20 YEARS OXBRIDGE APPLICATIONS

TEST OF MATHEMATICS FOR UNIVERSITY ADMISSION (TMUA)

MOCK EXAM 2

SECTION 1
MATHEMATICAL KNOWLEDGE

1 $f(x)$ is a quadratic function in x.

The graph of $y = f(x)$ has y-intercept $(0, -1)$ and its turning point is located at $(-1, -2)$.

Which of the following is an expression for $f(x)$?

A $2x^2 + x - 1$

B $x^2 + 2x - 1$

C $-x^2 + 4x + 1$

D $4x^2 - 2x + 1$

E $-2x^2 + 2x - 1$

2 The sequence x_n is given by

$$x_1 = 256$$

$$x_{n+1} = \frac{x_n}{2} \text{ for } n \geq 1$$

What is the value of x_{16}?

A 1

B 512

C $\frac{1}{2}$

D $\frac{1}{128}$

E $\frac{1}{256}$

3 Find the value of

$$\int_1^4 \frac{2x^2 - \sqrt{x}}{x\sqrt{x}} dx$$

A 0

B $2 \ln 2$

C $\frac{28}{3}$

D $\frac{28}{3} + 2 \ln 2$

E $\frac{28}{3} - 2 \ln 2$

4 Consider the following simultaneous equations:

$$\begin{aligned} x^2 + 2x - 1 - y^2 &= 4 \\ x + y &= a \end{aligned}$$

For which value(s) of a do the equations have exactly one real solution for x?

A All real values.

B All real values except -1.

C All real values except ± 2.

D All real values except -1 and ± 2.

E No values of a.

5 Find the number of solutions of the following equation, for $0 \leq x \leq 3\pi$:

$$x \sin 2x = 0$$

A 0

B 2

C 3

D 5

E 7

6 For which values of x does the following inequality hold?

$$2 - \frac{3x^2 + 2}{x + 1} < \frac{1}{2x}$$

A $x < 1$ and $x > 0$.

B $x < -\frac{1}{3}$

C $x > 0$

D $x < -1$ and $-\frac{1}{3} < x < 0$

E $-1 < x < -\frac{1}{3}$ and $x > 0$

7 Find the number of real roots of the equation

$$x^3 - x^2 - x - 2 = 0$$

A 0

B 1

C 2

D 3

E 4

8 Shop A always sells books at 80% the price of books in Shop B. In their summer sale, Shop B reduces the price of a book by 30%. The price of the book in Shop A is now reduced by £3.

What is the original price of the book in Shop B?

A £8.00

B £10.00

C £10.50

D £12.50

E £13.00

9 A box contains four different colours of pencils. There are n of each colour in the box. A pencil is chosen at random and is not replaced. A second pencil is then selected at random. Each individual pencil is equally likely to be chosen.

What is the probability that the two pencils are not of the same colour?

A $\frac{3}{4}$

B $\frac{3n}{4n-1}$

C $\frac{n-1}{4n-1}$

D $\frac{3n}{4(4n-1)}$

E $\frac{3n-1}{4n-1}$

10 Which of the following numbers is the largest?

A $e^{\log_\pi(2)}$

B $2^{\log_\pi(e)}$

C $2^{\log_e(\pi)}$

D $e^{\log_2(\pi)}$

E $2^{\log_\pi(\pi)}$

11 The variables v, w, x, y and z are related by the equation

$$z = \frac{w(x+2y)^3}{v^2}$$

The variable v is increased by 20%, w is tripled and x, y decrease by 20%.

What is the percentage change in z (to 2 decimal places)?

A 6.67% increase

B 6.67% decrease

C 6.25% increase

D 6.24% decrease

E No change

12 Find the product of the roots of the following equation:

$$\sqrt{x-1} + \sqrt{2x-1} = x\,.$$

A 0

B 1

C 5

D 6

E 8

13 The exterior angle of a regular polygon with n sides is $2°$ greater than the exterior angle of another regular polygon with $(n+2)$ sides.

Find the value of n.

A $n = 18$

B $n = 20$

C $n = 22$

D $n = 24$

E $n = 26$

14 What is the probability that a number chosen randomly from 1 to 1000 (inclusive) ends in 1?

A $\frac{101}{1000}$

B $\frac{99}{1000}$

C $\frac{34}{499}$

D $\frac{1}{9}$

E $\frac{1}{10}$

15 The hypotenuse of a right-angled triangle is twice as large as one of its sides, which itself is 1cm shorter than the third side.

Find the area of the triangle.

A $\frac{1+\sqrt{3}}{2}$ cm^2

B $\frac{2+\sqrt{3}}{2}$ cm^2

C $\frac{2+3\sqrt{3}}{4}$ cm^2

D $\frac{3+2\sqrt{3}}{4}$ cm^2

E No such triangle exists.

16 Which of the following statements about prime numbers is true?

A They are always odd.

B Aside from 2, they always leave a remainder of 1 or 5 upon division by 6.

C There are no primes p, q such that $p \neq q$ and p divides $q + 2$.

D No prime number ends in 9.

E No two prime numbers greater than 10 and less than 100 sum to 99.

17 Find the exact value of

$$1 + \frac{3\tan(30°)}{\sin(60°) + 3\cos(60°)} + \frac{5}{\cos(45°) + \sqrt{3}}$$

A $3\sqrt{3} - \sqrt{2}$

B $3\sqrt{3} + \sqrt{2}$

C $4 + \sqrt{3} - \sqrt{2}$

D $-2 + 5\sqrt{3} - \sqrt{2}$

E $-2 + 5\sqrt{3} + \sqrt{2}$

18 Groups A and B both consist of 10 people. The mean age of Group A is a and the mean age of Group B is a^2. One person of age a from Group A is exchanged with one person of age a^2 from Group B. The mean age of Group B is now four times that of Group A.

Find the value of a, given that $a > 0$.

A $a = 3$

B $a = 4$

C $a = 5$

D $a = 6$

E $a = 7$

19 If x and n are integers, then under which conditions is the following expression negative?

$$(1-x)^n(2-x)^{2n}(3-x)^{3n}(4-x)^{4n}(5-x)^{5n}$$

A $n > 5$ and $x < 5$

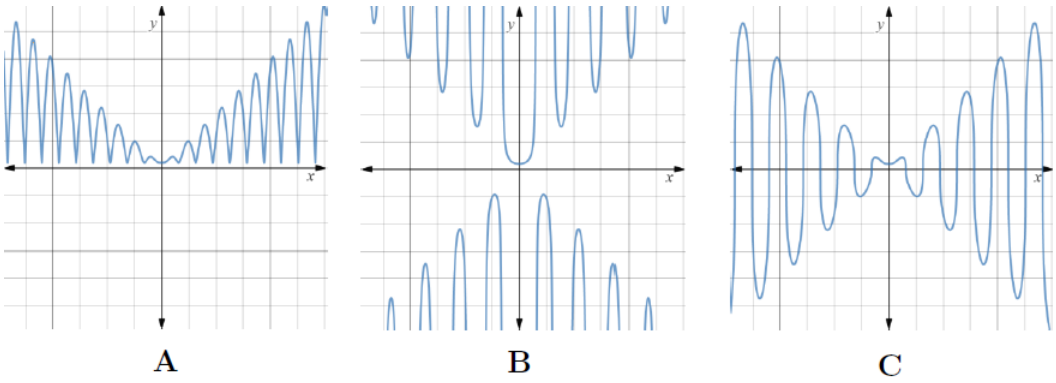
B n is odd and $x > 5$

C n is a multiple of 3 and $x > 5$

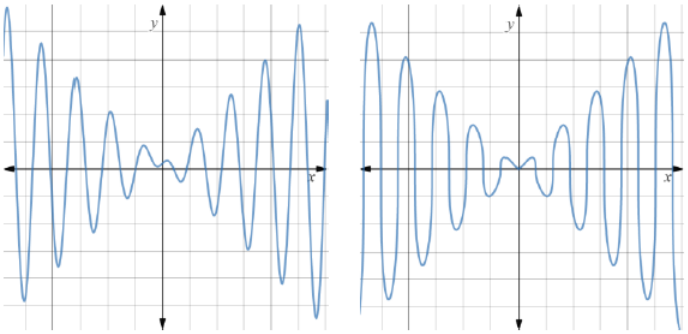
D n is even and $x > 5$

E n is odd and $x < 5$

20 Which of the following is a graph of $y = \left(x^3 \sin(x) + 1\right)^{\frac{1}{3}}$?



A **B** **C**



D **E**

SECTION 2
MATHEMATICAL THINKING

21 Find the coefficient of x^5 in the expansion of

$$3x^2\left(2x+\frac{1}{x}\right)^5$$

A 5

B 40

C 80

D 120

E 240

22 If $y=\dfrac{(x+2\sqrt{x})^3}{x}$, which of the following is a correct expression for $\dfrac{dy}{dx}$?

A $x^3+6x^{\frac{5}{2}}+12x^2+8x^{\frac{3}{2}}$

B $x^2+6x^{\frac{3}{2}}+12x+8x^{\frac{1}{2}}$

C $3x+10x^{\frac{1}{2}}+12x+4x^{\frac{1}{2}}$

D $2x+9x^{\frac{1}{2}}+12x+4x^{\frac{1}{2}}$

E $2x+9x^{\frac{1}{2}}+12+4x^{-\frac{1}{2}}$

23 A geometric progression has first term $7\sqrt{2}$ and fourth term $\frac{7}{2}$.

Find the sum to infinity of this geometric progression.

A $\frac{7\sqrt{2}}{1+\sqrt{2}}$

B $\frac{14}{1+\sqrt{2}}$

C $\frac{14}{1-\sqrt{2}}$

D $\frac{14}{-1+\sqrt{2}}$

E $\frac{14}{-1-\sqrt{2}}$

24 If m and n are non-zero integers, when is the following expression also an integer?

$$\frac{6^{n+2m} \times 3^{n+m} \times 18^{n-m}}{8^{3m} \times 12^{n+m}}$$

A $n > 0$ and $m > 0$

B $n > 0$ and $m < 0$

C $n < 0$ and $m < 0$

D $n < 0$ and $m > 0$

E No possible values of n and m

25 Consider the following conjecture:

> **If** N is a positive integer with three digits, such that the sum of its digits is a prime number, **then** N must also be a prime number.

Here are three numbers:

I $N = 101$

II $N = 205$

III $N = 3002$

Which of these provide(s) a counterexample to the above conjecture?

A none of them

B **I** only

C **II** only

D **III** only

E **I** and **II** only

F **I** and **II** only

G **II** and **III** only

H **I**, **II** and **III**

26 Which of the following expressions is largest for $0 < x < 1$?

A x^2

B $\log_4 x$

C $\log_7 x$

D e^x

E $\sin x$

27 Find the sum of the digits of all integers from 0 to 999 inclusive.

A 11500

B 12650

C 13500

D 325000

E 499500

28 Consider the following argument, for integers a and b:

$$a = b$$

I $\Rightarrow 2a^2 = a^2 + ab$

II $\Rightarrow 2(a^2 - ab) = a^2 - ab$

III $\Rightarrow 2 = 1$

Which of following is true?

A There is an error in line **I** only.

B There is an error in line **II** only.

C There is an error in line **III** only.

D There are errors in lines **I** and **II** only.

E There are errors in lines **I** and **III** only.

F There are errors in lines **II** and **III** only.

G There are errors in lines **I**, **II** and **III**.

29 A sequence is defined by:

$$\begin{aligned} u_1 &= 3 \\ u_{n+1} &= \frac{u_n - 3}{u_n - 2}, \quad \text{for } n \geq 1. \end{aligned}$$

Find the sum of the first 100 terms of this sequence.

A 0

B 50

C 72.5

D 100

E 151.5

30 A cube with sides of length 9cm is divided into smaller cubes, each of which has sides of length 3cm

By what factor has the total surface area increased?

A $\sqrt{3}$

B 3

C 9

D 12

E 27

31 Consider the equation

$$e^x = mx + c\,,$$

For what value(s) of m is $c = 1$ a sufficient condition for there to exist a unique solution?

A All values of m

B $m < 1$

C $m = 1$

D $m > 1$

E No values of m.

32 An equilateral triangle with perimeter p, a square with perimeter q and a circle with perimeter r all have the same area.

Which of the following is true?

A $p > q > r$

B $p > r > q$

C $q > p > r$

D $q > r > p$

E $r > q > p$

33 Let m, n and p be three non-negative integers such that $m^2 + mn = 6m + 18p$.

Which one of the following statements is true?

A At least one of m or $m + n$ is divisible by 3

B m, n and p can never be equal

C p can never equal 1

D n is always smaller than m

E All the above statements are false

34 Consider the following statements:

I $y = \sqrt{x}$

II $y^2 = x$

Which of the following is true?

A **I** is necessary (but not sufficient) for **II**

B **I** is sufficient (but not necessary) for **II**

C **II** is sufficient (but not necessaty) for **I**

D **I** is both necessary and sufficient for **II**

E None of the above

35 As x varies over the real numbers, find the largest value attained by the function

$$\left(9\cos^2(8x - 7) - 6\right)^2 .$$

A 9

B 36

C 45

D 81

E 225

36 An angle x is measured in radians and satisfies $0 \leq x \leq 2\pi$.

Find the total length of the intervals such that $\sin^2(x) \geq \frac{1}{4}$ and $\cos(x) \geq \frac{1}{2}$.

A $\frac{\pi}{6}$

B $\frac{\pi}{4}$

C $\frac{\pi}{3}$

D $\frac{\pi}{2}$

E $\frac{2\pi}{3}$

37 For all positive integers n, the function f satisfies

$$f(n+1) = 1 - f(n)^2 .$$

Which of the following functions correctly expresses $f(n+2)$ in terms of $f(n)$?

A $2f(n)^2$

B $2f(n) - 2f(n)^2$

C $2f(n) + 2f(n)^2$

D $2f(n)^2 - f(n)^4$

E $2f(n)^2 + f(n)^4$

38 Find the shortest distance between the circle with equation $(x-2)^2+(y+3)^2 = 9$ and the point $(6, 1)$.

A $4\sqrt{2}$

B $-3 + 2\sqrt{17}$

C $4\sqrt{2} - 3$

D $2\sqrt{17}$

E $\sqrt{37}$

39 A region in the (x, y)-plane is defined by the inequalities

$$y < \sqrt{x-1} \text{ and } xy > 1\,.$$

Which of the following points lies in the defined region?

A $(2, 2)$

B $(2, 6)$

C $(2, -2)$

D $(2, -6)$

E $(-2, -6)$

40 Consider the functions:

$$\begin{aligned} y &= \frac{1}{x} \\ y &= x^2 \\ y &= x^3 \end{aligned}$$

The graphs of these functions are plotted on the same set of axes.

How many regions is the (x, y)-plane divided into?

A 4

B 5

C 6

D 8

E 10

END OF EXAM