TMUA Logs and Exponentials

Syllabus

Graph of a^x for positive values of a; laws of logarithms; solving log equations and simultaneous equations.

1. Simplify the following expressions, giving the answer as a single logarithm:

a) $log_3 7 + log_3 5$

b) $log_5 24 - log_5 6$

c) $3log_5 2 + log_5 6$

d) $1 + 2log_n 3 + log_n 4$

e) $2log_4 3 + log_4 5 - log_4 0.5$

2. Simplify the following expressions, giving the answer as a single number:

a) $log_8 25 + log_8 10 - 3log_8 5$

b) $log_6 4 + log_6 9$

c) $log_2 5 + log_2 1.6$

d) $log_2(\frac{5}{2}) + log_2(\frac{4}{3}) - log_2(\frac{5}{3})$

e) $\frac{1}{3}log_{1.5}(\frac{8}{27}) + \frac{1}{2}log_{1.5}(\frac{4}{9})$

f) $log_a(a^2) - 4log_a(\frac{1}{a})$

3. Solve the following equations leaving your answer in terms of logarithms base 10 (*log*):

a) $2^x = 3^{x+1}$

b) $3^{y-1} = 2^{2y}$

c) $2^{x+3} = 6^{x-1}$

d) $8^{4-3y} = 7^y$

e) $2^{2x} - 2^x - 6 = 0$

f) $4^y - 3(2^y) - 10 = 0$

g) $3^{2y+1} - 11(3^y) - 4 = 0$

h) $x = 8^{log_2 x} - 9^{log_3 x} - 4^{log_2 x} + log_{0.5} 0.25$

4. Solve the following equations.

a) $log_2(x+1) - log_2 x = log_2 3$

b) $log_a y = log_a 3 + log_a(2y-1)$

c) $log_5(4w+3) - log_5(w-1) = 2$

d) $log_3(4x+1) - log_3(x-1) = 2$

e) $log_2(3y+4) - log_2 y = 3$

f) $log_2(4z+4) = 6$

g) $log_2(x^2+4x+3) = 4 + log_2(x^2+x)$

5. Find the difference between the solutions of the following equations:

a) $2^{2x} - 8.2^{x} + 15 = 0$

b) $4^{2x} + 12 = 2^{2x+3}$

c) $3^{x} - (\sqrt{3})^{x+4} + 20 = 0$

d) $2log_a x = log_a 18 + log_a(x - 4)$

e) $2log_a y - log_a(5y - 24) = log_a 4$

6. Given that $y = log_2 x$ write each expression in terms of y

a) $log_2 x^4$

b) $log_2(8x^2)$

c) $log_4 x$

d) $log_2(\frac{1}{2}x)$

7. Given that $p = log_a 4$ and $q = log_a 5$ write each expression in terms of p and q

a) $log_a 100$

b) $log_a 0.4$

c) $log_a 3.2$

d) $log_a 80a^2$

8. Rearrange the equation to make x the subject.

a) $y = -\frac{1}{2}log_{10}(10 - x)$

b) $y = a^x b^{2x} c^{3x}$

c) $y = log_3 8 - 3log_3 x$

d) $2 + log_a b + 3log_a x = 2log_a(a^2 x)$

9. Which is the largest of the following:

a) $log_2 4$ $log_4 2$ $log_3 5$ $log_8 2$

b) $log_2 3$ $log_4 8$ $log_3 2$ $log_5 10$

Which is the smallest of the following:

c) $log_{10}\pi$ $\sqrt{log_{10}\pi}$ $\left(\frac{1}{log_{10}\pi}\right)^3$ $\frac{1}{log_{10}\sqrt{\pi}}$

10. Solve the following simultaneous equations

a) $log_3(xy^2) = 1$ $(log_3 x)(log_3 y) = -3$

b) $2^x + 3(2^y) = 3$ $2^{2x} - 9(2^{2y}) = 6$

c) $2^{3x} = 8^{y+3}$ $4^{x+1} = \frac{16^{y+1}}{8^{y+3}}$

d) $log_y x = 3$ $log_3 x = 1 + log_3 y$

11. Find the solution of the following equations:

a) $log_\pi(log_2(log_7 x)) = 0$

b) $log_{99}(log_2(log_3 x)) = 0$

c) $log_a x = log_{a^2}(x + 20)$

12. The numbers a, b and c are each greater than or equal to 1.

a) The logarithms below are all to the same base. What is the base?

$log(ab^2c) = 6$

$log(a^2bc^4) = 9$

$log(a^5b^7c^5) = 25$

b) The logarithms below are all to the same base. What is the base?

$log(a^2b^3c^5) = 21$

$log(a^3b^6c^{15}) = 51$

$log(a^5b^4c^{10}) = 37$

c) The logarithms below are all to the same base. What is the base?

$log(\frac{ac^3}{b}) = 5$

$log(abc^6) = 12$

$log(\frac{a^3c^2}{b^2}) = 10$