Interview Questions - University of Oxford

1 Computer Science

1.1

1.1.1

(a) Take the 10 digit number 7396350874 and delete 3 of it's digits so that you are left with thehighest 7 digit number possible.

(b) Take an n-digit number and delete k (where $k < n$) of it's digits to make the highest possible $(n-k)$-digit number, and evaluate the number of steps taken to do so. Is there a more efficient way to do this?

1.1.2

(a) Let n be a perfect square. Out of the natural numbers $1,2,...,n$ choose $\lfloor \frac{n}{2} \rfloor$ numbers such that none of the numbers you chose are a perfect square and no pair of numbers can be chosen that add up to n. For what numbers, n, can this be achieved?

(b) What if n is a perfect cube? Does this generalise?

1.2

1.2.1

You have 6 counters: 4 red ones and 2 white ones. You cannot tell counters of the same colour apart.

(a) In how many ways can the counters be arranged?

(b) If the counters are put in a circle, in how many ways can they be arranged?

1.2.2

Given the rules below

$$00 \rightarrow \textit{nothing}$$

$$11 \rightarrow \textit{nothing}$$

$$0 \rightarrow 101$$

$$1 \rightarrow 010$$

(a) Convert 10 to 01.

(b) Convert 1001 to 0101.

(c) Simplify 10101010 as much as possible.

(d) What patterns can you spot? Can you justify them?

1.2.3

There are two people on an island, one of which is a Knight and the other is a Knave.

(a) One person comes up and says: ”both of us are Knaves”. What can you conclude?

(b) What could you conclude if one of them were to say ”I am a liar”?

(c) If one of them were to claim that they were both Knights, what could you conclude?

1.3

1.3.1

Given 10 random integers between 1 and 100 (inclusive), can you always find two disjoint sets whose sums are equal?

1.3.2

How many times do you need to break an 8x5 chocolate bar to get 40 1x1 chocolate pieces?

1.3.3

Given a set of n integers, prove that you can always find a subset whose sum is divisible by n. For example, given the set $A = \{1,2,4\}$ the subset $\{2,4\}$ has a sum of 6, which is divisible by $n(A) = 3$

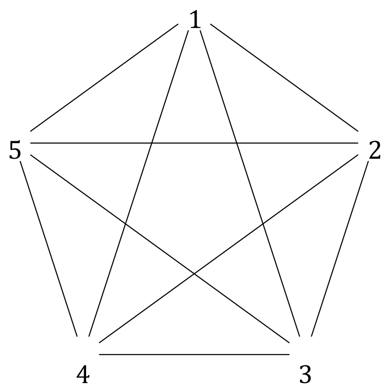
1.3.4

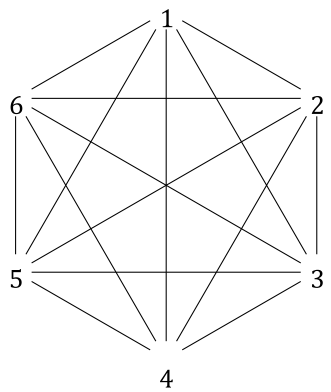
If you had a large supply of 2 types of eggs, each of which break when dropped from a certain floor. Find the most efficient method (in terms of the number of drops) to find out the number of floors each egg can be dropped from without breaking.

1.4

1.4.1

(a) Given a K_5 graph as shown below, colour the edges with the colours red and blue so that no monochromatic triangles are formed.





(b) Can this be done with a K_6 graph? Why or why not? Can you prove it?

(c) Given the following theorem:
$\forall n \geq 3\ \exists P$ s.t. $\forall p \in P$, K_p contains a subgraph, K_n, whose edges are all monochromatic.
Explain how you would extend this theorem to 3 colours - red, blue and green. Justify your answer.

(d) Prove the given theorem, by a method of your choice.

2 Mathematics

2.1

2.1.1

(a) Show that

$$\frac{1}{n+1} \le \sum_{r=1}^{n} \frac{1}{n^2+r} \le \frac{1}{n}$$

What happens as n → ∞?

(b) Show that

$$\frac{\sqrt{n}}{\sqrt{n+1}} \le \sum_{r=1}^{n} \frac{1}{\sqrt{n^2+r}} \le \frac{n}{\sqrt{n^2+1}}$$

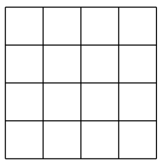
and find (c) Find

$$\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{\sqrt{n^2+r}}$$

$$\lim_{n\to\infty} (n!)^{\frac{2}{n}}$$

2.1.2

(a) Given below is a 4x4 grid. Can you tile the grid using L-shaped triominos if the top left corneris taken out?



(b) Can this be done on a 2^n *x* 2^n grid using L-shaped triominos if the top left corner is taken out?

(c) Can this be done on a 2^n x 2^n grid using L-shaped triominos if the top left *and bottom right* corners are taken out?

(d) Can this be done on a 2^n x 2^n grid using *a domino* if the top left and bottom right corners are taken out?

2.1.3

(a) Find the gradient of the line through the origin that is tangent to the curve $y = ln(x)$

(b) Find the minimum distance between the curve $y = ln(x)$ and the line $y = 2x$

2.1.4

Let the function $f: \mathrm{R} \to \mathrm{R}$ be defined as

$$f(a+b) = f(a)f(b)$$

(a) Show that $f(x) = 0\ \forall x$ *or* $f(x)$ 6= $0\ \forall x$

(b) Find a relationship between $f(m)$ and $f(1)$ where $m \in \mathrm{N}$

(c) Find a similar relationship between $f(\frac{k}{m})$ and $f(1)$ where $m,k \in \mathbb{N}$

(d) What can you say about $f(\frac{k}{m})$ if $m \in \mathbb{N}$ and $k \in \mathbb{Z}, k < 0$?

2.2

2.2.1

Let f be a function defined as

$$f(x,y) = \frac{1}{4}y^4 + y^3 + x^2y^2 + 2x^2 + 4x$$

Find the minimum of *f*.

2.2.2

Consider the following sequence

5,11,17,23,29

Make some conjectures about this sequence and prove as many as you can.

2.3

2.3.1

(a) Derive the double angle formulae for sin and cosine using rotational matrices.

(b) Describe the transformation $z \to (p + qi)z$, where $z \in \mathbb{C}$ as a matrix applied to a point in $\mathbb{R}^2$

2.3.2

Differentiate $ln(sin(e^{x^2}))$ with respect to x.

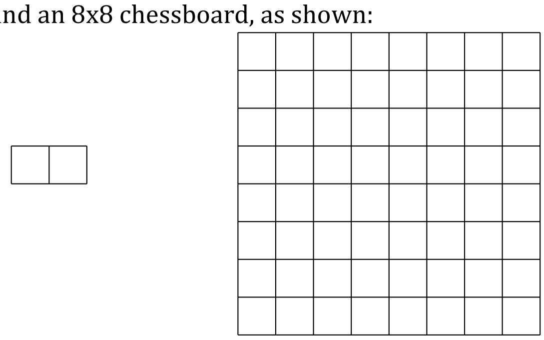
2.3.3

(a) Express $1.23\dot{0}\dot{3}$ as a fraction.

(b) does $\frac{p}{q}$ have a repeating decimal expansion if $p,q \in \mathbb{Z}$? If it does, why does it, and what does it look like?

2.3.4

Given set of 2x1 dominos and an 8x8 chessboard, as shown:



(a) is it possible to tile the chessboard with those dominos? What about any *m*x*n* board?

(b) what if the top-right and bottom-left corners are removed?

(c) If all the tiles on the chessboard were to be taken off, scrambled and put back on the boardunder the condition that each tile must touch the tiles it touched before it was scrambled (adjacent and diagonal). In how many ways can this be done?

2.3.5 Given that

$$I = \int_0^a \frac{sinx}{sinx + cosx} dx$$

and

$$J = \int_0^a \frac{cosx}{sinx + cosx} dx$$

Find expressions for *I* and *J* in terms of *a*.

2.4

2.4.1

Given that $g(n) = g(n + 10)$ and $g(n) = \frac{g(n+1) + g(n-1)}{2}$, describe $g(n)$.

2.4.2

Sketch $y = \arcsin \sqrt{1 - x}$

2.4.3

There are 50 ants on a table, with two bins on the sides. 20 blue ants travel along a straight line at a constant speed, *v* to the right. 30 red ants also travel along a straight line with the same speed, but towards the red ants, to the left. If there is nothing to stop an ant, it falls into the bin on the side of the table it is walking towards. How many ants, and of what colour fall into the left bin?

2.4.4

Let $f(x) = x + cos(x)$ and $g(x) = |x| + sin(x)$ and $x,y,z \in \mathbb{R}$. Prove or disprove the following statements:

(a) $g(x) \geq f(x) \ \forall x$

(b) $\exists x$ s.t. $g(x) \geq f(x)$

(c) $g(x) \geq f(x) \ \forall y \geq x$

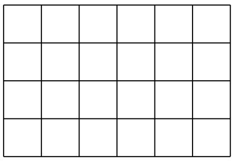
(d) $\forall x \geq y \ \exists y$ s.t. $g(x) \geq f(x)$

(e) $\forall y \ \exists x \geq y$ s.t. $g(x) \geq g(y)$

(f) $\exists z$ s.t. $\forall x,y \ x \geq y \geq z, g(x) \geq g(y)$

2.4.5

Consider a 6x4 grid as shown below, with 12 red tiles, 4 green tiles and 8 blue tiles. How many ways can the grid be tiled so that it has (at least 1) a line of symmetry.



2.5

2.5.1

Consider two circles: The first centred at the origin with radius R and the second centred at (1,0) with radius 1. A straight line, l is drawn through $(0,R)$ and the intersection of the two circles in the top-right quadrant. l intersects the x axis at p.

(a) Express p as a function of R.

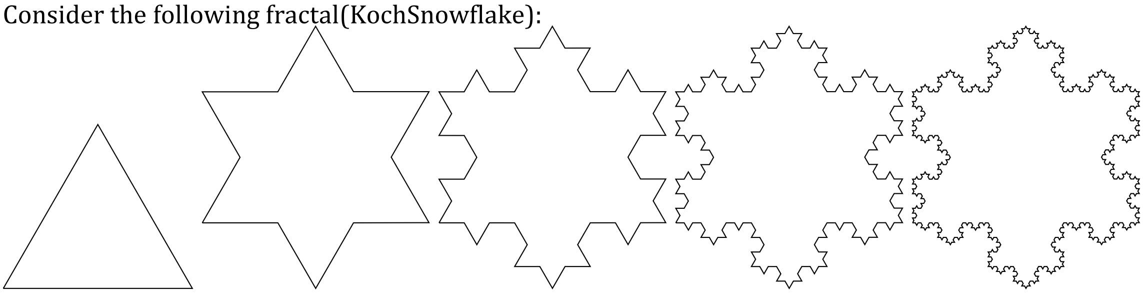
(b) Find the limit of p as $R \to 0$

2.5.2

Find the largest sphere of radius r that can fit in a cone of height H and radius R.

2.5.3

Consider the following fractal(KochSnowflake):



Let the area of the n^{th} iteration of the Koch Snowflake be equal to A_n, with $A_0 = a$

(a) Find a general expression for the area of the Koch Snowflake, A_n, in terms of a and find it's limit as $n \to \infty$

(b) Noting that A_n is finite, is the perimeter after the n^{th} iteration of the snowflake, P_n, bounded as $n \to \infty$?

2.6

2.6.1

Given two integers, $a,b \in \mathrm{N}$ with $a > b$,

(a) Prove that a can be written as $bq + r$ where $b,q \in \mathrm{N}$ with $r < b$

(b) Prove that $gcd(a,b) = gcd(b,r)$

(c) Find $gcd(120,54)$ using parts (a) and (b).

(d) Evaluate the continued fraction

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ldots}}}$$

(e) Find the general continued fraction for x, where $x^2 - \alpha x - \beta = 0$ ($\alpha, \beta \in \mathbb{R}$).

2.7

2.7.1

Prove that the only finite polynomial, $f(x)$ s.t. $f^0(x) = f(x)$ is $f(x) = 0$

2.7.2

Consider the function $f(x) = x^n + 1$ where $n \in \mathbb{N}$

(a) Sketch $f(x)$ where n is even.

(b) Sketch $f(x)$ where n is odd.

(c) Where n is odd, factorise $f(x)$.

(d) For what values of n is $f(2)$ *not* prime?