Dynamics of a rocket, no gravity or other external forces

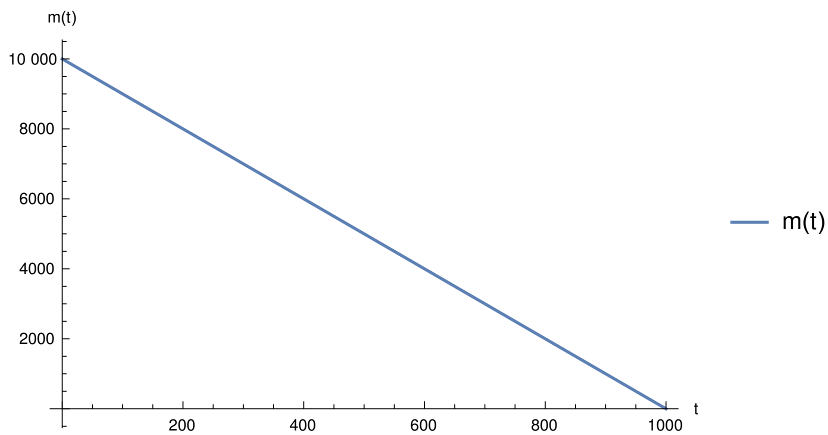
Numerical values used for graphs and comparisons

In[43]:=
```
params = {m0 → 10 000, R → 10, fthrust → 500, tmax → 500, v0 → 10}
```
Out[43]=
```
{m0 → 10 000, R → 10, fthrust → 500, tmax → 500, v0 → 10}
```

Mass vs time, m (t), with R=dm/dt=mass flow rate (exhausted mass per unit of time; R is assumed constant)

In[44]:=
```
m[t_] := m0 - R t
```

In[45]:=
```
Plot[m[t] /. params, {t, 0, 1000}, PlotLegends → {"m(t)"}, AxesLabel → {"t", "m(t)"}]
```

Out[45]=


Ideal rocket equation (Tsiolkovsky rocket equation): Δv = ve ln(m0 / mf), where ve = thrust / dmdt

```
(* note: in WolframMathematica, Log[...] is the natural logarithm *)
deltav[ve_, m0_, mf_] := ve Log[m0 / mf]
```

In[47]:=
$$\text{deltav}\left[\frac{\text{fthrust}}{\text{R}}, \text{m[0]}, \text{m[tmax]}\right]$$

Out[47]=
$$\frac{\text{fthrust Log}\left[\frac{\text{m0}}{\text{m0} - \text{R tmax}}\right]}{\text{R}}$$

In[48]:=
$$\text{deltav1} = \text{deltav}\left[\frac{\text{fthrust}}{\text{R}}, \text{m[0]}, \text{m[tmax]}\right] \text{/. params}$$

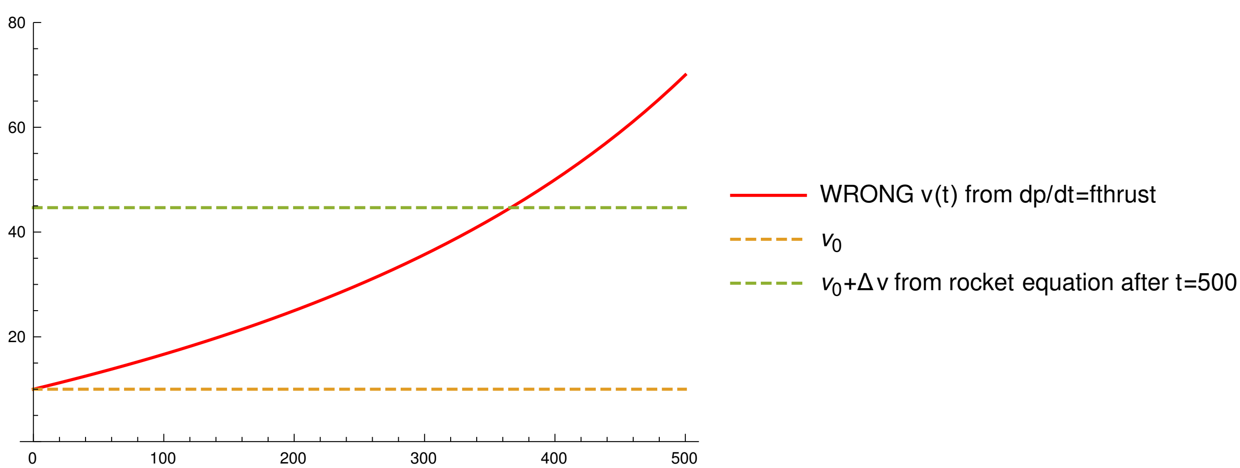
Out[48]=
```
50 Log[2]
```

In[49]:=
```
N[deltav1]
```
Out[49]= 34.6574

WARNING !!!! Newton's 2nd law, dp/dt=fthrust, with p=m(t)v(t) CANNOT BE DIRECTLY APPLIED !!!!

In[50]:=
```
(* to see that, let's try to solve dp/dt=fthrust : *)
sol1 = DSolve[{D[ m[t] × v[t], t] == fthrust, v[0] == v0}, v[t], t] // FullSimplify
```
Out[50]= $\left\{\left\{v[t] \to \frac{\text{fthrust } t + \text{m0 v0}}{\text{m0} - R\, t}\right\}\right\}$

In[51]:=
```
(* for our parametrs v(t) is: *)
v[t] /. sol1 /. params
```
Out[51]= $\left\{\frac{100\,000 + 500\, t}{10\,000 - 10\, t}\right\}$

In[68]:=
```
(* v(t) IS NOT CORRECT!!!! since v(tmax) is
 not equal Δv known from the rocket equation !!! *)
Plot[{v[t] /. sol1 /. params, (v0 /. params), (v0 /. params) + deltav1},
 {t, 0, tmax /. params}, PlotLegends →
  {"WRONG v(t) from dp/dt=fthrust", "v₀", "v₀+Δv from rocket equation after t=500"},
 PlotStyle -> {Red, Dashed, Dashed}, PlotRange → {0, 80}]
```
Out[68]=



```
(* let's compare with Newton's 2nd law, dp/dt=Fnet, for Fnet=f0=const and m=const
     correct: v(t) = a t + v0, where a=f0/m=const, but not useful for a rocket *)
```

In[53]:=
```
DSolve[{D[ m v[t], t] == f0, v[0] == v0}, v[t], t] // FullSimplify
```
Out[53]= $\left\{\left\{v[t] \to \frac{\text{f0 } t}{m} + \text{v0}\right\}\right\}$

How to find correct v(t)?

```
(*
    ax(t) = dvx/dt ax(t)=Fnet,x / m(t)   where  Fnet,x=
   Fthrust=(-ve,x)R=veR     note: minus because  ve vector opposite to Fthrust

                 and we need to integrate in [0;t] both sides of
```

$$\frac{dv(t)}{dt} = \frac{R\, v_e}{m(t)}$$

```
                where m(t) = m0 - R t, and ve=Fthrust/R
*)
```

(* hint: *) $\int \frac{1}{m0 - R\, t}\, dt$

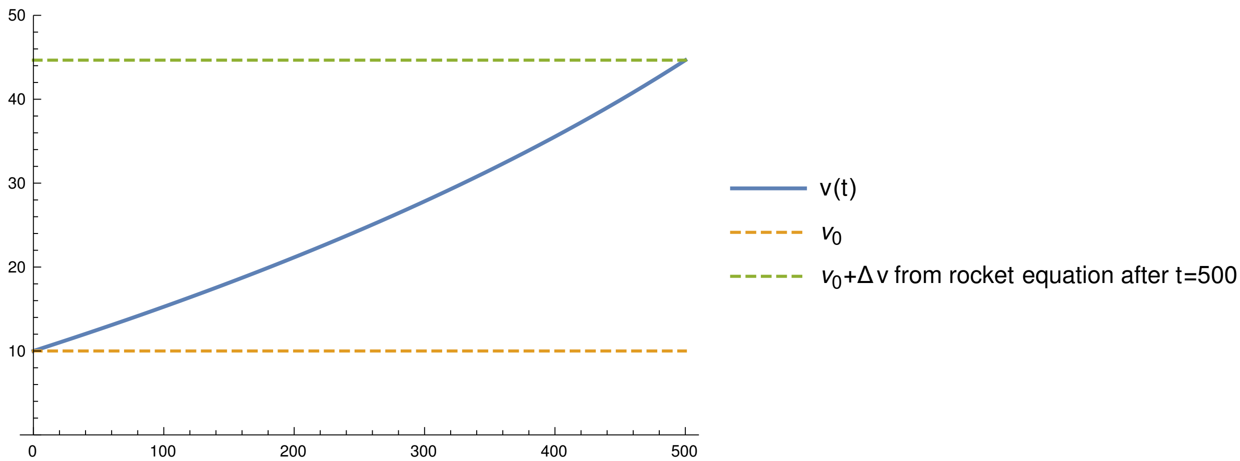
Out[59]= $-\frac{\text{Log}[m0 - R\, t]}{R}$

In[70]:= (* result: *)

$$v[t\_] := v0 + \frac{\text{fthrust}}{R}\,\text{Log}\left[\frac{m0}{m[t]}\right]$$

In[73]:=
```
Plot[{v[t] /. params, (v0 /. params), (v0 /. params) + deltav1}, {t, 0, tmax /. params},
 PlotLegends → {"v(t)", "v0", "v0+Δv from rocket equation after t=500"},
 PlotStyle -> {Thick, Dashed, Dashed}, PlotRange → {0, 50}]
```

Out[73]=



How to find x(t)?

(* $v=\frac{dx}{dt} \to \quad x(t_1) = x_0 + \int_0^{t_1} v(t)\, dt$ *)

In[74]:= (* hint: *) $\int \text{Log}\left[\frac{m0}{m0 - R\,t}\right] dt$

Out[74]= $t - \frac{(m0 - R\,t)\,\text{Log}\left[\frac{m0}{m0-R\,t}\right]}{R}$

In[75]:= $\int_0^{t1} \text{Log}\left[\frac{m0}{m0 - R\,t}\right] dt$

Out[75]= $\text{ConditionalExpression}\left[t1 + \left(-\frac{m0}{R} + t1\right)\text{Log}\left[\frac{m0}{m0 - R\,t1}\right], \text{Re}\left[\frac{m0}{R\,t1}\right] > 1 \,||\, \text{Re}\left[\frac{m0}{R\,t1}\right] < 0 \,||\, \frac{m0}{R\,t1} \notin \mathbb{R}\right]$

In[77]:= $\text{Integrate}\left[\text{Log}\left[\frac{m0}{m0 - R\,t}\right], \{t, 0, t1\}, \text{Assumptions} \to \{R > 0, t1 > 0, m0 > R\,t1\}\right]$

Out[77]= $t1 + \left(-\frac{m0}{R} + t1\right)\text{Log}\left[\frac{m0}{m0 - R\,t1}\right]$

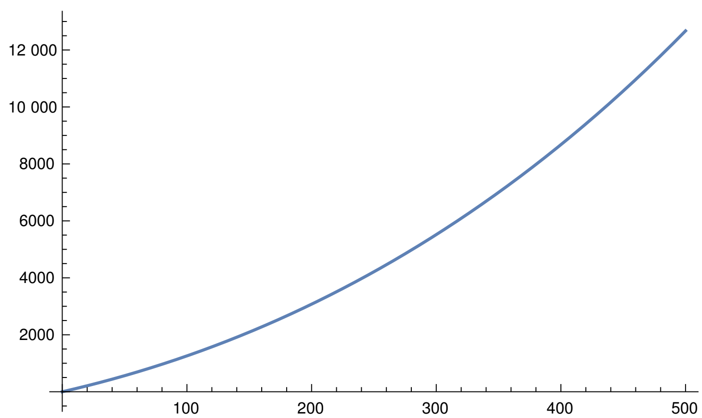
In[78]:= Integrate[v[t], {t, 0, t1}, Assumptions → {R > 0, t1 > 0, m0 > R t1}]

Out[78]= $t1\,v0 + \frac{\text{fthrust}\left(t1 + \left(-\frac{m0}{R} + t1\right)\text{Log}\left[\frac{m0}{m0-R\,t1}\right]\right)}{R}$

In[79]:= (* assuming x_0=0 : *)

$x[t\_] := t\,v0 + \frac{\text{fthrust}\left(t + \left(-\frac{m0}{R} + t\right)\text{Log}\left[\frac{m0}{m0-R\,t}\right]\right)}{R}$

In[80]:= Plot[{x[t] /. params}, {t, 0, tmax /. params}]

Out[80]= 

(* let's verify if x''(t) equals a(t) *)

In[82]:= x''[t] // FullSimplify

Out[82]= $\frac{\text{fthrust}}{m0 - R\,t}$

```
(* OK, a = F/m(t) *)
```