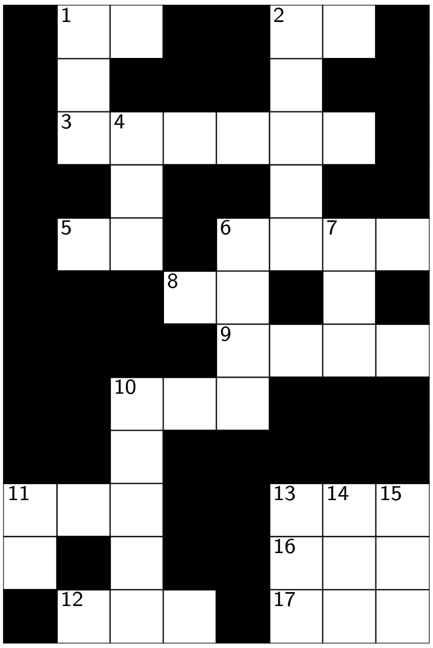
THE UK UNIVERSITY INTEGRATION BEE 2023/24

$\int$

ROUND TWO CROSSNUMBER

Sponsored by

Jane Street



Across

1. $\left|\left(\int_0^1 f(t)\,\mathrm{d}t\right)\left(\int_1^2 f(t)\,\mathrm{d}t\right)\right|$ where $3\int_0^2 f(t)\,\mathrm{d}t + 2\int_1^2 f(t)\,\mathrm{d}t = 5$ and $2\int_0^1 f(t)\,\mathrm{d}t + 3\int_1^2 f(t)\,\mathrm{d}t = 4$

2. $\int_0^1 \left(1-\sqrt{\ln(x)}\right)^a + \left(1+\sqrt{\ln(x)}\right)^a \,\mathrm{d}x$ where a is the final digit of 11 DOWN.

3. Let $A \cdot B \cdot C + D \cdot E - F = 28$ where $A, B, C, D, E,$ and F are digits 1 through to 6 with no digits repeated. If $A > B > C$ and $D > E$, what is the 6-digit number of $ABCDEF$.

5. $\lim_{n\to\infty} \left\lfloor \int_0^{24} \arctan(x^n)\,dx \right\rfloor$.

6. $\left\lfloor \dfrac{\text{Second largest clue on this crossnumber}}{\text{Second smallest clue on this crossnumber}} \right\rfloor$

8. Sum of the three smallest clues.

9. $a+b+c+d$ where $a, b, c, d \in \mathbb{N}$, $\gcd(a,b) = \gcd(c,d) = 1$ and $\int_0^\infty \frac{1}{x} \ln\left(\frac{20e^{1337x} + 24e^{420x}}{20e^{1337x} + 24e^{69x}}\right) \mathrm{d}x = \ln\left(\frac{a}{b}\right) \ln\left(\frac{c}{d}\right)$

10. A Fibonacci number.

11. Compute the value of $a+b-1$ if $\int_{-1}^1 \ln(x^2)\ln(1-x^2)\,\mathrm{d}x = a - \pi^2 - \ln(b)$.

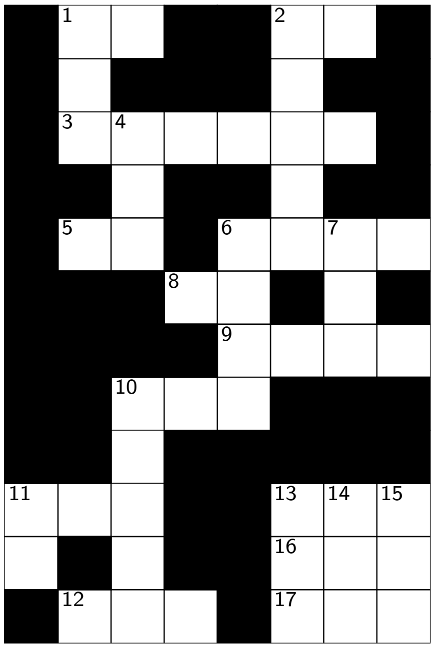
12. A power of 7.

The following clues must contain exactly one of each digit from 1 to 9.

13. One less than a prime.

16. The product of two primes.

17. Product of digits is 24 and sum of digits is 11.



Down

1. $\int_0^2 f(t)\,\mathrm{d}t$ where $3\int_0^2 f(t)\,\mathrm{d}t - 7\int_0^1 f(t)\,\mathrm{d}t = 23$ and $3\int_0^1 f(t)\,\mathrm{d}t - 2\int_1^2 f(t)\,\mathrm{d}t = 0$.

2. Compute $a+b$ where $\gcd(a,b)=1$ and $\int_0^1 x\ln(x)^c\,\mathrm{d}x = \frac{a}{b}$ where c is 1 ACROSS.

4. $f(11)$ where $f(x) + \int_{-1}^1 f(t)\,\mathrm{d}t = 3x^2 + x + 8$.

6. Product of the three smallest clues.

7. Compute the value of $f(0)$ if $\lim_{n\to 0^+} \int_{-1}^1 \frac{nf(x)}{25n^2 + x^2}\,\mathrm{d}x = 50\pi$.

10. Compute the value of $f(0)$ if $f(\pi) = 866$ and $\int_0^\pi \left[f(x) + f''(x)\right]\sin(x)\,\mathrm{d}x = 70000$.

11. Compute the value of A if $\int_0^1 \left(\int_0^a \left(\dots \left(\int_0^x \left(\int_0^y z\,\mathrm{d}z\right)\,\mathrm{d}y\right)\dots\right)\,\mathrm{d}b\right)\,\mathrm{d}a = \frac{1}{A!}$.

The following clues must contain exactly one of each digit from 1 to 9.

13. One more than a factorial.

14. Product of digits is 108 and sum of digits is 16.

15. Digits alternate between even and odd.