Dynamics of a rocket, with g=const

Numerical values used for graphs and comparisons

In[106]:=
```
params = {m0 → 1000, R → 10, fthrust → 15 000, tmax → 50, v0 → 0, g → 9.8}
```

Out[106]=
```
{m0 → 1000, R → 10, fthrust → 15 000, tmax → 50, v0 → 0, g → 9.8}
```

```
(* for comparison with g=0 *)
paramszerog = {m0 → 1000, R → 10, fthrust → 15 000, tmax → 50, v0 → 0, g → 0}
```

In[138]:=
```
{m0 → 1000, R → 10, fthrust → 15 000, tmax → 50, v0 → 0, g → 0}
```

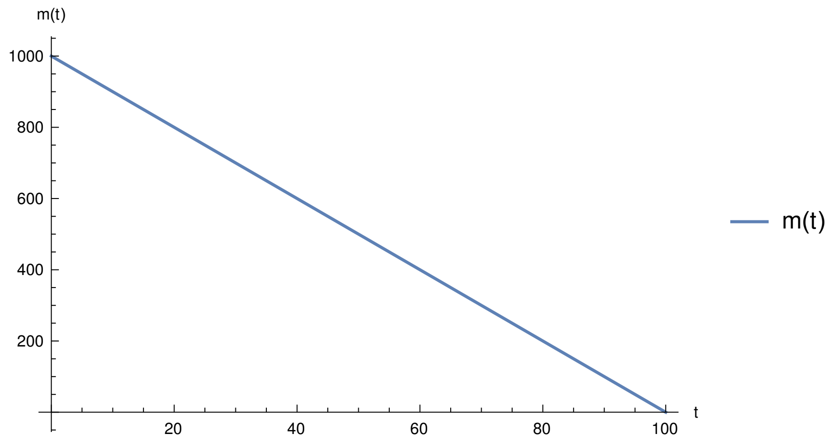
Out[138]=
```
{m0 → 1000, R → 10, fthrust → 15 000, tmax → 50, v0 → 0, g → 0}
```

Mass vs time, m (t), with R=dm/dt=mass flow rate (exhausted mass per unit of time; R is assumed constant)

In[107]:=
```
m[t_] := m0 - R t
```

In[108]:=
```
Plot[m[t] /. params, {t, 0, 100}, PlotLegends → {"m(t)"}, AxesLabel → {"t", "m(t)"}]
```

Out[108]=



Ideal rocket equation (Tsiolkovsky rocket equation): Δv = ve ln(m0 / mf) - g Δt, where ve = thrust / R

In[109]:=
```
(* see OpenStax Eq. 9.39 *)
deltav[ve_, m0_, mf_, dt_] := ve Log[m0 / mf] - g dt
```

In[110]:=
$$\text{deltav}\left[\frac{\text{fthrust}}{\text{R}}, \text{m[0]}, \text{m[tmax]}, \text{t1}\right]$$

Out[110]=
$$-\text{g t1} + \frac{\text{fthrust Log}\left[\frac{\text{m0}}{\text{m0}-\text{R tmax}}\right]}{\text{R}}$$

In[111]:=
$$\text{deltav1} = \text{deltav}\left[\frac{\text{fthrust}}{\text{R}}, \text{m[0]}, \text{m[tmax]}, \text{tmax}\right] \text{ /. params}$$

Out[111]=
```
549.721
```

In[142]:=
```
(* for g=0 *) deltav[fthrust/R, m[0], m[tmax], tmax] /. paramszerog // N
```
Out[142]= 1039.72

Velocity v(t)

In[112]:= (*

$a_y(t) = dv_y/dt$ $a_y(t)=F_{net,y} / m(t)$ where $F_{net,y}=F_{thrust}-m(t)g=v_e R-m(t)g$

and we need to integrate in [0;t] both sides of

$$\frac{dv(t)}{dt} = \frac{R\,v_e}{m(t)}-g$$

where $m(t) = m_0 - R\,t$, and $v_e=F_{thrust}/R$

*)

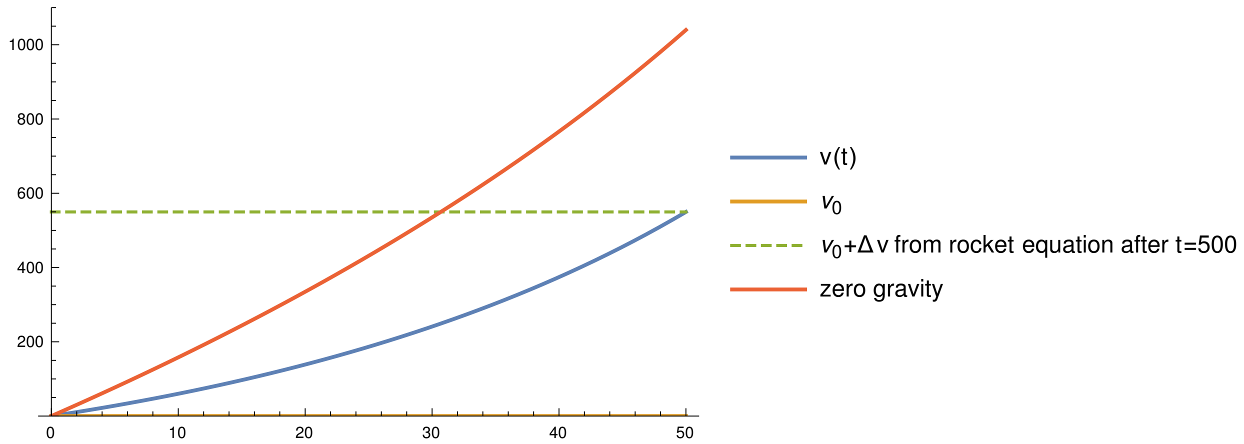
In[113]:= (* hint: *) $\int \frac{1}{m0 - R\,t}\,dt$

Out[113]= $-\frac{\text{Log}[m0 - R\,t]}{R}$

In[114]:= (* result: *)

$$v[t\_] := v0 + \frac{fthrust}{R}\,\text{Log}\left[\frac{m0}{m[t]}\right] - g\,t$$

In[140]:=
```
Plot[{v[t] /. params, (v0 /. params), (v0 /. params) + deltav1, v[t] /. paramszerog},
 {t, 0, tmax /. params},
 PlotLegends → {"v(t)", "v0", "v0+Δv from rocket equation after t=500", "zero gravity"},
 PlotStyle -> {Thick, Thick, Dashed}, PlotRange → {0, 1100}]
```

Out[140]= 

Position y(t)

(* $v=\frac{dy}{dt}$ → $y(t_1) = y_0 + \int_0^{t_1} v(t)\,dt$ *)

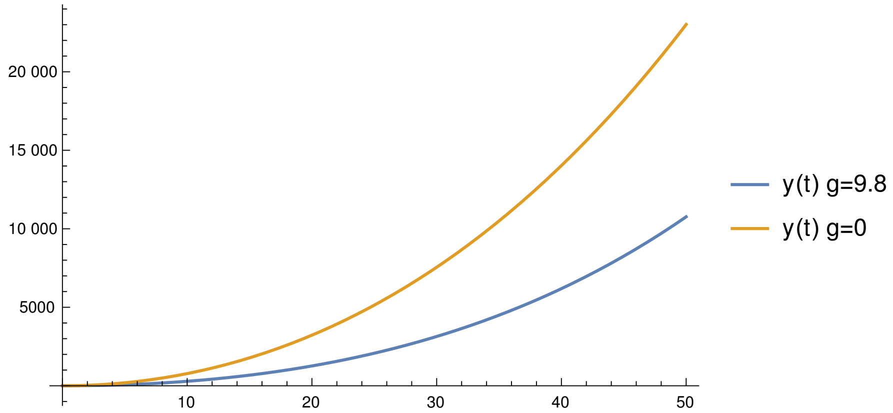
In[128]:=
```
Integrate[v[t], {t, 0, t1}, Assumptions → {R > 0, t1 > 0, m0 > R t1}]
```

Out[128]= $-\frac{g\,t1^2}{2} + t1\,v0 + \frac{fthrust\left(t1 + \left(-\frac{m0}{R} + t1\right)\text{Log}\left[\frac{m0}{m0-R\,t1}\right]\right)}{R}$

In[129]:=
```
(* assuming y0=0 : *)
```
$$y[t1\_] := -\frac{g\, t1^2}{2} + t1\, v0 + \frac{fthrust\left(t1 + \left(-\frac{m0}{R} + t1\right) Log\left[\frac{m0}{m0 - R\, t1}\right]\right)}{R}$$

In[146]:=
```
Plot[{y[t] /. params, y[t] /. paramszerog},
 {t, 0, tmax /. params}, PlotLegends → {"y(t) g=9.8", "y(t) g=0"}]
```

Out[146]=



In[123]:=
```
(* let's verify if x''(t) equals a(t) *)
```

In[147]:=
```
y''[t] // FullSimplify
```

Out[147]= $-g + \frac{fthrust}{m0 - R\, t}$

In[125]:=
```
(* OK, a = F/m(t) *)
```