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Experimental and Simulation Studies on Water Discharge Coefficients of Rectangular Piano Key Weirs

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Abstract. The piano key weir (PKW) is a developed type of labyrinth spillway with the ability to transfer large amounts of discharge by keeping executive costs constant. In this study, the parameters affecting the discharge coefficient of nine models were evaluated using physical models and simulations by Flow-3D software. The PKW models included: PK1.0, PK1.1, PK1.2, PK1.3, PK1.4, PK1.5, and PK1.6 representing the width ratios of the inlet (W_i) to outlet (W_o) keys of 1.0, 1.1, 1.2, 1.3, 1.4, 1.5 and 1.6 respectively and the other two models were PKT (PK1.1 with a thicker wall) and PKTP (PK1.1 with a thicker wall and an enhanced crown). According to the results of experimental and simulation evaluations, the model of PK1.4 was selected as the optimal model, which increased the discharge rate by 30% compared to the control weir. Moreover, increasing wall thickness (PKT model) led to an increase in the discharge and installing a parapet wall (PKTP model) resulted in an increase in discharge and a uniform distribution of flow lines on the weir. Considering the superiority of models PK1.4, PKT, and PKTP, the geometric properties of these models can be used to optimize the design of PKWs.

Keywords: Flow-3D; Overflow; Parapet wall; Physical models; Piano key weir; Water flow

1. Introduction

Recent technological advances have created vast facilities for constructing large dams, reservoirs, and canals. Increasing the water discharge has a decisive role in increasing the reliability of water storage structures such as dams, and in this case, spillways are designed to pass large discharges through a hydraulic structure without causing major damage to the structure and its surroundings (Karimi Chahartaghi et al., 2020).

Based on dam failure reports, one-third of failures were caused by low overflow discharge capacity (Kabiri-Samani & Javaheri, 2012). By conducting numerous studies, the researchers concluded that overflows should be constructed non-linearly to achieve high-performance economic structures. One of the simple and affordable solutions is to design piano key weirs (PKWs) (Lempérière & Ouamane, 2003; Erpicum et al., 2014). The PKW is a new shape of labyrinth spillway presented by the Hydrocoop research institute in collaboration with the University of Biskra, Algeria, in 2003 (Lempérière & Ouamane,

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2003; Pralong et al., 2011). This type of weirs has the advantages of having a high discharge capacity, being slightly affected by ground constraints, and being economically efficient (Li et al., 2020). It can easily be used in irrigation and drainage networks to increase the water head and reduce the water's extra energy. According to the conducted studies, the flow passing through a PKW is at least four times that of traditional spillways. Furthermore, these types of overflows are applicable for modifying the vortex in circular vertical overflows, which sometimes reduce the flow rate or cause vibration, crash, cavitation, distribution, and separation of flow lines and in many cases, endanger the safety of structures. In fact, the anti-vortex blades created by these overflows can slow down the flow (Shemshi & Kabiri-Samani, 2017). Many studies have been carried out on the PKWs, all of which have concluded that, at low head pressures, increasing the number of overflow openings increases the efficiency of the overflow (Machiels et al., 2012; Ribeiro et al., 2012; Anderson & Tullis, 2013). The studies conducted by Hien et al. (2006) indicated that six openings in low head pressures and five to seven openings in high head pressures increase the discharge coefficient. Noui and Ouamane (2011) and Hien et al. (2006) found that if the inlet opening is chosen larger than the outlet opening, it will have resulted in an increase in the discharge of the overflow.

Studies by Eslinger and Crookston (2020) showed that increasing the ratio of the inlet to outlet key width (W_i/W_o) increased the hydraulic efficiency of the piano key weirs significantly and had very little effect on energy dissipation. The experiments of Khassaf and Al-Baghdadi (2018) showed that the effect of increasing the ratio of W_i/W_o to 2.5 reduced the discharge coefficient of piano key weirs by 12%.

Although most studies (Seyedjavad et al., 2019; Feili et al., 2020; Kumar et al., 2020) have now shifted to trapezoidal piano key weirs, the benefits of rectangular piano key weirs, such as simple physical and software modeling, simple execution, and uniform distribution of incoming loads, has led to studies to improve quality and quantity of these types of overflows.

Today, computer models based on numerical solutions are increasingly used in a wide range of applied research (Šimůnek et al., 2008; Syaiful et al., 2017; Agrebi et al., 2019; Yanuar et al., 2020). Flow-3D is one of the most powerful 3D software packages for computational fluid dynamics with a wide range of applications and capabilities (Parsaie et al., 2015) due to its critical features such as user-friendly specifications, high simulation efficiency, and strong graphical interface (Taghavi & Ghodousi, 2015).

Despite the extensive studies on PKWs, there is still no comprehensive and accurate information on flow characteristics in this kind of overflow, as well as the relationships illustrating the straightforward changes of the inlet (W_i) and outlet (W_o) openings ratios, wall thickness and the height or flanging of outlet keys have not been provided till now. Therefore, the overall aim of this study was to improve the discharge coefficient of piano key weirs, and to achieve this aim; experiments were performed with the following objectives: (i) providing an optimum ratio of the inlet to outlet keys widths (Wi/Wo) of rectangular PKWs in the range of 1-1.6 through experimental models and Flow-3D simulations, (ii) maintaining the effects of changing in wall thickness and height of the outlet keys on weir performance.

2. Methods

2.1. Piano key weir traits

The main geometric parameters of a rectangular piano key weir shown in Figure 1 as defined by Machiels et al. (2011) are the transverse width W, the thickness T_s, the outlet key height P_o, the inlet key height P_i, the outlet key width W_o, the inlet key width W_i, the inlet overhang length B_i, the outlet overhang length B_o, the base length B_b and the lateral crest length B.

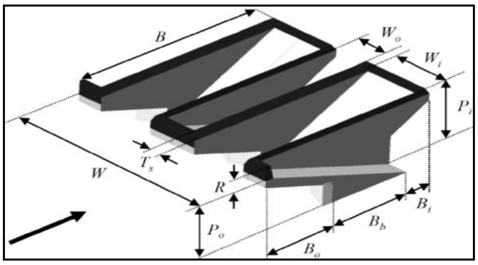

Figure 1 Main parameters of a PK weir, adapted from Machiels et al. (2011)

2.2. Dimensional analyses

The general equation for sharp edge rectangular weir is presented as equation (1):

$$Q = \frac{2}{3}\sqrt{2g}.C_d.L.H^{\frac{3}{2}} \tag{1}$$

C_d is the discharge coefficient, g is the gravitational constant, H relates to the upstream water level, and L refers to the spillway length. It may be assumed the discharge coefficient of piano key weirs can be obtained from the general relation of the spillways which is calculated by equation (2):

$$C_d = \frac{Q}{\frac{2}{3}\sqrt{2g}.L.H^{\frac{3}{2}}} \tag{2}$$

The discharge coefficient can be indicated by using dimensional analysis according to equation (3):

$$C_d = f\left(\frac{H}{P}, \frac{B}{w}, \frac{w_i}{w}, \frac{w_o}{w}, \frac{B_i}{B}, \frac{B_o}{B}, n_c, w_b, Re\right) \tag{3}$$

W_b and Re refer to the *Weber* and the *Reynolds* numbers, respectively. Usually, on channels, the Reynolds number is large enough so the effect of viscosity can be neglected. Salmasi and Abraham's (2022) studies showed that the effect of the surface tension could also be ignored if the water head on the weir is higher than 3 to 4 cm. Thus, the *Weber* and the *Reynolds* numbers are removed from Equation (3). Furthermore, because n is an input and output width function, it can be removed. As a result, Equation (3) can be simplified as Equation (4):

$$C_d = f\left(\frac{H}{P}, \frac{B}{w}, \frac{w_i}{w}, \frac{w_o}{w}, \frac{B_i}{B}, \frac{B_o}{B}\right) \tag{4}$$

Because of the three-dimensional nature and complexity of the flow on the piano key weirs, the simplest way of calculating the discharge coefficient for these weirs is as laboratory models.

2.3. Designing of models

Nine physical models were studied in this experiment. Physical modelling was performed in the Water Engineering Laboratory of Azad University's Kermanshah Branch. Facilities and laboratory space devoted to the study are as follows.

- Laboratory area: 200 m^2
- Flume length 10 m, flume width 60 cm, wall height 70 cm
- Feeding system included three Electro-pumps having a flow rate of 100 L/s and two tanks made of galvanized iron with a capacity of 15 m^3
- Ultrasonic flow meter with digital display and PC connectivity and data recording
- Ultrasonic altimeter with digital display and flume-specific carriage

Considering the overflow dimensions, nine plexiglass models were created, each with six openings (inlet and outlet) that were geometrically different in key widths. Overflows were placed on a 35-centimeter-high bench. The ratio between inlet and outlet widths in seven of the models were 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, and 1.6 which were named PK1.0, PK1.1, PK1, PK1.3, PK1.4, PK1.5, and PK1.6 respectively as displayed in Table 1. In the other two models, two types of modifications were made on PK1.1 including 1) The wall thickness of the PK1.1 model was increased about three times (from 2.8 to 8.5cm), and the new model was named PKT weir, 2). As illustrated in Table 1 and Figure 2, the crest of PK1.1 was raised by a 2 cm vertical parapet wall to increase the wall thickness, and the new model was named PKTP weir.

Table 1 Geometry characteristics of the PK weirs

Model	W_i/W_o	L (cm)	P/W_u	L/W	B_i=B_o (cm)	B_b (cm)	W_o (cm)	W_i (cm)	P (cm)	T_s (mm)	W (cm)
PK1.0	1.0	274.2	0.66	4.57	13	10	10	10.0	13.3	2.8	60
PK1.1	1.1	274.2	0.66	4.57	13	10	9.5	10.5	13.3	2.8	60
PK1.2	1.2	274.2	0.66	4.57	13	10	9.1	10.9	13.3	2.8	60
PK1.3	1.3	274.2	0.66	4.57	13	10	8.7	11.3	13.3	2.8	60
PK1.4	1.4	274.2	0.66	4.57	13	10	8.3	11.7	13.3	2.8	60
PK1.5	1.5	274.2	0.66	4.57	13	10	8.0	12.0	13.3	2.8	60
PK1.6	1.6	274.2	0.66	4.57	13	10	7.7	12.3	13.3	2.8	60
PKT	1.1	270.6	0.66	4.51	13	10	9.5	10.5	13.3	8.5	60
PKTP	1.1	247.1	0.66	4.12	13	10	9.5	10.5	13.3	8.5	60

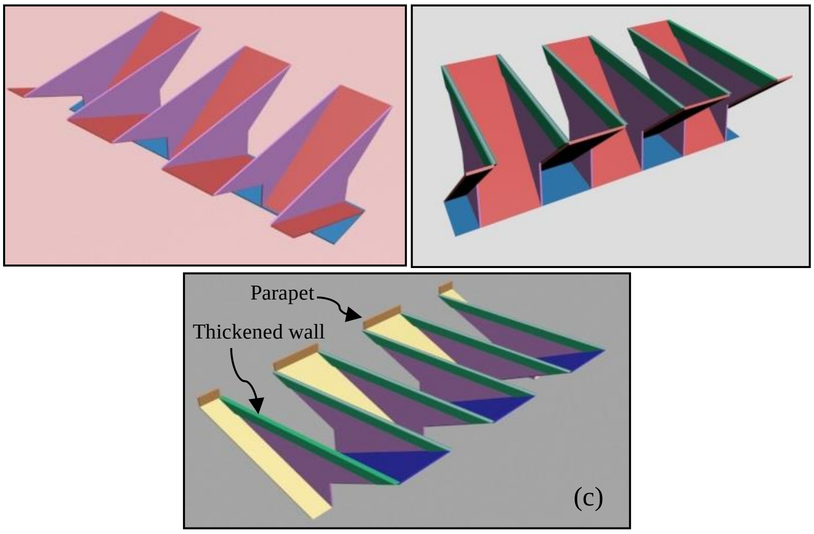

Figure 2 Schematic views of PK1.1 (a), PKT (b), and PKTP (c)

2.4. Flow rate measurement

Three scales, including a scale before the overflow and two on the overflow with an ultrasonic altimeter, were used in the required points to measure the flow depth with the accuracy of 0.1 mm. A triangular weir with an apex angle of 54° was installed at the end of the flume, which could be applicable by reading the scale and using the equation (5) for flume flow rate.

$$Q = \frac{8}{15}.\sqrt{2g}.C.tan\frac{\theta}{2}.H^{\frac{5}{2}} \tag{5}$$

In this equation, Q refers to flow rate (L/s), C is the overflow coefficient which was 0.576 for this overflow, and H is the water blade depth (cm) (note that the overflow was volumetrically calibrated across the range of tested discharges). Another way to measure the flow rate was using of ultrasonic flow meter, which could easily measure the volume of input water with a high accuracy.

2.5. Simulation by software

In this research, Flow-3D software was used to simulate the three-dimensional physical models and their hydraulic specifications, such as pressure distribution and flow depth along the parts of each weir. The important features of Flow-3D software include the use of a simple rectangular cube solution network, proper order and low memory requirement. Table 2 depicts the conditions and specifications used through Flow-3D in the current study. The simulation performance of the Flow-3D program was evaluated using the normalized root mean square error (NRMSE), which was calculated by equation (6) (Honari et al., 2017):

$$NRMSE\ (\%) = \sqrt{\frac{\sum_{i=0}^{n}(P_i - O_i)^2}{n}} \cdot \frac{100}{\bar{O}} \tag{6}$$

Where P_i and O_i are simulated and experimental values, n is the number of samples, and $\bar{O}$ is the mean of the experimental data.

Table 2 Networking specifications, boundary conditions, and equations used in numerical simulation

Networking	Model type	VOF
	Network type	Adaptive Rectangular Cube
	Number of blocks	3
	Number of cells	100 000
Boundary conditions	Overflow body	Solid (standard)
	Input boundaries	Volume
	Output boundaries	Output
	Side boundaries	Symmetry
	Floor boundaries	Wall
Equations	Turbulence model	RNG
	Free surface model	Fluid volume pattern

where VOF: volume of fluid, RNG: re-normalization group

3. Results and Discussion

3.1. The ratio of the inlet to outlet keys width (W_i/W_o)

The rating curves of Q (discharge) for seven weirs of PK1.0, PK1.1, PK1.2, PK1.3, PK1.4, PK1.5, and PK1.6 in Figure 3a indicate that the Q trend in different PK weirs is linearly and highly correlated (R^2=0.9674) with the trend of changing the height (H) or depth of the water blade. The trend of discharge *versus* H in different weirs has been similarly reported by Kabiri-Samani and Javaheri (2012) and Anderson and Tullis (2013). The process of improving the discharge rate started with increasing the width ratio (Wi/Wo) from 1.0 to 1.4, and it reached its peak in the PK1.4 model, where the Q increased by about 30% compared to the control. This increase in discharge capacity can be attributed to the improvement of flow conditions approaching the crown's sidewalls, increasing the usable width of the inlet keys, reducing the compression of the flow in the inlet keys, regular flow lines, and preventing local immersion on the overflow.

Discharge coefficient (C_d) is influenced by some factors, including flow depth upstream, flow shape on the overflow, crest length developed on the overflow, wall thickness, and the grade of weir submergence. According to the results of this experiment, changing the width of the inlet and outlet keys by keeping the developed length of the crest constantly led to changing the discharge coefficient. Figure 3b reveals that the highest discharge efficiency in the H/P range of 0.1-0.3 was obtained by PK1.4 (W_i/W_o = 1.4). However, the C_d differences among PK weirs for H/P ratios greater than 0.3 are negligible. Similarly, Anderson and Tullis (2013) declared that the influence of the W_i/W_o ratio decreases as H/P increases.

The following explains the impact of the Wi/Wo ratio on the discharge performance of the piano key weir; when the inlet cycle width is increased, the overall effect of head loss associated with flow entering the inlet cycles is reduced, and the flow area entering the inlet key increases. The inlet cycle's flow carrying capacity is increased as well.

The above results are consistent with those of Anderson and Tullis (2013), who found that among PKWs with W_i/W_o ratios of 0.67, 0.8, 1.0, 1.25, and 1.5, the PK1.25 had a more satisfactory performance compared with the other ones. Erpicum et al. (2014) studied the geometric parameters affecting the hydraulic performance of PKWs and showed that the optimal range of the ratio W_i/W_o was in the range of 1.29-1.57 as per the finding of this research.

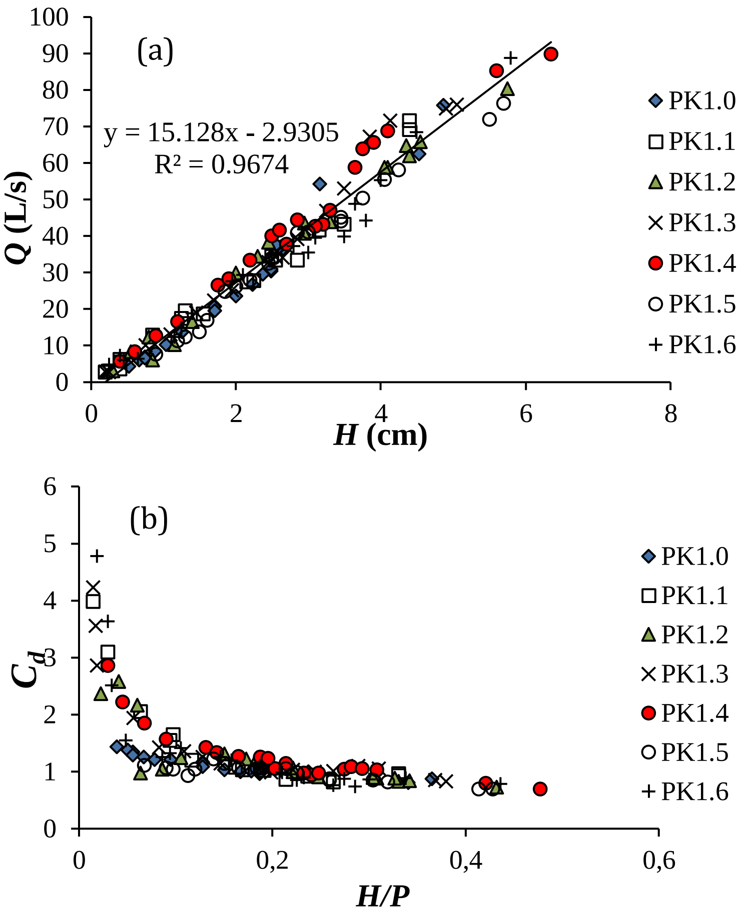

Figure 3 Q versus H rating curves for different PK weirs (a), and C_d versus H/P for different PK weirs (b)

3.2. Wall thickness and parapet wall

The plotted data in Figure 4a show considerable differences among the three weirs of PK1.1, PKT, and PKTP that can be an indication of the influence of the performed geometric modifications of "wall thickening" and "parapet wall" on discharge changing versus H.

Figure 4b illustrates that both modified weirs, namely PKT and PKTP, were more efficient than PK1.1. According to Figure 4b, PKT produced higher discharge efficiency than PKTP for $H/P < 0.1$, whereas PKTP produced higher discharge efficiency for $H/P > 0.1$. As can be seen from the trend lines in Figure 4b, PKT was expressively more efficient than

PK1.1 at low values of *H/P*, but its efficiency decreased as the H/P ratio increased, as shown by the convergent trend lines of PK1.1 and PKT. The trend lines also show that PKTP was moderately more efficient than PK1.1 for all *H/*P rates. The increased efficiency of PKT compared with PK1.1 may be attributed to the fact that an increase in wall thickness resulted in a decrease in vibration, a more regular shape water blade, and subsequently an increased water discharge capacity. These findings are consistent with the studies of Noui and Ouamane (2011).

The increase in discharge efficiency as the result of installing parapet walls on PK weir was similarly reported by Ribeiro et al. (2012), Machiels et al. (2013), and Anderson and Tullis (2013). The efficiency of the parapet wall model (PKTP) could be explained as follows. In the models without a parapet wall, turbulent flows with helix-shaped vortices are created in outlet keys, which asymmetrically load the weir. This loading type can lead to reduced weir discharge capacity, increased water head, and severe vibrations in the flume and upstream reservoir. Installing a parapet wall on the outlet keys effectively removes flow disturbances and eliminates the vortexes. In addition to increasing discharge capacity and lowering the weir water head, a gentle stream was created the upstream of the weir. Thus, it can be concluded that excessive flow compression, an improper form of foundations, and acute fractures in the walls may cause flow level disturbances, ultimately reducing the discharge coefficient.

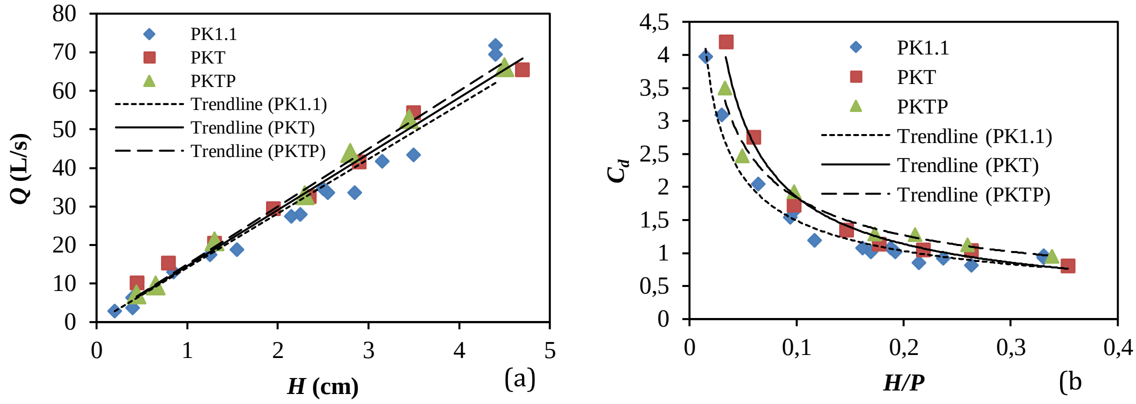

Figure 4 *Q* versus *H* rating curves for three weirs of PK1.1, PKT, and PKTP (a), and C_d versus *H/P* for three weirs of PK1.1, PKT, and PKTP (b)

3.3. Flow-3D simulation

The results of model simulation in the Flow-3D program revealed that the trend of flow coefficient changes is a function of different key width ratios (W_i/W_o). The changing trends of the net water height on the overflow, as simulated by Flow-3D software, were almost consistent with the experimental results from the physical models, as displayed in Table 3. According to the simulated values, the water load in models with a width ratio smaller than PK1.4 and a constant flow of 25 L/s gradually decreased from 19 (in PK1.0) to 17 (in PK1.4 It can be seen in Table 3, which indicates an increase in the discharge coefficient. Therefore, it can be concluded that increasing the width ratio to PK1.4, increased the uniformity of the approaching flow lines, reduced the turbulence of the upstream flow, and increased the overflow rate by 15%.

Table 3 shows that it returned to 18.3 (in PK1.6), which can be attributed to the reduction of the overflow coefficient due to flow line interference, energy-loss return, and the increase of negative parameters affecting the overflow coefficient. Flow-3D was used to simulate the various characteristics of the studied weirs. As illustrated in Figure 6, the pressure distribution on the sides of the overflow as well as the flow depth in different parts of the weir in the PK1.4 model was more optimal than in the other two models when Flow-3D simulated pressure and flow depth in three models of PK1.0, PK1.4, and PK1.6.

Table 3 Net height of water on the overflow obtained from experimental models and software simulations

Model	Experimental value (mm)	Simulated value (mm)
PK1.0	21	19
PK1.1	20	18.2
PK1.2	19	17.5
PK1.3	19.6	17.3
PK1.4	16.9	17
PK1.5	19	17.3
PK1.6	19.3	18.3
PKT	16.9	17.5
PKTP	19	17.3

The simulated values of discharge coefficient (C_d) obtained from the Flow-3D program were in good agreement with their relevant experimental values because the coefficient of determination (R^2) was high enough for all PK weirs, and the normalized root means square error (NRMSE) values were less than 14% as it is shown in Figure 5.

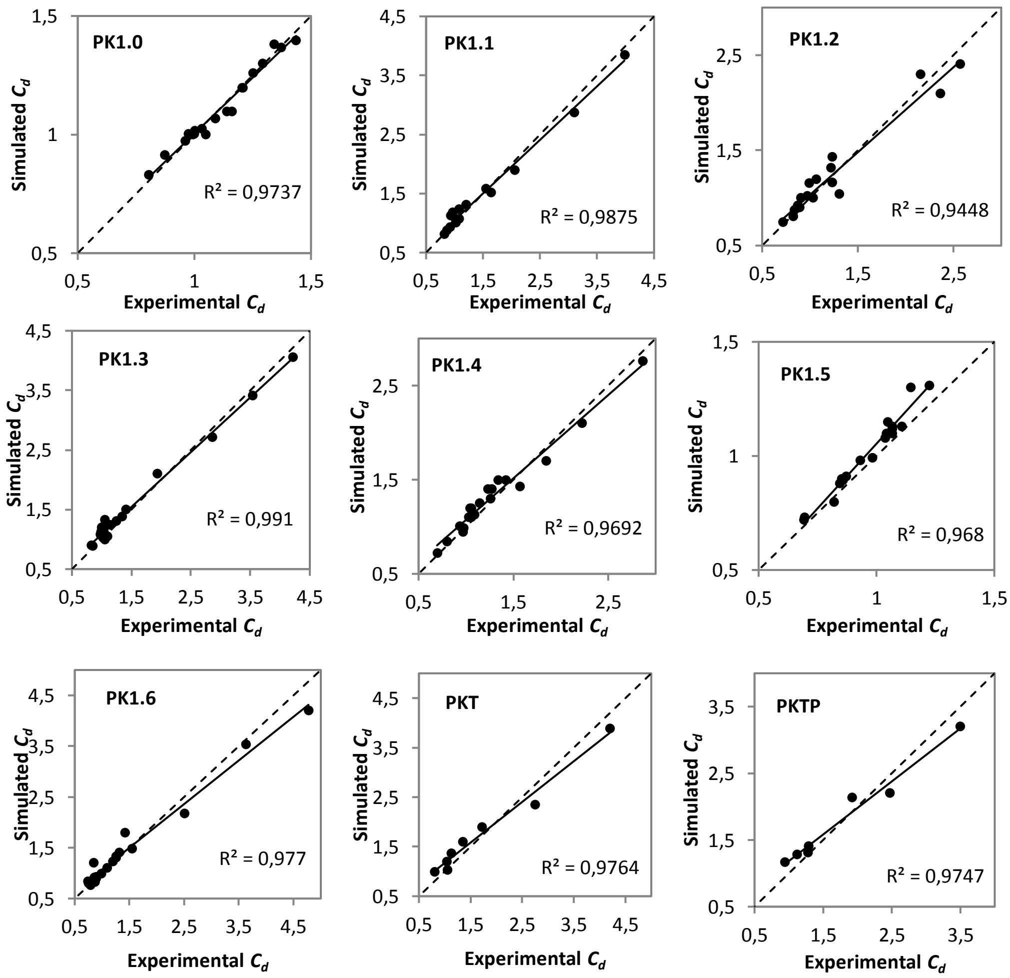

Figure 5 Simulated Flow-3D *versus* experimental values of C_d (discharge coefficient) in the studied PK weirs. Dotted lines represent the 1:1line; NRMSE – normalized root mean square error

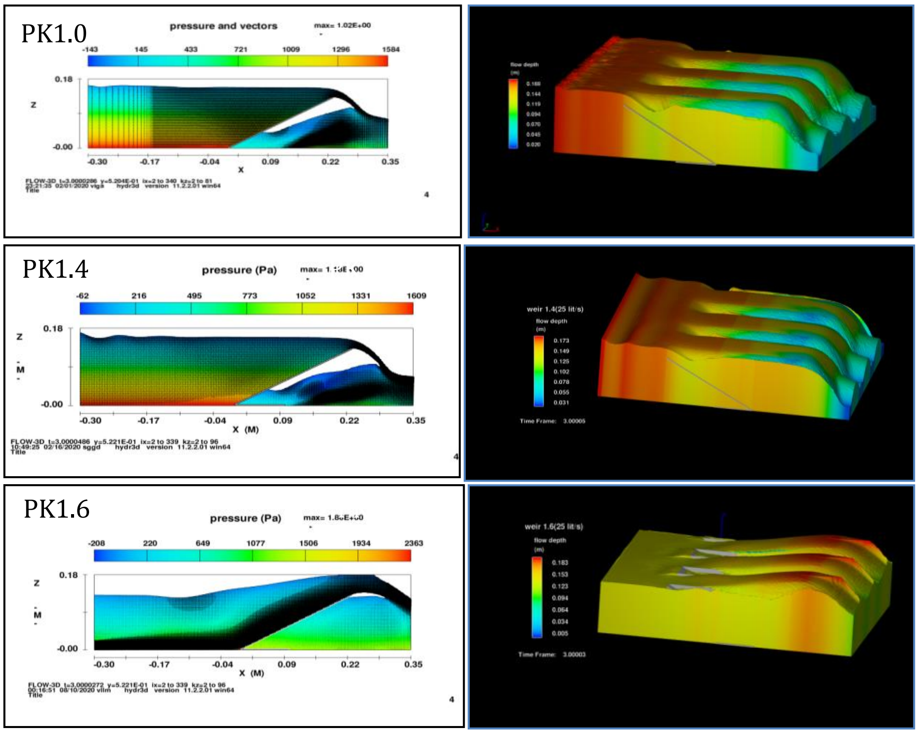

Figure 6 Trends of pressure (left panels) and flow depth (right panels) in PK1.0, PK1.4 and PK1.6 weirs simulated by Flow-3D software

4. Conclusions

In the present study, the optimal condition of geometrical parameters affecting the discharge coefficient was determined by physical models and the Flow-3D software. The results of Flow-3D simulations were consistent with the experimental results. A geometrical analysis of the models revealed that the discharge coefficient could be improved without changing the keys' length or width. By increasing the wall thickness in the study range, vibrations on the weir were reduced, and the shape of the water blade became more regular, while the flow rate was significantly increased. The crest heightening by adding a parapet wall increased discharge and uniform distribution of flow lines on the weir, in addition also to removing the turbulent flows and snail-shaped vortices. In general, modifying the geometry of the weirs should be taken to increase the useful width of the inlet keys and reduce the local submergence in outlet keys. The outcomes of this research can be used to optimize the parameters and design of rectangular piano key weirs.

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