20 YEARS OXBRIDGE APPLICATIONS

TEST FOR MATHEMATICS FOR UNIVERSITY ADMISSION (TMUA)

MOCK TEST 5 QUESTIONS

SECTION 1
MATHEMATICAL KNOWLEDGE

1 Find the value of

$$\int_0^1 \frac{3x^2 - 2\sqrt{x}}{\sqrt[3]{x^4}} dx$$

A $\frac{10}{3}$

B $\frac{21}{4}$

C 1

D $-\frac{10}{3}$

E $-\frac{51}{3}$

2 A room has four walls, each of which can be painted with one of three colours. Each colour has an equal likelihood of being chosen.

What is the probability that the room will be monochromatic?

A $\frac{1}{27}$

B $\frac{1}{81}$

C $\frac{4}{81}$

D $\frac{1}{9}$

E $\frac{4}{9}$

3 A circle of radius 3 is inscribed within an equilateral triangle.

Find the area of the equilateral triangle.

A $\frac{9}{2}$

B $\frac{27\sqrt{3}}{4}$

C $9\sqrt{3}$

D $27\sqrt{3}$

E 27

4 Each of the positive real numbers a, b, c, d and e is decreased by 20%.

Find the resulting percentage change in the value of the following expression:

$$\frac{a-b}{cd} - \frac{b^2+c^2}{d^3+ae^2}$$

A No change.

B Decrease of 20%

C Increase of 20%

D Decrease of 25%

E Increase of 20%

5 The real numbers x and y satisfy $|x+1| \leq 6$ and $|y+4| \leq 5$.

Find the greatest possible value of xy.

A No greatest value

B 63

C 30

D 5

E 4

6 What is the largest value attained by

$$\left(9\cos^2(8x-7)-6\right)^2$$

as x varies over the real numbers?

A 9

B 36

C 45

D 81

E 225

7 The height h of a right circular cylinder is inversely proportional to r^3, where r is the radius of its base. When $r = 1$, the volume of the cylinder is 3π.

Given that the volume of the cylinder is π, find it's height.

A 3

B 9

C $\frac{1}{3}$

D $\frac{1}{9}$

E $\frac{1}{27}$

8 The graph of $y = \cos(x)$ is reflected in the line $y = 1$ and is then translated by $\frac{\pi}{2}$ units in the negative x-direction.

What is the equation of the new graph?

A $y = 2 + \sin(x)$

B $y = 1 + \sin(x)$

C $y = 2 + \cos(x)$

D $y = 1 - \sin(x)$

E $y = 2 - \sin(x)$

9 Let x and y be positive integers, with $x < y$. The least common multiple of x and y is 138, and their greatest common divisor is 23.

Given that y is an odd number, find $x + y$.

A 161

B 115

C 92

D 207

E 69

10 Which of the following numbers is the smallest?

A $\sin(45°)^2$

B $\dfrac{\sqrt[3]{28}}{6}$

C $2^{-\frac{\pi}{4}}$

D $\dfrac{3^3 - 2^3}{3^3 + 3^2 + 3 + 1}$

E $\dfrac{\tan(60°) - \tan(30°)}{2}$

11 A rowing team consists of eight rowers. The current weights of the rowers (in kilograms) are:

$$72, 74, 75, 76, 79, 80, 83, 85.$$

The rowers each aim to increase their body weight by 2.5% by race day.

If they reach their intended weight, what will the mean weight of the rowers be?

A 82 kg

B 79.95 kg

C 78 kg

D 76.05 kg

E 77.90 kg

12 A sequence is defined by

$$x_n = (-1)^n + (-1)^{n+1} + (-1)^{n+2} \text{ for } n \geq 0.$$

What is the value of $\sum_{n=0}^{100} x_n$?

A -101

B -1

C 0

D 1

E 101

13 For how many values of the positive integer n is the following fraction an integer?

$$\frac{2n+38}{n+1}$$

A None

B 1

C 2

D 6

E 8

14 Two fair dice are rolled one after the other. What is the probability that the outcome of the second die is strictly greater than the outcome of the first one?

A $\frac{1}{2}$

B $\frac{2}{3}$

C $\frac{7}{18}$

D $\frac{5}{12}$

E $\frac{4}{9}$

15 Which of the following statements is false?

A $\frac{1}{x} - \frac{1}{x+1} = \frac{1}{x(x+1)}$

B $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} = 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3}$

C $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} = 1 - \frac{1}{3}$

D $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \ldots + \frac{1}{9\cdot 10} = \frac{9}{10}$

E $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \ldots + \frac{1}{9\cdot 10} = \frac{945}{10!}$

16 Solve the following inequality:

$$x \geq \sqrt{x} + 2\,.$$

A $x \geq 4$

B $x \leq 1$ or $x \geq 4$

C $2 \leq x \leq 4$

D $1 \leq x \leq 4$

E No solution

17 A particle situated at $(1, 0)$ is first rotated anticlockwise by $45°$ about the origin. It is then translated by $(1, 0)$ and is finally reflected in the line $y = x$. What is the new position vector of the particle?

A $\left(\frac{-1}{\sqrt{2}} + 1, \frac{-1}{\sqrt{2}}\right)$

B $\left(\frac{-1}{\sqrt{2}} + 1, \frac{1}{\sqrt{2}}\right)$

C $\left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} + 1\right)$

D $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} + 1\right)$

E $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} - 1\right)$

18 What is the value of the product

$$\frac{4}{3} \cdot \frac{6}{4} \cdot \frac{8}{5} \cdot \frac{10}{6} \cdot \ldots \cdot \frac{2(n+1)}{n+2} \quad ?$$

A $\frac{2^{n+2}}{3(n+1)}$

B $\frac{2(n+1)}{3}$

C $\frac{2^{n+1}}{n+2}$

D $\frac{2^{n+1}n}{3}$

E $\frac{2^{n}n}{3}$

19 The graph of $y = x^2 + ax + b$ meets the straight line $y = x + 1$ when $x = 2$ and $x = 4$.

Find the values of a and of b.

A $a = 5,\ b = 9$

B $a = -5,\ b = 9$

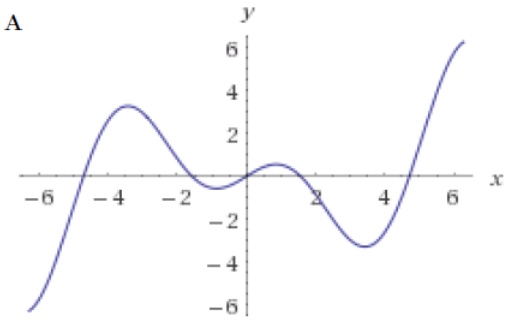
C $a = 5,\ b = 11$

D $a = -5,\ b = 11$

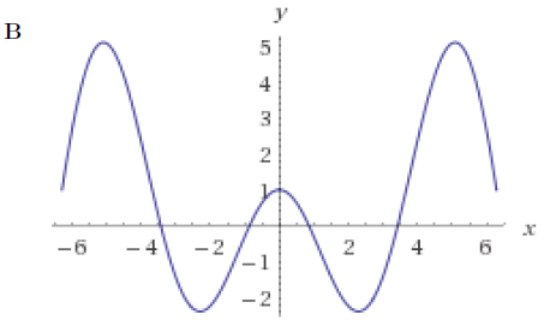
E $a = 5,\ b = 13$

20 Which of the following is a graph of the derivative of $y = x\cos(x)$?

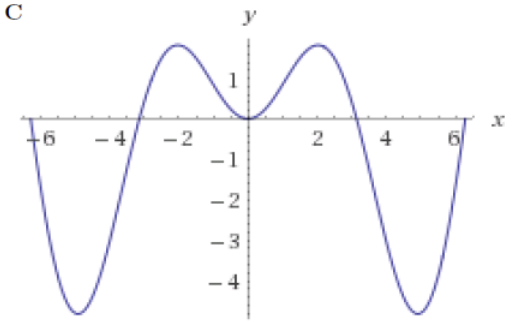
A



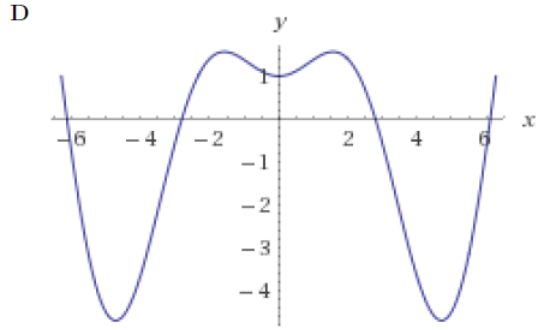
B



C



D



SECTION 2
MATHEMATICAL THINKING

21 Three consecutive terms of a geometric sequence are

$$2x + 4,\ 3x + 2,\ x^2 - 11.$$

The solution x satisfies a cubic equation of the form $(x - 6)g(x)$.

What is the discriminant of $g(x)$?

A 15

B 13

C -9

D -13

E -15

22 Find the sum of the solutions to the following equation:

$$8^x + \left(4 \times 2^{\frac{1}{x}}\right)^x - 2^x - 2 = 0.$$

A -2

B -1

C 0

D 1

E 2

23 The line $y = 2x$ intersects the curve $y = 2x^2 - 3x + 5$ at the points (x_1, y_1) and (x_2, y_2). What is the area of the right-angled triangle whose hypotenuse is the line segment joining (x_1, y_1) and (x_2, y_2)?

A $\frac{5}{2}$

B 5

C 10

D $2\sqrt{5}$

E 6

24 What is the coefficient of x^2 in the expansion of

$$(2x+3)^5(x-1)^4 \ ?$$

A -1080

B -702

C 40

D 504

E 2160

25 Consider the following conjecture:

If p is an odd prime number, then $p+2$ is also a prime number.

Here are three numbers:

I $p = 5$

II $N = 15$

III $N = 19$

Which of these is not a counterexample to the above conjecture?

A none of them

B **I** only

C **II** only

D **III** only

E **I** and **II** only

F **I** and **III** only

G **II** and **III** only

H **I**, **II** and **III**

26 Which of the following statements about $x = \log_2(3)$ is false?

A $x > 1$

B $x < 2$

C $5 < 2^x + 3^x \leq 6$

D $2 \leq x^2 + x^3 < 12$

E $\left(2^{\frac{x}{2}} - 1\right)\left(2^{\frac{x}{2}} + 1\right) = 3^{\frac{1}{x}}$

27 Which of the following expressions is a simplification of

$$\frac{(x+1)^2+1}{x^4+4} \quad ?$$

A $\frac{1}{x^2+2}$

B $\frac{1}{x^2-2}$

C $\frac{1}{x^2-2x+2}$

D $\frac{1}{x^2+2x+2}$

E None of the above.

28 Consider the following argument, for integers a and b:

$a = b = 1$

I $\Rightarrow a^2 = b^2$

II $\Rightarrow (a-b)(a+b) = 0$

III $\Rightarrow a + b = 1 + 1 = 0$

Which of following is true?

A There is an error in line **I** only.

B There is an error in line **II** only.

C There is an error in line **III** only.

D There are errors in lines **I** and **II** only.

E There are errors in lines **I** and **III** only.

F There are errors in lines **II** and **III** only.

G There are errors in lines **I**, **II** and **III**.

29 The equation $\sin(x) = x^2 - 2\pi x + \pi^2 + 1$ has

A no solutions

B one solution

C two solutions

D three solutions

E infinitely many solutions

30 The following equations describe two distinct circles in the plane:

$$\begin{aligned}(x-4)^2+(y-7)^2 &= 5^2\\ x^2+y^2-2x-4y-20 &= 0\end{aligned}$$

Find an equation of the line which passes through the two points of intersection of these circles.

A $3x+3y=10$

B $3x+5y=10$

C $3x+y=30$

D $3x+5y=30$

E $x+5y=10$

31 Consider the following statement about a positive integer n:

($\star$) n^2 is even

The statement 'n is even' is

A **necessary** but **not sufficient** for ($\star$)

B **sufficient** but **not necessary** for ($\star$)

C **necessary** and **sufficient** for ($\star$)

D **not necessary** and **not sufficient** for ($\star$)

32 For $n \geq 3$, let a_n be a solution of the equation

$$3^{a_n} = na_n$$

Calculate the value of the infinite sum

$$3^{-3^{a_3-\log_3(a_3)}} + 3^{-3^{a_4-\log_3(a_4)}} + 3^{-3^{a_5-\log_3(a_5)}} + \ldots$$

A $\frac{1}{3}$

B $\frac{1}{6}$

C $\frac{1}{9}$

D $\frac{1}{18}$

E $\log_3\left(\frac{2}{3}\right)$

33 Consider the sequence $(I_n)_{n\geq 1}$ with general term

$$I_n = \int_0^1 \frac{x^n}{x+1} dx\,.$$

Which of the following is false for $n \geq 2$?

A $I_{n-1} + I_n = \frac{1}{n}$

B $I_n \leq \frac{1}{n}$

C $I_{n-1} \geq I_n$

D $I_n \leq \frac{1}{2n}$

E $I_n < \frac{1}{2(n+1)}$

34 Consider the following statements:

I $x^2 \geq 4$

II $x \geq 2$

Which of the following is true?

A **II** is necessary (but not sufficient) for **I**

B **II** is sufficient (but not necessary) for **I**

C **I** is sufficient (but not necessaty) for **II**

D **II** is both necessary and sufficient for **I**

E None of the above

35 The difference between the roots of the equation

$$3x^2 + 13x - c = 0$$

is 7. Find the value of c.

A $\frac{123}{45}$

B $\frac{209}{16}$

C $\frac{122}{55}$

D $\frac{3353}{21}$

E $\frac{68}{3}$

36 The function $f(x)$ is a cubic of the form

$$f(x) = x^3 + ax^2 + bx + c,$$

where a, b, c are real numbers. The graph of $y = f(x)$ intersects the x-axis at $(0, 8)$ and has exactly one stationary point at $x = 1$.

Find the value of $a + b + c$.

A -1

B 3

C 8

D 11

E 14

37 We are given that

$$1, 5, 9, \ldots, x$$

are consecutive terms of an arithmetic sequence. The sum of these terms is 325.

What is the product of the digits of x?

A 12

B 18

C 24

D 36

E 48

38 For what values of the non-zero real number a does the equation

$$ax^2 = \frac{1}{a}x + 1$$

have distinct real solutions?

A $a > 0$

B $a > 4^{-\frac{1}{3}}$

C $a > 2^{\frac{1}{3}}$

D $a < -2^{\frac{1}{3}}$

E No value of a

39 Consider the following statements:

I The curves $y = x^3 + 1$ and $y = (x-1)^{\frac{1}{3}}$ are symmetric about $y = x$,

II If there is a real solution to $(x-1)^{\frac{1}{3}} = x^3 + 1$, then the same solution satisfies $x^3 = x - 1$.

III If there is a real solution to $(x-1)^{\frac{1}{3}} = x$, then the same solution satisfies $x^3 = x - 1$.

Which of these statements is true?

A None of them

B **I** only

C **II** only

D **III** only

E **I** and **II** only

F **I** and **III** only

G **II** and **III** only

H **I**, **II** and **III**

40 Two points x and y are selected at random in the interval $[0, 1]$.

What is the probability that $x^2 + y$ will be less than 1?

A $\frac{1}{4}$

B $\frac{1}{3}$

C $\frac{1}{2}$

D $\frac{2}{3}$

E $\frac{3}{4}$

END OF EXAM