Integration Competition Problems

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1. $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{1+e^x}\,\mathrm{d}x$

2. $\displaystyle\int_0^\infty \frac{\mathrm{d}x}{(x+\frac{1}{x})^2}$

3. $\displaystyle\int_0^{\frac{1}{2}} \frac{1+\sqrt{3}}{\sqrt[4]{(1+x)^2(1-x)^6}}\,\mathrm{d}x$

4. $\displaystyle\int_0^\infty \frac{\cos(\pi x)-\cos(ex)}{x}\,\mathrm{d}x$

5. $\displaystyle\int \sqrt{\tan x}\,\mathrm{d}x$

6. $\displaystyle\int_0^{\frac{\pi}{2}} \cot x \ln(\sec x)\,\mathrm{d}x$

7. $\displaystyle\int \frac{x^{-\frac{1}{2}}}{1+x^{\frac{1}{3}}}\,\mathrm{d}x$

8. $\displaystyle\int_0^1 \ln(x)\sin(\ln(x))\,\mathrm{d}x$

9. $\displaystyle\int \frac{\mathrm{d}x}{x^2-x\sqrt{x^2-1}}$

10. $\displaystyle\int_2^4 \frac{\sqrt{\ln(9-x)}\,\mathrm{d}x}{\sqrt{\ln(9-x)}+\sqrt{\ln(x+3)}}$

11. $\displaystyle\int \frac{(3x^{10}+2x^8-2)\sqrt[4]{x^{10}+x^8+1}}{x^6}\,\mathrm{d}x$

12. $\displaystyle\int_0^\infty \frac{\sin^2 x}{x^2(x^2+1)}\,\mathrm{d}x$

13. $\displaystyle\int \sqrt{1+e^x}\,\mathrm{d}x$

14. $\displaystyle\int_0^\infty \frac{\ln(x^2+1)}{x^2+1}\,\mathrm{d}x$

15. $\displaystyle\int \frac{1}{\sqrt{1+e^{2x}}}\,\mathrm{d}x$

16. $\displaystyle\int_0^\pi \frac{\ln(1+k\cos x)}{\cos x}\,\mathrm{d}x$ where $0 < k < 1$.

17. $\displaystyle\int_0^\infty \frac{\ln(1+x)}{x\sqrt{x}}\,\mathrm{d}x$

18. $\displaystyle\int \frac{\mathrm{d}x}{2+2\sin x+\cos x}$

19. $\displaystyle\int_0^1 x^{-x}\,\mathrm{d}x$

20. $\displaystyle\int_0^{\frac{\pi}{2}} (\ln(\tan\theta))^2\,\mathrm{d}\theta$

21. $\displaystyle\int_0^\pi \ln(1-2\pi\cos x+\pi^2)\,\mathrm{d}x$

22. $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{\sin x+\cos x}\,\mathrm{d}x$

23. $\displaystyle\int_0^1 x\left\{\frac{1}{x}\right\}\left\lfloor\frac{1}{x}\right\rfloor\,\mathrm{d}x$ where $\{x\} = x - \lfloor x \rfloor$ is the fractional part of x.

24. $\displaystyle\int \sqrt{x-\sqrt{x+\sqrt{x-\sqrt{x+\ldots}}}}\,\mathrm{d}x$

25. $\displaystyle\int_0^\pi \ln(a+b\cos x)\,\mathrm{d}x$ where $a > b$

26. $\displaystyle\int \cos(\ln x)\,\mathrm{d}x$

27. $\displaystyle\int_0^\infty \frac{\arctan x}{1+x}\frac{\mathrm{d}x}{\sqrt{x}}$

28. $\displaystyle\int_0^1 \ln x \ln(1-x)\,\mathrm{d}x$

29. $\displaystyle\int \frac{x \ln(x + \sqrt{x^2+1})}{\sqrt{1+x^2}}\,\mathrm{d}x$

30. $\displaystyle\int_0^{2\pi} \frac{1}{\sin^4(x) + \cos^4(x)}\,\mathrm{d}x$

31. $\displaystyle\int \sqrt{1 + \sin\frac{x}{2}}\,\mathrm{d}x$

32. $\displaystyle\int_0^1 \ln(1+x)\ln(1-x)\,\mathrm{d}x$

33. $\displaystyle\int_0^1 \arctan\left(\frac{1}{x^2 - x + 1}\right)\,\mathrm{d}x$

34. $\displaystyle\int e^{x+e^x}\,\mathrm{d}x$

35. $\displaystyle\int_0^\pi \arctan(3^{\cos x})\,\mathrm{d}x$

36. $\displaystyle\int \frac{x}{x^4 + 4}\,\mathrm{d}x$

37. $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{x-1} + \sqrt{(x-1)^3}}$

38. $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos x}{2 - \sin 2x}\,\mathrm{d}x$

39. $\displaystyle\int_0^1 \ln\left(\frac{2+x}{2-x}\right)\frac{\mathrm{d}x}{x\sqrt{1-x^2}}$

40. $\displaystyle\int_0^{\frac{\pi}{4}} \ln\left(\frac{\sec^2 x - 2}{\tan x - 1}\right)\,\mathrm{d}x$