

# THE UK UNIVERSITY INTEGRATION BEE

2021/22



## ROUND ONE

Sunday, 17 October 2021



UNIVERSITY OF  
CAMBRIDGE

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**Jane Street**

1.  $\int \sqrt{x^3 \sqrt{x^4 \sqrt{x^5 \sqrt{x \cdots}}}} dx$
2.  $\int_0^{2\pi} \cos^{420}(x) dx$
3.  $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos \theta}}{(\sqrt{\cos \theta} + \sqrt{\sin \theta})^5} d\theta$
4.  $\int_0^1 \frac{\sin(\ln x)}{\ln x} dx$
5.  $\int_0^4 \frac{\ln(x)}{\sqrt{4x - x^2}} dx$
6.  $\int_0^\infty \left( \frac{\ln x}{1+x} \right)^2 dx$
7.  $\int_0^\infty \frac{\ln(x)}{x^2 + 2x + 2} dx$
8.  $\int_{-\infty}^\infty \frac{dx}{(x^3 + \frac{1}{x^3})^2}$
9.  $\int \frac{x-1}{(x+1)\sqrt{x^3+x^2+x}} dx$
10.  $\int_0^{\frac{\pi}{2}} \frac{x}{\sin(x)} dx$
11.  $\int_1^\infty \left( \frac{\ln x}{x} \right)^{2011} dx$
12.  $\int_0^\infty \frac{\sin x}{x^n} dx \quad (0 < n < 2)$
13.  $\int_0^{\frac{\pi}{2}} \frac{\cos x}{(1 + \sqrt{\sin(2x)})^2} dx$
14.  $\int_0^{\frac{\pi}{2}} \ln(7997 \sin^2 \theta + 7945 \cos^2 \theta) d\theta$
15.  $\int \frac{dx}{\csc x + 1}$
16.  $\int_0^{\frac{\pi}{2}} \frac{\{\tan(x)\}}{\tan(x)} dx$ , where  $\{x\}$  is the fractional part of  $x$
17.  $\int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{2 + \sin x} dx$
18.  $\int \sqrt{\frac{1}{x} - 1} dx$
19.  $\int_0^{2\pi} e^{3\cos \theta} \cos(3 \sin \theta) d\theta$

20.  $\int \sqrt{x^2 - 1} dx$
21.  $\int_0^a \frac{x dx}{\cos(x) \cos(a - x)}$
22.  $\int_0^1 \frac{\arctan x}{1 + x} dx$
23.  $\int_0^{\frac{\pi}{2}} \frac{\ln(\cos x)}{\sin x} dx$
24.  $\int \frac{x \arcsin x}{\sqrt{1 - x^2}} dx$
25.  $\int_0^\infty \frac{\arctan(x)}{x(\log(x)^2 + 1)} dx$
26.  $\int_1^e \frac{x - \ln x + 1}{x(x + 1)^2 + x \ln^2 x} dx$
27.  $\int_0^\infty \frac{\cos(ax)}{x^2 + b^2} dx$
28.  $\int_1^\infty \frac{dx}{x + x^{\sqrt{2}}}$
29.  $\int_0^\infty \lfloor x \rfloor e^{-x} dx$
30.  $\int_0^1 \frac{\arctan(x^2)}{1 + x^2} dx$
31.  $\int_0^1 \frac{dx}{1 + \lfloor \frac{1}{x} \rfloor}$
32.  $\int_0^\infty \frac{\cos(ax) - \cos(bx)}{x^2} dx$
33.  $\int_0^{2\pi} e^{\cos x} \cos(\sin x) \cos(5x) dx$
34.  $\int_{-\infty}^\infty \frac{dx}{\cosh^2(x)}$
35.  $\int_0^e W(x) dx$ , where  $W$  is the Lambert  $W$  function, the solution to  $W(x)e^{W(x)} = x$
36.  $\int_0^1 \frac{\arctan^2 x}{x} dx$
37.  $\int_0^{\frac{\pi}{2}} \cos x^{\sin x \cos x} - \sin x^{\cos x \sin x} dx$
38.  $\int_3^5 \ln \Gamma(x) dx$
39.  $\int \frac{dx}{\sin^4(x) + \cos^4(x)}$
40.  $\int_0^\infty \frac{\cos(\ln x)}{(1 + x)^2} dx$