THE UK UNIVERSITY INTEGRATION BEE 2024/2025

$$\int$$

ROUND ONE

1. $\displaystyle\fint_{-1}^{1} e^{in}\,\mathrm{d}n$, where $\displaystyle\fint_{a}^{b} f(x)\,\mathrm{d}x = \frac{1}{b-a}\int_{a}^{b} f(x)\,\mathrm{d}x$

2. $\displaystyle\int (1 + 2024x^{2024})e^{x^{2024}}\,\mathrm{d}x$

3. $\displaystyle\int \frac{\ln(x)+1}{x^x + x^{-x}}\,\mathrm{d}x$

4. $\displaystyle\int \sin^2 + \cos^2\ \mathrm{d}s$

5. $\displaystyle\int_{\frac{1}{4}}^{4} \frac{\tan^{-1}(x)}{x}\,\mathrm{d}x$

6. $\displaystyle\int_{-\infty}^{\infty} \frac{e^{-x^2}}{e^x + 1}\,\mathrm{d}x$

7. $\displaystyle\int_{0}^{1} \sqrt[3]{x}\left\lfloor \frac{1}{\sqrt[3]{x}} \right\rfloor \mathrm{d}x$

8. $\displaystyle\int_{0}^{\infty} \frac{1}{(x+1)(\ln(x)^2+1)}\,\mathrm{d}x$

9. $\displaystyle\int_{0}^{2\pi} \frac{1}{1+\sqrt{1-\sin^2(x)}}\,\mathrm{d}x$

10. $\displaystyle\int_{0}^{\infty} e^{-t^2}\cos(2xt)\,\mathrm{d}t$

11. $\displaystyle\int_{0}^{\infty} \frac{\mathrm{Si}(x)\sin(x)}{x}\,\mathrm{d}x$, where $\mathrm{Si}(x) = \displaystyle\int_{0}^{x} \frac{\sin(t)}{t}\,\mathrm{d}t$ is the sine integral

12. $\displaystyle\int_{-\infty}^{\infty} \frac{1}{(e^x + e^{-x} + 2)^2}\,\mathrm{d}x$

13. $\displaystyle\int_{0}^{\infty} \frac{\sin(x) + \sin\left(\frac{1}{x}\right)}{x(1+x^2)}\,\mathrm{d}x$

14. $\displaystyle\int_{0}^{1} \frac{\tan^{-1}(x^n)}{x}\,\mathrm{d}x$, where $n \geq 1$ is an integer

15. $\displaystyle\int_{-1}^{1} \frac{\sin(\sqrt{x})}{\sinh(\sqrt{x}) + \sin(\sqrt{x})}\,\mathrm{d}x$

16. $\displaystyle\int_{0}^{\infty} \frac{e^{-px^2} - e^{-qx^2}}{x^2}\,\mathrm{d}x$, where $p, q > 0$

17. $\displaystyle\int_{0}^{\frac{\pi}{4}+1} \tan\left(x - \tan\left(x - \ldots\right)\right)\,\mathrm{d}x$

18. $\displaystyle\int_{0}^{\frac{\pi}{2}} \ln(\sin(x))\,\mathrm{d}x$

19. $\displaystyle\int_{0}^{\frac{\pi}{4}} \tan^{-1}\left(\frac{1+\tan(x)}{\sqrt{2}}\right)\,\mathrm{d}x$

20. $\displaystyle\int_0^{100} x^{\{x\}-1}\left(\ln(x^x) + \{x\}\right)\,\mathrm{d}x$

21. $\displaystyle\int_1^{2024} \frac{e^{x^x}}{\frac{1}{x} \cdot \frac{1}{x} \cdot \frac{2}{x} \cdot \frac{3}{x} \cdot \frac{5}{x} \cdot \frac{8}{x} \cdot \frac{13}{x} \cdot \frac{21}{x} \cdots}\,\mathrm{d}x$

22. $\displaystyle\int_a^b \frac{\tan^{-1}\left(\frac{x}{a}\right) + \tan^{-1}\left(\frac{x}{b}\right)}{x}\,\mathrm{d}x, \qquad \text{where } a, b > 0$

23. $\displaystyle\int_0^\infty \frac{\sin(x)\sin(2x)}{x}\,\mathrm{d}x$

24. $\displaystyle\int_{-1}^1 \frac{\ln\left((1+x)(1+x^2)(1+x^4)(1+x^8)\ldots(1+x^{2^{2024}})\right)}{x}\,\mathrm{d}x$

25. $\displaystyle\int_0^1 \sin(\pi x)\ln(\Gamma(x))\,\mathrm{d}x$

26. $\displaystyle\max_{k\in\mathbb{R}}\left(\int_0^{\sin^2(k)} \sin^{-1}(\sqrt{x})\,\mathrm{d}x + \int_0^{\cos^2(k)} \cos^{-1}(\sqrt{x})\,\mathrm{d}x\right)$

27. $\displaystyle\int_0^\infty \frac{e^{-x}}{\sqrt{x}\,\sqrt[x]{e}}\,\mathrm{d}x$

28. $\displaystyle\int_0^1 \frac{\sin^{-1}(x^2) + \sin^{-1}(\sqrt{x})}{\sqrt{1-x^2}}\,\mathrm{d}x$

29. $\displaystyle\int_{\frac{1}{\ln(2)}}^{2024} \lim_{n\to\infty}\sum_{k=1}^n \frac{\sqrt[n]{2^{kx}}kx^4}{nkx^3+1}\,\mathrm{d}x$

30. $\displaystyle\int_0^{\frac{\pi}{2}} \cosh^{-1}(\sin(x)+\cos(x))\,\mathrm{d}x$