TMUA Multiple Choice Practice - Differentiation

1. The gradient of the curve $y = \dfrac{(4x + \sqrt{x})(x^2 - 3)}{3\sqrt{x}}$ at the point where $x = 1$ is

A $\dfrac{1}{3}$ B 2 C $\dfrac{10}{3}$ D 4 E $\dfrac{20}{3}$

2. A curve C has equation $y = \dfrac{x^2 - 2}{\sqrt{x}}$. Find the gradient of C at the point $(2,\sqrt{2})$.

A $\sqrt{2}$ B $\dfrac{7}{4}\sqrt{2}$ C $\dfrac{7}{2}\sqrt{2}$ D $4\sqrt{2}$ E $\dfrac{9}{2}\sqrt{2}$

3. The curve $y = x^3 + 3\sqrt{5}px^2 + 3px + 13$ has two distinct turning points.

Find the possible values of p.

A $p < 0, p > 0.2$
B $p \leq 0, p \geq 0.2$
C $0 < p < 0.2$
D $0 \leq p \leq 0.2$
E $p < 0, p > 1.2$
F $p \leq 0, p \geq 1.2$

4. Find the complete set of values of k for which the graph $y = x^3 - 2kx^2 + 4x - k$ has two distinct real stationary points.

A $-3 < k < 3$

B $k < -3$ or $k > 3$

C $-\sqrt{3} < k < \sqrt{3}$

D $k < -\sqrt{3}$ or $k > \sqrt{3}$

E all values of k

5. Given that the cubic equation $f(x) = p^{\frac{2}{3}}x^3 + px^2 + p^{\frac{1}{3}}x + 3$ where p is a positive constant has exactly one point where $f'(x) = 0$, find the value of p.

A $\frac{1}{4}$ B $\frac{3}{4}$ C 1 D 3 E 6

6. Consider the function given by $f(x) = x^{\frac{1}{5}}(x^2 - 2x + 1)$

The fraction of the interval $0 < x < 2$ for which $f(x)$ is decreasing is

A. $\frac{5}{11}$ B $\frac{1}{2}$ C $\frac{3}{5}$ D $\frac{5}{6}$ E $\frac{10}{11}$

7. A curve has equation $y = 3x^2 + 2$ and a line has equation $y = 5x - 6$.
What is the shortest distance parallel to the y-axis between the curve and the line?

A $\frac{5}{6}$ B $\frac{6}{5}$ C $\frac{49}{12}$ D $\frac{71}{12}$ E 8

8. A curve C has equation $y = 2x^3 - 5x^2 + a$ where a is a constant.

The tangent to C at $x = 2$ and the normal to C at $x = 1$ meet on the x-axis.

The value of a is

A $\frac{1}{4}$ B $\frac{2}{3}$ C 4 D 6 E $\frac{8}{3}$

9. The point P lies on the curve with equation $y = x^2$ so that its distance from the point $Q\ (-5, -1)$ is least. Find the distance PQ.

A $2\sqrt{5}$ B $\sqrt{26}$ C $4\sqrt{5}$ D 20 E 26

10. How many real roots does the equation $y = 3x^4 - 16x^3 + 18x^2 - 5$ have?

A 1 B 2 C 3 D 4 E 5

11. What is the highest term in x of the following polynomial

$$\frac{d^2}{dx^2}\left[(x^6+2)^2(x^4-3)^4\right] - \frac{d}{dx}\left[(3x^5-1)^3(x^2+4)^6\right]$$

A $26x^{25}$ B $27x^{26}$ C $28x^{26}$ D x^{27} E $28x^{28}$

12. A water tank, with volume $500m^3$, is to be made in the shape of a cuboid with a square base and no top. What is the least amount of metal in m^2 required to make this tank?

A $100\sqrt{2}$

B $100 + 50\sqrt{2}$

C 200

D 300

E $50 + 200\sqrt{2}$

13. A curve C has equation given by $f(x) = 2p^3 + 3p^2x - 2px^2 + x^3$ where p is real.

The gradient of the normal to C at $x = 1$ is M.

What is the least possible value of M as p varies?

A $-\frac{7}{2}$ B $-\frac{5}{2}$ C $-\frac{5}{3}$ D $-\frac{3}{5}$ E $\frac{2}{3}$

14. How many real roots does the equation $y = x^5 + 5x^3 - 20x + 14$ have?

A 1 B 2 C 3 D 4 E 5

15. A cubic curve has equation $y = ax^3 + bx^2 + cx + d$ where a, b, c, d are non-zero constants. Given that this curve has one local maximum and one local minimum, which of the following statements is necessarily true:

A $b^2 > 3ac$

B $b^2 > 4ac$

C $c^2 > 4bd$

D If $a > 0$, then $d > 0$

E If $a > 0$, then $d < 0$

16. The function $f(x) = \dfrac{3x-2}{\sqrt[3]{x^2}}$ is defined for all $x \neq 0$

The complete set of values of x for which the function is decreasing is

A $x < -\dfrac{4}{3},\ x > 0$

B $-\dfrac{4}{3} < x < 0$

C $-\dfrac{4}{3} < x < \dfrac{4}{3},\ x \neq 0$

D $-\dfrac{2}{3} < x < 0$

E $-\dfrac{2}{3} < x < \dfrac{2}{3},\ x \neq 0$

17. The volume V, of a soap bubble is modelled by the formula $V = (p - qt)^2 \quad t \leq 0$

where p and q are positive constants and t is the time in seconds after a certain instant.

When $t = 1$, the volume of a soap bubble is $9cm^3$ and at that instant its volume is decreasing at the rate of $6cm^3$ per second. What is the value of $p + q$?

A 2 B 3 C 5 D 6 E 9

18. The least possible value of the gradient of the curve $y = (x + a)^2(3x - a)$ at the point where $x = \dfrac{1}{2}$, as a varies is

A -9 B $-\dfrac{25}{4}$ C -4 D $-\dfrac{5}{2}$ E $\dfrac{5}{4}$

19. Consider the function $f(x) = x^3 - 3x^2 - 144x$

Which of the following statements are true?

I The gradient of the function is negative for $x < 0$.

II There is a local maximum at $x = 8$.

III There is a point of inflexion at $x = 1$.

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

20. A curve has equation $y = 3x^4 - 4x^3 - 12x^2 + 20$

What is the complete set of values of the constant k for which the equation

$3x^4 - 4x^3 - 12x^2 + 20 = k$ has four distinct real roots

A no values of k

B $-12 < x < 15$

C $15 < x < 20$

D $k > 15$

E $7 < x < 20$