

# THE UK UNIVERSITY INTEGRATION BEE

2021/22



## ROUND TWO SHUTTLE ROUND

Saturday, 20 November 2021



UNIVERSITY OF  
CAMBRIDGE

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**Jane Street**

## SHUTTLE A

A1

Evaluate

$$\int_0^1 \ln^2(x) dx.$$

Pass on your answer squared.

*a is the number you will receive.*

A2

Evaluate

$$\int_0^{\frac{\pi}{2}} \frac{\sin(x)}{\sin(x + \frac{\pi}{a})} dx.$$

Your answer will be of the form  $\pi/b$ , pass on  $b$ .*b is the number you will receive.*

A3

Evaluate

$$\int_0^{\frac{\pi}{2}} \frac{dx}{1 + \tan^b(x)}.$$

Pass on 4 times your answer.

*c is the number you will receive.*

A4

Evaluate

$$\int_0^c x \ln(\sin x) dx.$$

## SHUTTLE B

B1

Evaluate

$$\int_0^{2\pi} \sin(\sin(x) - x) dx.$$

Pass on the answer.

*a is the number you will receive.*

B2

Evaluate

$$\int_a^\infty \ln\left(1 + \frac{169}{x^2}\right) dx.$$

Pass on the second digit of your answer.

*b is the number you will receive.*

B3

Evaluate

$$\int_0^1 \frac{\ln(b(x+1))}{x^2+1} dx.$$

Pass on your answer.

*c is the number you will receive.*

B4

Evaluate

$$\int_{\frac{1}{c}}^\infty \frac{\ln x}{x^2 + \pi^2} dx$$

## SHUTTLE C

C1

Evaluate

$$\int_{-1}^1 \frac{\sin(\cot^{-1} x) + \cos(\tan^{-1} x)}{x^2 + 1} dx$$

Pass on your answer squared minus 2.

*a is the number you will receive.*

C2

Evaluate

$$b = \int_0^a x^3 e^{-x} dx$$

Pass on 2 minus your answer.

*b is the number you will receive.*

C3

Evaluate

$$\int_0^\infty \frac{\ln(x^{\frac{1}{b}})}{x^{\frac{1}{b}}(x+1)^2} dx$$

Pass on the smallest integer greater than the absolute value of your answer.

*c is the number you will receive.*

C4

Evaluate

$$\int_0^{\frac{\pi}{2}} \frac{\cos \theta}{c - \sin 2\theta} dx$$

## SHUTTLE D

D1

Evaluate

$$\lim_{n \rightarrow \infty} \int_{\frac{5}{6}}^{\infty} e^{-x^n} dx$$

Your answer should be of the form  $\frac{p}{q}$  for coprime  $p, q \in \mathbb{Z}^+$ , pass on the prime factors of  $q$ .

$a, b$  are the numbers you will receive where  $a > b$ .

D2

Evaluate

$$\int_0^1 \frac{x^a - x^b}{\ln x} dx$$

Pass on the least integer bigger than your answer.

$c$  is the number you will receive.

D3

Evaluate

$$\int_0^1 \frac{1-x}{x^c + (x^c + 1)^c} dx$$

Your answer will be of the form  $\frac{r}{s}(\ln(t) - u)$  where  $r, s, t, u \in \mathbb{Z}^+$  and  $r, s$  are coprime.

Pass on  $\frac{r + \sqrt{r+s}}{u}$ .

$d$  is the number you will receive.

D4

Evaluate

$$\int_0^{2\pi} \frac{x}{d - \cos^2 x} dx$$

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