Integration Competition Problems

1. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{1+e^x}\mathrm{d}x$

2. $\int_0^{\infty} \frac{\mathrm{d}x}{(x+\frac{1}{x})^2}$

3. $\int_0^{\frac{1}{2}} \frac{1+\sqrt{3}}{\sqrt[4]{(1+x)^2(1-x)^6}}\mathrm{d}x$

4. $\int_0^{\infty} \frac{\cos(\pi x)-\cos(ex)}{x}\mathrm{d}x$

5. $\int \sqrt{\tan x}\mathrm{d}x$

6. $\int_0^{\frac{\pi}{2}} \cot x \ln(\sec x)\mathrm{d}x$

7. $\int \frac{x^{-\frac{1}{2}}}{1+x^{\frac{1}{3}}}\mathrm{d}x$

8. $\int_0^1 \ln(x)\sin(\ln(x))\mathrm{d}x$

9. $\int \frac{\mathrm{d}x}{x^2-x\sqrt{x^2-1}}$

10. $\int_2^4 \frac{\sqrt{\ln(9-x)}\mathrm{d}x}{\sqrt{\ln(9-x)}+\sqrt{\ln(x+3)}}$

11. $\int \frac{(3x^{10}+2x^8-2)\sqrt[4]{x^{10}+x^8+1}}{x^6}\mathrm{d}x$

12. $\int_0^{\infty} \frac{\sin^2 x}{x^2(x^2+1)}\mathrm{d}x$

13. $\int \sqrt{1+e^x}\mathrm{d}x$

14. $\int_0^{\infty} \frac{\ln(x^2+1)}{x^2+1}\mathrm{d}x$

15. $\displaystyle\int \frac{1}{\sqrt{1+e^{2x}}}\mathrm{d}x$

16. $\displaystyle\int_0^{\pi} \frac{\ln(1+k\cos x)}{\cos x}\mathrm{d}x$ where $0<k<1$.

17. $\displaystyle\int_0^{\infty} \frac{\ln(1+x)}{x\sqrt{x}}\mathrm{d}x$

18. $\displaystyle\int \frac{\mathrm{d}x}{2+2\sin x+\cos x}$

19. $\displaystyle\int_0^1 x^{-x}\mathrm{d}x$

20. $\displaystyle\int_0^{\frac{\pi}{2}} (\ln(\tan\theta))^2\mathrm{d}\theta$

21. $\displaystyle\int_0^{\pi} \ln(1-2\pi\cos x+\pi^2)\mathrm{d}x$

22. $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{\sin x+\cos x}\mathrm{d}x$

23. $\displaystyle\int_0^1 x\left\{\frac{1}{x}\right\}\left\lfloor\frac{1}{x}\right\rfloor\mathrm{d}x$ where $\{x\}=x-\lfloor x\rfloor$ is the fractional part of x.

24. $\displaystyle\int \sqrt{x-\sqrt{x+\sqrt{x-\sqrt{x+\ldots}}}}\mathrm{d}x$

25. $\displaystyle\int_0^{\pi} \ln(a+b\cos x)\mathrm{d}x$ where $a>b$

26. $\displaystyle\int \cos(\ln x)\mathrm{d}x$

27. $\displaystyle\int_0^{\infty} \frac{\arctan x}{1+x}\frac{\mathrm{d}x}{\sqrt{x}}$

28. $\int_0^1 \ln x \ln(1-x)\mathrm{d}x$

29. $\int \frac{x\ln(x+\sqrt{x^2+1})}{\sqrt{1+x^2}}\mathrm{d}x$

30. $\int_0^{2\pi} \frac{1}{\sin^4(x)+\cos^4(x)}\mathrm{d}x$

31. $\int \sqrt{1+\sin\frac{x}{2}}\mathrm{d}x$

32. $\int_0^1 \ln(1+x)\ln(1-x)\mathrm{d}x$

33. $\int_0^1 \arctan\left(\frac{1}{x^2-x+1}\right)\mathrm{d}x$

34. $\int e^{x+e^x}\mathrm{d}x$

35. $\int_0^{\pi} \arctan(3^{\cos x})\mathrm{d}x$

36. $\int \frac{x}{x^4+4}\mathrm{d}x$

37. $\int \frac{\mathrm{d}x}{\sqrt{x-1}+\sqrt{(x-1)^3}}$

38. $\int_0^{\frac{\pi}{2}} \frac{\cos x}{2-\sin 2x}\mathrm{d}x$

39. $\int_0^1 \ln\left(\frac{2+x}{2-x}\right)\frac{\mathrm{d}x}{x\sqrt{1-x^2}}$

40. $\int_0^{\frac{\pi}{4}} \ln\left(\frac{\sec^2 x-2}{\tan x-1}\right)\mathrm{d}x$