TMUA Practice - Logarithms and Exponentials

1. Given that $5^a = 32$ and $2^b = 125$, find the value of ab

A $\frac{log5}{log2}$ B $\frac{5}{2}$ C $\frac{15}{2}$ D 10 E 15

$\log 5^a = \log 2^5$
$a \log 5 = 5 \log 2$
$a = \frac{5 \log 2}{\log 5}$

$\log 2^b = \log 5^3$
$b \log 2 = 3 \log 5$
$b = \frac{3 \log 5}{\log 2}$

$ab = \frac{5 \log 2}{\log 5} \times \frac{3 \log 5}{\log 2} = 15$

2. Find the product of the real roots of the equation

$$\left(log_{10}x^2\right)^2 + log_{10}x = 3$$

A $-\frac{3}{4}$ B 10^{-1} C $10^{-\frac{1}{4}}$ D $\frac{3}{4}$ E $10^{\frac{1}{3}}$

Let $k = \log_{10} x$ $2k = \log_{10} x^2$
$(2k)^2 + k = 3$
$4k^2 + k - 3 = 0$
$(4k - 3)(k + 1) = 0$

$k = \frac{3}{4}$
$\log_{10} x = \frac{3}{4}$
$x = 10^{3/4}$

$k = -1$
$\log_{10} x = -1$
$x = 10^{-1}$

$10^{3/4} \times 10^{-1} = 10^{-1/4}$

3. Given $log_a y = \frac{1}{3}$ and $log_8 a = x + 1$ Express y in terms of x

A $y = x^{1/3}$ B $y = x^3 + 2$ C $y = 2^{x+1}$ D $y = 8^{x+1}$ E $y = 2^{x+\frac{1}{3}}$

$y = a^{1/3}$

$a = 8^{x+1}$
$a = 2^{3(x+1)}$

$y = 2^{\frac{1}{3}3(x+1)}$
$= 2^{x+1}$

4. In the following equation $x log 9 + log 28 = log 12 + x log 49$ the value of x is

A $\frac{log 7}{log 3}$ B $\frac{1}{2}$ C 2 D $\frac{7}{3}$ E 3

$$\log 28 - \log 12 = x(\log 49 - \log 9)$$
$$\log \frac{7}{3} = x \log \left(\frac{7}{3}\right)^2$$
$$1 = 2x$$
$$x = \frac{1}{2}$$

5. Given that x and y satisfy the following simultaneous equations

$$log_y x = 5 \qquad log_2 x = 2 + log_2 y$$

what is the value of $x + y$

A $\sqrt{2}$ B 2 C $4\sqrt{2}$ D $5\sqrt{2}$ E $2+\sqrt{2}$

$x = y^5$

$\log_2 x - \log_2 y = \log_2 4$

$\frac{x}{y} = 4$

$4y = y^5$

$y \neq 0 \quad y^4 = 4$

$y = \sqrt{2}$

$x = (\sqrt{2})^5 = 4\sqrt{2}$

$x + y = 5\sqrt{2}$

6. Given that x and y satisfy the following simultaneous equations

$$log_2(y-1) = 1 + log_2 x \qquad 2 log_3 y = 2 + log_3 x$$

the sum of the smallest solutions for x and y is

A $\frac{1}{4}$ B $\frac{5}{4}$ C $\frac{3}{2}$ D $\frac{7}{4}$ E 4

$\log_2 \frac{y-1}{x} = \log_2 2$

$y - 1 = 2x$

$y = 2x + 1$

$\log_3 \frac{y^2}{x} = \log_3 9$

$y^2 = 9x$

$4x^2 + 4x + 1 = 9x$

$4x^2 - 5x + 1 = 0$

$(4x - 1)(x - 1) = 0$

$x = \frac{1}{4} \quad x = 1$

$y = \frac{3}{2} \quad y = 3$

$\frac{1}{4} + \frac{3}{2} = \frac{7}{4}$

7. Find the sum of the real solutions of the equation

$$log_2 x = \frac{2}{log_2 x} + 1$$

A $\frac{9}{2}$ B 4 C $\frac{7}{2}$ D 2 E $\frac{3}{2}$

Let $a = \log_2 x$

$a^2 = 2 + a$

$a^2 - a - 2 = 0$

$(a+1)(a-2) = 0$

$a = -1 \quad a = 2$

$\log_2 x = -1$

$x = \frac{1}{2}$

$\log_2 x = 2$

$x = 4$

$4 + \frac{1}{2} = \frac{9}{2}$

8. Given that 6^{4x-3} can be written as 216^a what is a in terms of x

A $12x - 9$ B $\frac{4x-3}{3}$ C $4x - 1$ D $\sqrt[3]{4x-3}$

$216 = 6^3$

$6^{4x-3} = 6^{3a}$

$4x - 3 = 3a$

$a = \frac{4x-3}{3}$

9. $(log_{\frac{1}{2}}2)(log_{\frac{1}{3}}3)(log_{\frac{1}{4}}4).....(log_{\frac{1}{1000}}1000)$ is equal to:

A 2 B 1 C 0 D ± 1 E -1

$\log_{\frac{1}{2}} 2 = k$

$(\frac{1}{2})^k = 2$

$k = -1$

$\log_{\frac{1}{n}} n = k$

$(\frac{1}{n})^k = n$

$k = -1$

2 ... 1000 – odd number of brackets

$(-1)^{odd} = -1$

10. The following three numbers are consecutive terms in an arithmetic progression

$$\log_{10}2 \qquad \log_{10}(2^x - 1) \qquad \log_{10}(2^x + 3)$$

what is the value of x

A 2^5 B 5 C $\log_2 5$ D $\log_5 2$ E $\log_{10}\frac{5}{2}$

$\log_{10}(2^x-1) - \log_{10}2 = \log_{10}(2^x+3) - \log_{10}(2^x-1)$

$\frac{2^x-1}{2} = \frac{2^x+3}{2^x-1}$

Let $a = 2^x$

$a^2 - 2a + 1 = 2a + 6$

$a^2 - 4a - 5 = 0$

$(a-5)(a+1) = 0$

$a = 5 \quad a = -1$ (no solutions)

$2^x = 5$

$x = \log_2 5$

11. The positive real numbers a and b satisfy the following simultaneous equations

$$\log_2 4a - \log_2 b = 4 \qquad \log_2 a + \log_2 2b = 3$$

what is the value of $2a + b$

A 2 B 4 C 5 D 9 E 12

Let $A = \log_2 a \quad B = \log_2 b$

$\log_2 4 + A - B = 4$

$2 + A - B = 4$

$A - B = 2$

$A + \log_2 2 + B = 3$

$A + 1 + B = 3$

$A + B = 2$

$2A = 4$

$A = 2 \quad B = 0$

$a = 4 \quad b = 1$

$2a + b = 9$

12. The number of positive solutions x to the equation

$$\log_2 x = \log_2(x + a) + b$$

where a, b are non-zero real numbers, is

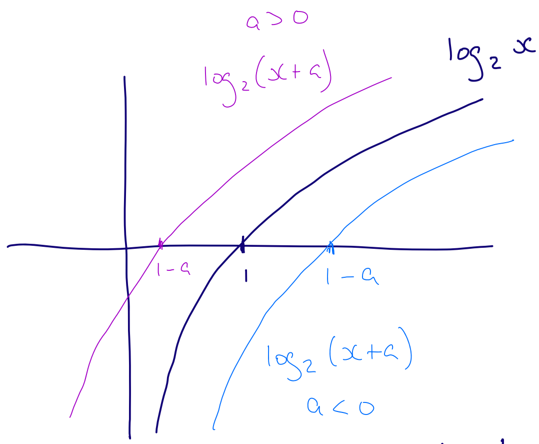
A zero if $ab < 1$, or ~~one~~ if $ab > 1$

B one if $ab < 1$, or ~~two~~ if $ab > 1$

C one if $ab < 0$, or zero if $ab > 0$

D zero if $ab < 0$, or ~~one~~ if $ab > 0$

E one if $ab < 1$, or ~~zero~~ if $ab > 1$



Translation 'a' left & 'b' up

no solutions if ab both -ve
both +ve

$\Rightarrow$ zero if $ab > 0$

13. Let $a, b, c > 0$. The equations: $log_a b = c$ $\quad log_b a = c + \frac{3}{2}$ $\quad log_c a = b$

A specify a, b and c uniquely

B specify c uniquely but have infinitely many solutions for a and b

C specify a and b uniquely but have infinitely many solutions for c

D have no solutions for a, b and c

E have infinitely many solutions for a, b and c

$b = a^c \qquad a = b^{c+3/2} \qquad a = c^b$

$b = a^c = b^{c^2 + \frac{3c}{2}}$

$1 = c^2 + \frac{3c}{2}$

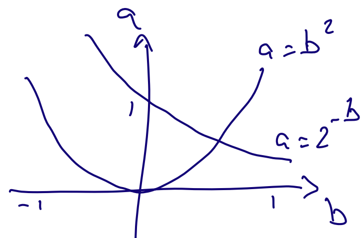
$2c^2 + 3c - 2 = 0$

$(2c-1)(c+2) = 0 \qquad c = \frac{1}{2}$

$a = b^2$

$a = (\frac{1}{2})^b$

$a = 2^{-b}$



single solution $b^2 = 2^{-b}$

14. The equation $log_b((b^x)^x) + log_a(\frac{c^x}{b^x}) + log_a(\frac{1}{b}) log_a c = 0$ has a repeated root when:

A $b^2 = 4ac$ $\qquad$ B $b = \frac{1}{a}$ $\qquad$ C $c = \frac{1}{b}$ $\qquad$ D $c = \frac{b}{a}$

$x^2 + x(\log_a c - \log_a b) - \log_a b \log_a c = 0$

$(x + \log_a c)(x - \log_a b) = 0$

$-\log_a c = \log_a b \qquad b = \frac{1}{c}$

15. If $\frac{loga}{b-c} = \frac{logb}{c-a} = \frac{logc}{a-b} = k$ then $(a^{b+c})(b^{c+a})(c^{a+b}) =$

A -1 $\qquad$ B 1 $\qquad$ C abc $\qquad$ D 0 $\qquad$ E $a+b+c$

$\log a = k(b-c)$

$\log b = k(c-a)$

$\log c = k(a-b)$

consider $\log(\_\_\_)$

$= \log a^{b+c} + \log b^{c+a} + \log c^{a+b}$

$= (b+c)k(b-c) + (c+a)k(c-a) + (a+b)k(a-b)$

$= k(b^2 - c^2 + c^2 - a^2 + a^2 - b^2) = 0$

$\log(\_\_\_) = 0$ so expression $= 1$