Oxford Interview Answers 2019

1 Interview 1

- Assume towards a contradiction that the shortest walk is not a path. Then at some point that walk will visit some vertex A, go to some other vertices which we shall call B, and return to A. We can skip the visiting of B and we have a shorter walk, which contradicts our original assumption.
- Only C4 is bipartite. For these small graphs, this can be shown by demonstrating a way to split the vertices of C4 into two groups, then doing case by case analysis on C3 and C5 to show there are no possible ways to split the vertices into two groups.
- This was a bad question, in my opinion. The answer wanted was that the lengths of the cycles are even.
- Yes. This can be shown by labelling some initial vertex A, then labelling adjacent vertices B, then label adjacent ones to those A again and so on. A brief, non-rigorous justification that no two A or B will be adjacent will be sufficient.

2 Interview 2

- There are n coordinates, each of which can be a zero or a one. The answer is 2^n. Alternatively, the coordinates are exactly all the binary numbers of n bits, of which there are 2^n. Another way to prove it is using induction. To go from a n-square to a $(n+1)$-square, we double the number of vertices. We can use the fact that a 0-square (a point) has 1 vertex as our basis case.
- $n!$
- For the first step, we must choose two of the $2n$ zeros to change to ones. After that, we are left with $2n-2$ zeros. Again we must choose two to change to ones. Repeating the process, we have:

$$\prod_{i=1}^{n} \binom{2i}{2} = \prod_{i=1}^{n} \frac{(2i)!}{(2i-2)!2!} = \frac{(2n)!}{2^n}$$

- We can think of this as two separate increasing one walks, first from the origin to the favoured point, then from the favoured point to the vertex of all ones. The answer is therefore $n!(n-s)!$.
- Imagine we decide the walk using a sequence of U and R commands. There are $2m$ commands in total and m of each command. The answer is therefore $\binom{2m}{m}$.