THE UK UNIVERSITY INTEGRATION BEE 2021/22

$$\int$$

ROUND TWO

Saturday, 20 November 2021

Sponsored by

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Shuttle 1

A1

Evaluate

$$\int_0^1 \ln^2(x)\,\mathrm{d}x$$

Pass on the answer squared.

2. *a is the number you will receive*
Evaluate

$$\int_0^{\frac{\pi}{2}} \frac{\sin(x)}{\sin(x + \frac{\pi}{a})}\,\mathrm{d}x$$

Your answer will be of the form $\dfrac{\pi}{b}$, pass on b.

3. *a is the number you will receive*
Evaluate

$$\int_0^{\frac{\pi}{2}} \frac{\mathrm{d}x}{1 + \tan^a(x)}$$

Pass on the 4 times the answer.

4. *a is the number you will receive*
Evaluate

$$\int_0^a x \ln(\sin x)\,\mathrm{d}x$$

Shuttle 2

1. *a is the number you will receive*
Evaluate

$$\int_0^{2\pi} \sin(\sin(x) - x)\mathrm{d}x$$

. Pass on the answer.

2. *a is the number you will receive*
Evaluate

$$b = \int_{a=0}^{\infty} \ln(1 + \frac{169}{x^2})\mathrm{d}x$$

. Pass on the second digit of b.

3. *a is the number you will receive*
Evaluate

$$b = \int_0^1 \frac{\ln(a = 0(x+1))}{x^2 + 1}\mathrm{d}x$$

. Pass on your answer.

4. *a is the number you will receive*
Evaluate

$$\int_{\frac{1}{a}}^{\infty} \frac{\ln x}{x^2 + \pi^2}\mathrm{d}x$$

.

Shuttle 3

1. *a is the number you will receive*
Evaluate
$$b = \int_{-1}^{1} \frac{\sin(\cot^{-1} x) + \cos(\tan^{-1} x)}{x^2 + 1} \, dx$$
. Pass on $b^2 - 2$.

2. *a is the number you will receive*
Evaluate
$$b = \int_{0}^{a} x^3 e^{-x} \, dx$$
. Pass on $2 - b$.

3. *a is the number you will receive*
Evaluate
$$b = \int_{0}^{\infty} \frac{\ln(x^{\frac{1}{a}})}{x^{\frac{1}{a}}(x + 1)^2} \, dx$$
. Pass on the least integer greater than $|b|$.

4. *a is the number you will receive*
Evaluate
$$b = \int_{0}^{\frac{\pi}{2}} \frac{\cos \theta}{a - \sin 2\theta} \, dx$$
.

Shuttle 4

1. *a is the number you will receive*
Evaluate

$$b = \lim_{n \to \infty} \int_{\frac{5}{6}}^{\infty} e^{-x^n} \, dx$$

. Your answer should be of the form $\dfrac{c}{d}$ for $c, d \in \mathbb{Z}$ are coprime, pass on the prime factors of d.

2. *a, b are the numbers you will receive where $a > b$*
Evaluate

$$c = \int_0^1 \frac{x^{a=3} - x^{b=2}}{\ln x} \, dx$$

. Pass on the greatest integer less than c.

3. *a is the number you will receive*
Evaluate

$$b = \int_0^1 \frac{1-x}{x^a + (x^a + 1)^a} \, dx$$

. Your answer will be of the form $b = \dfrac{x}{y}(\ln(w) - z)$ where $x, y, z, w \in \mathbb{Z}^+$ and x, y are coprime. Pass on $\dfrac{x + \sqrt{x+y}}{z}$.

4. *a is the number you will receive*
Evaluate

$$\int_0^{2\pi} \frac{x}{a - \cos^2 x} \, dx$$

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