20 YEARS OXBRIDGE APPLICATIONS

TEST FOR MATHEMATICS FOR UNIVERSITY ADMISSION (TMUA)

MOCK TEST 4 ANSWERS

Section 1

1. D
Apply ln to both sides and use implicit differentiation

2. D
Pepper has 6 letters, 2 repeat es and 3 ps. $\frac{6!}{3!2!}$

3. C
Use the formula for the sum of an arithmetic sequence for 6 and 8 equating both sides, then isolate a/d

4. B
Simplify the fraction using rules of exponentiation and notice the exponent is a polynomial in 5^x. Since one of the solutions is negative it can be discarded

5. A
Simply apply the transformations in order being careful with where the negative signs go

6. E
Expand out the formula for f (n+2). Since f(n) >= 1, $(\sqrt{1-f(n)})^2 = f(n) - 1$, then simplify to get the final answer

7. B
Notice how the root bounds the sign function touching where sin is 1

8. E
For this question replace liking a subject with a variable for simplicity. Then apply the rule $(P \to Q) \to (\neg Q \to \neg P)$ to check which of the 5 statements is logically necessary.

9. C
This is a simple application of the law of conditional probability.
$P(Chose\ two\ headed\ coin \mid Heads) = \frac{P(Heads\ and\ Choosing\ two\ headed)}{P(Heads)} = \frac{\frac{2}{3}}{\frac{1}{3}} = \frac{1}{3}$

10. D
A point plus a vertex uniquely determines a parabola. Use the formula for the x value of the vertex -b/2a.

11. C
Each of the x coefficients in the polynomial is the degree so the answer is simply the sum of 1 to 7, 0.5*7*8 = 28

12. C
Simplify the expression by elevation (x-1)^2 to both sides of the inequality and simply. The resulting polynomial has two solutions.

13. E
A correct bracketing is formed by x(y) where x and y are correct bracketing. Since you only have one combination with 2 brackets you can recursively build up to 8. See Catalan numbers.

14. C
Calculate the probability of getting no coins in a row and subtract away from 1. We do not care what the first coin is but after that all outcomes are determined. So, the final result is 1 – 2^-4.

15. C
The minimum is obtained when all lines are parallel and the maximum when the gradients are all different and there are no triple intersects.

16. B
Calculate the distance between the centre of the circles using Pythagoras and subtract the radii.

17. B
Same method as question 1

18. A
Complete the square on both summands to get rid of the outer radicals

19. C
Use the formula of x-value of the vertex being -b/2a and plug it back in to the formula to get the y-value, then require it to be greater or equal to -4

20. E
Calculate the set of solutions for both sin 2x and cos 2x and take the union

Section 2

1. B
$y = \frac{x}{x+y}$ $since\ x > 0 \ \rightarrow y(x+y) = x$ then use implicit differentiation to get the result

2. E
$$\frac{(14C10) + (16C10)}{(30C10)}$$

3. A
The hyperbola and the circle intersect at 4 points. The restrictions on k and c uniquely

determine the point in the upper left.

4. A
Denote x the segment of the base from the bottom left corner to the first square. You can extract two equations involving x and a from the left triangle with sides x and a, and by noticing the same triangle on the right-hand side is similar so $x + a + 2a + 3a + 3x = 1$.

5. E
r^3 is the fourth term divided by the first. Then use the formula for the sum to infinity of a/(1-r).

6. A
5x5 – 7x3 = 4. Therefore, the coefficient is 12C5.

7. C
Use the formula for the sum to infinity to formulate two simultaneous equations obtaining value for a and r. Plug them back in to get the result for the cube of the terms.

8. B
1 is a stable attractor of f so any value between (0,2) will give 1 when you apply f infinite many times.

9. D
Taking the square root on both sides does not always work as you have to select which solution you want.

10. D
Calculate the derivative using the product rule to determine the maxima and plug the values back in to check which y value is maximal.

11. D
The important part here is to notice some of the singularities of tan 2x are removable in the overall formula. Expand tan 2x in terms of sin x and cos x then calculate the roots.

12. B
Write out the remainders of the first 10 square numbers when divided by 7. By properties of congruences this will also be true for the next 10 numbers and so forth. Since 3 of the 10 have a remainder of 4 the chance to randomly pick one with remainder 4 is 3/10.

13. B
Decompose each of the numbers into their prime factors and obtain a simplification as a product of powers of 2, 3 and 5. Then require all exponents to be positive.

14. D
A 0 implies the number is divisible by 10. To calculate the number of 0s we need to know the largest power of 10 than we can divide 143! by. Since more numbers are even than divisible by 5

it is sufficient to find the power of 5 in its prime decomposition. This can be calculated using the formula $\left\lfloor\frac{143!}{5}\right\rfloor + \left\lfloor\frac{143!}{25}\right\rfloor + \left\lfloor\frac{143!}{125}\right\rfloor$

15. A
Since only one is correct it must be the most general condition.

16. A
Apply chain rule multiple times

17. C
Minima represent points where the derivative goes from being negative to positive

18. A
Use integration by parts and a variable substitution

19. E
The statement is true therefore none of them are a counter example

20. B
At x = 0, there are infinitely many possibilities for y so C, D are discarded. Since no transformation is done inside the sine function it is not possible to obtain the bending in A so the answer is B