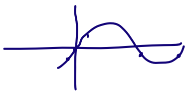
1. What is the largest solution for x in the range $0 \leq x < 2\pi$ for the following equation:



$$2sin\left(2x - \frac{\pi}{3}\right) + 1 = 0$$

$-\frac{\pi}{3} \leq 2x - \frac{\pi}{3} \leq \frac{11\pi}{3}$

A $\frac{\pi}{12}$ B $\frac{3\pi}{4}$ C $\frac{13\pi}{12}$ D $\frac{7\pi}{4}$ E $\frac{23\pi}{12}$

$\sin\left(2x - \frac{\pi}{3}\right) = -\frac{1}{2}$

largest

$2x - \frac{\pi}{3} = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$

out of range

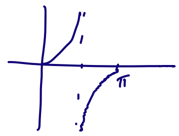
$2x = \frac{19\pi}{6} + \frac{\pi}{3} = \frac{21\pi}{6} = \frac{7\pi}{2}$

$x = \frac{7\pi}{4}$

2. What is the sum of the solutions for x in the range $0 \leq x < \pi$ for the following equation:

$$tan\left(2x - \pi\right) = 1$$

$-\pi \leq 2x - \pi < \pi$



A $\frac{\pi}{8}$ B $\frac{5\pi}{8}$ C $\frac{3\pi}{4}$ D $\frac{5\pi}{4}$ E $\frac{7\pi}{4}$

$2x - \pi = -\frac{3\pi}{4}, \frac{\pi}{4}$

$2x = \frac{\pi}{4}, \frac{5\pi}{4}$

$x = \frac{\pi}{8}, \frac{5\pi}{8}$

$\frac{\pi}{8} + \frac{5\pi}{8} = \frac{6\pi}{8} = \frac{3\pi}{4}$

3. How many solutions does the following equation have in the range $0 \leq x < 2\pi$

$$2sin(cosx) = \sqrt{2}$$

A 0

B 1

C 2

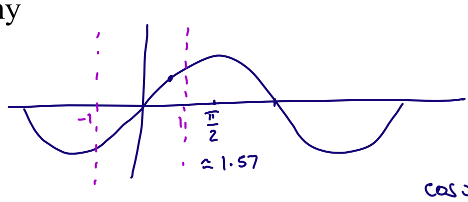
D 3

E infinitely many

Let $A = \cos x$ $\quad -1 \leq \cos x < 1$

Solve $2\sin A = \sqrt{2}$ for $-1 \leq A < 1$

$\sin A = \frac{\sqrt{2}}{2}$



$\frac{\pi}{4} < 1$

1 solution $A = \frac{\pi}{4}$

$\cos x = \frac{\pi}{4}$ has 2 solutions $0 \leq x < 2\pi$

4. x satisfies the simultaneous equations $2\sqrt{2}sin3x - tan3x = 3$

$\sqrt{2}tan3x + 4sin3x = \sqrt{2}$

where $0 \le x \le 180$.

Find the sum of the possible values of x

A 150

B 210

C 315

D 360

E 540

$2\sqrt{2}S - T = 3 \quad (1)$

$4S + \sqrt{2}T = \sqrt{2}$

$\div\sqrt{2}$

$2\sqrt{2}S + T = 1 \quad (2)$

Let $S = \sin 3x \quad T = \tan 3x$

$(1)+(2) \quad 4\sqrt{2}S = 4 \quad S = \frac{\sqrt{2}}{2}$

$(1)-(2) \quad -2T = 2 \quad T = -1$

$\sin 3x = \frac{1}{2}\sqrt{2}$

$\tan 3x = -1$

$0 \le 3x \le 540$

$\sin 3x = \frac{1}{2}\sqrt{2} \quad : \quad 3x = 45, 135, 405, 495$

$\tan 3x = -1 \quad : \quad 3x = 135, 495$

$x = 45, 165$

$\text{Sum} = 45 + 165 = 210$

5. Consider the inequality $sin(x + \frac{\pi}{3}) \ge \frac{1}{2}$

The fraction of the interval $0 \le x \le 2\pi$ for which this is true, is:

$\frac{\pi}{3} \le x + \frac{\pi}{3} \le \frac{7\pi}{3}$

A $\frac{1}{6}$ B $\frac{1}{4}$ C $\frac{1}{3}$ D $\frac{5}{12}$ E $\frac{1}{2}$

*Same fraction as $y = \sin x$

$\Rightarrow \frac{\pi}{6} \le x \le \frac{5\pi}{6}$

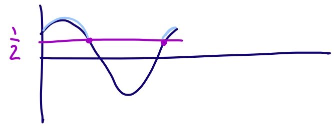
$\frac{\frac{4\pi}{6}}{2\pi} = \frac{1}{3}$

$\sin\left(x + \frac{\pi}{3}\right) = \frac{1}{2}$

$x + \frac{\pi}{3} = \left(\frac{\pi}{6}\right) \quad \frac{5\pi}{6} \quad \frac{13\pi}{6}$

$x = \frac{3\pi}{6} \quad \frac{11\pi}{6}$

$= \frac{\pi}{2}, \frac{11\pi}{6}$



$0 \to \frac{\pi}{2} \quad \frac{11\pi}{6} \to 2\pi$

$\frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3}$

$\frac{2\pi/3}{2\pi} = \frac{1}{3}$

6. Find the greatest value of the function $f(x) = \left(3sin^2(2x-5) - 7\right)^2$

A 16 B 25 C 36 D 49 E 100

$0 \le \sin^2 A \le 1$

$0 \le 3\sin^2 A \le 3$

$-7 \le 3\sin^2 A \le 4$

$\text{Max} \quad (-7)^2 = 49$

7. Find the maximum value of $3(4^{sinx}) - 10(2^{sinx}) + 9$

A $\frac{2}{3}$ B 1 C 2 D $\frac{19}{4}$ E 9

Let $y = 2^{\sin x}$

$\frac{1}{2} \le y \le 2$

$3y^2 - 10y + 9$

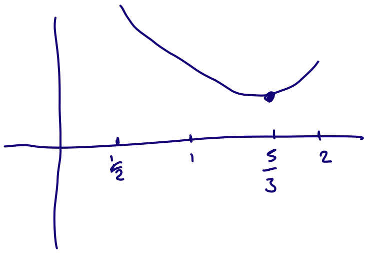
$3(y^2 - \frac{10}{3}y) + 9$

$3[(y - \frac{5}{3})^2 - \frac{25}{9}] + 9$

$3(y - \frac{5}{3})^2 - \frac{25}{3} + \frac{27}{3}$

$3(y - \frac{5}{3})^2 + \frac{2}{3}$

Min $(\frac{5}{3}, \frac{2}{3})$



Max at $y = \frac{1}{2}$ $\sin x = -1$

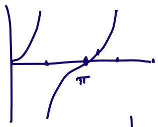
$\frac{3}{4} - 5 + 9 = \frac{19}{4}$

or $3(\frac{1}{2} - \frac{5}{3})^2 + \frac{2}{3} = 3(-\frac{7}{6})^2 + \frac{2}{3}$

$= 3(\frac{49}{36}) + \frac{8}{12} = \frac{57}{12} = \frac{19}{4}$

8. Which of the following is the largest?

A $tan(\frac{5\pi}{4})$ B $sin^2(\frac{3\pi}{4})$ C $log_{10}(\frac{5\pi}{4})$ D $log_2(\frac{3\pi}{4})$



$= 1$

$\sin\frac{3\pi}{4} < 1$

$\sin^2\frac{3\pi}{4} < 1$

✗ $< A$

$\log_{10}(\underbrace{1.25 \times 3.14}_{<10})$

< 1

✗ $< A$

$\log_2(\underbrace{\frac{1}{4} \times 9.\ldots}_{>2})$

> 1

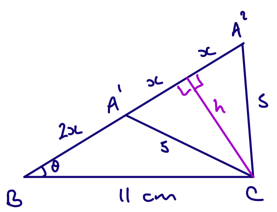
$> A$

9. A triangle ABC is drawn with $AC = 5$cm and $BC = 11$cm and the angle at B equal to a specified angle θ.

Of the two possible triangles that could be drawn, the larger triangle has double the area of the smaller one.

What is the value of $cos\theta$?

A $\frac{10}{11}$ B $\frac{3\sqrt{12}}{11}$ C $\frac{\sqrt{13}}{11}$ D $\frac{\sqrt{6}}{5}$ E $\frac{3\sqrt{6}}{25}$



Area $= \frac{1}{2} AB \times 11 \times \sin\theta$

Area $A^2BC = 2 \times$ Area A^1BC

$A^2B = 2A^1B$

Let $A^1B = 2x$ $A^2B = 4x$

$x^2 + h^2 = 25$

$9x^2 + h^2 = 121$

$8x^2 = 96$

$x^2 = 12$

$h^2 = 13$

$\cos\theta = \frac{3x}{11}$

$= \frac{3\sqrt{12}}{11}$

10. A triangle ABC is to be drawn with the following measurements.

AB = 10cm and angle BAC = 60°.

Which of the following statements is/are true ?

I   No such triangle can be drawn if $BC = 7cm$

II   Exactly one distinct triangle can be drawn if $BC = 5\sqrt{3}cm$

III   Exactly two distinct triangles can be drawn if $BC = 12cm$

A   none of them

B   I only

C   II only

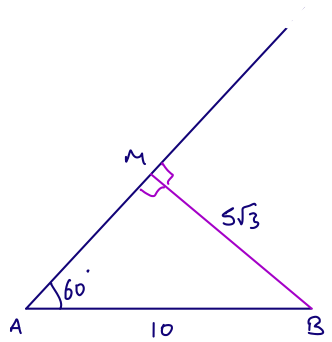
D   III only

E   I and II only

F   II and III only

G   I and III only

H   I, II and III



$MB = 10 \sin 60$
$= 5\sqrt{3} \simeq 8.5$

I $7 < 5\sqrt{3}$ true

II   true

III $12 > AB$ false
only 1 triangle