Cambridge Integration Bee Syllabus

Please note that this syllabus is more of a guideline of content that will allow you to be able to solve each problem rather than a strict requirement for every problem - a lot of the time advanced techniques/special functions can be avoided with clever substitutions and tricks!

- Everything which is on the STEP/A level Maths and Further Maths syllabus for Integration (substitution, integration by parts).
- Differentiation under the integral sign (DUTIS):

$$\frac{d}{dt}\left(\int_a^b f(x,t)\mathrm{d}x\right) = \int_a^b \frac{\partial}{\partial t}\left(f(x,t)\right)\mathrm{d}x$$

- The Weierstrass substitution, $t = \tan\left(\frac{x}{2}\right)$ (also known as t substitution)
- Infinite series and their use in evaluating integrals, swapping an integral and an infinite sum. Convergence issues won't be considered.
- The reflection property of integrals:

$$\int_a^b f(x)\mathrm{d}x = \int_a^b f(a+b-x)\mathrm{d}x$$

- The floor function $\lfloor x \rfloor$ which rounds down to the integer less than or equal to x.
- The Gamma function

$$\Gamma(n) = \int_0^\infty x^{n-1}e^{-x}\mathrm{d}x$$

- The Beta function

$$\mathrm{B}(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}\mathrm{d}t = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

- The Riemann zeta function $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$ for $s > 1$.
- Some useful infinite series include

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)^2} = G \quad \text{(Catalan's Constant)}$$

- Odd functions, functions such that $f(-x) = -f(x)$ and even functions, functions such that $f(-x) = f(x)$.