Paper B1

20 questions
75 minutes
No calculator allowed

1. Find the value of $\int_1^2 6\sqrt{x} + \frac{16}{x^3}\ dx$

A $8\sqrt{2} - 10$
B $4\sqrt{8} - \frac{1}{4}$
C $8\sqrt{2} + 2$
D $4\sqrt{8} + \frac{1}{4}$
E $4\sqrt{2} + 6$

2. How many solutions does the following equation have in the range $0 \leq x \leq 2\pi$

$$sin2x + sin^2x = 1$$

A 2
B 3
C 4
D 6
E 8
F infinitely many

3. What is the shortest distance from the point A (5 , 0) to the curve with equation $y = x^2 + 1$

A $\sqrt{5}$
B $2\sqrt{2}$
C $2\sqrt{5}$
D 5
E $5\sqrt{2}$

4. Consider an arithmetic sequence with $(2n + 1)$ terms.

What is the ratio of the sum of odd terms to the sum of even terms?

A $n^2 : n^2 - 1$

B $n + 1 : n$

C $n : n - 1$

D $n + 1 : n - 1$

E $n(n + 1) : 2n + 1$

5. Find the sum of the solutions of the equation $x^2 + 2\sqrt{x^2 + 6x} = 24 - 6x$

A - 6

B - 2

C 2

D 6

E 10

6. The function f is given by $f(x) = \left(\frac{1}{x} - \frac{2}{x^2}\right)^2 \qquad x \neq 0$

What is the value of $f''(1)$

A - 6

B - 2

C 26

D 38

E 122

7. A sequence is defined by $u_n = \sum_{r=0}^{n-1} u_r$ and $u_0 = 1$

Evaluate $\sum_{r=0}^{\infty} \frac{1}{u_r}$

A the sum does not converge

B 1

C 2

D 3

E ⅔

8. Given that $3^a = 16$ and $2^b = 27$, find the value of ab

A 3 B $\frac{7}{2}$ C 4 D $\frac{9}{2}$ E 12

9. In a set of k consecutive integers, the largest number is 23. What is the mean of the set?

A $\frac{1}{2}(k + 45)$

B $25 - 2k$

C $\frac{23}{2}k$

D $\frac{1}{2}(25 - k)$

E $\frac{1}{2}(47 - k)$

10. Evaluate the following integral $\int_0^4 x\,|x-4|\;dx$

A $-\frac{32}{3}$ B $-\frac{16}{3}$ C $\frac{16}{3}$ D $\frac{32}{3}$ E 0

11. Which of the following is the largest?

A $2^{\frac{1}{2}}$ B $3^{\frac{1}{3}}$ C $4^{\frac{1}{4}}$ D $6^{\frac{1}{6}}$ E $12^{\frac{1}{12}}$

12. Two players take it in turns to throw a fair six-sided die until one of them scores a six. What is the probability that the first player to throw the die is the first to score a six?

A $\frac{3}{5}$ B $\frac{4}{7}$ C $\frac{5}{9}$ D $\frac{6}{11}$ E $\frac{7}{12}$

13. How many real roots does the equation $12x^5 - 45x^4 + 40x^3 - 10$ have?

A 1 B 2 C 3 D 4 E 5

14. Find the coefficient of x^4y^2 in the expansion of $(1 + x^2 + y)^6$

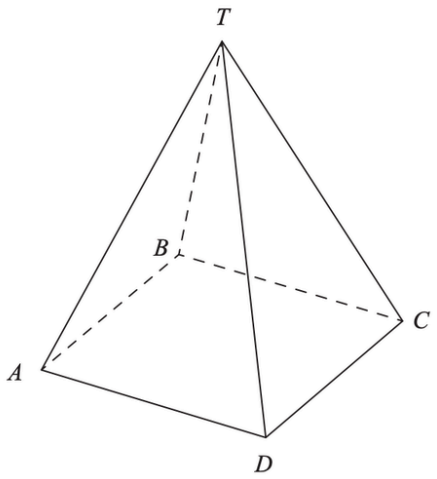
A 6
B 15
C 30
D 90
E 120

15. Find the minimum value of $f(x) = 25sin^4x - 30sin^2x + 11$

A 1
B 2
C 5
D 8
E 11

16. A square based pyramid, with base ABCD, and vertex T has all edges of length 2m.

Find the shortest distance, in metres, along the outer surface of the prism from the midpoint of AB to the midpoint of CT.



A $\sqrt{3}-1$

B 2

C $\sqrt{2}+1$

D $\sqrt{4+\sqrt{3}}$

E $2\sqrt{2}$

17. Three geometric transformations are defined as follows:

R is a reflection in the y-axis

S is a stretch parallel to the x-axis, scale factor 1/2

T is a translation by 3 units in the negative x direction

These three transformations are applied to the graph of $y=\sqrt{x}$ resulting in the graph of $y=\sqrt{3-2x}$

In which order were the transformations applied?

A R then S then T

B R then T then S

C S then R then T

D S then T then R

E T then S then R

18. The curve C has equation $y = x^2 + bx + 3$ where $b \geq 0$

Find the value of b that minimises the distance between the origin and the stationary point of the curve C.

A $b = 0$

B $b = 2$

C $b = \sqrt{6}$

D $b = \sqrt{10}$

E $b = 10$

19. How many solutions does the following equation have in the range $0 \leq x < 2\pi$

$$2 = sinx + sin^2x + sin^3x + sin^4x + \ldots$$

A 0

B 1

C 2

D 3

E infinitely many

20. The polynomial function $f(x)$ is such that $f(x) > 0$ for all values of x.

Given that $\int_2^4 f(x)\ dx = A$ and $\int_4^6 f(x)\ dx = B$

evaluate $\int_{-2}^{2} f(x+4) + 2\ dx$ in terms of A and B

A $A + B$

B $2(A + B)$

C $A + B + 2$

D $A + B + 8$

E $4(A + B) + 2$

Paper B2

20 questions
75 minutes
No calculator allowed

1. For how many values of the constant k does the following equation have only one real solution

$$kx^2 - (k-1)x + k = 0$$

A no values of k
B one value of k
C two values of k
D all values of k except $k = 1$
E all values of k

2. How many solutions does the following equation have in the range $0 \leq x \leq \pi$

$$sin2x = cosx$$

A 1
B 2
C 3
D 4
E infinitely many

3. For which real numbers x does the following inequality hold

$$\frac{x}{x^2+1} \leq \frac{1}{2}$$

A all real numbers x
B only for real numbers $x \leq \frac{1}{2}$
C only for real numbers $x \geq 1$
D none of the above

4. Consider the following attempt at solving the equation $3sin2x + 7cosx = 0$ for $0 \leq x \leq 360$.

I $6sinxcosx + 7cosx = 0$

II $6sinx + 7 = 0$

III $sinx = -\frac{6}{7}$

IV There are two real solutions to this equation.

Which statement describes this attempt?

A It is completely correct

B It is incorrect and the first mistake occurs on line I

C It is incorrect and the first mistake occurs on line II

D It is incorrect and the first mistake occurs on line III

E It is incorrect and the first mistake occurs on line IV

5. The function f is defined for positive integers and satisfies

$f(1) = 1$ and $f(2n) = f(n)$ and $f(2n + 1) = f(n) + 1$

What is the value of $f(9)$

A 1

B 2

C 3

D 4

E 5

6. Consider the following two statements about the polynomial $p(x)$

P: $p(x)$ has at least one real root.

Q: $p(x)$ is a polynomial of order n, where n is an odd integer.

Which of the following is correct?

A P is **necessary** and **sufficient** for Q

B P is **not necessary** and **not sufficient** for Q

C P is **necessary** but **not sufficient** for Q

D P is **sufficient** but **not necessary** for Q

7. The real numbers a and b are such that exactly one of the following statements is true. Which is the true statement?

A $a \geq 0$

B $a < b$

C $a^2 > b^2$

D $|a| \leq |b|$

8. Consider the following statement about the polynomial $p(x)$ where a and b are real numbers with $a < b$.

(*) There exists a number c with $a < c < b$ such that $p(c) = 0$

Which of the following is true?

A The condition $p(a)p(b) < 0$ is **necessary** and **sufficient** for (*)

B The condition $p(a)p(b) < 0$ is **not necessary** and **not sufficient** for (*)

C The condition $p(a)p(b) < 0$ is **necessary** but **not sufficient** for (*)

D The condition $p(a)p(b) < 0$ is **sufficient** but **not necessary** for (*)

9. A student wishes to evaluate the function $f(x) = \dfrac{tan\,x}{x}$ where x is in radians, but only has a calculator that works in degrees.

What can the student type into their calculator in order to correctly evaluate $f(5)$

A $\dfrac{1}{5} \times tan(\pi \times 5 \div 180)$

B $5 \times tan(\pi \times 5 \div 180)$

C $tan(\pi \times 5 \div 180) \div (\pi \times 5 \div 180)$

D $tan(180 \times 5 \div \pi) \div (180 \times 5 \div \pi)$

E $\dfrac{1}{5} \times tan(180 \times 5 \div \pi)$

F $5 \times tan(180 \times 5 \div \pi)$

10. A sequence is such that $u_1 = 6$ and $u_2 = 3$ and $u_{n+1} = \dfrac{u_n}{u_{n-1}}$ for $n > 1$

What is the value of u_{2023} ?

A $\dfrac{1}{2}$

B $\dfrac{1}{6}$

C 2

D 3

E 6

11. The fact that $5 \times 6 = 30$,
is a counter example to which of the following statements:

A the product of any two odd integers is odd

B if the product of two integers is a multiple of 4 then the integers are not consecutive

C if the product of two integers is not a multiple of 4 then the integers are not consecutive

D any even integer can be written as the product of two even integers.

12. What is the reflection of the point (3,4) in the line $3x + 4y = 50$

A (9 , 12)

B (12 , 9)

C (6 , 8)

D (8 , 6)

E (12 , 16)

F (16 , 12)

13. Consider the four options below about a particular statement:

A The statement is true if $x^2 < 1$

B The statement is true if and only if $x^2 < 1$

C The statement is true if $x^2 < 2$

D The statement is true if and only if $x^2 < 2$

Given that exactly one of these options is correct, which one is it?

14. Find the minimum value of $f(x) = 2x^3 - 9x^2 + 12x + 3$ for $0 \leq x \leq 2$

A 1

B 2

C 3

D 5

E 7

15. Given that the numbers x and y satisfy $(x-1)^2 + y^2 \leq 1$.

What is the largest value that $x + y$ can be?

A 1

B $\sqrt{2}$

C 2

D $1 + \sqrt{2}$

E 4

16. Given that $x = -b$ is a root of the equation
$f(x) = ax^3 + ax^2 + ax + b$ where a and b are constants.

Find the range of possible value of a.

A There are no possible values of a

B $a < 1$

C $0 \leq a \leq \frac{4}{3}$

D $a \geq 1$

E a can be any real number

17. Let $a, b, c > 0$. The equations: $log_b a = 2$ $log_a(4c - 5) = 2$ $log_b(c - 1) = 3$

A specify a uniquely

B are satisfied by two values of a

C are satisfied by infinitely many solutions for a

D are contradictory

18. Which of the following integrals has the largest value?
You are not expected to calculate the exact values of any of these.

A $\int_0^{8\pi} sin^{64}x \ dx$

B $\int_0^{4\pi} 64(cos^6 x - 1) \ dx$

C $\int_0^1 (x^2 - 2)sin^6(\pi x) \ dx$

D $\int_0^{4\pi} (3 + sin x)^6 \ dx$

19. Which of the following statements are true?

I **There exists** a real y such that **for all** real x, $y > x$

II **There exists** a real x such that **for all** real y, $x + y > xy$

III **For all** real x, **there exists** real y such th*at* $x - y = xy^2 + 1$

A none of them

B I only

C II only

D III only

E I and II only

F II and III only

G I and III only

H I, II and III

20. f is a function and a, b are real numbers.
Given that exactly one of the following statements is true, which one is it?

A $f(a) \geq f(b)$ if and only if $a \geq b$

B $f(a) \geq f(b)$ only if $a < b$

C $f(a) < f(b)$ if $a \geq b$

D $a \geq b$ if $f(a) \geq f(b)$

E $a < b$ only if $f(a) \geq f(b)$

F $a < b$ only if $f(a) < f(b)$