TMUA Practice - Algebra & Functions

1. Given that p and q are non-zero integers, the expression $\dfrac{(36^{p-q})(3^q)}{(12^{2p-q})(6^p)}$

is an integer if:

A $\quad p < 0$
B $\quad q < 0$
C $\quad p > 0$ and $q < 0$
D $\quad p > 0$ and $q > 0$
E $\quad p > 0$
F $\quad q > 0$

$$= \frac{2^{2p-2q} \cdot 3^{2p-2q} \cdot 3^{q}}{2^{4p-2q} \cdot 3^{2p-q} \cdot 2^{p} \cdot 3^{p}}$$

$$= 2^{2p-2q-4p+2q-p} \cdot 3^{2p-2q+q-2p+q-p}$$

$$= 2^{-3p} \cdot 3^{-p}$$ integer for $p < 0$ all q

2. Given that $m = 7^8$ and $n = 8^7$ which expression represents 56^{56}

A $\quad mn$
B $\quad (mn)^{56}$
C $\quad m^7 n^8$
D $\quad 8m^7 + 7n^8$
E $\quad (8m)^7(7n)^8$

$7^8 = m \quad 8^7 = n$

$7^{56} = m^7 \quad 8^{56} = n^8$

$56^{56} = 7^{56} \times 8^{56}$

$= m^7 n^8$

3. Find the set of values of x that satisfy both the following inequalities:

$$\frac{4x+1}{x-1} < 3 \qquad (x+2)(x-4) > 0$$

A $\quad x < -4$
B $\quad x > -4$
C $\quad -2 < x < 1$
D $\quad -4 < x < 4$
E $\quad -4 < x < -2$

$\dfrac{4x+1}{x-1} - 3 < 0$

$\dfrac{4x+1-3x+3}{x-1} < 0$

$\dfrac{x+4}{x-1} < 0$

either $x+4 > 0$, $x-1 < 0$

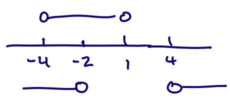
$-4 < x < 1$

or $x+4 < 0$, $x-1 > 0$
no solutions

$(x+2)(x-4) > 0$

$x^2 - 2x > 8$

$x < -2 \quad x > 4$



together
$-4 < x < -2$

4. Find the set of values of x that satisfy the following inequality:

$$\frac{3}{x+3} > \frac{x-4}{x}$$

-ve quadratic

$3(x+3)x^2 > (x-4)(x+3)^2 x$

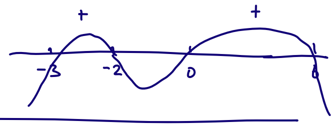
$(x+3)x\left(3x - (x^2-x-12)\right) > 0$

$(x+3)x\left(12+4x-x^2\right) > 0$

$(x+3)x(6-x)(x+2) > 0$

$-3 < x < -2$

$0 < x < 6$



A $-3 < x < 6$

B $-2 < x < 6$

C $-3 < x < -2$ and $0 < x < 6$

D $0 < x < 2$ and $3 < x < 6$

E $2 < x < 3$ and $5 < x < 6$

OR $\frac{3}{x+3} - \frac{x-4}{x} > 0$

$\frac{3x - x^2 + x + 12}{x(x+3)} > 0$

$\frac{x^2 - 4x - 12}{x(x+3)} < 0$

$\frac{(x-6)(x+2)}{x(x+3)} < 0$

-3	-2	0	6

$+ \quad - \quad + \quad - \quad +$

5. Find the set of values of x that satisfy the following inequality, where p is a positive constant:

$$\frac{x+p}{x+4p} < \frac{p}{x}$$

A $-2p < x < 2p$

B $0 < x < 2p$

C $x < -4p\ ,\ x > 0$

D $-4p < x < -2p\ ,\ 0 < x < 2p$

E $-4p < x < 0\ ,\ x > 2p$

$\frac{x+p}{x+4p} - \frac{p}{x} < 0$

$\frac{x^2+px-px-4p^2}{x(x+4p)} < 0$

$\frac{(x-2p)(x+2p)}{x(x+4p)} < 0$

$-4p$	$-2p$	0	$2p$

$+ \quad - \quad + \quad - \quad +$

$-4p < x < -2p$

$0 < x < 2p$

6. A cubic curve has equation $y = x^3 + kx - 2$ where k is a constant.
What value of k gives this curve exactly two distinct real roots

A -3

B -2

C -1

D 1

E 3

2 distinct roots $\Rightarrow$ repeated root (A) and single root (B)

$(x-A)^2(x-B) = (x^2 - 2Ax + A^2)(x-B)$

$= x^3 - x^2(2A+B) + x(A^2+2AB) - A^2B$

Comparing coefficients:

$2A + B = 0$

$B = -2A$ (1)

$A^2 + 2AB = k$ (2)

$A^2B = 2$ (3)

(1) in (3) $-2A^3 = 2$

$A^3 = -1$

$A = -1 \quad B = 2$

$(x+1)^2(x-2)$

(2) $k = 1 + 2(-1)(2) = -3$

7. The equation $2x^2 + 9x - k = 0$ where k is a constant has two distinct real roots.

One root is 4 more than the other root. The value of k is

A $\frac{55}{8}$ B $\frac{9}{2}$ C $-\frac{17}{8}$ D $-\frac{17}{4}$ E $-\frac{55}{8}$

Let roots $= a, a+4$

$(x-a)(x-a-4)=0$

$x^2-(a+a+4)x+a(a+4)=0$

$2x^2-(4a+8)x+2a(a+4)=0$

$4a+8=-9$

$a=-\frac{17}{4}$

$-k=-\frac{17}{2}\left(-\frac{1}{4}\right)$

$k=-\frac{17}{8}$

8. Find the minimum value of $2(2^{sinx}) - 4^{sinx} + \frac{10}{3}$

A $\frac{10}{3}$ B $\frac{13}{3}$ C $\frac{49}{12}$ D $\frac{20}{3}$ E 0

Let $y=2^{\sin x}$

$-1 \le \sin x \le 1$

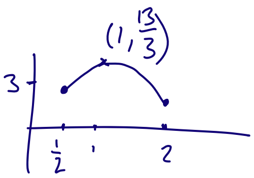
$\frac{1}{2} \le y \le 2$

$2y-y^2+\frac{10}{3}$

$\frac{10}{3}-(y^2-2y)$

$\frac{10}{3}-\left[(y-1)^2-1\right]$

$\frac{13}{3}-(y-1)^2$



Min at $y=2$

$\frac{13}{3}-1=\frac{10}{3}$

9. When $(2x^2 + 6x - 3)$ is multiplied by $(px - 1)$ and the resulting product is divided by $(x + 1)$ the remainder is 28.

The value of p is

A 3 B 2 C $\frac{7}{4}$ D $\frac{3}{2}$ E $\frac{28}{5}$

Let $f(x)=(2x^2+6x-3)(px-1)$

$f(-1)=28$

$(2-6-3)(-p-1)=28$

$7(p+1)=28$

$p=3$

10. The simultaneous equations below have two distinct real solutions

$$3x^2 - xy = 4 \text{ and } 2x - y = p \quad \text{where } p \text{ is a real constant}$$

What are the values that p can take

A there are no possible values for p

B $p < -4, p > 4$

C $-4 < p < 4$

D p can take any value

$y = 2x - p$

$3x^2 - 2x^2 + xp = 4$

$x^2 + px - 4 = 0$

$\Delta > 0$

$p^2 + 4(4) > 0$

$p^2 + 16 > 0$

true for all p

11. What is the sum of the solutions of the following equation

$$|x| - 3 = |2x + 12|$$

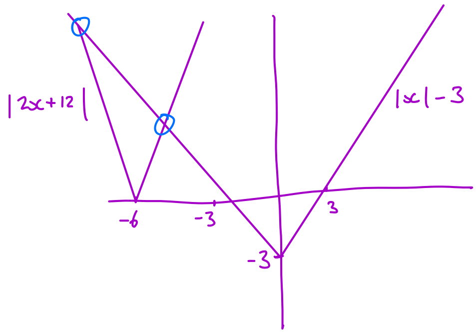
A - 14

B - 4

C 0

D 4

E 14



$-x - 3 = 2x + 12$

$3x = -15$

$x = -5$

$-x - 3 = -2x - 12$

$x = -9$

$-5 - 9 = -14$

12. How many solutions are there to the following equation:

$$|x| + |x - 1| = |x^3|$$

A 0

B 1

C 2

D 3

E 4

$|x-1| = |x^3| - |x|$

$x > 0 \quad |x^3| - |x| = x^3 - x$

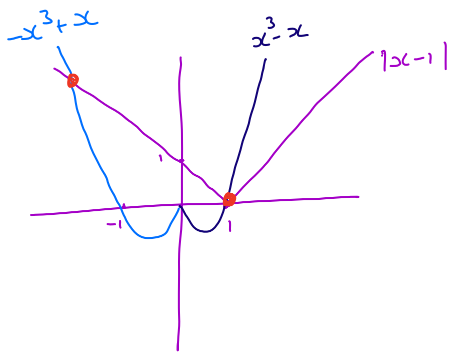
$= x(x^2 - 1)$

$= x(x-1)(x+1)$

$x < 0 \quad |x^3| - |x| = -x^3 + x$

$= -x(x^2 - 1)$

$= -x(x-1)(x+1)$



13. Given that $\left(a^3 + \frac{3}{b^3}\right)\left(b^3 - \frac{3}{a^3}\right) = 2\sqrt{3}$ where a, b are real numbers,

then the least value of ab is

A $-\sqrt{3}$

B $\sqrt{3}$

C $-3\sqrt{3}$

D $3\sqrt{3}$

E $-3^{\frac{1}{6}}$

F $3^{\frac{1}{6}}$

$a^3b^3 + 3 - 3 - \frac{9}{a^3b^3} = 2\sqrt{3}$

$x = ab$ $\quad x^3 - \frac{9}{x^3} = 2\sqrt{3}$

$y = x^3$

$y = (ab)^3$ $\quad y - \frac{9}{y} = 2\sqrt{3}$

$y^2 - 2\sqrt{3}y - 9 = 0$

$(y - 3\sqrt{3})(y + \sqrt{3}) = 0$

$y = 3\sqrt{3}$ $\quad y = -\sqrt{3}$

$x = \sqrt{3}$ $\quad = -3^{1/6}$

min value $ab = -3^{1/6}$

14. The function f is defined such that $3f(x) + 2f(-x) = 5x - 10$
find the value of $f(1)$

A 0

B 1

C 2

D 3

E 4

$3f(x) + 2f(-x) = 5x - 10$

$3f(-x) + 2f(x) = -5x - 10$

$9f(x) + 6f(-x) = 15x - 30$

$4f(x) + 6f(-x) = -10x - 20$

$5f(x) = 25x - 10$

$f(x) = 5x - 2$

$f(1) = 3$

15. The function f satisfies $2f(x) - f\left(\frac{2x+3}{x-2}\right) = 2x - 2, \quad x \in \mathbb{R}$

What is the value of $f(9)$

A 16

B 12

C 8

D -12

E -16

$x = 9$ $\quad 2f(9) - f(3) = 16$ ①

$x = 3$ $\quad 2f(3) - f(9) = 4$

① ×2 $\quad 4f(9) - 2f(3) = 32$

$2f(3) - f(9) = 4$

$3f(9) = 36$

$f(9) = 12$

$f(3) = 8$