TMUA Practice - Graphs of Functions

1. Given that $f(x) = x^2 - 5x + 7$ $= (x - \frac{5}{2})^2 - \frac{25}{4} + \frac{28}{4} = (x - \frac{5}{2})^2 + \frac{3}{4}$

Find the sum of the x- and y- coordinates of the minimum point of $y = f(x-2)$

translation 2 units right

A $\frac{21}{4}$ B $\frac{13}{4}$ C $\frac{5}{4}$ D $\frac{1}{4}$ E $-\frac{7}{4}$

Min $f(x) = (\frac{5}{2}, \frac{3}{4})$

$f(x-2) = (\frac{9}{2}, \frac{3}{4})$

$\frac{9}{2} + \frac{3}{4} = \frac{21}{4}$

2. The curve with equation $x^9 + x^7 + y^4 + y^8 = 2$ has

A neither the x-axis nor y-axis as a line of symmetry
B the x-axis but not the y-axis as a line of symmetry
C the y-axis but not the x-axis as a line of symmetry
D both axes as lines of symmetry

$y \leftrightarrow -y$ same, as even powers of y

$x \not\leftrightarrow -x$ not same, as odd powers of x

3. How many solutions does the following equation have (where x is given in degrees)?

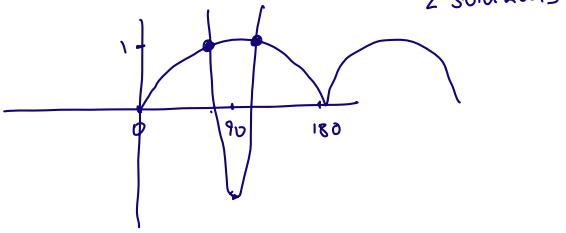
$$\sin^2 x = x^2 - 180x + 8099$$

A 0
B 1
C 2
D 4
E 8
F infinitely many

$\sin^2 x = x^2 - 180x + 8099$

$= (x - 90)^2 - 1$

2 solutions



4. How many regions are there (excluding the coordinate axes) when the following curves are drawn?

$y = x^2$ $\quad y = x^2 - 3x$ $\quad y = x^2 + 3x + 6$

$= x(x-3)$ $\quad = (x+\frac{3}{2})^2 + \frac{15}{4}$

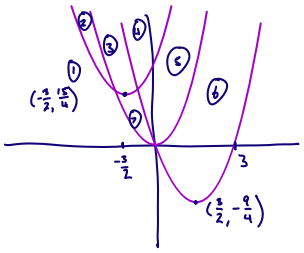
A 4

B 5

C 6

D 7

E 8

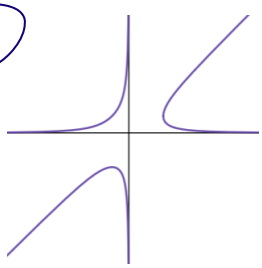


5. A sketch of the curve with equation $xy(x-y) = 1$ is drawn in:

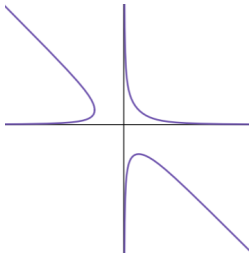
asymptotes
$x=0$, $y=0$, $x=y$
so not B, D

$x>0, y<0$
$x-y>0$
$xy<0$ X
so not C

A



B



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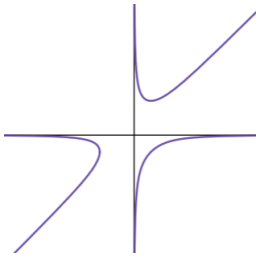
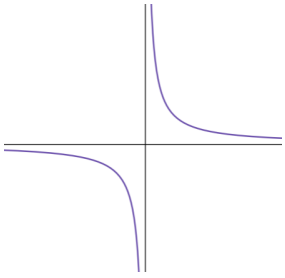


D



6. How many solutions does the following equation have

$$cos^2 x = x^3$$

A 0

B 1

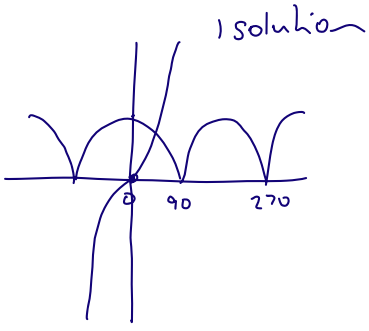
C 2

D 4

E 8

F infinitely many

1 solution



7. The graph of $y = 2^{x^2}$ has a series of transformations applied, resulting in the graph of $y = 2^{x^2+2x+4}$

Which of the following could be the sequence of transformations?

A a translation parallel to the x-axis, followed by a stretch parallel to the y-axis

B a translation parallel to the x-axis, followed by a translation parallel to the y-axis

C a translation parallel to the y-axis, followed by a stretch parallel to the y-axis

D a stretch parallel to the x-axis, followed by a translation parallel to the x-axis

E a stretch parallel to the x-axis, followed by a translation parallel to the y-axis

F a stretch parallel to the x-axis, followed by a stretch parallel to the y-axis

$y = 2^{(x+1)^2+3} = 2^3 \cdot 2^{(x+1)^2}$

2^{x^2} → translation 1 unit left x direction $2^{(x+1)^2}$ → stretch SF $2^3 = 8$ // b y axis

8. The graph of $y = f(x)$ interests the x-axis at exactly two distinct points.

Consider the following five graphs:

$y = f(x) - 3$ (×) translation down

$y = f(x-3)$ (✓) translation right

$y = 3f(x)$ (✓) stretch in y

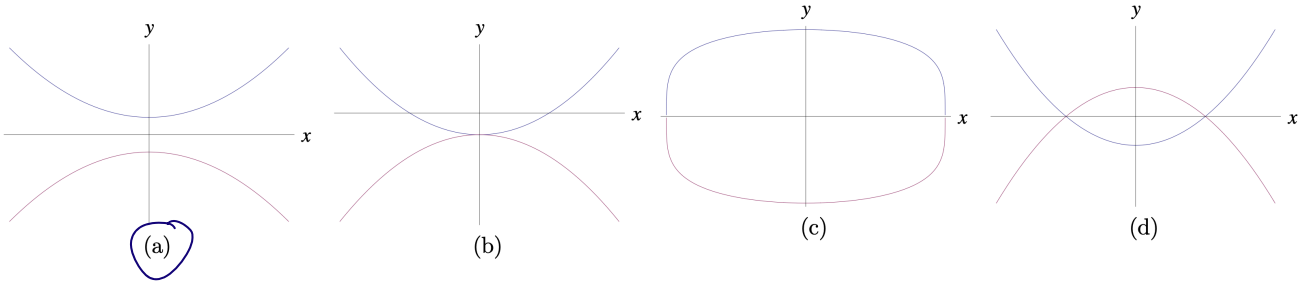
$y = 3 - f(x)$ (×) reflection in x-axis and translation up

$y = f(-3x)$ (✓) reflection in y-axis and stretch in x

How many of these graphs necessarily intersect the x-axis at exactly two distinct points?

A 0 B 1 C 2 D 3 E 4 F 5

9. Which of the following is a sketch of $y^2 - x^4 = 4$



$x^4 = y^2 - 4$

$x = 0$ $y = \pm 2$

$x^4 \geq 0$ $y^2 \geq 4$ $y \geq 2$, $y \leq -2$ so no solutions $-2 < y < 2$

10. The graph of a quadratic curve has equation $y = a + bx - x^2$

The image of the curve when reflected in the y-axis is identical to the image of the curve when translated 3 units in the negative x-direction. What is the value of b?

A $b = -3$

B $b = -1$

C $b = 1$

D $b = 3$

E $b = 9$

$f(-x) = f(x+3)$

$a - bx - x^2 = a + bx + 3b - (x+3)^2$

$0 = 2bx + 3b - 6x - 9$

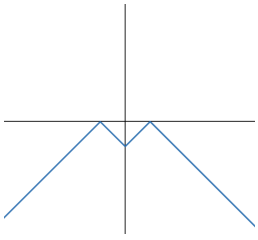
$0 = (2b-6)x + (3b-9)$

$b = 3$

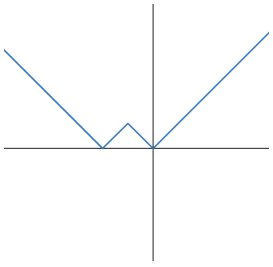
11. A sketch of the curve with equation $y = 1 - \left|1 - |x|\right|$ is drawn in:

(-1,1) (0,0) (1,1) (2,0)

A



B



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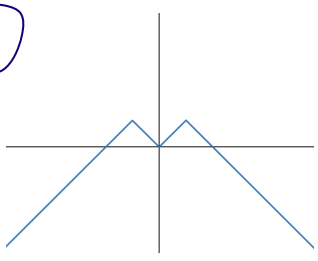
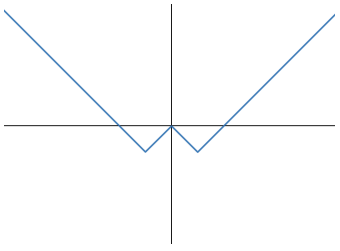
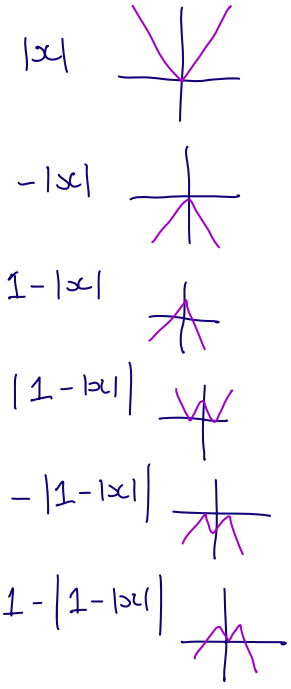


D





12. The function f is such that $f(x) = \dfrac{x-k}{x^2-4x-k}$, $x \in \mathbb{R}$ where k is a constant.

$x^2 - 4x - k \neq 0$

Given that the **range** of $f(x)$ is all the real numbers, what are the possible values of k?

A $-5 \leq k \leq 5$

B $k \leq -5, k \geq 5$

C $0 \leq k \leq 5$

D $k \leq 0, k \geq 5$

E $k = 0$ *or* $k = 5$

Let $y = \dfrac{x-k}{x^2-4x-k}$

$x^2y - 4xy - ky = x - k$

$x^2y - (4y+1)x + (k - ky) = 0$

quadratic in x must have $\Delta \geq 0$ in order that there is always a solution

$\Delta = (4y+1)^2 - 4y(k-ky) \geq 0$

$16y^2 + 8y + 1 - 4ky + 4ky^2 \geq 0$

$(16+4k)y^2 + (8-4k)y + 1 \geq 0$

quadratic in $y \geq 0$ must have $\Delta \leq 0$

$\Delta = (8-4k)^2 - 4(16+4k)(1) \leq 0$

$(2-k)^2 - 4 - k \leq 0$

$4 - 4k + k^2 - 4 - k \leq 0$

$k^2 - 5k \leq 0$

$k(k-5) \leq 0$

$0 \leq k \leq 5$

13. Consider the following function $f(x) = \dfrac{x^2 + 3x + 2}{x + 4}$ $= x - 1 + \dfrac{6}{x+4}$

What can be said about the asymptote(s) of the graph of this function?

$x \to \infty$
$f(x) \to x - 1$

A The graph has an asymptote at $x = -4$

B The graph has asymptotes at $x = -4$ and at $y = x - 1$

C The graph has asymptotes at $x = -4$ and at $y = x$

D The graph has asymptotes at $x = 0$ and at $y = x$

E The graph has asymptotes at $x = 0$ and at $y = \dfrac{1}{2}$

$x^2 + 3x + 2 = (x+4)(x-1) + 6$

14. The graph of a quadratic function $f(x)$ has a maximum point at (3,5)

The graph $y = f(x)$ is transformed onto the graph of $y = g(x)$ so that the graph of $g(x)$ has a minimum point at the origin. What is the equation for $g(x)$?

A $5 - f(x - 3)$

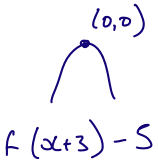
B $5 - f(x + 3)$

C $5 - f(3 - x)$

D $f(x + 3) - 5$

E $f(3 - x) - 5$

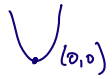
Translate left 3 down 5



Reflect in x-axis

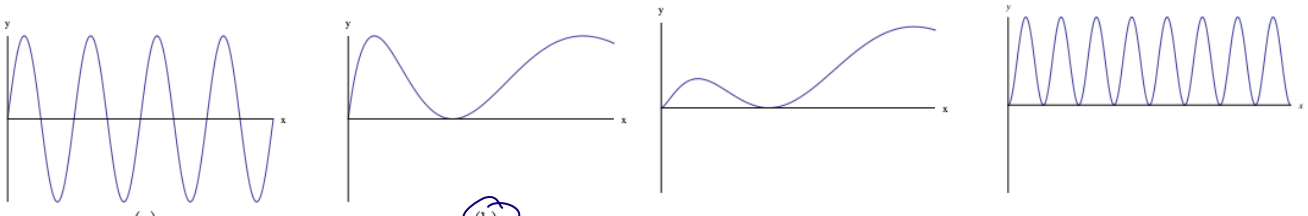
$-[f(x+3) - 5]$

$5 - f(x+3)$



15. Which is the graph of $y = sin^2\sqrt{x}$

$y > 0$ $(0,0)$ $y = 0$ $\sqrt{x} = 0, \pi, 2\pi$ $x = 0, \pi^2, 4\pi^2, 9\pi^2$



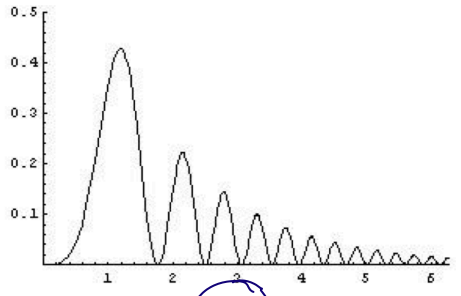
× $y > 0$

× peaks not same

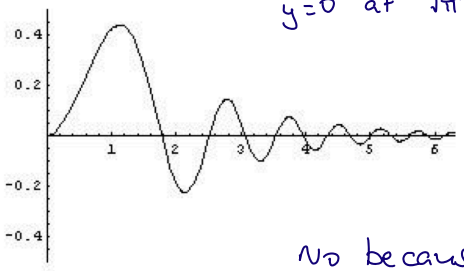
× roots too regular

16. Which of the following is a sketch of the graph $y = 2^{-x}sin^2(x^2)$

$y > 0$ $y(0)=0$
$y=0$ at $\sqrt{\pi}, \sqrt{2\pi}, \sqrt{3\pi}$

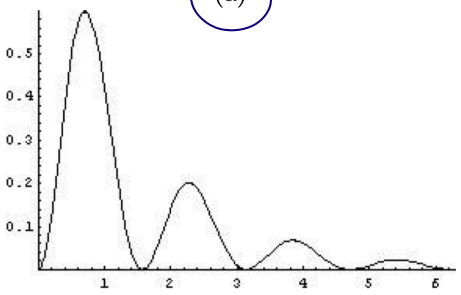


(a)



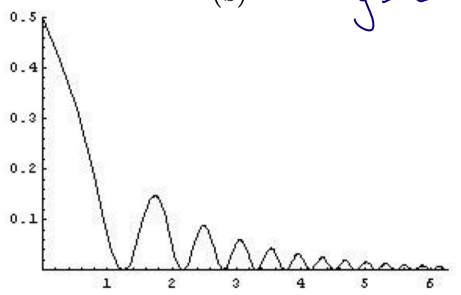
(b)

No because $y > 0$



(c)

looks too regular

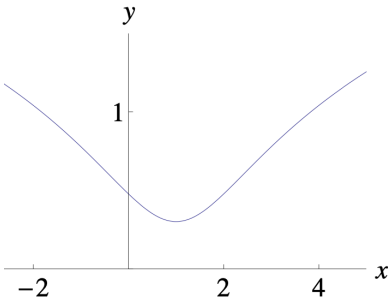


(d)

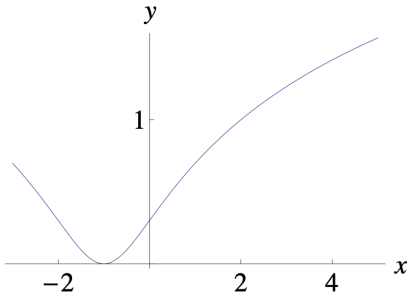
not (0,0)

17. Which of the following is a sketch of $y = log_{10}(x^2 - 2x + 2)$ $= \log\left[(x-1)^2+1\right]$ valid for all x

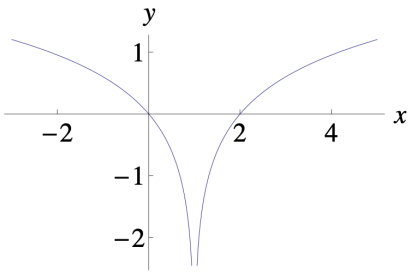
min at $x=1$ $(1,0)$



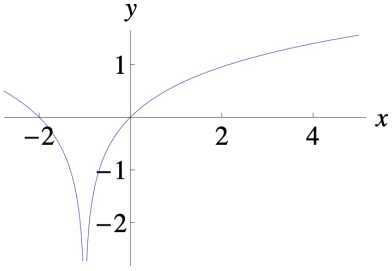
(a)



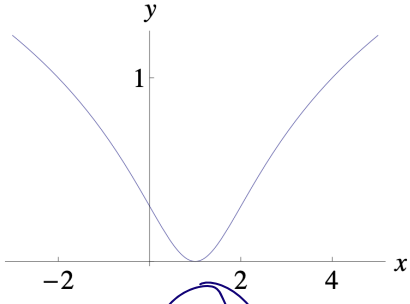
(b)



(c)



(d)



(e)