Q1. By factorising LHS, show that

a) $\left(\frac{x+y}{2}\right)^2 - \left(\frac{x-y}{2}\right)^2 = xy$

b) Use a) to show $\sqrt{xy} \leq \frac{x+y}{2}$

Q2. Show that the product of 4 consecutive integers is 1 less than a perfect square.

Q3. Sketch $f(x) = x^3 + Ax^2 + B$

a) If $A > 0, B > 0$

b) If $A < 0, B > 0$

c) Show that if $x^3 + ax^2 + b = 0$ has 3 solutions then $27b^2 + 4a^3b < 0$

Q4. Find all pairs of positive integers for which

$$(u+v)^2 = u^3 + v^3$$