Announcements

- Pset-2 due on March 6th.

Last time

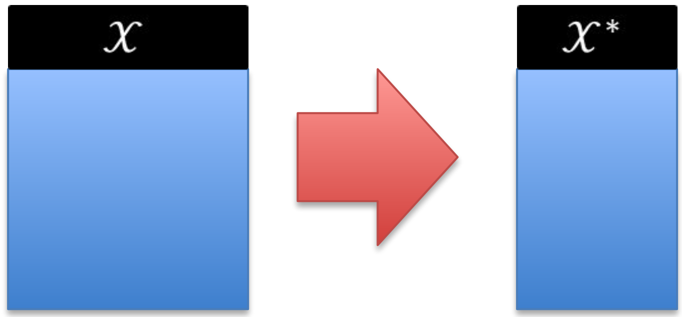
- Model Selection using AIC/BIC
- Robust Learning
  - Different loss functions
  - Boosting
  - Weak learners

Today

- Regression Trees
- Markov Chain
- Hidden Markov Model
- Decoding HMMs

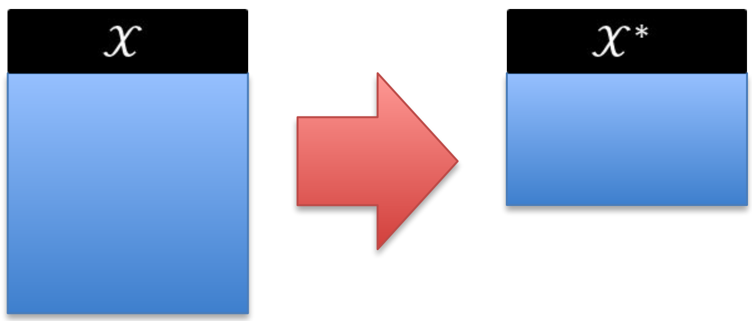
Regression on large datasets

PCA



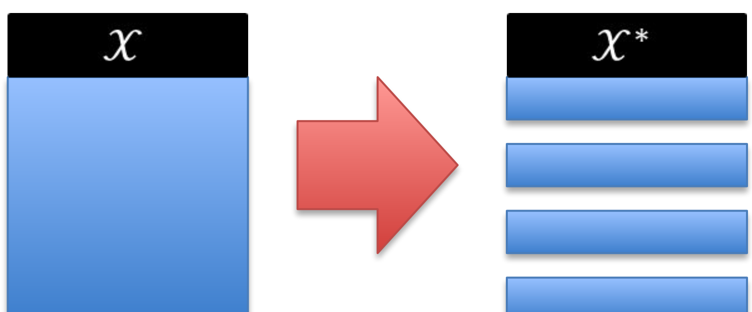
May lose important ***features***

Sub-sample



May lose important ***samples***

Stochastic Gradient Descent



Sees only ***few samples*** in each batch

Greedy Stagewise Linear Regression

- **Main idea:** segment the features and train a model on those features, i.e., we minimize

$$\mathcal{L}^{(j)}(\beta) = \left\| e^{(j-1)} - \mathcal{X}^{(j)}\beta \right\|^2$$

Greedy Stagewise Linear Regression

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Start $e^{(0)} = y$ and $j = 1$, then

1. Select a subset of features for $\mathcal{X}^{(j)}$
2. Learn $\hat{\beta}^{(j)}$ by minimizing $\mathcal{L}^{(j)}(\beta)$

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1. Select a subset of features for $\mathcal{X}^{(j)}$
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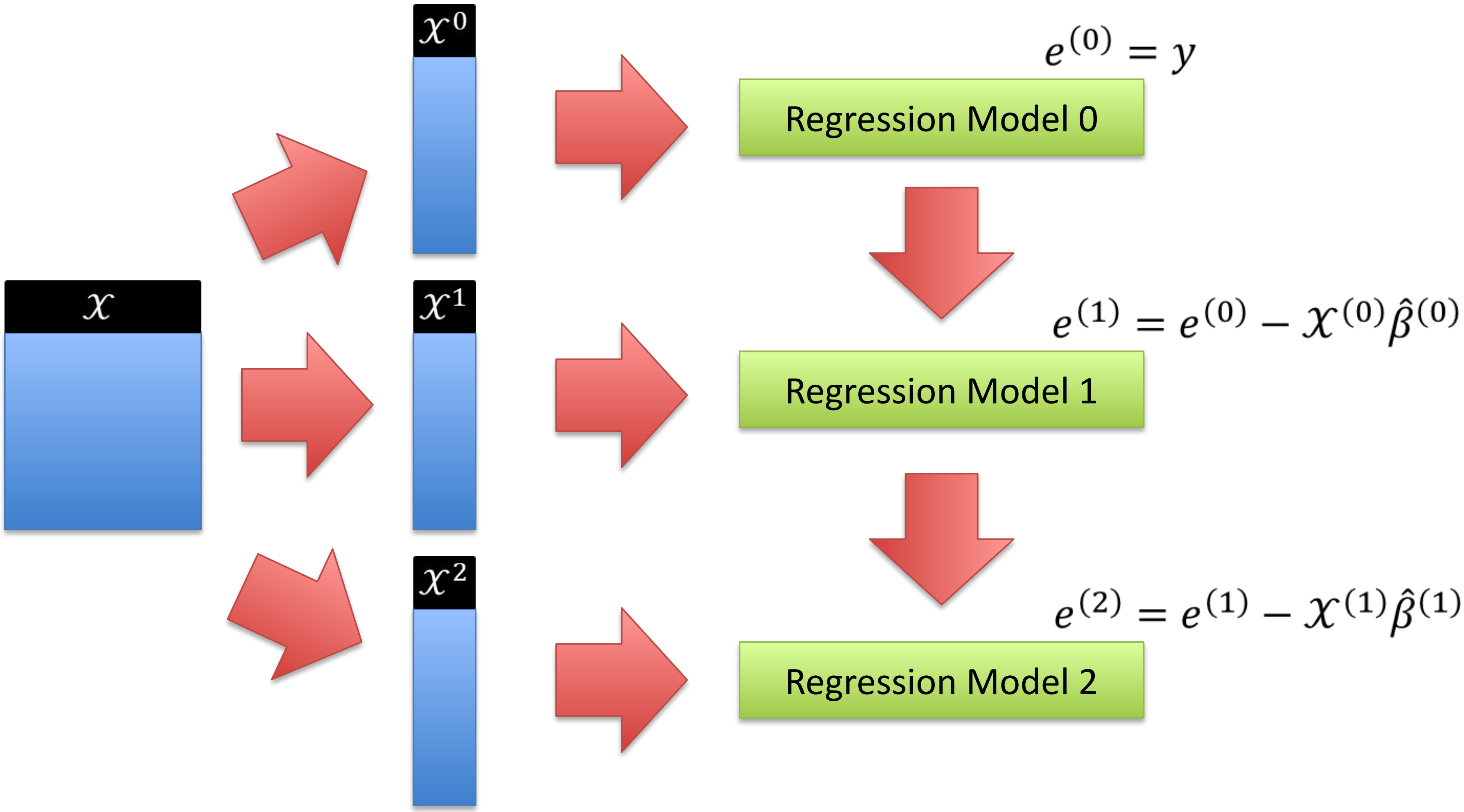
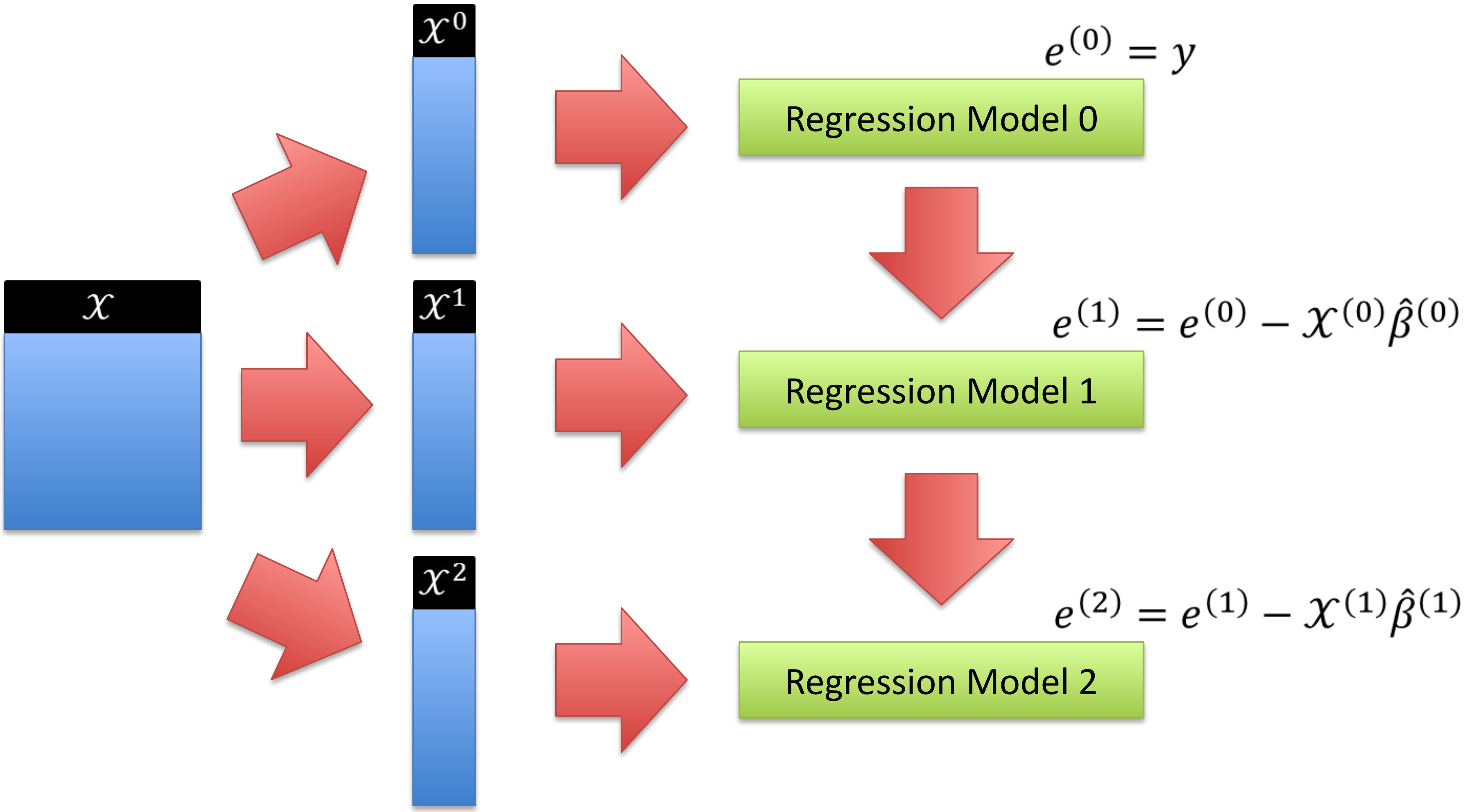
3. $e^{(j)} = e^{(j-1)} - \mathcal{X}^{(j)}\hat{\beta}^{(j)}$
4. Repeat for $j + 1$

Greedy Stagewise Linear Regression



Visual depiction of an extreme case - where a model is trained on each feature

Greedy Stagewise Linear Regression



At each stage, pick the feature that is most informative of **y**

When to stop adding weak learners?

- Recall are minimizing:

$$\mathcal{L}^{(j)}(\beta) = \left\| e^{(j-1)} - \mathcal{X}^{(j)}\beta \right\|^2$$

Any weak leaner cannot result in an increase in the residual, i.e., $\left\| e^{(j)} \right\|^2 \leq \left\| e^{(j-1)} \right\|^2$

Stop adding weak learners when the residual is high

Greedy Stagewise Linear Regression

- **Pros:**
  - Simple to implement
  - Computationally fast
- **Cons:**
  - Potential for overfitting.
    - Greedy selection can lead to selecting features that may not perform well on new data
  - Greedy selection might not lead to optimal solution

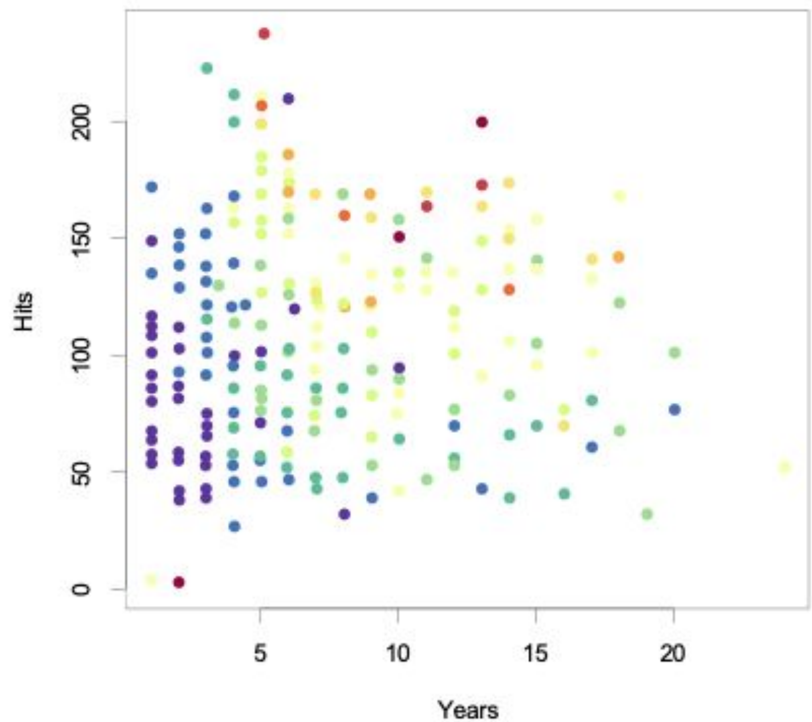
Today

- Model Selection using AIC/BIC
- **Robust Learning**
  - Different loss functions
  - Boosting
  - Weak learners
  - **Regression Trees**

An example: Regression Trees

Baseball salary data: how would you segment it?

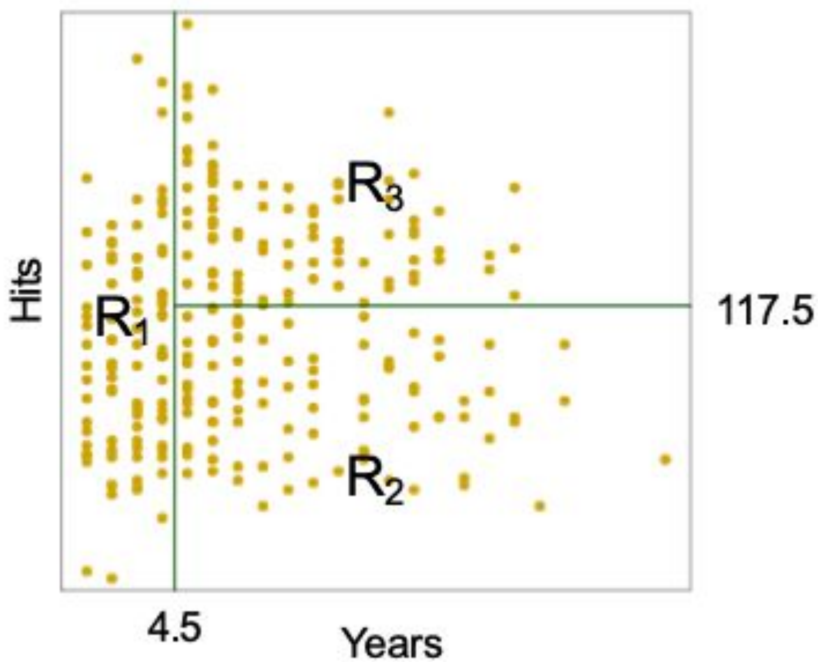
Salary is color-coded from low (blue, green) to high (yellow,red)



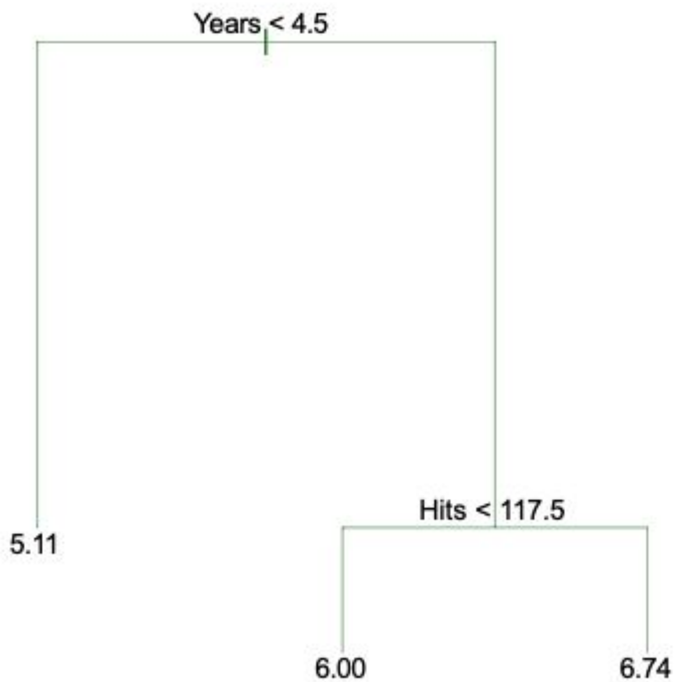
An example: Regression Trees

Results

Overall, the tree segments the players into three regions of predictor space: $R_1 = \{X \mid \text{Years} < 4.5\}$, $R_2 = \{X \mid \text{Years} >= 4.5, \text{Hits} < 117.5\}$, and $R_3 = \{X \mid \text{Years} >= 4.5, \text{Hits} >= 117.5\}$.



Example Decision Tree



Slide Credit: Saravanan Thirumuruganathan

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Greedy Stagewise Regression w/Trees

- Given regression tree $f(x;\theta)$, our stagewise regression will be the sum of trees, i.e.,

$$F(x;\theta) = \sum_j f\left(x;\theta^{(j)}\right)$$

We will learn this by minimizing:

$$\mathcal{L}^{(j)}(\theta) = \left\| e^{(j-1)} - f(x;\theta) \right\|^2$$

Follow same procedure as for Greedy Stagewise Linear Regression

Weak learners for classification vs. regression

- A primary difference between classification and regression is the training loss $\mathcal{L}$, i.e., given a predictor F:

For least squares regression we have,

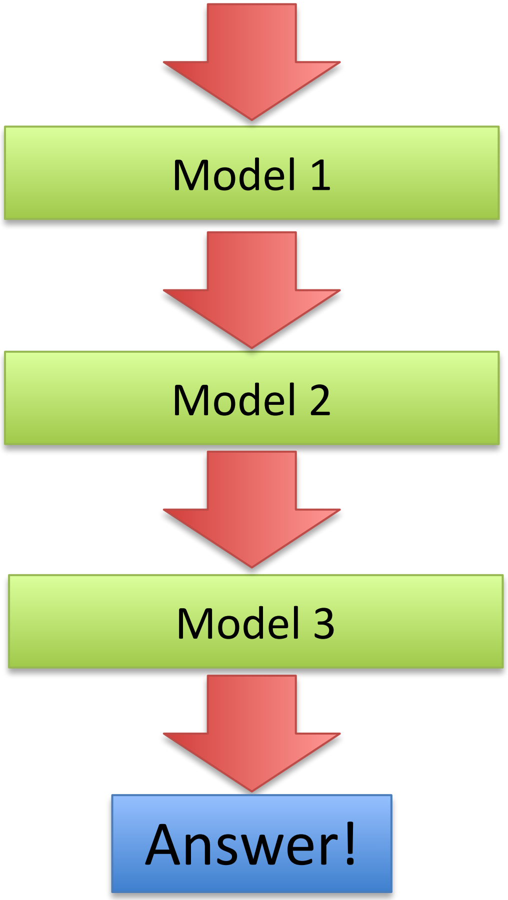
$$\mathcal{L}_{ls}(F) = \frac{1}{N}\sum_{i}\big(y_i - F(x_i)\big)^2$$

For a linear SVM we minimize the hinge loss,

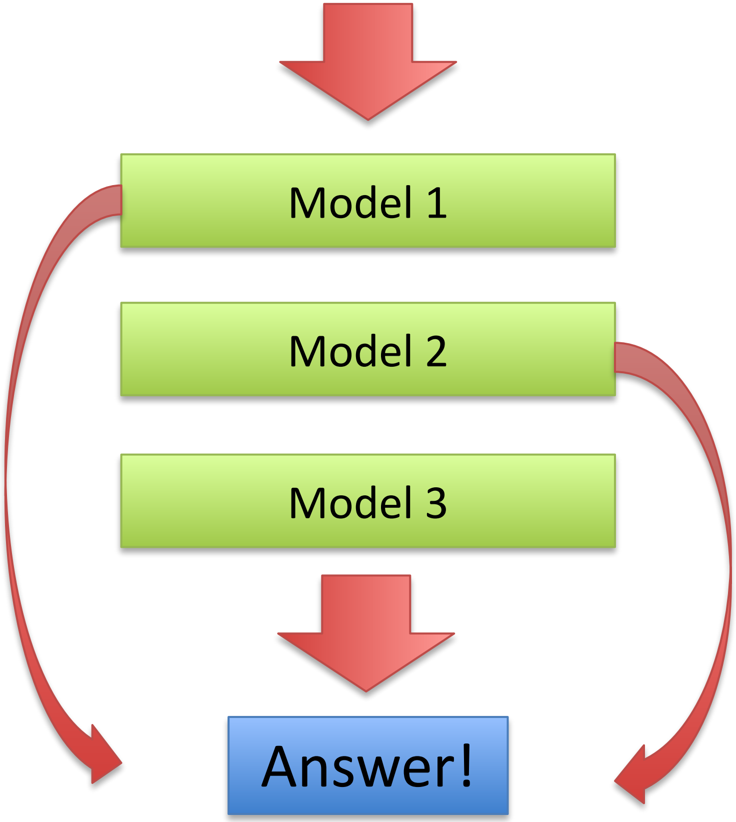
$$\mathcal{L}_h(F) = \frac{1}{N}\sum_{i}\max\big(0, 1 - y_i F(x_i)\big)$$

Boosting vs. Bagging Training

Boosting (Sequential)



Bagging (Parallel)

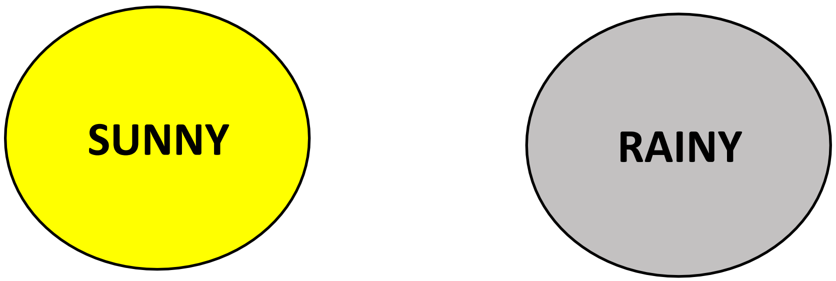


Today

- **Markov Chain**
- Hidden Markov Model
- Decoding HMMs

Representing transitions

Example setting: Based on many observations, the chance of a rainy day occurring after a rainy day is 50% and that the chance of a rainy day after a sunny day is 10%.



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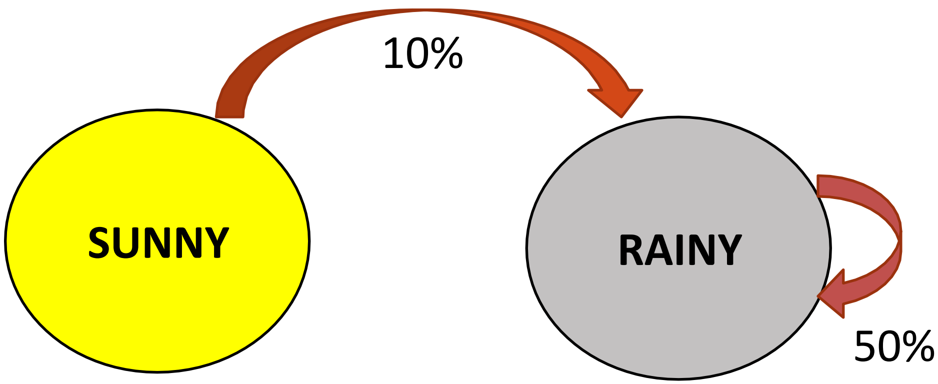

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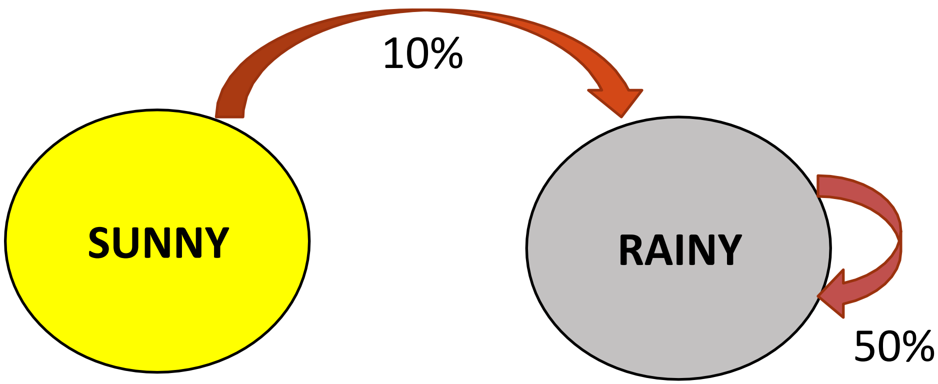

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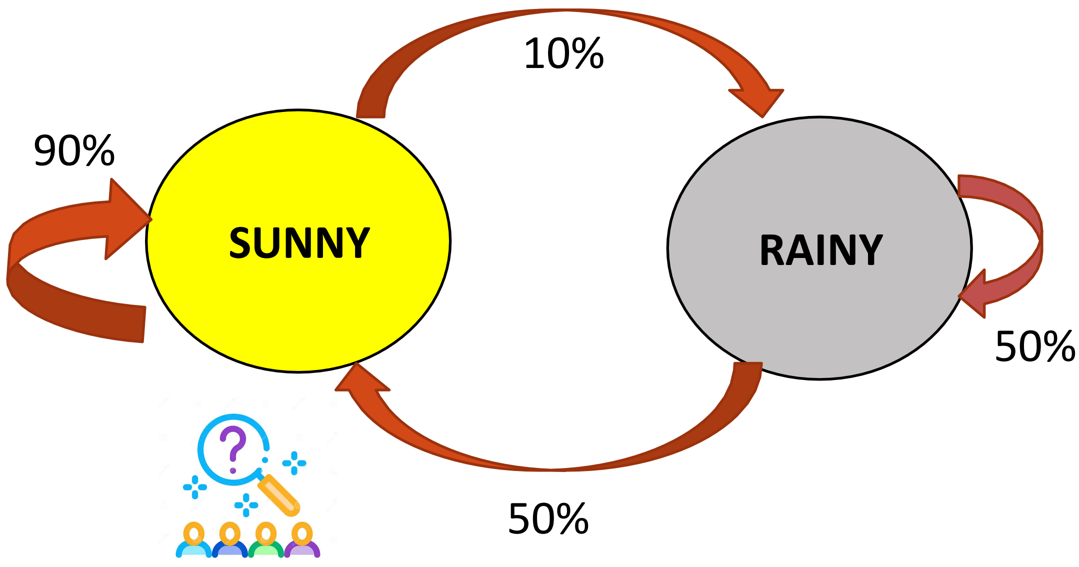

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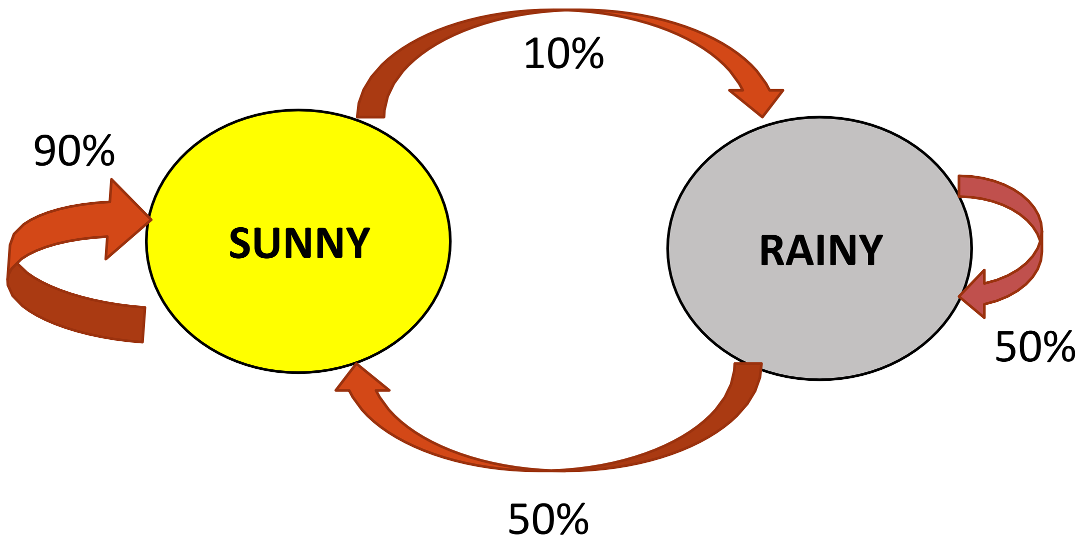
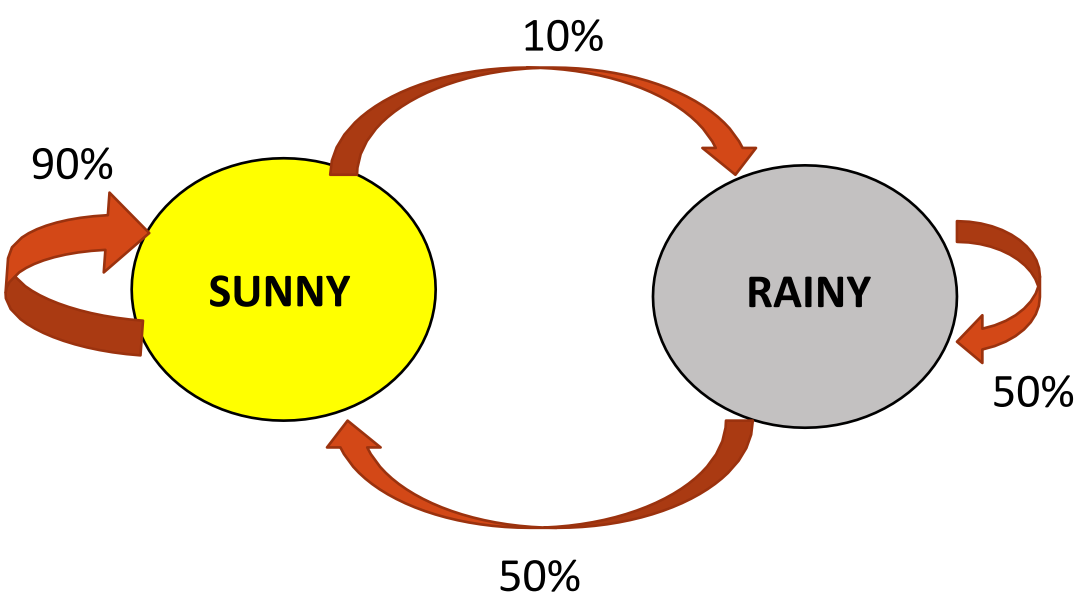


- **Graph:** Vertices, Edges.
- Represented using **adjacency matrix.**
- **Edge weights:** probabilities of weather conditions.

	Sunny	Rainy
Sunny		
Rainy		

Transition (or Markov) matrices

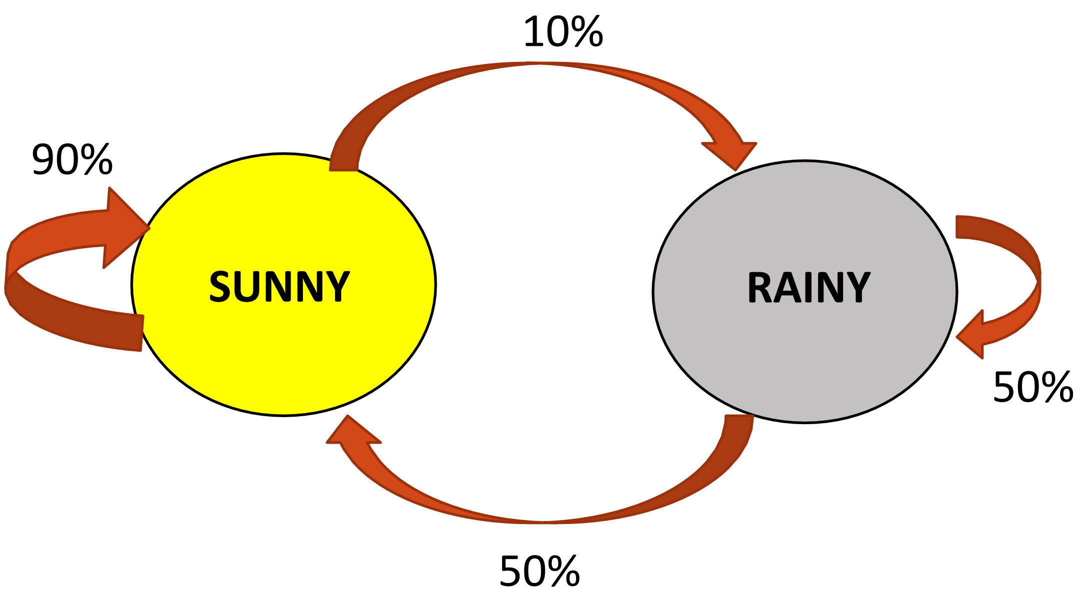
- Each entry is a non-negative real number representing a probability.
- (I,J) entry of the transition matrix has the probability of transitioning from state J to state I.
- Columns add up to one.



	Sunny	Rainy
Sunny	0.9	0.5
Rainy	0.1	0.5

Transition (or Markov) matrices

- Probability of being in one state at time t+1: depends on the probability of being in the current state (at time t).
  - memory less process.



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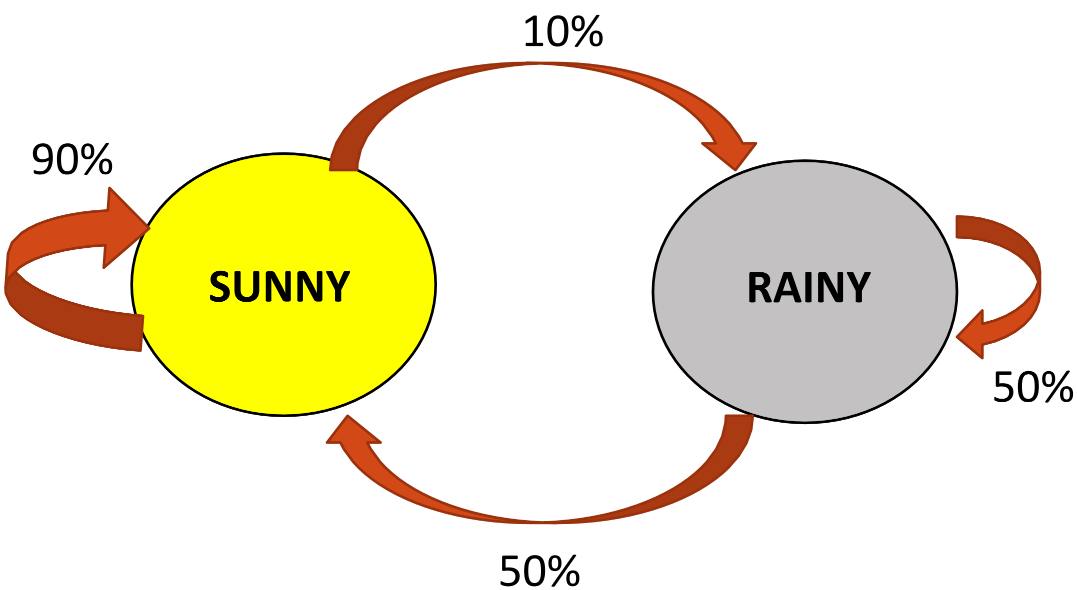
$$P(X_n = j | \text{values of all previous states}) = P(X_n = j | X_{n-1})$$



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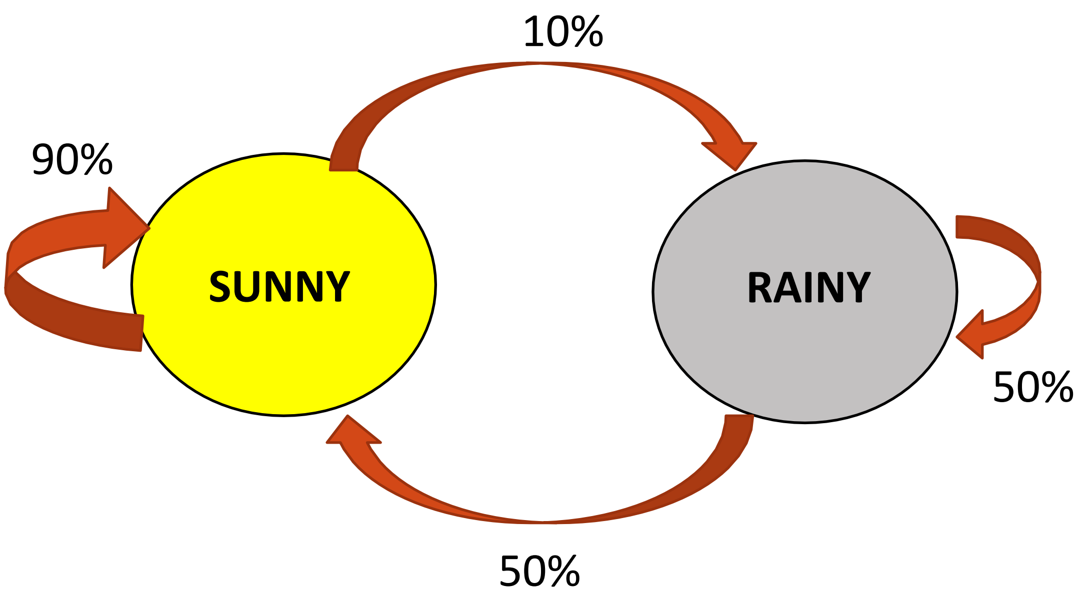
$$P(X_n = j | \text{values of all previous states}) = P(X_n = j | X_{n-1})$$

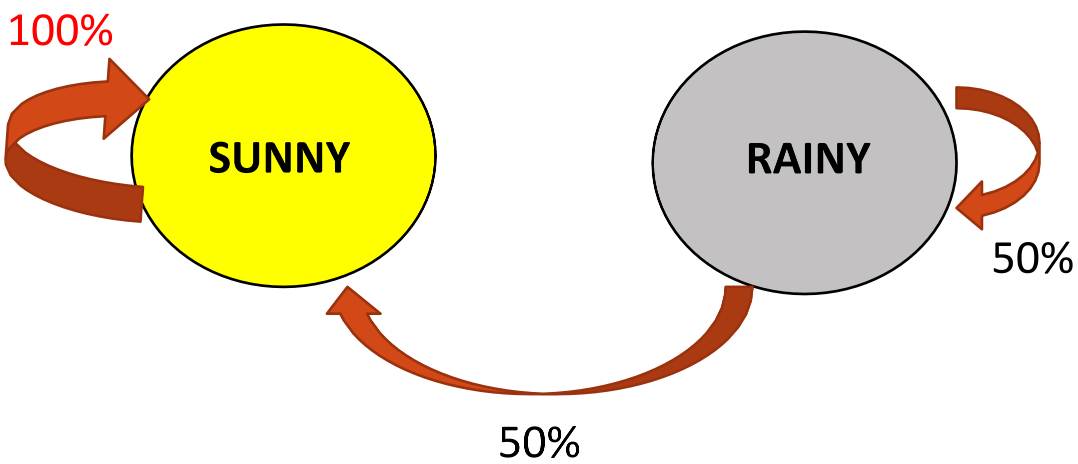
- This is called the **Markov property**, and the model is called a **Markov chain**



	Sunny	Rainy
Sunny	0.9	0.5
Rainy	0.1	0.5

Absorbing state

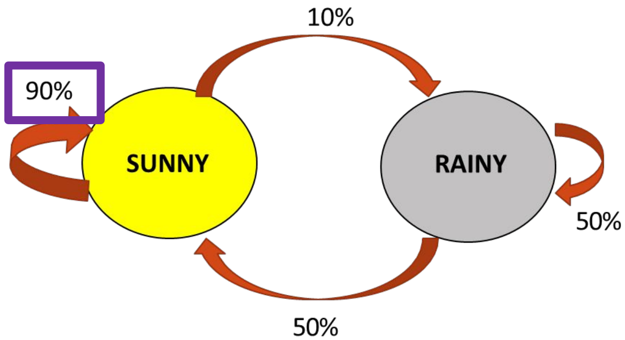
States in a Markov chain that it can never leave



	Sunny	Rainy
Sunny	1.0	0.5
Rainy	0.0	0.5

Transitions with biased random walk

Transition Matrix:



	Sunny	Rainy
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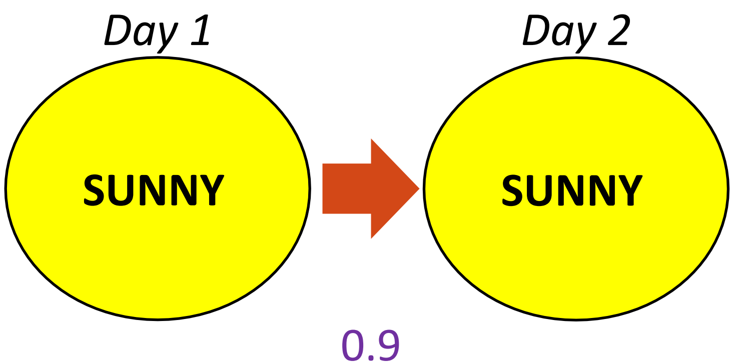
Sequence A:



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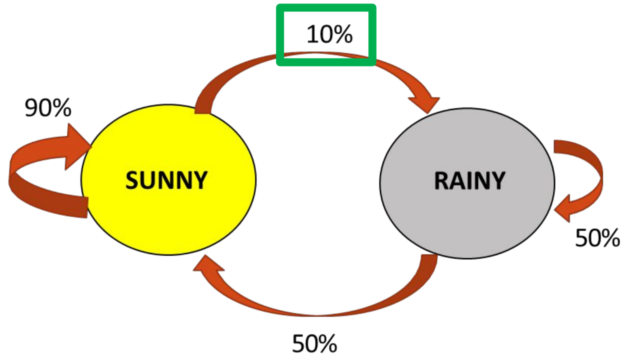


	Sunny	Rainy
Sunny	0.9	0.5
Rainy	$\boxed{0.1}$	0.5

Sequence A:

Day 1 **SUNNY** → *Day 2* **SUNNY** → *Day 3* **RAINY**

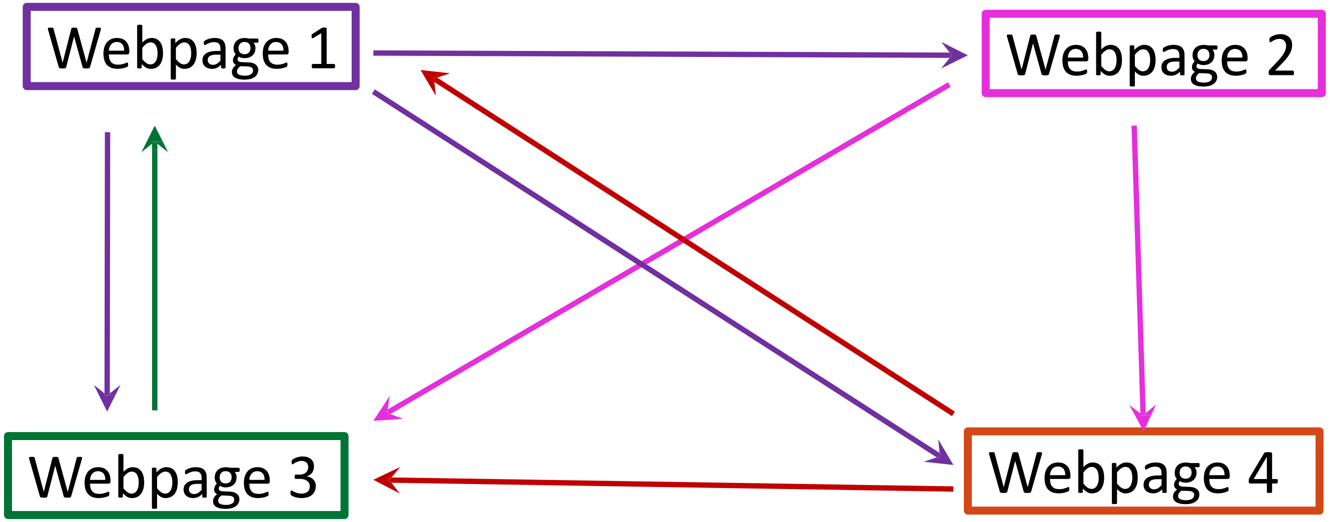
0.9 x 0.1 = 0.09

Sequence B:

Day 1 **SUNNY** → *Day 2* **RAINY** → *Day 3* **SUNNY**

0.1 x 0.5 = 0.05

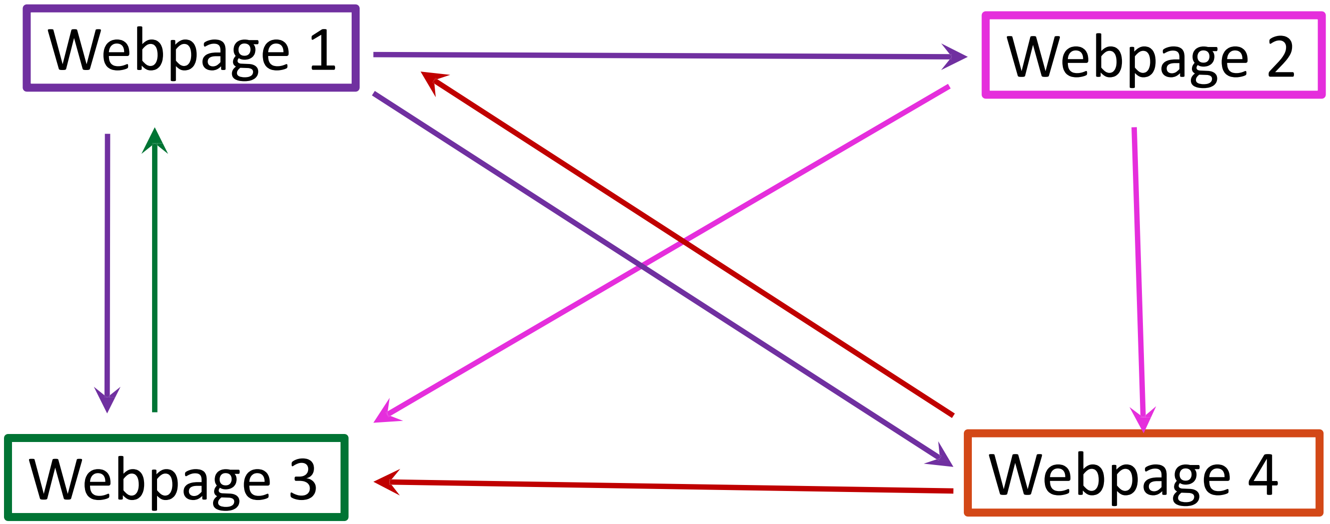
Random Walk Applications: Page Rank



Problem: Consider n linked web pages (above we have n = 4). Rank them.

- A link to a page increases the perceived ***importance*** of a webpage
- We can represent the ***importance*** of each webpage k with the scalar x_1

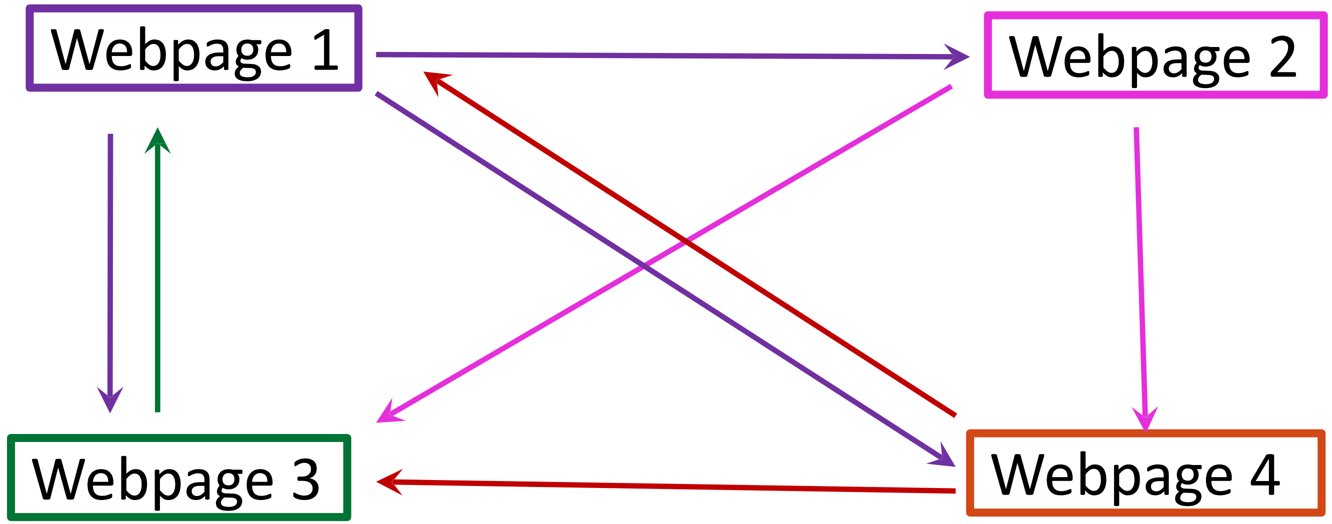
Page Rank



A possible way to rank web pages

- x_k is the number of links to page k (**incoming links**)
- $x_1 = 2$, $x_2 = 1$, $x_3 = 3$, $x_4 = 2$
- **Issue:** Doesn't take into account popularity / credibility of certain sources over others.

Page Rank



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•**Alternatively,** importance of a web page $\propto$ frequency of page visits

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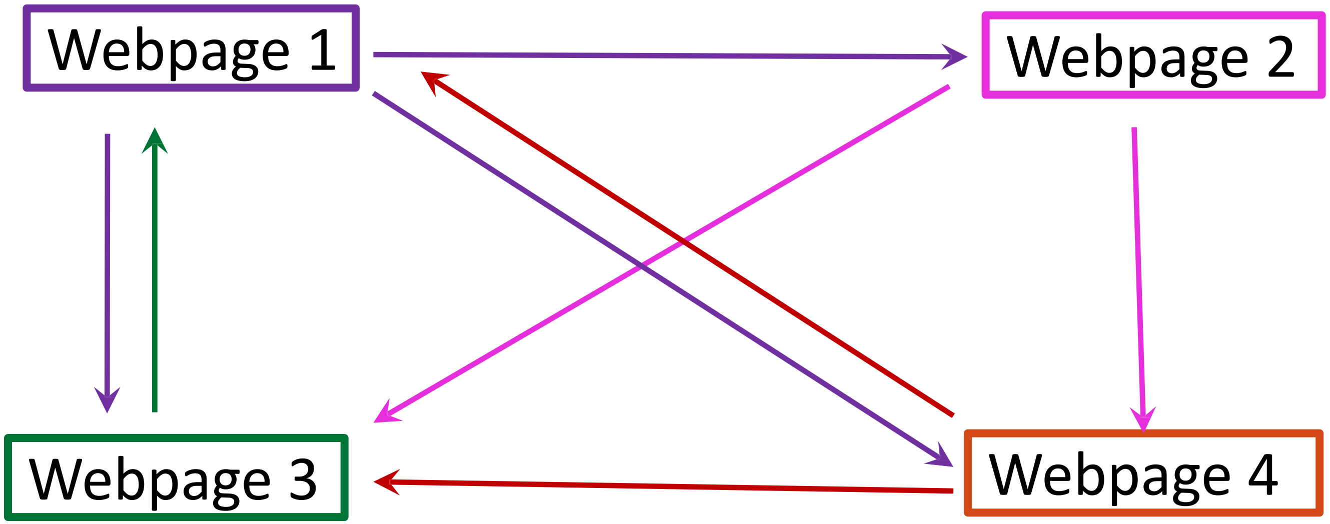

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- **Issue:** Doesn't take into account popularity / credibility of certain sources over others.

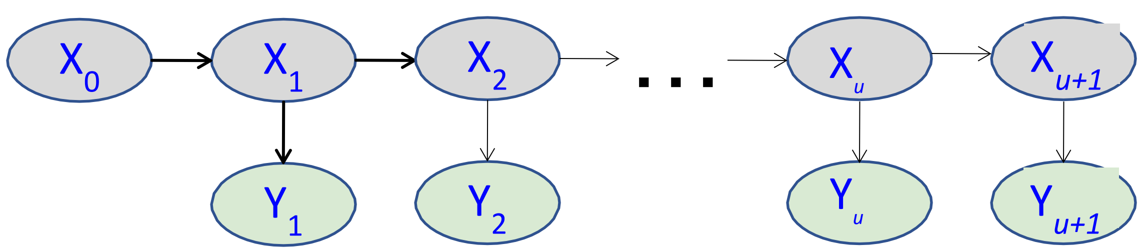
•**Alternatively,** importance of a web page $\propto$ number of outgoing links

Today

- Markov Chain
- **Hidden Markov Model**
- Decoding HMMs

Hidden Markov Models

- At each time slice *t*, the state of the world is described by an unobservable variable X_u and an observable *evidence* variable Y_u



Hidden Markov Models

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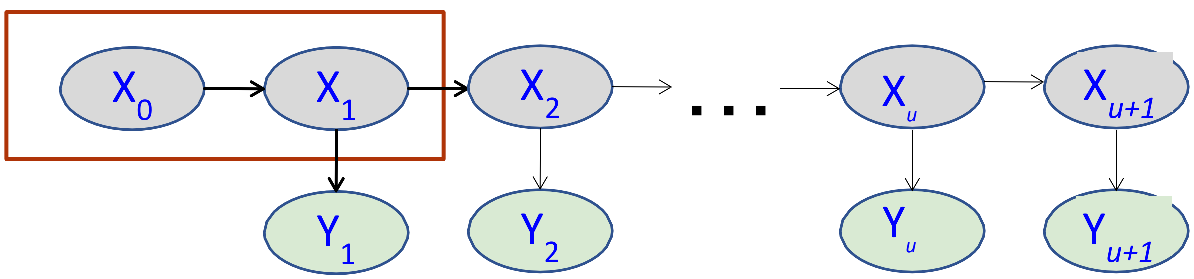
$$p_{ij} = P(X_{u+1} = j | X_u = i)$$



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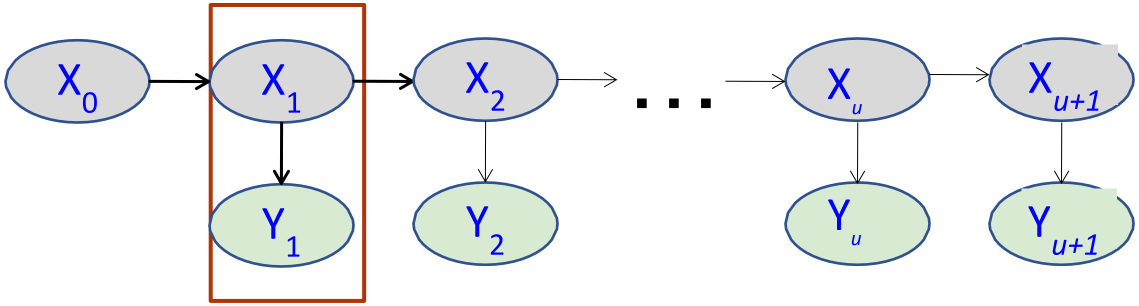
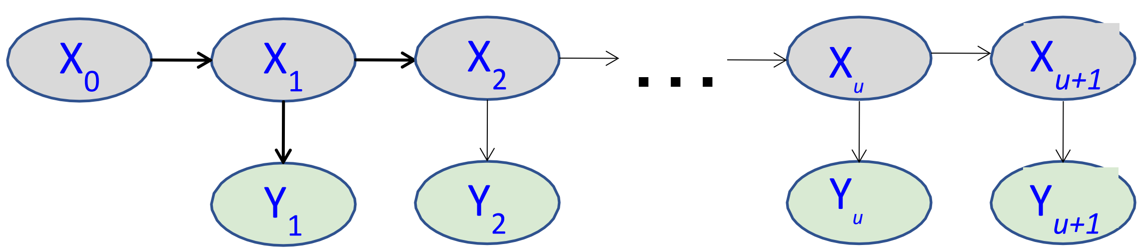
$$p_{ij} = P(X_{u+1} = j | X_u = i)$$

- **Observation model:** $P(Y_u | X_u = i) = q_i(Y_u)$



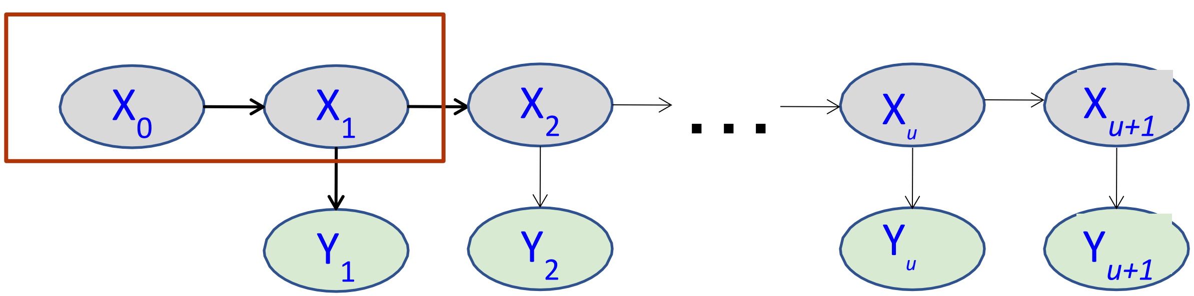
Hidden Markov Models

- **Markov assumption** (first order)
  - The current state is conditionally independent of all the other states given the state in the previous time step



Hidden Markov Models

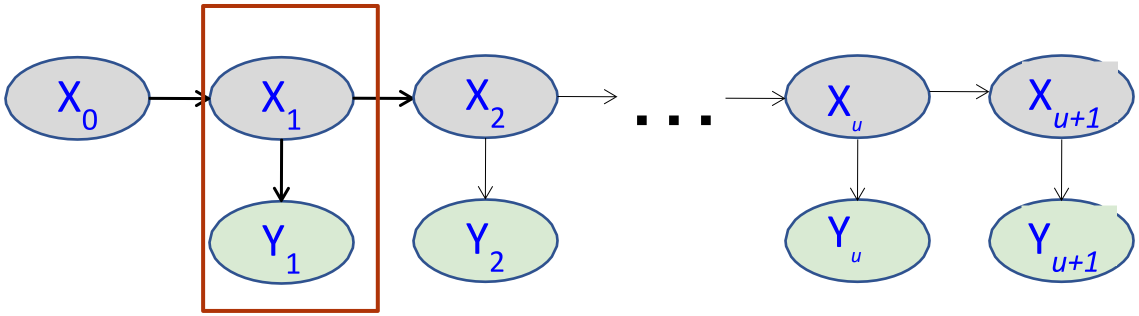
- **Markov assumption** (first order)
  - The current state is conditionally independent of all the other states given the state in the previous time step
  - What does $P(X_{u+1}|X_{o:u})$ simplify to?
    $P(X_{u+1}|X_{o:u}) = P(X_{u+1}|X_u)$



Hidden Markov Models

- Markov assumption for observations
  - The evidence at time *t* depends only on the state at time *t*
  - What does $P(Y_{u+1}|X_{u+1}, X_{0:u})$ simplify to?

$$P(Y_{u+1}|X_{u+1}, X_{0:u}) \qquad = P(Y_{u+1}|X_{u+1})$$



Comparing frameworks

Markov Chain

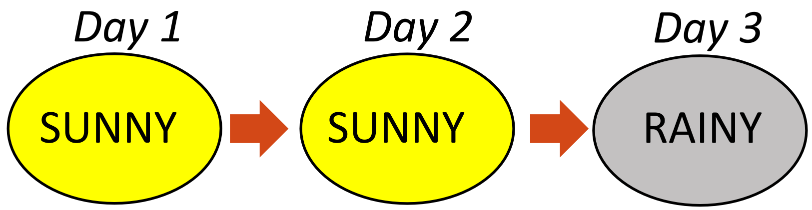
- Finite states
- Probabilistic formulation for transitions between states
- Markov property- next state determined only by current state

Hidden Markov Model

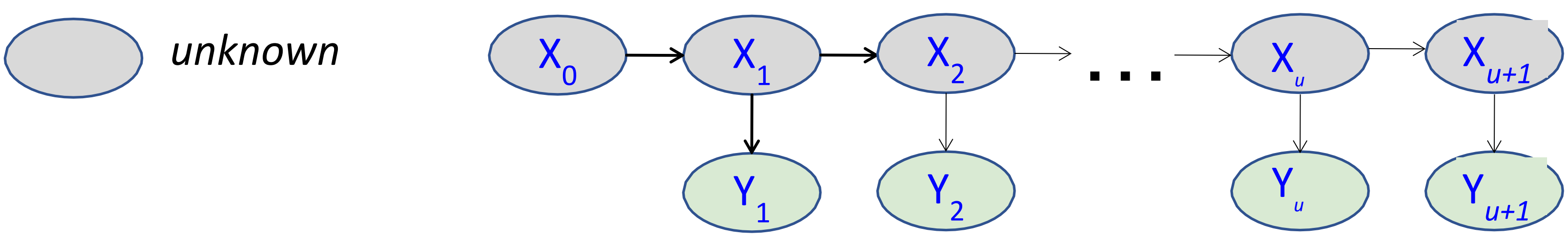
- Finite states
- Probabilistic formulation for transitions between states
- Markov property- next state determined only by current state
- **Current states are not observed.**

Markov vs Hidden

Markov



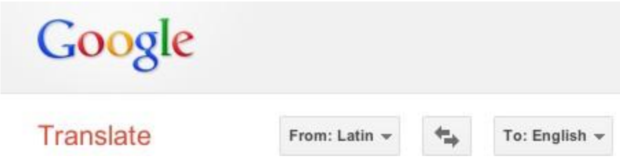
Hidden



Example HMM Applications

- Speech recognition:
  - Observations are acoustic signals (continuous valued)
  - States are specific positions in specific words (so, tens of thousands)

- Machine translation:
  - Observations are words (tens of thousands)
  - States are translation options



- Robot tracking:
  - Observations are range readings (continuous)
  - States are positions on a map (continuous)

Source: Tamara Berg

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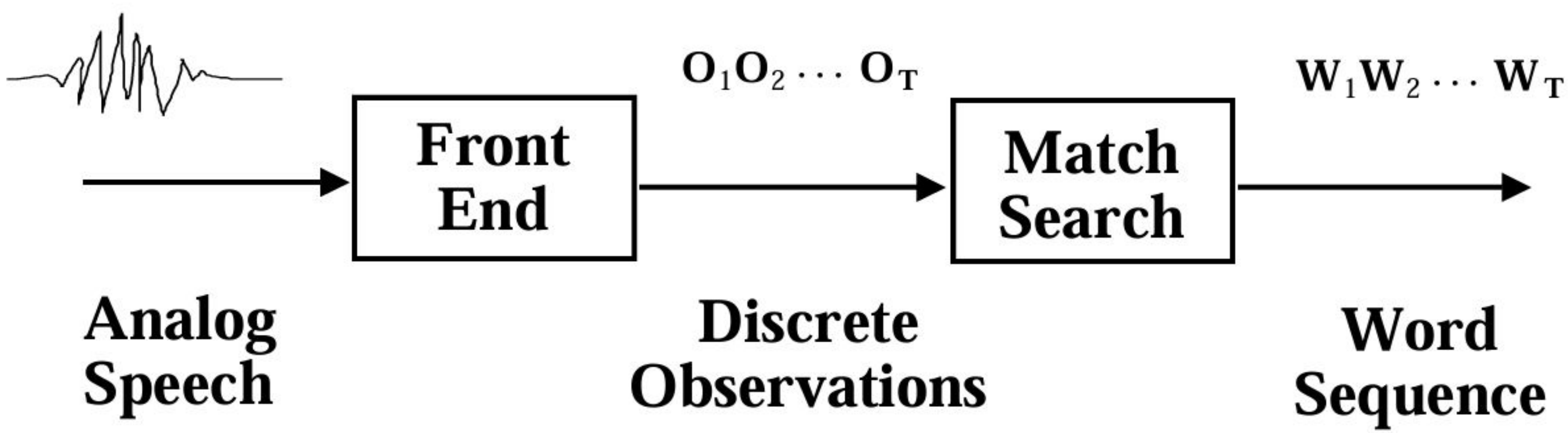
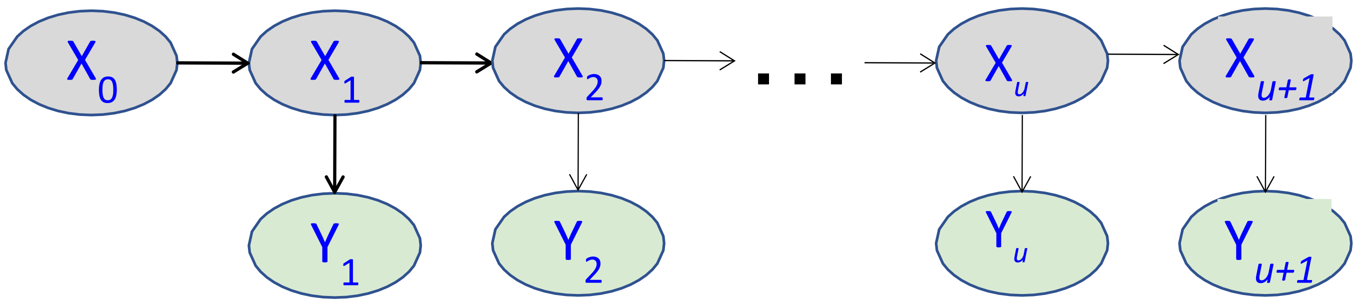
Speech recognition



Speech recognition



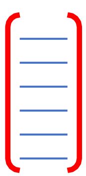
- **X:** Phones (in phonetics), i.e., concrete sound realizations.
  - **Unobserved**
- **Y:** audio utterances
  - **Observed**
  - Can extract features and represent them.

Example: Speech Recognition

- **Representing observations:** FFT of of the speech signal.

Acoustic wave form
Sampled at 16KHz, quantized to 8-12 bits

Fast Fourier Transform (FFT) **of one frame (10ms)** is the HMM observation, once per 10ms

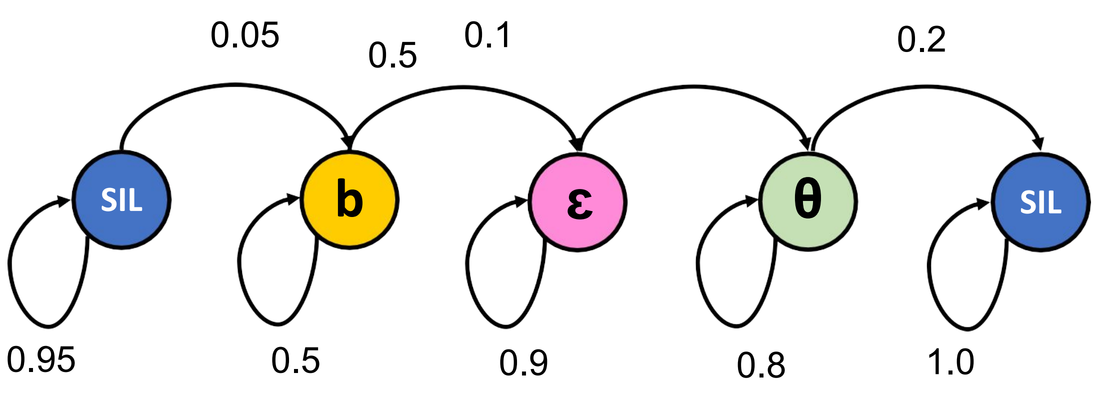


Observation = compressed version of the log magnitude FFT, from one 10ms frame

Example: Speech Recognition

- Observations: FFT of 10ms frame of the speech signal.
- Unobserved variables: a specific position in a specific word, coded using the international phonetic alphabet:
  - b = first sound of the word "Beth"
  - ε = second sound of the word "Beth"
  - θ = third sound in the word "Beth"

Finite State Machine model of the word "Beth"



Which of the following statement(s) is true? Select all that apply.

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Slicing the continuous FFT signal every 10ms helps us extract discrete features ✓

96%

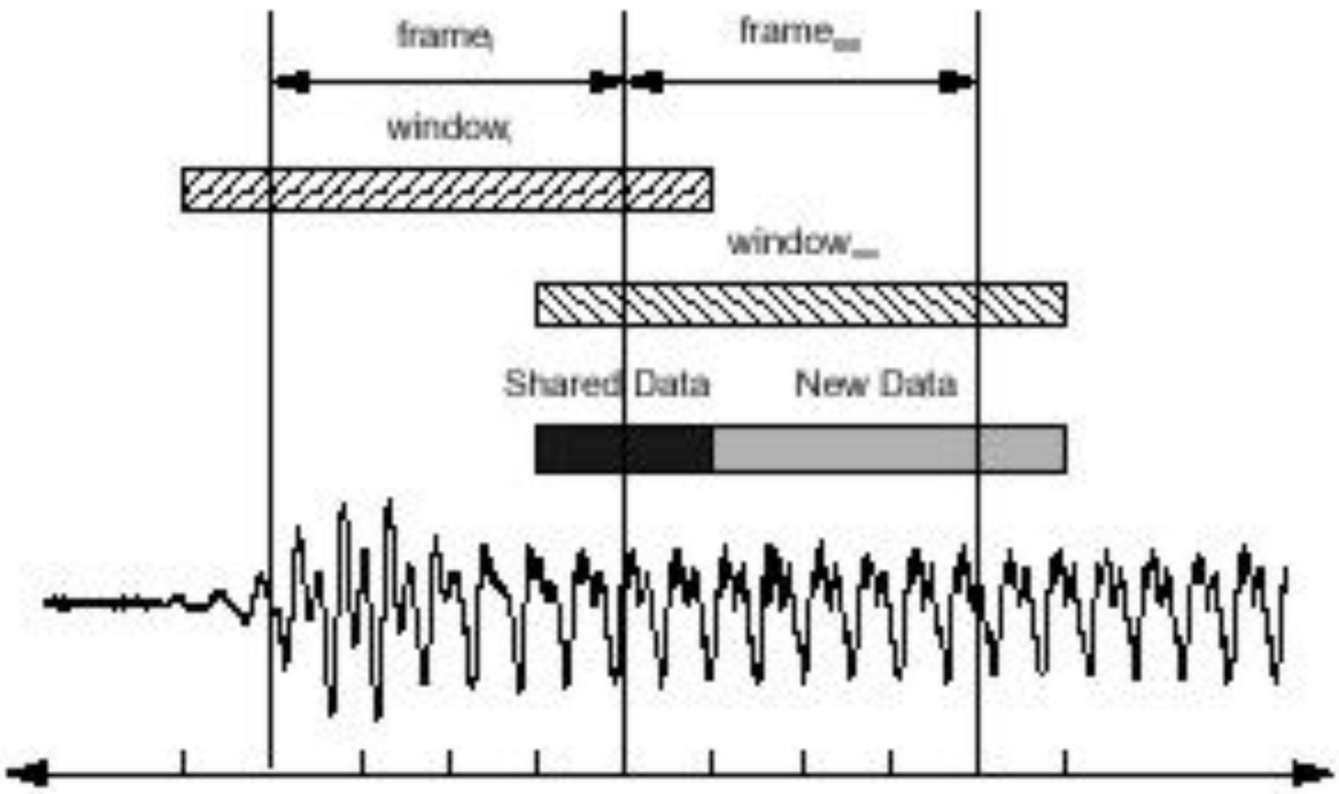
Slicing is ideal since most phones fall within the 10ms window

39%

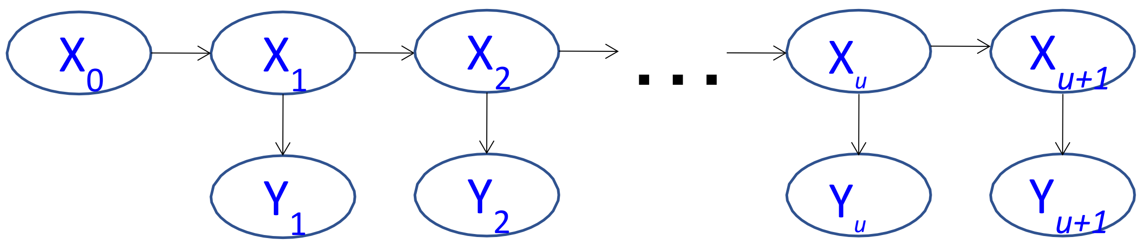
Slicing introduces noise because most phones may not fall within the 10ms window leading to incomplete or overlapping acoustic signals. ✓

63%

Compute features with a sliding window



The Joint Distribution

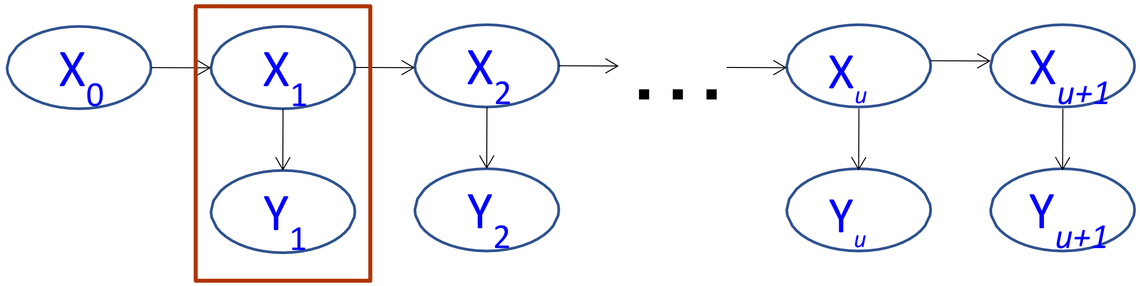


- Transition model: $P(X_{u+1} = j | X_u = i)$
- Observation model: $P(Y_u | X_u = i)$
- How do we compute the full joint probability table $P(X_{0:u+1} | Y_{0:u+1})$?

Bayes' Theorem

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

The Joint Distribution



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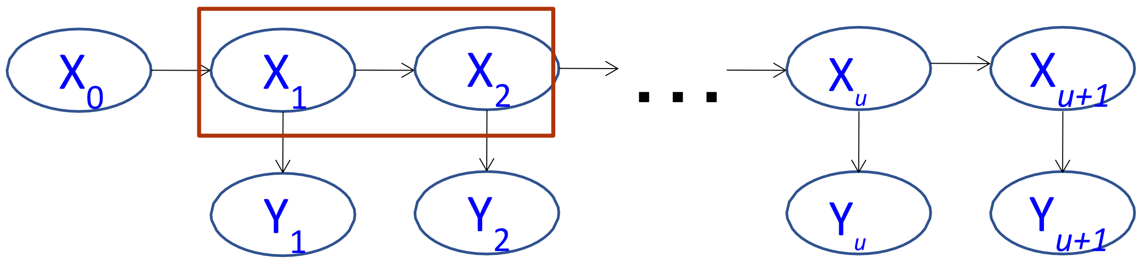
$$\prod_{i=1}^{u+1} \qquad P(Y_i | X_i)$$

The Joint Distribution



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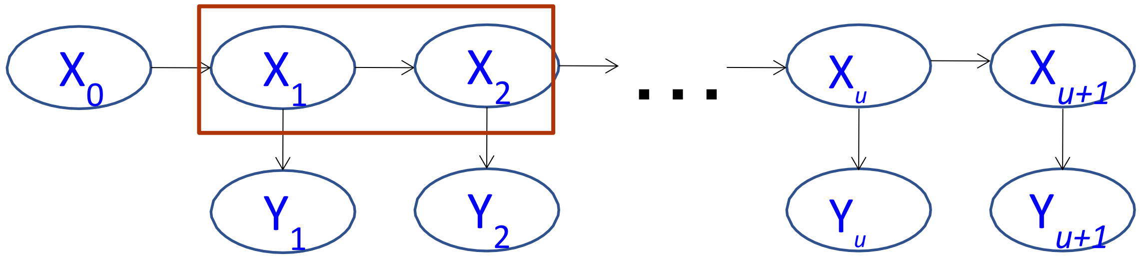
- How do we compute the full joint probability table $P(X_{0:u+1} | Y_{0:u+1})$?

$$\prod_{i=1}^{u+1} P(X_i | X_{i-1}) P(Y_i | X_i)$$

The Joint Distribution



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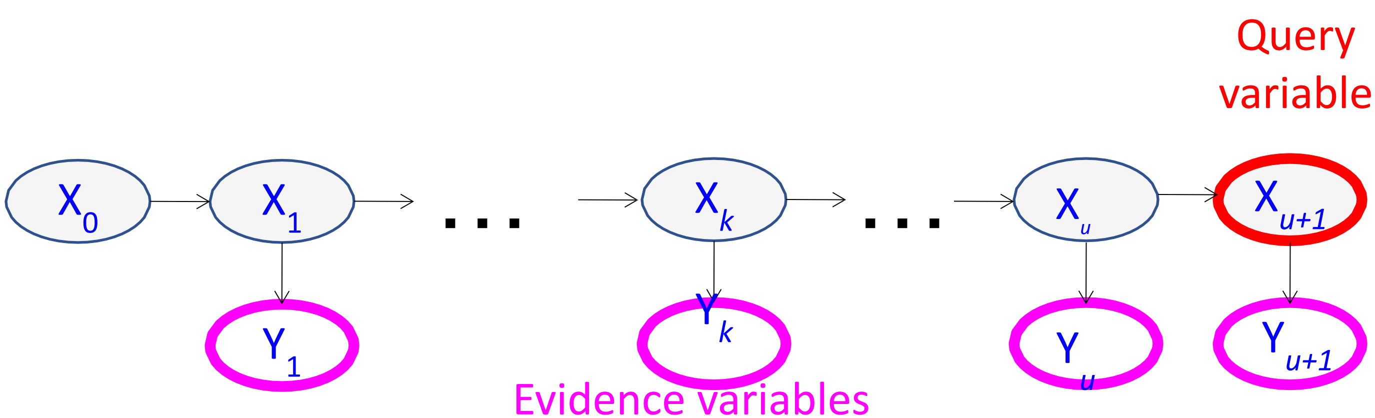
Bayes' Theorem

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

$$P(X_{0:u+1} | Y_{0:u+1}) = P(X_0) \prod_{i=1}^{u+1} P(X_i | X_{i-1}) P(Y_i | X_i)$$

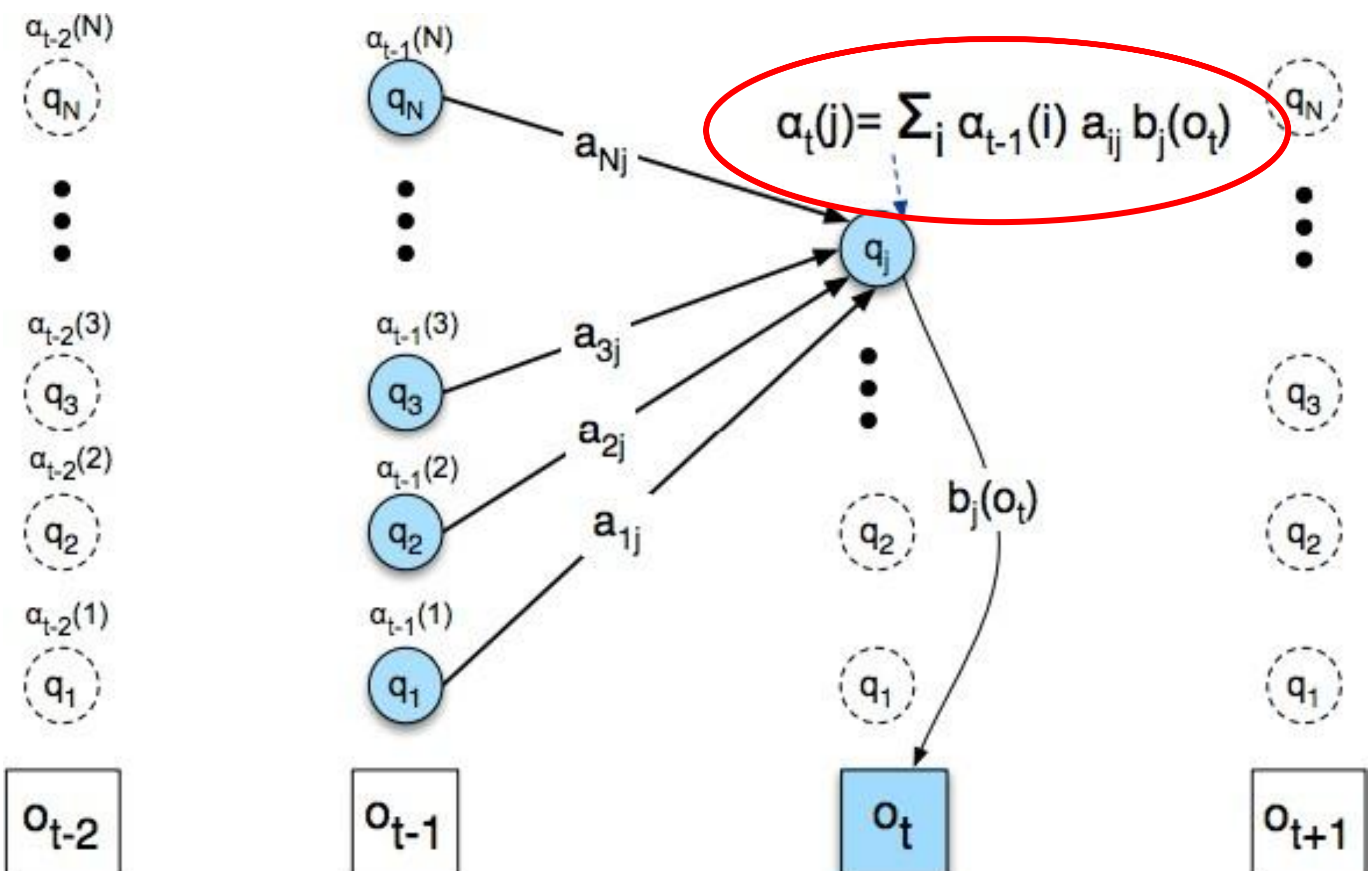
HMM inference tasks

- **Filtering:** what is the distribution over the current state X_t given all the evidence so far, $\mathbf{Y}_{1:t}$? (example: is it currently raining?)



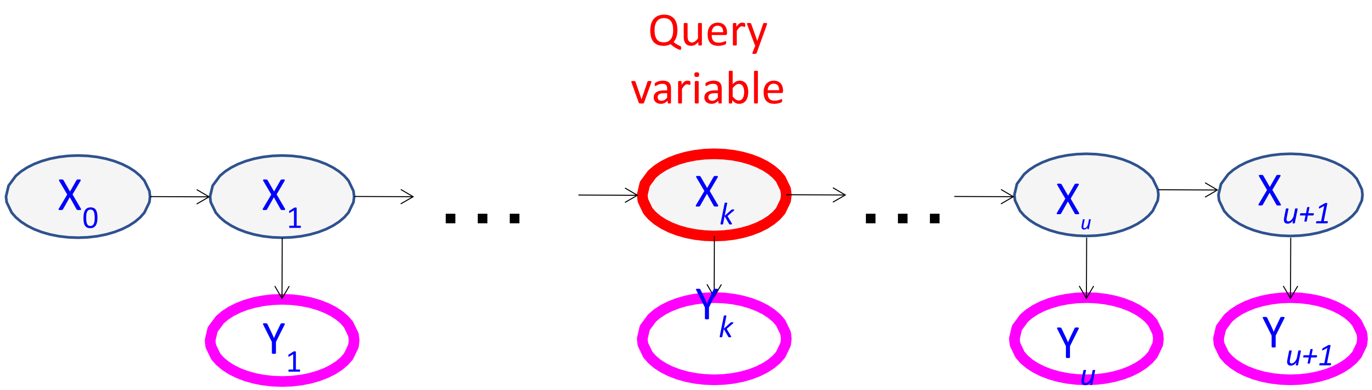
Forward algorithm

$\alpha_{t-1}(i)$	the **previous forward path probability** from the previous time step
a_{ij}	the **transition probability** from previous state q_i to current state q_j
$b_j(o_t)$	the **state observation likelihood** of the observation symbol o_t given the current state j



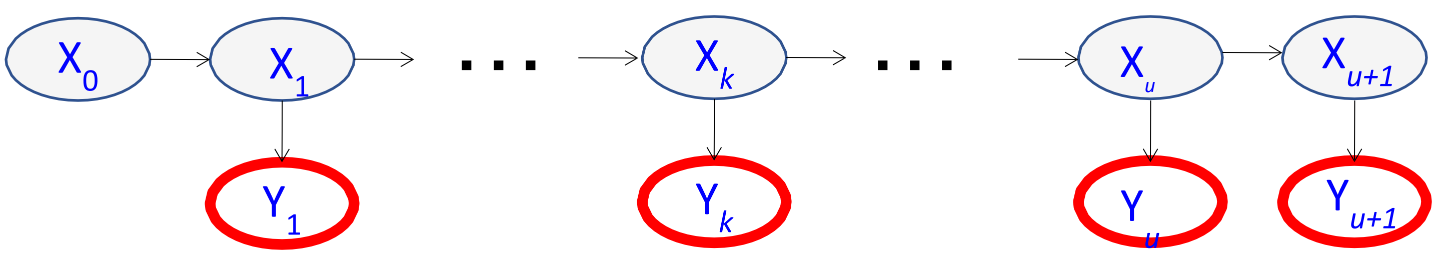
HMM inference tasks

- **Filtering:** what is the distribution over the current state X_t given all the evidence so far, $\mathbf{Y}_{1:t}$?
- **Smoothing:** what is the distribution of some state X_k (k<t) given the entire observation sequence $\mathbf{Y}_{1:t}$? (example: did it rain on Sunday?)



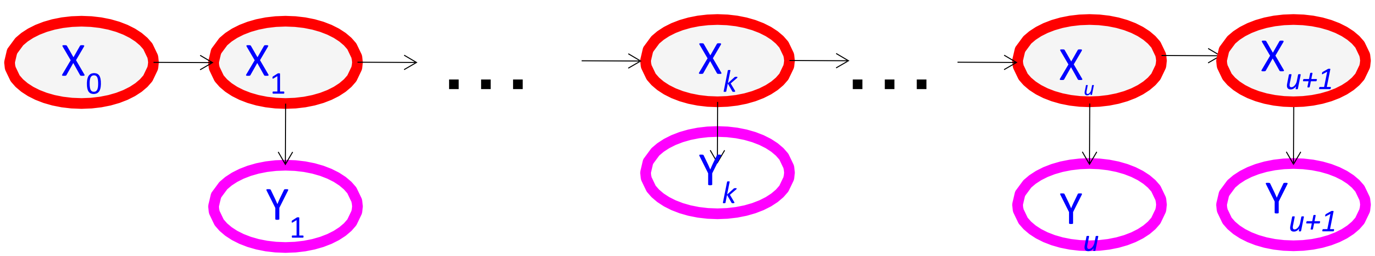
HMM inference tasks

- **Filtering:** what is the distribution over the current state X_t given all the evidence so far, $\mathbf{Y}_{1:t}$?
- **Smoothing:** what is the distribution of some state X_k (k<t) given the entire observation sequence $\mathbf{Y}_{1:t}$?
- **Evaluation:** compute the probability of a given observation sequence $\mathbf{Y}_{1:t}$



HMM inference tasks

- **Filtering:** what is the distribution over the current state X_t given all the evidence so far, $\mathbf{Y}_{1:t}$
- **Smoothing:** what is the distribution of some state X_k (k<t) given the entire observation sequence $\mathbf{Y}_{1:t}$?
- **Evaluation:** compute the probability of a given observation sequence $\mathbf{Y}_{1:t}$
- **Decoding:** what is the most likely state sequence $\mathbf{X}_{0:t}$ given the observation sequence $\mathbf{Y}_{1:t}$? (example: what's the weather every day?)



HMM Learning and Inference

- **Inference tasks**
  - **Filtering:** what is the distribution over the current state X_t given all the evidence so far, $\mathbf{Y}_{1:t}$
  - **Smoothing:** what is the distribution of some state X_k ($k<t$) given the entire observation sequence $\mathbf{Y}_{1:t}$?
  - **Evaluation:** compute the probability of a given observation sequence $\mathbf{Y}_{1:t}$
  - **Decoding:** what is the most likely state sequence $\mathbf{X}_{0:t}$ given the observation sequence $\mathbf{Y}_{1:t}$?
- **Learning**
  - Given a training sample of sequences, learn the model parameters (transition and emission probabilities)

HMM inference tasks

- **Filtering:** what is the distribution over the current state X_t given all the evidence so far, $\mathbf{Y}_{1:t}$
- **Smoothing:** what is the distribution of some state X_k ($k<t$) given the entire observation sequence $\mathbf{Y}_{1:t}$?
- **Evaluation:** compute the probability of a given observation sequence $\mathbf{Y}_{1:t}$
- **Decoding:** what is the most likely state sequence $\mathbf{X}_{0:t}$ given the observation sequence $\mathbf{Y}_{1:t}$? (example: what's the weather every day?)