

## TMUA Trigonometry

### **Syllabus**

Sine, cosine rule, area of triangle; radian measure; arcs and sectors; exact values; sin/cos/tan functions and graphs; basic trig identities ( $\tan x = \sin x / \cos x$  and  $\sin^2 x + \cos^2 x = 1$ ); solution of trig equations

1. Solve the following trigonometric equations in the range given

a)  $\cos(2\theta + 30) = 0.5 \quad 0 \leq \theta \leq 360$

b)  $\tan(5\theta - 35) = \sqrt{3} \quad 0 \leq \theta \leq 90$

c)  $2\sin^2\theta + 3\cos\theta = 0 \quad 0 \leq \theta \leq 360$

d)  $\sin(3\theta + 72) = \cos 48 \quad 0 \leq \theta \leq 180$

e)  $\frac{3 + \sin^2\theta}{\cos\theta - 2} = 3\cos\theta \quad 0 \leq \theta \leq 360$

f)  $\cos^2(2x) + \sqrt{3}\sin(2x) - \frac{7}{4} = 0 \quad 0 \leq x \leq 360$

2. Solve, in radians, the following trigonometric equations, giving your answers in terms of  $\pi$  .

a)  $\cos 2x = \cos \frac{2\pi}{5} \quad 0 \leq x \leq 2\pi$

b)  $8\sin\left(\frac{\pi}{3} - 2x\right) = 4 \quad 0 \leq x \leq 2\pi$

c)  $\sin^2 \frac{3\theta}{2} = \frac{1}{2} \quad 0 \leq \theta \leq 2\pi$

d)  $2\sin^2 y - 5\cos y + 1 = 0 \quad 0 \leq y \leq 2\pi$

e)  $\tan^4 x - \tan^2 x = 6 \quad 0 \leq x \leq 2\pi$

3. Find the number of solutions to the following equations:

a)  $x \tan x = 2$   $-2\pi \leq x \leq 2\pi$

b)  $x \sin 3x = \cos 3x$   $0 \leq x \leq 2\pi$

c)  $\sin^2 \theta + 3 \cos \theta = 3$   $0 \leq \theta \leq 4\pi$

d)  $(x + 3)(3 - x) = 9 \cos \frac{\pi}{2} x$

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4. Find the complete set of values for which:

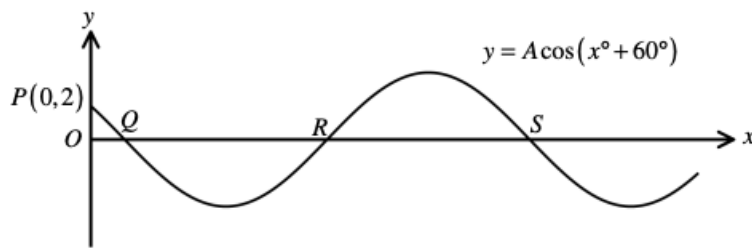
a)  $y \geq 0$  where  $y = \tan x \cos 2x$   $0 \leq x \leq \pi$

b)  $y \geq 0$  where  $y = (1 - 2 \sin x) \cos x$   $0 \leq x \leq \pi$

c)  $y \leq 0$  where  $y = (1 + \cos 2x) \cos 2x$   $0 \leq x \leq \pi$

d)  $\tan x \leq \sin 2x$   $0 \leq x \leq \pi$

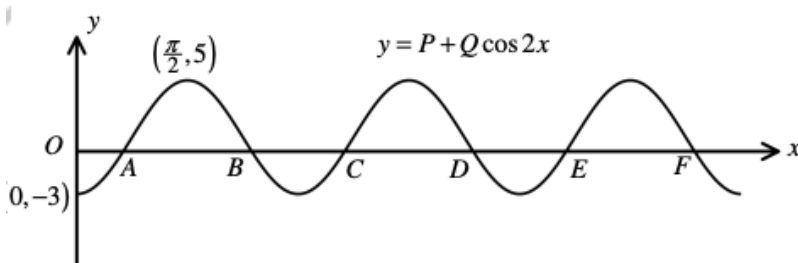
5. The figure shows part of the graph of the curve with equation  $y = A \cos(x + 60^\circ)$



The point  $P(0, 2)$  lies on the curve. Find the value of  $A$  and the coordinates of  $Q$ ,  $R$ , and  $S$ .

Find the coordinates of the points where the straight line with equation  $y = -2\sqrt{3}$  meets this graph.

6. The figure shows part of the graph of the curve with equation  $y = P + Q \cos 2x$   $x \geq 0$



The points  $(0, -3)$  and  $(\frac{\pi}{2}, 5)$  lie on the curve. Find the value of  $P$  and  $Q$ .

Find the coordinates of the points where the straight line with equation  $y = 3$  meets this graph for  $0 \leq x \leq 2\pi$ .

7. Sketch the graph of  $f(x) = 2 \cos 2x$  for  $0 \leq x \leq 2\pi$  and hence solve  $f(x) \leq 1$

8. A curve has equation  $y = A + k \sin x$ ,  $0 \leq x \leq 2\pi$ , where  $A$  and  $k$  are non-zero constants.

Given that the curve passes through  $(\frac{\pi}{6}, 1)$  and  $(\frac{7\pi}{6}, 5)$ , find the minimum and maximum value of  $y$ .

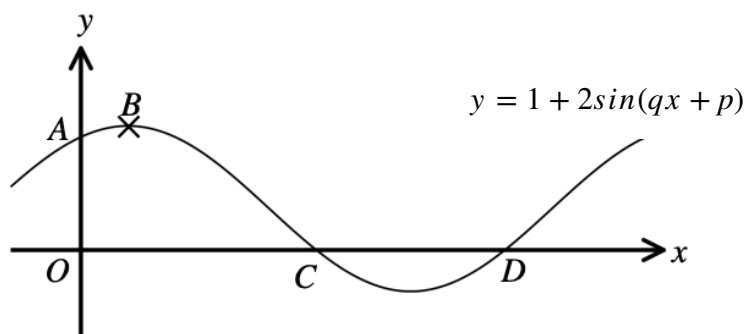
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9. The figure shows part of the graph of  $y = 1 + 2\sin(qx + p)$ ,

where  $q$  and  $p$  are positive constants, with  $0 < p < 90$ ,  $0 < q < 5$

The graph crosses the  $y$ -axis at the point  $A(0, 1 + \sqrt{3})$  and the  $x$ -axis at the points  $C(50, 0)$  and  $D$ , and point  $B$  is a maximum point on the curve.

Find the values of  $p$  and  $q$ , and the coordinates of  $B$  and  $D$



10.

a) Find the greatest value of the function  $f(x) = (2\cos^2(4x + 9) - 5)^2$  for  $x \in \mathbb{R}$

b) Find the largest value achieved by  $f(x) = 3\cos^2x + 2\sin x + 1$  for  $x \in \mathbb{R}$

c) Find the largest value of  $f(x) = (4\sin^2x + 4\cos x + 1)^2$  for  $x \in \mathbb{R}$

d) Find the minimum value of the function  $f(x) = 9\cos^4x - 12\cos^2x + 7$  for  $x \in \mathbb{R}$

e) Find the maximum value of  $9^{\sin x} - 4(3^{\sin x}) + \frac{13}{3}$  for  $x \in \mathbb{R}$

f) Find the minimum value of  $f(x) = \frac{\cos x + 4}{9 + 6\cos x - \sin^2 x}$  for  $x \in \mathbb{R}$

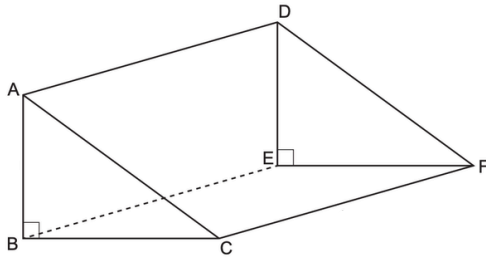
11. Find the equation of the new curves after the following transformations:

- a) The curve  $y = \cos x$  is stretched in the horizontal direction by a scale factor of  $\frac{1}{2}$ , and the resulting curve is translated by 4 units in the positive y-direction.
- b) The curve  $y = \sin x$  is stretched in the vertical direction by a scale factor of 2, followed by a translation by  $\frac{\pi}{3}$  units in the positive x-direction.
- c) The curve  $y = \tan x$  is stretched in the horizontal direction by a scale factor of 2, followed by a reflection in the x-axis, followed by a translation by 3 units in the positive y-direction.
- d) The curve  $y = \sin x$  is reflected in the y-axis, translated by 2 units in the negative y-direction and then translated by  $\frac{\pi}{4}$  units in the positive x-direction.
- e) The curve  $y = \cos x$  is reflected in the line  $y = 1$
- f) The curve  $y = \sin x$  is reflected in the line  $y = 2$  followed by a translation by  $\frac{\pi}{3}$  units in the negative x-direction.

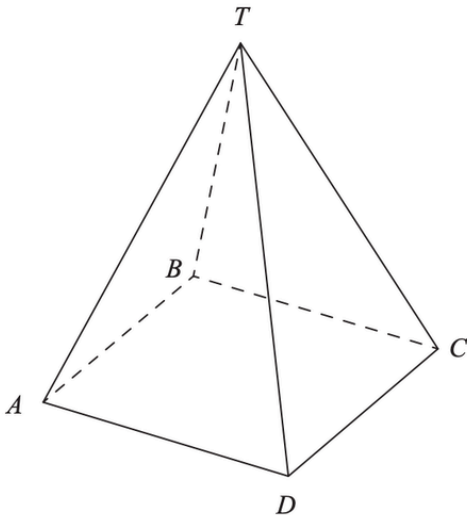
- 12a) The lengths of the sides QR, RP and PQ in triangle PQR are  $s + 1$ ,  $s$ , and  $\sqrt{3}s$ .  
Find the full range of values of  $s$  that make angle PRQ an obtuse angle.

- b) A triangle ABC has  $AB=10\text{cm}$ , angle  $BAC = 30^\circ$  and  $BC = k \text{ cm}$ , where  $k$  is a constant.  
Of the two possible triangles that could be drawn, the larger triangle has side AC twice as long as side AC in the smaller triangle. Find the value of  $k$ .

13. A solid right angled triangular prism, with base BEFC, has  $AB = BC = 2\text{m}$ , and  $AD = 5\text{m}$ .  
Find the shortest distance, in metres, along the outside of the prism from B to the midpoint of DF.

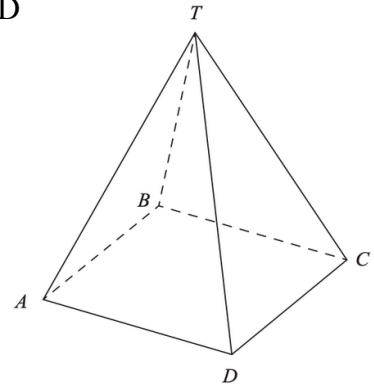


14. A square based pyramid, with base ABCD, and vertex T has all edges of length  $2\text{m}$ .  
Find the shortest distance, in metres, along the outer surface of the prism from the midpoint of AB to the midpoint of CT





15. A square based pyramid has base ABCD, where all the sides of the square are 12cm in length. The diagonals of the square intersect at O, and the vertex of the pyramid is at T, directly above O. Each of the sloping edges of the pyramid makes an angle of  $60^\circ$  with the base. Find the tangent of the angle between the face CDT and the base ABCD



16. A triangular-based pyramid PQRS has horizontal base PQR where  $PQ=PR=8\text{m}$  and  $RQ=12\text{m}$ . The vertex of the pyramid S lies directly above the level of PQR so that  $SQ=SR=10\text{m}$  and  $SP=8\text{m}$ . Find the shortest distance of S from the base PQR.