Dynamics of a rocket, no gravity or other external forces

Numerical values used for graphs and comparisons

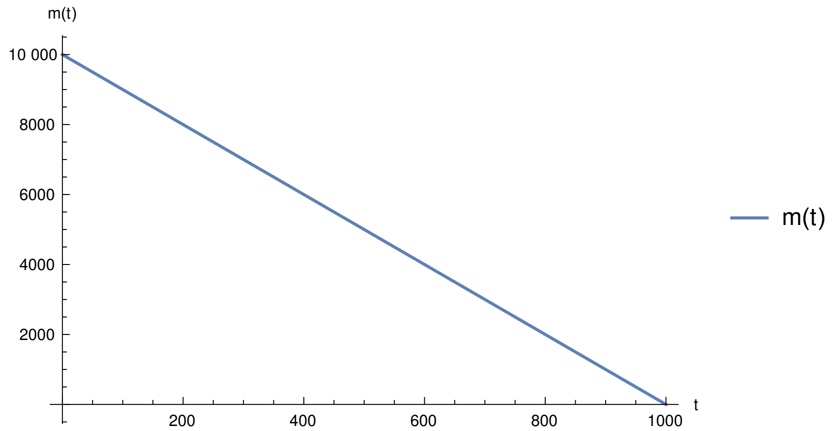
In[43]:=
```
params = {m0 → 10 000, R → 10, fthrust → 500, tmax → 500, v0 → 10}
```
Out[43]=
```
{m0 → 10 000, R → 10, fthrust → 500, tmax → 500, v0 → 10}
```

Mass vs time, m (t), with R=dm/dt=mass flow rate (exhausted mass per unit of time; R is assumed constant)

In[44]:=
```
m[t_] := m0 - R t
```

In[45]:=
```
Plot[m[t] /. params, {t, 0, 1000}, PlotLegends → {"m(t)"}, AxesLabel → {"t", "m(t)"}]
```

Out[45]=



Ideal rocket equation (Tsiolkovsky rocket equation): Δv = ve ln(m0 / mf), where ve = thrust / R

```
(* note: in WolframMathematica, Log[...] is the natural logarithm *)
deltav[ve_, m0_, mf_] := ve Log[m0 / mf]
```

In[47]:=
$$\text{deltav}\left[\frac{\text{fthrust}}{\text{R}}, \text{m[0]}, \text{m[tmax]}\right]$$

Out[47]=
$$\frac{\text{fthrust Log}\left[\frac{\text{m0}}{\text{m0}-\text{R tmax}}\right]}{\text{R}}$$

In[48]:=
$$\text{deltav1} = \text{deltav}\left[\frac{\text{fthrust}}{\text{R}}, \text{m[0]}, \text{m[tmax]}\right] \text{ /. params}$$

Out[48]=
```
50 Log[2]
```

In[49]:=
```
N[deltav1]
```
Out[49]=
```
34.6574
```

WARNING !!!! Newton's 2nd law, dp/dt=fthrust, with p=m(t)v(t) CANNOT BE DIRECTLY APPLIED !!!!

In[50]:=
```
(* to see that, let's try to solve dp/dt=fthrust : *)
sol1 = DSolve[{D[ m[t] × v[t], t] == fthrust, v[0] == v0}, v[t], t] // FullSimplify
```

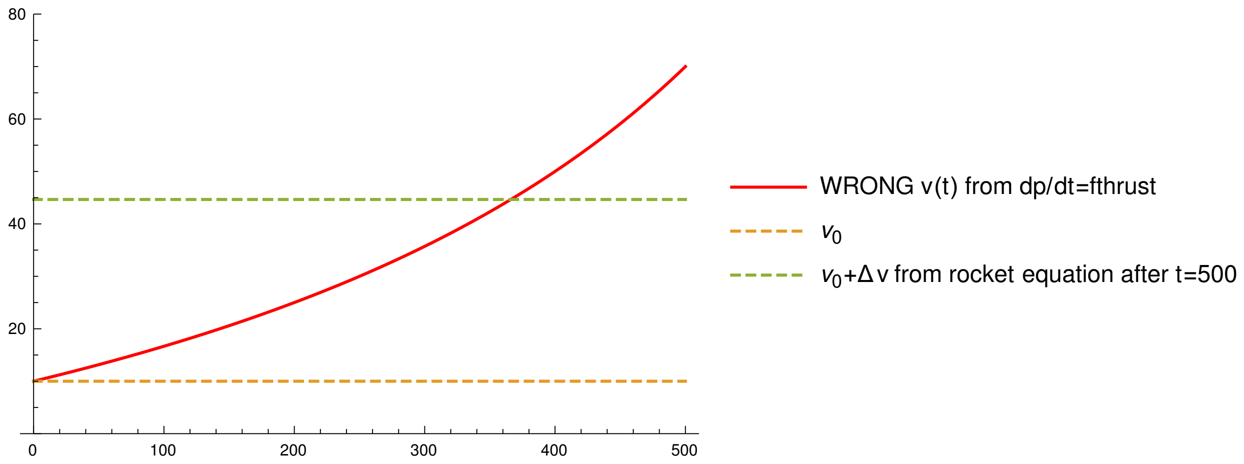
Out[50]= $\left\{\left\{v[t] \to \frac{\text{fthrust } t + m0\ v0}{m0 - R\ t}\right\}\right\}$

In[51]:=
```
(* for our parametrs v(t) is: *)
v[t] /. sol1 /. params
```

Out[51]= $\left\{\frac{100\,000 + 500\ t}{10\,000 - 10\ t}\right\}$

In[68]:=
```
(* v(t) IS NOT CORRECT!!!! since v(tmax) is not equal Δv known from the rocket equation !!! *)
Plot[{v[t] /. sol1 /. params, (v0 /. params), (v0 /. params) + deltav1},
 {t, 0, tmax /. params}, PlotLegends →
  {"WRONG v(t) from dp/dt=fthrust", "v0", "v0+Δv from rocket equation after t=500"},
 PlotStyle -> {Red, Dashed, Dashed}, PlotRange → {0, 80}]
```

Out[68]=




```
(* let's compare with Newton's 2nd law, dp/dt=Fnet, for Fnet=f0=const and m=const
     correct: v(t) = a t + v0, where a=f0/m=const, but not useful for a rocket *)
```

In[53]:=
```
DSolve[{D[ m v[t], t] == f0, v[0] == v0}, v[t], t] // FullSimplify
```

Out[53]= $\left\{\left\{v[t] \to \frac{f0\ t}{m} + v0\right\}\right\}$

How to find correct v(t)?

(*

$a_x(t)$ = dv_x/dt $a_x(t)=F_{net,x}$ / m(t) where $F_{net,x}$=

$F_{thrust}=(-v_{e,x})R=v_eR$ note: minus because v_e vector opposite to F_{thrust}

and we need to integrate in [0;t] both sides of

$$\frac{dv(t)}{dt} = \frac{R\ v_e}{m(t)}$$

where $m(t) = m_0 - R\ t$, and $v_e=F_{thrust}/R$

*)

(* hint: *) $\int \frac{1}{\text{m0} - \text{R t}} \, dt$

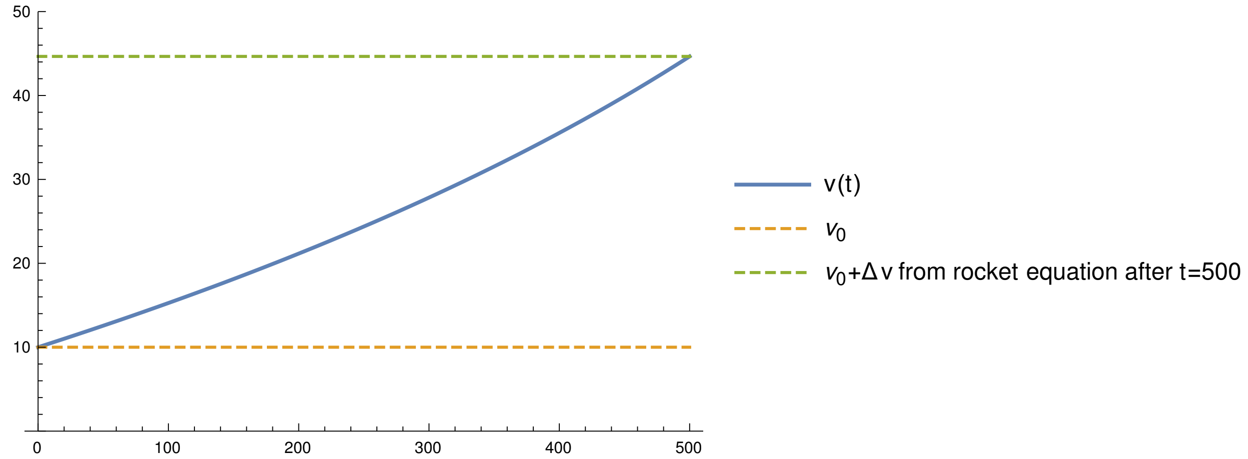
Out[59]= $-\frac{\text{Log[m0} - \text{R t]}}{\text{R}}$

In[70]:= (* result: *)

$\text{v[t\_]} := \text{v0} + \frac{\text{fthrust}}{\text{R}} \text{Log}\left[\frac{\text{m0}}{\text{m[t]}}\right]$

In[73]:=
```
Plot[{v[t] /. params, (v0 /. params), (v0 /. params) + deltav1}, {t, 0, tmax /. params},
 PlotLegends → {"v(t)", "v0", "v0+Δv from rocket equation after t=500"},
 PlotStyle -> {Thick, Dashed, Dashed}, PlotRange → {0, 50}]
```

Out[73]=



How to find x(t)?

(* $v = \frac{dx}{dt} \rightarrow x(t_1) = x_0 + \int_0^{t_1} v(t) \, dt$ *)

In[74]:= (* hint: *) $\int \text{Log}\left[\frac{\text{m0}}{\text{m0} - \text{R t}}\right] dt$

Out[74]= $\text{t} - \frac{(\text{m0} - \text{R t}) \, \text{Log}\left[\frac{\text{m0}}{\text{m0}-\text{R t}}\right]}{\text{R}}$

In[75]:= $\int_0^{\text{t1}} \text{Log}\left[\frac{\text{m0}}{\text{m0} - \text{R t}}\right] dt$

Out[75]= $\text{ConditionalExpression}\left[\text{t1} + \left(-\frac{\text{m0}}{\text{R}} + \text{t1}\right) \text{Log}\left[\frac{\text{m0}}{\text{m0} - \text{R t1}}\right], \text{Re}\left[\frac{\text{m0}}{\text{R t1}}\right] > 1 \,||\, \text{Re}\left[\frac{\text{m0}}{\text{R t1}}\right] < 0 \,||\, \frac{\text{m0}}{\text{R t1}} \notin \mathbb{R}\right]$

In[77]:= $\text{Integrate}\left[\text{Log}\left[\frac{\text{m0}}{\text{m0} - \text{R t}}\right], \{\text{t}, 0, \text{t1}\}, \text{Assumptions} \rightarrow \{\text{R} > 0, \text{t1} > 0, \text{m0} > \text{R t1}\}\right]$

Out[77]= $\text{t1} + \left(-\frac{\text{m0}}{\text{R}} + \text{t1}\right) \text{Log}\left[\frac{\text{m0}}{\text{m0} - \text{R t1}}\right]$

In[78]:=
```
Integrate[v[t], {t, 0, t1}, Assumptions → {R > 0, t1 > 0, m0 > R t1}]
```

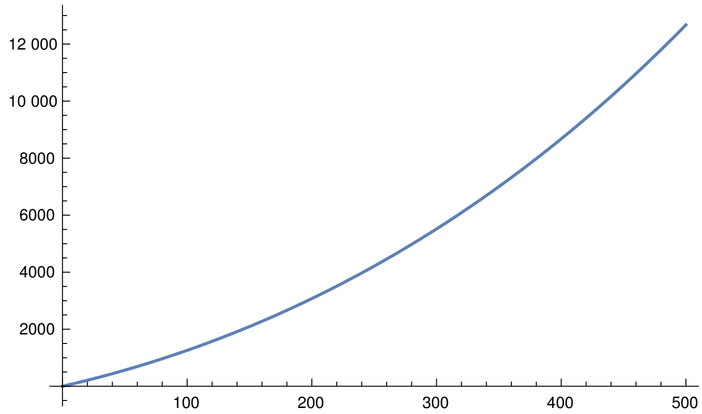
Out[78]= $\text{t1 v0} + \frac{\text{fthrust} \left(\text{t1} + \left(-\frac{\text{m0}}{\text{R}} + \text{t1}\right) \text{Log}\left[\frac{\text{m0}}{\text{m0}-\text{R t1}}\right]\right)}{\text{R}}$

In[79]:=
```
(* assuming x_0=0 : *)
```

$$x[t\_] := t\,v0 + \frac{fthrust\left(t + \left(-\frac{m0}{R} + t\right) Log\left[\frac{m0}{m0 - R\,t}\right]\right)}{R}$$

In[80]:=
```
Plot[{x[t] /. params}, {t, 0, tmax /. params}]
```

Out[80]=




```
(* let's verify if x''(t) equals a(t) *)
```

In[82]:=
```
x''[t] // FullSimplify
```

Out[82]=

$$\frac{fthrust}{m0 - R\,t}$$

```
(* OK, a = F/m(t) *)
```