TMUA/MAT Algebra and Functions

Syllabus
Laws of indices; surds; quadratic functions, graphs, discriminant, completing the square; solving equations; simultaneous equations; inequalities; polynomials (factorising, Factor & Remainder Theorem); $\sqrt{x}$ and modulus function.

1a) Solve the equation $2^{x+2} = 4\sqrt{2}$

b) Solve the equation $\dfrac{27^a}{3^{a-1}} = 3\sqrt{3}$

c) Solve the equation $4^x - 2^{x+2} = 32$

d) How many real solutions does the following equation have $8^x + 4 = 4^x + 2^{x+2}$

e) Find the values of k such that the equation $9^x - 3^{x+1} = k$ has one or more real solutions.

2a) Simplify the following expression giving your answer in the form $a + b\sqrt{3}$ $\dfrac{2\sqrt{3} - 1}{2 - \sqrt{3}}$

b) The area of a triangle is $(3 + \sqrt{3})$ cm^2 . Given that the base is $\sqrt{3}$ cm, find the height as a surd.

c) What positive integer does this expression simplify to $\dfrac{1 + \sqrt{7}}{3 - \sqrt{7}} - \dfrac{8 - \sqrt{7}}{\sqrt{7} - 2}$

d) Write this expression as a single fraction in its simplest form $\dfrac{1}{x - \sqrt{y}} + \dfrac{1}{x + \sqrt{y}}$

3a) The quadratic equation $x^2 + ax + b = 0$ where a and b are constants, is satisfied by $x = -2$ and $x = 5$. Find the values of a and b.

b) $f(x) = ax^2 + bx + c$ where a, b and c are non-zero constants,

Given $f(-1) = f(5) = 30$ and the minimum value of $f(x)$ is -6, solve the equation $f(x) = 3$

c) Solve the equation $x - \frac{14}{x} = 6\sqrt{2}$ giving your answers in the form $p\sqrt{2}$

d) Solve the equation $\sqrt{3}\left(x + \frac{6}{x}\right) = 9$ giving your answers in the form $p\sqrt{3}$

e) Solve the inequality $x^4 < 8x^2 + 9$

f) Given that $f(x) = x^2 + 10x + 27$ find k, such that the graph of $f(x) - k$ touches the x-axis

g) A quadratic curve meets the coordinate axis at (-2,0), (4,0), and (0,-20).

Find the equation of the curve.

h) A quadratic curve meets the coordinate axis at (2,0), (6,0), and (0,3).

Find the minimum point of the curve.

i) Find the constant k, such that the quadratic curves with equations

$y = k(2x^2 + 1)$ and $y = x^2 - 2x$ touch each other

j) Find the range of values of the constant k, such that the curve C with equation

$y = 4x^2 - 7x + 11$ and straight line L with equation $y = 5x + k$ do not intersect

k) The straight line L crosses the y-axis at (0,-1). The curve with equation $y = x^2 + 2x$ does not intersect with L. Determine the range of possible values of the gradient of L.

l) Given that f(n) is a square number for all values of n, find the possible values of the constant $\underline{k}$.

$f(n) = n^2 - 2kn + k + 12, \quad n \in \mathbb{N}$

m) The roots of $2x^2 - 7x + c = 0$ where c is a constant, differ by 3. Find the value of c.

n) The roots of $2x^2 + 5x + c = 0$ where c is a constant, differ by 2. Find the value of c.

4a) Given that the equation below has two distinct real roots,

$x^2 + 3ax + a = 0$ determine the range of values of *a*, where *a* is a constant.

b) Given that the equation below has two distinct real roots,

$x^2 + 6mx - 2m = 0$ determine the range of values of *m*, where *m* is a constant.

c) Given that the equation below has no real roots,

$x^2 + (k-1)x + (k+2) = 0$ determine the range of values of k, where k is a constant.

d) Given that the equation below has two different real roots,

$2x^2 + (3k-1)x + (3k^2-1) = 0$ determine the range of values of k, where k is a constant.

e) Given that the equation below has no real roots,

$kx^2 - x + (3k-1) = 0$ determine the range of values of k, where *k* is a non-zero constant.

f) Given that the equation below has two distinct real roots,

$mx^2 + (2m-3)x + 2m + 1 = 0$ determine the range of values of the non-zero constant *m*.

5a) The polynomial $x^3 + 4x^2 + 7x + a$ where a is a constant, has a factor of $(x + 2)$.

Find the value of a.

b) $f(x) = ax^3 - x^2 - 5x + b$ where a and b are constants.

When f(x) is divided by $(x - 2)$ the remainder is 36

When f(x) is divided by $(x + 2)$ the remainder is 40. Find the value of a and the value of b.

c) $f(x) = px^3 - 32x^2 - 10x + q$ where p and q are constants.

When f(x) is divided by $(x - 2)$ the remainder is exactly the same as when it is divided by $(2x + 3)$. Find the value of p.

d) $f(x) = 6x^2 + x + 7$

The remainder when f(x) is divided by $(x - a)$ is the same as the remainder when f(x) is divided by $(x + 2a)$, where a is a non-zero constant. Find the value of a.

e) $g(x) = x^3 + kx^2 - x + 12$

The remainder when g(x) is divided by $(x - 4)$ is 8 times the remainder when g(x) is divided by $(x - 1)$, where k is a constant. Find the value of k.

f) $f(x) = ax^2 + bx + c$ where *a, b* and *c* are non-zero constants

When f(x) is divided by $(x - 1)$ the remainder is 1.

When f(x) is divided by $(x - 2)$, the remainder is 2.

When f(x) is divided by $(x + 2)$, the remainder is 70. Find the values of *a, b* and *c.*

g) $f(x) = 2x^2 + 9x - 5$ Find k such that when f(x) is divided by $(2x - k)$ the remainder is 13.

h) $f(x) = x^3 + (a + 2)x^2 - 2x + b$ where a and b are non-zero constants, and $a > 0$.

Given that $(x - 2)$ and $(x + a)$ are factors of f(x), find the values of a and b.

i) When the polynomial $p(x) = x^2 - 2ax + a^4$ is divided by $(x + b)$ the remainder is 1

The polynomial $q(x) = bx^2 + x + 1$ has $(ax - 1)$ as a factor. Find the possible value(s) of b

j) Find the remainder when $1 + 3x + 5x^2 + 7x^3 + \ldots + 99x^{49}$ is divided by $(x - 1)$.

6a) Solve the equation $|3x + 2| = 1$

b) Solve the inequality $12 - 2|2x - 3| > 7$

c) Solve the equation $|x^2 - 2x - 4| = 4$

d) Solve the equation $f(x) = g(x)$ where $f(x) = |2x - 4|$ and $g(x) = |x|$ for $x \in \mathbb{R}$

e) Solve the inequality $fg(x) > 12$ where $f(x) = x + 4$ and $g(x) = |2x + 1| + 3$ for $x \in \mathbb{R}$

f) Solve the equation $|x^2 - 1| = |3x + 3|$

g) Find the set of values of x for which $|x^2 - 4| < 3x$

7a) The function f(n) is defined for positive integers n by

$f(1) = 5$ and for $n > 1$, $f(n+1) = 3f(n) - 1$ if $f(n)$ is odd and

$f(n+1) = \dfrac{f(n)}{2}$ if $f(n)$ is even

a) Find $f(99)$

b) How many numbers n in the interval $1 \leq n \leq 50$ satisfy $f(n) \leq 12$

b) The function f(n) is defined for positive integers n by

$f(1) = 2$ and for $n \geq 1$, $f(n+1) = 5f(n) + 1$ if $f(n)$ is odd and

$f(n+1) = \dfrac{1}{2}f(n)$ if $f(n)$ is even

a) Find $f(100)$

b) Find the value of $\displaystyle\sum_{r=1}^{50} f(r)$

c) The function f(n) is defined for positive integers n by

$f(1) = 4$ and for $n \geq 1$, $f(n+1) = \dfrac{1}{2}(f(n) + 3)$ if $f(n)$ is even and

$f(n+1) = 2f(n) + 3$ otherwise

Find the value of $f(99) + f(100)$

d) The function f is defined such that $f(mn) = \begin{cases} f(m)f(n) & \text{if mn is a multiple of 5} \\ mn & \text{if mn is not a multiple of 5} \end{cases}$

Given that $f(25) + f(9) - f(30) = 0$ find the value of $f(5)$