THE UK UNIVERSITY INTEGRATION BEE 2023/24

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ROUND TWO RELAY

Sponsored by

Jane Street

1 Relay Problems

1. $\int_0^{\frac{\pi}{2}} \cos(x)^{\sin(x)^{\cos(x)}} - \sin(x)^{\cos(x)^{\sin(x)}}$
2. $\int_0^{\pi} \sup_{n\in\mathbb{N}} \left(\sin(2^n x)\right)\ \mathrm{d}x$
3. $\int_0^2 \ln(\Gamma(2-x))\,\mathrm{d}x$
4. Suppose $4f''(x) + 4f'(x) + f(x) = \dfrac{1}{e^x - 1}$, with $e^{\frac{x}{2}} f(x) \to 0$ as $x \to \infty$. Evaluate $\int_0^{\infty} f(x)\,\mathrm{d}x$.
5. $\lim_{n\to\infty} \sqrt[n]{\int_0^2 (1+x^4)^n\ \mathrm{d}x}$
6. $\int_0^{2\pi} \dfrac{\sin^2(5x)\sin^2(4x)}{\sin^2 x}\,\mathrm{d}x$
7. $\lim_{n\to\infty} \dfrac{1}{n^2} \int_0^{n^2} \sqrt{\dfrac{n^2 - \left\lfloor\sqrt{x}\right\rfloor^2}{x}}\,\mathrm{d}x$
8. Evaluate $\int e^x\,\mathrm{d}e$
9. $\int \sqrt{x\sqrt[3]{x\sqrt[4]{x\ldots}}}\,\mathrm{d}x$
10. $\int_{-r}^{r} \max\left\{\sqrt{r^2 - x^2}, |x|\right\}\ \mathrm{d}x$
11. Find $\lim_{n\to\infty} \dfrac{\sqrt[n]{(n+1)(n+2)\ldots(n+n)}}{n}$
12. $\int_{\frac{1}{8}}^{\frac{1}{4}} \dfrac{\ln(2x)\ln(4x)\ln(8x)}{x}\,\mathrm{d}x$
13. Evaluate $\lim_{x\to 0} \int_x^{2x} \dfrac{\tan^{n-1}(x)}{\sin^n(x)}\,\mathrm{d}x$
14. $\int_0^1 e^{x+e^x}\,\mathrm{d}x$
15. Find the volume of a unit 4 dimensional ball.
16. $\int_0^{\infty} \lfloor x \rfloor e^{-x}\,\mathrm{d}x$
17. $\int_0^1 \left\lfloor \cos\left(\dfrac{\pi}{x}\right)\right\rfloor\ \mathrm{d}x$
18. $\int_0^{\frac{\pi}{2}} \dfrac{\mathrm{d}x}{1 + \tan^{2024}(x)}$
19. $\lim_{n\to\infty} \int_0^n \cos^n\left(\dfrac{x}{\sqrt{n}}\right)\ \mathrm{d}x$

20. If $(f(x))^2 = \int_0^x \frac{tf(t)}{1+t^2}\,\mathrm{d}t$, compute: $\int_{-1}^{1} f(x)\,\mathrm{d}x$.