Paper B1

20 questions

75 minutes

No calculator allowed

1. Find the value of $$\int_1^2 6\sqrt{x} + \frac{16}{x^3}\; dx$$

A $8\sqrt{2} - 10$

B $4\sqrt{8} - \frac{1}{4}$

C $8\sqrt{2} + 2$

D $4\sqrt{8} + \frac{1}{4}$

E $4\sqrt{2} + 6$

$= \int_1^2 6x^{1/2} + 16x^{-3}\; dx$

$= \left[4x^{3/2} - 8x^{-2}\right]_1^2$

$= (4\sqrt{8} - 2) - (4 - 8)$

$= 8\sqrt{2} + 2$

2. How many solutions does the following equation have in the range $0 \le x \le 2\pi$

$$\sin 2x + \sin^2 x = 1$$

$\sin 2x = 1 - \sin^2 x$

A 2

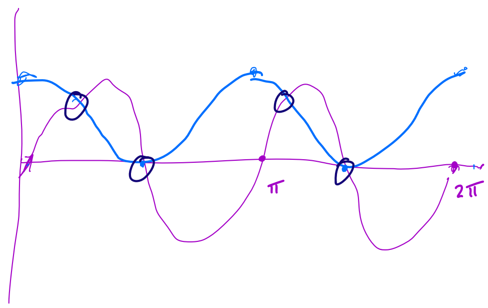
B 3

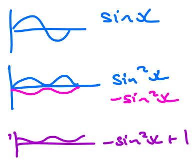
C 4

D 6

E 8

F infinitely many





3. What is the shortest distance from the point A (5 , 0) to the curve with equation $y = x^2 + 1$

A $\sqrt{5}$

B $2\sqrt{2}$

C $2\sqrt{5}$

D 5

E $5\sqrt{2}$

$\frac{dy}{dx} = 2x$

At B $\frac{dy}{dx} = 2p$

$y - 0 = -\frac{1}{2p}(x - 5)$

$y = \frac{1}{2p}(5 - x)$

At B $p^2 + 1 = \frac{1}{2p}(5 - p)$

$2p^3 + 2p + p - 5 = 0$

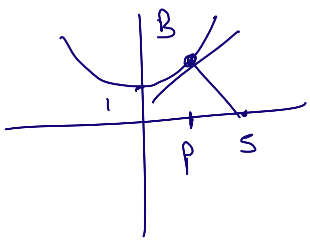
$2p^3 + 3p - 5 = 0$

$(p - 1)(2p^2 + 2p + 5)$

no soln

$p = 1$

$B(1, 2)$ $(5, 0)$



$\sqrt{4^2 + 2^2}$

$= \sqrt{20} = 2\sqrt{5}$

Tyler Tutoring

4. Consider an arithmetic sequence with $(2n + 1)$ terms.

What is the ratio of the sum of odd terms to the sum of even terms?

A $n^2 : n^2 - 1$

B $n + 1 : n$

C $n : n - 1$

D $n + 1 : n - 1$

E $n(n + 1) : 2n + 1$

$a \quad a+d \quad a+2d \quad \ldots \quad a+2nd$

$a \quad a+2d \quad a+4d \quad \ldots \quad a+2nd$

$S_{n+1} = \frac{n+1}{2}(a + a + 2nd)$

$= (n+1)(a+nd)$

$a+d \quad a+3d \quad \ldots \quad a+(2n-1)d$

$S_n = \frac{1}{2}(a+d + a + (2n-1)d$

$= n(a+nd)$

5. Find the sum of the solutions of the equation $\quad x^2 + 2\sqrt{x^2 + 6x} = 24 - 6x$

A - 6

B - 2

C 2

D 6

E 10

$x^2 + 6x + 2\sqrt{x^2+6x} - 24 = 0$

Let $y = \sqrt{x^2+6x}$

$y^2 + 2y - 24 = 0$

$(y-4)(y+6) = 0$

$\sqrt{x^2+6x} = 4$

$x^2 + 6x - 16 = 0$

$(x+8)(x-2) = 0$

$x = -8, 2$

6. The function f is given by $\quad f(x) = (\frac{1}{x} - \frac{2}{x^2})^2 \quad x \neq 0$

What is the value of $f''(1)$

A - 6

B - 2

C 26

D 38

E 122

$= \frac{1}{x^2} - \frac{4}{x^3} + \frac{4}{x^4}$

$= x^{-2} - 4x^{-3} + 4x^{-4}$

$f'(x) = -2x^{-3} + 12x^{-4} - 16x^{-5}$

$f''(x) = 6x^{-4} - 48x^{-5} + 80x^{-6}$

$f''(1) = 6 - 48 + 80$

$= 38$

7. A sequence is defined by $u_n = \sum_{r=0}^{n-1} u_r$ and $u_0 = 1$

Evaluate $\sum_{r=0}^{\infty} \frac{1}{u_r}$

$u_0 = 1$
$u_1 = u_0 = 1$
$u_2 = u_0 + u_1 = 2$
$u_3 = u_0 + u_1 + u_2 = 4$

A the sum does not converge

B 1

C 2

D 3

E ⅔

$$\sum_{k=0}^{\infty} \frac{1}{u_k} = \frac{1}{1} + \frac{1}{1} + \underbrace{\frac{1}{2} + \frac{1}{4} + \ldots}_{=1}$$

$$= 3$$

8. Given that $3^a = 16$ and $2^b = 27$, find the value of ab

A 3 B $\frac{7}{2}$ C 4 D $\frac{9}{2}$ E 12

$3^a = 2^4$ $2^b = 3^3$

$a \log 3 = 4 \log 2$ $b \log 2 = 3 \log 3$

$ab = \frac{4 \log 2}{\log 3} \times \frac{3 \log 3}{\log 2} = 12$

9. In a set of k consecutive integers, the largest number is 23. What is the mean of the set?

A $\frac{1}{2}(k + 45)$

B $25 - 2k$

C $\frac{23}{2}k$

D $\frac{1}{2}(25 - k)$

E $\frac{1}{2}(47 - k)$

. 23

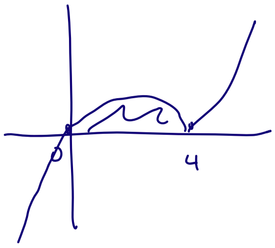
k

1st term is $23 - k + 1 = 24 - k$

Mean $= \frac{1}{2}(24 - k + 23) = \frac{1}{2}(47 - k)$

10. Evaluate the following integral $\int_0^4 x\,|x-4|\ dx$

A $-\dfrac{32}{3}$ B $-\dfrac{16}{3}$ C $\dfrac{16}{3}$ D $\dfrac{32}{3}$ E 0



$$\int_0^4 4x - x^2\ dx$$

$$\left[2x^2 - \tfrac{1}{3}x^3\right]_0^4 = \left(32 - \tfrac{64}{3}\right) - (0) = \frac{32}{3}$$

11. Which of the following is the largest?

A $2^{\frac{1}{2}}$ B $3^{\frac{1}{3}}$ C $4^{\frac{1}{4}}$ D $6^{\frac{1}{6}}$ E $12^{\frac{1}{12}}$

$2^{6/12}$ $3^{4/12}$ $4^{3/12}$ $6^{2/12}$ $12^{1/12}$

$64^{1/12}$ $81^{1/12}$ $64^{1/12}$ $36^{1/12}$

12. Two players take it in turns to throw a fair six-sided die until one of them scores a six. What is the probability that the first player to throw the die is the first to score a six?

A $\dfrac{3}{5}$ B $\dfrac{4}{7}$ C $\dfrac{5}{9}$ D $\dfrac{6}{11}$ E $\dfrac{7}{12}$

$$\frac{1}{6} + \frac{5}{6}\times\frac{5}{6}\times\frac{1}{6} + \frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6} + \dots$$

$a = \frac{1}{6}$ $r = \frac{25}{36}$ $S_\infty = \dfrac{1/6}{11/36} = \dfrac{6}{11}$

13. How many real roots does the equation $12x^5 - 45x^4 + 40x^3 - 10$ have?

A 1 B 2 C 3 D 4 E 5

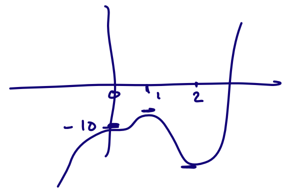
$\frac{d}{dx} = 60x^4 - 180x^3 + 120x^2 = 0$

$x^4 - 3x^3 + 2x^2 = 0$

$(x-2)(x-1)x^2 = 0$

$x = 0, 1, 2$

$(0; -10)$ repeated $(1, -3)$ $(2, -26)$



14. Find the coefficient of x^4y^2 in the expansion of $(1 + x^2 + y)^6$

A 6

B 15

C 30

D 90

E 120

$\binom{6}{r}(1+x^2)^r y^{6-r}$ $r = 4$

$15(1+x^2)^4 y^2$

$(1+x^2)^4$

$\binom{4}{2}(x^2)^2$

$6x^4$

$15 \times 6 = 90$

or $\binom{6}{4}(1+y)^4(x^2)^2$

$15x^4(1+y)^4$

$(1+y)^4 = 1 + 4y + 6y^2 + \dots$

$15 \times 6 = 90$

15. Find the minimum value of $f(x) = 25sin^4x - 30sin^2x + 11$

A 1

B 2

C 5

D 8

E 11

Let $y = \sin^2 x$

$25y^2 - 30y + 11$

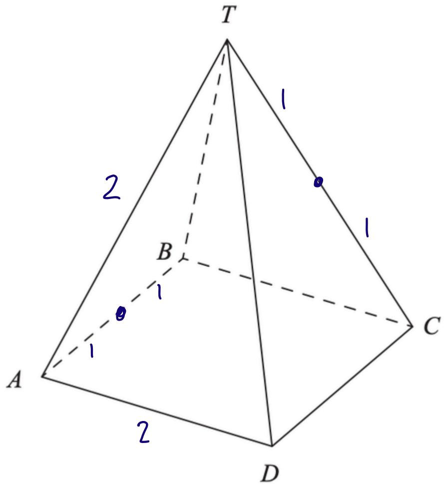
$25\left(y^2 - \frac{6}{5}y\right) + 11$

$25\left(y - \frac{3}{5}\right)^2 - 25\left(\frac{9}{25}\right) + 11$

$25\left(y - \frac{3}{5}\right)^2 + 2$

16. A square based pyramid, with base ABCD, and vertex T has all edges of length 2m.

Find the shortest distance, in metres, along the outer surface of the prism from the midpoint of AB to the midpoint of CT.



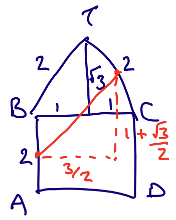
A $\sqrt{3}-1$

B 2

C $\sqrt{2}+1$

D $\sqrt{4+\sqrt{3}}$

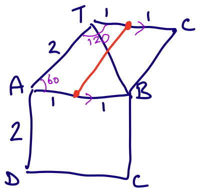
E $2\sqrt{2}$



$\left(\frac{3}{2}\right)^2 + \left(1+\frac{\sqrt{3}}{2}\right)^2$

$\frac{9}{4} + 1 + \sqrt{3} + \frac{3}{4} = 4 + \sqrt{3}$

$CT = \sqrt{4+\sqrt{3}}$



CT = 2 shortest distance

17. Three geometric transformations are defined as follows:

R is a reflection in the y-axis

S is a stretch parallel to the x-axis, scale factor 1/2

T is a translation by 3 units in the negative x direction

These three transformations are applied to the graph of $y = \sqrt{x}$ resulting in the graph of $y = \sqrt{3-2x}$

In which order were the transformations applied?

A R then S then T

B R then T then S

C S then R then T

D S then T then R

E T then S then R

$\sqrt{x} \xrightarrow{R} \sqrt{-x} \xrightarrow{T} \sqrt{-(x+3)} = \sqrt{-x-3}$

$\sqrt{x} \xrightarrow{S} \sqrt{2x} \xrightarrow{R} \sqrt{-2x}$

$\sqrt{2x} \xrightarrow{T} \sqrt{2(x+3)}$

$\sqrt{x} \xrightarrow{T} \sqrt{x+3} \xrightarrow{S} \sqrt{2x+3} \xrightarrow{R} \sqrt{-2x+3}$

18. The curve C has equation $y = x^2 + bx + 3$ where $b \geq 0$

Find the value of b that minimises the distance between the origin and the stationary point of the curve C.

A $b = 0$

B $b = 2$

C $b = \sqrt{6}$

D $b = \sqrt{10}$

E $b = 10$

$y = x^2 + bx + 3$

$= (x + \frac{1}{2}b)^2 + 3 - \frac{1}{4}b^2$ T.P. $(-\frac{1}{2}b, 3 - \frac{1}{4}b^2)$

$\text{Dist}^2 = \frac{b^2}{4} + 9 - \frac{3}{2}b^2 + \frac{1}{16}b^4$

$\frac{1}{4} - \frac{36}{24} = -\frac{5}{4}$

$= \frac{1}{16}(b^4 - 20b^2 + 144)$

$= \frac{1}{16}[(b^2 - 10)^2 + 44]$

Min when $b^2 = 10$ $b = \sqrt{10}$

19. How many solutions does the following equation have in the range $0 \leq x < 2\pi$

$$2 = sin x + sin^2 x + sin^3 x + sin^4 x + \ldots$$

A 0

B 1

C 2

D 3

E infinitely many

G.P. $a = \sin x$, $r = \sin x$

$S_\infty = \frac{\sin x}{1 - \sin x} = 2$

$\sin x = 2 - 2\sin x$

$3\sin x = 2$

$\sin x = 2/3$ 2 solutions

20. The polynomial function $f(x)$ is such that $f(x) > 0$ for all values of x.

Given that $\int_2^4 f(x)\ dx = A$ and $\int_4^6 f(x)\ dx = B$

evaluate $\int_{-2}^{2} f(x+4) + 2\ dx$ in terms of A and B

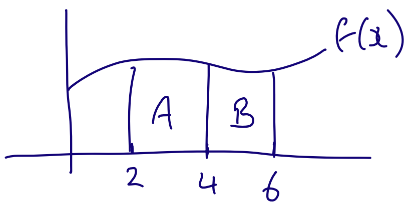
A $A + B$

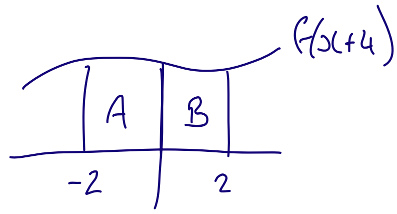
B $2(A + B)$

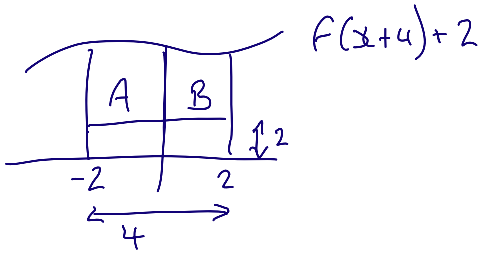
C $A + B + 2$

D $A + B + 8$

E $4(A + B) + 2$







Paper B2

20 questions

75 minutes

No calculator allowed

1. For how many values of the constant k does the following equation have only one real solution

$$kx^2 - (k-1)x + k = 0$$

A no values of k

B one value of k

C two values of k

D all values of k except $k = 1$

E all values of k

$\Delta = (k-1)^2 - 4k^2 = 0$

$k - 1 = 2k$ $\quad k = -1$

$k - 1 = -2k$ $\quad k = \frac{1}{3}$

2. How many solutions does the following equation have in the range $0 \leq x \leq \pi$

$$sin2x = cosx$$

A 1

B 2

C 3

D 4

E infinitely many

$2\sin x\cos x - \cos x = 0$

$\cos x = 0$ $\quad x = \frac{\pi}{2}$

$\sin x = \frac{1}{2}$ $\quad x = \frac{\pi}{6}, \frac{5\pi}{6}$

3. For which real numbers x does the following inequality hold

$$\frac{x}{x^2+1} \leq \frac{1}{2}$$

A all real numbers x

B only for real numbers $x \leq \dfrac{1}{2}$

C only for real numbers $x \geq 1$

D none of the above

$2x \leq x^2 + 1$

$x^2 - 2x + 1 \geq 0$

$(x-1)^2 \geq 0$

4. Consider the following attempt at solving the equation $3sin2x + 7cosx = 0$ for $0 \le x \le 360$.

I $\quad 6sinxcosx + 7cosx = 0$

II $\quad 6sinx + 7 = 0$

III $\quad sinx = -\frac{6}{7}$

IV $\quad$ There are two real solutions to this equation.

$\cos x = 0$

$\sin x = -\frac{7}{6}$

Which statement describes this attempt?

2nd mistake on line III

A It is completely correct

B It is incorrect and the first mistake occurs on line I

C It is incorrect and the first mistake occurs on line II

D It is incorrect and the first mistake occurs on line III

E It is incorrect and the first mistake occurs on line IV

5. The function f is defined for positive integers and satisfies

$f(1) = 1$ and $\quad f(2n) = f(n)$ and $\quad f(2n+1) = f(n) + 1$

What is the value of $f(9)$

A 1

B 2

C 3

D 4

E 5

$f(1) = 1 \quad f(2) = 1 \quad f(3) = 2 \quad f(4) = 1$

$f(5) = 2 \quad f(6) = 2 \quad f(7) = 3 \quad f(8) = 1$

$f(9) = 2$

6. Consider the following two statements about the polynomial $p(x)$

P: $\quad p(x)$ has at least one real root.

Q: $\quad p(x)$ is a polynomial of order n, where n is an odd integer.

Which of the following is correct?

A P is **necessary** and **sufficient** for Q

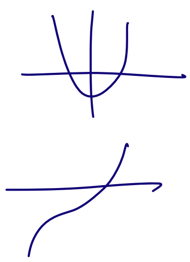
B P is **not necessary** and **not sufficient** for Q

C P is **necessary** but **not sufficient** for Q

D P is **sufficient** but **not necessary** for Q

$P \not\Rightarrow Q$

$Q \Rightarrow P$



P ✓

Q ✗

7. The real numbers a and b are such that exactly one of the following statements is true. Which is the true statement?

If A is only true statement

A	$a \geq 0$	✓
B	$a < b$	$b \leq a$
C	$a^2 > b^2$	$b^2 \geq a^2$ $\quad (b-a)(b+a) \geq 0 \quad b < -a$
D	$\lvert a \rvert \leq \lvert b \rvert$	$\lvert a\rvert > \lvert b\rvert$ contradiction so not A

eg $a = -\frac{1}{2}$, $b = -1$

✗

✗

✗

✓

If B true $\quad a < b \Rightarrow a < 0$

8. Consider the following statement about the polynomial $p(x)$ where a and b are real numbers with $a < b$.

(*) There exists a number c with $a < c < b$ such that $p(c) = 0$

Which of the following is true?

A The condition $p(a)p(b) < 0$ is **necessary** and **sufficient** for (*)

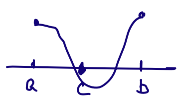
B The condition $p(a)p(b) < 0$ is **not necessary** and **not sufficient** for (*)

C The condition $p(a)p(b) < 0$ is **necessary** but **not sufficient** for (*)

D The condition $p(a)p(b) < 0$ is **sufficient** but **not necessary** for (*)

$p(a)p(b) < 0 \Rightarrow$ function crosses x-axis (sufficient)

$\Rightarrow$ * true

* true $\not\Rightarrow p(a)p(b) < 0$ eg  (not necessary)

9. A student wishes to evaluate the function $f(x) = \dfrac{tan\,x}{x}$ where x is in radians, but only has a calculator that works in degrees.

What can the student type into their calculator in order to correctly evaluate $f(5)$

A $\quad \frac{1}{5} \times tan(\pi \times 5 \div 180)$

B $\quad 5 \times tan(\pi \times 5 \div 180)$

C $\quad tan(\pi \times 5 \div 180) \div (\pi \times 5 \div 180)$

D $\quad tan(180 \times 5 \div \pi) \div (180 \times 5 \div \pi)$

E $\quad \frac{1}{5} \times tan(180 \times 5 \div \pi)$

F $\quad 5 \times tan(180 \times 5 \div \pi)$

$$f(5) = \frac{\tan\left(\frac{180 \times 5}{\pi}\right)}{5}$$

10. A sequence is such that $u_1 = 6$ and $u_2 = 3$ and $u_{n+1} = \dfrac{u_n}{u_{n-1}}$ for $n > 1$

What is the value of u_{2023} ?

A $\frac{1}{2}$

B $\frac{1}{6}$

C 2

D 3

E 6

$u_3 = \frac{3}{6} = \frac{1}{2}$

$u_4 = \frac{1/2}{3} = \frac{1}{6}$

$u_5 = \frac{1/6}{1/2} = \frac{1}{3}$

$u_6 = \frac{1/3}{1/6} = 2$

$u_7 = \frac{2}{1/3} = 6 = u_1 = u_{2023}$

sequence repeats with period 6

$6 \overline{)2023}$ = 337 r 1

11. The fact that $5 \times 6 = 30$,
is a counter example to which of the following statements:

A the product of any two odd integers is odd

B if the product of two integers is a multiple of 4 then the integers are not consecutive

C if the product of two integers is not a multiple of 4 then the integers are not consecutive

D any even integer can be written as the product of two even integers.

12. What is the reflection of the point (3,4) in the line $3x + 4y = 50$

A (9 , 12)

B (12 , 9)

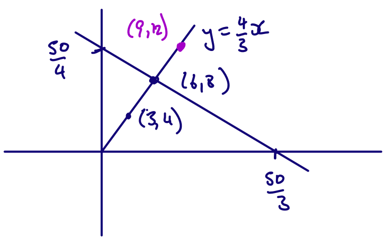
C (6 , 8)

D (8 , 6)

E (12 , 16)

F (16 , 12)

$y = -\frac{3}{4}x + \frac{25}{2}$



need normal passing through (3,4)

$y = \frac{4}{3}x$

$\frac{4}{3}x + \frac{3}{4}x = \frac{25}{2}$

$\frac{25x}{12} = \frac{25}{2}$

$x = 6 \quad y = 8$

$\begin{pmatrix}3\\4\end{pmatrix} \to \begin{pmatrix}6\\8\end{pmatrix} \to \begin{pmatrix}9\\12\end{pmatrix}$

13. Consider the four options below about a particular statement:

A The statement is true if $x^2 < 1$

B The statement is true if and only if $x^2 < 1$

C The statement is true if $x^2 < 2$

D The statement is true if and only if $x^2 < 2$

$B \Rightarrow A$

$C \Rightarrow A$

$D \Rightarrow C$

Given that exactly one of these options is correct, which one is it?

14. Find the minimum value of $f(x) = 2x^3 - 9x^2 + 12x + 3$ for $0 \leq x \leq 2$

A 1

B 2

C 3

D 5

E 7

$f(0) = 3$

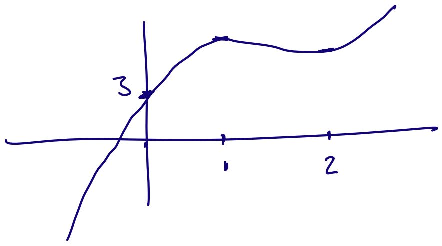
$f(1) = 8$

$f(2) = 16 - 36 + 27 = 7$

$f'(x) = 6x^2 - 18x + 12 = 0$

$x^2 - 3x + 2 = 0$

$(x-1)(x-2) = 0$



15. Given that the numbers x and y satisfy $(x-1)^2 + y^2 \leq 1$.

What is the largest value that $x + y$ can be?

A 1

B $\sqrt{2}$

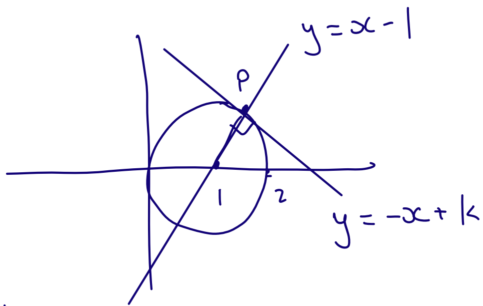
C 2

D $1 + \sqrt{2}$

E 4

Max $x + y = k$

$y = -x + k$



$y = x - 1$

At P: $2(x-1)^2 = 1$

$(x-1) = \pm\sqrt{\frac{1}{2}}$

$x = 1 \pm \sqrt{\frac{1}{2}}$

$y = \pm\sqrt{\frac{1}{2}}$

$\sqrt{\frac{1}{2}} = -1 - \sqrt{\frac{1}{2}} + k$

$k = 1 + \sqrt{2}$

16. Given that $x = -b$ is a root of the equation
$f(x) = ax^3 + ax^2 + ax + b$ where a and b are constants.

Find the range of possible value of a.

A There are no possible values of a

B $a < 1$

C $0 \le a \le \frac{4}{3}$

D $a \ge 1$

E a can be any real number

$ax^3 + ax^2 + ax + b = 0$ $x = -b$ is root

$-ab^3 + ab^2 - ab + b = 0$

$b \neq 0$ $-ab^2 + ab + (1-a) = 0$

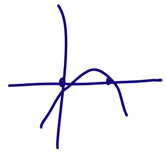
$ab^2 - ab + (a-1) = 0$

$\Delta \geq 0$ $a^2 - 4a(a-1) \geq 0$

$4a - 3a^2 \geq 0$

$a(4-3a) \geq 0$

$0 \leq a \leq \frac{4}{3}$



17. Let $a, b, c > 0$. The equations: $log_b a = 2$ $log_a(4c-5) = 2$ $log_b(c-1) = 3$

A specify a uniquely

B are satisfied by two values of a

C are satisfied by infinitely many solutions for a

D are contradictory

$b^2 = a$ $a^2 = 4c - 5$ $b^3 = c - 1$

$b^4 = a^2$ $4c = a^2 + 5$ $4c = 4b^3 + 4$

$b^4 + 5 = 4b^3 + 4$

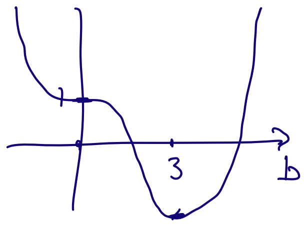
$b^4 - 4b^3 + 1 = 0$

T.P. $4b^3 - 12b^2 = 0$

$4b^2(b-3) = 0$

$b = 0, 3$

$3^4 - 4(3^3) + 1 < 0$



2 real solutions for b

$\Rightarrow$ 2 solutions for a

18. Which of the following integrals has the largest value?
You are not expected to calculate the exact values of any of these.

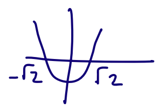
A $\int_0^{8\pi} sin^{64}x \; dx$

B $\int_0^{4\pi} 64(cos^6 x - 1) \; dx$

C $\int_0^{1} (x^2-2)sin^6(\pi x) \; dx$

D $\int_0^{4\pi} (3 + sin x)^6 \; dx$

c) $x^2 - 2$

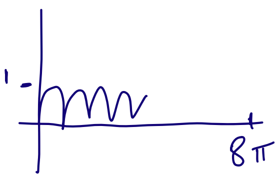


negative from 0 - 1

$\sin^6(\pi x) > 0$

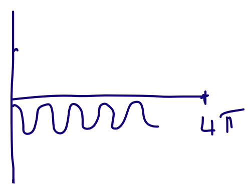
so integral is negative

A)



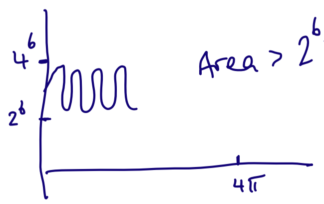
area 8π

B)



integral is negative

D)



Area $> 2^6 \times 4\pi$

$-1 < \sin x < 1$

$2 < 3 + \sin x < 4$

$2^6 < (3 + \sin x)^6 < 4^6$

Tyler Tutoring

19. Which of the following statements are true?

I **There exists** a real y such that **for all** real x, $y > x$ — False – There is no largest number

II **There exists** a real x such that **for all** real y, $x + y > xy$ — True $x = 1$

III **For all** real x, **there exists** real y such that $x - y = xy^2 + 1$ — True if $\Delta \geq 0$

A none of them

B I only

C II only

D III only

E I and II only

F II and III only

G I and III only

H I, II and III

III

$$x - y = xy^2 + 1$$
$$xy^2 + y + 1 - x = 0$$
$$\Delta = 1 - 4x(1-x) = 1 - 4x + 4x^2$$
$$= (2x-1)^2 \geq 0$$

$\Rightarrow$ always a solution for y

20. f is a function and a, b are real numbers.
Given that exactly one of the following statements is true, which one is it?

* same

A $f(a) \geq f(b)$ if and only if $a \geq b$ — If $f(a) \geq f(b)$ then $a \geq b$; If $a \geq b$ then $f(a) \geq f(b)$

B $f(a) \geq f(b)$ only if $a < b$ — If $f(a) \geq f(b)$ then $a < b$] contrapositive

C $f(a) < f(b)$ if $a \geq b$ — If $a \geq b$ then $f(a) < f(b)$

D $a \geq b$ if $f(a) \geq f(b)$ — If $f(a) \geq f(b)$ then $a \geq b$ *

E $a < b$ only if $f(a) \geq f(b)$ — If $a < b$ then $f(a) \geq f(b)$] contrapositive

F $a < b$ only if $f(a) < f(b)$ — If $a < b$ then $f(a) < f(b)$

$A \Rightarrow B$

not $B \Rightarrow$ not A

A, D equivalent

B, C D, F contrapositive