Sequences

TMUA / MAT Syllabus

*N*th term and recurrence relations; arithmetic series including S_n; geometric series including S_n and S_∞; binomial expansion of $(1+x)^n$; n! and ^nC_r

Arithmetic Sequences $\quad u_n = a + (n-1)d \quad$ *a* is the first term, *d* is the common difference

1. For each sequence, write down the first four terms, and find *a* and *d*.

a) $u_n = 5n + 3$

$8, 13, 18, 23$ $\quad a = 8$, $d = 5$

b) $u_n = 7 - 3n$

$4, 1, -2, -5$ $\quad a = 4$, $d = -3$

c) $u_n = 6 + \frac{1}{2}n$

$6\frac{1}{2}, 7, 7\frac{1}{2}, 8$ $\quad a = 6$, $d = \frac{1}{2}$

d) $u_n = 2 - n$

$1, 0, -1, -2$ $\quad a = 1$, $d = -1$

2. For each sequence, find the nth term, and the 10th term

a) 5, 8, 11, 14, …

$u_n = 3n + 2 \quad u_{10} = 32$

b) -1, 3, 7, 11, …

$u_n = 4n - 5 \quad u_{10} = 35$

c) 25, 18, 11, 4, …

$u_n = -7n + 32 \quad u_{10} = -38$

d) -2, -7, -12, -17, …

$u_n = -5n + 3 \quad u_{10} = -47$

3.

a) The first term of an arithmetic sequence is 12. The fourth term is 30. find the common difference.

$a = 12 \quad a + 3d = 30 \quad 3d = 30 - 12 = 18 \quad d = 6$

b) $u_3 = 30$ and $u_6 = 9$. Find the first negative term in the sequence.

$a + 2d = 30$
$a + 5d = 9$

$3d = -21$
$d = -7$

$a = 44$
$u_n = 51 - 7n$

$51 - 7n < 0$
$7n > 51$

$n > \frac{51}{7}$
$n = 8$

$u_8 = -5$

c) The 7th term and twelfth term of an arithmetic sequence are 28 and 73. Find the first term and common difference.

$u_7 = a + 6d = 28$
$u_{12} = a + 11d = 73$

$5d = 45$
$d = 9$

$a + 54 = 28$
$a = -26$

d) The first three terms of an arithmetic sequence are 5p, 20, 3p where p is a constant. Find the 20th term in the sequence

$20 - 5p = 3p - 20$
$40 = 8p$
$p = 5$

or $a = 5p$
$a + d = 20$
$a + 2d = 3p$

$a = 25$
$d = -5$

$u_{20} = 25 + 19(-5)$
$= -70$

$u_n = -5n$

e) The first three terms of an arithmetic sequence are -8, k^2, $17k$. Find two possible values of *k*.

$k^2 + 8 = 17k - k^2$
$2k^2 - 17k + 8 = 0$
$(2k - 1)(k - 8) = 0$

$k = \frac{1}{2}, 8$

f) The seventh term of an arithmetic series is 6 . The sum of its fifth term and its tenth term is 16. Find the first term and the common difference of the series.

$a + 6d = 6$
$a + 4d + a + 9d = 16$
$2a + 13d = 16$
$2a + 12d = 12$

$d = 4$
$a = -18$

Arithmetic Series $\qquad$ a is the first term, l is the last term, d is the common difference

$$S_n = \frac{n}{2}(2a + (n-1)d) \qquad S_n = \frac{n}{2}(a + l)$$

4. Find the sum of the following series:

a) $\quad 2 + 6 + 10 + 14 + \ldots$ (15 terms) $\quad S_{15} = \frac{15}{2}\left(4 + 14(4)\right) = \frac{15}{2} \times 60 = 450$

$a = 2,\ d = 4,\ n = 15$

b) $\quad 5 + 1 + -3 + -7 + \ldots$ (20 terms) $\quad S_{20} = \frac{20}{2}\left(10 + 19(-4)\right) = 10(10 - 76) = -660$

$a = 5 \quad d = -4 \quad n = 20$

c) $\quad 5 + 7 + 9 + \ldots + 75 \quad 5 + (n-1)2 = 75 \quad S_{36} = 18(5 + 75) = 1440$

$a = 5 \quad l = 75 \quad d = 2 \qquad n = 36$

5. Find how many terms are needed to make the given sums:

a) $\quad 5 + 6 + 7 + 8 + \ldots = 290 \quad \frac{n}{2}(10 + n - 1) = 290 \quad n^2 + 9n - 580 = 0$

$a = 5 \quad d = 1 \qquad (n - 20)(n + 29) \quad n = 20$

b) $\quad 64 + 62 + 60 + \ldots = 0 \quad \frac{n}{2}\left(128 - 2(n-1)\right) = 0 \quad 128 - 2n + 2 = 0$

$a = 64 \quad d = -2 \qquad n = 65$

6.

a) Find the sum of the first 50 positive even numbers. $\quad S_{50} = 25\left(4 + 49(2)\right)$

$a = 2 \quad d = 2 \quad n = 50 \qquad = 25 \times 102 = 2550$

b) Prove that the sum of the first n odd numbers is n^2

$a = 1 \quad d = 2 \quad S_n = \frac{n}{2}\left(2 + 2(n-1)\right) = \frac{n}{2}(2n)$

$= \frac{n}{2}(2 + 2n - 2) \qquad = n^2$

c) The 4th term of an arithmetic series is 15. The sum of its first three terms is 9.

Find the first term and common difference of the series.

$u_4 = a + 3d = 15 \quad 2a = -6 \quad d = 6$

$S_3 = 3a + 3d = 9 \quad a = -3$

d) The nth term of an arithmetic series is given by $u_n = 6n + 11$

Find the sum of the first twenty terms of the series

$a = 17 \quad S_{20} = 10\left(34 + 19(6)\right) = 1480$

$d = 6 \qquad = 10(34 + 114)$

e) The sixteenth term of an arithmetic series is 6. The sum of the first sixteen terms is 456.

The sum of the first k terms of the series is zero. Determine the value of k .

$a + 15d = 6 \qquad \frac{n}{2}\left(102 - 3(n-1)\right) = 0$

$S_{16} = 8(2a + 15d) = 456 \qquad 102 - 3n + 3 = 0$

$2a + 15d = 57 \qquad 3n = 105$

$a = 51 \quad d = -3 \qquad n = 35$

Geometric Sequences $u_n = ar^{n-1}$ a is the first term, r is the common ratio

7. Find the sixth and nth terms of the following geometric sequences:

a) 2, 6, 18, 54, … 162, 486 $u_6 = 486$ $u_n = 2(3)^{n-1}$

$a=2$ $r=3$

b) 1, -2, 4, -8, … 16, -32 $u_6 = -32$ $u_n = (-2)^{n-1}$

$a=1$ $r=-2$

c) 100, 50, 25, 12.5, … 6.25 3.125 $u_6 = 3.125$ $u_n = 100\left(\frac{1}{2}\right)^{n-1} = \frac{100}{(2)^{n-1}}$

$a=100$ $r=\frac{1}{2}$

8.

a) The first and second terms of a geometric sequence are 90 and 30 respectively. Find the common ratio and the 5th term.

$a = 90$
$r = \frac{1}{3}$

$u_5 = 90\left(\frac{1}{3}\right)^4 = \frac{90}{81} = \frac{10}{9}$

b) The third and fourth terms of a geometric sequence are 36 and 27 respectively. Find the common ratio and the 5th term.

$ar^2 = 36$
$ar^3 = 27$

$r = \frac{27}{36} = \frac{3}{4}$ $a = 36 \times \frac{4^2}{3^2} = 64$ $u_5 = 64\left(\frac{3}{4}\right)^4 = \frac{64 \times 81}{64 \times 4} = \frac{81}{4}$

c) The second and fifth terms of a geometric sequence are 12 and -1.5 respectively. Find the common ratio and the first term.

$ar = 12$
$ar^4 = -1.5$

$r^3 = -\frac{1.5}{12} = -\frac{1}{8}$
$r = -\frac{1}{2}$ ✓

$a\left(-\frac{1}{2}\right) = 12$
$a = -24$ ✓

d) The numbers 3, x and $(x + 6)$ are the first three terms of a geometric sequence with all positive terms. Find the possible values of x.

$a=3$
$x = 3r$
$x+6 = 3r^2$

$\frac{x}{3} = \frac{x+6}{x}$

$x^2 = 3x + 18$
$x^2 - 3x - 18 = 0$
$(x-6)(x+3) = 0$

$x = \underline{\underline{6}}, \cancel{-3}$

e) The first three terms of a geometric sequence are given by 8-a, 2a, and a^2 where $a > 0$. Find the value of a, and hence the sixth term.

$\frac{2a}{8-a} = \frac{a^2}{2a}$

$4a^2 = 8a^2 - a^3$
$a^3 - 4a^2 = 0$

$a = \cancel{0}, 4$
$a = 4$

$u_6 = 4(2)^5 = 128$

f) A population of ants is growing at a rate of 10% a year. If there were 200 ants in the initial population, how many ants will there be after 3 years.

$a = 200$ $r = 1.1$

$u_3 = 200 \times 1.1 \times 1.1$
$= 220 \times 1.1$
$= 242$

Geometric Series $S_n = \dfrac{a(1-r^n)}{1-r} = \dfrac{a(r^n-1)}{r-1}$ a is the first term, r is the common ratio

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$ (series is convergent)

9. Find the sum of the following series:

a) 1, 2, 4, 8, … (8 terms)

$a=1 \quad r=2 \quad n=8$

$S_8 = \dfrac{1(2^8-1)}{1} = 255$

b) 30, 15, 7.5, … (5 terms)

$a=30 \quad r=\frac{1}{2} \quad n=5$

$S_5 = \dfrac{30(1-(\frac{1}{2})^5)}{1/2} = 60\left(\dfrac{32-1}{32}\right) = \dfrac{15}{8} \times 31 = \dfrac{465}{8}$

c) 4, -4, 4, -4, … (25 terms)

$\underbrace{4-4+4-4+\ldots}_{0} + 4 = 4$

10. Work out whether the following series are convergent, and if so, find the sum to infinity:

a) 1 + 0.1 + 0.01 + 0.001 + …

$r=0.1$ convergent

$S_\infty = \dfrac{1}{1-0.1} = \dfrac{1}{0.9} = \dfrac{10}{9}$

b) 0.4 + 0.8 + 1.6 + 3.2 + …

NO

c) 16 + 8 + 4 + 2 + …

$r=\frac{1}{2}$ convergent

$S_\infty = \dfrac{16}{(1-\frac{1}{2})} = 32$

11.

a) A geometric series has common ratio $\dfrac{1}{3}$. Find the first term, given that the sum of the first four terms is 36.

$r=\frac{1}{3}$

$S_4 = \dfrac{a(1-(\frac{1}{3})^4)}{2/3} = 36$

$a\left(1-\frac{1}{81}\right) = 24$

$\frac{80}{81}a = 24$

$a = \dfrac{81 \times 24}{80} = \dfrac{243}{10}$

b) Find the least value of n such that the series with first term 5 and common ratio $\dfrac{1}{2}$ exceeds 9.

$S_n = \dfrac{5(1-(\frac{1}{2})^n)}{1-\frac{1}{2}} > 9$

$10\left(1-(\frac{1}{2})^n\right) > 9$

$1 - \frac{1}{2^n} > \frac{9}{10}$

$\frac{1}{2^n} < \frac{1}{10}$

$n = 4$

c) The first and second term of a geometric series are 90 and 15. Find the sum to infinity.

$a = 90$

$r = \frac{1}{6}$

$S_\infty = \dfrac{90}{5/6} = 18 \times 6 = 108$

d) The common ratio of a geometric series is twice as large as its first term. Given that the sum to infinity is 1, find the exact value of the fifth term of the series.

$r = 2a$

$S_\infty = \dfrac{a}{1-r} = 1$

$a = 1 - r$

$a = 1 - 2a$

$a = \frac{1}{3}$

$r = \frac{2}{3}$

$u_5 = \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^4 = \dfrac{16}{243}$

Sigma Notation

$\sum_{n=1}^{5}(2n-1)$ this is the sum of ($2n$ - 1) with the values of n substituted from 1 to 5 (=1+3+5+7+9)

12. Calculate

a) $\sum_{n=1}^{4}(3n+1)$

$= 4+7+10+13 = 34$

e) $\sum_{r=1}^{4}(r^2+1)$

$= 2+5+10+17 = 34$

b) $\sum_{n=4}^{6}(2n+5)$

$= 13+15+17 = 45$

f) $\sum_{r=1}^{20}1 = 20$

c) $\sum_{r=1}^{20}(13r+4)$ 17, 30, ...

$a=17 \quad d=13 \quad l=264$

$= 10(264+17) = 2810$

g) $\sum_{k=10}^{30}(4k+11)$ 51, 55 ... 131

$= \frac{21}{2}(51+131) = 21\times 91 = \frac{1820}{\underline{91}} = 1911$

d) $\sum_{n=1}^{50} sin(90n)$

$= \underbrace{1+0-1+0}_{=0} + \dots = 1+0 = 1$

h) $\sum_{r=1}^{5}(2(3^{r-1}))$ 2, 6, 18, 54, 162

$= \frac{2(3^5-1)}{2} = 242$

13. Show that

a) $\sum_{r=1}^{n}(2r) = n+n^2$

2, 4, 6 ...

$a=2$

$d=2$

$S_n = \frac{n}{2}(4+2(n-1))$

$= \frac{n}{2}(2n+2)$

$= n(n+1) = n+n^2$

or $S_n = \frac{n}{2}(2+2n)$

$= n+n^2$

b) $\sum_{k=1}^{n}(\frac{k+5}{3}) = \frac{1}{6}n(n+11)$

$a=2$

$d=1/3$

$2, \frac{7}{3}, \frac{8}{3} \dots$

or $S_n = \frac{n}{2}\left(2+\frac{n+5}{3}\right)$

$= \frac{n}{6}(6+n+5)$

$S_n = \frac{n}{2}\left(4+\frac{1}{3}(n-1)\right) = \frac{n}{2}\left(\frac{n}{3}+\frac{11}{3}\right)$

$= \frac{1}{6}n(n+11)$

c) $\sum_{k=1}^{n}(100-2k) = 99n-n^2$ 98, 96 ...

$a=98$

$d=-2$

or $S_n = \frac{n}{2}(98+100-2n)$

$= 99n-n^2$

$S_n = \frac{n}{2}(196-2(n-1))$

$= \frac{n}{2}(198-2n)$

$= n(99-n)$

$= 99n-n^2$

Recurrence Relations

$u_{n+1} = f(u_n)$ for example $u_{n+1} = 2u_n + 1$ and $u_1 = 3$ give the sequence 3, 7, 15, 31, …

14. Find the first four terms of the following sequences:

a) $u_{n+1} = u_n - 5$ and $u_1 = 16$

16 11 6 1

b) $u_{n+1} = 3u_n + 2$ and $u_1 = 1$

1 5 17 53

c) $u_{n+1} = (u_n)^2 - 1$ and $u_1 = 2$

2 3 8 63

d) $u_{n+1} = 3u_n$ and $u_1 = 1$

1 3 9 27

15.

a) Given $u_n = u_{n-1} + n$ and $u_0 = 4$, find an expression for u_n in terms of n

$u_1 = 4 + 1 = 5$
$u_2 = (4+1) + 2 = 7$
$u_3 = (4+1+2) + 3 = 10$

$u_n = 4 + \sum_{r=1}^{n} r = 4 + \frac{n(n+1)}{2} = \frac{8+n^2+n}{2}$

b) If $u_{n+1} = (3 - u_n)^2$ and $u_1 = 4$, find u_{10}

$u_2 = 1$
$u_3 = 4 \Rightarrow 4, 1, 4, 1 \ldots$

$u_{10} = 1$

c) If $a_{n+1} = 3a_n + 4$ and $a_1 = k$ and $\sum_{r=1}^{4} a_r = 32$ find k.

$a_2 = 3k + 4$
$a_3 = 9k + 12 + 4 = 9k + 16$
$a_4 = 27k + 52$

$40k + 72 = 32$
$k = -1$

d) A recurrence relation is defined by $a_{n+1} = 7a_n - n^3 - 3$ and $a_1 = 1$

Find $\sum_{r=1}^{5} a_r$

$a_2 = 7 - 1 - 3 = 3$
$a_3 = 21 - 8 - 3 = 10$
$a_4 = 70 - 27 - 3 = 40$

$a_5 = 280 - 64 - 3 = 213$

$\sum_{r=1}^{5} a_r = 1 + 3 + 10 + 40 + 213$
$= 267$

e) A sequence of numbers is given by the recurrence relation $u_{n+1} = \frac{1}{1 - u_n}$ $u_1 = 2$

Find u_{12} and show that $\sum_{r=1}^{12} u_r = 6$

$\sum_{r=1}^{12} u_r = 4\left(2 - 1 + \frac{1}{2}\right)$
$= 4\left(\frac{3}{2}\right) = 6$

$u_1 = 2$
$u_2 = -1$

$u_3 = \frac{1}{2}$
$u_4 = 2$

f) A sequence of numbers is given by the recurrence relation $u_{n+1} = 5 - \frac{18}{4 + u_n}$ $u_2 = 0$

Find u_3, u_4, and u_1

$u_3 = 5 - \frac{18}{4} = \frac{1}{2}$

$u_4 = 5 - \frac{18}{9/2} = 1$

$0 = 5 - \frac{18}{4+u_1}$

$4 + u_1 = \frac{18}{5}$

$u_1 = -\frac{2}{5}$

g) A sequence of numbers is given by the recurrence relation $u_{n+1} = \dfrac{k - 5u_n}{u_n}$ $u_1 = 1$

If $u_3 > 6$, find the range of values of k.

$u_2 = k - 5$

$u_3 = \dfrac{k - 5k + 25}{k - 5} > 6$

$(25 - 4k)(k-5) > 6(k^2 - 10k + 25)$

$25k - 4k^2 + 20k - 125 >$

$6k^2 - 60k + 150$

$0 > 10k^2 - 105k + 275$

$2k^2 - 21k + 55 < 0$

$(2k - 11)(k - 5) < 0$

$k = 5, 5.5$

$5 < k < 5.5$

h) A recurrence relation is defined for $n \geq 1$ by $a_{n+1} = k + \dfrac{1}{2}a_n$

If $a_1 = 520$ and $a_4 = 72$ find the value of k.

$a_2 = k + 260$

$a_3 = k + \frac{1}{2}k + 130$

$a_4 = k + \frac{1}{2}k + \frac{1}{4}k + 65 = 72$

$\frac{7}{4}k = 7$

$k = 4$

i) A recurrence relation is defined for $n \geq 1$ by $u_{n+1} = k + (-1)^n u_n$ $u_1 = 4$

Show that $u_5 = 4$ and given that $\sum_{r=1}^{4} u_r = 6$ find the value of k.

$u_2 = k - 4$

$u_3 = k + k - 4 = 2k - 4$

$u_4 = k - 2k + 4 = 4 - k$

$u_5 = k + 4 - k = 4$

$4 + k - 4 + 2k - 4 + 4 - k = 6$

$2k = 6$ $k = 3$

j) A sequence of numbers is given by the recurrence relation $u_{n+1} = k - \dfrac{12}{u_n}$ $u_1 = 1$

If $4u_2 = u_3 + 1$, find the possible values of k.

$u_2 = k - 12$

$u_3 = k - \dfrac{12}{k-12}$

$4k - 48 = k - \dfrac{12}{k-12} + 1$

$3k - 49 = \dfrac{-12}{k-12}$

$3k^2 - 36k - 49k + 588 + 12 = 0$

$3k^2 - 85k + 600 = 0$

$(3k - 40)(k - 15)$

$k = \frac{40}{3}, 15$

$\begin{array}{r} 49 \\ 12 \\ \hline 98 \\ 490 \\ \hline 588 \end{array}$

k) A sequence of numbers is defined by the recurrence relation $u_{n+1} = ku_n + 6$ $u_1 = 4$

If $u_3 = 10$, find the possible values of k.

$k = -2$
$4, -2, 10, -14, 34$
$k = \frac{1}{2}$
$4, 8, 10, 11, 11.5$

$u_2 = 4k + 6$

$u_3 = k(4k + 6) + 6 = 10$

$4k^2 + 6k - 4 = 0$

$2k^2 + 3k - 2 = 0$

$(2k - 1)(k + 2) = 0$

$k = \frac{1}{2}, -2$

Given that the sequence converges to a limit L, determine the value of L. $k = \frac{1}{2}$

$L = \frac{1}{2}L + 6$ $\frac{1}{2}L = 6$ $L = 12$

l) A recurrence relation is defined for $n \geq 1$ by $a_{n+1} = \sqrt{a_n + 12}$ $a_1 = k$

Given that the sequence converges to a limit L, determine the value of L.

$L = \sqrt{L + 12}$

$L^2 = L + 12$

$L^2 - L - 12 = 0$

$(L - 4)(L + 3) = 0$

$L = 4, -3$

Binomial Expansion

$$(1+x)^n = 1 + nx + \frac{1}{2}n(n-1)x^2 + \ldots + \frac{1}{r!}n(n-1)\ldots(n-r+1)x^r \quad = \sum_{r=0}^{n}\binom{n}{r}x^r$$

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \quad = \sum_{r=0}^{n}\binom{n}{r}a^{n-r}b^r$$

16.

a) Find the coefficient of x^3 in the series expansion of $(1-5x)^4$

$\binom{4}{3}(-5x)^3 = 4 \times (-125)x^3 = -500x^3$

b) Find the coefficient of x^6 in the series expansion of $(1+3x)^8$

$\binom{8}{6}(3x)^6 \qquad \frac{8\times 7}{2} \times 3^6$

c) Find the coefficient of x^2 in the expansion of $(1+2x)^2(1-3x+3x^2+4x^3)$

$(1+4x+4x^2)(1-3x+3x^2+4x^3)$

$x^2: 3-12+4 = -5$

d) In the binomial expansion of $(1+kx)^6$, the coefficient of x^3 is twice as large as the coefficient of x^2. Find the value of k.

$\binom{6}{3}k^3 = 2\binom{6}{2}k^2 \qquad k = 3/2$

$20k^3 = 30k^2$

e) It is given that $(1-2x)(2+kx)^5 = A + Bx + 240x^2 + \ldots$ where k, A, and B are constants.

Find the possible values of k.

$(1-2x)(2+kx)^5 = (1-2x)(2^5 + 5(2^4)(kx) + 10(2^3)(kx)^2 + \ldots)$

$A=32 \qquad B = -64+80k \qquad 240 = -160k + 80k^2$

$k^2 - 2k - 3 = 0 \qquad (k-3)(k+1) = 0 \qquad k = 3, -1$

f) Given that k is a non zero constant and n is a positive integer, then

$$(1+kx)^n = 1 + 40x + 120k^2x^2 + \ldots$$

Find the value of k and the value of n

$1 + nkx + \frac{n(n-1)}{2}k^2x^2 + \ldots$

$240 = 40\times 6$
$= 20\times 12$
$= 15\times 16$

$nk = 40$

$\frac{n(n-1)}{2} = 120 \qquad n(n-1) = 240 \qquad n = 16 \qquad k = \frac{5}{2}$

4 6 4
5 10 10 5
6 15 20
7 21 35

Trinomial Expansion

Coefficient of $a^pb^qc^r$ in $(a+b+c)^n$ where $n=p+q+r$ is $\dfrac{n!}{p!q!r!}$

17.

(i) Find the coefficients of the following terms of the series expansion of $(1+x+y^2)^7$

7 a) x^6

21 b) y^4

105 c) x^4y^2

42 d) xy^{10}

$u=1+x$ $\quad (u+y^2)^7 = u^7+7u^6y^2+21u^5y^4+35u^4y^6+35u^3y^8+21u^2y^{10}+7uy^{12}+y^{14}$

$x^6: (1+x)^7$ $\quad y^4: 21u^5=21$ $\quad x^4y^2: 7u^6y^2=7\times15=105$ $\quad xy^{10}: 21u^2=2\times21$

OR

a) $(1)^1(x)^6(y^2)^0$ $\quad \frac{7!}{1!6!0!}=7$

b) $(1)^5(x)^0(y^2)^2$ $\quad \frac{7!}{5!2!0!}=21$

c) $(1)^2(x)^4(y^2)^1$ $\quad \frac{7!}{2!4!1!}=\frac{7\times6\times5}{2}=105$

d) $(1)^1(x)^1(y^2)^5$ $\quad \frac{7!}{1!1!5!}=42$

check

(ii) Find the coefficients of the following terms of the series expansion of $(1+2x-3y^2)^4$

32 a) x^3

-12 b) y^2

216 c) xy^4

-144 d) x^2y^2

$=(1+2x)^4+4(1+2x)^3(-3y^2)+6(1+2x)^2(-3y^2)^2+4(1+2x)(-3y^2)^3+(-3y^2)^4$

a) $x^3: \binom{4}{3}(2x)^3=32$ $\quad$ b) $y^2: 4(-3)=-12$ $\quad$ c) $xy^4: 6(4)(9)=216$ $\quad$ d) $4(12)(-3)=-144$

OR

$(1)^1(2x)^3(-3y^2)^0$ $\quad 2^3\times\frac{4!}{3!}=32$

$(1)^3(2x)^0(-3y^2)^1$ $\quad (-3)\times\frac{4!}{3!}=-12$

$(1)^1(2x)^1(-3y^2)^2$ $\quad 2\times9\times\frac{4!}{2!}=216$

$(1)^1(2x)^2(-3y^2)^1$ $\quad 2^2\times(-3)\times\frac{4!}{2!}=-144$

(iii) Find the coefficients of the following terms of the series expansion of $(1+2x+3x^2)^5$

10 a) x

55 b) x^2

810 c) x^9

530 d) x^4

a) $x: (1)^4(2x)^1(3x^2)^0$ $\quad 2\times\frac{5!}{4!}=10$

b) $x^2: (1)^4(2x)^0(3x^2)^1$ $\quad 3\times\frac{5!}{4!}=15$

$(1)^3(2x)^2(3x^2)^0$ $\quad 2^2\times\frac{5!}{3!2!}=40$

$15+40=55$

c) $x^9: (1)^0(2x)^1(3x^2)^4$ $\quad 2\times3^4\times\frac{5!}{4!}=810$

$(1)(2x)^3(3x^2)^3$ ×

d) $x^4: (1)^3(2x)^0(3x^2)^2$ $\quad 3^2\times\frac{5!}{3!2!}=90$

$(1)^2(2x)^2(3x^2)^1$ $\quad 2^2\times3\times\frac{6!}{2!2!}=360$

$(1)^1(2x)^4(3x^2)^0$ $\quad 2^4\times\frac{5!}{4!}=80$

$90+360+80=530$

(iv) Find the coefficients of the following terms of the series expansion of $(1+2x-x^2)^6$

54 a) x^2

100 b) x^3

15 c) x^8

-100 d) x^9

a) $(1)^5(2x)^0(-x^2)^1$ $\quad (-1)\frac{6!}{5!}=-6$

$(1)^4(2x)^2(-x^2)^0$ $\quad 2^2\times\frac{6!}{4!2!}=60$

$60-6=54$

b) $(1)^4(2x)^1(-x^2)^1$ $\quad 2(-1)\times\frac{6!}{4!}=-60$

$(1)^3(2x)^3(-x^2)^0$ $\quad 2^3\times\frac{6!}{3!3!}\overset{20\times8}{=}160$

$160-60=100$

c) $(1)^2(2x)^0(-x^2)^4$ $\quad \frac{6!}{2!4!}=15$

$(1)^1(2x)^2(-x^2)^3$ $\quad 2^2\times(-1)^3\times\frac{6!}{2!3!}=-240$

$(1)^0(2x)^4(-x^2)^2$ $\quad 2^4\times\frac{6!}{4!2!}=240$

$15-240+240=15$

d) $(1)^1(2x)^1(-x^2)^4$ $\quad 2\times\frac{6!}{4!}=60$

$(1)^0(2x)^3(-x^2)^3$ $\quad 2^3\times(-1)^3\times\frac{6!}{3!3!}=-160$

$(1)(2x)^5(-x^2)^2$ ×

$60-160=-100$