Task 2 - filtration in spatial domain (histogram modifications, linear and non-linear operations, convolution).

This excercise is composed of three elements:

- Implement the histogram calculation algorithm. The application should be able to save the histogram of the chosen channel as an image (`--histogram`). Next, implement the given method of image quality improvement basing on image histogram and calculate all the image characteristics. It should be analyzed what is the influence of quality improvement methods on those characteristics for different sample images. The available variants are:

  - (H1) Uniform final probability density function (`--huniform`).
  - (H2) Exponential final probability density function (`--hexponent`).
  - (H3) Raleigh final probability density function (`--hraleigh`).
  - (H4) Power 2/3 final probability density function (`--hpower`).
  - (H5) Hyperbolic final probability density function (`--hhyper`).

  Image characteristics (each team should implement all of them):

  - (C1) Mean (`--cmean`). Variance (`--cvariance`).
  - (C2) Standard deviation (`--cstdev`). Variation coefficient I (`--cvarcoi`).
  - (C3) Asymmetry coefficient (`--casyco`).
  - (C4) Flattening coefficient (`--casyco`).
  - (C5) Variation coefficient II (`--cvarcoii`).
  - (C6) Information source entropy (`--centropy`).

- Implement linear image filtration algorithm in spatial domain basing on convolution. This algorithm should be implemented twice. First implementation should be universal and should work for each mask. Second implementation should be optimized for a chosen mask. The optimization should consider reduction of operations number and memory needed for image filtration. The available variants are:

  - (S1) Low-pass filter (`--slowpass`).
  - (S2) Edge sharpening (`--sedgesharp`).
  - (S3) Extraction of deteials I. N, NE, E, SE filters (`--sexdeti`).
  - (S4) Extraction of deteials II. S, SW, W, NW filters (`--sexdetii`).
  - (S5) Extraction of deteials III. Without direcition, laplacian filter (`--slaplace`).
  - (S6) Line identification (`--slineid`).

- Implement non-linear image filtration algorithm in spatial domain. It should be analyzed what is the result of the filtration for different sample images. The available variants are:

  - (O1) Roberts operator I (`--orobertsi`).
  - (O2) Roberts operator (`--orobertsii`).
  - (O3) Sobel operator (`--osobel`).
  - (O4) Kirsh operator (`--okirsf`).
  - (O5) Rosenfeld operator (`--orosenfeld`).
  - (O6) Uolis oprator (`--ouolis`).

In all the elements of the excercise the following issues will be taken into account:

- Correctness of the implementation. In the report the detailed description of code optimization should be placed. In case of memory reduction the needed memory (depending on the size of the image) should be estimated and in case of operations number reduction the number of additions and multiplications as well as execution time should be analyzed. The methods should be also applied for images with noise and with noise removed. The conclusions should be placed in the report. The approximate weight of this part of the exercise is 0.5.
- Analysis of the obtained results. The approximate weight of this part of the exercise is 0.5.

The report template:

- **report_2.doc** - Microsoft Word

Instructions and remarks:

- Image quality improvement basing on histogram. Further g denotes a new brightness (as a functions of old brightness calculated basing on the histogram), f jasność starą, zaś N is a number of pixels in the image (

f_{min} , f_{max} , g_{min} i g_{max} denote minimum and maximum brightness in input and output image, respectively - the latter are specified by the user). The histogram of the input image is denoted as H . In all the equations using logarithms the proper range of output values should be assured (the proper parameters should be chosen). All the equations below are specified for gray scale images (L - gray scale levels).

- Uniform final probability density function

$$g(f)=g_{min}+(g_{max}-g_{min})\frac{1}{N}\sum_{m=0}^{f}H(m)$$.

- Exponential final probability density function

$$g(f)=g_{min}-\frac{1}{\alpha}\ln\left(1-\frac{1}{N}\sum_{m=0}^{f}H(m)\right)$$.

- Raleigh final probability density function

$$g(f)=g_{\min}+\left[2\alpha^2\ln\left(\frac{1}{N}\sum_{m=0}^{f}H(m)\right)^{-1}\right]^{1/2}$$.

- Power 2/3 final probability density function

$$g(f)=\left(g_{min}^{\frac{1}{3}}+(g_{max}^{\frac{1}{3}}-g_{min}^{\frac{1}{3}})\frac{1}{N}\sum_{m=0}^{f}H(m)\right)^{3}$$.

- Hyperbolic final probability density function

$$g(f)=g_{min}\left(\frac{g_{max}}{g_{min}}\right)^{\frac{1}{N}\sum_{m=0}^{f}H(m)}$$.

Image charcteristics:

- Mean

$$\overline{b}=1/N\sum_{m=0}^{L-1}m\,H_A(m)$$.

- Variance

$$D^2=1/N\sum_{m=0}^{L-1}(m-\overline{b})^2H_A(m)$$.

- Standard deviation

$$\sigma_X=\sqrt{D^2}$$,

gdzie $D^2=1/N\sum_{m=0}^{L-1}(m-\overline{b})^2H_A(m)$.

- Variation coefficient I

$$b_Z=\sigma_X/\overline{b}$$,

gdzie

$$\overline{b} = 1/N\sum_{m=0}^{L-1} mH_A(m)$$
.

- Asymmetry coefficient

  $$b_S = (1/\sigma^3)1/N\sum_{m=0}^{L-1}(m-\overline{b})^3 H_A(m)$$
  ,

  gdzie $\overline{b} = 1/N\sum_{m=0}^{L-1} mH_A(m)$.

- Flattening coefficient

  $$b_K = (1/\sigma^4)1/N\sum_{m=0}^{L-1}(m-\overline{b})^4 H_A(m) - 3$$
  ,

  gdzie $\overline{b} = 1/N\sum_{m=0}^{L-1} mH_A(m)$.

- Variation coefficient II

  $$b_N = (1/N)^2\sum_{m=0}^{L-1}[H_A(m)]^2$$
  .

- Information source entropy

  $$b_E = -1/N\sum_{m=0}^{L-1} H_A(m)\,\log_2[H_A(m)/N]$$
  .

- Linear filtration in spatial domain (convolution):

  Convolution:

  $$g(p,q) = \sum_{i=-M}^{M}\sum_{j=-M}^{M} h(i,j)x(p+i,q+j), \quad p = M,2,\ldots,P-M-1,\ q = M,2,\ldots,Q-M-1$$

  where x(p,q) and g(p,q), for p=0,1,...,P-1, q=0,1,...,Q-1 - denote input and output images, respecitvely, whereas h(i, j), for i=-M,-M+1,...,M, j=-M,-M+1,...,M - denotes the filter impulse response. For p and q at the image border g(p,q) = x(p,q) (it is one of possible aproaches).

  Below the sample convolution maska are presented. For optimization only one should be chosen.

  - Low-pass filter.

    $$h(,) = \frac{1}{9}\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}, \qquad h(,) = \frac{1}{10}\begin{pmatrix}1 & 1 & 1\\ 1 & 2 & 1\\ 1 & 1 & 1\end{pmatrix}, \qquad h(,) = \frac{1}{16}\begin{pmatrix}1 & 2 & 1\\ 2 & 4 & 2\\ 1 & 2 & 1\end{pmatrix}.$$

  - Edge sharpening.

    $$h(,) = \begin{pmatrix}0 & -1 & 0\\ -1 & 5 & -1\\ 0 & -1 & 0\end{pmatrix}, \qquad h(,) = \begin{pmatrix}-1 & -1 & -1\\ -1 & 9 & -1\\ -1 & -1 & -1\end{pmatrix}, \qquad h(,) = \begin{pmatrix}1 & -2 & 1\\ -2 & 5 & -2\\ 1 & -2 & 1\end{pmatrix}.$$

  - Extraction of deteials I. N, NE, E, SE filters.

$$h(,)=\begin{pmatrix}1 & 1 & 1\\ 1 & -2 & 1\\ -1 & -1 & -1\end{pmatrix},\qquad h(,)=\begin{pmatrix}1 & 1 & 1\\ -1 & -2 & 1\\ -1 & -1 & 1\end{pmatrix},$$

$$h(,)=\begin{pmatrix}-1 & 1 & 1\\ -1 & -2 & 1\\ -1 & 1 & 1\end{pmatrix},\qquad h(,)=\begin{pmatrix}-1 & -1 & 1\\ -1 & -2 & 1\\ 1 & 1 & 1\end{pmatrix}.$$

- Extraction of deteials II. S, SW, W, NW filters.

$$h(,)=\begin{pmatrix}-1 & -1 & -1\\ 1 & -2 & 1\\ 1 & 1 & 1\end{pmatrix},\qquad h(,)=\begin{pmatrix}1 & -1 & -1\\ 1 & -2 & -1\\ 1 & 1 & 1\end{pmatrix},$$

$$h(,)=\begin{pmatrix}1 & 1 & -1\\ 1 & -2 & -1\\ 1 & 1 & -1\end{pmatrix},\qquad h(,)=\begin{pmatrix}1 & 1 & 1\\ 1 & -2 & -1\\ 1 & -1 & -1\end{pmatrix}.$$

- Extraction of deteials III. Without direcition, laplacian filter.

$$h(,)=\begin{pmatrix}0 & -1 & 0\\ -1 & 4 & -1\\ 0 & -1 & 0\end{pmatrix},\qquad h(,)=\begin{pmatrix}-1 & -1 & -1\\ -1 & 8 & -1\\ -1 & -1 & -1\end{pmatrix},\qquad h(,)=\begin{pmatrix}1 & -2 & 1\\ -2 & 4 & -2\\ 1 & -2 & 1\end{pmatrix}.$$

- Line identification.

$$h(,)=\begin{pmatrix}-1 & 2 & -1\\ -1 & 2 & -1\\ -1 & 2 & -1\end{pmatrix},\qquad h(,)=\begin{pmatrix}-1 & -1 & -1\\ 2 & 2 & 2\\ -1 & -1 & -1\end{pmatrix},$$

$$h(,)=\begin{pmatrix}-1 & -1 & 2\\ -1 & 2 & -1\\ 2 & -1 & -1\end{pmatrix},\qquad h(,)=\begin{pmatrix}2 & -1 & -1\\ -1 & 2 & -1\\ -1 & -1 & 2\end{pmatrix}.$$

- Non-linear filtration in spatial domain. Further $x(n, m)$ represents the currently considered image element , whereas $g(n, m)$ is an operator output for $n = 0, \dots , N - 1$ and $m = 0, \dots, M - 1$, where N and M denote the image width and height, respectively. The neighborhood of pixel $x(n, m)$ is presented below.

A_0	A_1	A_2
A_7	$x(n, m)$	A_3
A_6	A_5	A_4

- Roberts operator I

$$g(n,m)=[(x(n,m)-x(n+1,m+1))^2+(x(n,m+1)-x(n+1,m))^2]^{1/2}.$$

- Roberts operator II

$$g(n,m)=|\,x(n,m)-x(n+1,m+1)\,|+|\,x(n,m+1)-x(n+1,m)\,|\,.$$

- Sobel operator

  $$g(n,m) = \sqrt{X^2 + Y^2},$$

  gdzie $X = (A_2 + 2A_3 + A_4) - (A_0 + 2A_7 + A_6)$, $Y = (A_0 + 2A_1 + A_2) - (A_6 + 2A_5 + A_4)$.

- Kirsh operator

  $$g(n,m) = \max(1, \max_{i=0,1,2,..7}\{|\,5S_i - 3T_i\,|\}),$$

  where $S_i = A_i + A_{i+1} + A_{i+2}$, $T_i = A_{i+3} + A_{i+4} + A_{i+5} + A_{i+6} + A_{i+7}$.

- Rosenfeld operator

  $$g_P(n,m) = (1/P)\{x(n+P-1,m) + x(n+P-2,m) + \ldots + x(n,m) - x(n-1,m) - x(n-2,m) - \ldots - x(n-P,m)\},$$

  where P=1, 2, 4, 8, 16, ...

- Uolis operator

  $$g(n,m) = \frac{1}{4}\log\left\{\frac{|\,x(n,m)\,|^4}{A_1A_3A_5A_7}\right\}.$$