

THE UK UNIVERSITY  
INTEGRATION BEE

2022/23



**ROUND TWO RELAY ROUND**

Monday, 6 February 2023

Sponsored by



**Jane Street**

## Q1

Define

$$I = \int_0^1 \sqrt[a]{1-x^b} dx, J = \int_0^1 \sqrt[b]{1-x^a} dx$$

Is  $I > J$ ,  $J > I$  or  $I = J$ ?

## Q2

Evaluate

$$\int_{-\infty}^{\infty} \frac{1}{(x + \frac{1}{x})^2} dx$$

## Q3

Evaluate

$$\int_0^{\frac{\pi}{2}} x \ln(\sin x) dx$$

## Q4

Evaluate

$$\int_0^{\frac{\pi}{2}} \frac{dx}{a^2 - \sin^2 x}$$

Q5

Evaluate

$$\int_0^1 \frac{x^{2023} - x^{23}}{\ln x} dx$$

Q6

Evaluate

$$\int_1^3 \frac{\ln(x)}{x^2 + 3} dx$$

Q7

Evaluate

$$\int_0^\infty \frac{\sinh x - x}{x^2 \sinh x} dx$$

Q8

Evaluate

$$\int_{-1}^1 \frac{\cos x}{1 + e^{-\frac{1}{x}}} dx$$

Q9

Solve

$$f(x) - \int_0^x f(t) dt = 1$$

Q10

Evaluate

$$\int_0^\infty e^{-ax} Ei(x) dx$$

where

$$Ei(x) = \int_x^\infty \frac{e^{-t}}{t} dt$$

Q11

Evaluate

$$\lim_{x \rightarrow 0^+} \int_x^{2x} \frac{\sin^{n-1} t}{t^n} dt$$

Q12

Evaluate

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{k+n}$$

## Q13

How many solutions does the equation

$$\int_0^x \sin(\sin t) dt = 0$$

have for  $0 < x \leq 2\pi$ ?

## Q14

Evaluate

$$\int_0^{\frac{1}{2}} \left\lfloor \frac{1}{x} \right\rfloor \left\lceil \frac{1}{x} \right\rceil \ln \left( 1 - \frac{1}{\left\lfloor \frac{1}{x} \right\rfloor^2} \right) dx$$

## Q15

Evaluate

$$\int_0^\infty \cot^{-1}(x^{\frac{n}{2}}) dx$$

## Q16

Evaluate

$$\int_0^\infty e^{-x} \sqrt{1 - e^{-2x}} dx$$

## Q17

Evaluate

$$\int_0^\infty \frac{dx}{x^4 + 2ax^2 + 1}$$

Q18

Evaluate

$$\int_0^1 \sin(\cos^{-1}(x)) dx$$

Q19

Evaluate

$$\int_{-\infty}^{\infty} x^3 e^{-x^2} dx$$

Q20

Evaluate

$$\lim_{A \rightarrow \infty} \frac{1}{A} \int_1^A A^{\frac{1}{x}} dx$$