sqrt{5}$ B $\sqrt{26}$ C $4\sqrt{5}$ D 20 E 26

$(-5,-1)$ (x, x^2)

$D^2 = (x+5)^2 + (x^2+1)^2$

$= x^2+10x+25+x^4+2x^2+1$

$= x^4+3x^2+10x+26$

$\frac{dD}{dx} = 4x^3+6x+10 = 0$

$2x^3+3x+5=0$

$(x+1)(2x^2-2x+5)=0$

$(x+1)($ $\Delta < 0$ $)$

$x=-1$ no solutions

$D^2 = 4^2 + 4 = 20$

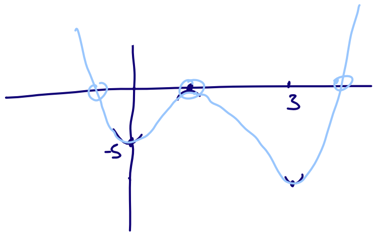
$D = \sqrt{20} = 2\sqrt{5}$

10. How many real roots does the equation $y = 3x^4 - 16x^3 + 18x^2 - 5$ have?

A 1 B 2 [✓ C 3] D 4 E 5

$$\frac{dy}{dx} = 12x^3 - 48x^2 + 36x = 0$$
$$x^3 - 4x^2 + 3x = 0$$
$$x(x^2 - 4x + 3) = 0$$
$$x(x-1)(x-3) = 0$$

W
+ve quartic



$x = 0$ $y = -5$
$x = 1$ $y = 3 - 16 + 18 - 5 = 0$
$x = 3$ $y = \underset{9(27)}{243} - 16(27) + \underset{6(27)}{18(9)} - 5 < 0$

$$\frac{d^2y}{dx^2} = 36x^2 - 96x + 36$$

>0 $x = 0$ min
<0 $x = 1$ max
>0 $x = 3$ min

11. What is the highest term in x of the following polynomial

$$\frac{d^2}{dx^2}\left[(x^6+2)^2(x^4-3)^4\right] - \frac{d}{dx}\left[(3x^5-1)^3(x^2+4)^6\right]$$

A $26x^{25}$ [✓ B $27x^{26}$] C $28x^{26}$ D x^{27} E $28x^{28}$

$(x^6)^2(x^4)^4 = x^{28}$
$12+16$
$28x^{27}$
$28.27x^{26}$

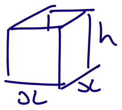
$(3x^5)^3(x^2)^6$
$27x^{15}\ x^{12}$
$27x^{27}$
27^2x^{26}

$27(28-27)x^{26}$
$\underline{\underline{27x^{26}}}$

12. A water tank, with volume $500m^3$, is to be made in the shape of a cuboid with a square base and no top. What is the least amount of metal in m^3 required to make this tank?

A $100\sqrt{2}$
B $100 + 50\sqrt{2}$
C 200
[✓ D 300]
E $50 + 200\sqrt{2}$

$V = 500$
$x^2h = 500$
$SA = x^2 + 4xh$
$= x^2 + \frac{2000}{x}$



$$\frac{dA}{dx} = 2x - \frac{2000}{x^2} = 0$$
$x^3 = 1000$
$x = 10$
$h = 5$

$SA = 100 + 200$
$= 300$

13. A curve C has equation given by $f(x) = 2p^3 + 3p^2x - 2px^2 + x^3$ where p is real.

The gradient of the normal to C at $x = 1$ is M.

What is the least possible value of M as p varies?

A $-\frac{7}{2}$ B $-\frac{5}{2}$ C $-\frac{5}{3}$ [✓ D] $-\frac{3}{5}$ E $\frac{2}{3}$

$f'(x) = 0 + 3p^2 - 4px + 3x^2$

$f'(1) = 3p^2 - 4p + 3$ - grad of tang.

$M = \frac{1}{-3p^2+4p-3}$ least M when max $(-3p^2+4p-3)$

$-3(p^2 - \frac{4}{3}p + 1)$

$-3[(p-\frac{2}{3})^2 + \frac{5}{9}]$

$-3(p-\frac{2}{3})^2 - \frac{5}{3}$

Max $= -\frac{5}{3}$ so $M = -\frac{3}{5}$

OR

$\frac{d}{dp} = -6p + 4 = 0$

$p = 2/3$

$-3(\frac{4}{9}) + \frac{8}{3} - \frac{9}{3} = -\frac{5}{3}$

14. How many real roots does the equation $y = x^5 + 5x^3 - 20x + 14$ have?

A 1 [✓ B] 2 C 3 D 4 E 5

$\frac{dy}{dx} = 5x^4 + 15x^2 - 20 = 0$

$x^4 + 3x^2 - 4 = 0$

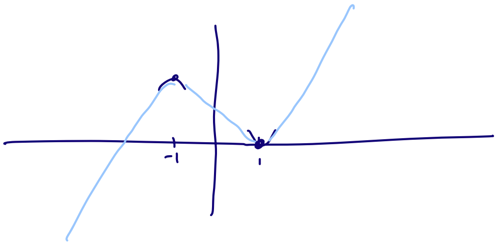
$(x^2 - 1)(x^2 + 4) = 0$

$x = \pm 1$ no solns

$x = 1$ $y = 1 + 5 - 20 + 14 = 0$ min

$x = -1$ $y = -1 - 5 + 20 + 14 > 0$ max

$\frac{d^2y}{dx^2} = 20x^3 + 30x = 10x(2x^2+3)$



15. A cubic curve has equation $y = ax^3 + bx^2 + cx + d$ where a, b, c, d are non-zero constants. Given that this curve has one local maximum and one local minimum, which of the following statements is necessarily true:

[✓ A] $b^2 > 3ac$

B $b^2 > 4ac$

C $c^2 > 4bd$

D If $a > 0$, then $d > 0$

E If $a > 0$, then $d < 0$

$\frac{dy}{dx} = 3ax^2 + 2bx + c = 0$

2 distinct solutions $\Delta > 0$

$4b^2 - 4(3a)c > 0$

$b^2 - 3ac > 0$

16. The function $f(x) = \dfrac{3x - 2}{\sqrt[3]{x^2}}$ is defined for all $x \neq 0$

The complete set of values of x for which the function is decreasing is

A $x < -\frac{4}{3},\ x > 0$

[✓ B] $-\frac{4}{3} < x < 0$

C $-\frac{4}{3} < x < \frac{4}{3},\ x \neq 0$

D $-\frac{2}{3} < x < 0$

E $-\frac{2}{3} < x < \frac{2}{3},\ x \neq 0$

$\frac{3x-2}{\sqrt[3]{x^2}} = 3x^{1/3} - 2x^{-2/3}$

$x^{-2/3} + \frac{4}{3}x^{-5/3} < 0$

$\times x^2$ $\quad x^{4/3} + \frac{4}{3}x^{1/3} < 0$

$x^{1/3}\left(x + \frac{4}{3}\right) < 0$

if $x > 0$ $\quad x < -4/3$ $\quad$ × contradiction

if $x < 0$ $\quad x > -4/3$ $\quad -\frac{4}{3} < x < 0$

17. The volume V, of a soap bubble is modelled by the formula $V = (p - qt)^2 \quad t \leq 0$

where p and q are positive constants and t is the time in seconds after a certain instant.

When $t = 1$, the volume of a soap bubble is $9cm^3$ and at that instant its volume is decreasing at the rate of $6cm^3$ per second. What is the value of $p + q$?

A 2 B 3 [✓ C] 5 D 6 E 9

$V = (p-qt)^2 = p^2 - 2pqt + q^2t^2$

$9 = (p-q)^2 \qquad p - q = \pm 3$

$V' = -2pq + 2q^2t$

$-2pq + 2q^2 = -6$

$q^2 - pq = -3$

$q(q-p) = -3$

$q - p = 3 \quad q = -1 \quad p = -4$ ×

$q - p = -3 \quad q = 1 \quad p = 4$ ✓

18. The least possible value of the gradient of the curve $y = (x + a)^2(3x - a)$ at the point where $x = \frac{1}{2}$, as a varies is

A -9 B $-\frac{25}{4}$ [✓ C] -4 D $-\frac{5}{2}$ E $\frac{5}{4}$

$= (x^2 + 2ax + a^2)(3x - a)$

$= 3x^3 + 6ax^2 + a^2 3x - ax^2 - 2a^2x - a^3$

$= 3x^3 + 5ax^2 + a^2x - a^3$

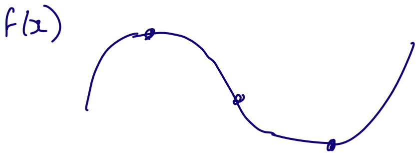
$y' = 9x^2 + 10ax + a^2$

$x = \frac{1}{2} \qquad y' = \frac{9}{4} + 5a + a^2$

$= \left(a + \frac{5}{2}\right)^2 - \frac{25}{4} + \frac{9}{4}$

$= \left(a + \frac{5}{2}\right)^2 - 4$

19. Consider the function $f(x) = x^3 - 3x^2 - 144x$

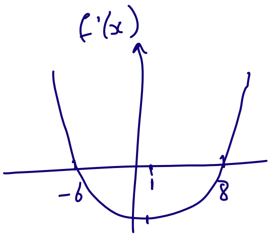


Which of the following statements are true?

I The gradient of the function is negative for $x < 0$.

II There is a local maximum at $x = 8$.

III There is a point of inflexion at $x = 1$.



A none of them
B I only
C II only
[✓ D III only]
E I and II only
F I and III only
G II and III only
H I, II and III

$f'(x) = 3x^2 - 6x - 144$

$f''(x) = 6x - 6$

$f'(x) = 3(x^2 - 2x - 48)$

$= 3(x-8)(x+6)$

Stat points $x = 8$ $x = -6$

$f''(8) = 48 - 6 > 0$ so minimum

$f'(x) > 0$ for $x < -6$

I is false

II is false

$f''(1) = 0$ - could be point of inflexion (even though not a stat. point)

$f''(0) < 0$ concave $f''(2) > 0$ convex / from sketch $f'(x) < 0$ immediately either side of $x = 1$

change of sign so III is true

20. A curve has equation $y = 3x^4 - 4x^3 - 12x^2 + 20$

What is the complete set of values of the constant k for which the equation

$3x^4 - 4x^3 - 12x^2 + 20 = k$ has four distinct real roots

A no values of k
B $-12 < x < 15$
[✓ C $15 < x < 20$]
D $k > 15$
E $7 < x < 20$

$y = 3x^4 - 4x^3 - 12x^2 + 20$

$y' = 12x^3 - 12x^2 + 24x = 0$

$x(x^2 - x + 2) = 0$

$x(x-2)(x+1) = 0$

$3 + 4 - 12 + 20$

$48 - 32 - 48 + 20$

T.Ps (0, 20)
(-1, 15)
(2, -12)

$y = k$ 4 solns

$15 < k < 20$