20 YEARS OXBRIDGE APPLICATIONS

Test of Mathematics For University Admission (TMUA)

QUESTION PAPER 3

DUKES EDUCATION

OXBRIDGE APPLICATIONS

TEST OF MATHEMATICS FOR UNIVERSITY ADMISSION

Time Allowed: 2½ hours

Instructions to Candidates

There are 40 questions in this paper. For each question, choose the one answer you consider correct and record your choice by circling the question letter. If you make a mistake, erase thoroughly and try again.

In the real TMUA, you will be provided with a separate answer sheet and the test will be split into two halves, each with 20 questions in the paper.

There are no penalties for incorrect responses, only points for correct answers, so you should attempt all questions. Each question is worth one mark.

All rough work should be done on this question paper. No extra paper is allowed.

- No calculators, formula sheets or dictionaries are permitted during the test.
- Only answers written in the booklet will be marked. There are spare blank pages at the end of the test paper, but working on these pages will not be marked.

Please wait to be told you may begin before turning this page

1. Which quadratic has the largest root?

(a) $x^2 + 5x - 1$

(b) $x^2 + 5x + 1$

(c) $3x^2 + 15x - 6$

(d) $2x^2 + 10x - 14$

(e) $x^2 - 4x - 9$

2. The equation $sin(x) = x^2 - 2\pi x + \pi^2 + 1$ has:

(a) no solutions

(b) one solution

(c) two solutions

(d) three solutions

(e) infinitely many solutions

3. Consider

$$N_n = \frac{4}{3} \cdot \frac{6}{4} \cdot \frac{8}{5} \cdot \frac{10}{6} \cdots \frac{2(n+1)}{n+2}$$

What does N_n equal?

(a) $\frac{2^{n+2}}{3(n+1)}$

(b) $\frac{2(n+1)}{3}$

(c) $\frac{2^{n+1}}{n+2}$

(d) $\frac{2^{n+1}n}{3}$

(e) $\frac{2^n n}{3}$

4. Which of the following numbers is the largest?

(a) $e^{\log_\pi 2}$

(b) $2^{\log_\pi e}$

(c) $2^{\log_e \pi}$

(d) $e^{\log_2 \pi}$

(e) $2^{\log_\pi \pi}$

5. Suppose a, b, c and d are real numbers such that $a < b < c < d$. Given the integral

$$I = \int_a^c f(x)dx - \int_b^d f(x)dx$$

What does I also equal?

(a) $\int_a^b f(x)dx$

(b) $\int_{a-b}^{c-d} f(x)dx$

(c) $\int_a^b f(x)\,dx + \int_c^d f(x)dx$

(d) $\int_a^b f(x)dx - \int_c^d f(x)dx$

(e) None of the above

6. Which value can the following expression not take?

$$3\sin\left(\left(e^{-x^2}+1\right)\frac{\pi}{2}\right) - 2$$

(a) 0

(b) -2

(c) 1

(d) -1

(e) ½

7. If n people must shake hands with each other exactly once how many handshakes will take place?

(a) n^2

(b) $\frac{(n-1)}{2}$

(c) $(n-1)$

(d) $\frac{n^2}{2}$

(e) $n(n+1)$

8. $\sqrt{2}+\left(2-\sqrt{2}\right)=2$ is a counterexample to which one of the following statements?

(a) A rational number plus a rational number is irrational number.

(b) A rational number minus an irrational number is an irrational number.

(c) An irrational number plus an irrational number is an irrational number.

(d) A rational number minus an irrational number plus a rational number is a rational number.

(e) A rational number plus a rational number is a rational number.

[Assume $\sqrt{2}$ and $\sqrt{3}$ are irrational]

9. Which of the following pairs of functions' graphs intersect perpendicularly?

(a) $y=\sin x$ and $y=\cos x$

(b) $y=4+\sqrt{x}$ and $y=4-\sqrt{x}$

(c) $y=\sqrt{x}$ and $y=4-\sqrt{x}$

(d) $y=ln(x)$ and $y=ln(3)+3x-x^2$

(e) $y=3/x$ and $y=x$

10. The function $y = sin(x)cos(x)$

Satisfies

(a) $\frac{dy}{dx} = 1 - 2cos^2 x$ (and this part only)

(b) $y = \frac{\sqrt{3}}{2}$ for infinitely many x (and this part only)

(c) $\frac{d^2y}{dx^2} + 4y = 0$ (and this part only)

(d) all of the above

(e) none of the above.

11. Which of the following is a factor of $x^n - y^n$ for integer $n \geq 1$?

(a) $(x - y)$

(b) $(x + y)$

(c) $(x + y)$ only when n is odd

(d) $(x - y)$ only when n is even

(e) None of the above

12. How many solutions does the equation $100^x = 10^x + 110$ have, for real x?

(a) No solutions

(b) One solution

(c) Two solutions

(d) Three solutions

(e) More than three solutions

13. Suppose *a*, *b*, *c* and *d* are real numbers such that $a < b$ and $< d$. What is an equivalent expression for $\int_a^b f(x)dx$?

(a) $\int_c^d f\left(\left(\frac{x-c}{d-c}\right)(b-a)+a\right)dx$

(b) $\left(\frac{b-a}{d-c}\right)\int_c^d f\left(\left(\frac{b-a}{d-c}\right)(x-c)+a\right)dx$

(c) $\left(\frac{d-b}{c-a}\right)\int_c^d f\left(\left(\frac{d-b}{c-a}\right)(x-a)+b\right)dx$

(d) $\int_c^d f\left(\left(\frac{b-a}{d-c}\right)(x-c)+a\right)dx$

(e) None of the above

14. Which is the largest of the following numbers?

(a) $6^{76} \times \frac{7^{75}}{3\times 21^2}$

(b) $\frac{42^{75}}{196}$

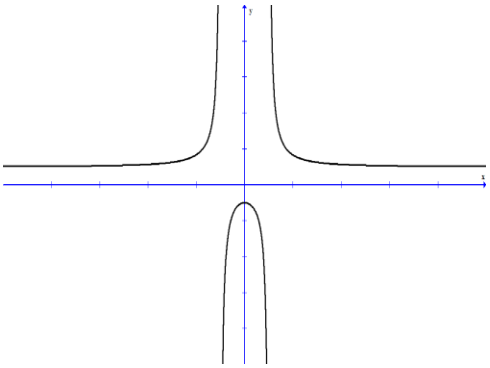
(c) $\frac{1764^{37}}{7}$

(d) 7×42^{73}

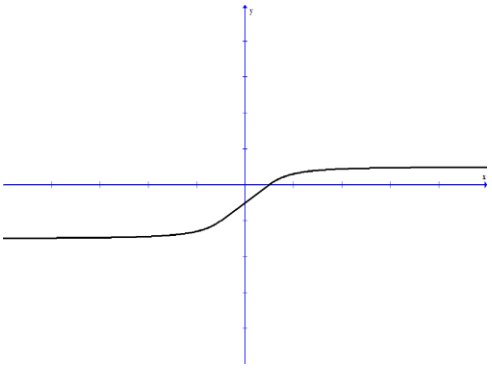
(e) 1764^{35}

15. Which of the following is a sketch of the function $y = \frac{x^2-1}{x^2+1}$?

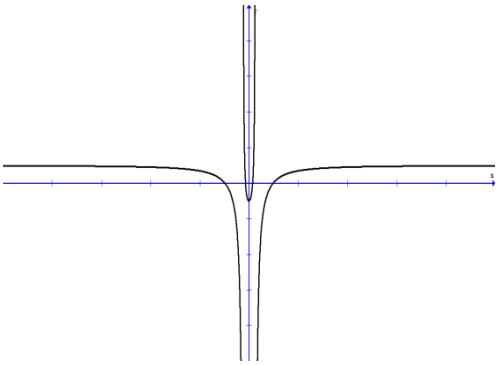
(a)



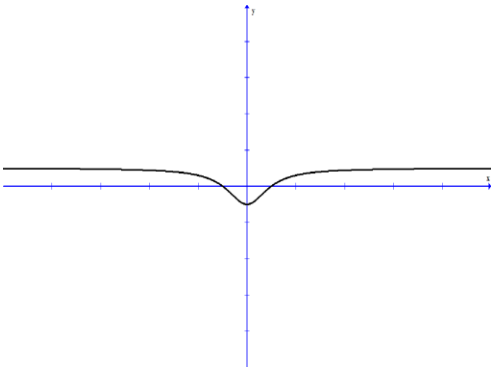
(b)



(c)



(d)



(e) None of the above

16. Consider the following statement: If the penny is in the jar then the penny is in the box. Which of the following statements logically follows?

(a) The penny is in the box so it is in the jar.

(b) The penny is not in the box so it is not in the jar.

(c) The penny is not in the jar so it is not in the box.

(d) The penny is not in the jar so it is in the box.

(e) None of the above

17. Which of the following is a factor of the polynomial $x^5 - x^4 + 2x^3 - 2x^2 + x - 1$?

(a) $(x + 1)$

(b) $(x^2 + 1)$

(c) $(x^2 + x + 1)$

(d) $(x^2 - 1)$

(e) (x – 1)

18. Suppose that *u* and *v* are both real numbers that aren't equal to zero. Given

$$2u + 2v = 20$$

$$u = 6/v$$

What is the sum of the reciprocals of *u* and *v*?

(a) $\frac{3}{5}$

(b) $5 + \sqrt{19}$

(c) $5 - \sqrt{19}$

(d) $1\frac{2}{3}$

(e) None of the above

19. Which of the following numbers ends in the highest number of consecutive zeros (e.g. 103000 ends in 3 consecutive zeros)?

(a) $101!$

(b) $((10^3)^3)^3$

(c) $2^{25} \times 3^{38} \times 5^{48} \times 7^{13}$

(d) $112!$

(e) 10^9

20. The equation $\frac{2x^2+4x-6}{x-1} = x - 1$ has:

(a) no solutions

(b) one solution

(c) two positive solutions

(d) one negative and one positive solution

(e) 2 negative solutions

21. Find the coefficient of x in $\left(3-\frac{5}{x^2}\right)\left(x+\frac{2}{x^2}\right)^6$

(a) -60

(b) -5

(c) -12

(d) 3

(e) 12

22. Find α so that $f(x)=\alpha x^3+x^2+1$ has a maximum at $(-\frac{1}{2}, 7/12)$

(a) $\alpha=4$

(b) $\alpha=2/3$

(c) $\alpha=-4/3$

(d) $\alpha=-4$

(e) None of the above

23. What is the largest interval for r such that $4r+8r^2+16r^3+32r^4+\cdots$ converges?

(a) $-1<r<1$

(b) $-\frac{1}{2}<r<\frac{1}{2}$

(c) $0<r<\frac{1}{2}$

(d) $-2<r<2$

(e) $0<r<1$

24. A differentiable function is called *concave up* on an interval (a, b) if its derivative is strictly increasing on (a, b). It is *concave down* if the derivative is strictly decreasing.

For $0 \leq x \leq \pi$, describe the concavity of $f(x) = -\frac{1}{4}\left(\sqrt{3}x^2 + sin2x\right)$:

(a) Concave down for $0 \leq x < \frac{\pi}{6}$; concave up for $\frac{\pi}{6} < x < \frac{\pi}{3}$; concave up for $\frac{\pi}{3} < x \leq \pi$

(b) Concave down for $0 \leq x < \frac{\pi}{6}$; concave up for $\frac{\pi}{6} < x < \frac{\pi}{3}$; concave down for $\frac{\pi}{3} < x \leq \pi$

(c) Concave down for $0 \leq x < \frac{\pi}{3}$; concave up for $\frac{\pi}{3} < x < \frac{2\pi}{3}$; concave down for $\frac{2\pi}{3} < x \leq \pi$

(d) Concave up for $0 \leq x < \frac{\pi}{6}$; concave down for $\frac{\pi}{6} < x < \frac{\pi}{3}$; concave down for $\frac{\pi}{3} < x \leq \pi$

(e) None of the above.

25. Sum the following series for an integer $n > 0$

$$ln\,1 + ln\,e + ln\,e^2 + \cdots + ln\,e^n$$

(a) n^2

(b) $\frac{1}{2}n(n-1)$

(c) $\frac{1}{2}n(n+1)$

(d) $1 \cdot 2 \cdot 3 \cdots n$

(e) $n^2 - n$

26. A student is asked to differentiate a function from first principles. Below is a line from their working:

$$\frac{df}{dx} = \lim_{h \to 0} \frac{6xh^2 + 2x^3 + 6x^2h - 2x^3 + 2h^3}{h}$$

Assuming the student is correct, which of the following is a possible candidate for being the original function?

(a) $f(x) = 2x(x^2 + 3)$

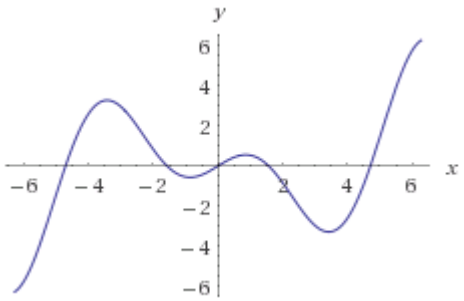
(b) $f(x) = 6x^2$

(c) $f(x) = 2xh^3$

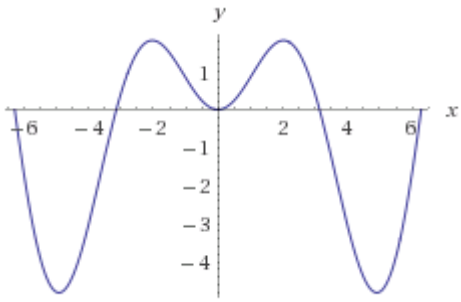
(d) $f(x) = 6x^3$

(e) $f(x) = 2x^3 + 1$

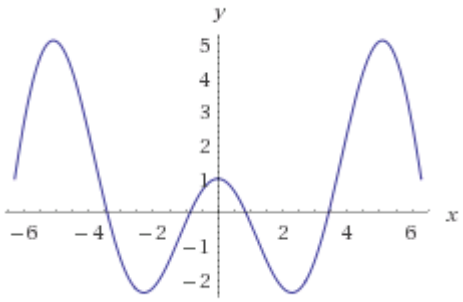
27. Which of the following graphs represents the derivative o f $xcosx$?



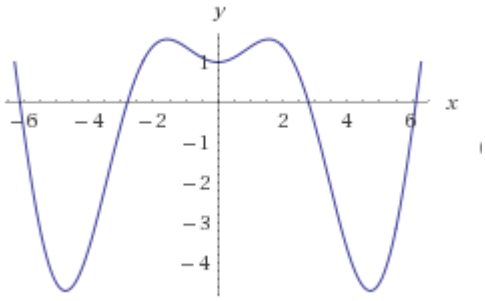
(a)



(b)



(c)



(d)

(e) None of the above

28. Which of the following statements is necessarily true if 'If $2^n - 1$ is prime, then *n* is prime' is true?

(a) 'If *n* is prime, then $2^n - 1$ is prime'

(b) 'If $2^n - 1$ is not prime, then *n* is not prime'

(c) 'If *n* is not prime, then $2^n - 1$ is not prime'

(d) 'If $2^n - 1$ is not prime, then *n* is prime'

(e) None of the above.

29. Let a be a fixed number, and let k be the minimum of $x^4 + a + x^{-4}$.

What is the minimum of $x^2 + x^{-2}$?

(a) $(k-a)^2$

(b) $\sqrt{k-a-2}$

(c) $\sqrt{k-a}$

(d) $\sqrt{k+2-a}$

(e) $\sqrt{k/a}$

30. If we are given that $ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2$ then we know which of the following to be true?

(a)$c = 0$

(b) $ax^2 + bx + c$ has only one distinct root

(c) $ax^2 + bx + c$ has two distinct roots

(d) the discriminant of the quadratic is negative

(e) c < 0

31. How many real roots does $y = |x^3 - x|$ have?

(a) 1

(b) 2

(c) 3

(d) 4

(e) Infinitely many

32. What is the highest power of x in:

$$\left[\frac{dx}{dy}(x^3 sinx)^7\right] - 7x^{18}cosx[(xsin^2x)^3 + x^2 sinx]$$

(a) 20

(b) 21

(c) 7

(d) 3

(e) 14

33. Find the coefficient of x^6 in $(3 + 2x - x^2 + 4x^3 + 5x^5)^3$

(a) 125

(b) 227

(c) 291

(d) 275

(e) 175

34. Evaluate $(3.6)^5$ exactly.

(a) 602.68776

(b) 604.66176

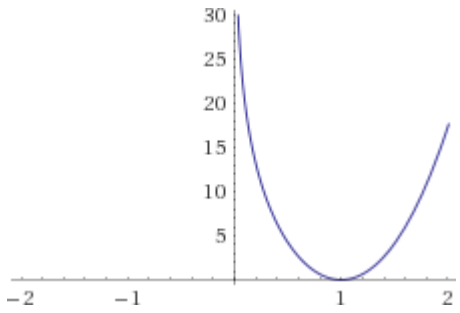
(c) 602.68196

(d) 602.67966

(e) 603.60276

[Hint: either simplify and use a binomial expansion, or consider digit sums]

35. Which of the following functions could be represented by the following sketch on the real plane?



(a) $y = (x-1)(x+3)^2\log(x)$

(b) $y = (x+1)(x+3)\log(x)$

(c) $y = (1-x)(x+3)^2\log(x)$

(d) $y = (x+1)(x+3)^2\log(x)$

(e) $y = (x+1)^2(x+3)\log(x)$

36. If a function maps each element from a set of *M* points to one of *N* elements, how many possible pairings could be created?

(a) M^N
(b) N^M
(c) MN
(d) $M+N$
(e) 2^{M+N}

37. Let $f(x) = ax^2 + bx + c$ such that $f(-b/2a) = 0$. We can deduce that:

(a) $b/2a$ is a root of f.

(b) f has a point of inflection at $x = -b/2a$.

(c) $b > a$

(d) the discriminant of f is 0

(e) none of the above.

38. You are given the following identities,

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

what then does $\tan(3\alpha)$ equal in its most simplified form?

(a) $\frac{tan\alpha + tan2\alpha}{1 - tan\alpha tan2\alpha}$

(b) $\frac{3tan\alpha - tan^3\alpha}{1 - 3tan^2\alpha}$

(c) $\frac{tan\alpha - tan^3\alpha}{1 - tan^2\alpha}$

(d) $\frac{tan\alpha - tan^3\alpha}{1 - 3tan^2\alpha}$

(e) None of the above.

39. How many digits does the number 2^{1000} have?

(a) 302

(b) 31

(c) 301

(d) 32

(e) 30

$[log 2 \approx 0.30103 \ldots]$

40. Given that $x = log_5 2$, find $log_8 25$ in terms of x.

[Hint: find $log_2 5$ first]

(a) $\frac{12}{5x}$

(b) $\frac{8}{5x}$

(c) $\frac{1}{x}$

(d) $\frac{2}{3x}$

(e) none of the above