Task 3 - morphological operations, segmentation.

This excercise is composed of two elements:

- Implement basic morphological operations (dilation, erosion, opening, closing, HMT transformation) and analyze them. Next, one of the following variants should be implemented:

    - (M1) Apply for the whole image A the following operations: $(A \oplus B)/A$, $A/(A \ominus B)$, $(A \oplus B)/(A \ominus B)$ (basic structural element B here is (iii) but other can also be tested).
    - (M2) Ask the user to choose any point p in the image A and apply the following iterative operation $X_k = (X_{k-1} \oplus B) \cap A^c, X_0 = \{p\}, p \in \mathbb{Z}^2$ until $X_k = X_{k-1}$. As the result $Y = X_k \cup A$ should be given (basic structural element B here is (iv) but other can also be tested).
    - (M3) Ask the user to choose any point p in the image A and apply the following iterative operation $X_k = (X_{k-1} \oplus B) \cap A, X_0 = \{p\}, p \in \mathbb{Z}^2$ until $X_k = X_{k-1}$. As the result $Y = X_k$ should be given (basic structural element B here is (iii) but other can also be tested).
    - (M4) For the whole image, for the four successive (i=1,2,3,4) structural elements (xi) (other can also be tested) using the following operations iteratively $X_k^i = (X_{k-1} \otimes B^i) \cup A$ starting from $X_0^i = A$ find $D^i = X_k^i$ where k is such iteration where $X_k = X_{k-1}$. As a result $H(A) = D^1 \cup D^2 \cup D^3 \cup D^4$ should be given (A denotes image, B denotes structural element - it should be bold in all the equations in this variant).
    - (M5) Assuming that $N(A, \boldsymbol{B}) = A/(A \otimes \boldsymbol{B}) = A \cap (A \otimes \boldsymbol{B})^c$ for the successive eight structural elements (xii) (other can also be tested) calculate $N(A, \{\boldsymbol{B_1}, \ldots, \boldsymbol{B_n}\}) = N(N(\ldots N(N(A, \boldsymbol{B_1}), \boldsymbol{B_2}) \ldots), \boldsymbol{B_n})$ (n=8). This operaions should be repeated as long as there are any changes (A denotes image, B denotes structural element).
    - (M6) Assuming that $C(A, \boldsymbol{B}) = A \cup (A \otimes \boldsymbol{B})$ for the successive eight structural elements (xii) (other can also be tested) where 1 and 0 should be swapped calculate $C(A, \{\boldsymbol{B_1}, \ldots, \boldsymbol{B_n}\}) = C(C(\ldots C(C(A, \boldsymbol{B_1}), \boldsymbol{B_2}) \ldots), \boldsymbol{B_n})$ (n=8). Czynność tę powtarzać tak długo jak natępują jeszcze jakieś zmiany This operaions should be repeated as long as there are any changes (A denotes image, B denotes structural element).
    - (M7) Assuming that $S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$ where $A \ominus kB$ denotes erosion applied k times (basic structural element B here is (iii) but other can also be tested) find $S(A) = S_0(A) \cup S_1(A) \cup \ldots \cup S_K(A)$ where $K = max\{k \in \mathbb{N} : A \ominus kB \neq \emptyset\}$ (A denotes image, B denotes structural element).

    Those methods should be tested using 1-bit images and basing on the analysis of the results it should be pointed out where they can be of use. Results for different types of structural elements should also be discussed.

- Implement the given method of image segmentation. Implementation should be flexible and should allow to use the segmentation technique for different segmentation tasks. The available variants are:

    - (R1) Region growing (merging).

In all the elements of the excercise the following issues will be taken into account:

- Correctness of the implementation of morphological operations. The approximate weight of this part of the exercise is 0.25.
- Analysis of the morphological operations results. The approximate weight of this part of the exercise is 0.25.
- Correctness and flexibility of the implementantation of image segmentation method. The approximate weight of this part of the exercise is 0.25.
- Analysis of the segmentation method. The approximate weight of this part of the exercise is 0.25.

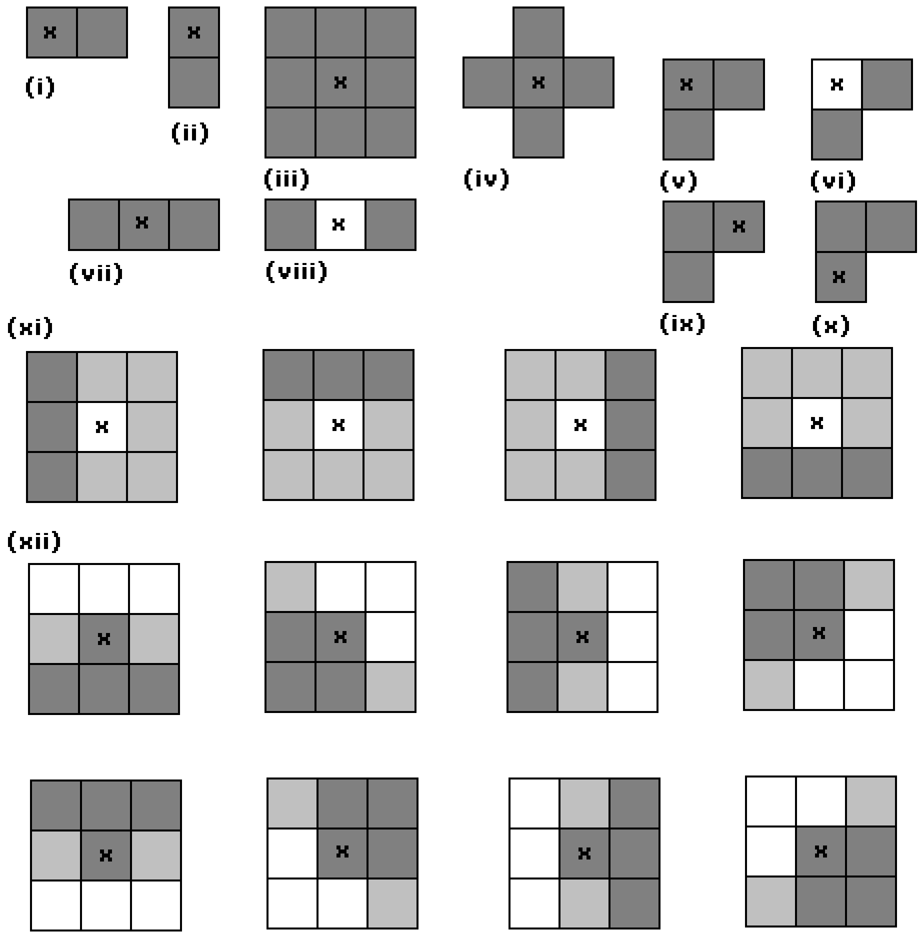
In all the variants the special attention should be paid to implementation efficiency (reduction of needed memory, reduction of execution time).

The report template:

- **report_3.doc** - Microsoft Word

Instructions and remarks:

- Morphological operations.
  - Basic notation. Let $A, B, C \subseteq \mathbb{Z}^2$ then:
    $A \cup B$ - sum
    $A \cap B$ - intersection
    $A^c = \{a \in \mathbb{Z}^2 : a \notin A\}$ - complement
    $\hat{A} = \{-a : a \in A\}$ - reflection
    $A_p = \{a + p : a \in A\}\ p \in \mathbb{Z}^2$ - translation using vector p
  - Basic operations:
    $A \oplus B = \{p \in \mathbb{Z}^2 : p = a + b, a \in A, b \in B\}$ - dilation
    $A \ominus B = \{p \in \mathbb{Z}^2 : p + b \in A\ \forall b \in B\}$ - erosion
    $A \circ B = (A \ominus B) \oplus B$ - opening
    $A \bullet B = (A \oplus B) \ominus B$ - closing
    $A \otimes \boldsymbol{B} = \{p \in \mathbb{Z}^2 : p + b_1 \in B_1 \wedge p + b_2 \in A^c\ \forall b_1 \in B_1, b_2 \in B_2\}$ - HMT transformation, where $\boldsymbol{B} = (B_{1,} B_2)$
  - Basic properties:
    $A \oplus B = \{p \in \mathbb{Z}^2 : \hat{B}_p \cap A \neq \emptyset\}$
    $A \ominus B = \{p \in \mathbb{Z}^2 : B_p \subseteq A\}$
    $(A \oplus B) \oplus C = A \oplus (B \oplus C)$
    $(A \ominus B) \ominus C = A \ominus (B \oplus C)$
    $A \oplus B = (A^c \ominus \hat{B})^c$
    $A \ominus B = (A^c \oplus \hat{B})^c$
    $A \oplus B = B \oplus A$
    $A_p \oplus B = (B \oplus A)_p$
    $A \circ B \subseteq A$
    $A \subseteq A \bullet B$
    $A \circ B = (A \circ B) \circ B$
    $A \bullet B = (A \bullet B) \bullet B$
    $A \otimes \boldsymbol{B} = (A \ominus B_1) \cap (A^c \ominus B_2)$
    $A \otimes \boldsymbol{B} = (A \ominus B_1) / (A \oplus \hat{B}_2)$
  - Sample structural elements:



Dark gray color represents 1, white 0, and light gray in elements for HMT transformation represents inactive points. Additionally, for HMT transformation and structural elements (xi) and (xii), B1 is shown in the image and B2 is its complement (the inactive elements are inactive before and after complement).

- Naturally the morphological operations presented here are only the subset of more developed thoeory. There is the possibility to extend them for gray scale images. Then dilation is defined as a maximum filter and erosion is defined as a minimum filter inside the srtructural element. Those aspects, however, are not considered here.

- Image segmentation task can be defined as a task where pixels representing some object in the image are to be groupped together. Those pixels are groupped together when they fulfill the given criterion which. of course, depends on the segmentation task. Below a short characteristic of one of the methods is presented:

  - Region growing is a method where regions are joined together. In the begining the elementary regions must be defined (usually these are pixels) and some of them must be chosen as, so called, seed regions. Next, the adjacent regions should be linked with the seed regions if the assumed criterion is fulfilled. That process should be repeated until there are no regions that could be joined together. At the end it should be checked whether two regions with different seed regions should be joined or not. The special attention should be paid to: the choice of the criterion, to the choice of the basic regions, to the choice seed regions, etc.