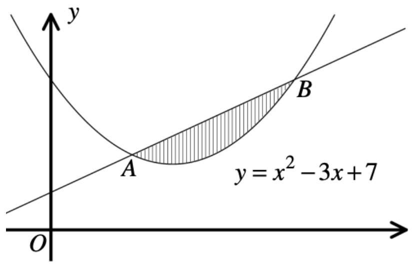
1) The area of the region bounded by the curve $y = \sqrt{x}$, the line $y = x - 2$ and the x-axis is:

A 2 B $\frac{5}{2}$ C 3 D $\frac{10}{3}$ E $\frac{16}{3}$

2) The graph shows a quadratic curve with equation $y = x^2 - 3x + 7$ and a straight line $y = x + 4$.
What is the value of the shaded area ?



A $\frac{1}{3}$ B $\frac{4}{3}$ C $\frac{7}{3}$ D $\frac{32}{3}$ E 12

3) The area of the region bounded by the curves $y = x^2$, $y = x + 2$ is:

A $\frac{9}{2}$ B $\frac{7}{3}$ C $\frac{7}{2}$ D $\frac{9}{4}$ E $\frac{11}{2}$

4) Find the area of the finite region between the curves with equations

$$y = x^2 + x - 1 \qquad \text{and} \qquad y = x$$

A $\dfrac{2}{3}$ B 1 C $\dfrac{4}{3}$ D $\dfrac{5}{3}$ E 2

5) A line is tangent to the parabola $y = x^2$ at the point (a, a^2) where $a > 0$.

The area of the finite region bounded by the parabola, the tangent line and the x-axis equals:

A $\dfrac{a^2}{3}$ B $\dfrac{2a^2}{3}$ C $\dfrac{a^3}{12}$ D $\dfrac{5a^3}{6}$ E $\dfrac{a^4}{10}$

6) The area of the finite region between the parabolas with equations

$$y = x^2 + 2ax + a \qquad \text{and} \qquad y = a - x^2 \qquad \text{equals } 9.$$

The possible values of a are:

A $a = 1$ B $a = \pm 3$ C $a = -3$ D $a = \pm 1$ E $a = 3$

7) Find the area of the finite region between the curves with equations

$$y = 5 - x^2 \qquad \text{and} \qquad y = |x| - 1$$

A $\dfrac{19}{3}$ B $\dfrac{22}{3}$ C $\dfrac{25}{3}$ D $\dfrac{28}{3}$ E $\dfrac{44}{3}$

8) Evaluate the following integral $\displaystyle\int_{-1}^{1} 2(x + |x|) - 7x|x| \ dx$

A 2 B $\dfrac{7}{3}$ C $\dfrac{5}{2}$ D 4 E $\dfrac{9}{2}$

9) The positive number k satisfies $\displaystyle\int_0^k (\sqrt{x} + x^2) \ dx = 5$ for which value of k ?

A $k = (\sqrt{21} - 1)^{\frac{1}{3}}$

B $k = \sqrt{3}$

C $k = 3^{\frac{2}{3}}$

D $k = (\sqrt{6} - 1)^{\frac{2}{3}}$

E $k = 5^{\frac{2}{3}}$

10) Let $f(x) = \int_{-x}^{x} \frac{1}{2}t^2 \; dt$ $\qquad g(x) = \int_{0}^{1} x^2 t \; dt$

Which of the following statements is true?

A $gf(A) > fg(A)$ for all $A > 0$

B $gf(A) < fg(A)$ for all $A > 0$

C $gf(A) = fg(A)$ for all $A > 0$

D $gf(A) > fg(A)$ for $A > 1$ and $gf(A) < fg(A)$ for $A < 1$

E $gf(A) < fg(A)$ for $A > 1$ and $gf(A) > fg(A)$ for $A < 1$

11) Find the minimum value of the function $f(t)$ where $f(t) \equiv \int_{0}^{1} (x - t)^2 + t^2 \; dx \quad t \geq 0$

A 0 B $\frac{5}{24}$ C $\frac{1}{4}$ D $\frac{1}{3}$ E $\frac{7}{12}$

12) The trapezium rule approximation using four trapezia for:

$$\int_{0}^{6} |x(x-3)(x-6)| \; dx$$

A $\frac{3^5}{2^3}$ B $\frac{3^5}{4}$ C $\frac{3^3}{2^5}$ D 6 E $\left(\frac{3}{2}\right)^5$

13) Find the area of the finite region between the curve with equation

$y = (x - a)(x - b)$ where $0 < a < b$ and the x-axis

A $\frac{1}{3}(b-a)^3$ B $\frac{1}{6}(b-a)^3$ C $\frac{1}{2}(b+a)^2$ D $\frac{1}{3}(b+a)^2$ E $\frac{1}{2}(b+a)^3$

14) The function $f(x)$ is such that $f(x) + 4f(-x) \equiv 1 + x^2 \int_{-1}^{1} f(u)\ du$

Determine the value of $\int_{-1}^{1} f(x)\ dx$

A $\frac{6}{13}$ B $\frac{5}{6}$ C 2 D $\frac{5}{2}$ E $\frac{25}{9}$

15) Place the following integrals in order of size from smallest to largest.

$K = \int_{1}^{4} log_4\sqrt{x}\ dx$ $L = \int_{1}^{4} log_4 x\ dx$ $M = \int_{1}^{4} \sqrt{log_4 x}\ dx$

A K < L < M

B K < M < L

C L < M < K

D L < K < M

E M < K < L

F M < L < K

Long Questions

For each positive integer k, let $f_k(x) = x^{\frac{1}{k}}$ for x ⩾ 0.

(i) On the same axes, labelling each curve clearly, sketch $y = f_k(x)$ for $k = 1, 2, 3$ indicating the intersection points.

(ii) Between the two points in (i), the curves $y = f_k(x)$ enclose several regions. What is the area of the region between the graphs of $y = f_k(x)$ and $y = f_{k+1}(x)$?

Verify that the area of the region between f_1 and f_2 is $\dfrac{1}{6}$

Let c be a constant where $0 < c < 1$.

(iii) Find the x-coordinates of the points of intersection of the line $y = c$ with $y = f_1(x)$ and of $y = c$ with $y = f_2(x)$.

(iv) The constant c is chosen so that the line $y = c$ divides the region between $y = f_1(x)$ and $y = f_2(x)$ into two regions of equal area.

Show that c satisfies the cubic equation $4c^3 - 6c^2 + 1 = 0$. Hence find c.