THE UK UNIVERSITY INTEGRATION BEE

2021/22

$$\int$$

ROUND TWO SHUTTLE ROUND
Saturday, 20 November 2021

UNIVERSITY OF OXFORD

Durham University

WARWICK
THE UNIVERSITY OF WARWICK

Imperial College London

UNIVERSITY OF CAMBRIDGE

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SHUTTLE A   A1

Evaluate
$$\int_0^1 \ln^2(x)\mathrm{d}x.$$
Pass on your answer squared.

a is the number you will receive.   A2

Evaluate
$$\int_0^{\frac{\pi}{2}} \frac{\sin(x)}{\sin(x + \frac{\pi}{a})}\mathrm{d}x.$$
Your answer will be of the form π/b, pass on b.

b is the number you will receive.   A3

Evaluate
$$\int_0^{\frac{\pi}{2}} \frac{\mathrm{d}x}{1 + \tan^b(x)}.$$
Pass on 4 times your answer.

c is the number you will receive.   A4

Evaluate
$$\int_0^c x\ln(\sin x)\mathrm{d}x.$$

SHUTTLE B   B1

Evaluate
$$\int_0^{2\pi} \sin(\sin(x) - x)\mathrm{d}x.$$
Pass on the answer.

a is the number you will receive.   B2

Evaluate
$$\int_a^\infty \ln\left(1 + \frac{169}{x^2}\right)\mathrm{d}x.$$
Pass on the second digit of your answer.

b is the number you will receive.   B3

Evaluate
$$\int_0^1 \frac{\ln(b(x+1))}{x^2+1}\mathrm{d}x.$$
Pass on your answer.

c is the number you will receive.   B4

Evaluate
$$\int_{\frac{1}{c}}^\infty \frac{\ln x}{x^2 + \pi^2}\mathrm{d}x$$

SHUTTLE C   C1

Evaluate

$$\int_{-1}^{1} \frac{\sin(\cot^{-1} x) + \cos(\tan^{-1} x)}{x^2 + 1} \mathrm{d}x$$

Pass on your answer squared minus 2.

a is the number you will receive.   C2

Evaluate

$$b = \int_{0}^{a} x^3 e^{-x} \mathrm{d}x$$

Pass on 2 minus your answer.

b is the number you will receive.   C3

Evaluate

$$\int_{0}^{\infty} \frac{\ln(x^{\frac{1}{b}})}{x^{\frac{1}{b}}(x+1)^2} \mathrm{d}x$$

Pass on the smallest integer greater than the absolute value of your answer.

c is the number you will receive.   C4

Evaluate

$$\int_{0}^{\frac{\pi}{2}} \frac{\cos x}{c - \sin 2x} \mathrm{d}x$$

SHUTTLE D   D1

Evaluate

$$\lim_{n \to \infty} \int_{\frac{5}{6}}^{\infty} e^{-x^n} \, \mathrm{d}x$$

Your answer should be of the form $\frac{p}{q}$ for coprime $p, q \in \mathbb{Z}^+$, pass on the prime factors of q.

a, b are the numbers you will receive where $a > b$.   D2

Evaluate

$$\int_0^1 \frac{x^a - x^b}{\ln x} \, \mathrm{d}x$$

Pass on the least integer bigger than your answer.

c is the number you will receive.   D3

Evaluate

$$\int_0^1 \frac{1 - x}{x^c + (x^c + 1)^c} \, \mathrm{d}x$$

Your answer will be of the form $\frac{r}{s}(\ln(t) - u)$ where $r, s, t, u \in \mathbb{Z}^+$ and r, s are coprime.

Pass on $\dfrac{r + \sqrt{r + s}}{u}$.

d is the number you will receive.   D4

Evaluate

$$\int_0^{2\pi} \frac{x}{d - \cos^2 x} \, \mathrm{d}x$$

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