MOCK TEST OF MATHEMATICS FOR UNIVERSITY ADMISSION

Paper 2

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1.

Sketching the curve $(x^2 + y^2 - 1)^2 = 1$ in the real plane gives:

A a pair of quadratic parabolas

B a pair of quartic parabolas

C a pair of lines

D a circle

E a pair of circles

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2.

The number of solutions to $\sin(x^{-1}) = 0$ when $0 < x < 1$ is:

A 0

B 1

C 2

D 4

E infinitely many

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3.

Suppose the integer n^3 is divisible by 3. Consider the following the sentences

(1) n is divisible by 3

(2) 27 is a factor of n^3

(3) if d is a multiple of 3 which divides n^3, then n^3/d is a multiple of 3.

Which of the above sentences are **implications** of n^3 being divisible by 3?

A none

B (1) only

C (1) and (2) only

D (1) and (3) only

E (1), (2), and (3)

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4.

Consider a quadratic $q(x) = x^2 + ax + b$.

How many points belonging to q would be **sufficient** to determine the values of a and b?

A 1

B 2

C 3

D it is not possible to determine q only with points

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5.

A function is said be *strictly convex* on an interval (a, b) if, in the interval, **every** line segment joining two points on the graph of f lies **above** the rest of the graph between those points.

A sufficient condition, if f is assumed to be twice-differentiable, is that $f''(x) > 0$ on (a, b).

Which of the following functions illustrates that this condition *on twice-differentiable functions* is sufficient but **not** necessary?

A $x^2 + \cos x$

B $x + \cos x$

C $|x|$

D x^4

E $x^2(1 + x^2)^{-1}$

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6.

Let μ be a positive real number and consider the following simultaneous equations:

$$\begin{cases} a+b=\mu \\ a^2+b^2=\mu \end{cases}$$

What is the full range of values for μ such that the system has no solutions?

A $\mu > \sqrt{2}$

B $\mu > 2$

C $\mu > 1$

D $1 < \mu < 2$

E $0 < \mu < 1$ or $\mu > \sqrt{2}$

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7.

Suppose (α), (β), (γ) and (δ) are statements for which:

$$\begin{array}{rcr} (\alpha) & \text{implies} & (\beta) \\ (\delta) & \text{implies} & (\beta) \\ (\text{not } \gamma) & \text{implies} & (\delta) \\ (\gamma) & \text{implies} & (\alpha) \end{array}$$

This means:

A (α) implies (γ)

B (not α) implies (not β)

C (β) implies (δ)

D (α) implies (not δ)

E the set of statements is inconsistent (i.e. contains a contradiction)

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8.

Which of the following is **sufficient** for the polynomial $z(x) = x^4 + px^3 + qx^3 + rx^2 + sx + t$ to have 4 (real) roots?

A $z'(x)$ has 3 roots

B z has either two maxima or two minima

C z has two stationary points, one of which is positive and the other is negative

D $z(x) = (x^2 + ax + b)(x^2 + cx + d)$ for some a, b, c, d such that $(ac)^2 > 16bd$

E there are 4 distinct polynomials (up to scaling) that divide z

F none of these conditions are sufficient

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9.

A *word* is a finite string of letters. A *permutation* of a word is a word made of the same letters, including multiplicity. For instance, a permutation of 'txiotw' is 'xoittw'.

Which of the following is **necessary** for a 5-letter word to have **exactly** 20 permutations (including itself)?

A it contains exactly two identical consecutive letters

B it contains exactly two identical letters

C it contains two identical letters and three other identical letters, in some order

D it contains exactly three identical letters

E none of these conditions are necessary

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10.

A student claims they have found two real numbers α and β such that they can integrate any cubic over the interval $(-1, 1)$ by simply plugging in α and β and adding, i.e.

$$\int_{-1}^{1} p(x)\,\mathrm{d}x = p(\alpha) + p(\beta)$$

where p is any cubic polynomial.

The student's proof:

Assume there are such values.

(1) Since integration is linear, we can ignore coefficients and consider terms separately:

$$\int_{-1}^{1} x^3\,\mathrm{d}x = \alpha^3 + \beta^3; \qquad \int_{-1}^{1} x^2\,\mathrm{d}x = \alpha^2 + \beta^2; \qquad \int_{-1}^{1} x\,\mathrm{d}x = \alpha + \beta$$

(2) The last equation tells us that $\alpha = -\beta$.

(3) Applying this to the middle integral gives $\alpha = \pm\dfrac{1}{\sqrt{3}}$ and $\beta = \mp\dfrac{1}{\sqrt{3}}$.

(4) These pairs agree with the first integral, hence the conjecture is true.

A the proof is correct

B there is a flaw at line (1)

C there is a flaw in line (2)

D there is a flaw in line (3)

E there is a flaw in line (4)

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11.

We are given the following functions:

$$a(x) = x + 1; \quad b(x) = x - 1, \quad c(x) = x^{-1}; \quad d(x) = 2x$$

Call a function '*abcd*-composite' if it can be written as a composition of a, b, c and d in some order, using each exactly once. For instance, $b(d(c(a(x))))$ is *abcd*-composite.

Which of the following functions **cannot** be *abcd*-composite?

A $(1 - x)(1 + x)^{-1}$

B $(x + 2)(1 - x)^{-1}$

C $2x(2x - 1)^{-1}$

D $(2x)^{-1}$

E $(x + 3)(x - 1)^{-1}$

F $-\frac{1}{2}(2x + 1)(x + 1)^{-1}$

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12.

a, b, c are positive integers such that a, b are coprime (that is, they share no common factors other than 1) and $7a = bc$.

Which of the following is **necessary** for this to hold?

A $b = 7$

B $c = 7$

C either b or c is equal to 7

D b is either 1 or 7

E c is either 1 or 7

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13.

Let $f(x)$ be the function defined by:

$$f(x) = \begin{cases} 2^{-k} & \text{when } 2k \leqslant x < 2k+1 \text{ for each integer } k \\ -2^{-k} & \text{when } 2k-1 \leqslant x < 2k \text{ for each integer } k \end{cases}$$

Let F be the integral function defined for $x > 0$ by:

$$F(x) = \int_0^x f(y)\,\mathrm{d}y$$

Consider the following (false) claims, with h being any integrable function:

(1) if $\int_0^x h(y)\,\mathrm{d}y > 0$ for all $x > 0$ then $h(x) \geqslant 0$ for all $x > 0$.

(2) for any integrable function h, we have $\int_0^x \left|h(y)\right| \mathrm{d}y > 0$.

(3) if the integral function $H(x) = \int_0^x h(y)\,\mathrm{d}y$ is not increasing, then $h(x)$ is not increasing.

F is a **counterexample** to:

A (1) only

B (2) only

C (3) only

D (1) and (2)

E (2) and (3)

F (1) and (3)

G (1), (2), and (3)

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14.

In logic, the *principle of explosion* postulates that a statement beginning with a false premise must be read as true, regardless of what follows this premise. For instance, the *statement* "if $0 = 1$ then the Earth is flat" is a *logically true.*

Consider now the following two sentences:

(i) "it is raining in Paris **or** it is raining in New York" [*inclusive* **or**]

(ii) "**if** it is **not** raining in Paris, **then** it is raining in New York"

Assuming we know nothing about the weather in either city:

A these statements are logically equivalent (i.e. they express the same thing)

B statement (i) implies (ii) but not the other way around

C statement (ii) implies (i) but not the other way around

D these statements are logically incomparable

15.

Consider the sequence (ξ_n) defined by:

$$\xi_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

all $n \geqslant 0$. Given that, for all $k \geqslant 0$, the following identities hold for this sequence:

$$1 + \xi_0 + \xi_2 + \cdots + \xi_{2k} = \xi_{2k+1}$$

$$\xi_1 + \xi_3 + \cdots + \xi_{2k+1} = \xi_{2k+2}$$

one can deduce:

A ξ_n is always an integer

B ξ_n is always an integer and if $n > 1$, ξ_n is odd when n is even and ξ_n is even when n is odd

C ξ_n alternates between being an integer and not being an integer

D ξ_n is never an integer but the sequence is increasing

E the subsequences (ξ_{2n}) and (ξ_{2n+1}) are increasing but (ξ_n) is not

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16.

Consider the inequality

$$|x + a|x|| > |x + |x||$$

and the following solution attempt:

(1) The only instances in which both sides are simultaneously equal to 0 are when $a \neq 1$ and $x = 0$, or when $a = 1$ and $x < 0$, so we can rule these out of our solution set.

(2) When $x > 0$ both expressions within the absolute values are positive, hence we can remove them and the inequality becomes $(a + 1)x > 2x$, which is satisfied exactly when $a > 1$ (since we have assumed $x > 0$).

(3) When $x < 0$, the right hand side is equal to 0, so the inequality becomes $|x + a|x|| > 0$ which is always true since we have assumed $x \neq 0$ and $a \neq 1$.

(4) Hence the full solution set is:

$$\begin{cases} a > 1 \text{ and } x \neq 0 \\ a < 1 \text{ and } x < 0 \end{cases}$$

A the solution is correct

B the solution is incomplete and there is a flaw in line (1)

C the solution is incomplete and there is a flaw in line (2)

D the solution is incomplete and there is a flaw in line (3)

E the solution is incomplete and there are flaws in lines (1) and (2)

F the solution is incomplete and there are flaws in lines (2) and (3)

G the solution is incomplete and there are flaws in lines (1) and (3)

H the solution is incomplete because (4) does not take into account a result from (1), (2) or (3)

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17.

Consider the following statements:

(1) if a function has a vertical asymptote at 0, then its integral on $(0, 1)$ is not finite (i.e. diverges).

(2) for any $\alpha \neq -1$, the function x^α is integrable on $(0, 1)$.

(3) for a function to be integrated on $(0, 1)$, it needs to be defined at 0 and 1.

(4) if a function is $\geqslant 1$ for infinitely many x in $(0, 1)$ then its integral on $(0, 1)$ is $\geqslant 1$.

The integral

$$\int_0^1 \frac{\mathrm{d}x}{\sqrt{x}}$$

is a **counterexample** to:

A (1) only

B (2) only

C (3) only

D (4) only

E (1) and (3)

F (2) and (4)

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18.

A sequence of real numbers (x_n) **converges** to a value a if the following statement is true:

For every $\epsilon > 0$ there exists a positive integer N such that for all $n \geqslant N$, $|x_n - a| < \epsilon$

Which of the following sentences expresses the **negation** of convergence to a value a?

A For every $\epsilon > 0$ and every positive integer N, if $n > N$ then $|x_n - a| \geqslant \epsilon$

B For every $\epsilon > 0$ there is no integer N such that if $n > N$ then $|x_n - a| < \epsilon$

C There exists $\epsilon > 0$ such that for all integers N, if $n > N$ then $|x_n - a| \geqslant \epsilon$

D There exists $\epsilon > 0$ such that for all integers N, there exists $n > N$ where $|x_n - a| \geqslant \epsilon$

E There exists $\epsilon > 0$ such that for all integers N, if $|x_n - a| < \epsilon$ then $n > N$

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19.

Consider the following (true) statements:

(1) $\sqrt{2}$ is irrational.
(2) $\sqrt{2}^{\sqrt{2}}$ is either rational or irrational.
(3) $(\sqrt{2}^{\sqrt{2}})^{\sqrt{2}}$ is rational since it is equal to 2.

These statements combined **prove** which of the following:

A an irrational number to an irrational power can be rational

B an irrational number to an irrational power is always irrational

C a rational number to an irrational power is always irrational

D a rational number to a rational power is never irrational

[*a number is rational if and only if it can be written as a fraction of integers*]

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20.

Let T_1, T_2, $T_3 \dots$ be the sequence defined by $T_n = 2^n$, and let V_1, V_2, $V_3 \dots$ be the sequence resulting from taking the ordered positive integers and removing all powers of 2 other than $2^0 = 1$.

Define now the sequence (Φ_n) by:

$$\begin{cases} \Phi_0 = 0,\ \Phi_1 = 1 \\ \Phi_{T_k} = k \text{ for each } k \geqslant 1 \\ \Phi_{V_{k+1}} = \Phi_{V_k} + 1 \text{ for each } k \geqslant 1 \end{cases}$$

The sequence (Φ_n) has which of the following properties?

(i) (Φ_n) is non-increasing but (Φ_{A_n}) is increasing for every arithmetic sequence of positive integers (A_n) with difference $\geqslant 2$ and first term > 0

(ii) (Φ_n) is non-increasing but the sequence (μ_n) defined by $\mu_n = \Phi_n + \Phi_{n+1}$ is increasing

(iii) (Φ_n) is non-increasing but (Φ_{G_n}) is increasing for every geometric sequence of positive integers (G_n) with ratio $\geqslant 2$ and first term > 1

(iv) (Φ_n) is non-increasing but the sequence (ρ_n) defined by $\rho_n = \Phi_n \Phi_{n+1}$ is increasing

A (i) only

B (ii) only

C (i) and (ii)

D (iii) only

E (iv) only

F (iii) and (iv)

G (Φ_n) has none of the above properties

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