Differentiation

Syllabus

Derivative as gradient of tangent (rate of change, second order derivatives, notation); differentiation of x^n; application to tangents, normals, stationary points, strictly increasing and decreasing functions.

1. Differentiate the following expressions with respect to x

a) $\dfrac{2x + 4x^2}{\sqrt{x}}$

b) $\dfrac{1 - \sqrt{x}}{4x^3}$

c) $2\sqrt{x}\left(\dfrac{5}{x} + x^2\right)$

d) $\dfrac{(3 + 2\sqrt{x})^2}{4x}$

e) $\dfrac{(2x - 1)(x^2 + 4)}{2\sqrt[3]{x}}$

2. Find the equation of the tangent to the curve at the point given

a) $y = 2\sqrt{x} - \dfrac{6}{\sqrt{x}}$ where $x = 4$

b) $y = 3x^{\frac{3}{2}} - \dfrac{32}{x}$ where $x = 4$

Find the equation of the normal to the curve at the point given

c) $y = x^2(x - 6) + \dfrac{5}{x} - 1$ where $x = 1$

d) $y = 2x^2 - 4x^{\frac{3}{2}} - \dfrac{8}{x} - 1$ where $x = 4$

e) The tangent to the curve $y = x^3 - x$ at the point P (1,0) meets the curve again at the point Q.

What is the distance PQ?

f) The normal to the curve $y = (x - 1)(x^2 + 4x + 5)$ at the point where $x = -1$ meets the coordinate axes at the points P and Q.

What is the area of triangle OPQ, where O is the origin?

3. Find the coordinates of the stationary point(s) of the following equations, and determine if they are maximums, minimums, or points of inflexion.

a) $y = x^3 - 3x^2 - 9x + 3$

b) $y = x^2 + \frac{16}{x}$

c) $y = 3x^4 + 16x^3 + 24x^2 + 3$

4 Find the range of values of x, for which y is a decreasing function

a) $y = x^3 - 3x^2 - 9x + 10$

b) $y = 6x + 3x^2 - 4x^3$