1. caligraphic ⠠⠎

   $\mathcal{S}$

   `\mathcal S`

2. ⠒⠬56⠤13⠬⠦⠌⠆

   $3+56-13+8/2$

   `3 + 56 - 13 + 8/2`

3. ⠆⠬⠒ ⠭ ⠢

   $2+3=5$

   `2 + 3 = 5`

4. ⠆⠭ ⠭ ⠖

   $2x=6$

   `2x = 6`

5. ⠭ ⠭ ⠲

   $x=4$

   `x = 4`

6. ⠁⠭⠘⠆⠐⠬⠃⠭⠬⠉ ⠭ ⠴

   $ax^2+bx+c=0$

   `ax^2 + bx + c = 0`

7. ⠁ ⠌⠭ ⠴

   $a \neq 0$

   `a \neq 0`

8. ⠭ ⠭ ⠹⠤⠃⠬⠤⠜⠃⠘⠆⠐⠤⠲⠁⠉⠻⠌⠆⠁⠼

   $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

   `x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}`

9. ⠭⠘⠒⠐⠤⠲⠭⠘⠆⠐⠬⠢⠭⠤⠖

   x^3-4x^2+5x-6

   `x^3 - 4x^2 + 5 x - 6`

10. ⠆⠡⠲

    $2 \cdot 4$

    `2 \cdot 4`

11. ⠖ ⠌⠭ ⠦

    $6 \neq 8$

    `6 \neq 8`

12. 10⠌⠢ ⠭ ⠆

    $10/5=2$

    `10/5 = 2`

13. p

    p

14. q

    q

15. r

    r

16. s

    s

17. ⠗⠀⠨⠅⠀⠎

    $r = s$

    r = s

18. x

    x

19. A

    A

20. X

    X

21. a

    a

22. ⠁⠀⠈⠑⠀⠠⠁

    $a \in A$

    a \in A

23. ⠠⠭⠀⠨⠅⠀StartSet⠭⠂⠀⠠⠭⠆⠀⠤⠀⠠⠭⠰⠝⠐EndSet

    $X = \{x_1, x_2, \ldots, x_n\}$

    X = \{ x_1, x_2, \ldots, x_n \}

24. ⠭⠂⠀⠠⠭⠆⠀⠤⠀⠠⠭⠰⠝⠀⠐

    $x_1, x_2, \ldots, x_n$

    x_1, x_2, \ldots, x_n

25. ⠠⠭⠀⠨⠅⠀StartSet⠭⠈⠆⠭satisfiescaligraphic ⠠⠏EndSet

    $X = \{x : x \text{ satisfies } \mathcal{P}\}$

    X = \{ x :x \text{ satisfies }{\mathcal P}\}

26. caligraphic ⠠⠏

    $\mathcal{P}$

    {\mathcal P}

27. E

    E

28. ⠠⠑ ⠨⠅ StartSet⠆ ⠠⠲ ⠠⠖ ⠄⠄ EndSetor ⠠⠑ ⠨⠅ StartSet⠭⠈⠒⠭isanevenintegerand⠭ ⠨⠂ ⠠⠴EndS

    $E = \{2, 4, 6, \ldots\}$ or $E = \{x : x \text{ is an even integer and } x > 0\}$

    `E = \{2, 4, 6, \ldots \} \quad \text{or} \quad E = \{ x : x \text{`
    `is an even integer and } x \gt 0 \}`

29. ⠆ ⠈⠑ ⠠⠑

    $2 \in E$

    `2 \in E`

30. ⠤⠒⠌⠈⠑ ⠠⠑

    $-3 \notin E$

    `-3 \notin E`

31. ⠤⠒

    -3

    `-3`

32. B

    `B`

33. ⠠⠁ ⠸⠐⠅ ⠠⠃

    $A \subset B$

    `A \subset B`

34. ⠠⠃ ⠸⠨⠂ ⠠⠁

    $B \supset A$

    `B \supset A`

35. StartSet⠲ ⠠⠢ ⠠⠦EndSet ⠸⠐⠅ StartSet⠆ ⠠⠒ ⠠⠲ ⠠⠢ ⠠⠖ ⠠⠶ ⠠⠦ ⠠⠔EndSet

    $\{4, 5, 8\} \subset \{2, 3, 4, 5, 6, 7, 8, 9\}$

    `\{4,5,8\} \subset \{2, 3, 4, 5, 6, 7, 8, 9 \}`

36. ⠈⠠⠝ ⠸⠐⠅ ⠈⠠⠵ ⠸⠐⠅ ⠈⠠⠟ ⠸⠐⠅ ⠈⠠⠗ ⠸⠐⠅ ⠈⠠⠉

    $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$

    `{\mathbb N} \subset {\mathbb Z} \subset {\mathbb Q} \subset {\mathbb`
    `R} \subset {\mathbb C}`

37. ⠠⠃ ⠸⠐⠅ ⠠⠁

    $B \subset A$

    `B \subset A`

38. ⠠⠃ ⠌⠨⠅ ⠠⠁

    $B \neq A$

    `B \neq A`

39. ⠠⠁ ⠌⠸⠐⠅ ⠠⠃

    $A \not\subset B$

    `A \notsubset B`

40. StartSet⠲ ⠠⠶ ⠠⠔EndSet⠌⠸⠐⠅ StartSet⠆ ⠠⠲ ⠠⠢ ⠠⠦ ⠠⠔EndSet

$\{4, 7, 9\} \not\subset \{2, 4, 5, 8, 9\}$

`\{4, 7, 9\} \notsubset \{2, 4, 5, 8, 9 \}`

41. ⠠⠁ ⠨⠅ ⠠⠃

$A = B$

`A = B`

42. ⠸⠴

$\emptyset$

`\emptyset`

43. ⠠⠁ ⠨⠬ ⠠⠃

$A \cup B$

`A \cup B`

44. ⠠⠁ ⠨⠬ ⠠⠃ ⠨⠅ StartSet⠭⠩⠭ ⠈⠑ ⠠⠁ or⠭ ⠈⠑ ⠠⠃ EndSet⠸⠆

$A \cup B = \{x : x \in A \text{ or } x \in B\};$

`A \cup B = \{x : x \in A \text{ or } x \in B \};`

45. ⠠⠁ ⠨⠩ ⠠⠃ ⠨⠅ StartSet⠭⠩⠭ ⠈⠑ ⠠⠁ and⠭ ⠈⠑ ⠠⠃ EndSet

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

`A \cap B = \{x : x \in A \text{ and } x \in B \}`

46. ⠠⠁ ⠨⠅ StartSet⠂ ⠠⠒ ⠠⠢EndSet

$A = \{1, 3, 5\}$

`A = \{1, 3, 5\}`

47. ⠠⠃ ⠨⠅ StartSet⠂ ⠠⠆ ⠠⠒ ⠠⠔EndSet

$B = \{1, 2, 3, 9\}$

`B = \{ 1, 2, 3, 9 \}`

48. ⠠⠁ ⠨⠬ ⠠⠃ ⠨⠅ StartSet⠂ ⠠⠆ ⠠⠒ ⠠⠢ ⠠⠔EndSetand ⠠⠁ ⠨⠩ ⠠⠃ ⠨⠅ StartSet⠂ ⠠⠒EndSet

$A \cup B = \{1, 2, 3, 5, 9\} \quad \text{and} \quad A \cap B = \{1, 3\}$

`A \cup B = \{1, 2, 3, 5, 9 \} \quad \text{and} \quad A \cap B =`
`\{ 1, 3 \}`

49. ⠐⠄⠸⠩⠅⠅ ⠉⠤⠄ ⠟⠌ ⠨⠅ ⠐⠹⠻⠄⠈ ⠅⠌⠐ ⠨⠅ ⠠⠁⠐ ⠨⠬⠄ ⠨⠬⠠⠁ ⠅⠟

$\bigcup_{i=1}^{n} A_i = A_1 \cup \ldots \cup A_n$

`\bigcup_{i = 1}^{n} A_{i} = A_{1} \cup \ldots \cup A_n`

50. ⠐⠄⠸⠩⠅⠅ ⠉⠅⠄ ⠟⠌ ⠨⠅ ⠐⠹⠻⠄⠈ ⠅⠌⠐ ⠨⠅ ⠠⠁⠐ ⠨⠩⠄ ⠨⠩⠠⠁ ⠅⠟

$\bigcap_{i=1}^{n} A_i = A_1 \cap \ldots \cap A_n$

`\bigcap_{i = 1}^{n} A_{i} = A_{1} \cap \ldots \cap A_n`

51. ⠠⠁⠂ ⠤ ⠠⠠⠈ ⠅⠟⠂

$A_1, \ldots, A_n$

`A_1, \ldots, A_n`

52. O

`O`

53. ⠠⠁ ⠨⠩⠠⠃ ⠨⠅ ⠸⠴

$A \cap B = \emptyset$

```
A \cap B = \emptyset
```

54. U

```
U
```

55. ⠠⠁ ⠸⠐⠅ ⠠⠥

$A \subset U$

```
A \subset U
```

56. ⠠⠁⠄

A'

```
A'
```

57. ⠠⠁⠄ ⠨⠅ StartSet⠭⠸⠒⠭ ⠈⠑ ⠠⠥and⠭⠌ ⠈⠑ ⠠⠁ EndSet

$A' = \{x : x \in U \text{ and } x \notin A\}$

```
A' = \{ x : x \in U \text{ and } x \notin A \}
```

58. ⠠⠁⠄⡳⠭⠆⠆⠂⠖⠄⠠⠃ ⠨⠅ ⠠⠁ ⠨⠩⠠⠃⠄⠂ ⠨⠅ StartSet⠭⠸⠒⠭ ⠈⠑ ⠠⠁ and⠭⠌ ⠈⠑ ⠠⠃ EndSet

$A \setminus B = A \cap B' = \{x : x \in A \text{ and } x \notin B\}$

```
A \setminus B = A \cap B' = \{ x : x \in A \text{ and } x \notin
B \}
```

59. ⠈⠠⠗

$\mathbb{R}$

```
{\mathbb R}
```

60. ⠠⠁ ⠨⠅ StartSet⠭ ⠈⠑ ⠈⠠⠗⠸⠒⠴ ⠐⠅ ⠭ ⠐⠅⠱ ⠒EndSetand ⠠⠃ ⠨⠅ StartSet⠭ ⠈⠑ ⠈⠠⠗⠸⠒⠆ ⠐⠅⠱

$A = \{x \in \mathbb{R} : 0 < x \leq 3\} \quad \text{and} \quad B = \{x \in \mathbb{R} : 2 \leq x < 4\}$

```
A = \{ x \in {\mathbb R} : 0 \lt x \leq 3 \} \quad \text{and} \quad
B = \{ x \in {\mathbb R} : 2 \leq x \lt 4 \}
```

61. C

```
C
```

62. ⠠⠁ ⠨⠬⠠⠁ ⠨⠅ ⠠⠁

$A \cup A = A$

```
A \cup A = A
```

63. ⠠⠁ ⠨⠩⠠⠁ ⠨⠅ ⠠⠁

$A \cap A = A$

```
A \cap A = A
```

64. ⠠⠁⠄⡳⠭⠆⠆⠂⠖⠄⠠⠁ ⠨⠅ ⠸⠴

$A \setminus A = \emptyset$

```
A \setminus A = \emptyset
```

65. ⠠⠁ ⠨⠬⠸⠴ ⠨⠅ ⠠⠁

$A \cup \emptyset = A$

```
A \cup \emptyset = A
```

66. ⠠⠁⠨⠩⠸⠴⠀⠨⠅⠀⠸⠴

$A \cap \emptyset = \emptyset$

```
A \cap \emptyset = \emptyset
```

67. ⠠⠁⠨⠬⠷⠠⠃⠨⠬⠠⠉⠾⠀⠨⠅⠀⠷⠠⠁⠨⠬⠠⠃⠾⠨⠬⠠⠉

$A \cup (B \cup C) = (A \cup B) \cup C$

```
A \cup (B \cup C) = (A \cup B) \cup C
```

68. ⠠⠁⠨⠩⠷⠠⠃⠨⠩⠠⠉⠾⠀⠨⠅⠀⠷⠠⠁⠨⠩⠠⠃⠾⠨⠩⠠⠉

$A \cap (B \cap C) = (A \cap B) \cap C$

```
A \cap (B \cap C) = (A \cap B) \cap C
```

69. ⠠⠁⠨⠬⠠⠃⠀⠨⠅⠀⠠⠃⠨⠬⠠⠁

$A \cup B = B \cup A$

```
A \cup B = B \cup A
```

70. ⠠⠁⠨⠩⠠⠃⠀⠨⠅⠀⠠⠃⠨⠩⠠⠁

$A \cap B = B \cap A$

```
A \cap B = B \cap A
```

71. ⠠⠁⠨⠬⠷⠠⠃⠨⠩⠠⠉⠾⠀⠨⠅⠀⠷⠠⠁⠨⠬⠠⠃⠾⠨⠩⠷⠠⠁⠨⠬⠠⠉⠾

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

```
A \cup (B \cap C) = (A \cup B) \cap (A \cup C)
```

72. ⠠⠁⠨⠩⠷⠠⠃⠨⠬⠠⠉⠾⠀⠨⠅⠀⠷⠠⠁⠨⠩⠠⠃⠾⠨⠬⠷⠠⠁⠨⠩⠠⠉⠾

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

```
A \cap (B \cup C) = (A \cap B) \cup (A \cap C)
```

73. ⠠⠁⠄⡷⠭⠆⠆⠂⠖⠄⠠⠁⠀⠨⠅⠀⠠⠁⠨⠩⠠⠁⠘⠄⠐⠀⠨⠅⠀⠸⠴

$A \setminus A = A \cap A' = \emptyset$

```
A \setminus A = A \cap A' = \emptyset
```

74. ⠷⠠⠁⠨⠬⠠⠃⠾⠄⠀⠨⠅⠀⠠⠁⠄⠨⠩⠠⠃⠄

$(A \cup B)' = A' \cap B'$

```
(A \cup B)' = A' \cap B'
```

75. ⠷⠠⠁⠨⠩⠠⠃⠾⠄⠀⠨⠅⠀⠠⠁⠄⠨⠬⠠⠃⠄

$(A \cap B)' = A' \cup B'$

```
(A \cap B)' = A' \cup B'
```

76. ⠠⠁⠨⠬⠠⠃⠀⠨⠅⠀⠸⠴

$A \cup B = \emptyset$

```
A \cup B = \emptyset
```

77. ⠷⠠⠁⠨⠬⠠⠃⠾⠄⠸⠐⠅⠠⠁⠄⠨⠩⠠⠃⠄

$(A \cup B)' \subset A' \cap B'$

```
(A \cup B)' \subset A' \cap B'
```

78. ⠷⠠⠁⠨⠬⠠⠃⠾⠄⠸⠨⠂⠠⠁⠄⠨⠩⠠⠃⠄

$(A \cup B)' \supset A' \cap B'$

```
(A \cup B)' \supset A' \cap B'
```

79. ⠭ ⠈⠑ ⠷⠠⠁⠨⠬⠠⠃⠾⠄

$x \in (A \cup B)'$

x \in (A \cup B)'

80. ⠭⠌⠈⠑ ⠠⠁⠨⠬⠠⠃

$x \notin A \cup B$

x \notin A \cup B

81. ⠭ ⠈⠑ ⠠⠁⠄

$x \in A'$

x \in A'

82. ⠭ ⠈⠑ ⠠⠃⠄

$x \in B'$

x \in B'

83. ⠭ ⠈⠑ ⠠⠁⠄⠨⠩⠠⠃⠄

$x \in A' \cap B'$

x \in A' \cap B'

84. ⠭⠌⠈⠑ ⠠⠁

$x \notin A$

x \notin A

85. ⠭⠌⠈⠑ ⠠⠃

$x \notin B$

x \notin B

86. ⠷⠠⠁⠠⡷⠭⠆⠆⠂⠖⠠⠠⠃⠾⠨⠩⠷⠠⠃⠠⡷⠭⠆⠆⠂⠖⠠⠠⠁⠾ ⠨⠅ ⠸⠴

$(A \setminus B) \cap (B \setminus A) = \emptyset$

(A \setminus B) \cap (B \setminus A) = \emptyset

87. ⠠⠁⠈⠡⠠⠃

$A \times B$

A \times B

88. ⠠⠁⠈⠡⠠⠃ ⠨⠅ StartSet⠷⠁⠠⠃⠾⠈⠆⠁ ⠈⠑⠠⠁and⠃ ⠈⠑⠠⠃EndSet

$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$

A \times B = \{ (a,b) : a \in A \text{ and } b \in B \}

89. ⠠⠁ ⠨⠅ StartSet⠭⠠⠽EndSet

$A = \{x, y\}$

A = \{ x, y \}

90. ⠠⠃ ⠨⠅ StartSet⠂⠠⠆⠠⠒EndSet

$B = \{1, 2, 3\}$

B = \{ 1, 2, 3 \}

91. ⠠⠉ ⠨⠅ ⠸⠴

$C = \emptyset$

C = \emptyset

92. StartSet⠷⠭ ⠠⠂ ⠾ ⠠⠷⠭ ⠠⠆ ⠾ ⠠⠷⠭ ⠠⠒⠾ ⠠⠷⠽ ⠠⠂ ⠾ ⠠⠷⠽ ⠠⠆ ⠾ ⠠⠷⠽ ⠠⠒⠾EndSet

$\{(x,1),(x,2),(x,3),(y,1),(y,2),(y,3)\}$

\{ (x, 1), (x, 2), (x, 3), (y, 1), (y, 2), (y, 3) \}

93. ⠠⠁⠈⠡⠠⠉ ⠨⠅ ⠸⠴

$A \times C = \emptyset$

A \times C = \emptyset

94. n

n

95. ⠠⠁⠂⠈⠡⠄⠄⠄⠈⠡⠠⠁⠰⠝ ⠨⠅ ⠸⠷⠷⠁⠂⠠ ⠄⠄ ⠠⠁⠰⠝⠾⠸⠒⠁⠰⠊ ⠈⠑ ⠠⠁⠰⠊ ⠐for⠐⠊ ⠨⠅ ⠂⠠ ⠄⠄ ⠠⠝⠸⠾

$A_1 \times \cdots \times A_n = \{(a_1, \ldots, a_n) : a_i \in A_i \text{ for } i = 1, \ldots, n\}$

A_1 \times \cdots \times A_n = \{ (a_1, \ldots, a_n): a_i \in A_i
\text{ for } i = 1, \ldots, n \}

96. ⠠⠁ ⠨⠅ ⠠⠁⠂ ⠨⠅ ⠠⠁⠆ ⠨⠅ ⠄⠄⠄ ⠨⠅ ⠠⠁⠰⠝

$A = A_1 = A_2 = \cdots = A_n$

A = A_1 = A_2 = \cdots = A_n

97. ⠠⠁ ⠘⠝

A^n

A^n

98. ⠠⠁⠈⠡⠄⠄⠄⠈⠡⠠⠁

$A \times \cdots \times A$

A \times \cdots \times A

99. ⠈⠠⠗ ⠘⠒

$\mathbb{R}^3$

{\mathbb R}^3

100. ⠋ ⠸⠐⠅ ⠠⠁⠈⠡⠠⠃

$f \subset A \times B$

f \subset A \times B

101. ⠷⠁ ⠠⠃ ⠾ ⠈⠑⠋

$(a,b) \in f$

(a, b) \in f

102. ⠃ ⠈⠑ ⠠⠃

$b \in B$

b \in B

103. f

f

104. ⠋⠸⠒⠠⠁ ⠫⠕ ⠠⠃

$f : A \to B$

f:A \rightarrow B

105. ⠠⠁ ⠐ ⠫⠕ ⠣⠋⠻⠠⠃

$A \stackrel{f}{\rightarrow} B$

`A \stackrel{f}{\rightarrow} B`

106. ⠷⠁ ⠠⠃⠾ ⠈⠑ ⠠⠁ ⠡ ⠠⠃

$(a,b) \in A \times B$

`(a,b) \in A \times B`

107. ⠋⠷⠁⠾ ⠨⠅ ⠃

$f(a) = b$

`f(a) = b`

108. ⠋⠱⠁ ⠫⠳⠒⠒⠕ ⠃

$f : a \mapsto b$

`f : a \mapsto b`

109. ⠋⠷⠠⠁⠾ ⠨⠅ StartSet⠋⠷⠁⠾⠱⠁ ⠈⠑ ⠠⠁ EndSet⠸⠐⠅ ⠠⠃

$f(A) = \{ f(a) : a \in A \} \subset B$

`f(A) = \{ f(a) : a \in A \} \subset B`

110. ⠠⠁ ⠨⠅ StartSet⠂ ⠠⠆ ⠠⠒EndSet

$A = \{1, 2, 3\}$

`A = \{1, 2, 3 \}`

111. ⠠⠃ ⠨⠅ StartSet⠁ ⠠⠃ ⠠⠉EndSet

$B = \{a, b, c\}$

`B = \{a, b, c \}`

112. g

`g`

113. ⠂ ⠈⠑ ⠠⠁

$1 \in A$

`1 \in A`

114. ⠛⠷⠂⠾ ⠨⠅ ⠁

$g(1) = a$

`g(1) = a`

115. ⠛⠷⠂⠾ ⠨⠅ ⠃

$g(1) = b$

`g(1) = b`

116. ⠋⠱⠠⠁ ⠫⠕ ⠠⠃

$f : A \rightarrow B$

`f : A \rightarrow B`

117. ⠋⠱⠈⠠⠗ ⠫⠕ ⠈⠠⠗

$f : {\mathbb R} \rightarrow {\mathbb R}$

`f: {\mathbb R} \rightarrow {\mathbb R}`

118. ⠋⠷⠭⠾ ⠶ ⠭⠘⠒

$f(x) = x^3$

f(x) = x^3

119. ⠋⠱⠭ ⠫⠳⠒⠒⠕ ⠭⠘⠒

$f : x \mapsto x^3$

f:x \mapsto x^3

120. ⠋⠱⠈⠠⠟ ⠫⠕ ⠈⠠⠵

$f : \mathbb{Q} \to \mathbb{Z}$

f : {\mathbb Q} \rightarrow {\mathbb Z}

121. ⠋⠷⠏⠌⠟⠾ ⠶ ⠏

$f(p/q) = p$

f(p/q) = p

122. ⠂⠌⠆ ⠶ ⠆⠌⠲

$1/2 = 2/4$

1/2 = 2/4

123. ⠋⠷⠂⠌⠆⠾ ⠶ ⠂

$f(1/2) = 1$

f(1/2) = 1

124. ⠼⠆

2

2

125. ⠋⠷⠠⠁⠾ ⠶ ⠠⠃

$f(A) = B$

f(A) = B

126. ⠁⠂ ⠌⠶ ⠁⠆

$a_1 \neq a_2$

a_1 \neq a_2

127. ⠋⠷⠁⠂⠾ ⠌⠶ ⠋⠷⠁⠆⠾

$f(a_1) \neq f(a_2)$

f(a_1) \neq f(a_2)

128. ⠋⠷⠁⠂⠾ ⠶ ⠋⠷⠁⠆⠾

$f(a_1) = f(a_2)$

f(a_1) = f(a_2)

129. ⠁⠂ ⠶ ⠁⠆

$a_1 = a_2$

a_1 = a_2

130. ⠋⠱⠈⠠⠵ ⠫⠕ ⠈⠠⠟

$f : \mathbb{Z} \to \mathbb{Q}$

f:{\mathbb Z} \rightarrow {\mathbb Q}

131. ⠋⠷⠝⠾⠀⠨⠅⠀⠝⠌⠂

$f(n) = n/1$

f(n) = n/1

132. ⠛⠱⠀⠈⠠⠟⠀⠫⠕⠀⠈⠠⠵

$g : \mathbb{Q} \rightarrow \mathbb{Z}$

g : {\mathbb Q} \rightarrow {\mathbb Z}

133. ⠛⠷⠏⠌⠟⠾⠀⠨⠅⠀⠏

$g(p/q) = p$

g(p/q) = p

134. ⠏⠌⠟

p/q

p/q

135. ⠛⠱⠀⠠⠃⠀⠫⠕⠀⠠⠉

$g : B \rightarrow C$

g : B \rightarrow C

136. ⠷⠛⠘⠨⠡⠋⠾⠷⠭⠾⠀⠨⠅⠀⠛⠷⠋⠷⠭⠾⠾

$(g \circ f)(x) = g(f(x))$

(g \circ f)(x) = g(f(x))

137. ⠋⠱⠀⠠⠁⠀⠫⠕⠀⠠⠃

$f : A \rightarrow B$

f: A \rightarrow B

138. ⠛⠱⠀⠠⠃⠀⠫⠕⠀⠠⠉

$g : B \rightarrow C$

g: B \rightarrow C

139. ⠛⠘⠨⠡⠋⠱⠠⠁⠀⠫⠕⠀⠠⠉

$g \circ f : A \rightarrow C$

g \circ f: A \rightarrow C

140. ⠋⠷⠭⠾⠀⠨⠅⠀⠭⠘⠆

$f(x) = x^2$

f(x) = x^2

141. ⠛⠷⠭⠾⠀⠨⠅⠀⠆⠭⠬⠢

$g(x) = 2x + 5$

g(x) = 2x + 5

142. ⠷⠋⠘⠨⠡⠛⠾⠷⠭⠾⠀⠨⠅⠀⠋⠷⠛⠷⠭⠾⠾⠀⠨⠅⠀⠷⠆⠭⠬⠢⠾⠘⠆⠐⠀⠨⠅⠀⠲⠭⠘⠆⠐⠬20⠭⠬25

$(f \circ g)(x) = f(g(x)) = (2x + 5)^2 = 4x^2 + 20x + 25$

(f \circ g)(x) = f(g(x)) = (2x + 5)^2 = 4x^2 + 20x + 25

143. ⠷⠛⠘⠨⠡⠋⠾⠷⠭⠾⠀⠨⠅⠀⠛⠷⠋⠷⠭⠾⠾⠀⠨⠅⠀⠆⠭⠘⠆⠐⠬⠢

$(g \circ f)(x) = g(f(x)) = 2x^2 + 5$

(g \circ f)(x) = g(f(x)) = 2x^2 + 5

144. ⠋⠘⠨⠡⠛ ⠌⠨⠅ ⠛⠘⠨⠡⠋

$f \circ g \neq g \circ f$

f \circ g \neq g \circ f

145. ⠋⠘⠨⠡⠛ ⠨⠅ ⠛⠘⠨⠡⠋

$f \circ g = g \circ f$

f \circ g= g \circ f

146. ⠛⠷⠭⠾ ⠨⠅ ⠣⠒⠜⠭⠻

$g(x) = \sqrt[3]{x}$

g(x) = \sqrt[3]{x}

147. ⠷⠋⠘⠨⠡⠛⠾⠷⠭⠾ ⠨⠅ ⠋⠷⠛⠷⠭⠾⠾ ⠨⠅ ⠋⠷⠣⠒⠜⠭⠻⠾ ⠨⠅ ⠷⠣⠒⠜⠭⠻⠾⠘⠒⠐ ⠨⠅ ⠭

$(f \circ g)(x) = f(g(x)) = f(\sqrt[3]{x}\,) = (\sqrt[3]{x}\,)^3 = x$

(f \circ g)(x) = f(g(x)) = f(\sqrt[3]{x}\,) = (\sqrt[3]{x}\,
)^3 = x

148. ⠷⠛⠘⠨⠡⠋⠾⠷⠭⠾ ⠨⠅ ⠛⠷⠋⠷⠭⠾⠾ ⠨⠅ ⠛⠷⠭⠘⠒⠐⠾ ⠨⠅ ⠣⠒⠜⠭⠘⠒⠐⠻ ⠨⠅ ⠭

$(g \circ f)(x) = g(f(x)) = g(x^3) = \sqrt[3]{x^3} = x$

(g \circ f)(x) = g(f(x)) = g(x^3) = \sqrt[3]{ x^3} = x

149. ⠆⠈⠡⠆

2×2

2 \times 2

150. ⠠⠁ ⠨⠅ ⠠⠷⠁ ⠃⠠⠾ ⠠⠷⠉ ⠙⠠⠾

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}

151. ⠠⠞⠰⠠⠁⠐⠱⠈⠠⠗⠘⠆ ⠐ ⠫⠕ ⠈⠠⠗⠘⠆

$T_A : \mathbb{R}^2 \rightarrow \mathbb{R}^2$

T_A : {\mathbb R}^2 \rightarrow {\mathbb R}^2

152. ⠠⠞⠰⠠⠁⠐⠷⠭⠠⠽⠾ ⠨⠅ ⠷⠁⠭⠬⠃⠽⠠⠉⠭⠬⠙⠽⠾

$T_A(x, y) = (ax + by, cx + dy)$

T_A (x,y) = (ax + by, cx +dy)

153. ⠷⠭⠠⠽⠾

(x, y)

(x,y)

154. ⠈⠠⠗⠘⠆

$\mathbb{R}^2$

{\mathbb R}^2

155. ⠠⠷⠁ ⠃⠠⠾ ⠠⠷⠉ ⠙⠠⠾⠭⠽ ⠨⠅ ⠁⠭⠬⠃⠽⠉⠭⠬⠙⠽

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$$

\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\
y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx +dy \end{pmatrix}

156. ⠈⠠⠗⠘⠝

$\mathbb{R}^n$

`{\mathbb R}^n`

157. ⠈⠠⠗⠘⠍

$\mathbb{R}^m$

`{\mathbb R}^m`

158. ⠠⠎ ⠨⠅ StartSet⠂⠠⠆⠠⠒EndSet

$S = \{1, 2, 3\}$

`S = \{ 1,2,3 \}`

159. ⠨⠏⠱⠠⠎ ⠫⠕ ⠠⠎

$\pi : S \rightarrow S$

`\pi :S\rightarrow S`

160. ⠨⠏⠷⠂⠾ ⠨⠅ ⠆⠠⠨⠏⠷⠆⠾ ⠨⠅ ⠂⠠⠨⠏⠷⠒⠾ ⠨⠅ ⠒

$\pi(1) = 2, \qquad \pi(2) = 1, \qquad \pi(3) = 3$

`\pi( 1 ) = 2, \qquad \pi( 2 ) = 1, \qquad \pi( 3 ) = 3`

161. ⠨⠏

π

`\pi`

162. ⠠⠷⠼⠂ ⠼⠆ ⠼⠒⠠⠾⠠⠷⠨⠏⠷⠂⠾ ⠨⠏⠷⠆⠾ ⠨⠏⠷⠒⠾⠠⠾ ⠨⠅ ⠠⠷⠼⠂ ⠼⠆ ⠼⠒⠠⠾
⠠⠷⠼⠆ ⠼⠂ ⠼⠒⠠⠾

$$\begin{pmatrix} 1 & 2 & 3 \\ \pi(1) & \pi(2) & \pi(3) \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$$

```
\begin{pmatrix} 1 & 2 & 3 \\ \pi(1) & \pi(2) & \pi(3) \end{pmatrix}
= \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}
```

163. S

`S`

164. ⠨⠏⠱⠠⠎ ⠫⠕ ⠠⠎

$\pi : S \rightarrow S$

`\pi : S \rightarrow S`

165. ⠓⠱⠠⠉ ⠫⠕ ⠠⠙

$h : C \rightarrow D$

`h : C \rightarrow D`

166. ⠷⠓⠘⠨⠡⠛⠾⠘⠨⠡⠋ ⠨⠅ ⠓⠘⠨⠡⠷⠛⠘⠨⠡⠋⠾

$(h \circ g) \circ f = h \circ (g \circ f)$

`(h \circ g) \circ f = h \circ (g \circ f)`

167. ⠛⠘⠨⠡⠋

$g \circ f$

`g \circ f`

168. ⠓⠘⠨⠡⠷⠛⠘⠨⠡⠋⠾ ⠨⠅ ⠷⠓⠘⠨⠡⠛⠾⠘⠨⠡⠋

$h \circ (g \circ f) = (h \circ g) \circ f$

`h \circ (g \circ f) = (h \circ g) \circ f`

169. ⠉ ⠈⠑ ⠠⠉

$c \in C$

`c \in C`

170. ⠷⠛⠘⠨⠡⠋⠾⠷⠁⠾ ⠨⠅ ⠛⠷⠋⠷⠁⠾⠾ ⠨⠅ ⠉

$(g \circ f)(a) = g(f(a)) = c$

`(g \circ f)(a) = g(f(a)) = c`

171. ⠛⠷⠃⠾ ⠨⠅ ⠉

$g(b) = c$

`g(b) = c`

172. ⠷⠛⠘⠨⠡⠋⠾⠷⠁⠾ ⠨⠅ ⠛⠷⠋⠷⠁⠾⠾ ⠨⠅ ⠛⠷⠃⠾ ⠨⠅ ⠉

$(g \circ f)(a) = g(f(a)) = g(b) = c$

`(g \circ f)(a) = g(f(a)) = g(b) = c`

173. ⠊⠙⠰⠠⠎

id_S

`id_S`

174. ⠊⠙

id

`id`

175. ⠊⠙⠷⠎⠾ ⠨⠅ ⠎

$id(s) = s$

`id(s) = s`

176. ⠎ ⠈⠑ ⠠⠎

$s \in S$

`s \in S`

177. ⠛⠱⠠⠃ ⠫⠕ ⠠⠁

$g : B \rightarrow A$

`g: B \rightarrow A`

178. ⠛⠘⠨⠡⠋ ⠨⠅ ⠊⠙⠰⠠⠁

$g \circ f = id_A$

`g \circ f = id_A`

179. ⠋⠘⠨⠡⠛ ⠨⠅ ⠊⠙⠰⠠⠃

$f \circ g = id_B$

`f \circ g = id_B`

180. ⠋⠘⠤⠂

f^{-1}

`f^{-1}`

181. ⠋⠘⠤⠂⠐⠷⠭⠾ ⠨⠅ ⠣⠒⠜⠭⠻

$f^{-1}(x) = \sqrt[3]{x}$

f^{-1}(x) = \sqrt[3]{x}

182. ⠋⠷⠭⠾ ⠨⠅ ⠇⠝ ⠭

$f(x) = \ln x$

f(x) = \ln x

183. ⠋⠘⠤⠂⠐⠷⠭⠾ ⠨⠅ ⠑⠘⠭

$f^{-1}(x) = e^x$

f^{-1}(x) = e^x

184. ⠋⠷⠋⠘⠤⠂⠐⠷⠭⠾⠾ ⠨⠅ ⠋⠷⠑⠘⠭⠐⠾ ⠨⠅ ⠇⠝ ⠑⠘⠭⠐ ⠨⠅ ⠭

$f(f^{-1}(x)) = f(e^x) = \ln e^x = x$

f(f^{-1}(x)) = f(e^x) = \ln e^x = x

185. ⠋⠘⠤⠂⠐⠷⠋⠷⠭⠾⠾ ⠨⠅ ⠋⠘⠤⠂⠐⠷⠇⠝ ⠭⠾ ⠨⠅ ⠑⠘⠇⠝ ⠭⠐ ⠨⠅ ⠭

$f^{-1}(f(x)) = f^{-1}(\ln x) = e^{\ln x} = x$

f^{-1}(f(x)) = f^{-1}(\ln x) = e^{\ln x} = x

186. ⠠⠁ ⠨⠅ ⠠⠷⠼⠒ ⠼⠂⠠⠾ ⠠⠷⠼⠢ ⠼⠆⠠⠾

$A = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$

A = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}

187. ⠠⠞⠰⠠⠁⠐⠷⠭⠠⠽⠾ ⠨⠅ ⠷⠒⠭⠬⠽⠠⠢⠭⠬⠆⠽⠾

$T_A(x, y) = (3x + y, 5x + 2y)$

T_A (x,y) = (3x + y, 5x + 2y)

188. ⠠⠞⠰⠠⠁

T_A

T_A

189. ⠠⠞⠰⠠⠁⠘⠤⠂⠐ ⠨⠅ ⠠⠞⠰⠠⠁⠰⠘⠤⠂

$T_A^{-1} = T_{A^{-1}}$

T_A^{-1} = T_{A^{-1}}

190. ⠠⠁⠘⠤⠂⠐ ⠨⠅ ⠠⠷⠼⠆ ⠤⠂⠠⠾ ⠠⠷⠤⠢ ⠼⠒⠠⠾⠷

$A^{-1} = \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix};$

A^{-1} = \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix};

191. ⠠⠞⠰⠠⠁⠘⠤⠂⠐⠷⠭⠠⠽⠾ ⠨⠅ ⠷⠆⠭⠤⠽⠠⠤⠢⠭⠬⠒⠽⠾

$T_A^{-1}(x, y) = (2x - y, -5x + 3y)$

T_A^{-1} (x,y) = (2x - y, -5x + 3y)

192. ⠠⠞⠰⠠⠁⠘⠤⠂⠐⠘⠨⠡⠠⠞⠰⠠⠁⠐⠷⠭⠠⠽⠾ ⠨⠅ ⠠⠞⠰⠠⠁⠐⠘⠨⠡⠠⠞⠰⠠⠁⠘⠤⠂⠐⠷⠭⠠⠽⠾ ⠨⠅ ⠷⠭⠠⠽⠾

$T_A^{-1} \circ T_A(x, y) = T_A \circ T_A^{-1}(x, y) = (x, y)$

T^{-1}_A \circ T_A (x,y) = T_A \circ T_A^{-1} (x,y) = (x,y)

193. ⠠⠞⠰⠠⠃⠐⠷⠭⠠⠽⠾ ⠨⠅ ⠷⠒⠭⠠⠴⠾

$T_B(x,y) = (3x,0)$

`T_B (x,y) = (3x , 0 )`

194. ⠠⠃ ⠨⠅ ⠠⠷⠼⠒ ⠼⠴⠠⠾ ⠠⠷⠼⠴ ⠼⠴⠠⠾

$B = \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix}$

`B = \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix}`

195. ⠠⠞⠰⠠⠃⠘⠤⠂⠐⠷⠭⠠⠽⠾ ⠨⠅ ⠷⠁⠭⠬⠃⠽⠠⠉⠭⠬⠙⠽⠾

$T_B^{-1}(x,y) = (ax+by, cx+dy)$

`T_B^{-1} (x,y) = (ax + by, cx +dy)`

196. ⠷⠭⠠⠽⠾ ⠨⠅ ⠠⠞⠰⠠⠃⠐⠘⠨⠡⠠⠞⠰⠠⠃⠘⠤⠂⠐⠷⠭⠠⠽⠾ ⠨⠅ ⠷⠒⠁⠭⠬⠒⠃⠽⠠⠴⠾

$(x,y) = T_B \circ T_B^{-1}(x,y) = (3ax+3by,0)$

`(x,y) = T_B \circ T_B^{-1} (x,y) = (3ax + 3by, 0)`

197. y

`y`

198. ⠼⠴

0

`0`

199. ⠨⠏ ⠨⠅ ⠠⠷⠼⠂ ⠼⠆ ⠼⠒⠠⠾ ⠠⠷⠼⠆ ⠼⠒ ⠼⠂⠠⠾

$\pi = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$

`\pi = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}`

200. ⠨⠏⠘⠤⠂⠐ ⠨⠅ ⠠⠷⠼⠂ ⠼⠆ ⠼⠒⠠⠾ ⠠⠷⠼⠒ ⠼⠂ ⠼⠆⠠⠾

$\pi^{-1} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$

`\pi^{-1} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}`

201. ⠛⠷⠋⠷⠁⠾⠾ ⠨⠅ ⠁

$g(f(a)) = a$

`g(f(a)) = a`

202. ⠁⠂⠠⠁⠆ ⠈⠑⠠⠁

$a_1, a_2 \in A$

`a_1, a_2 \in A`

203. ⠁⠂ ⠨⠅ ⠛⠷⠋⠷⠁⠂⠾⠾ ⠨⠅ ⠛⠷⠋⠷⠁⠆⠾⠾ ⠨⠅ ⠁⠆

$a_1 = g(f(a_1)) = g(f(a_2)) = a_2$

`a_1 = g(f(a_1)) = g(f(a_2)) = a_2`

204. ⠋⠷⠛⠷⠃⠾⠾ ⠨⠅ ⠃

$f(g(b)) = b$

`f(g(b)) = b`

205. ⠛⠷⠃⠾ ⠈⠑ ⠠⠁

$g(b) \in A$

`g(b) \in A`

206. ⠁ ⠨⠅ ⠛⠷⠃⠾

$a = g(b)$

`a = g(b)`

207. ⠛⠷⠃⠾ ⠨⠅ ⠁

$g(b) = a$

`g(b) = a`

208. ⠠⠗ ⠸⠐⠅ ⠠⠭⠈⠡⠠⠭

$R \subset X \times X$

`R \subset X \times X`

209. ⠷⠭⠠⠭⠾ ⠈⠑ ⠠⠗

$(x, x) \in R$

`(x, x) \in R`

210. ⠭ ⠈⠑ ⠠⠭

$x \in X$

`x \in X`

211. ⠷⠭⠠⠽⠾ ⠈⠑ ⠠⠗

$(x, y) \in R$

`(x, y) \in R`

212. ⠷⠽⠠⠭⠾ ⠈⠑ ⠠⠗

$(y, x) \in R$

`(y, x) \in R`

213. ⠷⠭⠠⠽⠾

(x, y)

`(x, y)`

214. ⠷⠽⠠⠵⠾ ⠈⠑ ⠠⠗

$(y, z) \in R$

`(y, z) \in R`

215. ⠷⠭⠠⠵⠾ ⠈⠑ ⠠⠗

$(x, z) \in R$

`(x, z) \in R`

216. R

`R`

217. ⠭⠄⡷⠭⠆⠆⠒⠉⠄⠽

$x \sim y$

`x \sim y`

218. ⠀⠶

$=$

`=`

219. ⠿

$\equiv$

`\equiv`

220. ⠀⠉⠆⠉⠆

$\cong$

`\cong`

221. ⠏⠄⠌⠟⠄⡳⠭⠆⠆⠒⠉⠄⠗⠄⠌⠎

$p/q \sim r/s$

`p/q \sim r/s`

222. ⠏⠎ ⠶ ⠟⠗

$ps = qr$

`ps = qr`

223. ⠄⡳⠭⠆⠆⠒⠉⠄

$\sim$

`\sim`

224. ⠗⠄⠌⠎⠄⡳⠭⠆⠆⠒⠉⠄⠞⠄⠌⠥

$r/s \sim t/u$

`r/s \sim t/u`

225. u

`u`

226. ⠗⠥ ⠶ ⠎⠞

$ru = st$

`ru = st`

227. ⠏⠎⠥ ⠶ ⠟⠗⠥ ⠶ ⠟⠎⠞

$psu = qru = qst$

`psu = qru = qst`

228. ⠎ ⠌⠶ ⠴

$s \neq 0$

`s \neq 0`

229. ⠏⠥ ⠶ ⠟⠞

$pu = qt$

`pu = qt`

230. ⠏⠄⠌⠟⠄⡳⠭⠆⠆⠒⠉⠄⠞⠄⠌⠥

$p/q \sim t/u$

`p/q \sim t/u`

231. ⠋⠷⠭⠾⠄⡷⠭⠆⠆⠒⠉⠄⠛⠷⠭⠾

$f(x) \sim g(x)$

f(x) \sim g(x)

232. ⠋⠄⠷⠭⠾ ⠶ ⠛⠄⠷⠭⠾

$f'(x) = g'(x)$

f'(x) = g'(x)

233. ⠛⠷⠭⠾⠄⡷⠭⠆⠆⠒⠉⠄⠓⠷⠭⠾

$g(x) \sim h(x)$

g(x) \sim h(x)

234. ⠋⠷⠭⠾⠤⠛⠷⠭⠾ ⠶ ⠉⠂

$f(x) - g(x) = c_1$

f(x) - g(x) = c_1

235. ⠛⠷⠭⠾⠤⠓⠷⠭⠾ ⠶ ⠉⠆

$g(x) - h(x) = c_2$

g(x)- h(x) = c_2

236. ⠉⠂

c_1

c_1

237. ⠉⠆

c_2

c_2

238. ⠋⠷⠭⠾⠤⠓⠷⠭⠾ ⠶ ⠷⠋⠷⠭⠾⠤⠛⠷⠭⠾⠾⠬⠷⠛⠷⠭⠾⠤⠓⠷⠭⠾⠾ ⠶ ⠉⠂⠬⠉⠆

$f(x) - h(x) = (f(x) - g(x)) + (g(x) - h(x)) = c_1 + c_2$

f(x) - h(x) = (f(x) - g(x)) + (g(x)- h(x)) = c_1 + c_2

239. ⠋⠄⠷⠭⠾⠤⠓⠄⠷⠭⠾ ⠶ ⠴

$f'(x) - h'(x) = 0$

f'(x) - h'(x) = 0

240. ⠋⠷⠭⠾⠄⡷⠭⠆⠆⠒⠉⠄⠓⠷⠭⠾

$f(x) \sim h(x)$

f(x) \sim h(x)

241. ⠷⠭⠂⠠⠽⠂⠾

(x_1, y_1)

(x_1, y_1)

242. ⠷⠭⠆⠠⠽⠆⠾

(x_2, y_2)

(x_2, y_2)

243. ⠷⠭⠂⠠⠽⠂⠾⠄⡷⠭⠆⠆⠒⠉⠄⠷⠭⠆⠠⠽⠆⠾

$(x_1, y_1) \sim (x_2, y_2)$

(x_1, y_1) \sim (x_2, y_2)

244. ⠭⠂⠘⠆⠐⠬⠽⠂⠘⠆⠐⠀⠨⠅⠀⠭⠆⠘⠆⠐⠬⠽⠆⠘⠆

$x_1^2 + y_1^2 = x_2^2 + y_2^2$

```
x_1^2 + y_1^2 = x_2^2 + y_2^2
```

245. ⠠⠁⠄⡷⠭⠆⠆⠒⠉⠄⠠⠃

$A \sim B$

```
A \sim B
```

246. P

```
P
```

247. ⠠⠏⠠⠁⠠⠏⠘⠤⠂⠐⠀⠨⠅⠀⠠⠃

$PAP^{-1} = B$

```
PAP^{-1} = B
```

248. ⠠⠁⠀⠨⠅⠀⠠⠷⠼⠂⠀⠼⠆⠠⠾⠠⠷⠤⠂⠀⠼⠂⠠⠾and⠠⠃⠀⠨⠅⠀⠠⠷⠤18⠀⠼33⠠⠾
⠠⠷⠤11⠀⠼20⠠⠾

$A = \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -18 & 33 \\ -11 & 20 \end{pmatrix}$

```
A = \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix} \quad \text{and}
\quad B = \begin{pmatrix} -18 & 33 \\ -11 & 20 \end{pmatrix}
```

249. ⠠⠏⠀⠨⠅⠀⠠⠷⠼⠆⠀⠼⠢⠠⠾⠠⠷⠼⠂⠀⠼⠒⠠⠾

$P = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$

```
P = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}
```

250. I

```
I
```

251. ⠠⠊⠀⠨⠅⠀⠠⠷⠼⠂⠀⠼⠴⠠⠾⠠⠷⠼⠴⠀⠼⠂⠠⠾

$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

```
I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}
```

252. ⠠⠊⠠⠁⠠⠊⠘⠤⠂⠐⠀⠨⠅⠀⠠⠊⠠⠁⠠⠊⠀⠨⠅⠀⠠⠁

$IAI^{-1} = IAI = A$

```
IAI^{-1} = IAI = A
```

253. ⠠⠁⠀⠨⠅⠀⠠⠏⠘⠤⠂⠐⠠⠃⠠⠏⠀⠨⠅⠀⠠⠏⠘⠤⠂⠐⠠⠃⠷⠠⠏⠘⠤⠂⠐⠾⠘⠤⠂

$A = P^{-1}BP = P^{-1}B(P^{-1})^{-1}$

```
A = P^{-1} B P = P^{-1} B (P^{-1})^{-1}
```

254. ⠠⠃⠄⡷⠭⠆⠆⠒⠉⠄⠠⠉

$B \sim C$

```
B \sim C
```

255. Q

```
Q
```

256. ⠠⠟⠠⠃⠠⠟⠘⠤⠂⠐⠀⠨⠅⠀⠠⠉

$QBQ^{-1} = C$

```
QBQ^{-1} = C
```

257. ⠠⠉ ⠶ ⠠⠟⠠⠃⠠⠟⠘⠤⠂⠐ ⠶ ⠠⠟⠠⠏⠠⠁⠠⠏⠘⠤⠂⠐⠠⠟⠘⠤⠂⠐ ⠶ ⠷⠠⠟⠠⠏⠾⠠⠁⠷⠠⠟⠠⠏⠾⠘⠤⠂

$C = QBQ^{-1} = QPAP^{-1}Q^{-1} = (QP)A(QP)^{-1}$

```
C = QBQ^{-1} = QPAP^{-1} Q^{-1} = (QP)A(QP)^{-1}
```

258. ⠠⠭⠂⠠ ⠠⠭⠆ ⠤

$X_1, X_2, \ldots$

```
X_1, X_2, \ldots
```

259. ⠠⠭⠰⠊⠐⠨⠩⠠⠭⠰⠚⠐ ⠶ ⠸⠴

$X_i \cap X_j = \emptyset$

```
X_i \cap X_j = \emptyset
```

260. ⠊ ⠌⠶ ⠚

$i \neq j$

```
i \neq j
```

261. ⠢⡸⠭⠆⠆ ⠉⠤⠄ ⠙⠭ ⠻⠠⠭⠰⠅⠐ ⠶ ⠠⠭

$\bigcup_k X_k = X$

```
\bigcup_k X_k = X
```

262. ⠈⠷⠭⠈⠾ ⠶ StartSet⠽ ⠈⠑ ⠠⠭⠈⠒⠸⠐ ⡸⠽⠒ ⠒ ⠤⠈⠐ ⠭EndSet

$[x] = \{y \in X : y \sim x\}$

```
[x] = \{ y \in X : y \sim x \}
```

263. caligraphic ⠠⠏ ⠶ ⠨⠷⠠⠭⠰⠊⠐⠨⠾

$\mathcal{P} = \{X_i\}$

```
{\mathcal P} = \{ X_i\}
```

264. ⠠⠭⠰⠊

X_i

```
X_i
```

265. ⠭ ⠈⠑ ⠈⠷⠭⠈⠾

$x \in [x]$

```
x \in [x]
```

266. ⠈⠷⠭⠈⠾

$[x]$

```
[x]
```

267. ⠠⠭ ⠶ ⠢⡸⠭⠆⠆ ⠉⠤⠄ ⠙⠭⠭ ⠈⠑ ⠠⠭⠻ ⠈⠷⠭⠈⠾

$X = \bigcup_{x \in X} [x]$

```
X = \bigcup_{x \in X} [x]
```

268. ⠭ ⠠⠽ ⠈⠑ ⠠⠭

$x, y \in X$

```
x, y \in X
```

269. ⠈⠷⠭⠈⠾ ⠶ ⠈⠷⠽⠈⠾

$[x] = [y]$

```
[x] = [y]
```

270. ⠈⠷⠭⠈⠾⠨⠩⠈⠷⠽⠈⠾ ⠨⠅ ⠸⠴

$[x] \cap [y] = \emptyset$

[x] \cap [y] = \emptyset

271. ⠈⠷⠽⠈⠾

$[y]$

[y]

272. ⠵⠈⠑⠈⠷⠭⠈⠾⠨⠩⠈⠷⠽⠈⠾

$z \in [x] \cap [y]$

z \in [x] \cap [y]

273. ⠵⠄⡗⠭⠆⠆⠒⠉⠄⠭

$z \sim x$

z \sim x

274. ⠵⠄⡗⠭⠆⠆⠒⠉⠄⠽

$z \sim y$

z \sim y

275. ⠈⠷⠭⠈⠾⠸⠐⠅⠈⠷⠽⠈⠾

$[x] \subset [y]$

[x] \subset [y]

276. ⠈⠷⠽⠈⠾⠸⠐⠅⠈⠷⠭⠈⠾

$[y] \subset [x]$

[y] \subset [x]

277. caligraphic ⠠⠏ ⠨⠅ ⠨⠷⠠⠭⠰⠊⠐⠨⠾

$\mathcal{P} = \{X_i\}$

{\mathcal P} = \{X_i\}

278. ⠽⠄⡗⠭⠆⠆⠒⠉⠄⠭

$y \sim x$

y \sim x

279. z

z

280. ⠷⠏⠠⠟⠾

(p, q)

(p,q)

281. ⠷⠗⠠⠎⠾

(r, s)

(r,s)

282. ⠋⠷⠭⠾

$f(x)$

f(x)

283. ⠛⠷⠭⠾

$g(x)$

g(x)

284. ⠝⠈⠑⠈⠠⠝

$n \in \mathbb{N}$

n \in {\mathbb N}

285. ⠗⠤⠎

$r - s$

r - s

286. ⠗⠤⠎ ⠨⠅ ⠝⠅

$r - s = nk$

r - s = nk

287. ⠅⠈⠑⠈⠠⠵

$k \in \mathbb{Z}$

k \in {\mathbb Z}

288. ⠗⠸⠇⠎⠷⠍⠕⠙⠝⠾

$r \equiv s \pmod{n}$

r \equiv s \pmod{n}

289. b

b

290. 41 ⠸⠇ 17⠷⠍⠕⠙⠦⠾

$41 \equiv 17 \pmod{8}$

41 \equiv 17 \pmod{ 8}

291. 41⠤17 ⠨⠅ 24

$41 - 17 = 24$

41 - 17=24

292. ⠼⠦

8

8

293. ⠈⠠⠵

$\mathbb{Z}$

{\mathbb Z}

294. ⠗⠤⠗ ⠨⠅ ⠴

$r - r = 0$

r - r = 0

295. ⠗⠸⠇⠎⠷⠍⠕⠙⠝⠾

$r \equiv s \pmod{n}$

r \equiv s \pmod{ n}

296. ⠗⠤⠎ ⠭ ⠤⠷⠎⠤⠗⠾

$r - s = -(s - r)$

r - s = -(s -r)

297. ⠎⠤⠗

$s - r$

s - r

298. ⠎ ⠿ ⠗⠷⠍⠕⠙⠝⠾

$s \equiv r \pmod{n}$

s \equiv r \pmod{ n}

299. ⠎ ⠿ ⠞⠷⠍⠕⠙⠝⠾

$s \equiv t \pmod{n}$

s \equiv t \pmod{ n}

300. k

k

301. l

l

302. ⠗⠤⠎ ⠭ ⠅⠝

$r - s = kn$

r -s = kn

303. ⠎⠤⠞ ⠭ ⠇⠝

$s - t = ln$

s - t = ln

304. ⠗⠤⠞

$r - t$

r - t

305. ⠗⠤⠞ ⠭ ⠗⠤⠎⠬⠎⠤⠞ ⠭ ⠅⠝⠬⠇⠝ ⠭ ⠷⠅⠬⠇⠾⠝

$r - t = r - s + s - t = kn + ln = (k + l)n$

r - t = r - s + s - t = kn + ln = (k + l)n

306. ⠼⠒

3

3

307. ⠈⠷⠴⠈⠾⠨⠬⠈⠷⠂⠈⠾⠨⠬⠈⠷⠆⠈⠾ ⠭ ⠈⠠⠵

$[0] \cup [1] \cup [2] = \mathbb{Z}$

[0] \cup [1] \cup [2] = {\mathbb Z}

308. ⠈⠷⠴⠈⠾

$[0]$

[0]

309. ⠈⠷⠂ ⠈⠾

$[1]$

`[1]`

310. ⠈⠷⠆ ⠈⠾

$[2]$

`[2]`

311. ⠠⠁ ⠨⠩⠠⠃

$A \cap B$

`A \cap B`

312. ⠠⠃ ⠨⠩⠠⠉

$B \cap C$

`B \cap C`

313. ⠠⠁ ⠨⠩⠷⠠⠃ ⠨⠬⠠⠉⠾

$A \cap (B \cup C)$

`A \cap (B \cup C)`

314. ⠠⠁ ⠨⠩⠠⠃ ⠨⠅ StartSet⠆ EndSet

$A \cap B = \{2\}$

`A \cap B = \{ 2 \}`

315. ⠠⠃ ⠨⠩⠠⠉ ⠨⠅ StartSet⠢EndSet

$B \cap C = \{5\}$

`B \cap C = \{ 5 \}`

316. ⠠⠁ ⠨⠅ StartSet⠁ ⠠⠃ ⠠⠉EndSet

$A = \{a, b, c\}$

`A = \{ a, b, c \}`

317. ⠠⠉ ⠨⠅ StartSet⠭EndSet

$C = \{x\}$

`C = \{ x \}`

318. ⠠⠙ ⠨⠅ ⠸⠴

$D = \emptyset$

`D = \emptyset`

319. ⠠⠃⠈⠡⠠⠁

$B \times A$

`B \times A`

320. ⠠⠁⠈⠡⠠⠃⠈⠡⠠⠉

$A \times B \times C$

`A \times B \times C`

321. ⠠⠁⠈⠡⠠⠙

$A \times D$

`A \times D`

322. ⠠⠁⠡⠠⠃ ⠨⠅ StartSet⠷⠁⠠⠂⠾⠠⠷⠁⠠⠆⠾⠠⠷⠁⠠⠒⠾⠠⠷⠃⠠⠂⠾⠠⠷⠃⠠⠆⠾⠠⠷⠃⠠⠒⠾⠠⠷⠉⠠⠂⠾⠠⠷⠉⠠⠆⠾

$$A \times B = \{(a,1),(a,2),(a,3),(b,1),(b,2),(b,3),(c,1),(c,2),(c,3)\}$$

`A \times B = \{ (a,1), (a,2), (a,3), (b,1), (b,2), (b,3), (c,1),`
`(c,2), (c,3) \}`

323. ⠠⠁⠡⠠⠙ ⠨⠅ ⠸⠴

$$A \times D = \emptyset$$

`A \times D = \emptyset`

324. ⠠⠁⠡⠠⠃ ⠨⠅ ⠠⠃⠡⠠⠁

$$A \times B = B \times A$$

`A \times B = B \times A`

325. ⠭⠈⠑⠠⠁⠨⠬⠷⠠⠃⠨⠩⠠⠉⠾

$$x \in A \cup (B \cap C)$$

`x \in A \cup (B \cap C)`

326. ⠭⠈⠑⠠⠁

$$x \in A$$

`x \in A`

327. ⠭⠈⠑⠠⠃⠨⠩⠠⠉

$$x \in B \cap C$$

`x \in B \cap C`

328. ⠭⠈⠑⠠⠁⠨⠬⠠⠃

$$x \in A \cup B$$

`x \in A \cup B`

329. ⠠⠁⠨⠬⠠⠉

$$A \cup C$$

`A \cup C`

330. ⠭⠈⠑⠷⠠⠁⠨⠬⠠⠃⠾⠨⠩⠷⠠⠁⠨⠬⠠⠉⠾

$$x \in (A \cup B) \cap (A \cup C)$$

`x \in (A \cup B) \cap (A \cup C)`

331. ⠠⠁⠨⠬⠷⠠⠃⠨⠩⠠⠉⠾⠸⠐⠅⠷⠠⠁⠨⠬⠠⠃⠾⠨⠩⠷⠠⠁⠨⠬⠠⠉⠾

$$A \cup (B \cap C) \subset (A \cup B) \cap (A \cup C)$$

`A \cup (B \cap C) \subset (A \cup B) \cap (A \cup C)`

332. ⠷⠠⠁⠨⠬⠠⠃⠾⠨⠩⠷⠠⠁⠨⠬⠠⠉⠾⠸⠐⠅⠠⠁⠨⠬⠷⠠⠃⠨⠩⠠⠉⠾

$$(A \cup B) \cap (A \cup C) \subset A \cup (B \cap C)$$

`(A \cup B) \cap (A \cup C) \subset A \cup (B \cap C)`

333. ⠠⠁⠨⠩⠠⠃ ⠨⠅ ⠠⠁

$$A \cap B = A$$

`A \cap B = A`

334. $A \cup B = (A \cap B) \cup (A \setminus B) \cup (B \setminus A)$

```
A \cup B = (A \cap B) \cup (A \setminus B) \cup (B \setminus A)
```

335. $(A \cap B) \cup (A \setminus B) \cup (B \setminus A) = (A \cap B) \cup (A \cap B') \cup (B \cap A') = [A \cap (B \cup B')] \cup (B \cap A') = A \cup (B \cap A') = (A \cup B) \cap (A \cup A') = A \cup B$

```
(A \cap B) \cup (A \setminus B) \cup (B \setminus A) = (A \cap B)
\cup (A \cap B') \cup (B \cap A') = [A \cap (B \cup B')] \cup (B \cap
A') = A \cup (B \cap A') = (A \cup B) \cap (A \cup A') = A \cup B
```

336. $(A \cup B) \times C = (A \times C) \cup (B \times C)$

```
(A \cup B) \times C = (A \times C ) \cup (B \times C)
```

337. $(A \cap B) \setminus B = \emptyset$

```
(A \cap B) \setminus B = \emptyset
```

338. $(A \cup B) \setminus B = A \setminus B$

```
(A \cup B) \setminus B = A \setminus B
```

339. $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$

```
A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)
```

340. $A \setminus (B \cup C) = A \cap (B \cup C)' = (A \cap A) \cap (B' \cap C') = (A \cap B') \cap (A \cap C') = (A \setminus B) \cap (A \setminus C)$

```
A \setminus (B \cup C) = A \cap (B \cup C)' = (A \cap A) \cap (B'
\cap C') = (A \cap B') \cap (A \cap C') = (A \setminus B) \cap (A
\setminus C)
```

341. $A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C)$

```
A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C)
```

342. $(A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B)$

```
(A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap
B)
```

343. $f : \mathbb{Q} \to \mathbb{Q}$

```
f: {\mathbb Q} \rightarrow {\mathbb Q}
```

344. $f(p/q) = \dfrac{p+1}{p-2}$

```
\displaystyle f(p/q) = \frac{p+ 1}{p - 2}
```

345. ⠋⠷⠏⠌⠟⠾ ⠶ ⠹⠒⠏⠌⠒⠟⠼

$$f(p/q) = \frac{3p}{3q}$$

\displaystyle f(p/q) = \frac{3p}{3q}

346. ⠋⠷⠏⠌⠟⠾ ⠶ ⠹⠏⠬⠟⠌⠟⠘⠆⠐⠼

$$f(p/q) = \frac{p+q}{q^2}$$

\displaystyle f(p/q) = \frac{p+q}{q^2}

347. ⠋⠷⠏⠌⠟⠾ ⠶ ⠹⠒⠏⠘⠆⠐⠌⠶⠟⠘⠆⠐⠼⠤⠹⠏⠌⠟⠼

$$f(p/q) = \frac{3p^2}{7q^2} - \frac{p}{q}$$

\displaystyle f(p/q) = \frac{3 p^2}{7 q^2} - \frac{p}{q}

348. ⠋⠷⠆⠌⠒⠾

$$f(2/3)$$

f(2/3)

349. ⠋⠷⠂⠌⠆⠾ ⠶ ⠒⠌⠲

$$f(1/2) = 3/4$$

f(1/2) = 3/4

350. ⠋⠷⠆⠌⠲⠾ ⠶ ⠒⠌⠦

$$f(2/4) = 3/8$$

f(2/4)=3/8

351. ⠋⠷⠭⠾ ⠶ ⠑⠘⠭

$$f(x) = e^x$$

f(x) = e^x

352. ⠋⠱ ⠈⠠⠵ ⠫⠕ ⠈⠠⠵

$$f : \mathbb{Z} \rightarrow \mathbb{Z}$$

f: {\mathbb Z} \rightarrow {\mathbb Z}

353. ⠋⠷⠝⠾ ⠶ ⠝⠘⠆⠐⠬⠒

$$f(n) = n^2 + 3$$

f(n) = n^2 + 3

354. ⠋⠷⠭⠾ ⠶ ⠎⠊⠝ ⠭

$$f(x) = \sin x$$

f(x) = \sin x

355. ⠋⠷⠈⠠⠗⠾ ⠶ StartSet⠭ ⠈⠑ ⠈⠠⠗⠱⠭ ⠨⠂ ⠴EndSet

$$f(\mathbb{R}) = \{x \in \mathbb{R} : x > 0\}$$

f({\mathbb R}) = \{ x \in {\mathbb R} : x \gt 0 \}

356. ⠋⠷⠈⠠⠗⠾ ⠶ StartSet⠭⠱⠤⠂ ⠆⠱ ⠭ ⠆⠱ ⠂EndSet

$$f(\mathbb{R}) = \{x : -1 \leq x \leq 1\}$$

f(\mathbb R) = \{ x : -1 \leq x \leq 1 \}

357. ⠋⠱⠠⠁⠀⠫⠕⠀⠠⠃

$f : A \rightarrow B$

f :A \rightarrow B

358. ⠛⠘⠤⠂

g^{-1}

g^{-1}

359. ⠷⠛⠘⠨⠡⠋⠾⠘⠤⠂⠐⠀⠨⠅⠀⠋⠘⠤⠂⠐⠘⠨⠡⠛⠘⠤⠂

$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

(g \circ f)^{-1} =f^{-1} \circ g^{-1}

360. ⠋⠱⠈⠠⠝⠀⠫⠕⠀⠈⠠⠝

$f : \mathbb{N} \rightarrow \mathbb{N}$

f: {\mathbb N} \rightarrow {\mathbb N}

361. ⠋⠷⠝⠾⠀⠨⠅⠀⠝⠬⠂

$f(n) = n + 1$

f(n) = n + 1

362. ⠭⠠⠽⠈⠑⠠⠁

$x, y \in A$

x, y \in A

363. ⠛⠷⠋⠷⠭⠾⠾⠀⠨⠅⠀⠷⠛⠘⠨⠡⠋⠾⠷⠭⠾⠀⠨⠅⠀⠷⠛⠘⠨⠡⠋⠾⠷⠽⠾⠀⠨⠅⠀⠛⠷⠋⠷⠽⠾⠾

$g(f(x)) = (g \circ f)(x) = (g \circ f)(y) = g(f(y))$

g(f(x)) = (g \circ f)(x) = (g \circ f)(y) = g(f(y))

364. ⠋⠷⠭⠾⠀⠨⠅⠀⠋⠷⠽⠾

$f(x) = f(y)$

f(x) = f(y)

365. ⠭⠀⠨⠅⠀⠽

$x = y$

x = y

366. ⠉⠀⠨⠅⠀⠷⠛⠘⠨⠡⠋⠾⠷⠭⠾⠀⠨⠅⠀⠛⠷⠋⠷⠭⠾⠾

$c = (g \circ f)(x) = g(f(x))$

c = (g \circ f)(x) = g(f(x))

367. ⠋⠷⠭⠾⠈⠑⠠⠃

$f(x) \in B$

f(x) \in B

368. ⠋⠷⠭⠾⠀⠨⠅⠀⠹⠭⠬⠂⠌⠭⠤⠂⠼

$f(x) = \frac{x+1}{x-1}$

f(x) = \frac{x + 1}{x - 1}

369. ⠋⠘⠨⠡⠋⠘⠤⠂

$f \circ f^{-1}$

f \circ f^{-1}

370. ⠋⠘⠤⠂⠐⠘⠨⠡⠋

$f^{-1} \circ f$

f^{-1} \circ f

371. ⠋⠘⠤⠂⠐⠷⠭⠾⠀⠶⠀⠷⠭⠬⠂⠾⠸⠌⠷⠭⠤⠂⠾

$f^{-1}(x) = (x+1)/(x-1)$

f^{-1}(x) = (x+1)/(x-1)

372. ⠋⠱⠠⠭⠀⠫⠕⠀⠠⠽

$f: X \rightarrow Y$

f: X \rightarrow Y

373. ⠠⠁⠂⠠⠀⠠⠁⠆⠀⠸⠐⠅⠀⠠⠭

$A_1, A_2 \subset X$

A_1, A_2 \subset X

374. ⠠⠃⠂⠠⠀⠠⠃⠆⠀⠸⠐⠅⠀⠠⠽

$B_1, B_2 \subset Y$

B_1, B_2 \subset Y

375. ⠋⠷⠠⠁⠂⠨⠬⠠⠁⠆⠾⠀⠶⠀⠋⠷⠠⠁⠂⠾⠨⠬⠋⠷⠠⠁⠆⠾

$f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$

f(A_1 \cup A_2) = f(A_1) \cup f(A_2)

376. ⠋⠷⠠⠁⠂⠨⠩⠠⠁⠆⠾⠸⠐⠅⠋⠷⠠⠁⠂⠾⠨⠩⠋⠷⠠⠁⠆⠾

$f(A_1 \cap A_2) \subset f(A_1) \cap f(A_2)$

f(A_1 \cap A_2) \subset f(A_1) \cap f(A_2)

377. ⠋⠘⠤⠂⠐⠷⠠⠃⠂⠨⠬⠠⠃⠆⠾⠀⠶⠀⠋⠘⠤⠂⠐⠷⠠⠃⠂⠾⠨⠬⠋⠘⠤⠂⠐⠷⠠⠃⠆⠾

$f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$

f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)

378. ⠋⠘⠤⠂⠐⠷⠠⠃⠾⠀⠶⠀StartSet⠭⠀⠈⠑⠀⠠⠭⠱⠋⠷⠭⠾⠀⠈⠑⠀⠠⠃ EndSet

$f^{-1}(B) = \{ x \in X : f(x) \in B \}$

f^{-1}(B) = \{ x \in X : f(x) \in B \}

379. ⠋⠘⠤⠂⠐⠷⠠⠃⠂⠨⠩⠠⠃⠆⠾⠀⠶⠀⠋⠘⠤⠂⠐⠷⠠⠃⠂⠾⠨⠩⠋⠘⠤⠂⠐⠷⠠⠃⠆⠾

$f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)$

f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)

380. ⠋⠘⠤⠂⠐⠷⠠⠽⠸⠡⠠⠃⠂⠾⠀⠶⠀⠠⠭⠸⠡⠋⠘⠤⠂⠐⠷⠠⠃⠂⠾

$f^{-1}(Y \setminus B_1) = X \setminus f^{-1}(B_1)$

f^{-1}(Y \setminus B_1) = X \setminus f^{-1}(B_1)

381. ⠽⠈⠑⠋⠷⠠⠁⠂⠨⠬⠠⠁⠆⠾

$y \in f(A_1 \cup A_2)$

y \in f(A_1 \cup A_2)

382. ⠭⠈⠑⠠⠁⠂⠨⠬⠠⠁⠆

$x \in A_1 \cup A_2$

x \in A_1 \cup A_2

383. ⠋⠷⠭⠾⠀⠨⠅⠀⠽

$f(x) = y$

f(x) = y

384. ⠽⠈⠑⠋⠷⠠⠁⠂⠾

$y \in f(A_1)$

y \in f(A_1)

385. ⠋⠷⠠⠁⠆⠾

$f(A_2)$

f(A_2)

386. ⠽⠈⠑⠋⠷⠠⠁⠂⠾⠨⠬⠋⠷⠠⠁⠆⠾

$y \in f(A_1) \cup f(A_2)$

y \in f(A_1) \cup f(A_2)

387. ⠋⠷⠠⠁⠂⠨⠬⠠⠁⠆⠾⠸⠐⠅⠋⠷⠠⠁⠂⠾⠨⠬⠋⠷⠠⠁⠆⠾

$f(A_1 \cup A_2) \subset f(A_1) \cup f(A_2)$

f(A_1 \cup A_2) \subset f(A_1) \cup f(A_2)

388. ⠠⠁⠂

A_1

A_1

389. ⠠⠁⠆

A_2

A_2

390. ⠋⠷⠠⠁⠂⠾⠨⠬⠋⠷⠠⠁⠆⠾⠸⠐⠅⠋⠷⠠⠁⠂⠨⠬⠠⠁⠆⠾

$f(A_1) \cup f(A_2) \subset f(A_1 \cup A_2)$

f(A_1) \cup f(A_2) \subset f(A_1 \cup A_2)

391. ⠋⠷⠠⠁⠂⠨⠬⠠⠁⠆⠾⠀⠨⠅⠀⠋⠷⠠⠁⠂⠾⠨⠬⠋⠷⠠⠁⠆⠾

$f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$

f(A_1 \cup A_2) = f(A_1) \cup f(A_2)

392. ⠭⠀⠨⠂⠱⠀⠽

$x \geq y$

x \geq y

393. ⠍⠄⡵⠍⠆⠆⠤⠉⠄⠝

$m \sim n$

m \sim n

394. ⠍⠝⠀⠨⠂⠀⠴

$mn > 0$

mn > 0

395. ⠳⠭⠤⠽⠳⠀⠐⠅⠱⠀⠲

$|x - y| \leq 4$

|x - y| \leq 4

396. ⠍ ⠸⠇ ⠝⠷⠍⠕⠙⠖⠾

$m \equiv n \pmod{6}$

`m \equiv n \pmod{6}`

397. ⠷⠁⠠⠃⠾⠄⡷⠭⠆⠆⠒⠉⠄⠷⠉⠠⠙⠾

$(a, b) \sim (c, d)$

`(a, b) \sim (c, d)`

398. ⠁⠘⠆⠐⠬⠃⠘⠆⠐ ⠅⠱ ⠉⠘⠆⠐⠬⠙⠘⠆

$a^2 + b^2 \leq c^2 + d^2$

`a^2 + b^2 \leq c^2 + d^2`

399. ⠍⠡⠝

$m \times n$

`m \times n`

400. ⠭⠄⡷⠭⠆⠆⠒⠉⠄⠭

$x \sim x$

`x \sim x`

401. ⠠⠭ ⠨⠅ ⠈⠠⠝⠨⠬⠨⠷⠜⠆⠻⠨⠾

$X = {\mathbb N} \cup \{ \sqrt{2}\, \}$

`X = {\mathbb N} \cup \{ \sqrt{2}\, \}`

402. ⠭⠬⠽⠈⠑⠈⠠⠝

$x + y \in {\mathbb N}$

`x + y \in {\mathbb N}`

403. ⠈⠠⠗⠘⠆⠐⠄⡷⠭⠆⠆⠂⠒⠄⠨⠷⠷⠴⠠⠴⠾⠨⠾

${\mathbb R}^2 \setminus \{ (0,0) \}$

`{\mathbb R}^2 \setminus \{ (0,0) \}`

404. ⠷⠭⠂⠠⠽⠂⠾⠄⡷⠭⠆⠆⠒⠉⠄⠷⠭⠆⠠⠽⠆⠾

$(x_1, y_1) \sim (x_2, y_2)$

`(x_1, y_1) \sim (x_2, y_2)`

405. ⠨⠇

λ

`\lambda`

406. ⠷⠭⠂⠠⠽⠂⠾ ⠨⠅ ⠷⠨⠇⠭⠆⠠⠨⠇⠽⠆⠾

$(x_1, y_1) = (\lambda x_2, \lambda y_2)$

`(x_1, y_1) = ( \lambda x_2, \lambda y_2)`

407. ⠈⠠⠗⠘⠆⠐⠄⡷⠭⠆⠆⠂⠒⠄⠷⠴⠠⠴⠾

${\mathbb R}^2 \setminus (0,0)$

`{\mathbb R}^2 \setminus (0,0)`

408. ⠈⠠⠏⠷⠈⠠⠗⠾

${\mathbb P}({\mathbb R})$

`{\mathbb P}({\mathbb R})`

409. ⠼300⠯

$300!$

`300!`

410. ⠼10

10

`10`

411. ⠼66

66

`66`

412. ⠼46

46

`46`

413. ⠒⠤⠂ ⠨⠅ ⠆

$3 - 1 = 2$

`3-1=2`

414. ⠂⠬⠆⠬⠄⠄⠄⠬⠝ ⠨⠅ ⠹⠝⠷⠝⠬⠂⠾⠌⠆⠼

$1 + 2 + \cdots + n = \frac{n(n+1)}{2}$

`1 + 2 + \cdots + n = \frac{n(n + 1)}{2}`

415. ⠝ ⠨⠅ ⠂

$n = 1$

`n = 1`

416. ⠼⠲

4

`4`

417. ⠷⠝⠬⠂⠾

$(n + 1)$

`(n + 1)`

418. ⠂ ⠨⠅ ⠹⠂⠷⠂⠬⠂⠾⠌⠆⠼

$1 = \frac{1(1+1)}{2}$

`1 = \frac{1(1 + 1)}{2}`

419. ⠈⠠⠝

$\mathbb{N}$

`{\mathbb N}`

420. ⠠⠎⠷⠝⠾

$S(n)$

`S(n)`

421. ⠠⠎⠷⠝⠴⠾

$S(n_0)$

`S(n_0)`

422. ⠝⠴

n_0

`n_0`

423. ⠅ ⠨⠂⠱ ⠝⠴

$k \geq n_0$

`k \geq n_0`

424. ⠠⠎⠷⠅⠾

$S(k)$

`S(k)`

425. ⠠⠎⠷⠅⠬⠂⠾

$S(k+1)$

`S(k+1)`

426. ⠝ ⠨⠂⠱ ⠒

$n \geq 3$

`n \geq 3`

427. ⠆⠘⠝⠐ ⠨⠂ ⠝⠬⠲

$2^n > n + 4$

`2^n \gt n + 4`

428. ⠦ ⠨⠅ ⠆⠘⠒⠐ ⠨⠂ ⠒⠬⠲ ⠨⠅ ⠶

$8 = 2^3 > 3 + 4 = 7$

`8 = 2^3 \gt 3 + 4 = 7`

429. ⠝⠴ ⠨⠅ ⠒

$n_0 = 3$

`n_0 = 3`

430. ⠆⠘⠅⠐ ⠨⠂ ⠅⠬⠲

$2^k > k + 4$

`2^k \gt k + 4`

431. ⠅ ⠨⠂⠱ ⠒

$k \geq 3$

`k \geq 3`

432. ⠆⠘⠅⠬⠂⠐ ⠨⠅ ⠆⠡⠆⠘⠅⠐ ⠨⠂ ⠆⠷⠅⠬⠲⠾

$2^{k+1} = 2 \cdot 2^k > 2(k + 4)$

`2^{k + 1} = 2 \cdot 2^{k} \gt 2(k + 4)`

433. ⠆⠷⠅⠬⠲⠾ ⠨⠅ ⠆⠅⠬⠦ ⠨⠂ ⠅⠬⠢ ⠨⠅ ⠷⠅⠬⠂⠾⠬⠲

$2(k + 4) = 2k + 8 > k + 5 = (k + 1) + 4$

`2(k + 4) = 2k + 8 \gt k + 5 = (k + 1) + 4`

434. 10 ⠘⠝⠬⠂⠐ ⠐⠬⠒⠡10 ⠘⠝⠬⠢

$10^{n+1} + 3 \cdot 10^n + 5$

`10^{n + 1} + 3 \cdot 10^n + 5`

435. ⠼⠔

9

9

436. 10⠘⠂⠬⠂⠐⠬⠒⠡10⠬⠢ ⠨⠅ 135 ⠨⠅ ⠔⠡15

$10^{1+1} + 3 \cdot 10 + 5 = 135 = 9 \cdot 15$

10^{1 + 1} + 3 \cdot 10 + 5 = 135 = 9 \cdot 15

437. 10⠘⠅⠬⠂⠐⠬⠒⠡10⠘⠅⠬⠢

$10^{k+1} + 3 \cdot 10^k + 5$

10^{k + 1} + 3 \cdot 10^k + 5

438. ⠅ ⠨⠂⠱ ⠂

$k \geq 1$

k \geq 1

439. ⠷⠁⠬⠃⠾⠘⠝⠐ ⠨⠅ ⠐⠨⠠⠎⠩⠅ ⠨⠅ ⠴⠣⠝⠻⠝ ⠅ ⠁⠘⠅⠐⠃⠘⠝⠤⠅

$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$

(a + b)^n = \sum_{k = 0}^{n} \binom{n}{k} a^k b^{n - k}

440. ⠝ ⠈⠑ ⠈⠠⠝

$n \in \mathbb{N}$

n \in \mathbb{N}

441. ⠝ ⠅ ⠨⠅ ⠹⠝⠯⠌⠅⠯⠷⠝⠤⠅⠾⠯⠼

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$

\binom{n}{k} = \frac{n!}{k! (n - k)!}

442. ⠝⠯⠌⠷⠅⠯⠷⠝⠤⠅⠾⠯⠾

$n!/(k!(n-k)!)$

n!/(k!(n-k)!)

443. ⠝⠬⠂ ⠅ ⠨⠅ ⠝ ⠅⠬⠝ ⠅⠤⠂

$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$

\binom{n + 1}{k} = \binom{n}{k} + \binom{n}{k - 1}

444. ⠼⠂

1

1

445. ⠠⠎⠷⠝⠴⠾⠠⠠⠎⠷⠝⠴⠬⠂⠾⠠⠄ ⠠⠠⠎⠷⠅⠾

$S(n_0), S(n_0+1), \ldots, S(k)$

S(n_0), S(n_0 + 1), \ldots, S(k)

446. ⠠⠎⠷⠅⠬⠂⠾

$S(k+1)$

S(k + 1)

447. ⠝ ⠨⠂⠱ ⠝⠴

$n \geq n_0$

n \geq n_0

448. ⠠⠎ ⠨⠅ StartSet⠝ ⠈⠑ ⠈ ⠠⠝⠱⠝ ⠨⠂⠱ ⠂ EndSet

$S = \{n \in \mathbb{N} : n \geq 1\}$

S = \{ n \in {\mathbb N} : n \geq 1 \}

449. ⠂ ⠈⠑ ⠠⠎

$1 \in S$

1 \in S

450. ⠝ ⠈⠑ ⠠⠎

$n \in S$

n \in S

451. ⠴ ⠐⠅ ⠂

$0 < 1$

0 \lt 1

452. ⠝ ⠨⠅ ⠝⠬⠴ ⠐⠅ ⠝⠬⠂

$n = n + 0 < n + 1$

n = n + 0 \lt n + 1

453. ⠂ ⠅⠱ ⠝ ⠐⠅ ⠝⠬⠂

$1 \leq n < n + 1$

1 \leq n \lt n + 1

454. ⠝⠬⠂

$n + 1$

n + 1

455. ⠠⠎ ⠨⠅ ⠈ ⠠⠝

$S = \mathbb{N}$

S = \mathbb N

456. ⠈ ⠠⠝

$\mathbb{N}$

\mathbb N

457. ⠂ ⠅⠱ ⠅ ⠅⠱ ⠝

$1 \leq k \leq n$

1 \leq k \leq n

458. ⠝⠬⠂

$n + 1$

n+1

459. ⠝⠯

$n!$

n!

460. ⠝⠯ ⠨⠅ ⠂⠡⠆⠡⠒⠄⠄⠄⠷⠝⠤⠂⠾⠡⠝

$n! = 1 \cdot 2 \cdot 3 \cdots (n - 1) \cdot n$

n! = 1 \cdot 2 \cdot 3 \cdots (n - 1) \cdot n

461. ⠼⠂⠯⠀⠨⠅⠀⠂

$1! = 1$

1! = 1

462. ⠝⠯⠀⠨⠅⠀⠝⠷⠝⠤⠂⠾⠯

$n! = n(n-1)!$

n! = n(n - 1)!

463. ⠝⠀⠨⠂⠀⠂

$n > 1$

n \gt 1

464. ⠃⠀⠨⠂⠀⠴

$b > 0$

b \gt 0

465. ⠁⠀⠨⠅⠀⠃⠟⠬⠗

$a = bq + r$

a = bq + r

466. ⠴⠀⠐⠅⠱⠀⠗⠀⠐⠅⠀⠃

$0 \leq r < b$

0 \leq r \lt b

467. ⠟⠄

q'

q'

468. ⠗⠄

r'

r'

469. ⠟⠀⠨⠅⠀⠟⠄

$q = q'$

q = q'

470. ⠗⠀⠨⠅⠀⠗⠄

$r = r'$

r = r'

471. ⠠⠎⠀⠨⠅⠀StartSet⠁⠤⠃⠅⠀⠒⠀⠅⠀⠈⠑⠀⠨⠵and⠁⠤⠃⠅⠀⠨⠂⠱⠀⠴EndSet

$S = \{a - bk : k \in \mathbb{Z}$ and $a - bk \geq 0\}$

S = \{ a - bk : k \in {\mathbb Z} \text{ and } a - bk \geq 0 \}

472. ⠴⠈⠑⠠⠎

$0 \in S$

0 \in S

473. ⠟⠀⠨⠅⠀⠁⠸⠌⠃

$q = a/b$

q = a/b

474. ⠗⠀⠨⠅⠀⠴

$r = 0$

r = 0

475. ⠴⠌⠈⠑⠀⠠⠎

$0 \notin S$

0 \notin S

476. ⠁⠀⠨⠂⠀⠴

$a > 0$

a \gt 0

477. ⠁⠤⠃⠡⠴⠀⠈⠑⠀⠠⠎

$a - b \cdot 0 \in S$

a - b \cdot 0 \in S

478. ⠁⠀⠐⠅⠀⠴

$a < 0$

a \lt 0

479. ⠁⠤⠃⠷⠆⠁⠾⠀⠨⠅⠀⠁⠷⠂⠤⠆⠃⠾⠀⠈⠑⠀⠠⠎

$a - b(2a) = a(1 - 2b) \in S$

a - b(2a) = a(1 - 2b) \in S

480. ⠠⠎⠀⠌⠨⠅⠀⠸⠴

$S \neq \emptyset$

S \neq \emptyset

481. ⠗⠀⠨⠅⠀⠁⠤⠃⠟

$r = a - bq$

r = a - bq

482. ⠗⠀⠨⠂⠱⠀⠴

$r \geq 0$

r \geq 0

483. ⠗⠀⠐⠅⠀⠃

$r < b$

r \lt b

484. ⠗⠀⠨⠂⠀⠃

$r > b$

r \gt b

485. ⠁⠤⠃⠷⠟⠬⠂⠾⠀⠨⠅⠀⠁⠤⠃⠟⠤⠃⠀⠨⠅⠀⠗⠤⠃⠀⠨⠂⠀⠴

$a - b(q + 1) = a - bq - b = r - b > 0$

a - b(q + 1)= a - bq - b = r - b \gt 0

486. ⠁⠤⠃⠷⠟⠬⠂⠾

$a - b(q + 1)$

a - b(q + 1)

487. ⠁⠤⠃⠷⠟⠬⠂⠾ ⠐⠅ ⠁⠤⠃⠟

$a - b(q+1) < a - bq$

a - b(q + 1) \lt a - bq

488. ⠗ ⠅⠱ ⠃

$r \leq b$

r \leq b

489. ⠗ ⠌⠨⠅ ⠃

$r \neq b$

r \neq b

490. ⠁ ⠨⠅ ⠃⠟⠬⠗⠠⠴ ⠅⠱ ⠗ ⠐⠅ ⠃and⠁ ⠨⠅ ⠃⠟⠘⠄⠐⠬⠗⠘⠄⠐⠠⠴ ⠅⠱ ⠗⠄ ⠐⠅ ⠃

$a = bq + r, 0 \leq r < b \quad \text{and} \quad a = bq' + r', 0 \leq r' < b$

a = bq + r, 0 \leq r \lt b \quad \text{and}\quad a = bq' + r', 0
\leq r' \lt b

491. ⠃⠟⠬⠗ ⠨⠅ ⠃⠟⠘⠄⠐⠬⠗⠄

$bq + r = bq' + r'$

bq + r = bq' + r'

492. ⠗⠄ ⠨⠂⠱ ⠗

$r' \geq r$

r' \geq r

493. ⠃⠷⠟⠤⠟⠘⠄⠐⠾ ⠨⠅ ⠗⠄⠤⠗

$b(q - q') = r' - r$

b(q - q') = r' - r

494. ⠗⠄⠤⠗

$r' - r$

r' - r

495. ⠴ ⠅⠱ ⠗⠄⠤⠗ ⠅⠱ ⠗⠄ ⠐⠅ ⠃

$0 \leq r' - r \leq r' < b$

0 \leq r'- r \leq r' \lt b

496. ⠗⠄⠤⠗ ⠨⠅ ⠴

$r' - r = 0$

r' - r = 0

497. ⠃ ⠨⠅ ⠁⠅

$b = ak$

b = ak

498. ⠁⠳⠃

$a \mid b$

a \mid b

499. d

d

500. ⠙⠳⠁

$d \mid a$

d \mid a

501. ⠙⠳⠃

$d \mid b$

d \mid b

502. ⠙⠄

d'

d'

503. ⠙⠄ ⠳⠙

$d' \mid d$

d' \mid d

504. ⠙ ⠨⠅ ⠛⠉⠙⠷⠁ ⠠⠃ ⠾

$d = \gcd(a, b)$

d = \gcd(a, b)

505. ⠛⠉⠙⠷24 ⠠36⠾ ⠨⠅ 12

$\gcd(24, 36) = 12$

\gcd(24, 36) = 12

506. ⠛⠉⠙⠷120 ⠠102⠾ ⠨⠅ ⠖

$\gcd(120, 102) = 6$

\gcd(120, 102) = 6

507. ⠛⠉⠙⠷⠁ ⠠⠃ ⠾ ⠨⠅ ⠂

$\gcd(a, b) = 1$

\gcd(a, b) = 1

508. ⠛⠉⠙⠷⠁ ⠠⠃ ⠾ ⠨⠅ ⠁⠗⠬⠃⠎

$\gcd(a, b) = ar + bs$

\gcd(a, b) = ar + bs

509. ⠠⠎ ⠨⠅ ⠨⠷⠁⠍⠬⠃⠝⠸⠒⠍⠠⠝ ⠈⠑ ⠠⠵and⠁⠍⠬⠃⠝ ⠨⠂ ⠼⠨⠾

$S = \{am + bn : m, n \in \mathbb{Z} \text{ and } am + bn > 0\}$

S = \{ am + bn : m, n \in {\mathbb Z} \text{ and } am + bn \gt 0
\}

510. ⠙ ⠨⠅ ⠁⠗⠬⠃⠎

$d = ar + bs$

d = ar + bs

511. ⠙ ⠨⠅ ⠛⠉⠙⠷⠁ ⠠⠃ ⠾

$d = \gcd(a, b)$

d = \gcd(a, b)

512. ⠁⠀⠨⠅⠀⠙⠟⠬⠗⠄

$a = dq + r'$

a = dq + r'

513. ⠴⠀⠅⠱⠀⠗⠄⠀⠐⠅⠀⠙

$0 \leq r' < d$

0 \leq r' \lt d

514. ⠗⠄⠀⠨⠂⠀⠴

$r' > 0$

r' \gt 0

515. ⠗⠄⠀⠨⠅⠀⠴

$r' = 0$

r' = 0

516. ⠁⠀⠨⠅⠀⠙⠄⠓

$a = d'h$

a = d'h

517. ⠃⠀⠨⠅⠀⠙⠄⠅

$b = d'k$

b = d'k

518. ⠙⠀⠨⠅⠀⠁⠗⠬⠃⠎⠀⠨⠅⠀⠙⠄⠓⠗⠬⠙⠄⠅⠎⠀⠨⠅⠀⠙⠄⠷⠓⠗⠬⠅⠎⠾

$d = ar + bs = d'hr + d'ks = d'(hr + ks)$

d = ar + bs = d'hr + d'ks = d'(hr + ks)

519. ⠁⠗⠬⠃⠎⠀⠨⠅⠀⠂

$ar + bs = 1$

ar + bs = 1

520. ⠼945

945

945

521. ⠼2415

2415

2415

522. ⠼105

105

105

523. ⠼420

420

420

524. ⠼525

525

525

525. ⠛⠉⠙⠷945⠠2415⠾ ⠶ 105

$\gcd(945, 2415) = 105$

\gcd(945, 2415) = 105

526. 945⠗⠬2415⠎ ⠶ 105

$945r + 2415s = 105$

945 r + 2415 s = 105

527. ⠗ ⠶ ⠤⠢

$r = -5$

r = -5

528. ⠎ ⠶ ⠆

$s = 2$

s= 2

529. ⠗ ⠶ 41

$r = 41$

r = 41

530. ⠎ ⠶ ⠤16

$s = -16$

s = -16

531. ⠛⠉⠙⠷⠁⠠⠃⠾ ⠶ ⠙

$\gcd(a, b) = d$

\gcd(a,b) = d

532. ⠗⠂ ⠕ ⠗⠆ ⠕ ⠄⠄⠄ ⠕ ⠗⠰⠝⠐ ⠶ ⠙

$r_1 > r_2 > \cdots > r_n = d$

r_1 \gt r_2 \gt \cdots \gt r_n = d

533. ⠁⠗⠬⠃⠎ ⠶ ⠙

$ar + bs = d$

ar + bs = d

534. ⠏ ⠕ ⠂

$p > 1$

p \gt 1

535. ⠏⠳⠁⠃

$p \mid ab$

p \mid ab

536. ⠏⠳⠁

$p \mid a$

p \mid a

537. ⠏⠳⠃

$p \mid b$

p \mid b

538. ⠛⠉⠙⠷⠁ ⠠⠏⠾ ⠨⠅ ⠂

$\gcd(a, p) = 1$

\gcd(a, p) = 1

539. ⠁⠗⠬⠏⠎ ⠨⠅ ⠂

$ar + ps = 1$

ar + ps = 1

540. ⠃ ⠨⠅ ⠃⠷⠁⠗⠬⠏⠎⠾ ⠨⠅ ⠷⠁⠃⠾⠗⠬⠏⠷⠃⠎⠾

$b = b(ar + ps) = (ab)r + p(bs)$

b = b(ar + ps) = (ab)r + p(bs)

541. ⠁⠃

ab

ab

542. ⠃ ⠨⠅ ⠷⠁⠃⠾⠗⠬⠏⠷⠃⠎⠾

$b = (ab)r + p(bs)$

b = (ab)r + p(bs)

543. ⠏⠂ ⠠⠏⠆ ⠄⠄ ⠠⠏⠰⠝⠐

$p_1, p_2, \ldots, p_n$

p_1, p_2, \ldots, p_n

544. ⠠⠏ ⠨⠅ ⠏⠂⠏⠆⠄⠄⠄⠏⠰⠝⠐⠬⠂

$P = p_1 p_2 \cdots p_n + 1$

P = p_1 p_2 \cdots p_n + 1

545. ⠏⠰⠊

p_i

p_i

546. ⠂ ⠐⠅⠱ ⠊ ⠐⠅⠱ ⠝

$1 \leq i \leq n$

1 \leq i \leq n

547. ⠠⠏⠤⠏⠂⠏⠆⠄⠄⠄⠏⠰⠝⠐ ⠨⠅ ⠂

$P - p_1 p_2 \cdots p_n = 1$

P - p_1 p_2 \cdots p_n = 1

548. ⠏ ⠌⠨⠅ ⠏⠰⠊

$p \neq p_i$

p \neq p_i

549. ⠝ ⠨⠅ ⠏⠂⠏⠆⠄⠄⠄⠏⠰⠅

$n = p_1 p_2 \cdots p_k$

n = p_1 p_2 \cdots p_k

550. ⠏⠂ ⠄⠄ ⠠⠏⠰⠅⠐

$p_1, \ldots, p_k$

p_1, \ldots, p_k

551. ⠝ ⠶ ⠟⠂⠟⠆⠄⠄⠄⠟⠰⠇

$n = q_1 q_2 \cdots q_l$

n = q_1 q_2 \cdots q_l

552. ⠅ ⠶ ⠇

$k = l$

k = l

553. ⠟⠰⠊

q_i

q_i

554. ⠝ ⠶ ⠆

$n = 2$

n = 2

555. m

m

556. ⠂ ⠅⠱ ⠍ ⠐⠅ ⠝

$1 \leq m < n$

1 \leq m \lt n

557. ⠝ ⠶ ⠏⠂⠏⠆⠄⠄⠄⠏⠰⠅⠐ ⠶ ⠟⠂⠟⠆⠄⠄⠄⠟⠰⠇

$n = p_1 p_2 \cdots p_k = q_1 q_2 \cdots q_l$

n = p_1 p_2 \cdots p_k = q_1 q_2 \cdots q_l

558. ⠏⠂ ⠅⠱ ⠏⠆ ⠅⠱ ⠄⠄⠄ ⠅⠱ ⠏⠰⠅

$p_1 \leq p_2 \leq \cdots \leq p_k$

p_1 \leq p_2 \leq \cdots \leq p_k

559. ⠟⠂ ⠅⠱ ⠟⠆ ⠅⠱ ⠄⠄⠄ ⠅⠱ ⠟⠰⠇

$q_1 \leq q_2 \leq \cdots \leq q_l$

q_1 \leq q_2 \leq \cdots \leq q_l

560. ⠏⠂ ⠳⠟⠰⠊

$p_1 \mid q_i$

p_1 \mid q_i

561. ⠊ ⠶ ⠂ ⠤ ⠠⠇

$i = 1, \ldots, l$

i = 1, \ldots, l

562. ⠟⠂ ⠳⠏⠰⠚

$q_1 \mid p_j$

q_1 \mid p_j

563. ⠚ ⠶ ⠂ ⠤ ⠠⠅

$j = 1, \ldots, k$

j = 1, \ldots, k

564. ⠏⠂ ⠨⠅ ⠟⠰⠊

$p_1 = q_i$

p_1 = q_i

565. ⠟⠂ ⠨⠅ ⠏⠰⠚

$q_1 = p_j$

q_1 = p_j

566. ⠏⠂ ⠨⠅ ⠟⠂

$p_1 = q_1$

p_1 = q_1

567. ⠏⠂ ⠅⠱ ⠏⠰⠚⠐ ⠨⠅ ⠟⠂ ⠅⠱ ⠟⠰⠊⠐ ⠨⠅ ⠏⠂

$p_1 \leq p_j = q_1 \leq q_i = p_1$

p_1 \leq p_j = q_1 \leq q_i = p_1

568. ⠝⠄ ⠨⠅ ⠏⠆⠄⠄⠄⠏⠰⠅⠐ ⠨⠅ ⠟⠆⠄⠄⠄⠟⠰⠇

$n' = p_2 \cdots p_k = q_2 \cdots q_l$

n' = p_2 \cdots p_k = q_2 \cdots q_l

569. ⠟⠰⠊⠐ ⠨⠅ ⠏⠰⠊

$q_i = p_i$

q_i = p_i

570. ⠊ ⠨⠅ ⠂ ⠄⠄ ⠠⠅

$i = 1, \ldots, k$

i = 1, \ldots, k

571. ⠁ ⠨⠅ ⠁⠂⠁⠆

$a = a_1 a_2$

a = a_1 a_2

572. ⠂ ⠐⠅ ⠁⠂ ⠐⠅ ⠁

$1 < a_1 < a$

1 \lt a_1 \lt a

573. ⠂ ⠐⠅ ⠁⠆ ⠐⠅ ⠁

$1 < a_2 < a$

1 \lt a_2 \lt a

574. ⠁⠂ ⠈⠑ ⠠⠎

$a_1 \in S$

a_1\in S

575. ⠁⠆ ⠈⠑ ⠠⠎

$a_2 \in S$

a_2 \in S

576. ⠁ ⠨⠅ ⠁⠂⠁⠆ ⠨⠅ ⠏⠂⠄⠄⠄⠏⠰⠗⠐⠟⠂⠄⠄⠄⠟⠰⠎

$a = a_1 a_2 = p_1 \cdots p_r q_1 \cdots q_s$

a = a_1 a_2 = p_1 \cdots p_r q_1 \cdots q_s

577. ⠁⠌⠈⠑⠠⠎

$a \notin S$

a \notin S

578. ⠋⠷⠝⠾

$f(n)$

f(n)

579. ⠆⠘⠆⠘⠘⠝⠐⠬⠂

$2^{2^n} + 1$

2^{2^n} + 1

580. ⠆⠘⠆⠘⠘⠢⠐⠬⠂ ⠨⠅ 4,294,967,297

$2^{2^5} + 1 = 4{,}294{,}967{,}297$

2^{2^5} + 1 = 4{,}294{,}967{,}297

581. ⠲ ⠨⠅ ⠆⠬⠆

$4 = 2 + 2$

4 = 2 + 2

582. ⠖ ⠨⠅ ⠒⠬⠒

$6 = 3 + 3$

6 = 3 + 3

583. ⠦ ⠨⠅ ⠒⠬⠢

$8 = 3 + 5$

8 =3 + 5

584. ⠄

$\ldots$

\ldots

585. ⠲⠡10⠘18

4×10^{18}

4 \times 10^{18}

586. ⠂⠘⠆⠐⠬⠆⠘⠆⠐⠬⠄⠄⠄⠬⠝⠘⠆⠐ ⠨⠅ ⠹⠝⠷⠝⠬⠂⠾⠷⠆⠝⠬⠂⠾⠌⠖⠼

$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$

1^2 + 2^2 + \cdots + n^2 = \frac{n(n + 1)(2n + 1)}{6}

587. ⠠⠎⠷⠂⠾⠱⠈⠷⠂⠷⠂⠬⠂⠾⠷⠆⠷⠂⠾⠬⠂⠾⠈⠾⠌⠖ ⠨⠅ ⠂ ⠨⠅ ⠂⠘⠆

$S(1) : [1(1+1)(2(1)+1)]/6 = 1 = 1^2$

S(1): [1(1 + 1)(2(1) + 1)]/6 = 1 = 1^2

588. ⠠⠎⠷⠅⠾⠱⠂⠘⠆⠐⠬⠆⠘⠆⠐⠬⠄⠄⠄⠬⠅⠘⠆⠐ ⠨⠅ ⠈⠷⠅⠷⠅⠬⠂⠾⠷⠆⠅⠬⠂⠾⠈⠾⠌⠖

$S(k) : 1^2 + 2^2 + \cdots + k^2 = [k(k+1)(2k+1)]/6$

S(k): 1^2 + 2^2 + \cdots + k^2 = [k(k + 1)(2k + 1)]/6

589. ⠂⠘⠒⠐⠬⠆⠘⠒⠐⠬⠄⠄⠄⠬⠝⠘⠒⠐ ⠨⠅ ⠹⠝⠘⠆⠐⠷⠝⠬⠂⠾⠘⠆⠐⠌⠲⠼

$1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$

1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n + 1)^2}{4}

590. ⠝⠯ ⠨⠂ ⠆⠘⠝

$n! > 2^n$

n! \gt 2^n

591. ⠝ ⠨⠂⠱ ⠲

$n \geq 4$

n \geq 4

592. ⠠⠎⠷⠲⠾⠸⠒⠲⠯ ⠨⠅ 24 ⠨⠂ 16 ⠨⠅ ⠆⠘⠲

$S(4) : 4! = 24 > 16 = 2^4$

S(4): 4! = 24 \gt 16 =2^4

593. ⠠⠎⠷⠅⠾⠸⠒⠅⠯ ⠨⠂ ⠆⠘⠅

$S(k) : k! > 2^k$

S(k): k! \gt 2^k

594. ⠷⠅⠬⠂⠾⠯ ⠨⠅ ⠅⠯⠷⠅⠬⠂⠾ ⠨⠂ ⠆⠘⠅⠐⠡⠆ ⠨⠅ ⠆⠘⠅⠬⠂

$(k+1)! = k!(k+1) > 2^k \cdot 2 = 2^{k+1}$

(k + 1)! = k! (k + 1) \gt 2^k \cdot 2 = 2^{k + 1}

595. ⠭⠬⠲⠭⠬⠶⠭⠬⠄⠄⠄⠬⠷⠒⠝⠤⠆⠾⠭ ⠨⠅ ⠹⠝⠷⠒⠝⠤⠂⠾⠭⠌⠆⠼

$x + 4x + 7x + \cdots + (3n-2)x = \frac{n(3n-1)x}{2}$

x + 4x + 7x + \cdots + (3n - 2)x = \frac{n(3n - 1)x}{2}

596. 10⠘⠝⠬⠂⠐⠬10⠘⠝⠐⠬⠂

$10^{n+1} + 10^n + 1$

10^{n + 1} + 10^n + 1

597. ⠲⠡10⠘⠆⠝⠐⠬⠔⠡10⠘⠆⠝⠤⠂⠐⠬⠢

$4 \cdot 10^{2n} + 9 \cdot 10^{2n-1} + 5$

4 \cdot 10^{2n} + 9 \cdot 10^{2n - 1} + 5

598. ⠼99

99

99

599. ⠣⠝⠜⠁⠂⠁⠆⠄⠄⠄⠁⠰⠝⠻ ⠐⠅⠱ ⠹⠂⠌⠝⠼⠐⠎⠩⠅⠨⠅⠂⠣⠝⠻ ⠐⠁⠰⠅

$\sqrt[n]{a_1 a_2 \cdots a_n} \leq \frac{1}{n} \sum_{k=1}^{n} a_k$

\sqrt[n]{a_1 a_2 \cdots a_n} \leq \frac{1}{n} \sum_{k = 1}^{n} a_k

600. ⠋ ⠘⠷⠝⠾⠐⠷⠭⠾

$f^{(n)}(x)$

f^{(n)} (x)

601. ⠋ ⠘⠷⠝⠾

$f^{(n)}$

f^{(n)}

602. ⠷⠋⠛⠾⠘⠷⠝⠾⠐⠷⠭⠾ ⠨⠅ ⠐⠨⠠⠎⠩⠅ ⠨⠅ ⠴⠣⠝⠻⠷⠝⠩⠅⠾⠋⠘⠷⠅⠾⠐⠷⠭⠾⠛⠘⠷⠝⠤⠅⠾⠐⠷⠭⠾

$(fg)^{(n)}(x) = \sum_{k=0}^{n} \binom{n}{k} f^{(k)}(x) g^{(n-k)}(x)$

(fg)^{(n)}(x) = \sum_{k = 0}^{n} \binom{n}{k} f^{(k)}(x) g^{(n -
k)}(x)

603. ⠂⠬⠆⠬⠆⠘⠆⠐⠬⠄⠄⠄⠬⠆⠘⠝⠐ ⠨⠅ ⠆⠘⠝⠬⠂⠐⠤⠂

$1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$

1 + 2 + 2^2 + \cdots + 2^n = 2^{n + 1} - 1

604. one-half⠬one-sixth⠬⠄⠄⠄⠬⠹⠂⠌⠝⠷⠝⠬⠂⠾⠼ ⠨⠅ ⠹⠝⠌⠝⠬⠂⠼

$\frac{1}{2} + \frac{1}{6} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

\frac{1}{2}+ \frac{1}{6} + \cdots + \frac{1}{n(n + 1)} = \frac{n}{n
+ 1}

605. ⠷⠂⠬⠭⠾⠘⠝⠐⠤⠂ ⠨⠂⠨⠅ ⠝⠭

$(1 + x)^n - 1 \geq nx$

(1 + x)^n - 1 \geq nx

606. ⠝ ⠨⠅ ⠴⠠⠂⠠⠆⠠ ⠄⠄⠄

$n = 0, 1, 2, \ldots$

n = 0, 1, 2, \ldots

607. ⠠⠎⠷⠴⠾⠸⠒⠷⠂⠬⠭⠾⠘⠴⠐⠤⠂ ⠨⠅ ⠴ ⠨⠂⠨⠅ ⠴ ⠨⠅ ⠴⠡⠭

$S(0) : (1 + x)^0 - 1 = 0 \geq 0 = 0 \cdot x$

S(0): (1 + x)^0 - 1 = 0 \geq 0 = 0 \cdot x

608. ⠠⠎⠷⠅⠾⠸⠒⠷⠂⠬⠭⠾⠘⠅⠐⠤⠂ ⠨⠂⠨⠅ ⠅⠭

$S(k) : (1 + x)^k - 1 \geq kx$

S(k): (1 + x)^k -1 \geq kx

609. caligraphic ⠠⠏⠷⠠⠭⠾

$\mathcal{P}(X)$

{\mathcal P}(X)

610. caligraphic ⠠⠏⠷StartSet⠁ ⠠⠃ EndSet⠾ ⠨⠅ StartSet⠸⠴ ⠠StartSet⠁ EndSet ⠠StartSet⠃ EndSet

$\mathcal{P}(\{a, b\}) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

{\mathcal P}(\{a, b\}) = \{ \emptyset, \{a\}, \{b\}, \{a, b\}
\}

611. ⠆⠘⠝

2^n

2^n

612. ⠠⠎ ⠸⠐⠅ ⠈⠠⠝

$S \subset \mathbb{N}$

S \subset {\mathbb N}

613. ⠝⠬⠂ ⠈⠑ ⠠⠎

$n + 1 \in S$

n + 1 \in S

614. ⠠⠎ ⠨⠅ ⠈⠠⠝

$S = \mathbb{N}$

`S = {\mathbb N}`

615. ⠛⠉⠙⠷⠁⠠⠃⠾

$\gcd(a, b)$

`\gcd(a,b)`

616. ⠛⠉⠙⠷⠁⠠⠃⠾ ⠨⠅ ⠗⠁⠬⠎⠃

$\gcd(a, b) = ra + sb$

`\gcd(a,b) = ra + sb`

617. ⠼14

14

`14`

618. ⠼39

39

`39`

619. ⠼234

234

`234`

620. ⠼165

165

`165`

621. ⠼1739

1739

`1739`

622. ⠼9923

9923

`9923`

623. ⠼471

471

`471`

624. ⠼562

562

`562`

625. ⠼23771

23771

`23771`

626. ⠼19945

19945

`19945`

627. ⠤4357

-4357

-4357

628. ⠼3754

3754

3754

629. ⠁⠗⠬⠃⠎⠀⠨⠅⠀⠂

$ar + bs = 1$

ar + bs =1

630. ⠼⠂⠠⠼⠂⠠⠼⠆⠠⠼⠒⠠⠼⠢⠠⠼⠦⠠⠼13⠠⠼21⠠⠄

$1, 1, 2, 3, 5, 8, 13, 21, \ldots$

1, 1, 2, 3, 5, 8, 13, 21, \ldots

631. ⠋⠂⠀⠨⠅⠀⠂

$f_1 = 1$

f_1 = 1

632. ⠋⠆⠀⠨⠅⠀⠂

$f_2 = 1$

f_2 = 1

633. ⠋⠰⠝⠬⠆⠐⠀⠨⠅⠀⠋⠰⠝⠬⠂⠐⠬⠋⠰⠝

$f_{n+2} = f_{n+1} + f_n$

f_{n + 2} = f_{n + 1} + f_n

634. ⠋⠰⠝⠐⠀⠐⠅⠀⠆⠘⠝

$f_n < 2^n$

f_n \lt 2^n

635. ⠋⠰⠝⠬⠂⠐⠋⠰⠝⠤⠂⠐⠀⠨⠅⠀⠋⠰⠝⠘⠆⠐⠬⠷⠤⠂⠾⠘⠝

$f_{n+1} f_{n-1} = f_n^2 + (-1)^n$

f_{n + 1} f_{n - 1} = f^2_n + (-1)^n

636. ⠝⠀⠨⠂⠱⠀⠆

$n \geq 2$

n \geq 2

637. ⠋⠰⠝⠐⠀⠨⠅⠀⠈⠷⠷⠂⠬⠜⠢⠻⠾⠘⠝⠐⠤⠷⠂⠤⠜⠢⠻⠾⠘⠝⠐⠈⠾⠌⠆⠘⠝⠐⠜⠢⠻

$f_n = [(1 + \sqrt{5}\,)^n - (1 - \sqrt{5}\,)^n]/2^n \sqrt{5}$

f_n = [(1 + \sqrt{5}\,)^n - (1 - \sqrt{5}\,)^n]/ 2^n \sqrt{5}

638. ⠐⠇⠊⠍⠩⠝⠀⠫⠕⠀⠠⠿⠻⠋⠰⠝⠐⠌⠋⠰⠝⠬⠂⠐⠀⠨⠅⠀⠷⠜⠢⠻⠤⠂⠾⠌⠆

$\lim_{n \rightarrow \infty} f_n / f_{n+1} = (\sqrt{5} - 1)/2$

\lim_{n \rightarrow \infty} f_n / f_{n + 1} = (\sqrt{5} - 1)/2

639. ⠋⠰⠝

f_n

f_n

640. ⠋⠰⠝⠬⠂

f_{n+1}

`f_{n + 1}`

641. ⠛⠉⠙⠷⠁⠠⠃⠾ ⠨⠅ ⠂

$\gcd(a,b) = 1$

`\gcd(a,b) = 1`

642. ⠛⠉⠙⠷⠁⠠⠎⠾ ⠨⠅ ⠛⠉⠙⠷⠗⠠⠃⠾ ⠨⠅ ⠛⠉⠙⠷⠗⠠⠎⠾ ⠨⠅ ⠂

$\gcd(a,s) = \gcd(r,b) = \gcd(r,s) = 1$

`\gcd(a,s) = \gcd(r,b) = \gcd(r,s) = 1`

643. ⠭⠠⠽⠈⠑⠈⠠⠝

$x, y \in {\mathbb N}$

`x, y \in {\mathbb N}`

644. ⠭⠽

xy

`xy`

645. ⠲⠅

$4k$

`4k`

646. ⠲⠅⠬⠂

$4k + 1$

`4k + 1`

647. ⠁⠠⠃⠠⠗⠠⠎

a, b, r, s

`a, b, r, s`

648. ⠼⠴⠠⠼⠂⠠⠄⠠⠝⠤⠂

$0, 1, \ldots, n-1$

`0, 1, \ldots, n-1`

649. ⠴ ⠅⠱ ⠎ ⠐⠅ ⠝

$0 \leq s < n$

`0 \leq s \lt n`

650. ⠈⠷⠗⠈⠾ ⠨⠅ ⠈⠷⠎⠈⠾

$[r] = [s]$

`[r] = [s]`

651. ⠇⠉⠍⠷⠁⠠⠃⠾

$\operatorname{lcm}(a,b)$

`\lcm(a,b)`

652. ⠙ ⠨⠅ ⠛⠉⠙⠷⠁⠠⠃⠾

$d= \gcd(a, b)$

`d= \gcd(a, b)`

653. ⠍⠀⠨⠅⠀⠇⠉⠍⠷⠁⠠⠃⠾

$m = \operatorname{lcm}(a, b)$

`m = \lcm(a, b)`

654. ⠙⠍⠀⠨⠅⠀⠳⠁⠃⠳

$dm = |ab|$

`dm = |ab|`

655. ⠇⠉⠍⠷⠁⠠⠃⠾⠀⠨⠅⠀⠁⠃

$\operatorname{lcm}(a,b) = ab$

`\lcm(a,b) = ab`

656. ⠛⠉⠙⠷⠁⠠⠉⠾⠀⠨⠅⠀⠛⠉⠙⠷⠃⠠⠉⠾⠀⠨⠅⠀⠂

$\gcd(a,c) = \gcd(b,c) = 1$

`\gcd(a,c) = \gcd(b,c) =1`

657. ⠛⠉⠙⠷⠁⠃⠠⠉⠾⠀⠨⠅⠀⠂

$\gcd(ab,c) = 1$

`\gcd(ab,c) = 1`

658. c

`c`

659. ⠁⠠⠃⠠⠉⠈⠑⠈⠠⠵

$a, b, c \in \mathbb{Z}$

`a, b, c \in {\mathbb Z}`

660. ⠁⠳⠃⠉

$a \mid bc$

`a \mid bc`

661. ⠁⠳⠉

$a \mid c$

`a \mid c`

662. ⠁⠉⠗⠬⠃⠉⠎⠀⠨⠅⠀⠉

$acr + bcs = c$

`acr + bcs = c`

663. ⠏⠀⠨⠂⠱⠀⠆

$p \geq 2$

`p \geq 2`

664. ⠆⠘⠏⠐⠤⠂

$2^p - 1$

`2^p - 1`

665. ⠖⠝⠬⠢

$6n + 5$

`6n + 5`

666. ⠖⠝⠬⠂

$6n + 1$

`6n + 1`

667. ⠖⠅ ⠬⠢

$6k + 5$

`6k + 5`

668. ⠲⠝⠤⠂

$4n - 1$

`4n - 1`

669. ⠏⠘⠆⠐ ⠶ ⠆⠟⠘⠆

$p^2 = 2q^2$

`p^2 = 2 q^2`

670. ⠜⠆⠻

$\sqrt{2}$

`\sqrt{2}`

671. N

`N`

672. ⠂ ⠐⠅ ⠝ ⠐⠅ ⠠⠝

$1 < n < N$

`1 \lt n \lt N`

673. ⠼⠢

5

`5`

674. ⠜⠠⠝⠻

$\sqrt{N}$

`\sqrt{N}`

675. ⠠⠝ ⠶ 250

$N = 250$

`N = 250`

676. ⠠⠝ ⠶ 120

$N = 120$

`N= 120`

677. ⠈⠠⠝⠘⠴⠐ ⠶ ⠈⠠⠝⠨⠬StartSet⠴EndSet

$\mathbb{N}^0 = \mathbb{N} \cup \{0\}$

`{\mathbb N}^0 = {\mathbb N} \cup \{ 0 \}`

678. ⠠⠁⠱⠈⠠⠝⠘⠴⠐⠡⠈⠠⠝⠘⠴⠐ ⠫⠕ ⠈⠠⠝⠘⠴

$A : \mathbb{N}^0 \times \mathbb{N}^0 \rightarrow \mathbb{N}^0$

`A :{\mathbb N}^0 \times {\mathbb N}^0 \rightarrow {\mathbb N}^0`

679. ⠠⠁ ⠷⠒ ⠠⠂ ⠾

$A(3,1)$

`A(3, 1)`

680. ⠠⠁ ⠷⠲ ⠠⠂ ⠾

$A(4,1)$

`A(4, 1)`

681. ⠠⠁ ⠷⠢ ⠠⠂ ⠾

$A(5,1)$

`A(5, 1)`

682. ⠛⠉⠙⠷⠁ ⠠⠃ ⠾

$\gcd(a,b)$

`\gcd( a,b)`

683. ⠛⠉⠙⠷⠁ ⠠⠃ ⠾ ⠶ ⠗⠁ ⠬⠎⠃

$\gcd(a,b) = ra + sb$

`\gcd( a,b) = ra + sb`

684. ⠴ ⠅⠱ ⠗ ⠐⠅ ⠃

$0 \leq r < b$

`0\leq r\lt b`

685. ⠁ ⠶ ⠃⠟⠬⠗

$a = bq + r$

`a=bq+r`

686. ⠷⠁ ⠤⠗⠾⠌⠃

$(a-r)/b$

`(a-r)/b`

687. ⠗⠁ ⠬⠎⠃ ⠶ ⠛⠉⠙⠷⠁ ⠠⠃ ⠾

$ra + sb = \gcd(a,b)$

`ra+sb=\gcd(a,b)`

688. ⠃ ⠤⠂

$b - 1$

`b-1`

689. 2600 ⠶ ⠆ ⠘⠒ ⠐⠈⠡⠢ ⠘⠆ ⠐⠈⠡13

$2600 = 2^3 \times 5^2 \times 13$

`2600 = 2^3\times 5^2\times 13`

690. ⠼2600

2600

`2600`

691. ⠉ ⠶ ⠲598037234

$c = 4\,598\,037\,234$

`c=4\,598\,037\,234`

692. ⠙ ⠶ ⠶

$d = 7$

d=7

693. ⠙ ⠶ 11

$d = 11$

d=11

694. ⠁ ⠤⠃

$a - b$

a - b

695. ⠁ ⠠⠵ ⠰⠝

$\mathbb{Z}_n$

{\mathbb Z}_n

696. ⠼12

12

12

697. ⠼⠴⠠ ⠼⠂ ⠠⠄ ⠠⠼11

$0, 1, \ldots, 11$

0, 1, \ldots, 11

698. ⠈⠷⠴ ⠈⠾ ⠠ ⠈⠷⠂ ⠈⠾ ⠄⠄ ⠠ ⠈⠷11 ⠈⠾

$[0], [1], \ldots, [11]$

{[0]}, {[1]}, \ldots, {[11]}

699. ⠷⠁ ⠬⠃ ⠾⠷⠍⠕⠙⠝⠾

$(a + b) \pmod{n}$

(a + b) \pmod{n}

700. ⠁ ⠬⠃

$a + b$

a + b

701. ⠷⠁ ⠃ ⠾⠷⠍⠕⠙⠝⠾

$(ab) \pmod{n}$

(a b) \pmod{ n}

702. ⠁ ⠃

ab

a b

703. ⠁ ⠠⠵⠦

$\mathbb{Z}_8$

{\mathbb Z}_8

704. ⠼⠖

6

6

705. ⠅⠝ ⠸⠇ ⠂ ⠷⠍⠕⠙⠦⠾

$kn \equiv 1 \pmod{8}$

```
k n \equiv 1 \pmod{ 8}
```

706. ⠈ ⠠⠵⠦

$\mathbb{Z}_8$

```
{\mathbb Z_8}
```

707. StartLayout1stRow⠡ ⠼⠴ ⠼⠂ ⠼⠆ ⠼⠒ ⠼⠲ ⠼⠢ ⠼⠖ ⠼⠶2ndRow⠼⠴ ⠼⠴ ⠼⠴ ⠼⠴ ⠼⠴ ⠼⠴ ⠠

$\cdot$	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7
2	0	2	4	6	0	2	4	6
3	0	3	6	1	4	7	2	5
4	0	4	0	4	0	4	0	4
5	0	5	2	7	4	1	6	3
6	0	6	4	2	0	6	4	2
7	0	7	6	5	4	3	2	1

```
\begin{array}{c|cccccccc} \cdot & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7
\\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 2 & 3
& 4 & 5 & 6 & 7 \\ 2 & 0 & 2 & 4 & 6 & 0 & 2 & 4 & 6 \\ 3 & 0 &
3 & 6 & 1 & 4 & 7 & 2 & 5 \\ 4 & 0 & 4 & 0 & 4 & 0 & 4 & 0 & 4 \\
5 & 0 & 5 & 2 & 7 & 4 & 1 & 6 & 3 \\ 6 & 0 & 6 & 4 & 2 & 0 & 6 &
4 & 2 \\ 7 & 0 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \end{array}
```

708. ⠁⠠ ⠃⠠ ⠉ ⠈⠑ ⠈ ⠠⠵⠰⠝⠐

$a, b, c \in \mathbb{Z}_n$

```
a, b, c \in {\mathbb Z}_n
```

709. ⠁⠷⠃⠬⠉⠾⠸⠇ ⠁⠃⠬⠁⠉ ⠷⠍⠕⠙⠝⠾

$a(b + c) \equiv ab + ac \pmod{n}$

```
a (b + c) \equiv a b + a c \pmod{ n}
```

710. ⠤⠁

$-a$

```
-a
```

711. ⠁ ⠬⠷⠤⠁ ⠾ ⠸⠇ ⠴⠷⠍⠕⠙⠝⠾

$a + (-a) \equiv 0 \pmod{n}$

```
a + (-a) \equiv 0 \pmod{ n}
```

712. ⠛⠉⠙⠷⠁ ⠠⠝⠾ ⠨⠅ ⠂

$\gcd(a, n) = 1$

```
\gcd(a,n) = 1
```

713. ⠁ ⠷⠍⠕⠙⠝⠾

$a \pmod{n}$

```
a \pmod{n}
```

714. ⠁⠃ ⠸⠇ ⠂ ⠷⠍⠕⠙⠝⠾

$ab \equiv 1 \pmod{n}$

```
a b \equiv 1 \pmod{ n}
```

715. ⠃ ⠬⠁

$b + a$

`b + a`

716. ⠛⠉⠙⠷⠁ ⠠⠝⠾ ⠨⠅ ⠂

$\gcd(a, n) = 1$

`\gcd(a, n) = 1`

717. ⠁⠗⠬⠝⠎ ⠨⠅ ⠂

$ar + ns = 1$

`ar + ns = 1`

718. ⠝⠎ ⠨⠅ ⠂⠤⠁⠗

$ns = 1 - ar$

`ns = 1 - ar`

719. ⠁⠗ ⠸⠇⠂⠷⠍⠕⠙⠝⠾

$ar \equiv 1 \pmod{n}$

`ar \equiv 1 \pmod{n}`

720. ⠁⠃ ⠸⠇⠂⠷⠍⠕⠙⠝⠾

$ab \equiv 1 \pmod{n}$

`a b \equiv 1\pmod{n}`

721. ⠁⠃ ⠸⠇⠂⠷⠍⠕⠙⠝⠾

$ab \equiv 1 \pmod{n}$

`ab \equiv 1 \pmod{ n}`

722. ⠁⠃⠤⠂

$ab - 1$

`ab -1`

723. ⠁⠃⠤⠝⠅ ⠨⠅ ⠂

$ab - nk = 1$

`ab - nk = 1`

724. ⠙ ⠨⠅ ⠛⠉⠙⠷⠁ ⠠⠝⠾

$d = \gcd(a, n)$

`d = \gcd(a,n)`

725. ⠁⠃⠤⠝⠅

$ab - nk$

`ab - nk`

726. ⠙ ⠨⠅ ⠂

$d = 1$

`d = 1`

727. 180⠘⠘⠋⠄

180°

`180^{\circ}`

728. 360⠘⠘⠨⠡

360°

```
360^{\circ}
```

729. 90⠘⠘⠨⠡

90°

```
90^{\circ}
```

730. ⠫⠞⠠⠁⠠⠃⠠⠉

$\bigtriangleup ABC$

```
\bigtriangleup ABC
```

731. ⠨⠏⠱⠠⠎ ⠫⠕ ⠠⠎

$\pi :S \rightarrow S$

```
\pi :S \rightarrow S
```

732. ⠼⠒⠯ ⠶ ⠖

$3! = 6$

```
3! = 6
```

733. ⠒⠡⠆⠡⠂ ⠶ ⠒⠯ ⠶ ⠖

$3 \cdot 2 \cdot 1 = 3! = 6$

```
3 \cdot 2 \cdot 1 = 3! = 6
```

734. ⠠⠷⠠⠁ ⠠⠃ ⠠⠉⠠⠾⠠⠷⠠⠃ ⠠⠉ ⠠⠁⠠⠾

$\begin{pmatrix} A & B & C \\ B & C & A \end{pmatrix}$

```
\begin{pmatrix} A & B & C \\ B & C & A \end{pmatrix}
```

735. 120⠘⠘⠨⠡

120°

```
120^{\circ}
```

736. ⠨⠍⠂⠨⠗⠂

$\mu_1 \rho_1$

```
\mu_1 \rho_1
```

737. ⠨⠗⠂

ρ_1

```
\rho_1
```

738. ⠨⠍⠂

μ_1

```
\mu_1
```

739. ⠨⠍⠆

μ_2

```
\mu_2
```

740. ⠨⠍⠒

μ_3

\mu_3

741. ⠨⠗⠂ ⠨⠍⠂ ⠌⠨⠅ ⠨⠍⠂ ⠨⠗⠂

$\rho_1\mu_1 \neq \mu_1\rho_1$

\rho_1 \mu_1 \neq \mu_1 \rho_1

742. ⠨⠁

α

\alpha

743. ⠨⠃

β

\beta

744. ⠨⠁ ⠨⠃ ⠨⠅ ⠊⠙

$\alpha\beta = \mathrm{id}$

\alpha \beta = \identity

745. StartLayout1stRow ⠘⠨⠡ ⠊⠙ ⠨⠗⠂ ⠨⠗⠆ ⠨⠍⠂ ⠨⠍⠆ ⠨⠍⠒2ndRow ⠊⠙ ⠊⠙ ⠨⠗⠂ ⠨⠗⠆ ⠨⠍⠂

$\circ$	id	ρ_1	ρ_2	μ_1	μ_2	μ_3
id	id	ρ_1	ρ_2	μ_1	μ_2	μ_3
ρ_1	ρ_1	ρ_2	id	μ_3	μ_1	μ_2
ρ_2	ρ_2	id	ρ_1	μ_2	μ_3	μ_1
μ_1	μ_1	μ_2	μ_3	id	ρ_1	ρ_2
μ_2	μ_2	μ_3	μ_1	ρ_2	id	ρ_1
μ_3	μ_3	μ_1	μ_2	ρ_1	ρ_2	id

```
\begin{array}{c|cccccc} \circ & \identity & \rho_1 & \rho_2 & \mu_1
& \mu_2 & \mu_3 \\ \hline \identity & \identity & \rho_1 & \rho_2
& \mu_1 & \mu_2 & \mu_3 \\ \rho_1 & \rho_1 & \rho_2 & \identity
& \mu_3 & \mu_1 & \mu_2 \\ \rho_2 & \rho_2 & \identity & \rho_1
& \mu_2 & \mu_3 & \mu_1 \\ \mu_1 & \mu_1 & \mu_2 & \mu_3 & \identity
& \rho_1& \rho_2\\ \mu_2 & \mu_2 & \mu_3 & \mu_1 & \rho_2 & \identity
& \rho_1\\ \mu_3 & \mu_3 & \mu_1 & \mu_2 & \rho_1 & \rho_2& \identity
\end{array}
```

746. G

G

747. ⠠⠛⠡⠠⠛ ⠫⠕ ⠠⠛

$G \times G \rightarrow G$

G \times G \rightarrow G

748. ⠷⠁⠠⠃⠾ ⠈⠑ ⠠⠛⠡⠠⠛

$(a, b) \in G \times G$

(a,b) \in G \times G

749. ⠁ ⠘⠨⠡⠃

$a \circ b$

a \circ b

750. ⠷⠠⠛⠠⠘⠨⠡⠾

$(G, \circ)$

`(G, \circ )`

751. ⠷⠁⠠⠃⠾ ⠫⠳⠒⠒⠕ ⠁⠘⠨⠡⠃

$(a,b) \mapsto a \circ b$

`(a,b) \mapsto a \circ b`

752. ⠷⠁⠘⠨⠡⠃⠾⠘⠨⠡⠉ ⠶ ⠁⠘⠨⠡⠷⠃⠘⠨⠡⠉⠾

$(a \circ b) \circ c = a \circ (b \circ c)$

`(a \circ b) \circ c = a \circ (b \circ c)`

753. ⠁⠠⠃⠠⠉⠈⠑⠠⠛

$a, b, c \in G$

`a, b, c \in G`

754. ⠑⠈⠑⠠⠛

$e \in G$

`e \in G`

755. ⠁⠈⠑⠠⠛

$a \in G$

`a \in G`

756. ⠑⠘⠨⠡⠁ ⠶ ⠁⠘⠨⠡⠑ ⠶ ⠁

$e \circ a = a \circ e = a$

`e \circ a = a \circ e = a`

757. ⠁⠘⠤⠂

a^{-1}

`a^{-1}`

758. ⠁⠘⠨⠡⠁⠘⠤⠂⠐ ⠶ ⠁⠘⠤⠂⠐⠘⠨⠡⠁ ⠶ ⠑

$a \circ a^{-1} = a^{-1} \circ a = e$

`a \circ a^{-1} = a^{-1} \circ a = e`

759. ⠁⠘⠨⠡⠃ ⠶ ⠃⠘⠨⠡⠁

$a \circ b = b \circ a$

`a \circ b = b \circ a`

760. ⠁⠠⠃⠈⠑⠠⠛

$a, b \in G$

`a, b \in G`

761. ⠈⠠⠵ ⠶ StartSet⠄ ⠠⠤⠂ ⠠⠴ ⠠⠂ ⠠⠆ ⠠⠄ EndSet

$\mathbb{Z} = \{\ldots, -1, 0, 1, 2, \ldots\}$

`{\mathbb Z } = \{ \ldots , -1, 0, 1, 2, \ldots \}`

762. ⠍⠠⠝⠈⠑⠈⠠⠵

$m, n \in \mathbb{Z}$

`m, n \in {\mathbb Z}`

763. ⠬

$+$

+

764. ⠘⠨⠡

$\circ$

\circ

765. ⠍⠬⠝

$m + n$

m + n

766. ⠍⠘⠨⠡⠝

$m \circ n$

m \circ n

767. ⠝⠈⠑⠈⠠⠵

$n \in \mathbb{Z}$

n \in {\mathbb Z}

768. ⠤⠝

$-n$

-n

769. ⠝⠘⠤⠂

n^{-1}

n^{-1}

770. ⠍⠬⠝ ⠨⠅ ⠝⠬⠍

$m + n = n + m$

m + n = n + m

771. ⠍⠤⠝

$m - n$

m - n

772. ⠍⠬⠷⠤⠝⠾

$m + (-n)$

m + (-n)

773. ⠈⠠⠵⠢

$\mathbb{Z}_5$

{\mathbb Z}_5

774. ⠆⠬⠒ ⠨⠅ ⠒⠬⠆ ⠨⠅ ⠴

$2 + 3 = 3 + 2 = 0$

2 + 3 = 3 + 2 = 0

775. ⠈⠠⠵⠰⠝⠐ ⠨⠅ StartSet⠴ ⠠⠂ ⠄⠄ ⠄⠝⠤⠂ EndSet

$\mathbb{Z}_n = \{0, 1, \ldots, n - 1\}$

{\mathbb Z}_n = \{0, 1, \ldots, n-1 \}

776. ⠷⠈⠠⠵⠢⠠⠠⠬⠾

$(\mathbb{Z}_5, +)$

({\mathbb Z_5}, +)

777. StartLayout1stRow⠬ ⠼⠴ ⠼⠂ ⠼⠆ ⠼⠒ ⠼⠲2ndRow⠼⠴ ⠼⠴ ⠼⠂ ⠼⠆ ⠼⠒ ⠼⠲3rdRow⠼⠂ ⠼⠂

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

\begin{array}{c|ccccc} + & 0 & 1 & 2 & 3 & 4 \\ \hline 0 & 0 & 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 3 & 4 & 0 \\ 2 & 2 & 3 & 4 & 0 & 1 \\ 3 & 3 & 4 & 0 & 1 & 2 \\ 4 & 4 & 0 & 1 & 2 & 3 \end{array}

778. ⠂⠡⠅ ⠨⠅ ⠅⠡⠂ ⠨⠅ ⠅

$1 \cdot k = k \cdot 1 = k$

1 \cdot k = k \cdot 1 = k

779. ⠅ ⠈⠑ ⠈⠠⠵⠰⠝

$k \in \mathbb{Z}_n$

k \in {\mathbb Z}_n

780. ⠴⠡⠅ ⠨⠅ ⠅⠡⠴ ⠨⠅ ⠴

$0 \cdot k = k \cdot 0 = 0$

0 \cdot k = k \cdot 0 = 0

781. ⠈⠠⠵⠰⠝⠑⠸⠫⠱⠂⠐⠹⠄ StartSet⠴EndSet

$\mathbb{Z}_n \setminus \{0\}$

{\mathbb Z}_n \setminus \{0 \}

782. ⠆ ⠈⠑ ⠈⠠⠵⠖

$2 \in \mathbb{Z}_6$

2 \in {\mathbb Z}_6

783. ⠠⠥⠷⠝⠾

$U(n)$

U(n)

784. ⠈⠠⠵⠰⠝

$\mathbb{Z}_n$

\mathbb Z_n

785. ⠠⠥⠷⠦⠾

$U(8)$

U(8)

786. StartLayout1stRow⠡ ⠼⠂ ⠼⠒ ⠼⠢ ⠼⠶2ndRow⠼⠂ ⠼⠂ ⠼⠒ ⠼⠢ ⠼⠶3rdRow⠼⠒ ⠼⠒ ⠼⠂ ⠼⠶

$\cdot$	1	3	5	7
1	1	3	5	7
3	3	1	7	5
5	5	7	1	3
7	7	5	3	1

```
\begin{array}{c|cccc} \cdot & 1 & 3 & 5 & 7 \\ \hline 1 & 1 & 3
& 5 & 7 \\ 3 & 3 & 1 & 7 & 5 \\ 5 & 5 & 7 & 1 & 3 \\ 7 & 7 & 5 &
3 & 1 \end{array}
```

787. ⠨⠁⠨⠃ ⠨⠅ ⠨⠃⠨⠁

$\alpha\beta = \beta\alpha$

```
\alpha \beta = \beta \alpha
```

788. ⠠⠎⠒

S_3

```
S_3
```

789. ⠠⠙⠒

D_3

```
D_3
```

790. ⠈⠠⠍⠆⠷⠈⠠⠗⠾

$\mathbb{M}_2(\mathbb{R})$

```
{\mathbb M}_2 ( {\mathbb R})
```

791. ⠠⠛⠠⠇⠆⠷⠈⠠⠗⠾

$GL_2(\mathbb{R})$

```
GL_2({\mathbb R})
```

792. ⠝⠡⠝

$n \times n$

```
n \times n
```

793. ⠈⠠⠗

$\mathbb{R}$

```
\mathbb R
```

794. ⠠⠛⠠⠇⠆⠷⠈⠠⠗⠾

$GL_2(\mathbb{R})$

```
GL_2( {\mathbb R})
```

795. ⠠⠁⠘⠤⠂

A^{-1}

```
A^{-1}
```

796. ⠠⠁⠠⠁⠘⠤⠂⠐ ⠨⠅ ⠠⠁⠘⠤⠂⠐⠠⠁ ⠨⠅ ⠠⠊

$AA^{-1} = A^{-1}A = I$

```
A A^{-1} = A^{-1} A = I
```

797. ⠙⠑⠞⠀⠠⠁ ⠨⠅ ⠁⠙⠤⠃⠉ ⠌⠨⠅ ⠴

$\det A = ad - bc \neq 0$

`\det A = ad - bc \neq 0`

798. ⠠⠁ ⠈⠑ ⠠⠛⠠⠇⠆⠷⠈⠠⠗⠾

$A \in GL_2({\mathbb R})$

`A \in GL_2( {\mathbb R})`

799. ⠠⠁⠘⠤⠂⠐ ⠨⠅ ⠹⠂⠌⠁⠙⠤⠃⠉⠼ ⠠⠷⠙ ⠤⠃⠠⠾ ⠠⠷⠤⠉ ⠁⠠⠾

$A^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

`A^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}`

800. ⠠⠁⠠⠃ ⠨⠅ ⠠⠃⠠⠁

$AB = BA$

`AB = BA`

801. ⠊⠘⠆⠐ ⠨⠅ ⠤⠂

$i^2 = -1$

`i^2 = -1`

802. ⠠⠊⠘⠆⠐ ⠨⠅ ⠠⠚⠘⠆⠐ ⠨⠅ ⠠⠅⠘⠆⠐ ⠨⠅ ⠤⠂

$I^2 = J^2 = K^2 = -1$

`I^2 = J^2 = K^2 = -1`

803. ⠠⠊⠠⠚ ⠨⠅ ⠠⠅

$IJ=K$

`IJ=K`

804. ⠠⠚⠠⠅ ⠨⠅ ⠠⠊

$JK = I$

`JK = I`

805. ⠠⠅⠠⠊ ⠨⠅ ⠠⠚

$KI = J$

`KI = J`

806. ⠠⠚⠠⠊ ⠨⠅ ⠤⠠⠅

$JI = -K$

`JI = -K`

807. ⠠⠅⠠⠚ ⠨⠅ ⠤⠠⠊

$KJ = -I$

`KJ = -I`

808. ⠠⠊⠠⠅ ⠨⠅ ⠤⠠⠚

$IK = -J$

`IK = -J`

809. ⠠⠟⠦ ⠶ StartSet⠬⠤⠂ ⠬⠤⠠⠊ ⠬⠤⠠⠚ ⠬⠤⠠⠅ EndSet

$Q_8 = \{\pm 1, \pm I, \pm J, \pm K \}$

Q_8 = \{\pm 1, \pm I, \pm J, \pm K \}

810. ⠠⠟⠦

Q_8

Q_8

811. ⠈⠠⠉⠘⠈⠼

${\mathbb C}^\ast$

{\mathbb C}^\ast

812. ⠵ ⠶ ⠁⠬⠃⠊

$z = a+bi$

z = a+bi

813. ⠵⠘⠤⠂⠐ ⠶ ⠹⠁⠤⠃⠊⠌⠁⠘⠆⠐⠬⠃⠘⠆⠐⠼

$z^{-1} = \frac{a -bi}{a^2 +b^2}$

z^{-1} = \frac{a -bi}{a^2 +b^2}

814. ⠳⠠⠛⠳ ⠶ ⠝

$|G| = n$

|G| = n

815. ⠳⠈⠠⠵⠳ ⠶ ⠠⠿

$|{\mathbb Z}| = \infty$

|{\mathbb Z}| = \infty

816. ⠑⠛ ⠶ ⠛⠑ ⠶ ⠛

$eg = ge = g$

eg = ge = g

817. ⠛ ⠈⠑ ⠠⠛

$g \in G$

g \in G

818. e

e

819. ⠑⠄

e'

e'

820. ⠑⠄⠛ ⠶ ⠛⠑⠣⠐ ⠶ ⠛

$e'g = ge' = g$

e'g = ge' = g

821. ⠑ ⠶ ⠑⠄

$e = e'$

e = e'

822. ⠑⠑⠘⠄⠐ ⠨⠅ ⠑⠄

$ee' = e'$

ee' = e'

823. ⠑⠑⠘⠄⠐ ⠨⠅ ⠑

$ee' = e$

ee' = e

824. ⠑ ⠨⠅ ⠑⠑⠘⠄⠐ ⠨⠅ ⠑⠄

$e = ee' = e'$

e = ee' = e'

825. ⠛⠄

g'

g'

826. ⠛⠄⠄

g''

g''

827. ⠛⠛⠘⠄⠐ ⠨⠅ ⠛⠄⠛ ⠨⠅ ⠑

$gg' = g'g = e$

gg' = g'g = e

828. ⠛⠛⠘⠄⠄⠐ ⠨⠅ ⠛⠄⠄⠛ ⠨⠅ ⠑

$gg'' = g''g = e$

gg'' = g''g = e

829. ⠛⠄ ⠨⠅ ⠛⠄⠄

$g' = g''$

g' = g''

830. ⠛⠄ ⠨⠅ ⠛⠄⠑ ⠨⠅ ⠛⠄⠷⠛⠛⠘⠄⠄⠐⠾ ⠨⠅ ⠷⠛⠄⠛⠾⠛⠘⠄⠄⠐ ⠨⠅ ⠑⠛⠘⠄⠄⠐ ⠨⠅ ⠛⠄⠄

$g' = g'e = g'(gg'') = (g'g)g'' = eg'' = g''$

g' = g'e = g'(gg'') = (g'g)g'' = eg'' = g''

831. ⠷⠁⠃⠾⠘⠤⠂⠐ ⠨⠅ ⠃⠘⠤⠂⠐⠁⠘⠤⠂

$(ab)^{-1} = b^{-1}a^{-1}$

(ab)^{-1} = b^{-1}a^{-1}

832. ⠁⠃⠃⠘⠤⠂⠐⠁⠘⠤⠂⠐ ⠨⠅ ⠁⠑⠁⠘⠤⠂⠐ ⠨⠅ ⠁⠁⠘⠤⠂⠐ ⠨⠅ ⠑

$abb^{-1}a^{-1} = aea^{-1} = aa^{-1} = e$

abb^{-1}a^{-1} = aea^{-1} = aa^{-1} = e

833. ⠃⠘⠤⠂⠐⠁⠘⠤⠂⠐⠁⠃ ⠨⠅ ⠑

$b^{-1}a^{-1}ab = e$

b^{-1}a^{-1}ab = e

834. ⠷⠁⠘⠤⠂⠐⠾⠘⠤⠂⠐ ⠨⠅ ⠁

$(a^{-1})^{-1} = a$

(a^{-1})^{-1} = a

835. ⠁⠃⠤⠂⠐⠷⠁⠃⠤⠂⠐⠾⠃⠤⠂⠐ ⠭ ⠑

$a^{-1}(a^{-1})^{-1} = e$

a^{-1} (a^{-1})^{-1} = e

836. ⠷⠁⠃⠤⠂⠐⠾⠃⠤⠂⠐ ⠭ ⠑⠷⠁⠃⠤⠂⠐⠾⠃⠤⠂⠐ ⠭ ⠁⠁⠃⠤⠂⠐⠷⠁⠃⠤⠂⠐⠾⠃⠤⠂⠐ ⠭ ⠁⠑ ⠭ ⠁

$(a^{-1})^{-1} = e(a^{-1})^{-1} = aa^{-1}(a^{-1})^{-1} = ae = a$

(a^{-1})^{-1} = e (a^{-1})^{-1} = a a^{-1} (a^{-1})^{-1} = ae =
a

837. ⠭ ⠉⠑ ⠠⠛

$x \in G$

x \in G

838. ⠁⠭ ⠭ ⠃

$ax = b$

ax = b

839. ⠭⠁ ⠭ ⠃

$xa = b$

xa = b

840. ⠭ ⠭ ⠑⠭ ⠭ ⠁⠃⠤⠂⠐⠁⠭ ⠭ ⠁⠃⠤⠂⠐⠃

$x = ex = a^{-1}ax = a^{-1}b$

x = ex = a^{-1}ax = a^{-1}b

841. ⠭⠂

x_1

x_1

842. ⠭⠆

x_2

x_2

843. ⠁⠭⠂ ⠭ ⠃ ⠭ ⠁⠭⠆

$ax_1 = b = ax_2$

ax_1 = b = ax_2

844. ⠭⠂ ⠭ ⠁⠃⠤⠂⠐⠁⠭⠂ ⠭ ⠁⠃⠤⠂⠐⠁⠭⠆ ⠭ ⠭⠆

$x_1 = a^{-1}ax_1 = a^{-1}ax_2 = x_2$

x_1 = a^{-1}ax_1 = a^{-1}ax_2 = x_2

845. ⠃⠁ ⠭ ⠉⠁

$ba = ca$

ba = ca

846. ⠃ ⠭ ⠉

$b = c$

b = c

847. ⠁⠃ ⠨⠅ ⠁⠉

$ab = ac$

ab = ac

848. ⠛⠘⠴⠐ ⠨⠅ ⠑

$g^0 = e$

g^0 = e

849. ⠛⠘⠝⠐ ⠨⠅ ⠐ModifyingBelow⠛⠡⠛⠄⠄⠄⠛With⠄ ⡷⠩⠆ ⠤⠙⠙⠂ ⠙⠕times⠻

$g^n = \underbrace{g \cdot g \cdots g}_{n \; \text{times}}$

g^n = \underbrace{g \cdot g \cdots g}_{n \; \text{times}}

850. ⠛⠘⠤⠝⠐ ⠨⠅ ⠐ModifyingBelow⠛⠘⠤⠂⠐⠡⠛⠘⠤⠂⠐⠄⠄⠄⠛⠘⠤⠂⠐With⠄ ⡷⠩⠆ ⠤⠙⠙⠂ ⠙⠕times⠻

$g^{-n} = \underbrace{g^{-1} \cdot g^{-1} \cdots g^{-1}}_{n \; \text{times}}$

g^{-n} = \underbrace{g^{-1} \cdot g^{-1} \cdots g^{-1}}_{n \; \text{times}}

851. ⠛⠠⠓⠈⠑⠠⠛

$g, h \in G$

g, h \in G

852. ⠛⠘⠍⠐⠛⠘⠝⠐ ⠨⠅ ⠛⠘⠍⠬⠝

$g^m g^n = g^{m+n}$

g^mg^n = g^{m+n}

853. ⠷⠛⠘⠍⠐⠾⠘⠝⠐ ⠨⠅ ⠛⠘⠍⠝

$(g^m)^n = g^{mn}$

(g^m)^n = g^{mn}

854. ⠷⠛⠓⠾⠘⠝⠐ ⠨⠅ ⠷⠓⠘⠤⠂⠐⠛⠘⠤⠂⠐⠾⠘⠤⠝

$(gh)^n = (h^{-1}g^{-1})^{-n}$

(gh)^n = (h^{-1}g^{-1})^{-n}

855. ⠷⠛⠓⠾⠘⠝⠐ ⠨⠅ ⠛⠘⠝⠐⠓⠘⠝

$(gh)^n = g^n h^n$

(gh)^n = g^nh^n

856. ⠷⠛⠓⠾⠘⠝⠐ ⠌⠨⠅ ⠛⠘⠝⠐⠓⠘⠝

$(gh)^n \neq g^n h^n$

(gh)^n \neq g^nh^n

857. ⠝⠛

ng

ng

858. ⠛⠘⠝

g^n

g^n

859. ⠍⠛⠬⠝⠛ ⠨⠅ ⠷⠍⠬⠝⠾⠛

$mg + ng = (m + n)g$

mg + ng = (m+n)g

860. ⠍⠷⠝⠛⠾ ⠨⠅ ⠷⠍⠝⠾⠛

$m(ng) = (mn)g$

m(ng) = (mn)g

861. ⠍⠷⠛⠬⠓⠾ ⠨⠅ ⠍⠛⠬⠍⠓

$m(g + h) = mg + mh$

m(g + h) = mg + mh

862. ⠆ ⠈⠠⠵ ⠨⠅ StartSet⠄ ⠄⠄⠄⠆ ⠄⠤⠆ ⠴ ⠄⠲ ⠄⠄ EndSet

$2\mathbb{Z} = \{\ldots, -2, 0, 2, 4, \ldots\}$

2{\mathbb Z} = \{\ldots, -2, 0, 2, 4, \ldots \}

863. H

H

864. ⠠⠓ ⠨⠅ StartSet⠑EndSet

$H = \{e\}$

H = \{ e \}

865. ⠈ ⠠⠗ ⠘ ⠈⠼

$\mathbb{R}^*$

{\mathbb R}^*

866. ⠁ ⠈⠑ ⠈ ⠠⠗ ⠘ ⠈⠼

$a \in \mathbb{R}^*$

a \in {\mathbb R}^*

867. ⠂⠌⠁

$1/a$

1/a

868. ⠈ ⠠⠟ ⠘ ⠈⠼ ⠂ ⠨⠅ StartSet⠏⠌⠟⠈⠒⠏and⠟arenonzerointegersEndSet

$\mathbb{Q}^* = \{p/q : p$ and q are nonzero integers$\}$

{\mathbb Q}^* = \{ p/q : p \, \text{and}\, q\, \text{are nonzero
integers} \}

869. ⠂ ⠨⠅ ⠂⠌⠂

$1 = 1/1$

1 = 1/1

870. ⠈ ⠠⠟ ⠘ ⠈⠼

$\mathbb{Q}^*$

{\mathbb Q}^*

871. ⠗⠌⠎

r/s

r/s

872. ⠏⠗⠌⠟⠎

pr/qs

pr/qs

873. ⠏⠌⠟⠈⠑⠈⠠⠟⠘⠈⠼

$p/q \in \mathbb{Q}^*$

p/q \in {\mathbb Q}^*

874. ⠷⠏⠌⠟⠾⠘⠤⠂⠐ ⠨⠅ ⠟⠌⠏

$(p/q)^{-1} = q/p$

(p/q)^{-1} = q/p

875. ⠈⠠⠉⠘⠈⠼

$\mathbb{C}^*$

{\mathbb C}^{\ast}

876. ⠠⠓ ⠨⠅ StartSet⠂⠠⠤⠂⠠⠊⠠⠤⠊EndSet

$H = \{1, -1, i, -i\}$

H = \{ 1, -1, i, -i \}

877. ⠠⠓ ⠸⠐⠅ ⠈⠠⠉⠘⠈⠼

$H \subset \mathbb{C}^*$

H \subset {\mathbb C}^{\ast}

878. ⠠⠎⠠⠇⠆⠷⠈⠠⠗⠾

$SL_2(\mathbb{R})$

SL_2({\mathbb R})

879. ⠠⠛⠠⠇⠆⠷⠈⠠⠗⠾

$GL_2(\mathbb{R})$

GL_2({\mathbb R })

880. ⠁⠙⠤⠃⠉ ⠨⠅ ⠂

$ad - bc = 1$

ad - bc = 1

881. ⠠⠁⠘⠤⠂⠐ ⠨⠅ ⠠⠷⠙ ⠤⠃⠠⠾⠠⠷⠤⠉ ⠁⠠⠾

$A^{-1} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

A^{-1} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}

882. ⠠⠎⠠⠇⠆⠷⠈⠠⠗⠾

$SL_2(\mathbb{R})$

SL_2({\mathbb R})

883. ⠈⠠⠍⠆⠷⠈⠠⠗⠾

$\mathbb{M}_2(\mathbb{R})$

{\mathbb M}_2(\mathbb R)

884. ⠠⠷⠼⠂ ⠼⠴⠠⠾ ⠠⠷⠼⠴ ⠼⠂ ⠠⠾⠬⠠⠷⠤⠂ ⠼⠴⠠⠾ ⠠⠷⠼⠴ ⠤⠂ ⠠⠾ ⠶ ⠠⠷⠼⠴ ⠼⠴⠠⠾
⠠⠷⠼⠴ ⠼⠴⠠⠾

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

```
\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} -1
& 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}
```

885. ⠈ ⠠⠵⠲

$\mathbb{Z}_4$

```
{\mathbb Z}_4
```

886. ⠈ ⠠⠵⠆

$\mathbb{Z}_2$

```
{\mathbb Z}_2
```

887. ⠈ ⠠⠵⠆ ⠡ ⠈ ⠠⠵⠆

$\mathbb{Z}_2 \times \mathbb{Z}_2$

```
{\mathbb Z}_2 \times {\mathbb Z}_2
```

888. ⠷⠁ ⠠⠃ ⠾⠬⠷⠉ ⠠⠙⠾ ⠶ ⠷⠁ ⠬⠉ ⠠⠃ ⠬⠙⠾

$(a, b) + (c, d) = (a + c, b + d)$

```
(a,b) + (c,d) = (a+c, b+d)
```

889. ⠠⠓⠂ ⠶ StartSet⠷⠴ ⠠⠴⠾ ⠠⠷⠴ ⠠⠂ ⠾EndSet

$H_1 = \{(0, 0), (0, 1)\}$

```
H_1 = \{ (0,0), (0,1) \}
```

890. ⠠⠓⠆ ⠶ StartSet⠷⠴ ⠠⠴⠾ ⠠⠷⠂ ⠠⠴⠾EndSet

$H_2 = \{(0, 0), (1, 0)\}$

```
H_2 = \{ (0,0), (1,0) \}
```

891. ⠠⠓⠒ ⠶ StartSet⠷⠴ ⠠⠴⠾ ⠠⠷⠂ ⠠⠂ ⠾EndSet

$H_3 = \{(0, 0), (1, 1)\}$

```
H_3 = \{ (0,0), (1,1) \}
```

892. StartLayout1stRow⠬ ⠷⠴ ⠠⠴⠾ ⠷⠴ ⠠⠂ ⠾ ⠷⠂ ⠠⠴⠾ ⠷⠂ ⠠⠂ ⠾2ndRow⠷⠴ ⠠⠴⠾ ⠷⠴ ⠠⠴⠾ ⠷⠴ ⠠⠂ ⠾ ⠇

$+$	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,0)$	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,1)$	$(0,1)$	$(0,0)$	$(1,1)$	$(1,0)$
$(1,0)$	$(1,0)$	$(1,1)$	$(0,0)$	$(0,1)$
$(1,1)$	$(1,1)$	$(1,0)$	$(0,1)$	$(0,0)$

```
\begin{array}{c|cccc} + & (0,0) & (0,1) & (1,0) & (1,1) \\ \hline
(0,0) & (0,0) & (0,1) & (1,0) & (1,1) \\ (0,1) & (0,1) & (0,0) &
(1,1) & (1,0) \\ (1,0) & (1,0) & (1,1) & (0,0) & (0,1) \\ (1,1)
& (1,1) & (1,0) & (0,1) & (0,0) \end{array}
```

893. ⠓⠂ ⠠⠓⠆ ⠈⠑ ⠠⠓

$h_1, h_2 \in H$

```
h_1, h_2 \in H
```

894. ⠓⠂⠓⠆ ⠈⠑ ⠠⠓

$h_1h_2 \in H$

h_1h_2 \in H

895. ⠓ ⠈⠑ ⠠⠓

$h \in H$

h \in H

896. ⠓⠘⠤⠂ ⠐⠈⠑ ⠠⠓

$h^{-1} \in H$

h^{-1} \in H

897. ⠑⠰⠠⠓

e_H

e_H

898. ⠑⠰⠠⠓⠐ ⠨⠅ ⠑

$e_H = e$

e_H = e

899. ⠑⠰⠠⠓⠐⠑⠰⠠⠓⠐ ⠨⠅ ⠑⠰⠠⠓

$e_He_H = e_H$

e_H e_H = e_H

900. ⠑⠑⠰⠠⠓⠐ ⠨⠅ ⠑⠰⠠⠓⠐⠑ ⠨⠅ ⠑⠰⠠⠓

$ee_H = e_He = e_H$

ee_H = e_H e = e_H

901. ⠑⠑⠰⠠⠓⠐ ⠨⠅ ⠑⠰⠠⠓⠐⠑⠰⠠⠓

$ee_H = e_He_H$

ee_H = e_H e_H

902. ⠑ ⠨⠅ ⠑⠰⠠⠓

$e = e_H$

e =e_H

903. ⠓⠄ ⠈⠑ ⠠⠓

$h' \in H$

h' \in H

904. ⠓⠓⠄⠐ ⠨⠅ ⠓⠄⠓ ⠨⠅ ⠑

$hh' = h'h = e$

hh' = h'h = e

905. ⠓⠄ ⠨⠅ ⠓⠘⠤⠂

$h' = h^{-1}$

h' = h^{-1}

906. ⠠⠓ ⠌⠨⠅ ⠸⠴

$H \neq \emptyset$

H \neq \emptyset

907. ⠛⠠⠓⠈⠑⠠⠓

$g, h \in H$

g, h \in H

908. ⠛⠓⠘⠤⠂

gh^{-1}

gh^{-1}

909. ⠛⠓⠘⠤⠂⠈⠑⠠⠓

$gh^{-1} \in H$

gh^{-1} \in H

910. h

h

911. ⠓⠘⠤⠂

h^{-1}

h^{-1}

912. ⠠⠓⠸⠐⠅⠠⠛

$H \subset G$

H \subset G

913. ⠛⠓⠘⠤⠂⠈⠑⠠⠓

$gh^{-1} \in H$

g h^{-1} \in H

914. ⠛⠈⠑⠠⠓

$g \in H$

g \in H

915. ⠛⠛⠘⠤⠂⠐⠀⠨⠅⠀⠑

$gg^{-1} = e$

gg^{-1} = e

916. ⠑⠛⠘⠤⠂⠐⠀⠨⠅⠀⠛⠘⠤⠂

$eg^{-1} = g^{-1}$

eg^{-1} = g^{-1}

917. ⠓⠂⠷⠓⠆⠘⠤⠂⠐⠾⠘⠤⠂⠐⠀⠨⠅⠀⠓⠂⠓⠆⠈⠑⠠⠓

$h_1(h_2^{-1})^{-1} = h_1h_2 \in H$

h_1(h_2^{-1})^{-1} = h_1 h_2 \in H

918. ⠭⠈⠑⠈⠠⠵

$x \in \mathbb{Z}$

x \in {\mathbb Z}

919. ⠒⠭⠸⠇⠆⠷⠍⠕⠙⠶⠾

$3x \equiv 2 \pmod{7}$

3x \equiv 2 \pmod{7}

920. ⠢⠭⠬⠂ ⠿ 13⠷⠍⠕⠙23⠾

$5x + 1 \equiv 13 \pmod{23}$

5x + 1 \equiv 13 \pmod{23}

921. ⠢⠭⠬⠂ ⠿ 13⠷⠍⠕⠙26⠾

$5x + 1 \equiv 13 \pmod{26}$

5x + 1 \equiv 13 \pmod{26}

922. ⠔⠭ ⠿ ⠒⠷⠍⠕⠙⠢⠾

$9x \equiv 3 \pmod{5}$

9x \equiv 3 \pmod{5}

923. ⠢⠭ ⠿ ⠂ ⠷⠍⠕⠙⠖⠾

$5x \equiv 1 \pmod{6}$

5x \equiv 1 \pmod{6}

924. ⠒⠭ ⠿ ⠂ ⠷⠍⠕⠙⠖⠾

$3x \equiv 1 \pmod{6}$

3x \equiv 1 \pmod{6}

925. ⠒⠬⠶ ⠈ ⠠⠵ ⠶ StartSet⠠ ⠄⠄⠄⠠⠒ ⠠⠒ ⠠10 ⠄⠄ EndSet

$3 + 7\mathbb{Z} = \{\ldots, -4, 3, 10, \ldots\}$

3 + 7 \mathbb Z = \{ \ldots, -4, 3, 10, \ldots \}

926. 18⠬26 ⠈ ⠠⠵

$18 + 26\mathbb{Z}$

18 + 26 \mathbb Z

927. ⠢⠬⠖ ⠈ ⠠⠵

$5 + 6\mathbb{Z}$

5 + 6 \mathbb Z

928. ⠠⠛ ⠶ StartSet⠁ ⠠⠃ ⠠⠉ ⠠⠙EndSet

$G = \{a, b, c, d\}$

G = \{ a, b, c, d \}

929. StartLayout1stRow⠆⠨⠡ ⠁ ⠃ ⠉ ⠙2ndRow⠁ ⠁ ⠉ ⠙ ⠁ 3rdRow⠃ ⠃ ⠃ ⠉ ⠙4thRow⠉ ⠉

$\circ$	a	b	c	d
a	a	c	d	a
b	b	b	c	d
c	c	d	a	b
d	d	a	b	c

\begin{array}{c|cccc} \circ & a & b & c & d \\ \hline a & a & c & d & a \\ b & b & b & c & d \\ c & c & d & a & b \\ d & d & a & b & c \end{array}

930. StartLayout1stRow⠆⠨⠡ ⠁ ⠃ ⠉ ⠙2ndRow⠁ ⠁ ⠃ ⠉ ⠙3rdRow⠃ ⠃ ⠁ ⠙ ⠉4thRow⠉ ⠉

$\circ$	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	a	b
d	d	c	b	a

\begin{array}{c|cccc} \circ & a & b & c & d \\ \hline a & a & b & c & d \\ b & b & a & d & c \\ c & c & d & a & b \\ d & d & c & b & a \end{array}

931. StartLayout1stRow⠘⠩⠄ ⠁ ⠃ ⠉ ⠙2ndRow⠁ ⠁ ⠃ ⠉ ⠙3rdRow⠃ ⠃ ⠉ ⠙ ⠁ 4thRow⠉ ⠉

$$\begin{array}{c|cccc} \circ & a & b & c & d \\ \hline a & a & b & c & d \\ b & b & c & d & a \\ c & c & d & a & b \\ d & d & a & b & c \end{array}$$

\begin{array}{c|cccc} \circ & a & b & c & d \\ \hline a & a & b & c & d \\ b & b & c & d & a \\ c & c & d & a & b \\ d & d & a & b & c \end{array}

932. StartLayout1stRow⠘⠩⠄ ⠁ ⠃ ⠉ ⠙2ndRow⠁ ⠁ ⠃ ⠉ ⠙3rdRow⠃ ⠃ ⠁ ⠉ ⠙4thRow⠉ ⠉

$$\begin{array}{c|cccc} \circ & a & b & c & d \\ \hline a & a & b & c & d \\ b & b & a & c & d \\ c & c & b & a & d \\ d & d & d & b & c \end{array}$$

\begin{array}{c|cccc} \circ & a & b & c & d \\ \hline a & a & b & c & d \\ b & b & a & c & d \\ c & c & b & a & d \\ d & d & d & b & c \end{array}

933. ⠷ ⠈ ⠠⠵⠲ ⠠⠠⠬⠾

$(\mathbb{Z}_4, +)$

({\mathbb Z}_4, +)

934. ⠠⠙⠲

D_4

D_4

935. ⠠⠥⠷12⠾

$U(12)$

U(12)

936. StartLayout1stRow⠈⠄ ⠠⠼⠂ ⠠⠼⠢ ⠠⠼⠶ ⠠⠼112ndRow⠠⠼⠂ ⠠⠼⠂ ⠠⠼⠢ ⠠⠼⠶ ⠠⠼113rdRow⠠⠼⠢ ⠠⠼⠢ ⠠⠼⠂ ⠠⠼

$$\begin{array}{c|cccc} \cdot & 1 & 5 & 7 & 11 \\ \hline 1 & 1 & 5 & 7 & 11 \\ 5 & 5 & 1 & 11 & 7 \\ 7 & 7 & 11 & 1 & 5 \\ 11 & 11 & 7 & 5 & 1 \end{array}$$

\begin{array}{c|cccc} \cdot & 1 & 5 & 7 & 11 \\ \hline 1 & 1 & 5 & 7 & 11 \\ 5 & 5 & 1 & 11 & 7 \\ 7 & 7 & 11 & 1 & 5 \\ 11 & 11 & 7 & 5 & 1 \end{array}

937. ⠠⠎ ⠨⠅ ⠈⠠⠼⠄⠸⠫⠒⠂⠐⠂⠄ ⠸⠧⠤⠠⠂ ⠸⠾

$S = \mathbb{R} \setminus \{-1\}$

S = {\mathbb R} \setminus \{ -1 \}

938. ⠁ ⠈⠼⠃ ⠨⠅ ⠁ ⠬⠃ ⠬⠁⠃

$a \ast b = a + b + ab$

a \ast b = a + b + ab

939. ⠷⠠⠎⠠⠈⠼⠾

$(S, \ast)$

```
(S, \ast)
```

940. ⠠⠁⠠⠃ ⠌⠶ ⠠⠃⠠⠁

$AB \neq BA$

```
AB \neq BA
```

941. ⠠⠷⠠⠗⠂ ⠭ ⠽⠠⠾ ⠠⠷⠠⠗⠴ ⠠⠗⠂ ⠵⠠⠾ ⠠⠷⠠⠗⠴ ⠠⠗⠴ ⠠⠗⠂ ⠠⠾

$$\begin{pmatrix} 1 & x & y \\ 0 & 1 & z \\ 0 & 0 & 1 \end{pmatrix}$$

```
\begin{pmatrix} 1 & x & y \\ 0 & 1 & z \\ 0 & 0 & 1 \end{pmatrix}
```

942. ⠠⠷⠠⠗⠂ ⠭ ⠽⠠⠾ ⠠⠷⠠⠗⠴ ⠠⠗⠂ ⠵⠠⠾ ⠠⠷⠠⠗⠴ ⠠⠗⠴ ⠠⠗⠂ ⠠⠾⠠⠷⠠⠗⠂ ⠭⠃⠄ ⠂ ⠽⠃⠄ ⠂⠠⠾
⠠⠷⠠⠗⠴ ⠠⠗⠂ ⠵⠃⠄ ⠂⠠⠾ ⠠⠷⠠⠗⠴ ⠠⠗⠴ ⠠⠗⠂ ⠠⠾ ⠶ ⠠⠷⠠⠗⠂ ⠭⠤⠭⠃⠄ ⠂ ⠽⠤⠽⠤⠭⠵⠃⠄ ⠂⠠⠾
⠠⠷⠠⠗⠴ ⠠⠗⠂ ⠵⠤⠵⠃⠄ ⠂⠠⠾ ⠠⠷⠠⠗⠴ ⠠⠗⠴ ⠠⠗⠂ ⠠⠾

$$\begin{pmatrix} 1 & x & y \\ 0 & 1 & z \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & x' & y' \\ 0 & 1 & z' \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & x+x' & y+y'+xz' \\ 0 & 1 & z+z' \\ 0 & 0 & 1 \end{pmatrix}$$

```
\begin{pmatrix} 1 & x & y \\ 0 & 1 & z \\ 0 & 0 & 1 \end{pmatrix}
\begin{pmatrix} 1 & x' & y' \\ 0 & 1 & z' \\ 0 & 0 & 1 \end{pmatrix}
= \begin{pmatrix} 1 & x+x' & y+y'+xz' \\ 0 & 1 & z+z' \\ 0 & 0 & 1
\end{pmatrix}
```

943. ⠙⠑⠞⠷⠠⠁⠠⠃⠾ ⠶ ⠙⠑⠞⠷⠠⠁⠾⠙⠑⠞⠷⠠⠃⠾

$\det(AB) = \det(A) \det(B)$

```
\det(AB) = \det(A) \det(B)
```

944. ⠠⠁⠠⠃ ⠈⠑ ⠠⠛⠠⠇⠆⠷⠈⠠⠗⠾

$AB \in GL_2(\mathbb{R})$

```
AB \in GL_2({\mathbb R})
```

945. ⠈⠠⠵⠆ ⠘⠝⠂ ⠶ StartSet⠷⠁⠂ ⠠⠁⠆ ⠄⠄ ⠠⠁⠰⠝⠂⠾⠒⠁ ⠰⠊⠂ ⠈⠑ ⠈⠠⠵⠆ EndSet

$\mathbb{Z}_2^n = \{ (a_1, a_2, \ldots, a_n) : a_i \in \mathbb{Z}_2 \}$

```
{\mathbb Z}_2^n = \{ (a_1, a_2, \ldots, a_n) : a_i \in {\mathbb
Z}_2 \}
```

946. ⠈⠠⠵⠆ ⠘⠝

$\mathbb{Z}_2^n$

```
{\mathbb Z}_2^n
```

947. ⠷⠁⠂ ⠠⠁⠆ ⠄⠄ ⠠⠁ ⠰⠝⠂⠬⠷⠃⠂ ⠠⠃⠆ ⠄⠄ ⠠⠃ ⠰⠝⠂⠾ ⠶ ⠷⠁⠂⠬⠃⠂ ⠠⠁⠆⠬⠃⠆ ⠄⠄ ⠠⠁ ⠰⠝⠬⠃ ⠰⠝⠂⠾

$(a_1, a_2, \ldots, a_n) + (b_1, b_2, \ldots, b_n) = (a_1 + b_1, a_2 + b_2, \ldots, a_n + b_n)$

```
(a_1, a_2, \ldots, a_n) + (b_1, b_2, \ldots, b_n) = (a_1 + b_1,
a_2 + b_2, \ldots, a_n + b_n)
```

948. ⠈⠠⠗⠘⠈⠼⠂ ⠶ ⠈⠠⠗⠠⠸⠫⠆⠂⠈⠡⠄ StartSet⠴EndSet

$\mathbb{R}^{\ast} = \mathbb{R} \setminus \{0 \}$

```
{\mathbb R}^{\ast} = {\mathbb R} \setminus \{0 \}
```

949. ⠈⠠⠗⠘⠈⠼

$\mathbb{R}^{*}$

{\mathbb R}^{\ast}

950. ⠠⠛ ⠨⠅ ⠈⠠⠗⠘⠈⠼⠐⠡⠈⠠⠵

$G = \mathbb{R}^{*} \times \mathbb{Z}$

G = {\mathbb R}^{\ast} \times {\mathbb Z}

951. ⠷⠁⠠⠍⠾⠘⠨⠡⠷⠃⠠⠝⠾ ⠨⠅ ⠷⠁⠃⠠⠍⠬⠝⠾

$(a,m) \circ (b,n) = (ab, m+n)$

(a,m) \circ (b,n) = (ab, m + n)

952. ⠨⠎ ⠨⠅ ⠠⠷⠼⠂ ⠼⠆ ⠄⠄⠄ ⠝⠠⠾ ⠠⠷⠁⠂ ⠁⠆ ⠄⠄⠄ ⠁⠰⠝⠠⠾

$\sigma = \begin{pmatrix} 1 & 2 & \cdots & n \\ a_1 & a_2 & \cdots & a_n \end{pmatrix}$

\sigma = \begin{pmatrix} 1 & 2 & \cdots & n \\ a_1 & a_2 & \cdots & a_n \end{pmatrix}

953. ⠠⠎⠰⠝

S_n

S_n

954. ⠁⠰⠊

a_i

a_i

955. ⠁⠂

a_1

a_1

956. ⠝⠤⠂

$n - 1$

n - 1

957. ⠁⠆⠠⠄

$a_2, \ldots$

a_2, \ldots

958. ⠁⠰⠝⠤⠂

a_{n-1}

a_{n - 1}

959. ⠁⠰⠝

a_n

a_n

960. ⠨⠎

σ

\sigma

961. ⠝⠷⠝⠤⠂⠾⠄⠄⠄⠆⠡⠂ ⠶ ⠝⠫

$n(n-1)\cdots 2\cdot 1 = n!$

n(n - 1) \cdots 2 \cdot 1 = n!

962. ⠴⠬⠁ ⠿⠁ ⠬⠴ ⠿⠁⠷⠍⠕⠙⠝⠾

$0 + a \equiv a + 0 \equiv a \pmod{n}$

0 + a \equiv a + 0 \equiv a \pmod{ n }

963. ⠁ ⠈⠑⠈⠰⠵⠰⠝

$a \in \mathbb{Z}_n$

a \in {\mathbb Z}_n

964. ⠁⠡⠂ ⠿⠁⠷⠍⠕⠙⠝⠾

$a \cdot 1 \equiv a \pmod{n}$

a \cdot 1 \equiv a \pmod{n}

965. ⠃ ⠈⠑⠈⠰⠵⠰⠝

$b \in \mathbb{Z}_n$

b \in {\mathbb Z}_n

966. ⠁⠬⠃ ⠿⠃⠬⠁ ⠿⠴⠷⠍⠕⠙⠝⠾

$a + b \equiv b + a \equiv 0 \pmod{n}$

a + b \equiv b + a \equiv 0 \pmod{ n}

967. ⠁⠷⠃⠬⠉⠾ ⠿⠁⠃⠬⠁⠉⠷⠍⠕⠙⠝⠾

$a(b + c) \equiv ab + ac \pmod{n}$

a(b + c) \equiv ab + ac \pmod{n}

968. ⠁⠃⠘⠝⠐⠁⠘⠤⠂⠐ ⠶ ⠷⠁⠃⠁⠘⠤⠂⠐⠾⠘⠝

$ab^n a^{-1} = (aba^{-1})^n$

ab^na^{-1} = (aba^{-1})^n

969. ⠝ ⠈⠑⠈⠰⠵

$n \in \mathbb{Z}$

n \in \mathbb Z

970. ⠝ ⠨⠂ ⠆

$n > 2$

n \gt 2

971. ⠅ ⠈⠑⠠⠥⠷⠝⠾

$k \in U(n)$

k \in U(n)

972. ⠅⠘⠆⠐ ⠶ ⠂

$k^2 = 1$

k^2 = 1

973. ⠅ ⠌⠶ ⠂

$k \neq 1$

k \neq 1

974. ⠛⠂⠛⠆⠄⠄⠄⠛⠰⠝

$g_1 g_2 \cdots g_n$

g _1 g_2 \cdots g_n

975. ⠛⠰⠝⠘⠤⠂⠐⠛⠰⠝⠤⠂⠘⠤⠂⠐⠄⠄⠄⠛⠂⠘⠤⠂

$g_n^{-1} g_{n-1}^{-1} \cdots g_1^{-1}$

g_n^{-1} g_{n-1}^{-1} \cdots g_1^{-1}

976. ⠁⠘⠆⠐ ⠶ ⠑

$a^2 = e$

a^2 = e

977. ⠁⠃⠁⠃ ⠶ ⠷⠁⠃⠾⠘⠆⠐ ⠶ ⠑ ⠶ ⠁⠘⠆⠐⠃⠘⠆⠐ ⠶ ⠁⠁⠃⠃

$abab = (ab)^2 = e = a^2 b^2 = aabb$

abab = (ab)^2 = e = a^2 b^2 = aabb

978. ⠃⠁ ⠶ ⠁⠃

$ba = ab$

ba = ab

979. ⠷⠁⠃⠾⠘⠆⠐ ⠶ ⠁⠘⠆⠐⠃⠘⠆

$(ab)^2 = a^2b^2$

(ab)^2 = a^2b^2

980. ⠈⠠⠵⠒⠡⠈⠠⠵⠒

${\mathbb Z}_3 \times {\mathbb Z}_3$

{\mathbb Z}_3 \times {\mathbb Z}_3

981. ⠈⠠⠵⠔

${\mathbb Z}_9$

{\mathbb Z}_9

982. ⠠⠓⠂ ⠶ ⠰⠷⠊⠙⠰⠾

$H_1 = \{\mathrm{id}\}$

H_1 = \{ \identity \}

983. ⠠⠓⠆ ⠶ StartSet⠊⠙⠠⠨⠗⠂⠠⠨⠗⠆EndSet

$H_2 = \{\mathrm{id}, \rho_1, \rho_2\}$

H_2 = \{ \identity, \rho_1, \rho_2 \}

984. ⠠⠓⠒ ⠶ StartSet⠊⠙⠠⠨⠍⠂ EndSet

$H_3 = \{\mathrm{id}, \mu_1\}$

H_3 = \{ \identity, \mu_1 \}

985. ⠠⠓⠲ ⠶ StartSet⠊⠙⠠⠨⠍⠆EndSet

$H_4 = \{\mathrm{id}, \mu_2\}$

H_4 = \{ \identity, \mu_2 \}

986. ⠠⠓⠢ ⠶ StartSet⠊⠙⠠⠨⠍⠒EndSet

$H_5 = \{\mathrm{id}, \mu_3\}$

H_5 = \{ \identity, \mu_3 \}

987. ⠠⠓ ⠶ StartSet⠆ ⠫ ⠐⠣⠅ ⠈⠑ ⠈ ⠠⠵EndSet

$H = \{2^k : k \in \mathbb{Z}\}$

H = \{2^k : k \in {\mathbb Z} \}

988. ⠝ ⠈ ⠠⠵ ⠶ StartSet⠝⠅ ⠣⠅ ⠈⠑ ⠈ ⠠⠵EndSet

$n\mathbb{Z} = \{nk : k \in \mathbb{Z}\}$

n {\mathbb Z} = \{ nk : k \in {\mathbb Z} \}

989. ⠝ ⠈ ⠠⠵

$n\mathbb{Z}$

n {\mathbb Z}

990. ⠈ ⠠⠵

$\mathbb{Z}$

\mathbb{Z}

991. ⠈ ⠠⠞ ⠶ StartSet⠵ ⠈⠑ ⠈ ⠠⠉ ⠃ ⠈⠠⠼ ⠐⠳⠵⠳ ⠶ ⠂EndSet

$\mathbb{T} = \{z \in \mathbb{C}^* : |z| = 1\}$

{\mathbb T} = \{ z \in {\mathbb C}^* : |z| =1 \}

992. ⠈ ⠠⠞

$\mathbb{T}$

{\mathbb T}

993. ⠈ ⠠⠉ ⠃ ⠈⠠⠼

$\mathbb{C}^*$

{\mathbb C}^*

994. ⠠⠵⠉⠕⠎ ⠹ ⠤⠎⠊⠝ ⠹ ⠠⠻ ⠠⠵⠎⠊⠝ ⠹ ⠉⠕⠎ ⠹ ⠠⠻

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos
\theta \end{pmatrix}

995. ⠹ ⠈⠑ ⠈ ⠠⠗

$\theta \in \mathbb{R}$

\theta \in {\mathbb R}

996. ⠠⠛ ⠶ ⠸⠷⠁ ⠤⠖ ⠠⠖ ⠩⠊⠁ ⠠⠅ ⠈⠑ ⠈ ⠠⠟and⠁ and⠃ arenotbothzero ⠸⠾

$G = \{a + b\sqrt{2} : a, b \in \mathbb{Q}$ and a and b are not both zero$\}$

G = \{ a + b \sqrt{2} : a, b \in {\mathbb Q} \text{ and } a \text{
and } b \text{ are not both zero} \}

997. ⠂ ⠶ ⠂⠤⠴⠖ ⠩

$1 = 1 + 0\sqrt{2}$

1 = 1 + 0 \sqrt{2}

998. ⠷⠁ ⠤⠃ ⠖ ⠩⠾⠷⠉ ⠤⠙ ⠖ ⠩⠾ ⠶ ⠷⠁ ⠉⠤⠆ ⠃ ⠙⠾⠤⠷⠁ ⠙⠤⠃ ⠉⠾⠖ ⠩

$(a + b\sqrt{2}\,)(c + d\sqrt{2}\,) = (ac + 2bd) + (ad + bc)\sqrt{2}$

(a + b \sqrt{2}\,)(c + d \sqrt{2}\,) = (ac + 2bd) + (ad + bc)\sqrt{2}

999. ⠷⠁⠬⠃⠜⠆⠻⠾⠘⠤⠂⠐ ⠶ ⠁⠌⠷⠁⠘⠆⠐⠤⠆⠃⠘⠆⠐⠾⠤⠃⠜⠆⠻⠌⠷⠁⠘⠆⠐⠤⠆⠃⠘⠆⠐⠾

$(a + b\sqrt{2}\,)^{-1} = a/(a^2 - 2b^2) - b\sqrt{2}/(a^2 - 2b^2)$

`(a + b \sqrt{2}\, )^{-1} = a/(a^2 - 2b^2) - b\sqrt{2}/(a^2 - 2 b^2)`

1000. ⠠⠓ ⠶ StartSet ⠠⠷⠁ ⠃ ⠠⠾ ⠠⠷⠉ ⠙ ⠠⠾⠒⠁ ⠬⠙ ⠶ ⠴EndSet

$H = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a + d = 0 \right\}$

`H = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a + d`
`= 0 \right\}`

1001. ⠠⠎⠠⠇⠆⠷⠈⠠⠵⠾

$SL_2(\mathbb{Z})$

`SL_2( {\mathbb Z} )`

1002. ⠠⠎⠠⠇⠆⠷⠈⠠⠗⠾

$SL_2(\mathbb{R})$

`SL_2( {\mathbb R} )`

1003. K

`K`

1004. ⠠⠓⠨⠬⠠⠅

$H \cup K$

`H \cup K`

1005. ⠠⠓⠠⠅ ⠶ StartSet⠓⠅⠒⠓ ⠈⠑ ⠠⠓and⠅ ⠈⠑ ⠠⠅ EndSet

$HK = \{hk : h \in H \text{ and } k \in K\}$

`H K = \{hk : h \in H \text{ and } k \in K \}`

1006. ⠠⠵⠷⠠⠛⠾ ⠶ StartSet⠭ ⠈⠑ ⠠⠛⠒⠛⠭ ⠶ ⠭⠛forall⠛ ⠈⠑ ⠠⠛EndSet

$Z(G) = \{x \in G : gx = xg \text{ for all } g \in G\}$

`Z(G) = \{ x \in G : gx = xg \text{ for all } g \in G \}`

1007. ⠁⠘⠲⠐⠃ ⠶ ⠃⠁

$a^4b = ba$

`a^4 b = ba`

1008. ⠁⠘⠒⠐ ⠶ ⠑

$a^3 = e$

`a^3 = e`

1009. ⠁⠃ ⠶ ⠃⠁

$ab = ba$

`ab = ba`

1010. ⠃⠁ ⠶ ⠁⠘⠲⠐⠃ ⠶ ⠁⠘⠒⠐⠁⠃ ⠶ ⠁⠃

$ba = a^4b = a^3ab = ab$

`b a = a^4 b = a^3 a b = ab`

1011. ⠭⠽ ⠀⠨⠅⠀ ⠭⠘⠤⠂⠐⠽⠘⠤⠂

$xy = x^{-1}y^{-1}$

```
xy = x^{-1} y^{-1}
```

1012. ⠠⠉⠷⠠⠓⠾ ⠀⠨⠅⠀ StartSet⠛ ⠈⠑ ⠠⠛⠈⠆⠛⠓ ⠀⠨⠅⠀ ⠓⠛forall⠓ ⠈⠑ ⠠⠓EndSet

$C(H) = \{g \in G : gh = hg \text{ for all } h \in H\}$

```
C(H) = \{ g \in G : gh = hg \text{ for all } h \in H \}
```

1013. ⠠⠉⠷⠠⠓⠾

$C(H)$

```
C(H)
```

1014. ⠛ ⠠⠓⠛⠘⠤⠂ ⠐ ⠀⠨⠅⠀ StartSet⠛⠓⠛⠘⠤⠂ ⠐⠆⠓ ⠈⠑ ⠠⠓EndSet

$gHg^{-1} = \{ghg^{-1} : h \in H\}$

```
gHg^{-1} = \{ghg^{-1} : h\in H\}
```

1015. ⠙⠂ ⠙⠆ ⠄ ⠄ ⠄ ⠙12

$d_1 d_2 \cdots d_{12}$

```
d_1 d_2 \cdots d_{12}
```

1016. ⠒⠈⠄⠙⠂ ⠬⠂ ⠈⠄⠙⠆ ⠬⠒⠈⠄⠙⠒⠬⠄ ⠄ ⠄ ⠬⠒⠈⠄⠙11⠬⠂ ⠈⠄⠙12 ⠸⠸ ⠰⠷⠍⠕⠙⠙10⠾

$3 \cdot d_1 + 1 \cdot d_2 + 3 \cdot d_3 + \cdots + 3 \cdot d_{11} + 1 \cdot d_{12} \equiv 0 \pmod{10}$

```
3 \cdot d_1 + 1 \cdot d_2 + 3 \cdot d_3 + \cdots + 3 \cdot d_{11}
+ 1 \cdot d_{12} \equiv 0 \pmod{10}
```

1017. ⠙12

d_{12}

```
d_{12}
```

1018. ⠷⠙⠂ ⠠⠙⠆ ⠄⠄ ⠠⠙ ⠰⠅ ⠐⠾⠈⠡⠷⠺⠂ ⠠⠺⠆ ⠄⠄ ⠠⠺ ⠰⠅ ⠐⠾ ⠸⠸ ⠰⠷⠍⠕⠙⠝⠾

$(d_1, d_2, \ldots, d_k) \cdot (w_1, w_2, \ldots, w_k) \equiv 0 \pmod{n}$

```
(d_1, d_2, \ldots, d_k ) \cdot (w_1, w_2, \ldots, w_k ) \equiv 0
\pmod{ n }
```

1019. ⠙⠂ ⠺⠂ ⠬⠙⠆ ⠺⠆ ⠬⠄ ⠄ ⠄ ⠬⠙ ⠰⠅ ⠐⠺ ⠰⠅ ⠐ ⠸⠸ ⠰⠷⠍⠕⠙⠝⠾

$d_1w_1 + d_2w_2 + \cdots + d_kw_k \equiv 0 \pmod{n}$

```
d_1 w_1 + d_2 w_2 + \cdots + d_k w_k \equiv 0 \pmod{ n}
```

1020. ⠷⠙⠂ ⠠⠙⠆ ⠄⠄ ⠠⠙ ⠰⠅ ⠐⠾⠈⠡⠷⠺⠂ ⠠⠺⠆ ⠄⠄ ⠠⠺ ⠰⠅ ⠐⠾ ⠸⠸ ⠰⠷⠍⠕⠙⠝⠾

$(d_1, d_2, \ldots, d_k) \cdot (w_1, w_2, \ldots, w_k) \equiv 0 \pmod{n}$

```
(d_1, d_2, \ldots, d_k ) \cdot (w_1, w_2, \ldots, w_k ) \equiv 0
\pmod{ n}
```

1021. ⠙⠂ ⠙⠆ ⠄ ⠄ ⠄ ⠙ ⠰⠅

$d_1 d_2 \cdots d_k$

```
d_1 d_2 \cdots d_k
```

1022. ⠴ ⠀⠐⠅⠱ ⠙ ⠰⠊⠐ ⠀⠐⠅⠀ ⠝

$0 \leq d_i < n$

```
0 \leq d_i \lt n
```

1023. ⠛⠉⠙⠷⠺⠰⠊⠐⠠⠝⠾ ⠨⠅ ⠂

$\gcd(w_i, n) = 1$

\gcd(w_i, n) = 1

1024. ⠂ ⠅⠱ ⠊ ⠅⠱ ⠅

$1 \leq i \leq k$

1 \leq i \leq k

1025. ⠙⠰⠊

d_i

d_i

1026. ⠙⠰⠚

d_j

d_j

1027. ⠛⠉⠙⠷⠺⠰⠊⠐⠤⠺⠰⠚⠐⠠⠝⠾ ⠨⠅ ⠂

$\gcd(w_i - w_j, n) = 1$

\gcd(w_i - w_j, n) = 1

1028. i

i

1029. j

j

1030. ⠷⠙⠂ ⠠⠙⠆ ⠠⠄ ⠠⠙10⠾⠡⠷10⠠⠔ ⠠⠄ ⠠⠂⠾ ⠸⠇ ⠴⠷⠍⠕⠙11⠾

$(d_1, d_2, \ldots, d_{10}) \cdot (10, 9, \ldots, 1) \equiv 0 \pmod{11}$

(d_1, d_2, \ldots, d_{10}) \cdot (10, 9, \ldots, 1) \equiv 0 \pmod{11}

1031. ⠙10

d_{10}

d_{10}

1032. six-firsts

$\frac{6}{1}$

\frac{6}{1}

1033. ⠼6.00000

6.00000

6.00000

1034. 6.00000⠬0.00000⠊

$6.00000 + 0.00000i$

6.00000+0.00000i

1035. ⠨⠗⠆

ρ_2

\rho_2

1036. ⠨⠗⠆⠀⠨⠅⠀⠠⠷⠠⠁⠀⠠⠃⠀⠠⠉⠠⠾⠠⠷⠠⠉⠀⠠⠁⠀⠠⠃⠠⠾⠀⠨⠅⠀⠠⠷⠂⠀⠆⠀⠒⠠⠾
⠠⠷⠒⠀⠂⠀⠆⠠⠾

$\rho_2 = \begin{pmatrix} A & B & C\\ C & A & B \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3\\ 3 & 1 & 2 \end{pmatrix}$

\rho_2= \begin{pmatrix} A & B & C\\ C & A & B \end{pmatrix} = \begin{pmatrix}
1 & 2 & 3\\ 3 & 1 & 2 \end{pmatrix}

1037. ⠋⠛

fg

fg

1038. ⠷⠋⠛⠾⠷⠭⠾⠀⠨⠅⠀⠋⠷⠛⠷⠭⠾⠾

$(fg)(x)=f(g(x))$

(fg)(x)=f(g(x))

1039. ⠄

$\dots{}$

\dots{}

1040. ⠨⠍

μ

\mu

1041. ⠨⠗

ρ

\rho

1042. ⠤⠂

-1

-1

1043. J

J

1044. ⠤⠂⠡⠤⠂⠀⠨⠅⠀⠂

$-1\cdot -1= 1$

-1\cdot -1= 1

1045. ⠤⠀⠠⠊

$-I$

-I

1046. ⠊⠀⠨⠅⠀⠜⠤⠂⠻

$i=\sqrt{-1}$

i=\sqrt{-1}

1047. ⠠⠎⠦

S_8

S_8

1048. ⠈⠠⠵

$\mathbb{Z}$

`\mathbb Z`

1049. ⠒⠈⠑⠈⠠⠵

$3 \in \mathbb{Z}$

`3 \in {\mathbb Z}`

1050. ⠒⠈⠠⠵ ⠨⠅ StartSet⠄⠠⠤⠒⠠⠴⠠⠒⠠⠖⠠⠄EndSet

$3\mathbb{Z} = \{\ldots, -3, 0, 3, 6, \ldots\}$

`3 {\mathbb Z} = \{ \ldots, -3, 0, 3, 6, \ldots \}`

1051. ⠒⠈⠠⠵

$3\mathbb{Z}$

`3 {\mathbb Z}`

1052. ⠠⠓ ⠨⠅ StartSet⠆⠘⠝⠐⠱⠝⠈⠑⠈⠠⠵EndSet

$H = \{2^n : n \in \mathbb{Z}\}$

`H = \{ 2^n : n \in {\mathbb Z} \}`

1053. ⠁ ⠨⠅ ⠆⠘⠍

$a = 2^m$

`a = 2^m`

1054. ⠃ ⠨⠅ ⠆⠘⠝

$b = 2^n$

`b = 2^n`

1055. ⠁⠃⠘⠤⠂⠐ ⠨⠅ ⠆⠘⠍⠐⠆⠘⠤⠝⠐ ⠨⠅ ⠆⠘⠍⠤⠝

$ab^{-1} = 2^m 2^{-n} = 2^{m-n}$

`ab^{-1} = 2^m 2^{-n} = 2^{m-n}`

1056. ⠄⡓⠟⠆⠶⠈⠢⠤⠄⠈⠄⡓⠟⠆⠶⠈⠢⠤⠄ ⠨⠅ StartSet⠁⠘⠅⠐⠱⠅⠈⠑⠈⠠⠵EndSet

$\langle a \rangle = \{a^k : k \in \mathbb{Z}\}$

`\langle a \rangle = \{ a^k : k \in {\mathbb Z} \}`

1057. ⠄⡓⠟⠆⠶⠈⠢⠤⠄⠈⠄⡓⠟⠆⠶⠈⠢⠤⠄

$\langle a \rangle$

`\langle a \rangle`

1058. ⠁⠘⠴⠐ ⠨⠅ ⠑

$a^0 = e$

`a^0 = e`

1059. ⠛ ⠨⠅ ⠁⠘⠍

$g = a^m$

`g = a^m`

1060. ⠓ ⠨⠅ ⠁⠘⠝

$h = a^n$

`h = a^n`

1061. ⠛⠓ ⠨⠅ ⠁⠘⠍⠐⠁⠘⠝⠐ ⠨⠅ ⠁⠘⠍⠬⠝

$gh = a^m a^n = a^{m+n}$

gh = a^m a^n = a^{m+n}

1062. ⠛ ⠨⠅ ⠁⠘⠝

$g = a^n$

g = a^n

1063. ⠛⠘⠤⠂⠐ ⠨⠅ ⠁⠘⠤⠝

$g^{-1} = a^{-n}$

g^{-1} = a^{-n}

1064. ⠄⡷⠭⠆⠶⠑⠦⠄⠁⠄⡷⠭⠆⠶⠑⠔⠄ ⠨⠅ StartSet⠝⠁ ⠈⠆⠝ ⠈⠑ ⠈⠠⠵EndSet

$\langle a \rangle = \{ na : n \in \mathbb{Z} \}$

\langle a \rangle = \{ na : n \in {\mathbb Z} \}

1065. ⠠⠛ ⠨⠅ ⠄⡷⠭⠆⠶⠑⠦⠄⠁⠄⡷⠭⠆⠶⠑⠔⠄

$G = \langle a \rangle$

G = \langle a \rangle

1066. ⠁⠘⠝⠐ ⠨⠅ ⠑

$a^n = e$

a^n= e

1067. ⠳⠁⠳ ⠨⠅ ⠝

$|a| = n$

|a| = n

1068. ⠳⠁⠳ ⠨⠅ ⠠⠿

$|a| = \infty$

|a| = \infty

1069. ⠈⠠⠵⠖

$\mathbb{Z}_6$

{\mathbb Z}_6

1070. ⠄⡷⠭⠆⠶⠑⠦⠄⠆⠄⡷⠭⠆⠶⠑⠔⠄ ⠨⠅ StartSet⠴⠠⠆⠠⠲EndSet

$\langle 2 \rangle = \{ 0, 2, 4 \}$

\langle 2 \rangle = \{ 0, 2, 4 \}

1071. ⠠⠥⠷⠔⠾

$U(9)$

U(9)

1072. StartSet⠂⠠⠆⠠⠲⠠⠢⠠⠶⠠⠦EndSet

$\{ 1, 2, 4, 5, 7, 8 \}$

\{ 1, 2, 4, 5, 7, 8 \}

1073. ⠛ ⠨⠅ ⠁⠘⠗

$g = a^r$

g = a^r

1074. ⠓ ⠨⠅ ⠁⠘⠎

$h = a^s$

h = a^s

1075. ⠛⠓ ⠨⠅ ⠁⠘⠗⠐⠁⠘⠎⠐ ⠨⠅ ⠁⠘⠗⠬⠎⠐ ⠨⠅ ⠁⠘⠎⠬⠗⠐ ⠨⠅ ⠁⠘⠎⠐⠁⠘⠗⠐ ⠨⠅ ⠓⠛

$gh = a^r a^s = a^{r+s} = a^{s+r} = a^s a^r = hg$

g h = a^r a^s = a^{r+s} = a^{s+r} = a^s a^r = h g

1076. ⠁⠘⠝

a^n

a^n

1077. ⠝ ⠨⠂ ⠴

$n > 0$

n \gt 0

1078. ⠁⠘⠍⠐⠈⠑⠠⠓

$a^m \in H$

a^m \in H

1079. ⠓ ⠨⠅ ⠁⠘⠍

$h = a^m$

h = a^m

1080. ⠓⠄ ⠨⠅ ⠁⠘⠅

$h' = a^k$

h' = a^k

1081. ⠅ ⠨⠅ ⠍⠟⠬⠗

$k = mq + r$

k = mq +r

1082. ⠴ ⠅⠱ ⠗ ⠐⠅ ⠍

$0 \leq r < m$

0 \leq r \lt m

1083. ⠁⠘⠅⠐ ⠨⠅ ⠁⠘⠍⠟⠬⠗⠐ ⠨⠅ ⠷⠁⠘⠍⠐⠾⠘⠟⠐⠁⠘⠗⠐ ⠨⠅ ⠓⠘⠟⠐⠁⠘⠗

$a^k = a^{mq+r} = (a^m)^q a^r = h^q a^r$

a^k = a^{mq +r} = (a^m)^q a^r = h^q a^r

1084. ⠁⠘⠗⠐ ⠨⠅ ⠁⠘⠅⠐⠓⠘⠤⠟

$a^r = a^k h^{-q}$

a^r = a^k h^{-q}

1085. ⠁⠘⠅

a^k

a^k

1086. ⠓⠘⠤⠟

h^{-q}

h^{-q}

1087. ⠁⠘⠗

a^r

`a^r`

1088. ⠁⠘⠍

a^m

`a^m`

1089. ⠗ ⠨⠅ ⠴

$r = 0$

`r=0`

1090. ⠅ ⠨⠅ ⠍⠟

$k = mq$

`k=mq`

1091. ⠓⠄ ⠨⠅ ⠁⠘⠅⠐ ⠨⠅ ⠁⠘⠍⠟⠐ ⠨⠅ ⠓⠘⠟

$h' = a^k = a^{mq} = h^q$

`h' = a^k = a^{mq} = h^q`

1092. ⠝⠈⠠⠵

$n\mathbb{Z}$

`n{\mathbb Z}`

1093. ⠝ ⠨⠅ ⠴⠠⠼⠂⠠⠼⠆⠠⠄⠄

$n = 0, 1, 2, \ldots$

`n = 0, 1, 2,\ldots`

1094. ⠁⠘⠅⠐ ⠨⠅ ⠑

$a^k = e$

`a^k=e`

1095. ⠅ ⠨⠅ ⠝⠟⠬⠗

$k = nq + r$

`k = nq + r`

1096. ⠴ ⠅⠱ ⠗ ⠐⠅ ⠝

$0 \leq r < n$

`0 \leq r \lt n`

1097. ⠑ ⠨⠅ ⠁⠘⠅⠐ ⠨⠅ ⠁⠘⠝⠟⠬⠗⠐ ⠨⠅ ⠁⠘⠝⠟⠐⠁⠘⠗⠐ ⠨⠅ ⠑⠁⠘⠗⠐ ⠨⠅ ⠁⠘⠗

$e = a^k = a^{nq+r} = a^{nq} a^r = e a^r = a^r$

`e = a^k = a^{nq + r} = a^{nq} a^r = e a^r = a^r`

1098. ⠁⠘⠍⠐ ⠨⠅ ⠑

$a^m = e$

`a^m = e`

1099. ⠗ ⠨⠅ ⠴

$r = 0$

`r= 0`

1100. ⠅ ⠶ ⠝⠎

$k = ns$

k=ns

1101. ⠁⠘⠅⠐ ⠶ ⠁⠘⠝⠎⠐ ⠶ ⠷⠁⠘⠝⠐⠾⠘⠎⠐ ⠶ ⠑⠘⠎⠐ ⠶ ⠑

$a^k = a^{ns} = (a^n)^s = e^s = e$

a^k = a^{ns} = (a^n)^s = e^s = e

1102. ⠃ ⠶ ⠁⠘⠅

$b = a^k$

b = a^k

1103. ⠝⠌⠙

n/d

n/d

1104. ⠙ ⠶ ⠛⠉⠙⠷⠅⠠⠝⠾

$d = \gcd(k,n)$

d = \gcd(k,n)

1105. ⠑ ⠶ ⠃⠘⠍⠐ ⠶ ⠁⠘⠅⠍

$e = b^m = a^{km}$

e = b^m = a^{km}

1106. ⠅⠍

km

km

1107. ⠍⠷⠅⠌⠙⠾

$m(k/d)$

m(k/d)

1108. ⠅⠌⠙

k/d

k/d

1109. ⠂ ⠅⠱ ⠗ ⠐⠅ ⠝

$1 \leq r < n$

1 \leq r \lt n

1110. ⠛⠉⠙⠷⠗⠠⠝⠾ ⠶ ⠂

$\gcd(r,n) = 1$

\gcd(r,n) = 1

1111. ⠈⠠⠵16

${\mathbb Z}_{16}$

{\mathbb Z}_{16}

1112. ⠼⠶

7

7

1113. ⠼11

11

11

1114. ⠼13

13

13

1115. ⠼15

15

15

1116. ⠼16

16

16

1117. ⠈⠠⠉ ⠨⠅ ⠨⠷⠁⠬⠃⠊⠱⠁⠠⠃ ⠈⠑ ⠈⠠⠗⠨⠾

$\mathbb{C} = \{a + bi : a, b \in \mathbb{R}\}$

{\mathbb C} = \{ a + bi : a, b \in {\mathbb R} \}

1118. ⠵ ⠨⠅ ⠁⠬⠃⠊

$z = a + bi$

z = a + bi

1119. ⠵ ⠨⠅ ⠁⠬⠃⠊

$z = a + bi$

z=a+bi

1120. ⠺ ⠨⠅ ⠉⠬⠙⠊

$w = c + di$

w= c+di

1121. ⠵⠬⠺ ⠨⠅ ⠷⠁⠬⠃⠊⠾⠬⠷⠉⠬⠙⠊⠾ ⠨⠅ ⠷⠁⠬⠉⠾⠬⠷⠃⠬⠙⠾⠊

$z + w = (a + bi) + (c + di) = (a + c) + (b + d)i$

z + w=(a + bi) + (c + di) = (a + c) + (b + d)i

1122. w

w

1123. ⠷⠁⠬⠃⠊⠾⠷⠉⠬⠙⠊⠾ ⠨⠅ ⠁⠉⠬⠃⠙⠊⠘⠆⠐⠬⠁⠙⠊⠬⠃⠉⠊ ⠨⠅ ⠷⠁⠉⠤⠃⠙⠾⠬⠷⠁⠙⠬⠃⠉⠾⠊

$(a + bi)(c + di) = ac + bdi^2 + adi + bci = (ac - bd) + (ad + bc)i$

(a + bi)(c + di) = ac + bdi^2 + adi + bci = (ac -bd) +(ad + bc)i

1124. ⠵ ⠨⠅ ⠁⠬⠃⠊

$z = a + bi$

z = a +bi

1125. ⠵⠘⠤⠂ ⠐⠈⠑ ⠈⠠⠉⠘⠈⠼

$z^{-1} \in \mathbb{C}^*$

z^{-1} \in {\mathbb C}^\ast

1126. ⠵⠵⠘⠤⠂⠐ ⠨⠅ ⠵⠘⠤⠂⠐⠵ ⠨⠅ ⠂

$zz^{-1} = z^{-1}z = 1$

z z^{-1} = z^{-1} z = 1

1127. ⠵⠘⠤⠂⠐ ⠨⠅ ⠹⠁⠤⠃⠊⠌⠁⠘⠆⠐⠬⠃⠘⠆⠐⠼

$z^{-1} = \frac{a-bi}{a^2+b^2}$

z^{-1} = \frac{a-bi}{ a^2 + b^2 }

1128. ⠵overbar ⠨⠅ ⠁⠤⠃⠊

$\overline{z} = a - bi$

\overline{z} = a- bi

1129. ⠳⠵⠳ ⠨⠅ ⠜⠁⠘⠆⠐⠬⠃⠘⠆⠐⠻

$|z| = \sqrt{a^2 + b^2}$

|z| = \sqrt{a^2 + b^2}

1130. ⠵ ⠨⠅ ⠆⠬⠒⠊

$z = 2 + 3i$

z = 2 + 3i

1131. ⠺ ⠨⠅ ⠂⠤⠆⠊

$w = 1 - 2i$

w = 1-2i

1132. ⠵⠬⠺ ⠨⠅ ⠷⠆⠬⠒⠊⠾⠬⠷⠂⠤⠆⠊⠾ ⠨⠅ ⠒⠬⠊

$z + w = (2 + 3i) + (1 - 2i) = 3 + i$

z + w = (2 + 3i) + (1 - 2i) = 3 + i

1133. ⠵⠺ ⠨⠅ ⠷⠆⠬⠒⠊⠾⠷⠂⠤⠆⠊⠾ ⠨⠅ ⠦⠤⠊

$zw = (2 + 3i)(1 - 2i) = 8 - i$

z w = (2 + 3i)(1 - 2i) = 8 - i

1134. ⠵⠂ ⠨⠅ ⠆⠬⠒⠊

$z_1 = 2 + 3i$

z_1 = 2 + 3i

1135. ⠵⠆ ⠨⠅ ⠂⠤⠆⠊

$z_2 = 1 - 2i$

z_2 = 1 - 2i

1136. ⠵⠒ ⠨⠅ ⠤⠒⠬⠆⠊

$z_3 = -3 + 2i$

z_3 = - 3 + 2i

1137. ⠨⠹

θ

\theta

1138. ⠵ ⠨⠅ ⠁⠬⠃⠊ ⠨⠅ ⠗⠷⠉⠕⠎ ⠨⠹⠬⠊⠎⠊⠝ ⠨⠹⠾

$z = a + bi = r(\cos\theta + i\sin\theta)$

z = a + bi = r(\cos \theta + i \sin \theta)

1139. ⠗⠀⠶⠀⠳⠵⠳⠀⠶⠀⠜⠁⠘⠆⠐⠬⠃⠘⠆⠐⠻

$r = |z| = \sqrt{a^2 + b^2}$

```
r = |z| = \sqrt{a^2 + b^2}
```

1140. ⠗⠷⠉⠕⠎⠀⠨⠹⠬⠊⠎⠊⠝⠀⠨⠹⠾

$r(\cos\theta + i\sin\theta)$

```
r( \cos \theta + i \sin \theta)
```

1141. ⠗⠉⠊⠎⠀⠨⠹

$r\,\mathrm{cis}\,\theta$

```
r \cis \theta
```

1142. ⠉⠕⠎⠀⠨⠹⠬⠊⠎⠊⠝⠀⠨⠹

$\cos\theta + i\sin\theta$

```
\cos \theta + i \sin \theta
```

1143. ⠴⠘⠘⠨⠡⠐⠀⠅⠱⠀⠨⠹⠀⠐⠅⠀360⠘⠘⠨⠡

$0^{\circ} \leq \theta < 360^{\circ}$

```
0^{\circ} \leq \theta \lt 360^{\circ}
```

1144. ⠴⠀⠅⠱⠀⠨⠹⠀⠐⠅⠀⠆⠨⠏

$0 \leq \theta < 2\pi$

```
0 \leq \theta \lt2 \pi
```

1145. ⠵⠀⠶⠀⠆⠉⠊⠎60⠘⠘⠨⠡

$z = 2\,\mathrm{cis}\,60^{\circ}$

```
z = 2 \cis 60^{\circ}
```

1146. ⠁⠀⠶⠀⠆⠉⠕⠎⠀60⠘⠘⠨⠡⠐⠀⠶⠀⠂

$a = 2\cos 60^{\circ} = 1$

```
a = 2 \cos 60^{\circ} = 1
```

1147. ⠃⠀⠶⠀⠆⠎⠊⠝⠀60⠘⠘⠨⠡⠐⠀⠶⠀⠜⠒⠻

$b = 2\sin 60^{\circ} = \sqrt{3}$

```
b = 2 \sin 60^{\circ} = \sqrt{3}
```

1148. ⠵⠀⠶⠀⠂⠬⠜⠒⠻⠊

$z = 1+\sqrt{3}\, i$

```
z = 1+\sqrt{3}\, i
```

1149. ⠵⠀⠶⠀⠒⠜⠆⠻⠤⠒⠜⠆⠻⠊

$z = 3\sqrt{2} - 3\sqrt{2}\, i$

```
z = 3 \sqrt{2} - 3 \sqrt{2}\, i
```

1150. ⠗⠀⠶⠀⠜⠁⠘⠆⠐⠬⠃⠘⠆⠐⠻⠀⠶⠀⠜36⠻⠀⠶⠀⠖

$r = \sqrt{a^2 + b^2} = \sqrt{36} = 6$

```
r = \sqrt{a^2 + b^2} = \sqrt{36 } = 6
```

1151. ⠨⠹⠀⠶⠀⠁⠗⠉⠀⠞⠁⠝⠀⠷⠹⠃⠌⠁⠼⠾⠀⠶⠀⠁⠗⠉⠀⠞⠁⠝⠀⠷⠤⠂⠾⠀⠶⠀315⠘⠘⠨⠡

$\theta = \arctan\left(\frac{b}{a}\right) = \arctan(-1) = 315^{\circ}$

```
\theta = \arctan \left( \frac{b}{a} \right) = \arctan( - 1) = 315^{\circ}
```

1152. ⠒⠜⠆⠻⠤⠒⠜⠆⠻⠊ ⠨⠅ ⠖⠉⠊⠎315⠘⠘⠎⠄

$3\sqrt{2} - 3\sqrt{2}\, i=6 \operatorname{cis} 315^{\circ}$

```
3 \sqrt{2} - 3 \sqrt{2}\, i=6 \cis 315^{\circ}
```

1153. ⠵ ⠨⠅ ⠗⠉⠊⠎⠨⠹

$z = r \operatorname{cis} \theta$

```
z = r \cis \theta
```

1154. ⠺ ⠨⠅ ⠎⠉⠊⠎⠨⠋

$w = s \operatorname{cis} \phi$

```
w = s \cis \phi
```

1155. ⠵⠺ ⠨⠅ ⠗⠎⠉⠊⠎⠷⠨⠹⠬⠨⠋⠾

$zw = r s \operatorname{cis}(\theta + \phi)$

```
zw = r s \cis( \theta + \phi)
```

1156. ⠵ ⠨⠅ ⠒⠉⠊⠎⠷⠨⠏⠌⠒⠾

$z = 3 \operatorname{cis}(\pi / 3)$

```
z = 3 \cis( \pi / 3 )
```

1157. ⠺ ⠨⠅ ⠆⠉⠊⠎⠷⠨⠏⠌⠖⠾

$w = 2 \operatorname{cis}(\pi / 6)$

```
w = 2 \cis(\pi / 6 )
```

1158. ⠵⠺ ⠨⠅ ⠖⠉⠊⠎⠷⠨⠏⠌⠆⠾ ⠨⠅ ⠖⠊

$zw = 6 \operatorname{cis}(\pi / 2) = 6i$

```
zw = 6 \cis( \pi / 2 ) = 6i
```

1159. ⠈⠷⠗⠉⠊⠎⠨⠹⠈⠾⠘⠝⠐ ⠨⠅ ⠗⠘⠝⠐⠉⠊⠎⠷⠝⠨⠹⠾

$[r \operatorname{cis} \theta]^n = r^n \operatorname{cis}(n \theta)$

```
[r \cis \theta ]^n = r^n \cis( n \theta)
```

1160. ⠝ ⠨⠅ ⠂ ⠠⠼⠆ ⠠⠄

$n = 1, 2, \ldots$

```
n = 1, 2, \ldots
```

1161. ⠵ ⠨⠅ ⠂⠬⠊

$z= 1+i$

```
z= 1+i
```

1162. ⠵⠘10

z^{10}

```
z^{10}
```

1163. ⠷⠂⠬⠊⠾⠘10

$(1 + i)^{10}$

```
(1 + i)^{10}
```

1164. ⠈⠠⠞ ⠨⠅ StartSet⠵⠈⠑⠈⠠⠉⠱⠳⠵⠳ ⠨⠅ ⠂EndSet

${\mathbb T} = \{ z \in {\mathbb C} : |z| = 1 \}$

```
{\mathbb T} = \{ z \in {\mathbb C} : |z| = 1 \}
```

1165. ⠤⠊

$-i$

-i

1166. ⠵⠘⠲⠐ ⠨⠅ ⠂

$z^4 = 1$

z^4 = 1

1167. ⠵⠘⠝⠐ ⠨⠅ ⠂

$z^n = 1$

z^n=1

1168. ⠵⠘⠝⠐ ⠨⠅ ⠂

$z^n = 1$

z^n = 1

1169. ⠵ ⠨⠅ ⠉⠊⠎⠷⠹⠆⠅ ⠨⠏⠌⠝⠼⠾

$z = \operatorname{cis}\left(\frac{2k\pi}{n}\right)$

z = \cis\left(\frac{2 k \pi}{n } \right)

1170. ⠅ ⠨⠅ ⠴⠠⠂⠠ ⠄⠄ ⠠⠝⠤⠂

$k = 0, 1, \ldots, n-1$

k = 0, 1, \ldots, n-1

1171. ⠵⠘⠝⠐ ⠨⠅ ⠉⠊⠎⠷⠝⠹⠆⠅ ⠨⠏⠌⠝⠼⠾ ⠨⠅ ⠉⠊⠎⠷⠆⠅ ⠨⠏⠾ ⠨⠅ ⠂

$z^n = \operatorname{cis}\left(n\frac{2k\pi}{n}\right) = \operatorname{cis}(2k\pi) = 1$

z^n = \cis \left(n \frac{2 k \pi}{n } \right) = \cis(2 k \pi)
= 1

1172. ⠆⠅ ⠨⠏⠸⠌⠝

$2k\pi/n$

2 k \pi /n

1173. ⠆ ⠨⠏

2π

2 \pi

1174. ⠆ ⠘⠆

2^2

2^2

1175. ⠆ ⠘⠦

2^8

2^8

1176. ⠆ ⠘⠆ ⠘⠘1,000,000

$2^{2^{1{,}000{,}000}}$

2^{2^{1{,}000{,}000} }

1177. ⠆ ⠘37,398,332 ⠐⠷⠍⠕⠙⠙46,389⠾

$2^{37{,}398{,}332} \pmod{46{,}389}$

```
2^{37{,}398{,}332 } \pmod{ 46{,}389}
```

1178. ⠼46,388

$46{,}388$

```
46{,}388
```

1179. ⠁ ⠨⠅ ⠆⠘⠅⠂⠐⠬⠆⠘⠅⠆⠐⠬⠄⠄⠄⠬⠆⠘⠅⠘⠰⠝

$a = 2^{k_1} + 2^{k_2} + \cdots + 2^{k_n}$

```
a = 2^{k_1} + 2^{k_2} + \cdots + 2^{k_n}
```

1180. ⠅⠂ ⠐⠅ ⠅⠆ ⠐⠅ ⠄⠄⠄ ⠐⠅ ⠅⠰⠝

$k_1 < k_2 < \cdots < k_n$

```
k_1 \lt k_2 \lt \cdots \lt k_n
```

1181. 57 ⠨⠅ ⠆⠘⠴⠐⠬⠆⠘⠒⠐⠬⠆⠘⠲⠐⠬⠆⠘⠢

$57 = 2^0 + 2^3 + 2^4 + 2^5$

```
57 = 2^0 + 2^3 + 2^4 + 2^5
```

1182. ⠃ ⠸⠇ ⠁⠘⠭⠐⠷⠍⠕⠙⠝⠾

$b \equiv a^x \pmod{n}$

```
b \equiv a^x \pmod{ n}
```

1183. ⠉ ⠸⠇ ⠁⠘⠽⠐⠷⠍⠕⠙⠝⠾

$c \equiv a^y \pmod{n}$

```
c \equiv a^y \pmod{ n}
```

1184. ⠃⠉ ⠸⠇ ⠁⠘⠭⠬⠽⠐⠷⠍⠕⠙⠝⠾

$bc \equiv a^{x+y} \pmod{n}$

```
bc \equiv a^{x+y} \pmod{ n}
```

1185. ⠁⠘⠆⠘⠘⠅⠐⠷⠍⠕⠙⠝⠾

$a^{2^k} \pmod{n}$

```
a^{2^k} \pmod{ n}
```

1186. 271⠘321⠐⠷⠍⠕⠙481⠾

$271^{321} \pmod{481}$

```
271^{321} \pmod{ 481}
```

1187. 321 ⠨⠅ ⠆⠘⠴⠐⠬⠆⠘⠖⠐⠬⠆⠘⠦⠐⠸⠆

$321 = 2^0 + 2^6 + 2^8;$

```
321 = 2^0 +2^6 + 2^8;
```

1188. 271⠘321⠐⠷⠍⠕⠙481⠾

$271^{321} \pmod{481}$

```
271^{ 321} \pmod{ 481}
```

1189. 271⠘⠆⠘⠘⠴⠘⠬⠆⠘⠘⠖⠘⠬⠆⠘⠘⠦⠐⠸⠇271⠘⠆⠘⠘⠴⠐⠡271⠘⠆⠘⠘⠖⠐⠡271⠘⠆⠘⠘⠦⠐⠷⠍⠕⠙481⠾

$271^{2^0+2^6+2^8} \equiv 271^{2^0} \cdot 271^{2^6} \cdot 271^{2^8} \pmod{481}$

271^{ 2^0 +2^6 + 2^8 } \equiv 271^{ 2^0 } \cdot 271^{2^6 } \cdot
271^{ 2^8 } \pmod{ 481}

1190. 271⠘⠆⠘⠘⠊⠐⠷⠍⠕⠙481⠾

$271^{2^i} \pmod{481}$

271^{ 2^i } \pmod{ 481}

1191. ⠊ ⠨⠅ ⠴⠠⠼⠖⠠⠼⠦

$i = 0, 6, 8$

i = 0, 6, 8

1192. 271⠘⠆⠘⠘⠂⠐ ⠨⠅ 73,441 ⠸⠇ 329⠷⠍⠕⠙481⠾

$271^{2^1} = 73{,}441 \equiv 329 \pmod{481}$

271^{ 2^1} = 73{,}441 \equiv 329 \pmod{ 481}

1193. 271⠘⠆⠘⠘⠆⠐⠷⠍⠕⠙481⠾

$271^{2^2} \pmod{481}$

271^{ 2^2} \pmod{481}

1194. ⠷⠁⠘⠆⠘⠘⠝⠐⠾⠘⠆⠐⠸⠇⠁⠘⠆⠡⠆⠘⠘⠝⠐⠸⠇⠁⠘⠆⠘⠘⠝⠬⠂⠐⠷⠍⠕⠙⠝⠾

$(a^{2^n})^2 \equiv a^{2 \cdot 2^n} \equiv a^{2^{n+1}} \pmod{n}$

(a^{2^n})^2 \equiv a^{2 \cdot 2^n} \equiv a^{ 2^{n+1} } \pmod{ n}

1195. 271⠘⠆⠘⠘⠖⠐⠸⠇419⠷⠍⠕⠙481⠾

$271^{2^6} \equiv 419 \pmod{481}$

271^{ 2^6 } \equiv 419 \pmod{481}

1196. 271⠘⠆⠘⠘⠦⠐⠸⠇16⠷⠍⠕⠙481⠾

$271^{2^8} \equiv 16 \pmod{481}$

271^{ 2^8 } \equiv 16 \pmod{481}

1197. ⠈⠠⠵60

${\mathbb Z}_{60}$

{\mathbb Z}_{60}

1198. ⠈⠠⠟

${\mathbb Q}$

{\mathbb Q}

1199. ⠢⠈⠑⠈⠠⠵12

$5 \in {\mathbb Z}_{12}$

5 \in {\mathbb Z}_{12}

1200. ⠜⠒⠻⠈⠑⠈⠠⠗

$\sqrt{3} \in {\mathbb R}$

\sqrt{3} \in {\mathbb R}

1201. ⠜⠒⠻⠈⠑⠈⠠⠗⠘⠈⠼

$\sqrt{3} \in {\mathbb R}^\ast$

\sqrt{3} \in {\mathbb R}^\ast

1202. ⠤⠊⠈⠑⠈⠠⠉⠘⠈⠼

$-i \in {\mathbb C}^\ast$

-i \in {\mathbb C}^\ast

1203. 72 ⠈⠑⠈⠠⠵240

$72 \in {\mathbb Z}_{240}$

72 \in {\mathbb Z}_{240}

1204. 312 ⠈⠑⠈⠠⠵471

$312 \in {\mathbb Z}_{471}$

312 \in {\mathbb Z}_{471}

1205. ⠈⠠⠵24

${\mathbb Z}_{24}$

{\mathbb Z}_{24}

1206. ⠈⠠⠵12

${\mathbb Z}_{12}$

{\mathbb Z}_{12}

1207. ⠈⠠⠵13

${\mathbb Z}_{13}$

{\mathbb Z}_{13}

1208. ⠈⠠⠵48

${\mathbb Z}_{48}$

{\mathbb Z}_{48}

1209. ⠠⠥⠷20⠾

$U(20)$

U(20)

1210. ⠠⠥⠷18⠾

$U(18)$

U(18)

1211. ⠈⠠⠗⠘⠈⠼

${\mathbb R}^\ast$

{\mathbb R}^\ast

1212. ⠆⠊

$2i$

2i

1213. ⠷⠂⠬⠊⠾⠌⠜⠆⠻

$(1 + i) / \sqrt{2}$

(1 + i) / \sqrt{2}

1214. ⠷⠂ ⠬⠜⠒⠻⠊⠾⠌⠆

$(1 + \sqrt{3}\, i) / 2$

`(1 + \sqrt{3}\, i) / 2`

1215. ⠶ ⠈⠠⠵ ⠨⠅ StartSet⠠ ⠠⠤⠶ ⠠⠴ ⠠⠶ ⠠14 ⠠⠠ EndSet

$7 {\mathbb Z} = \{ \ldots, -7, 0, 7, 14, \ldots \}$

`7 {\mathbb Z} = \{ \ldots, -7, 0, 7, 14, \ldots \}`

1216. StartSet⠴ ⠠⠒ ⠠⠖ ⠠⠔ ⠠12 ⠠15 ⠠18 ⠠21EndSet

$\{ 0, 3, 6, 9, 12, 15, 18, 21 \}$

`\{ 0, 3, 6, 9, 12, 15, 18, 21 \}`

1217. StartSet⠴EndSet

$\{ 0 \}$

`\{ 0 \}`

1218. StartSet⠴ ⠠⠖EndSet

$\{ 0, 6 \}$

`\{ 0, 6 \}`

1219. StartSet⠴ ⠠⠲ ⠠⠦EndSet

$\{ 0, 4, 8 \}$

`\{ 0, 4, 8 \}`

1220. StartSet⠴ ⠠⠒ ⠠⠖ ⠠⠔EndSet

$\{ 0, 3, 6, 9 \}$

`\{ 0, 3, 6, 9 \}`

1221. StartSet⠴ ⠠⠆ ⠠⠲ ⠠⠖ ⠠⠦ ⠠10EndSet

$\{ 0, 2, 4, 6, 8, 10 \}$

`\{ 0, 2, 4, 6, 8, 10 \}`

1222. StartSet⠂ ⠠⠒ ⠠⠶ ⠠⠔EndSet

$\{ 1, 3, 7, 9 \}$

`\{ 1, 3, 7, 9 \}`

1223. StartSet⠂ ⠠⠤⠂ ⠠⠊ ⠠⠤⠊EndSet

$\{ 1, -1, i, -i \}$

`\{ 1, -1, i, -i \}`

1224. ⠠⠷⠼⠴ ⠼⠂ ⠠⠾ ⠠⠷⠤⠂ ⠼⠴ ⠠⠾

$$\displaystyle \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

`\displaystyle \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}`

1225. ⠠⠷⠼⠴ ⠂⠌⠒ ⠠⠾ ⠠⠷⠼⠒ ⠼⠴ ⠠⠾

$$\displaystyle \begin{pmatrix} 0 & 1/3 \\ 3 & 0 \end{pmatrix}$$

`\displaystyle \begin{pmatrix} 0 & 1/3 \\ 3 & 0 \end{pmatrix}`

1226. ⠠⠷⠼⠂ ⠤⠂ ⠠⠾ ⠠⠷⠼⠂ ⠼⠴ ⠠⠾

$$\displaystyle \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$$

\displaystyle \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}

1227. ⠠⠷⠼⠂ ⠤⠂ ⠠⠾ ⠠⠷⠼⠴ ⠼⠂ ⠠⠾

$$\displaystyle \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

\displaystyle \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}

1228. ⠠⠷⠼⠂ ⠤⠂ ⠠⠾ ⠠⠷⠤⠂ ⠼⠴ ⠠⠾

$$\displaystyle \begin{pmatrix} 1 & -1 \\ -1 & 0 \end{pmatrix}$$

\displaystyle \begin{pmatrix} 1 & -1 \\ -1 & 0 \end{pmatrix}

1229. ⠠⠷⠜⠒⠻⠌⠆ ⠂⠌⠆ ⠠⠾ ⠠⠷⠤⠂⠌⠆ ⠜⠒⠻⠌⠆ ⠠⠾

$$\displaystyle \begin{pmatrix} \sqrt{3}/ 2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{pmatrix}$$

\displaystyle \begin{pmatrix} \sqrt{3}/ 2 & 1/2 \\ -1/2 & \sqrt{3}/2
\end{pmatrix}

1230. ⠠⠷⠼⠂ ⠼⠴ ⠠⠾ ⠠⠷⠼⠴ ⠼⠂ ⠠⠾⠠⠠⠷⠤⠂ ⠼⠴ ⠠⠾ ⠠⠷⠼⠴ ⠤⠂ ⠠⠾⠠⠠⠷⠼⠴ ⠤⠂ ⠠⠾
⠠⠷⠼⠂ ⠼⠴ ⠠⠾⠠⠠⠷⠼⠴ ⠼⠂ ⠠⠾ ⠠⠷⠤⠂ ⠼⠴ ⠠⠾

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1
& 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix},
\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}

1231. ⠠⠷⠼⠂ ⠼⠴ ⠠⠾ ⠠⠷⠼⠴ ⠼⠂ ⠠⠾⠠⠠⠷⠼⠂ ⠤⠂ ⠠⠾ ⠠⠷⠼⠂ ⠼⠴ ⠠⠾⠠⠠⠷⠤⠂ ⠼⠂ ⠠⠾
⠠⠷⠤⠂ ⠼⠴ ⠠⠾⠠⠠⠷⠼⠴ ⠼⠂ ⠠⠾ ⠠⠷⠤⠂ ⠼⠂ ⠠⠾⠠⠠⠷⠼⠴ ⠤⠂ ⠠⠾ ⠠⠷⠼⠂ ⠤⠂ ⠠⠾⠠⠠⠷⠤⠂ ⠼⠴ ⠠⠾
⠠⠷⠼⠴ ⠤⠂ ⠠⠾

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} -1 & 1 \\ -1 & 0 \end{pmatrix},$$
$$\begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1
& -1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} -1 & 1 \\ -1 & 0 \end{pmatrix},
\\ \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}, \begin{pmatrix}
0 & -1 \\ 1 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1
\end{pmatrix}

1232. ⠈⠠⠵18

$\mathbb{Z}_{18}$

{\mathbb Z}_{18}

1233. ⠠⠥⠷30⠾

$U(30)$

U(30)

1234. ⠈⠠⠵32

$\mathbb{Z}_{32}$

{\mathbb Z}_{32}

1235. ⠈⠼

$*$

\ast

1236. ⠈⠠⠟⠘⠈⠼

$\mathbb{Q}^*$

{\mathbb Q}^\ast

1237. ⠼⠂⠠⠤⠂

$1, -1$

1, -1

1238. ⠁⠘24⠐ ⠨⠅ ⠑

$a^{24} = e$

a^{24} =e

1239. ⠼⠂⠠⠼⠆⠠⠼⠒⠠⠼⠲⠠⠼⠖⠠⠼⠦⠠⠼12⠠⠼24

$1, 2, 3, 4, 6, 8, 12, 24$

1, 2, 3, 4, 6, 8, 12, 24

1240. ⠝ ⠅⠱ 20

$n \leq 20$

n \leq 20

1241. ⠠⠁ ⠨⠅ ⠠⠷⠼⠴ ⠼⠂ ⠠⠾ ⠠⠷⠤⠂ ⠼⠴ ⠠⠾and⠠⠃ ⠨⠅ ⠠⠷⠼⠴ ⠤⠂ ⠠⠾ ⠠⠷⠼⠂ ⠤⠂ ⠠⠾

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad \text{and} \qquad B = \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix}$$

A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad \text{and}
\qquad B = \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix}

1242. ⠠⠛⠠⠇⠆⠷⠈⠠⠗⠾

$GL_2(\mathbb{R})$

GL_2({\mathbb R})

1243. ⠠⠁⠠⠃

AB

AB

1244. ⠷⠒⠤⠆⠊⠾⠬⠷⠢⠊⠤⠖⠾

$(3-2i)+(5i-6)$

(3-2i)+ (5i-6)

1245. ⠷⠲⠤⠢⠊⠾⠤ModifyingAbove⠷⠲⠊⠤⠲⠾With

$(4-5i)-\overline{(4i-4)}$

(4-5i)-\overline{(4i -4)}

1246. ⠷⠢⠤⠲⠊⠾⠷⠶⠬⠆⠊⠾

$(5-4i)(7+2i)$

(5-4i)(7+2i)

1247. ⠷⠔⠤⠊⠾ModifyingAbove⠷⠔⠤⠊⠾With

$(9-i)\overline{(9-i)}$

(9-i) \overline{(9-i)}

1248. ⠊⠘45

i^{45}

i^{45}

1249. ⠷⠂⠬⠊⠾⠬ModifyingAbove⠷⠂⠬⠊⠾With

$(1+i)+\overline{(1+i)}$

(1+i)+\overline{(1+i)}

1250. ⠤⠒⠬⠒⠊

$-3+3i$

-3 + 3i

1251. 43⠤18⠊

$43-18i$

43- 18i

1252. ⠁⠬⠃⠊

$a+bi$

a + bi

1253. ⠆⠉⠊⠎⠷⠨⠏⠌⠖⠾

$2\operatorname{cis}(\pi/6)$

2 \cis(\pi / 6)

1254. ⠢⠉⠊⠎⠷⠔⠨⠏⠌⠲⠾

$5\operatorname{cis}(9\pi/4)$

5 \cis(9\pi/4)

1255. ⠒⠉⠊⠎⠷⠨⠏⠾

$3\operatorname{cis}(\pi)$

3 \cis(\pi)

1256. ⠉⠊⠎⠷⠶⠨⠏⠌⠲⠾⠌⠆

$\operatorname{cis}(7\pi/4)/2$

\cis(7\pi/4) /2

1257. ⠜⠒⠻⠬⠊

$\sqrt{3}+i$

\sqrt{3} + i

1258. ⠂⠤⠊

$1-i$

1-i

1259. ⠤⠢

-5

-5

1260. ⠆ ⠬⠆ ⠊

$2+2i$

```
2+2i
```

1261. ⠤⠒⠊

$-3i$

```
-3i
```

1262. ⠆ ⠊⠬⠆ ⠜⠒⠻

$2i+2\sqrt{3}$

```
2i + 2 \sqrt{3}
```

1263. ⠜⠆ ⠻⠉⠊⠎⠷⠶ ⠨⠏⠌⠲⠾

$\sqrt{2}\operatorname{cis}(7\pi/4)$

```
\sqrt{2} \cis( 7 \pi /4)
```

1264. ⠆ ⠜⠆ ⠻⠉⠊⠎⠷ ⠨⠏⠌⠲⠾

$2\sqrt{2}\operatorname{cis}(\pi/4)$

```
2 \sqrt{2} \cis( \pi /4)
```

1265. ⠒⠉⠊⠎⠷⠒ ⠨⠏⠌⠆ ⠾

$3\operatorname{cis}(3\pi/2)$

```
3 \cis(3 \pi/2)
```

1266. ⠷⠂ ⠬⠊⠾ ⠘⠤⠂

$(1+i)^{-1}$

```
(1+i)^{-1}
```

1267. ⠷⠂ ⠤⠊⠾ ⠘⠖

$(1-i)^{6}$

```
(1 - i)^{6}
```

1268. ⠷⠜⠒⠻⠬⠊⠾ ⠘⠢

$(\sqrt{3}+i)^{5}$

```
(\sqrt{3} + i)^{5}
```

1269. ⠷⠤⠊⠾ ⠘10

$(-i)^{10}$

```
(-i)^{10}
```

1270. ⠷⠷⠂ ⠤⠊⠾⠌⠆ ⠾ ⠘⠲

$((1-i)/2)^{4}$

```
((1-i)/2)^{4}
```

1271. ⠷⠤⠜⠆ ⠻⠤⠜⠆ ⠻⠊⠾ ⠘12

$(-\sqrt{2}-\sqrt{2}\, i)^{12}$

```
(-\sqrt{2} - \sqrt{2}\, i)^{12}
```

1272. ⠷⠤⠆ ⠬⠆ ⠊⠾ ⠘⠤⠢

$(-2+2i)^{-5}$

```
(-2 + 2i)^{-5}
```

1273. ⠷⠂⠤⠊⠾⠌⠆

$(1 - i)/2$

`(1 - i)/2`

1274. 16⠷⠊⠤⠜⠒⠻⠾

$16(i - \sqrt{3}\,)$

`16(i - \sqrt{3}\, )`

1275. ⠤⠂⠌⠲

$-1/4$

`-1/4`

1276. ⠳⠵⠳ ⠨⠅ ⠳⠵overbar⠳

$|z| = | \overline{z}|$

`|z| = | \overline{z}|`

1277. ⠵⠵overbar ⠨⠅ ⠳⠵⠳⠘⠆

$z \overline{z} = |z|^2$

`z \overline{z} = |z|^2`

1278. ⠵⠘⠤⠂⠐ ⠨⠅ ⠵overbar⠌⠳⠵⠳⠘⠆

$z^{-1} = \overline{z} / |z|^2$

`z^{-1} = \overline{z} / |z|^2`

1279. ⠳⠵⠬⠺⠳ ⠅⠱ ⠳⠵⠳⠬⠳⠺⠳

$|z +w| \leq |z| + |w|$

`|z +w| \leq |z| + |w|`

1280. ⠳⠵⠤⠺⠳ ⠨⠂⠱ ⠳⠳⠵⠳⠤⠳⠺⠳⠳

$|z - w| \geq | |z| - |w||$

`|z - w| \geq | |z| - |w||`

1281. ⠳⠵⠺⠳ ⠨⠅ ⠳⠵⠳⠳⠺⠳

$|z w| = |z| |w|$

`|z w| = |z| |w|`

1282. 292 ⠘3171 ⠐⠷⠍⠕⠙582⠾

$292^{3171} \pmod{ 582}$

`292^{3171} \pmod{ 582}`

1283. 2557 ⠘341 ⠐⠷⠍⠕⠙5681⠾

$2557^{ 341} \pmod{ 5681}$

`2557^{ 341} \pmod{ 5681}`

1284. 2071 ⠘9521 ⠐⠷⠍⠕⠙4724⠾

$2071^{ 9521} \pmod{ 4724}$

`2071^{ 9521} \pmod{ 4724}`

1285. 971 ⠘321 ⠐⠷⠍⠕⠙765⠾

$971^{ 321} \pmod{ 765}$

`971^{ 321} \pmod{ 765}`

1286. ⠼292

292

292

1287. ⠼1523

1523

1523

1288. ⠳⠁⠳ ⠨⠅ ⠳⠛⠘⠤⠂⠐⠁⠛⠳

$|a| = |g^{-1}ag|$

|a| = |g^{-1}ag|

1289. ⠃⠁

ba

ba

1290. ⠈⠠⠵⠰⠏⠟

${\mathbb Z}_{pq}$

{\mathbb Z}_{pq}

1291. ⠈⠠⠵⠰⠏⠰⠘⠗

${\mathbb Z}_{p^r}$

{\mathbb Z}_{p^r}

1292. ⠈⠠⠵⠰⠏

${\mathbb Z}_{p}$

{\mathbb Z}_{p}

1293. ⠄⡷⠭⠆⠶⠑⠤⠄⠛⠄⡷⠭⠆⠶⠑⠔⠄⠨⠩⠄⡷⠭⠆⠶⠑⠤⠄⠓⠄⡷⠭⠆⠶⠑⠔⠄

$\langle g \rangle \cap \langle h \rangle$

\langle g \rangle \cap \langle h \rangle

1294. ⠳⠄⡷⠭⠆⠶⠑⠤⠄⠛⠄⡷⠭⠆⠶⠑⠔⠄⠨⠩⠄⡷⠭⠆⠶⠑⠤⠄⠓⠄⡷⠭⠆⠶⠑⠔⠄⠳ ⠨⠅ ⠂

$|\langle g \rangle \cap \langle h \rangle| = 1$

|\langle g \rangle \cap \langle h \rangle| = 1

1295. ⠄⡷⠭⠆⠶⠑⠤⠄⠁⠘⠍⠐⠄⡷⠭⠆⠶⠑⠔⠄⠨⠩⠄⡷⠭⠆⠶⠑⠤⠄⠁⠘⠝⠐⠄⡷⠭⠆⠶⠑⠔⠄

$\langle a^m \rangle \cap \langle a^n \rangle$

\langle a^m \rangle \cap \langle a^n \rangle

1296. ⠃ ⠈⠑ ⠠⠛

$b \in G$

b \in G

1297. ⠳⠁⠳ ⠨⠅ ⠍

$|a| = m$

|a| = m

1298. ⠳⠃⠳ ⠨⠅ ⠝

$|b| = n$

|b| = n

1299. ⠛⠉⠙⠷⠍⠠⠝⠾ ⠨⠅ ⠂

$\gcd(m,n) = 1$

\gcd(m,n) = 1

1300. ⠄⡗⠭⠆ ⠿⠈⠢⠤⠄⠁⠄⡗⠭⠆ ⠿⠈⠢⠔⠄ ⠫⠉⠄⡗⠭⠆ ⠿⠈⠢⠤⠄⠃⠄⡗⠭⠆ ⠿⠈⠢⠔⠄ ⠨⠅ StartSet⠑EndSet

$\langle a \rangle \cap \langle b \rangle = \{ e \}$

\langle a \rangle \cap \langle b \rangle = \{ e \}

1301. ⠷⠛⠘⠤⠂⠐⠾⠘⠍⠐ ⠨⠅ ⠑

$(g^{-1})^m = e$

(g^{-1})^m = e

1302. ⠷⠛⠓⠾⠘⠍⠝⠐ ⠨⠅ ⠑

$(gh)^{mn} = e$

(gh)^{mn} = e

1303. ⠽ ⠨⠅ ⠭⠘⠅

$y = x^k$

y = x^k

1304. ⠛⠉⠙⠷⠅⠠⠝⠾ ⠨⠅ ⠂

$\gcd(k,n) = 1$

\gcd(k,n) = 1

1305. ⠏⠟

pq

pq

1306. ⠛⠉⠙⠷⠏⠠⠟⠾ ⠨⠅ ⠂

$\gcd(p,q) = 1$

\gcd(p,q) = 1

1307. ⠄⡗⠭⠆ ⠿⠈⠢⠤⠄⠛⠄⡗⠭⠆ ⠿⠈⠢⠔⠄

$\langle g \rangle$

\langle g \rangle

1308. ⠙⠳⠍

$d \mid m$

d \mid m

1309. ⠵ ⠨⠅ ⠗⠷⠉⠕⠎ ⠨⠹⠬⠊⠎⠊⠝ ⠨⠹⠾

$z = r(\cos \theta + i \sin \theta)$

z = r(\cos \theta + i \sin \theta)

1310. ⠺ ⠨⠅ ⠎⠷⠉⠕⠎ ⠨⠋⠬⠊⠎⠊⠝ ⠨⠋⠾

$w = s(\cos \phi + i \sin \phi)$

w = s(\cos \phi + i \sin \phi)

1311. ⠵⠺ ⠨⠅ ⠗⠎⠈⠷⠉⠕⠎ ⠷⠨⠹⠬⠨⠋⠾⠬⠊⠎⠊⠝ ⠷⠨⠹⠬⠨⠋⠾⠈⠾

$zw = rs[\cos(\theta + \phi) + i \sin(\theta + \phi)]$

zw = rs[\cos(\theta + \phi) + i \sin(\theta + \phi)]

1312. ⠨⠁⠈⠑⠈⠠⠞

$\alpha \in \mathbb T$

`\alpha \in \mathbb T`

1313. ⠨⠁⠘⠍⠐ ⠨⠅ ⠂

$\alpha^m =1$

`\alpha^m =1`

1314. ⠨⠁⠘⠝⠐ ⠨⠅ ⠂

$\alpha^n = 1$

`\alpha^n = 1`

1315. ⠨⠁⠘⠙⠐ ⠨⠅ ⠂

$\alpha^d = 1$

`\alpha^d = 1`

1316. ⠙ ⠨⠅ ⠛⠉⠙⠷⠍⠠⠝⠾

$d = \gcd(m,n)$

`d = \gcd(m,n)`

1317. ⠵⠈⠑⠈⠠⠉⠘⠈⠼

$z \in {\mathbb C}^\ast$

`z \in {\mathbb C}^\ast`

1318. ⠳⠵⠳ ⠌⠨⠅ ⠂

$|z| \neq 1$

`|z| \neq 1`

1319. ⠵ ⠨⠅ ⠉⠕⠎ ⠨⠹⠬⠊⠎⠊⠝ ⠨⠹

$z =\cos \theta + i \sin \theta$

`z =\cos \theta + i \sin \theta`

1320. ⠨⠹⠈⠑⠈⠠⠟

$\theta \in {\mathbb Q}$

`\theta \in {\mathbb Q}`

1321. ⠁⠘⠭⠐⠷⠍⠕⠙⠝⠾

$a^x \pmod{ n}$

`a^x \pmod{ n}`

1322. ⠒⠈⠠⠵

$3\mathbb Z$

`3\mathbb Z`

1323. ⠈⠠⠵14

$\mathbb Z_{14}$

`\mathbb Z_{14}`

1324. ⠹⠆⠨⠏⠌14⠼

$\frac{2\pi}{14}$

`\frac{2\pi}{14}`

1325. ⠼40

40

40

1326. ⠠⠥⠷40⠾

$U(40)$

U(40)

1327. ⠠⠥⠷49⠾

$U(49)$

U(49)

1328. ⠠⠥⠷35⠾

$U(35)$

U(35)

1329. ⠠⠎ ⠶ StartSet ⠠⠁ ⠠ ⠠⠃ ⠠ ⠠⠉EndSet

$S = \{A, B, C\}$

S = \{ A, B, C \}

1330. ⠠⠎⠰⠠⠭

S_X

S_X

1331. ⠠⠭ ⠶ StartSet⠂ ⠠⠆ ⠄⠄ ⠠⠝EndSet

$X = \{1, 2, \ldots, n\}$

X=\{ 1, 2, \ldots, n\}

1332. ⠋⠱⠠⠎⠰⠝⠐ ⠫⠔ ⠠⠎⠰⠝⠐

$f : S_n \to S_n$

f : S_n \rightarrow S_n

1333. ⠳⠠⠎⠰⠝⠐⠳ ⠶ ⠝⠯

$|S_n| = n!$

|S_n|= n!

1334. ⠠⠎⠢

S_5

S_5

1335. ⠊⠙

id

\identity

1336. StartLayout1stRow ⠘⠨⠡ ⠊⠙ ⠨⠎ ⠨⠞ ⠨⠍2ndRow⠊⠙ ⠊⠙ ⠨⠎ ⠨⠞ ⠨⠍3rdRow⠨⠎ ⠨⠎ ⠊⠙

$\circ$	id	σ	τ	μ
id	id	σ	τ	μ
σ	σ	id	μ	τ
τ	τ	μ	id	σ
μ	μ	τ	σ	id

\begin{array}{c|cccc} \circ & \identity & \sigma & \tau & \mu \\ \hline \identity & \identity & \sigma & \tau & \mu \\ \sigma & \sigma & \identity & \mu & \tau \\ \tau & \tau & \mu & \identity & \sigma \\ \mu & \mu & \tau & \sigma & \identity \end{array}

1337. ⠨⠞

τ

\tau

1338. ⠷⠨⠎⠘⠨⠡⠨⠞⠾⠷⠭⠾ ⠨⠅ ⠨⠎⠷⠨⠞⠷⠭⠾⠾

$(\sigma \circ \tau)(x) = \sigma(\tau(x))$

(\sigma \circ \tau)(x) = \sigma(\tau(x))

1339. ⠨⠎⠨⠞

$\sigma\tau$

\sigma \tau

1340. ⠨⠎⠨⠞⠷⠭⠾

$\sigma\tau(x)$

\sigma \tau (x)

1341. ⠨⠎⠷⠨⠞⠷⠭⠾⠾

$\sigma(\tau(x))$

\sigma(\tau(x))

1342. ⠨⠎⠷⠭⠾

$\sigma(x)$

\sigma(x)

1343. ⠷⠭⠾⠨⠎

$(x)\sigma$

(x)\sigma

1344. ⠨⠎⠨⠞ ⠨⠅ ⠠⠷⠠⠂ ⠠⠆ ⠠⠒ ⠠⠲⠠⠾ ⠠⠷⠠⠂ ⠠⠲ ⠠⠒ ⠠⠆⠠⠾

$\sigma\tau = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 3 & 2 \end{pmatrix}$

\sigma \tau = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 3 & 2 \end{pmatrix}

1345. ⠨⠞⠨⠎ ⠨⠅ ⠠⠷⠠⠂ ⠠⠆ ⠠⠒ ⠠⠲⠠⠾ ⠠⠷⠠⠒ ⠠⠆ ⠠⠂ ⠠⠲⠠⠾

$\tau\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix}$

\tau \sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix}

1346. ⠨⠎ ⠈⠑ ⠠⠎⠰⠠⠭

$\sigma \in S_X$

\sigma \in S_X

1347. ⠁⠂ ⠠⠁⠆ ⠄⠄ ⠠⠁ ⠰⠅ ⠐⠈⠑ ⠠⠭

$a_1, a_2, \ldots, a_k \in X$

a_1, a_2, \ldots, a_k \in X

1348. ⠨⠎⠷⠭⠾ ⠭ ⠭

$\sigma(x) = x$

\sigma(x) = x

1349. ⠷⠁⠂ ⠠⠁⠆ ⠤ ⠠⠁ ⠱ ⠐⠾

$(a_1, a_2, \ldots, a_k)$

(a_1, a_2, \ldots, a_k)

1350. ⠨⠎ ⠭ ⠠⠷⠠⠐ ⠠⠂ ⠠⠆ ⠠⠒ ⠠⠲ ⠠⠢ ⠠⠖ ⠠⠶ ⠠⠷⠠⠖ ⠠⠆ ⠠⠒ ⠠⠢ ⠠⠂ ⠠⠲ ⠠⠆ ⠠⠶ ⠠⠾ ⠭ ⠷⠂⠖

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 3 & 5 & 1 & 4 & 2 & 7 \end{pmatrix} = (162354)$$

\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7\\ 6 & 3 & 5 &
1 & 4 & 2 & 7 \end{pmatrix} = (1 6 2 3 5 4)

1351. ⠨⠞ ⠭ ⠠⠷⠠⠐ ⠠⠂ ⠠⠆ ⠠⠒ ⠠⠲ ⠠⠢ ⠠⠾ ⠠⠷⠠⠐ ⠠⠒ ⠠⠂ ⠠⠆ ⠠⠲ ⠠⠢ ⠠⠾ ⠭ ⠷⠆⠲⠒⠾

$$\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 4 & 2 & 3 & 5 & 6 \end{pmatrix} = (243)$$

\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 4 & 2 & 3 &
5 & 6 \end{pmatrix} = (2 4 3)

1352. ⠠⠷⠠⠐ ⠠⠂ ⠠⠆ ⠠⠒ ⠠⠲ ⠠⠢ ⠠⠾ ⠠⠷⠠⠂ ⠠⠒ ⠠⠐ ⠠⠆ ⠠⠢ ⠠⠲ ⠠⠾ ⠭ ⠷⠐⠆⠲⠒⠾⠷⠢⠖⠾

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 3 & 6 & 5 \end{pmatrix} = (1243)(56)$$

\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 3 & 6 & 5 \end{pmatrix}
= (1 2 4 3)(5 6)

1353. ⠨⠎ ⠭ ⠷⠐⠒⠢⠆⠾and⠨⠞ ⠭ ⠷⠆⠢⠖⠾

$\sigma = (1352)$ and $\tau = (256)$

\sigma = (1 3 5 2) \quad \text{and} \quad \tau = (2 5 6)

1354. ⠐ ⠳⠕⠒⠒⠕ ⠒ ⠠⠒ ⠳⠕⠒⠒⠕ ⠢ ⠠⠢ ⠳⠕⠒⠒⠕ ⠆ ⠠⠆ ⠳⠕⠒⠒⠕ ⠐

$1 \mapsto 3, \qquad 3 \mapsto 5, \qquad 5 \mapsto 2, \qquad 2 \mapsto 1$

1 \mapsto 3, \qquad 3 \mapsto 5, \qquad 5 \mapsto 2, \qquad 2 \mapsto
1

1355. ⠆ ⠳⠕⠒⠒⠕ ⠢ ⠠⠢ ⠳⠕⠒⠒⠕ ⠖ ⠠⠖ ⠳⠕⠒⠒⠕ ⠆

$2 \mapsto 5, \qquad 5 \mapsto 6, \qquad 6 \mapsto 2$

2 \mapsto 5, \qquad 5 \mapsto 6, \qquad 6 \mapsto 2

1356. ⠐ ⠳⠕⠒⠒⠕ ⠒ ⠠⠒ ⠳⠕⠒⠒⠕ ⠢ ⠠⠢ ⠳⠕⠒⠒⠕ ⠖ ⠠⠖ ⠳⠕⠒⠒⠕ ⠆ ⠳⠕⠒⠒⠕ ⠐

$1 \mapsto 3, \qquad 3 \mapsto 5, \qquad 5 \mapsto 6, \qquad 6 \mapsto 2 \mapsto 1$

1 \mapsto 3, \qquad 3 \mapsto 5, \qquad 5 \mapsto 6, \qquad 6 \mapsto
2 \mapsto 1

1357. ⠨⠎⠨⠞ ⠭ ⠷⠐⠒⠢⠖⠾

$\sigma\tau = (1356)$

\sigma \tau = (1 3 5 6)

1358. ⠨⠍ ⠭ ⠷1634⠾

$\mu = (1634)$

\mu = (1634)

1359. ⠨⠎⠨⠍ ⠨⠅ ⠷⠂⠖⠢⠆⠾⠷⠒⠲⠾

$\sigma\mu = (1652)(34)$

`\sigma \mu = (1 6 5 2)(3 4)`

1360. ⠨⠎ ⠨⠅ ⠷⠁⠂⠠ ⠁⠆⠠ ⠄⠄⠄⠠ ⠁⠰⠅ ⠐⠾

$\sigma = (a_1, a_2, \ldots, a_k)$

`\sigma = (a_1, a_2, \ldots, a_k )`

1361. ⠨⠞ ⠨⠅ ⠷⠃⠂⠠ ⠃⠆⠠ ⠄⠄⠄⠠ ⠃⠰⠇ ⠐⠾

$\tau = (b_1, b_2, \ldots, b_l)$

`\tau = (b_1, b_2, \ldots, b_l )`

1362. ⠁⠰⠊ ⠌⠨⠅ ⠃⠰⠚

$a_i \neq b_j$

`a_i \neq b_j`

1363. ⠷⠂⠒⠢⠾

(135)

`(1 3 5)`

1364. ⠷⠆⠶⠾

(27)

`(2 7 )`

1365. ⠷⠒⠲⠶⠾

(347)

`(3 4 7 )`

1366. ⠨⠎⠨⠞ ⠨⠅ ⠨⠞⠨⠎

$\sigma\tau = \tau\sigma$

`\sigma \tau = \tau \sigma`

1367. ⠨⠎⠨⠞⠷⠭⠾ ⠨⠅ ⠨⠞⠨⠎⠷⠭⠾

$\sigma\tau(x) = \tau\sigma(x)$

`\sigma \tau(x) = \tau \sigma(x)`

1368. StartSet⠁⠂⠠ ⠁⠆⠠ ⠄⠄⠄⠠ ⠁⠰⠅ ⠐EndSet

$\{a_1, a_2, \ldots, a_k\}$

`\{ a_1, a_2, \ldots, a_k \}`

1369. StartSet⠃⠂⠠ ⠃⠆⠠ ⠄⠄⠄⠠ ⠃⠰⠇ ⠐EndSet

$\{b_1, b_2, \ldots, b_l\}$

`\{b_1, b_2, \ldots, b_l \}`

1370. ⠨⠎⠷⠭⠾ ⠨⠅ ⠭

$\sigma(x) = x$

`\sigma(x)=x`

1371. ⠨⠞⠷⠭⠾ ⠨⠅ ⠭

$\tau(x) = x$

`\tau(x)=x`

1372. ⠨⠎ ⠨⠞⠷⠭⠾ ⠨⠅ ⠨⠎⠷⠨⠞⠷⠭⠾⠾ ⠨⠅ ⠨⠎⠷⠭⠾ ⠨⠅ ⠭ ⠨⠅ ⠨⠞⠷⠭⠾ ⠨⠅ ⠨⠞⠷⠨⠎⠷⠭⠾⠾ ⠨⠅ ⠨⠞

$\sigma\tau(x) = \sigma(\tau(x)) = \sigma(x) = x = \tau(x) = \tau(\sigma(x)) = \tau\sigma(x)$

```
\sigma \tau(x) = \sigma( \tau(x)) = \sigma(x) = x = \tau(x) = \tau(
\sigma(x)) = \tau \sigma(x)
```

1373. ⠭ ⠈⠑StartSet⠁ ⠂ ⠠⠁ ⠆ ⠄⠄ ⠠⠁ ⠰⠅ ⠐EndSet

$x \in \{a_1, a_2, \ldots, a_k\}$

```
x \in \{ a_1, a_2, \ldots, a_k \}
```

1374. ⠨⠎⠷⠁ ⠰⠊⠐⠾ ⠨⠅ ⠁ ⠰⠷⠊⠍⠕⠙⠅⠾⠬⠂⠐

$\sigma(a_i) = a_{(i \bmod k)+1}$

```
\sigma( a_i ) = a_{(i \bmod k) + 1}
```

1375. ⠨⠞⠷⠁ ⠰⠊⠐⠾ ⠨⠅ ⠁ ⠰⠊

$\tau(a_i) = a_i$

```
\tau(a_i) = a_i
```

1376. ⠭ ⠈⠑StartSet⠃ ⠂ ⠠⠃ ⠆ ⠄⠄ ⠠⠃ ⠰⠇ ⠐EndSet

$x \in \{b_1, b_2, \ldots, b_l\}$

```
x \in \{b_1, b_2, \ldots, b_l \}
```

1377. ⠠⠭ ⠨⠅ StartSet⠂ ⠠⠆ ⠄⠄ ⠠⠝EndSet

$X = \{1, 2, \ldots, n\}$

```
X = \{ 1, 2, \ldots, n \}
```

1378. ⠨⠎ ⠈⠑ ⠠⠎ ⠰⠝

$\sigma \in S_n$

```
\sigma \in S_n
```

1379. ⠠⠭⠂

X_1

```
X_1
```

1380. StartSet ⠨⠎⠷⠂⠾ ⠠ ⠨⠎ ⠘⠆ ⠐⠷⠂⠾ ⠄⠄ EndSet

$\{\sigma(1), \sigma^2(1), \ldots\}$

```
\{ \sigma(1), \sigma^2(1), \ldots \}
```

1381. ⠠⠭⠆

X_2

```
X_2
```

1382. StartSet ⠨⠎⠷⠊⠾ ⠠ ⠨⠎ ⠘⠆ ⠐⠷⠊⠾ ⠄⠄ EndSet

$\{\sigma(i), \sigma^2(i), \ldots\}$

```
\{ \sigma(i), \sigma^2(i), \ldots \}
```

1383. ⠠⠭⠒ ⠠ ⠠⠭⠲ ⠄⠄

$X_3, X_4, \ldots$

```
X_3, X_4, \ldots
```

1384. ⠨⠎⠰⠊

σ_i

\sigma_i

1385. ⠨⠎⠰⠊⠐⠷⠭⠾ ⠨⠅ StartLayoutEnlarged ⠸⠳1stRow ⠸⠳ ⠨⠎⠷⠭⠾ ⠭ ⠈⠑ ⠠⠭ ⠰⠊ ⠐2ndRow ⠸⠳⠭ ⠭⠠⠈

$$\sigma_i(x) = \begin{cases} \sigma(x) & x \in X_i \\ x & x \notin X_i \end{cases}$$

\sigma_i(x) = \begin{cases} \sigma(x) & x \in X_i \\ x & x \notin
X_i \end{cases}

1386. ⠨⠎ ⠨⠅ ⠨⠎⠂ ⠨⠎⠆⠄⠄⠄⠨⠎⠰⠗

$\sigma = \sigma_1\sigma_2\cdots\sigma_r$

\sigma = \sigma_1 \sigma_2 \cdots \sigma_r

1387. ⠠⠭⠂ ⠠⠠⠭⠆ ⠄⠄ ⠠⠠⠭⠰⠗ ⠐

$X_1, X_2, \ldots, X_r$

X_1, X_2, \ldots, X_r

1388. ⠨⠎⠂ ⠠⠨⠎⠆ ⠄⠄ ⠠⠨⠎⠰⠗ ⠐

$\sigma_1, \sigma_2, \ldots, \sigma_r$

\sigma_1, \sigma_2, \ldots, \sigma_r

1389. ⠷⠂ ⠾

(1)

(1)

1390. ⠷⠁⠂ ⠠⠁⠆ ⠄⠄ ⠠⠁ ⠰⠝⠐⠾ ⠨⠅ ⠷⠁⠂⠁ ⠰⠝⠐⠾⠷⠁⠂⠁ ⠰⠝⠤⠂ ⠐⠾⠄⠄⠄⠷⠁⠂⠁⠒⠾⠷⠁⠂⠁⠆⠾

$(a_1, a_2, \ldots, a_n) = (a_1 a_n)(a_1 a_{n-1}) \cdots (a_1 a_3)(a_1 a_2)$

(a_1, a_2, \ldots, a_n) = (a_1 a_n) (a_1 a_{n-1}) \cdots (a_1
a_3) (a_1 a_2)

1391. ⠷⠂ ⠖⠾⠷⠆ ⠢⠒⠾ ⠨⠅ ⠷⠂ ⠖⠾⠷⠆ ⠒⠾⠷⠆ ⠢⠾ ⠨⠅ ⠷⠂ ⠖⠾⠷⠲⠢⠾⠷⠆ ⠒⠾⠷⠲⠢⠾⠷⠆ ⠢⠾

$(16)(253) = (16)(23)(25) = (16)(45)(23)(45)(25)$

(1 6) (2 5 3) = (1 6)(2 3)(2 5) = (1 6)(4 5)(2 3)(4
5)(2 5)

1392. ⠷⠂ ⠆⠾⠷⠂ ⠆⠾

$(12)(12)$

(1 2)(1 2)

1393. ⠷⠂ ⠒⠾⠷⠆ ⠲⠾⠷⠂ ⠒⠾⠷⠆ ⠲⠾

$(13)(24)(13)(24)$

(1 3)(2 4)(1 3)(2 4)

1394. ⠷⠂ ⠖⠾

(16)

(1 6)

1395. ⠷⠆ ⠒⠾⠷⠂ ⠖⠾⠷⠆ ⠒⠾

$(23)(16)(23)$

(2 3)(1 6)(2 3)

1396. ⠷⠒⠢⠾⠷⠂⠖⠾⠷⠂⠒⠾⠷⠂⠖⠾⠷⠂⠒⠾⠷⠒⠢⠾⠷⠢⠖⠾

$(35)(16)(13)(16)(13)(35)(56)$

(3 5) (1 6) (1 3) (1 6) (1 3) (3 5) (5 6)

1397. ⠊⠙⠀⠨⠅⠀⠨⠞⠂⠨⠞⠆⠄⠄⠄⠨⠞⠰⠗

$\mathrm{id} = \tau_1 \tau_2 \cdots \tau_r$

\identity = \tau_1 \tau_2 \cdots \tau_r

1398. ⠗⠀⠨⠂⠀⠂

$r > 1$

r \gt 1

1399. ⠗⠀⠨⠅⠀⠆

$r = 2$

r=2

1400. ⠗⠀⠨⠂⠀⠆

$r > 2$

r \gt 2

1401. ⠨⠞⠰⠗⠤⠂⠐⠨⠞⠰⠗

$\tau_{r-1}\tau_r$

\tau_{r-1} \tau_r

1402. ⠊⠙⠀⠨⠅⠀⠨⠞⠂⠨⠞⠆⠄⠄⠄⠨⠞⠰⠗⠤⠒⠐⠨⠞⠰⠗⠤⠆

$\mathrm{id} = \tau_1 \tau_2 \cdots \tau_{r-3}\tau_{r-2}$

\identity = \tau_1 \tau_2 \cdots \tau_{r - 3} \tau_{r - 2}

1403. ⠗⠤⠆

$r - 2$

r - 2

1404. ⠨⠞⠰⠗⠤⠂⠐⠨⠞⠰⠗

$\tau_{r-1}\tau_r$

\tau_{r - 1} \tau_r

1405. ⠨⠞⠰⠗⠤⠆⠐⠨⠞⠰⠗⠤⠂

$\tau_{r-2}\tau_{r-1}$

\tau_{r - 2} \tau_{r - 1}

1406. ⠨⠞⠰⠗⠤⠆

τ_{r-2}

\tau_{r - 2}

1407. ⠨⠞⠰⠗⠤⠒⠐⠨⠞⠰⠗⠤⠆

$\tau_{r-3}\tau_{r-2}$

\tau_{r - 3} \tau_{r - 2}

1408. ⠗⠤⠆

$r - 2$

r-2

1409. ⠨⠎ ⠨⠅ ⠨⠎⠂⠨⠎⠆⠄⠄⠄⠨⠎⠰⠍⠐ ⠨⠅ ⠨⠞⠂⠨⠞⠆⠄⠄⠄⠨⠞⠰⠝

$\sigma = \sigma_1 \sigma_2 \cdots \sigma_m = \tau_1 \tau_2 \cdots \tau_n$

\sigma = \sigma_1 \sigma_2 \cdots \sigma_m = \tau_1 \tau_2 \cdots \tau_n

1410. ⠨⠎⠰⠍⠐⠄⠄⠄⠨⠎⠂

$\sigma_m \cdots \sigma_1$

\sigma_m \cdots \sigma_1

1411. ⠊⠙ ⠨⠅ ⠨⠎⠨⠎⠰⠍⠐⠄⠄⠄⠨⠎⠂ ⠨⠅ ⠨⠞⠂⠄⠄⠄⠨⠞⠰⠝⠐⠨⠎⠰⠍⠐⠄⠄⠄⠨⠎⠂

$\mathrm{id} = \sigma \sigma_m \cdots \sigma_1 = \tau_1 \cdots \tau_n \sigma_m \cdots \sigma_1$

\identity = \sigma \sigma_m \cdots \sigma_1 = \tau_1 \cdots \tau_n \sigma_m \cdots \sigma_1

1412. ⠠⠁⠰⠝

A_n

A_n

1413. ⠨⠎⠘⠤⠂⠐ ⠨⠅ ⠨⠎⠰⠗⠐⠨⠎⠰⠗⠤⠂⠐⠄⠄⠄⠨⠎⠂

$\sigma^{-1} = \sigma_r \sigma_{r-1} \cdots \sigma_1$

\sigma^{-1} = \sigma_r \sigma_{r-1} \cdots \sigma_1

1414. ⠝⠯⠌⠆

$n!/2$

n!/2

1415. ⠠⠃⠰⠝

B_n

B_n

1416. ⠨⠇⠰⠨⠎⠐⠱⠠⠁⠰⠝⠐ ⠫⠕ ⠠⠃⠰⠝⠐

$\lambda_{\sigma} : A_n \rightarrow B_n$

\lambda_{\sigma} : A_n \rightarrow B_n

1417. ⠨⠇⠰⠨⠎⠐⠷⠨⠞⠾ ⠨⠅ ⠨⠎⠨⠞

$\lambda_{\sigma}(\tau) = \sigma \tau$

\lambda_{\sigma} (\tau) = \sigma \tau

1418. ⠨⠇⠰⠨⠎⠐⠷⠨⠞⠾ ⠨⠅ ⠨⠇⠰⠨⠎⠐⠷⠨⠍⠾

$\lambda_{\sigma}(\tau) = \lambda_{\sigma}(\mu)$

\lambda_{\sigma} (\tau) = \lambda_{\sigma} (\mu)

1419. ⠨⠎⠨⠞ ⠨⠅ ⠨⠎⠨⠍

$\sigma \tau = \sigma \mu$

\sigma \tau = \sigma \mu

1420. ⠨⠞ ⠨⠅ ⠨⠎⠘⠤⠂⠐⠨⠎⠨⠞ ⠨⠅ ⠨⠎⠘⠤⠂⠐⠨⠎⠨⠍ ⠨⠅ ⠨⠍

$\tau = \sigma^{-1} \sigma \tau = \sigma^{-1} \sigma \mu = \mu$

\tau = \sigma^{-1} \sigma \tau = \sigma^{-1} \sigma \mu = \mu

1421. ⠽ ⠆⠵⠈

λ_{σ}

\lambda_{\sigma}

1422. ⠌ ⠲

A_4

A_4

1423. ⠄⠆⠈⠲

S_4

S_4

1424. ⠝ ⠶ ⠒ ⠠⠼⠒ ⠄⠄

$n = 3, 4, \ldots$

n = 3, 4, \ldots

1425. ⠠⠙ ⠆⠝

D_n

D_n

1426. ⠼⠂ ⠠⠼⠆ ⠄⠄ ⠠⠝

$1, 2, \ldots, n$

1, 2, \ldots, n

1427. ⠅ ⠬⠂

$k+1$

k+1

1428. ⠅ ⠤⠂

$k-1$

k-1

1429. ⠆⠝

$2n$

2n

1430. ⠊⠙ ⠠⠹360⠘⠘⠣⠨⠌⠝⠼ ⠠⠆ ⠑⠹360⠘⠘⠣⠨⠌⠝⠼ ⠄⠄ ⠠⠷⠝⠤⠂⠾⠡⠹360⠘⠘⠣⠨⠌⠝⠼

$\mathrm{id}, \frac{360^{\circ}}{n}, 2 \cdot \frac{360^{\circ}}{n}, \ldots, (n-1) \cdot \frac{360^{\circ}}{n}$

\identity, \frac{360^{\circ} }{n}, 2 \cdot \frac{360^{\circ} }{n},
\ldots, (n-1) \cdot \frac{360^{\circ} }{n}

1431. 360⠘⠘⠣⠨⠌⠝

$360^{\circ}/n$

360^{\circ} /n

1432. ⠗⠘⠅ ⠐ ⠶ ⠅⠡⠹360⠘⠘⠣⠨⠌⠝⠼

$r^k = k \cdot \frac{360^{\circ}}{n}$

r^k = k \cdot \frac{360^{\circ} }{n}

1433. ⠎⠂ ⠠⠎⠆ ⠤ ⠠⠎⠰⠝⠐

$s_1, s_2, \ldots, s_n$

s_1, s_2, \ldots, s_n

1434. ⠎⠰⠅

s_k

s_k

1435. ⠎⠂ ⠨⠅ ⠎⠰⠝⠸⠌⠆⠬⠂⠐⠠⠎⠆ ⠨⠅ ⠎⠰⠝⠸⠌⠆⠬⠆⠐⠤ ⠠⠎⠰⠝⠸⠌⠆⠐ ⠨⠅ ⠎⠰⠝⠐

$s_1 = s_{n/2+1}, s_2 = s_{n/2+2}, \ldots, s_{n/2} = s_n$

s_1 = s_{n/2 + 1}, s_2 = s_{n/2 + 2}, \ldots, s_{n/2} = s_n

1436. ⠎ ⠨⠅ ⠎⠂

$s = s_1$

s = s_1

1437. ⠎⠘⠆⠐ ⠨⠅ ⠂

$s^2 = 1$

s^2 = 1

1438. ⠗⠘⠝⠐ ⠨⠅ ⠂

$r^n = 1$

r^n = 1

1439. t

t

1440. ⠞ ⠨⠅ ⠗⠘⠅

$t = r^k$

t = r^k

1441. ⠞ ⠨⠅ ⠎⠗⠘⠅

$t = sr^k$

t = s r^k

1442. ⠠⠙⠰⠝⠐ ⠨⠅ StartSet⠂ ⠠⠗ ⠠⠗⠘⠆ ⠐⠤ ⠠⠗⠘⠝⠤⠂ ⠐⠠⠎ ⠠⠎⠗ ⠠⠎⠗⠘⠆ ⠐⠤ ⠠⠎⠗⠘⠝⠤⠂ ⠐EndSet

$D_n = \{1, r, r^2, \ldots, r^{n-1}, s, sr, sr^2, \ldots, sr^{n-1}\}$

D_n = \{1, r, r^2, \ldots, r^{n-1}, s, sr, sr^2, \ldots, sr^{n-1}\}

1443. ⠎⠗⠎ ⠨⠅ ⠗⠘⠤⠂

$srs = r^{-1}$

srs = r^{-1}

1444. ⠖⠡⠲ ⠨⠅ 24

$6 \cdot 4 = 24$

6 \cdot 4 = 24

1445. ⠼24

24

24

1446. ⠠⠷⠼⠂ ⠼⠆ ⠼⠒ ⠼⠲ ⠼⠢⠠⠾ ⠠⠷⠼⠆ ⠼⠲ ⠼⠂ ⠼⠢ ⠼⠒⠠⠾

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 1 & 5 & 3 \end{pmatrix}$$

\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 1 & 5 & 3 \end{pmatrix}

1447. ⠠⠷⠼⠂ ⠼⠆ ⠼⠒ ⠼⠲ ⠼⠢⠠⠾ ⠠⠷⠼⠲ ⠼⠆ ⠼⠢ ⠼⠂ ⠼⠒⠠⠾

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 5 & 1 & 3 \end{pmatrix}$$

\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 5 & 1 & 3 \end{pmatrix}

1448. ⠠⠷⠼⠂ ⠼⠆ ⠼⠒ ⠼⠲ ⠼⠢⠠⠾ ⠠⠷⠼⠒ ⠼⠢ ⠼⠂ ⠼⠲ ⠼⠆⠠⠾

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 1 & 4 & 2 \end{pmatrix}$$

\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 1 & 4 & 2 \end{pmatrix}

1449. ⠠⠷⠼⠂ ⠼⠆ ⠼⠒ ⠼⠲ ⠼⠢⠠⠾ ⠠⠷⠼⠂ ⠼⠲ ⠼⠒ ⠼⠆ ⠼⠢⠠⠾

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 3 & 2 & 5 \end{pmatrix}$$

\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 3 & 2 & 5 \end{pmatrix}

1450. ⠷12453⠾

(12453)

(12453)

1451. ⠷13⠾⠷25⠾

$(13)(25)$

(13)(25)

1452. ⠷1345⠾⠷234⠾

$(1345)(234)$

(1345)(234)

1453. ⠷12⠾⠷1253⠾

$(12)(1253)$

(12)(1253)

1454. ⠷143⠾⠷23⠾⠷24⠾

$(143)(23)(24)$

(143)(23)(24)

1455. ⠷1423⠾⠷34⠾⠷56⠾⠷1324⠾

$(1423)(34)(56)(1324)$

(1423)(34)(56)(1324)

1456. ⠷1254⠾⠷13⠾⠷25⠾

$(1254)(13)(25)$

(1254)(13)(25)

1457. ⠷1254⠾⠷13⠾⠷25⠾ ⠘⠆

$(1254)(13)(25)^2$

(1254) (13)(25)^2

1458. ⠷1254⠾⠘⠤⠂⠐⠷123⠾⠷45⠾⠷1254⠾

$(1254)^{-1}(123)(45)(1254)$

`(1254)^{-1} (123)(45) (1254)`

1459. ⠷1254⠾⠘⠆⠐⠷123⠾⠷45⠾

$(1254)^2(123)(45)$

`(1254)^2 (123)(45)`

1460. ⠷123⠾⠷45⠾⠷1254⠾⠘⠤⠆

$(123)(45)(1254)^{-2}$

`(123)(45) (1254)^{-2}`

1461. ⠷1254⠾⠘100

$(1254)^{100}$

`(1254)^{100}`

1462. ⠳⠷1254⠾⠳

$|(1254)|$

`|(1254)|`

1463. ⠳⠷1254⠾⠘⠆⠐⠳

$|(1254)^2|$

`|(1254)^2|`

1464. ⠷12⠾⠘⠤⠂

$(12)^{-1}$

`(12)^{-1}`

1465. ⠷12537⠾⠘⠤⠂

$(12537)^{-1}$

`(12537)^{-1}`

1466. ⠈⠷⠷12⠾⠷34⠾⠷12⠾⠷47⠾⠈⠾⠘⠤⠂

$[(12)(34)(12)(47)]^{-1}$

`[(12)(34)(12)(47)]^{-1}`

1467. ⠈⠷⠷1235⠾⠷467⠾⠈⠾⠘⠤⠂

$[(1235)(467)]^{-1}$

`[(1235)(467)]^{-1}`

1468. ⠷135⠾⠷24⠾

$(135)(24)$

`(135)(24)`

1469. ⠷14⠾⠷23⠾

$(14)(23)$

`(14)(23)`

1470. ⠷1324⠾

(1324)

`(1324)`

1471. ⠷134⠾⠷25⠾

$(134)(25)$

(134)(25)

1472. ⠷17352⠾

(17352)

(17352)

1473. ⠷14356⠾

(14356)

(14356)

1474. ⠷156⠾⠷234⠾

$(156)(234)$

(156)(234)

1475. ⠷1426⠾⠷142⠾

$(1426)(142)$

(1426)(142)

1476. ⠷17254⠾⠷1423⠾⠷154632⠾

$(17254)(1423)(154632)$

(17254)(1423)(154632)

1477. ⠷142637⠾

(142637)

(142637)

1478. ⠷16⠾⠷15⠾⠷13⠾⠷14⠾

$(16)(15)(13)(14)$

(16)(15)(13)(14)

1479. ⠷16⠾⠷14⠾⠷12⠾

$(16)(14)(12)$

(16)(14)(12)

1480. ⠷⠁⠂⠠⠁⠆⠠⠄⠠⠁⠰⠝⠐⠾⠘⠤⠂

$(a_1, a_2, \ldots, a_n)^{-1}$

(a_1, a_2, \ldots, a_n)^{-1}

1481. ⠷⠁⠂⠠⠁⠆⠠⠄⠠⠁⠰⠝⠐⠾⠘⠤⠂⠐ ⠨⠅ ⠷⠁⠂⠠⠁⠰⠝⠐⠠⠁⠰⠝⠤⠂⠐⠠⠄⠠⠁⠆⠾

$(a_1, a_2, \ldots, a_n)^{-1} = (a_1, a_n, a_{n-1}, \ldots, a_2)$

(a_1, a_2, \ldots, a_n)^{-1} = (a_1, a_{n}, a_{n-1}, \ldots, a_2)

1482. StartSet ⠨⠎⠈⠑⠠⠎⠲⠱⠨⠎⠷⠂⠾ ⠨⠅ ⠒EndSet

$\{\sigma \in S_4 : \sigma(1) = 3\}$

\{ \sigma \in S_4 : \sigma(1) = 3 \}

1483. StartSet ⠨⠎⠈⠑⠠⠎⠲⠱⠨⠎⠷⠆⠾ ⠨⠅ ⠆EndSet

$\{\sigma \in S_4 : \sigma(2) = 2\}$

\{ \sigma \in S_4 : \sigma(2) = 2 \}

1484. ⠨⠷⠨⠎⠈⠑⠠⠎⠲⠱⠨⠎⠷⠂⠾⠀⠨⠅⠀⠒

$\{\sigma \in S_4 : \sigma(1) = 3$

\{ \sigma \in S_4 : \sigma(1) = 3

1485. ⠨⠎⠷⠆⠾⠀⠨⠅⠀⠆⠨⠾

$\sigma(2) = 2\}$

\sigma(2) = 2 \}

1486. StartSet⠷13⠾ ⠠⠷13⠾⠷24⠾ ⠠⠷132⠾ ⠠⠷134⠾ ⠠⠷1324⠾ ⠠⠷1342⠾EndSet

$\{(13), (13)(24), (132), (134), (1324), (1342)\}$

\{ (13), (13)(24), (132), (134), (1324), (1342) \}

1487. ⠠⠎⠶

S_7

S_7

1488. ⠠⠁⠶

A_7

A_7

1489. ⠠⠁10

A_{10}

A_{10}

1490. ⠷12345⠾⠷678⠾

$(12345)(678)$

(12345)(678)

1491. ⠠⠁⠦

A_8

A_8

1492. ⠼26

26

26

1493. ⠝⠀⠨⠅⠀⠒⠀⠠⠄⠀⠠⠼10

$n = 3, \ldots, 10$

n = 3, \ldots, 10

1494. ⠠⠁⠢

A_5

A_5

1495. ⠠⠁⠖

A_6

A_6

1496. ⠷⠂⠾⠠⠷⠁⠂⠠⠁⠆⠾⠷⠁⠒⠠⠁⠲⠾⠠⠷⠁⠂⠠⠁⠆⠠⠁⠒⠾⠠⠷⠁⠂⠠⠁⠆⠠⠁⠒⠠⠁⠲⠠⠁⠢⠾

$(1), (a_1, a_2)(a_3, a_4), (a_1, a_2, a_3), (a_1, a_2, a_3, a_4, a_5)$

(1), (a_1, a_2)(a_3, a_4), (a_1, a_2, a_3), (a_1, a_2, a_3, a_4,
a_5)

1497. ⠰⠎⠘⠊⠐⠀⠨⠅⠀⠰⠎⠘⠚

$\sigma^i = \sigma^j$

\sigma^i = \sigma^j

1498. ⠊⠸⠇⠚⠷⠍⠕⠙⠝⠾

$i \equiv j \pmod{n}$

i \equiv j \pmod{n}

1499. ⠰⠎⠀⠨⠅⠀⠰⠎⠂⠄⠄⠄⠰⠎⠰⠍⠐⠈⠑⠠⠎⠰⠝

$\sigma = \sigma_1 \cdots \sigma_m \in S_n$

\sigma = \sigma_1 \cdots \sigma_m \in S_n

1500. ⠰⠎⠂⠠⠄⠀⠄⠰⠎⠰⠍⠐

$\sigma_1, \ldots, \sigma_m$

\sigma_1, \ldots, \sigma_m

1501. ⠠⠙⠢

D_5

D_5

1502. ⠷12⠾⠷34⠾

$(12)(34)$

(12)(34)

1503. ⠷123⠾⠷12⠾

$(123)(12)$

(123)(12)

1504. ⠷12⠾⠷123⠾

$(12)(123)$

(12)(123)

1505. ⠝⠤⠂

$n - 1$

n-1

1506. ⠝⠤⠆

$n - 2$

n - 2

1507. ⠰⠎⠘⠆

σ^2

\sigma^2

1508. ⠷⠁⠃⠾⠷⠃⠉⠾

$(ab)(bc)$

(ab)(bc)

1509. ⠷⠁⠃⠾⠷⠉⠙⠾

$(ab)(cd)$

(ab)(cd)

1510. ⠷⠂⠆⠾⠠⠷13⠾⠄⠄ ⠠⠷⠂⠝⠾

$(12), (13), \ldots, (1n)$

(1 2), (13), \ldots, (1n)

1511. ⠷⠂⠆⠾⠠⠷23⠾⠄⠄ ⠠⠷⠝⠤⠂ ⠠⠝⠾

$(12), (23), \ldots, (n-1, n)$

(1 2), (23), \ldots, (n- 1,n)

1512. ⠷12⠾⠠⠷⠂⠆⠄⠝⠾

$(12), (12 \ldots n)$

(12), (1 2 \ldots n)

1513. ⠨⠇ ⠰⠛ ⠐⠒⠠⠛ ⠫⠕ ⠠⠛

$\lambda_g : G \rightarrow G$

\lambda_g : G \rightarrow G

1514. ⠨⠇ ⠰⠛ ⠐⠷⠁⠾ ⠨⠅ ⠛⠁

$\lambda_g(a) = ga$

\lambda_g(a) = g a

1515. ⠨⠇ ⠰⠛

λ_g

\lambda_g

1516. ⠠⠵⠷⠠⠛⠾ ⠨⠅ StartSet⠛ ⠈⠑ ⠠⠛⠐⠒⠛⠭ ⠨⠅ ⠭⠛forall⠭ ⠈⠑ ⠠⠛EndSet

$Z(G) = \{g \in G : gx = xg$ for all $x \in G\}$

Z(G) = \{ g \in G : gx = xg \text{ for all } x \in G \}

1517. ⠠⠙⠦

D_8

D_8

1518. ⠠⠙10

D_{10}

D_{10}

1519. ⠨⠞ ⠨⠅ ⠷⠁⠂⠠⠁⠆⠄⠄⠠⠁⠰⠅⠐⠾

$\tau = (a_1, a_2, \ldots, a_k)$

\tau = (a_1, a_2, \ldots, a_k)

1520. ⠨⠎⠨⠞⠨⠎⠘⠤⠂⠐ ⠨⠅ ⠷⠨⠎⠷⠁⠂⠐⠾⠠⠨⠎⠷⠁⠆⠐⠾⠄⠄⠠⠨⠎⠷⠁⠰⠅⠐⠾⠾

$\sigma\tau\sigma^{-1} = (\sigma(a_1), \sigma(a_2), \ldots, \sigma(a_k))$

\sigma \tau \sigma^{-1 } = (\sigma(a_1), \sigma(a_2), \ldots, \sigma(a_k))

1521. ⠨⠎⠨⠞⠨⠎⠘⠤⠂⠐ ⠨⠅ ⠨⠍

$\sigma \tau \sigma^{-1} = \mu$

`\sigma \tau \sigma^{-1 } = \mu`

1522. ⠨⠎⠨⠞⠨⠎⠘⠤⠂⠐⠷⠨⠎⠷⠁⠰⠊⠐⠾⠾ ⠨⠅ ⠨⠎⠷⠁⠰⠊⠬⠂⠐⠾

$\sigma \tau \sigma^{-1}(\sigma(a_i)) = \sigma(a_{i+1})$

`\sigma \tau \sigma^{-1 }(\sigma(a_i)) = \sigma(a_{i + 1})`

1523. ⠨⠁ ⠠⡳⠭⠆⠆⠒⠉⠄ ⠨⠃

$\alpha \sim \beta$

`\alpha \sim \beta`

1524. ⠨⠎⠨⠁⠨⠎⠘⠤⠂⠐ ⠨⠅ ⠨⠃

$\sigma \alpha \sigma^{-1} = \beta$

`\sigma \alpha \sigma^{-1} = \beta`

1525. ⠨⠎⠘⠝⠐⠷⠭⠾ ⠨⠅ ⠽

$\sigma^n(x) = y$

`\sigma^n(x) = y`

1526. ⠨⠎ ⠈⠑ ⠠⠁⠰⠝

$\sigma \in A_n$

`\sigma \in A_n`

1527. ⠨⠞ ⠈⠑ ⠠⠎⠰⠝

$\tau \in S_n$

`\tau \in S_n`

1528. ⠨⠞⠘⠤⠂⠐⠨⠎⠨⠞ ⠈⠑ ⠠⠁⠰⠝

$\tau^{-1} \sigma \tau \in A_n$

`\tau^{-1} \sigma \tau \in A_n`

1529. caligraphic ⠠⠕⠰⠭⠠⠨⠎⠐ ⠨⠅ StartSet⠽⠈⠒⠭⠠⡳⠭⠆⠆⠒⠉⠄⠽EndSet

${\mathcal O}_{x,\sigma} = \{y : x \sim y\}$

`{\mathcal O}_{x, \sigma} = \{ y : x \sim y \}`

1530. caligraphic ⠠⠕⠰⠭⠠⠨⠎⠐⠨⠩caligraphic ⠠⠕⠰⠽⠠⠨⠎⠐ ⠌⠨⠅ ⠸⠴

${\mathcal O}_{x,\sigma} \cap {\mathcal O}_{y,\sigma} \neq \emptyset$

`{\mathcal O}_{x, \sigma} \cap {\mathcal O}_{y, \sigma} \neq \emptyset`

1531. caligraphic ⠠⠕⠰⠭⠠⠨⠎⠐ ⠨⠅ caligraphic ⠠⠕⠰⠽⠠⠨⠎

${\mathcal O}_{x,\sigma} = {\mathcal O}_{y,\sigma}$

`{\mathcal O}_{x, \sigma} = {\mathcal O}_{y, \sigma}`

1532. ⠨⠎ ⠈⠑ ⠠⠓

$\sigma \in H$

`\sigma \in H`

1533. ⠨⠎⠷⠭⠾ ⠨⠅ ⠽

$\sigma(x) = y$

`\sigma(x) = y`

1534. ⠠⡗⠭⠆ ⠶⠈⠢⠄ ⠅⠋⠄⡗⠭⠆ ⠶⠈⠔⠄

$\langle \sigma \rangle$

`\langle \sigma \rangle`

1535. caligraphic ⠠⠕⠰⠭⠠⠨⠎⠐ ⠨⠅ ⠠⠭

$\mathcal{O}_{x,\sigma} = X$

`{\mathcal O}_{x, \sigma} = X`

1536. ⠨⠁ ⠈⠑⠠⠎⠰⠝

$\alpha \in S_n$

`\alpha \in S_n`

1537. ⠨⠃ ⠈⠑⠠⠎⠰⠝

$\beta \in S_n$

`\beta \in S_n`

1538. ⠨⠁ ⠘⠤⠂

α^{-1}

`\alpha^{-1}`

1539. ⠨⠁ ⠘⠤⠂ ⠐⠨⠃ ⠘⠤⠂ ⠐⠨⠁ ⠨⠃

$\alpha^{-1}\beta^{-1}\alpha\beta$

`\alpha^{-1} \beta^{-1} \alpha \beta`

1540. ⠨⠁ ⠠⠨⠃ ⠈⠑⠠⠎⠰⠝⠐

$\alpha, \beta \in S_n$

`\alpha, \beta \in S_n`

1541. ⠗⠘⠅ ⠐⠎ ⠨⠅ ⠎⠗⠘⠤⠅

$r^k s = s r^{-k}$

`r^k s = s r^{-k}`

1542. ⠗⠘⠅ ⠐⠈⠑⠠⠙⠰⠝

$r^k \in D_n$

`r^k \in D_n`

1543. ⠝⠸⠌⠛⠉⠙⠷⠅⠠ ⠝⠾

$n / \gcd(k,n)$

`n / \gcd(k,n)`

1544. ⠨⠞⠨⠎

$\tau\sigma$

`\tau\sigma`

1545. ⠠⠎10

S_{10}

`S_{10}`

1546. ⠁ ⠘⠒

a^3

`a^3`

1547. ⠃⠉

bc

bc

1548. ⠁⠙⠘⠤⠂⠐⠃

$ad^{-1}b$

ad^{-1}b

1549. ⠁⠠⠃⠠⠉⠠⠙

a, b, c, d

a,b,c,d

1550. L

L

1551. ⠠⠎⠖

S_6

S_6

1552. ⠼30

30

30

1553. ⠛⠠⠓ ⠨⠅ StartSet⠛⠓⠱⠓ ⠈⠑ ⠠⠓EndSet

$gH = \{gh : h \in H\}$

gH = \{ gh : h \in H \}

1554. ⠠⠓⠛ ⠨⠅ StartSet⠓⠛⠱⠓ ⠈⠑ ⠠⠓EndSet

$Hg = \{hg : h \in H\}$

Hg = \{ hg : h \in H \}

1555. StartSet⠷⠂⠾⠠⠷123⠾⠠⠷132⠾EndSet

$\{(1), (123), (132)\}$

\{(1), (123), (132) \}

1556. StartSet⠷⠂⠾⠠⠷⠂⠆⠾EndSet

$\{(1), (12)\}$

\{(1), (1 2)\}

1557. ⠛⠂⠠⠛⠆ ⠈⠑ ⠠⠛

$g_1, g_2 \in G$

g_1, g_2 \in G

1558. ⠛⠂⠠⠓ ⠨⠅ ⠛⠆⠠⠓

$g_1 H = g_2 H$

g_1 H = g_2 H

1559. ⠠⠓⠛⠂⠘⠤⠂⠐ ⠨⠅ ⠠⠓⠛⠆⠘⠤⠂

$Hg_1^{-1} = Hg_2^{-1}$

H g_1^{-1} = H g_2^{-1}

1560. ⠛⠂ ⠠⠓ ⠸⠐⠅ ⠛⠆ ⠠⠓

$g_1 H \subset g_2 H$

g_1 H \subset g_2 H

1561. ⠛⠆ ⠈⠑⠛⠂ ⠠⠓

$g_2 \in g_1 H$

g_2 \in g_1 H

1562. ⠛⠂ ⠘⠤⠂ ⠐⠛⠆ ⠈⠑ ⠠⠓

$g_1^{-1} g_2 \in H$

g_1^{-1} g_2 \in H

1563. ⠛⠂ ⠠⠓

$g_1 H$

g_1 H

1564. ⠛⠆ ⠠⠓

$g_2 H$

g_2 H

1565. ⠛⠂ ⠠⠓ ⠨⠩⠛⠆ ⠠⠓ ⠨⠅ ⠸⠴

$g_1 H \cap g_2 H = \emptyset$

g_1 H \cap g_2 H = \emptyset

1566. ⠛⠂ ⠠⠓ ⠨⠩⠛⠆ ⠠⠓ ⠌⠨⠅ ⠸⠴

$g_1 H \cap g_2 H \neq \emptyset$

g_1 H \cap g_2 H \neq \emptyset

1567. ⠁ ⠈⠑⠛⠂ ⠠⠓ ⠨⠩⠛⠆ ⠠⠓

$a \in g_1 H \cap g_2 H$

a \in g_1 H \cap g_2 H

1568. ⠁ ⠨⠅ ⠛⠂⠓⠂ ⠨⠅ ⠛⠆⠓⠆

$a = g_1 h_1 = g_2 h_2$

a = g_1 h_1 = g_2 h_2

1569. ⠓⠂

h_1

h_1

1570. ⠓⠆

h_2

h_2

1571. ⠛⠂ ⠨⠅ ⠛⠆⠓⠆⠓⠂⠘⠤⠂

$g_1 = g_2 h_2 h_1^{-1}$

g_1 = g_2 h_2 h_1^{-1}

1572. ⠛⠂ ⠈⠑⠛⠆ ⠠⠓

$g_1 \in g_2 H$

g_1 \in g_2 H

1573. ⠈⠷⠠⠛⠐⠂⠠⠓⠈⠾

$[G : H]$

[G:H]

1574. ⠠⠛ ⠨⠅ ⠈⠠⠵⠆

$G = \mathbb{Z}_6$

G= {\mathbb Z}_6

1575. ⠠⠓ ⠨⠅ StartSet⠴ ⠠⠒EndSet

$H = \{0, 3\}$

H = \{ 0, 3 \}

1576. ⠈⠷⠠⠛⠐⠂⠠⠓⠈⠾ ⠨⠅ ⠒

$[G : H] = 3$

[G:H] = 3

1577. ⠠⠛ ⠨⠅ ⠠⠎⠒

$G = S_3$

G= S_3

1578. ⠠⠓ ⠨⠅ StartSet⠷⠂ ⠾ ⠠⠷123⠾ ⠠⠷132⠾EndSet

$H = \{(1), (123), (132)\}$

H = \{ (1),(123), (132) \}

1579. ⠠⠅ ⠨⠅ StartSet⠷⠂ ⠾ ⠠⠷12⠾EndSet

$K = \{(1), (12)\}$

K= \{ (1), (12) \}

1580. ⠈⠷⠠⠛⠐⠂⠠⠓⠈⠾ ⠨⠅ ⠆

$[G : H] = 2$

[G:H] = 2

1581. ⠈⠷⠠⠛⠐⠂⠠⠅⠈⠾ ⠨⠅ ⠒

$[G : K] = 3$

[G:K] = 3

1582. caligraphic ⠠⠇ ⠰⠠⠓

$\mathcal{L}_H$

{\mathcal L}_H

1583. caligraphic ⠠⠗ ⠰⠠⠓

$\mathcal{R}_H$

{\mathcal R}_H

1584. ⠨⠋⠐⠂caligraphic ⠠⠇ ⠰⠠⠓⠐ ⠫⠕ caligraphic ⠠⠗ ⠰⠠⠓⠐

$\phi : \mathcal{L}_H \rightarrow \mathcal{R}_H$

\phi : {\mathcal L}_H \rightarrow {\mathcal R}_H

1585. ⠛ ⠠⠓ ⠈⠑caligraphic ⠠⠇ ⠰⠠⠓

$gH \in \mathcal{L}_H$

gH \in {\mathcal L}_H

1586. ⠨⠋⠷⠛⠠⠓⠾ ⠨⠅ ⠠⠓⠛⠘⠤⠂

$\phi(gH) = Hg^{-1}$

`\phi( gH ) = Hg^{-1}`

1587. ⠨⠋

ϕ

`\phi`

1588. ⠠⠓⠛⠂⠘⠤⠂⠐ ⠨⠅ ⠨⠋⠷⠛⠂⠠⠓⠾ ⠨⠅ ⠨⠋⠷⠛⠆⠠⠓⠾ ⠨⠅ ⠠⠓⠛⠆⠘⠤⠂

$H g_1^{-1} = \phi(g_1 H) = \phi(g_2 H) = H g_2^{-1}$

`H g_1^{-1} = \phi( g_1 H ) = \phi( g_2 H ) = H g_2^{-1}`

1589. ⠨⠋⠷⠛⠘⠤⠂⠐⠠⠓⠾ ⠨⠅ ⠠⠓⠛

$\phi(g^{-1} H) = H g$

`\phi(g^{-1} H ) = H g`

1590. ⠨⠋⠱⠠⠓ ⠫⠕ ⠛⠠⠓

$\phi:H \rightarrow gH$

`\phi:H \rightarrow gH`

1591. ⠨⠋⠷⠓⠾ ⠨⠅ ⠛⠓

$\phi(h) = gh$

`\phi(h) = gh`

1592. ⠛⠠⠓

gH

`gH`

1593. ⠨⠋⠷⠓⠂⠐⠾ ⠨⠅ ⠨⠋⠷⠓⠆⠐⠾

$\phi(h_1) = \phi(h_2)$

`\phi(h_1) = \phi(h_2)`

1594. ⠓⠂ ⠨⠅ ⠓⠆

$h_1 = h_2$

`h_1 = h_2`

1595. ⠨⠋⠷⠓⠂⠐⠾ ⠨⠅ ⠛⠓⠂

$\phi(h_1) = gh_1$

`\phi(h_1) = gh_1`

1596. ⠨⠋⠷⠓⠆⠐⠾ ⠨⠅ ⠛⠓⠆

$\phi(h_2) = gh_2$

`\phi(h_2) = gh_2`

1597. ⠛⠓⠂ ⠨⠅ ⠛⠓⠆

$gh_1 = gh_2$

`gh_1 = gh_2`

1598. ⠓⠂ ⠨⠅ ⠓⠆

$h_1= h_2$

`h_1= h_2`

1599. ⠛⠓

gh

gh

1600. ⠳⠠⠛⠳⠌⠳⠠⠓⠳ ⠨⠅ ⠈⠷⠠⠛⠱⠠⠓⠈⠾

$|G|/|H| = [G : H]$

|G|/|H| = [G : H]

1601. ⠈⠷⠠⠛⠱⠠⠓⠈⠾

$[G : H]$

[G : H]

1602. ⠳⠠⠓⠳

$|H|$

|H|

1603. ⠳⠠⠛⠳ ⠨⠅ ⠈⠷⠠⠛⠱⠠⠓⠈⠾⠳⠠⠓⠳

$|G| = [G : H]|H|$

|G| = [G : H] |H|

1604. ⠳⠠⠛⠳ ⠨⠅ ⠏

$|G| = p$

|G| = p

1605. ⠛ ⠌⠨⠅ ⠑

$g \neq e$

g \neq e

1606. ⠳⠄⡳⠭⠆⠶⠑⠦⠄⠛⠄⡳⠭⠆⠶⠑⠔⠄⠳ ⠨⠂ ⠂

$|\langle g \rangle| > 1$

|\langle g \rangle| \gt 1

1607. ⠈⠠⠵⠰⠏

$\mathbb{Z}_p$

{\mathbb Z}_p

1608. ⠠⠛ ⠸⠨⠂ ⠠⠓ ⠸⠨⠂ ⠠⠅

$G \supset H \supset K$

G \supset H \supset K

1609. ⠈⠷⠠⠛⠱⠠⠅⠈⠾ ⠨⠅ ⠈⠷⠠⠛⠱⠠⠓⠈⠾⠈⠷⠠⠓⠱⠠⠅⠈⠾

$[G : K] = [G : H][H : K]$

[G:K] = [G:H][H:K]

1610. ⠈⠷⠠⠛⠱⠠⠅⠈⠾ ⠨⠅ ⠹⠳⠠⠛⠳⠌⠳⠠⠅⠳⠼ ⠨⠅ ⠹⠳⠠⠛⠳⠌⠳⠠⠓⠳⠼⠈⠡⠹⠳⠠⠓⠳⠌⠳⠠⠅⠳⠼ ⠨⠅ ⠈⠷⠠⠛⠱⠠⠓

$[G : K] = \frac{|G|}{|K|} = \frac{|G|}{|H|} \cdot \frac{|H|}{|K|} = [G : H][H : K]$

[G:K] = \frac{|G|}{|K|} = \frac{|G|}{|H|} \cdot \frac{|H|}{|K|}
= [G:H][H:K]

1611. ⠈⠷⠠⠁⠲⠱⠠⠓⠈⠾ ⠨⠅ ⠆

$[A_4 : H] = 2$

`[A_4 : H] = 2`

1612. ⠛⠠⠓ ⠨⠅ ⠠⠓⠛

$gH = Hg$

`gH = Hg`

1613. ⠛⠠⠓⠛⠘⠤⠂⠐ ⠨⠅ ⠠⠓

$gHg^{-1} = H$

`g H g^{-1} = H`

1614. ⠛⠈⠑⠠⠁⠲

$g \in A_4$

`g \in A_4`

1615. ⠷123⠾

(123)

`(123)`

1616. ⠷123⠾⠘⠤⠂⠐ ⠨⠅ ⠷132⠾

$(123)^{-1} = (132)$

`(123)^{-1} = (132)`

1617. ⠛⠓⠛⠘⠤⠂⠈⠑⠠⠓

$ghg^{-1} \in H$

`g h g^{-1} \in H`

1618. ⠷⠂⠾⠠⠷123⠾⠠⠷132⠾⠠⠷243⠾⠠⠷243⠾⠘⠤⠂⠐ ⠨⠅ ⠷234⠾⠠⠷142⠾⠠⠷142⠾⠘⠤⠂⠐ ⠨⠅ ⠷124⠾

$(1), (123), (132), (243), (243)^{-1} = (234), (142), (142)^{-1} = (124)$

`(1), (123), (132), (243), (243)^{-1} = (234), (142), (142)^{-1}`
`= (124)`

1619. ⠨⠍ ⠨⠅ ⠨⠎⠨⠞⠨⠎⠘⠤⠂

$\mu = \sigma \tau \sigma^{-1}$

`\mu = \sigma \tau \sigma^{-1}`

1620. ⠨⠞ ⠨⠅ ⠷⠁⠂⠠⠁⠆⠠⠄⠠⠁⠰⠅⠐⠾

$\tau = (a_1, a_2, \ldots, a_k)$

`\tau = (a_1, a_2, \ldots, a_k )`

1621. ⠨⠎⠷⠁⠰⠊⠐⠾ ⠨⠅ ⠃

$\sigma(a_i) = b$

`\sigma( a_i ) = b`

1622. ⠨⠎⠷⠁⠰⠷⠊⠍⠕⠙⠅⠾⠬⠂⠐⠾ ⠨⠅ ⠃⠄

$\sigma(a_{(i \bmod k)+1}) = b'$

`\sigma( a_{(i \bmod k) + 1}) = b'`

1623. ⠨⠍⠷⠃⠾ ⠨⠅ ⠃⠄

$\mu(b) = b'$

`\mu( b) = b'`

1624. ⠨⠍ ⠨⠅ ⠷⠨⠎⠷⠁⠂⠾⠠⠨⠎⠷⠁⠆⠾⠠⠄⠠⠨⠎⠷⠁⠰⠅⠐⠾⠾

$\mu = (\sigma(a_1), \sigma(a_2), \ldots, \sigma(a_k))$

`\mu = ( \sigma(a_1), \sigma(a_2), \ldots, \sigma(a_k) )`

1625. ⠨⠋⠒⠈⠠⠝ ⠫⠕ ⠈⠠⠝

$\phi : \mathbb{N} \rightarrow \mathbb{N}$

`\phi : {\mathbb N } \rightarrow {\mathbb N}`

1626. ⠨⠋⠷⠝⠾ ⠨⠅ ⠂

$\phi(n) = 1$

`\phi(n) = 1`

1627. ⠝ ⠨⠅ ⠂

$n = 1$

`n=1`

1628. ⠨⠋⠷⠝⠾

$\phi(n)$

`\phi(n)`

1629. ⠳⠠⠥⠷12⠾⠳ ⠨⠅ ⠨⠋⠷12⠾ ⠨⠅ ⠲

$|U(12)| = \phi(12) = 4$

`|U(12)| = \phi(12) = 4`

1630. ⠨⠋⠷⠏⠾ ⠨⠅ ⠏⠤⠂

$\phi(p) = p - 1$

`\phi(p) = p-1`

1631. ⠳⠠⠥⠷⠝⠾⠳ ⠨⠅ ⠨⠋⠷⠝⠾

$|U(n)| = \phi(n)$

`|U(n)| = \phi(n)`

1632. ⠁ ⠘⠨⠋⠷⠝⠾⠐⠸⠇⠂⠷⠍⠕⠙⠝⠾

$a^{\phi(n)} \equiv 1 \pmod{n}$

`a^{\phi(n)} \equiv 1 \pmod{n}`

1633. ⠁ ⠘⠨⠋⠷⠝⠾⠐ ⠨⠅ ⠂

$a^{\phi(n)} = 1$

`a^{\phi(n)} = 1`

1634. ⠁ ⠈⠑⠠⠥⠷⠝⠾

$a \in U(n)$

`a \in U(n)`

1635. ⠁ ⠘⠨⠋⠷⠝⠾⠐⠤⠂

$a^{\phi(n)} - 1$

`a^{\phi(n)} - 1`

1636. ⠝⠰⠆⠏

$n = p$

`n = p`

1637. ⠨⠋⠷⠏⠾⠰⠆⠏⠤⠂

$\phi(p) = p - 1$

`\phi(p) = p - 1`

1638. ⠏⠌⠳⠁

$p \nmid a$

`p \notdivide a`

1639. ⠁⠘⠏⠤⠂⠐⠸⠇⠂⠷⠍⠕⠙⠏⠾

$a^{p-1} \equiv 1 \pmod{p}$

`a^{p-1} \equiv 1 \pmod{ p }`

1640. ⠃⠘⠏⠐⠸⠇⠃⠷⠍⠕⠙⠏⠾

$b^p \equiv b \pmod{p}$

`b^p \equiv b \pmod{ p}`

1641. ⠳⠠⠛⠳⠨⠂⠱ 35

$|G| \geq 35$

`|G| \geq 35`

1642. ⠼60

60

`60`

1643. ⠄⡳⠭⠆⠶⠑⠦⠄⠦⠄⡳⠭⠆⠶⠑⠔⠄

$\langle 8 \rangle$

`\langle 8 \rangle`

1644. ⠄⡳⠭⠆⠶⠑⠦⠄⠒⠄⡳⠭⠆⠶⠑⠔⠄

$\langle 3 \rangle$

`\langle 3 \rangle`

1645. ⠠⠓⠰⠆ StartSet⠷⠂⠾⠠⠷123⠾⠠⠷132⠾EndSet

$H = \{(1), (123), (132)\}$

`H = \{ (1), (123), (132) \}`

1646. ⠂⠬⠄⡳⠭⠆⠶⠑⠦⠄⠦⠄⡳⠭⠆⠶⠑⠔⠄

$1 + \langle 8 \rangle$

`1 + \langle 8 \rangle`

1647. ⠆⠬⠄⡳⠭⠆⠶⠑⠦⠄⠦⠄⡳⠭⠆⠶⠑⠔⠄

$2 + \langle 8 \rangle$

`2 + \langle 8 \rangle`

1648. ⠒⠬⠄⡳⠭⠆⠶⠑⠦⠄⠦⠄⡳⠭⠆⠶⠑⠔⠄

$3 + \langle 8 \rangle$

`3 + \langle 8 \rangle`

1649. ⠲⠬⠄⡳⠭⠆⠶⠑⠦⠄⠦⠄⡳⠭⠆⠶⠑⠔⠄

$4 + \langle 8 \rangle$

4 + \langle 8 \rangle

1650. ⠢⠬⠄⡳⠭⠆⠶⠑⠦⠄⠦⠄⡳⠭⠆⠶⠑⠔⠄

$5 + \langle 8 \rangle$

5 + \langle 8 \rangle

1651. ⠖⠬⠄⡳⠭⠆⠶⠑⠦⠄⠦⠄⡳⠭⠆⠶⠑⠔⠄

$6 + \langle 8 \rangle$

6 + \langle 8 \rangle

1652. ⠶⠬⠄⡳⠭⠆⠶⠑⠦⠄⠦⠄⡳⠭⠆⠶⠑⠔⠄

$7 + \langle 8 \rangle$

7 + \langle 8 \rangle

1653. ⠂⠬⠒⠈⠠⠵

$1 + 3\mathbb{Z}$

1 + 3 {\mathbb Z}

1654. ⠆⠬⠒⠈⠠⠵

$2 + 3\mathbb{Z}$

2 + 3 {\mathbb Z}

1655. ⠝ ⠨⠅ 15

$n = 15$

n = 15

1656. ⠁ ⠨⠅ ⠲

$a = 4$

a = 4

1657. ⠲⠘⠨⠋⠷15⠾⠐ ⠸⠇ ⠲⠘⠦⠐ ⠸⠇ ⠂ ⠷⠍⠕⠙15⠾

$4^{\phi(15)} \equiv 4^8 \equiv 1 \pmod{15}$

4^{\phi(15)} \equiv 4^8 \equiv 1 \pmod{15}

1658. ⠏ ⠨⠅ ⠲⠝⠬⠒

$p = 4n + 3$

p = 4n + 3

1659. ⠭⠘⠆⠐ ⠸⠇ ⠤⠂ ⠷⠍⠕⠙⠏⠾

$x^2 \equiv -1 \pmod{p}$

x^2 \equiv -1 \pmod{p}

1660. ⠛⠓⠛⠘⠤⠂ ⠈⠑ ⠠⠓

$ghg^{-1} \in H$

ghg^{-1} \in H

1661. ⠛⠂ ⠈⠑⠛⠠⠓

$g_1 \in gH$

g_1 \in gH

1662. ⠛⠂ ⠈⠑ ⠠⠓⠛

$g_1 \in Hg$

g_1 \in Hg

1663. ⠛⠠⠓ ⠸⠐⠅ ⠠⠓⠛

$gH \subset Hg$

gH \subset Hg

1664. ⠨⠋⠷⠛⠠⠓⠾ ⠨⠅ ⠠⠓⠛

$\phi(gH) = Hg$

\phi(gH) = Hg

1665. ⠛⠘⠝⠐ ⠨⠅ ⠑

$g^n = e$

g^n = e

1666. ⠨⠎ ⠨⠅ ⠷12⠾⠷345⠾⠷78⠾⠷⠔⠾

$\sigma = (12)(345)(78)(9)$

\sigma = (12)(345)(78)(9)

1667. ⠷⠆ ⠠⠒ ⠠⠆ ⠠⠂ ⠾

$(2, 3, 2, 1)$

(2,3,2,1)

1668. ⠷⠂ ⠠⠆ ⠠⠆ ⠠⠒⠾

$(1, 2, 2, 3)$

(1, 2, 2, 3)

1669. ⠨⠛

γ

\gamma

1670. ⠨⠃ ⠨⠅ ⠨⠛⠨⠁⠨⠛⠘⠤⠂

$\beta = \gamma\alpha\gamma^{-1}$

\beta = \gamma \alpha \gamma^{-1}

1671. ⠨⠛ ⠈⠑ ⠠⠎⠰⠝

$\gamma \in S_n$

\gamma \in S_n

1672. ⠳⠠⠛⠳ ⠨⠅ ⠆⠝

$|G| = 2n$

|G| = 2n

1673. ⠈⠷⠠⠛⠱⠠⠓⠈⠾ ⠨⠅ ⠆

$[G : H] = 2$

[G : H] = 2

1674. ⠁⠃ ⠈⠑ ⠠⠓

$ab \in H$

ab \in H

1675. ⠛⠠⠓⠨⠩⠛⠠⠅

$gH \cap gK$

```
gH \cap gK
```

1676. ⠠⠓⠨⠩⠠⠅

$H \cap K$

```
H \cap K
```

1677. ⠛⠷⠠⠓⠨⠩⠠⠅⠾ ⠨⠅ ⠛⠠⠓⠨⠩⠛⠠⠅

$g(H \cap K) = gH \cap gK$

```
g(H \cap K) = gH \cap gK
```

1678. ⠁⠄⡷⠽⠆⠆⠒⠉⠄⠃

$a \sim b$

```
a \sim b
```

1679. ⠅⠈⠑⠠⠅

$k \in K$

```
k \in K
```

1680. ⠓⠁⠅ ⠨⠅ ⠃

$hak = b$

```
hak = b
```

1681. ⠝ ⠨⠅ ⠏⠂⠘⠑⠂⠐⠏⠆⠘⠑⠆⠐⠄⠄⠄⠏⠰⠅⠘⠑⠘⠰⠅

$n = p_1^{e_1} p_2^{e_2} \cdots p_k^{e_k}$

```
n = p_1^{e_1} p_2^{e_2} \cdots p_k^{e_k}
```

1682. ⠏⠂⠠⠏⠆⠠⠄⠠⠏⠰⠅⠐

$p_1, p_2, \ldots, p_k$

```
p_1, p_2, \ldots, p_k
```

1683. ⠨⠋⠷⠝⠾ ⠨⠅ ⠝⠷⠂⠤⠹⠂⠌⠏⠂⠼⠾⠷⠂⠤⠹⠂⠌⠏⠆⠼⠾⠄⠄⠄⠷⠂⠤⠹⠂⠌⠏⠰⠅⠐⠼⠾

$\phi(n) = n \left(1 - \frac{1}{p_1} \right) \left(1 - \frac{1}{p_2} \right)\cdots \left(1 - \frac{1}{p_k} \right)$

```
\phi(n) = n \left( 1 - \frac{1}{p_1} \right) \left( 1 - \frac{1}{p_2}
\right)\cdots \left( 1 - \frac{1}{p_k} \right)
```

1684. ⠨⠋⠷⠍⠝⠾ ⠨⠅ ⠨⠋⠷⠍⠾⠨⠋⠷⠝⠾

$\phi(mn) = \phi(m)\phi(n)$

```
\phi(mn) = \phi(m)\phi(n)
```

1685. ⠝ ⠨⠅ ⠐⠨⠠⠎⠰⠙⠳⠝⠐⠨⠋⠷⠙⠾

$n = \sum_{d \mid n} \phi(d)$

```
n = \sum_{d \mid n} \phi(d)
```

1686. 391 ⠨⠅ 17⠡23

$391=17\cdot 23$

```
391=17\cdot 23
```

1687. ⠼100

100

```
100
```

1688. ⠼1000

1000

```
1000
```

1689. ⠼⠴ ⠐⠅ ⠝ ⠐⠅ 100

$0 < n < 100$

```
0\lt n\lt 100
```

1690. ⠂ ⠐⠅⠱ ⠁ ⠐⠅⠱ ⠝

$1 \leq a \leq n$

```
1\leq a \leq n
```

1691. ⠼⠶⠖ ⠨⠅ 5040

$7! = 5040$

```
7! = 5040
```

1692. ⠼5040

5040

```
5040
```

1693. ⠠⠁ ⠨⠅ 00⠠ ⠠⠃ ⠨⠅ 01⠠ ⠄⠄⠄⠠ ⠠⠵ ⠨⠅ 25

$\text{A} = 00, \text{B} = 01, \ldots, \text{Z} = 25$

```
\text{A} = 00, \text{B} = 01, \ldots, \text{Z} = 25
```

1694. ⠋⠷⠏⠾ ⠨⠅ ⠏⠬⠒⠍⠕⠙⠒26⠸⠆

$f(p) = p + 3 \bmod 26;$

```
f(p) = p + 3 \bmod 26;
```

1695. ⠠⠁ ⠫⠳⠒⠒⠕ ⠠⠙⠠ ⠠⠃ ⠫⠳⠒⠒⠕ ⠠⠑⠠ ⠄⠄⠄⠠ ⠠⠵ ⠫⠳⠒⠒⠕ ⠠⠉

$A \mapsto D, B \mapsto E, \ldots, Z \mapsto C$

```
A \mapsto D, B \mapsto E, \ldots, Z \mapsto C
```

1696. ⠋⠘⠤⠂ ⠐⠷⠏⠾ ⠨⠅ ⠏⠤⠒⠍⠕⠙⠒26 ⠨⠅ ⠏⠬23⠍⠕⠙⠒26

$f^{-1}(p) = p - 3 \bmod 26 = p + 23 \bmod 26$

```
f^{-1}(p) = p - 3 \bmod 26 = p + 23 \bmod 26
```

1697. ⠼⠒ ⠠⠼14 ⠠⠼⠔ ⠠⠼⠶ ⠠⠼⠲ ⠠⠼20 ⠠⠼⠒

$3, 14, 9, 7, 4, 20, 3$

```
3, 14, 9, 7, 4, 20, 3
```

1698. ⠼⠴ ⠠⠼11 ⠠⠼⠖ ⠠⠼⠲ ⠠⠼⠂ ⠠⠼17 ⠠⠼⠴

$0, 11, 6, 4, 1, 17, 0$

```
0, 11, 6, 4, 1, 17, 0
```

1699. ⠋⠷⠏⠾ ⠨⠅ ⠏⠬⠃ ⠍⠕⠙⠒26

$f(p) = p + b \bmod 26$

```
f(p) = p + b \bmod 26
```

1700. .⠑ ⠭ 04

E $= 04$

\text{E} = 04

1701. .⠎ ⠭ 18

S $= 18$

\text{S} = 18

1702. 18 ⠭ ⠲⠬⠃ ⠍⠕⠙26

$18 = 4 + b \bmod 26$

18 = 4 + b \bmod 26

1703. ⠃ ⠭ 14

$b = 14$

b= 14

1704. ⠋⠷⠏⠾ ⠭ ⠏⠬14⠍⠕⠙26

$f(p) = p + 14 \bmod 26$

f(p) = p + 14 \bmod 26

1705. ⠋ ⠘⠤⠂ ⠐⠷⠏⠾ ⠭ ⠏⠬12⠍⠕⠙26

$f^{-1}(p) = p + 12 \bmod 26$

f^{-1}(p) = p + 12 \bmod 26

1706. ⠋⠷⠏⠾ ⠭ ⠁⠏⠬⠃ ⠍⠕⠙26

$f(p) = ap + b \bmod 26$

f(p) = ap + b \bmod 26

1707. ⠉ ⠭ ⠁⠏⠬⠃ ⠍⠕⠙26

$c = ap + b \bmod 26$

c = ap + b \bmod 26

1708. ⠛⠉⠙⠷⠁ .26⠾ ⠭ ⠂

$\gcd(a, 26) = 1$

\gcd(a, 26) =1

1709. ⠋ ⠘⠤⠂ ⠐⠷⠏⠾ ⠭ ⠁ ⠘⠤⠂ ⠐⠏⠤⠁ ⠘⠤⠂ ⠐⠃ ⠍⠕⠙26

$f^{-1}(p) = a^{-1}p - a^{-1}b \bmod 26$

f^{-1}(p) = a^{-1} p - a^{-1} b \bmod 26

1710. ⠁ ⠈⠑⠈ .⠵26

$a \in \mathbb{Z}_{26}$

a \in {\mathbb Z}_{26}

1711. ⠛⠉⠙⠷⠁ .26⠾ ⠭ ⠂

$\gcd(a, 26) = 1$

\gcd(a, 26) = 1

1712. ⠁ ⠭ ⠢

$a = 5$

a = 5

1713. ⠛⠉⠙⠷⠢⠠26⠾ ⠨⠅ ⠂

$\gcd(5, 26) = 1$

```
\gcd(5, 26) = 1
```

1714. ⠁⠘⠤⠂⠐ ⠨⠅ 21

$a^{-1} = 21$

```
a^{-1} = 21
```

1715. ⠋⠷⠏⠾ ⠨⠅ ⠢⠏⠬⠒⠍⠕⠙26

$f(p) = 5p + 3 \bmod 26$

```
f(p) = 5p + 3 \bmod 26
```

1716. ⠠⠼⠒ ⠠⠠⠼⠖ ⠠⠠⠼⠶ ⠠⠠⠼23 ⠠⠠⠼⠦ ⠠⠠⠼10 ⠠⠠⠼⠒

$3, 6, 7, 23, 8, 10, 3$

```
3, 6, 7, 23, 8, 10, 3
```

1717. ⠋⠘⠤⠂⠐⠷⠏⠾ ⠨⠅ 21⠏⠤21⠡⠒⠍⠕⠙26 ⠨⠅ 21⠏⠬15⠍⠕⠙26

$f^{-1}(p) = 21 p - 21 \cdot 3 \bmod 26 = 21 p + 15 \bmod 26$

```
f^{-1}(p) = 21 p - 21 \cdot 3 \bmod 26 = 21 p + 15 \bmod 26
```

1718. ⠏⠂

p_1

```
p_1
```

1719. ⠏⠆

p_2

```
p_2
```

1720. ⠸⠏ ⠨⠅ ⠏⠂ ⠏⠆

$\mathbf{p} = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$

```
{\mathbf p} = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}
```

1721. ⠈⠠⠵26

$\mathbb{Z}_{26}$

```
{\mathbb Z}_{26}
```

1722. ⠋⠷⠸⠏⠾ ⠨⠅ ⠠⠁⠸⠏⠬⠸⠃

$f(\mathbf{p}) = A \mathbf{p} + \mathbf{b}$

```
f({\mathbf p}) = A {\mathbf p} + {\mathbf b}
```

1723. ⠸⠃

$\mathbf{b}$

```
{\mathbf b}
```

1724. ⠋⠘⠤⠂⠐⠷⠸⠏⠾ ⠨⠅ ⠠⠁⠘⠤⠂⠐⠸⠏⠤⠠⠁⠘⠤⠂⠐⠸⠃

$f^{-1}(\mathbf{p}) = A^{-1} \mathbf{p} - A^{-1} \mathbf{b}$

```
f^{-1}({\mathbf p}) = A^{-1} {\mathbf p} - A^{-1} {\mathbf b}
```

1725. ⠼⠶ ⠠⠼⠲ ⠠⠼11 ⠠⠼15

$7, 4, 11, 15$

7, 4, 11, 15

1726. ⠠⠁ ⠶ ⠠⠷⠼⠒ ⠼⠢⠠⠾ ⠠⠷⠼⠂ ⠼⠆⠠⠾

$A = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}$

A = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}

1727. ⠠⠁ ⠘⠤⠂ ⠐ ⠶ ⠠⠷⠼⠆ ⠼21⠠⠾ ⠠⠷⠼25 ⠼⠒⠠⠾

$A^{-1} = \begin{pmatrix} 2 & 21 \\ 25 & 3 \end{pmatrix}$

A^{-1} = \begin{pmatrix} 2 & 21 \\ 25 & 3 \end{pmatrix}

1728. ⠸⠃ ⠶ ⠷⠆ ⠠⠆ ⠾⠘⠜

${\mathbf b} = (2, 2)^\mathsf{t}$

{\mathbf b} = (2, 2)^\transpose

1729. ⠝ ⠶ ⠏⠟

$n= pq$

n= pq

1730. ⠨⠋⠷⠝⠾ ⠶ ⠍ ⠶ ⠷⠏⠤⠂⠾⠷⠟⠤⠂⠾

$\phi(n) = m = (p - 1)(q-1)$

\phi(n) = m = (p - 1)(q-1)

1731. ⠛⠉⠙⠷⠠⠑⠠⠍⠾ ⠶ ⠂

$\gcd(E, m) = 1$

\gcd(E, m) = 1

1732. D

D

1733. ⠠⠙⠠⠑ ⠿ ⠂ ⠷⠍⠕⠙⠍⠾

$DE \equiv 1 \pmod{m}$

DE \equiv 1 \pmod{m}

1734. ⠠⠁ ⠶ 00 ⠠⠠⠃ ⠶ 02⠤ ⠠⠠⠵ ⠶ 25

$\text{A} = 00, \text{B} = 02, \ldots, \text{Z}= 25$

\text{A} = 00, \text{B} = 02, \ldots, \text{Z}= 25

1735. ⠽ ⠶ ⠭⠘⠠⠑⠐⠍⠕⠙⠝

$y = x^E \mod n$

y = x^E \mod n

1736. ⠭ ⠶ ⠽⠘⠠⠙⠐⠍⠕⠙⠝

$x = y^D \bmod n$

x = y^D \bmod n

1737. ⠼25

25

25

1738. ⠏ ⠭ 23

$p = 23$

`p = 23`

1739. ⠟ ⠭ 29

$q = 29$

`q = 29`

1740. ⠝ ⠭ ⠏⠟ ⠭ 667

$n = pq = 667$

`n = pq = 667`

1741. ⠨⠋⠷⠝⠾ ⠭ ⠍ ⠭ ⠷⠏⠤⠂⠾⠷⠟⠤⠂⠾ ⠭ 616

$\phi(n) = m = (p - 1)(q - 1) = 616$

`\phi(n) = m = (p - 1)(q - 1) = 616`

1742. ⠠⠑ ⠭ 487

$E = 487$

`E = 487`

1743. ⠛⠉⠙⠷616⠠487⠾ ⠭ ⠂

$\gcd(616, 487) = 1$

`\gcd(616, 487) = 1`

1744. 25⠘487⠐⠍⠕⠙667 ⠭ 169

$25^{487} \bmod 667 = 169$

`25^{487} \bmod 667 = 169`

1745. 191⠠⠑ ⠭ ⠂⠬151⠍

$191 E = 1 + 151 m$

`191 E = 1 + 151 m`

1746. ⠷⠝⠠⠠⠙⠾ ⠭ ⠷667⠠191⠾

$(n, D) = (667, 191)$

`(n, D) = ( 667, 191)`

1747. 169⠘191⠐⠍⠕⠙667 ⠭ 25

$169^{191} \bmod 667 = 25$

`169^{191} \bmod 667 = 25`

1748. ⠠⠙⠠⠑ ⠿ ⠂ ⠷⠍⠕⠙⠍⠾

$DE \equiv 1 \pmod{ m}$

`DE \equiv 1 \pmod{ m}`

1749. ⠠⠙⠠⠑ ⠭ ⠅⠍⠬⠂ ⠭ ⠅⠨⠋⠷⠝⠾⠬⠂

$DE = km + 1 = k \phi(n) + 1$

`DE = km + 1 = k \phi(n) + 1`

1750. ⠛⠉⠙⠷⠭⠠⠝⠾ ⠭ ⠂

$\gcd(x, n) = 1$

`\gcd(x, n) = 1`

1751. ⠽⠘⠠⠙⠐ ⠶ ⠷⠭⠘⠠⠑⠐⠾⠘⠠⠙⠐ ⠶ ⠭⠘⠠⠙⠠⠑⠐ ⠶ ⠭⠘⠅⠍⠬⠂⠐ ⠶ ⠷⠭⠘⠨⠋⠷⠝⠾⠐⠾⠘⠅⠐⠭ ⠶

$y^D = (x^E)^D = x^{DE} = x^{km+1} = (x^{\phi(n)})^k x = (1)^k x = x \bmod n$

```
y^D = (x^E)^D = x^{DE} = x^{km + 1} = (x^{\phi(n)})^k x = (1)^k
x = x \bmod n
```

1752. ⠽⠘⠠⠙⠐⠍⠕⠙⠝

$y^D \bmod n$

```
y^D \bmod n
```

1753. ⠛⠉⠙⠷⠭⠠⠝⠾ ⠌⠶ ⠂

$\gcd(x, n) \neq 1$

```
\gcd(x, n) \neq 1
```

1754. ⠝ ⠶ ⠏⠟

$n = pq$

```
n = pq
```

1755. ⠭ ⠪ ⠝

$x < n$

```
x \lt n
```

1756. ⠗ ⠪ ⠟

$r < q$

```
r \lt q
```

1757. ⠭ ⠶ ⠗⠏

$x = rp$

```
x = rp
```

1758. ⠛⠉⠙⠷⠭⠠⠟⠾ ⠶ ⠂

$\gcd(x, q) = 1$

```
\gcd(x, q) = 1
```

1759. ⠍ ⠶ ⠨⠋⠷⠝⠾ ⠶ ⠷⠏⠤⠂⠾⠷⠟⠤⠂⠾ ⠶ ⠨⠋⠷⠏⠾⠨⠋⠷⠟⠾

$m = \phi(n) = (p - 1)(q - 1) = \phi(p)\phi(q)$

```
m=\phi(n)=(p - 1)(q - 1)=\phi(p)\phi(q)
```

1760. ⠭⠘⠅⠍⠐ ⠶ ⠭⠘⠅⠨⠋⠷⠏⠾⠨⠋⠷⠟⠾⠐ ⠶ ⠷⠭⠘⠨⠋⠷⠟⠾⠐⠾⠘⠅⠨⠋⠷⠏⠾⠐ ⠶ ⠷⠂⠾⠘⠅⠨⠋⠷⠏⠾⠐

$x^{km} = x^{k\phi(p)\phi(q)} = (x^{\phi(q)})^{k\phi(p)} = (1)^{k\phi(p)} = 1 \bmod q$

```
x^{km} = x^{k\phi(p)\phi(q)} = (x^{\phi(q)})^{k\phi(p)} = (1)^{k\phi(p)}
= 1 \bmod q
```

1761. ⠭⠘⠅⠍⠐ ⠶ ⠂⠬⠞⠟

$x^{km} = 1 + tq$

```
x^{km}=1 + tq
```

1762. ⠽⠘⠠⠙⠐ ⠶ ⠭⠘⠅⠍⠬⠂⠐ ⠶ ⠭⠘⠅⠍⠐⠭ ⠶ ⠷⠂⠬⠞⠟⠾⠭ ⠶ ⠭⠬⠞⠟⠷⠗⠏⠾ ⠶ ⠭⠬⠞⠗⠝ ⠶

$y^D = x^{km+1} = x^{km}x = (1 + tq)x = x + tq(rp) = x + trn = x \bmod n$

```
y^D = x^{km + 1} = x^{km} x = (1 + tq) x = x + tq(rp) = x + trn
= x \bmod n
```

1763. 667 ⠨⠅ 23⠡29

$667 = 23 \cdot 29$

667 = 23 \cdot 29

1764. ⠷⠝⠄⠠⠠⠑⠘⠄⠐⠾

(n', E')

(n', E')

1765. ⠷⠝⠄⠠⠠⠙⠘⠄⠐⠾

(n', D')

(n', D')

1766. ⠷⠝⠠⠠⠑⠾

(n, E)

(n, E)

1767. ⠷⠝⠠⠠⠙⠾

(n, D)

(n, D)

1768. ⠭⠄ ⠨⠅ ⠭⠘⠠⠙⠄⠐⠍⠕⠙⠝⠄

$x' = x^{D'} \bmod n'$

x' = x ^{D'} \bmod n'

1769. ⠭⠄

x'

x'

1770. ⠽⠄ ⠨⠅ ⠭⠄⠘⠠⠑⠐⠍⠕⠙⠝

$y' = {x'}^E \bmod n$

y' = {x'}^E \bmod n

1771. ⠼26⠯⠤⠂

$26! - 1$

26! - 1

1772. ⠛⠉⠙⠶⠙⠑⠞⠶⠠⠁⠾⠠26⠾ ⠨⠅ ⠂

$\gcd(\det(A), 26) = 1$

\gcd(\det(A), 26) = 1

1773. ⠠⠁ ⠨⠅ ⠠⠷⠼⠒ ⠼⠲ ⠠⠾ ⠠⠷⠼⠆ ⠼⠒ ⠠⠾

$A = \begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}$

A = \begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}

1774. ⠸⠃ ⠨⠅ ⠷⠆⠠⠢⠾⠘⠞

$\mathbf{b} = (2, 5)^{\mathsf{t}}$

{\mathbf b} = (2, 5)^\transpose

1775. ⠭ ⠶ 142528

$x = 142528$

x = 142528

1776. ⠼28

28

28

1777. ⠝ ⠶ 3551⠠⠠⠑ ⠶ 629⠠⠭ ⠶ 31

$n = 3551, E = 629, x = 31$

n = 3551, E = 629, x = 31

1778. ⠝ ⠶ 2257⠠⠠⠑ ⠶ 47⠠⠭ ⠶ 23

$n = 2257, E = 47, x = 23$

n = 2257, E = 47, x = 23

1779. ⠝ ⠶ 120979⠠⠠⠑ ⠶ 13251⠠⠭ ⠶ 142371

$n = 120979, E = 13251, x = 142371$

n = 120979, E = 13251, x = 142371

1780. ⠝ ⠶ 45629⠠⠠⠑ ⠶ 781⠠⠭ ⠶ 231561

$n = 45629, E = 781, x = 231561$

n = 45629, E = 781, x = 231561

1781. ⠼2791

2791

2791

1782. 11213525032442

11213525032442

112135 25032 442

1783. ⠝ ⠶ 3551⠠⠠⠙ ⠶ 1997⠠⠽ ⠶ 2791

$n = 3551, D = 1997, y = 2791$

n = 3551, D = 1997, y = 2791

1784. ⠝ ⠶ 5893⠠⠠⠙ ⠶ 81⠠⠽ ⠶ 34

$n = 5893, D = 81, y = 34$

n = 5893, D = 81, y = 34

1785. ⠝ ⠶ 120979⠠⠠⠙ ⠶ 27331⠠⠽ ⠶ 112135

$n = 120979, D = 27331, y = 112135$

n = 120979, D = 27331, y = 112135

1786. ⠝ ⠶ 79403⠠⠠⠙ ⠶ 671⠠⠽ ⠶ 129381

$n = 79403, D = 671, y = 129381$

n = 79403, D = 671, y = 129381

1787. ⠼31

31

31

1788. ⠷⠝⠠⠠⠑⠾ ⠨⠅ ⠷451⠠231⠾

$(n, E) = (451, 231)$

(n, E) = (451, 231)

1789. ⠷⠝⠠⠠⠑⠾ ⠨⠅ ⠷3053⠠1921⠾

$(n, E) = (3053, 1921)$

(n, E) = (3053, 1921)

1790. ⠷⠝⠠⠠⠑⠾ ⠨⠅ ⠷37986733⠠12371⠾

$(n, E) = (37986733, 12371)$

(n, E) = (37986733, 12371)

1791. ⠷⠝⠠⠠⠑⠾ ⠨⠅ ⠷16394854313⠠34578451⠾

$(n, E) = (16394854313, 34578451)$

(n, E) = (16394854313, 34578451)

1792. ⠝ ⠨⠅ 11⠡41

$n = 11 \cdot 41$

n = 11 \cdot 41

1793. ⠝ ⠨⠅ 8779⠡4327

$n = 8779 \cdot 4327$

n = 8779 \cdot 4327

1794. ⠠⠭⠘⠠⠑⠐⠸⠇⠠⠭⠷⠍⠕⠙⠝⠾

$X^E \equiv X \pmod{n}$

X^E \equiv X \pmod{n}

1795. ⠙ ⠨⠅ ⠆⠠⠼⠒⠠⠄⠠⠜⠝⠻

$d = 2, 3, \ldots, \sqrt{n}$

d = 2, 3, \ldots, \sqrt{n}

1796. ⠝ ⠨⠅ ⠁⠃

$n = ab$

n= ab

1797. ⠝ ⠨⠅ ⠭⠘⠆⠐⠤⠽⠘⠆⠐ ⠨⠅ ⠷⠭⠤⠽⠾⠷⠭⠬⠽⠾

$n = x^2 - y^2 = (x - y)(x + y)$

n = x^2 - y^2 = (x - y)(x + y)

1798. ⠝ ⠨⠅ ⠭⠘⠆⠐⠤⠽⠘⠆

$n = x^2 - y^2$

n = x^2 - y^2

1799. ⠛⠉⠙⠷⠁⠠⠏⠾ ⠨⠅ ⠂

$\gcd(a, p) = 1$

\gcd(a, p) = 1

1800. ⠁⠘⠏⠤⠂⠐⠸⠇⠂⠷⠍⠕⠙⠏⠾

$a^{p-1} \equiv 1 \pmod{p}$

a^{p-1} \equiv 1 \pmod{p}

1801. ⠆ ⠘15⠤⠂ ⠐⠿ ⠆ ⠘14⠐⠿ ⠲⠷⠍⠕⠙15⠾

$2^{15-1} \equiv 2^{14} \equiv 4 \pmod{15}$

`2^{15-1} \equiv 2^{14} \equiv 4 \pmod{15}`

1802. ⠼17

17

17

1803. ⠆ ⠘17⠤⠂ ⠐⠿ ⠆ ⠘16⠐⠿ ⠂ ⠷⠍⠕⠙17⠾

$2^{17-1} \equiv 2^{16} \equiv 1 \pmod{17}$

`2^{17-1} \equiv 2^{16} \equiv 1 \pmod{17}`

1804. ⠆ ⠘⠝⠤⠂ ⠐⠿ ⠂ ⠷⠍⠕⠙⠝⠾

$2^{n-1} \equiv 1 \pmod{n}$

`2^{n-1} \equiv 1 \pmod{n}`

1805. ⠼342

342

342

1806. ⠼811

811

811

1807. ⠼561

561

561

1808. ⠼771

771

771

1809. ⠼631

631

631

1810. ⠛⠉⠙⠷⠃ ⠠⠝⠾ ⠨⠅ ⠂

$\gcd(b, n) = 1$

`\gcd(b, n) = 1`

1811. ⠃ ⠘⠝⠤⠂ ⠐⠿ ⠂ ⠷⠍⠕⠙⠝⠾

$b^{n-1} \equiv 1 \pmod{n}$

`b^{n-1} \equiv 1 \pmod{n}`

1812. ⠼341

341

341

1813. ⠼2000

2000

2000

1814. 561 ⠶ ⠒⠡11⠡17

$561 = 3 \cdot 11 \cdot 17$

561 = 3 \cdot 11 \cdot 17

1815. 25⠡10⠘⠔

25×10^9

25 \times 10^9

1816. ⠼21

21

21

1817. ⠷⠏⠤⠂⠾⠷⠟⠤⠂⠾

$(p-1)(q-1)$

(p-1)(q-1)

1818. 128⠘⠲⠐ ⠶ 268⠠⠼435⠠⠼456

$128^4=268,435,456$

128^4=268,435,456

1819. 10⠘12

10^{12}

10^{12}

1820. ⠆⠡10⠘12

$2\cdot 10^{12}$

2\cdot 10^{12}

1821. ⠷⠭⠂ ⠠⠭⠆ ⠄⠄ ⠠⠭⠰⠝ ⠐⠾

$(x_{1}, x_{2}, \ldots, x_{n})$

(x_{1}, x_{2}, \ldots, x_{n})

1822. ⠒⠝

$3n$

3n

1823. ⠷⠭⠂ ⠠⠭⠆ ⠄⠄ ⠠⠭⠰⠝ ⠐⠾ ⠫⠳⠒⠒⠕ ⠷⠭⠂ ⠠⠭⠆ ⠄⠄ ⠠⠭⠰⠝ ⠐⠠⠭⠂ ⠠⠭⠆ ⠄⠄ ⠠⠭⠰⠝ ⠐⠠⠭⠂ ⠠⠭⠆ ⠄⠄ ⠠⠭⠰⠝ ⠐⠾

$(x_{1}, x_{2}, \ldots, x_{n}) \mapsto (x_{1}, x_{2}, \ldots, x_{n}, x_{1}, x_{2}, \ldots, x_{n}, x_{1}, x_{2}, \ldots, x_{n})$

(x_{1}, x_{2}, \ldots, x_{n}) \mapsto (x_{1}, x_{2}, \ldots, x_{n},
x_{1}, x_{2}, \ldots, x_{n}, x_{1}, x_{2}, \ldots, x_{n})

1824. ⠷0110⠾

(0110)

(0110)

1825. ⠷011001100110⠾

$(0110\; 0110\; 0110)$

(0110\; 0110\; 0110)

1826. ⠷011011100110⠾

$(0110\; 1110\; 0110)$

`(0110\; 1110\; 0110)`

1827. ⠆⠘⠦⠐ ⠨⠅ 256

$2^{8} = 256$

`2^{8} = 256`

1828. ⠆⠘⠶⠐ ⠨⠅ 128

$2^7 = 128$

`2^7 = 128`

1829. ⠼128

128

`128`

1830. ⠷01000101⠾

$(0100\; 0101)$

`(0100\; 0101)`

1831. ⠼32

32

`32`

1832. ⠷10011000⠾

$(1001\; 1000)$

`(1001\; 1000)`

1833. ⠷000⠾

(000)

`(000)`

1834. ⠷111⠾

(111)

`(111)`

1835. ⠷101⠾

(101)

`(101)`

1836. ⠼000

000

`000`

1837. ⠼001

001

`001`

1838. ⠼010

010

`010`

1839. ⠼011

011

011

1840. ⠼101

101

101

1841. ⠼110

110

110

1842. ⠼111

111

111

1843. ⠟ ⠶ ⠂⠤⠏

$q = 1 - p$

q = 1 - p

1844. ⠏⠘⠝

p^n

p^{n}

1845. ⠏ ⠶ 0.999

$p = 0.999$

p=0.999

1846. ⠷0.999⠾⠘⠼10⠠⠼000⠐⠠ ⡷⠭⠆ ⠆ ⠐⠆⠆⠄⠄ 0.00005

$(0.999)^{10,000} \approx 0.00005$

(0.999)^{10,000} \approx 0.00005

1847. ⠷⠭⠂ ⠄⠄ ⠄⠭⠆⠝⠐⠾

$(x_1, \ldots, x_n)$

(x_{1}, \ldots, x_{n})

1848. ⠝ ⠅ ⠟ ⠘⠅ ⠐⠏ ⠘⠝⠤⠅

$\binom{n}{k} q^k p^{n-k}$

\binom{n}{k} q^kp^{n - k}

1849. ⠝⠤⠅

$n - k$

n-k

1850. ⠟ ⠘⠅ ⠐⠏ ⠘⠝⠤⠅

$q^k p^{n-k}$

q^{k}p^{n-k}

1851. ⠝ ⠅ ⠶ ⠹⠝⠯⠌⠅⠯⠷⠝⠤⠅ ⠾⠯⠼

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$

\binom{n}{k} = \frac{n!}{k!(n - k)!}

1852. ⠹⠅⠿⠣ ⠠⠏ ⠳⠝⠤⠨

$\binom{n}{k}q^kp^{n-k}$

\binom{n}{k} q^{k}p^{n - k}

1853. ⠏ ⠭ 0.995

$p = 0.995$

p = 0.995

1854. ⠼500

500

500

1855. ⠏ ⠳⠝⠂ ⠭ ⠷0.995⠾⠈500⠐⠂⡷⠭⠆ ⠆ ⠒⠖⠄ 0.082

$p^n = (0.995)^{500} \approx 0.082$

p^{n} = (0.995)^{500} \approx 0.082

1856. ⠹⠂⠿⠏ ⠳⠝⠤⠂ ⠂ ⠭ 500⠷0.005⠾⠷0.995⠾⠈499⠐⠂⡷⠭⠆ ⠆ ⠒⠖⠄ 0.204

$\binom{n}{1}qp^{n-1} = 500(0.005)(0.995)^{499} \approx 0.204$

\binom{n}{1} qp^{n - 1}= 500(0.005)(0.995)^{499} \approx 0.204

1857. ⠹⠆⠿⠣ ⠠⠏ ⠳⠝⠤⠆ ⠂ ⠭ ⠹500⠈⠄499⠠⠆⠼⠷0.005⠾⠘⠆ ⠐⠷0.995⠾⠈498⠐⠂⡷⠭⠆ ⠆ ⠒⠖⠄ 0.257

$\binom{n}{2}q^2p^{n-2} = \frac{500 \cdot 499}{2}(0.005)^2(0.995)^{498} \approx 0.257$

\binom{n}{2} q^{2}p^{n - 2}= \frac{500 \cdot 499}{2}(0.005)^{2}(0.995)^{498} \approx 0.257

1858. ⠂ ⠤0.082⠤0.204⠤0.257 ⠭ 0.457

$1 - 0.082 - 0.204 - 0.257 = 0.457$

1 - 0.082 - 0.204 - 0.257 = 0.457

1859. ⠷⠝ ⠠⠍⠾

(n, m)

(n, m)

1860. ⠠⠑⠒⠈⠠⠵⠆ ⠳⠂ ⠫⠕ ⠈⠠⠵⠆ ⠳

$E : \mathbb{Z}_2^m \to \mathbb{Z}_2^n$

E:{\mathbb Z}^{m}_{2} \rightarrow {\mathbb Z}^{n}_{2}

1861. ⠠⠙⠒⠈⠠⠵⠆ ⠳⠂ ⠫⠕ ⠈⠠⠵⠆ ⠳

$D : \mathbb{Z}_2^n \to \mathbb{Z}_2^m$

D:{\mathbb Z}^{n}_{2} \rightarrow {\mathbb Z}^{m}_{2}

1862. ⠷⠦ ⠠⠶⠾

$(8, 7)$

(8,7)

1863. ⠠⠑⠷⠭⠶ ⠠⠭⠖ ⠤ ⠠⠭⠂⠾ ⠭ ⠷⠭⠦ ⠠⠭⠶ ⠤ ⠠⠭⠂⠾

$E(x_7, x_6, \ldots, x_1) = (x_8, x_7, \ldots, x_1)$

E(x_7, x_6, \ldots, x_1) = (x_8, x_7, \ldots, x_1)

1864. ⠭⠦ ⠨⠅ ⠭⠶⠬⠭⠖⠬⠄⠄⠄⠬⠭⠂

$x_8 = x_7 + x_6 + \cdots + x_1$

x_8 = x_7 + x_6 + \cdots + x_1

1865. ⠸⠭ ⠨⠅ ⠷⠭⠂⠠⠄⠠⠭⠰⠝⠐⠾

${\mathbf x} = (x_1, \ldots, x_n)$

{\mathbf x} = (x_1, \ldots, x_n)

1866. ⠸⠽ ⠨⠅ ⠷⠽⠂⠠⠄⠠⠽⠰⠝⠐⠾

${\mathbf y} = (y_1, \ldots, y_n)$

{\mathbf y} = (y_1, \ldots, y_n)

1867. ⠙⠷ ⠸⠭⠠⠸⠽⠾

$d({\mathbf x}, {\mathbf y})$

d({\mathbf x}, {\mathbf y})

1868. ⠸⠭

${\mathbf x}$

{\mathbf x}

1869. ⠸⠽

${\mathbf y}$

{\mathbf y}

1870. ⠙ ⠰⠍⠊⠝

$d_{\min}$

d_{\min}

1871. ⠺⠷ ⠸⠭⠾

$w({\mathbf x})$

w({\mathbf x})

1872. ⠺⠷ ⠸⠭⠾ ⠨⠅ ⠙⠷ ⠸⠭⠠⠸⠴⠾

$w({\mathbf x}) = d({\mathbf x}, {\mathbf 0})$

w({\mathbf x}) = d({\mathbf x}, {\mathbf 0})

1873. ⠸⠴ ⠨⠅ ⠷00⠄⠄⠄⠴⠾

${\mathbf 0} = (00 \cdots 0)$

{\mathbf 0} = (00 \cdots 0)

1874. ⠸⠭

$\mathbf x$

\mathbf x

1875. ⠸⠽

$\mathbf y$

\mathbf y

1876. ⠸⠭ ⠨⠅ ⠷10101⠾

${\mathbf x} = (10101)$

{\mathbf x} = (10101)

1877. ⠸⠽ ⠨⠅ ⠷11010⠾

$\mathbf{y} = (11010)$

{\mathbf y} = (11010)

1878. ⠸⠵ ⠨⠅ ⠷00011⠾

$\mathbf{z} = (00011)$

{\mathbf z} = (00011)

1879. ⠙⠷⠸⠭⠠⠸⠽⠾ ⠨⠅ ⠲⠠⠙⠷⠸⠭⠠⠸⠵⠾ ⠨⠅ ⠒⠠⠙⠷⠸⠽⠠⠸⠵⠾ ⠨⠅ ⠒

$d(\mathbf{x},\mathbf{y}) = 4, \qquad d(\mathbf{x},\mathbf{z}) = 3, \qquad d(\mathbf{y},\mathbf{z}) = 3$

d({\mathbf x},{\mathbf y}) = 4, \qquad d({\mathbf x},{\mathbf z})
= 3, \qquad d({\mathbf y},{\mathbf z}) = 3

1880. ⠺⠷⠸⠭⠾ ⠨⠅ ⠒⠠⠺⠷⠸⠽⠾ ⠨⠅ ⠒⠠⠺⠷⠸⠵⠾ ⠨⠅ ⠆

$w(\mathbf{x}) = 3, \qquad w(\mathbf{y}) = 3, \qquad w(\mathbf{z}) = 2$

w({\mathbf x}) = 3, \qquad w({\mathbf y}) = 3, \qquad w({\mathbf
z}) = 2

1881. ⠸⠵

$\mathbf{z}$

{\mathbf z}

1882. ⠺⠷⠸⠭⠾ ⠨⠅ ⠙⠷⠸⠭⠠⠸⠴⠾

$w(\mathbf{x}) = d(\mathbf{x}, \mathbf{0})$

w({\mathbf x}) = d({\mathbf x}, {\mathbf 0})

1883. ⠙⠷⠸⠭⠠⠸⠽⠾ ⠨⠂⠱ ⠴

$d(\mathbf{x},\mathbf{y}) \geq 0$

d({\mathbf x}, {\mathbf y}) \geq 0

1884. ⠙⠷⠸⠭⠠⠸⠽⠾ ⠨⠅ ⠴

$d(\mathbf{x},\mathbf{y}) = 0$

d({\mathbf x}, {\mathbf y}) = 0

1885. ⠸⠭ ⠨⠅ ⠸⠽

$\mathbf{x} = \mathbf{y}$

{\mathbf x} = {\mathbf y}

1886. ⠙⠷⠸⠭⠠⠸⠽⠾ ⠨⠅ ⠙⠷⠸⠽⠠⠸⠭⠾

$d(\mathbf{x},\mathbf{y}) = d(\mathbf{y},\mathbf{x})$

d({\mathbf x}, {\mathbf y})= d({\mathbf y}, {\mathbf x})

1887. ⠙⠷⠸⠭⠠⠸⠽⠾ ⠅⠱ ⠙⠷⠸⠭⠠⠸⠵⠾⠬⠙⠷⠸⠵⠠⠸⠽⠾

$d(\mathbf{x},\mathbf{y}) \leq d(\mathbf{x},\mathbf{z}) + d(\mathbf{z},\mathbf{y})$

d({\mathbf x}, {\mathbf y}) \leq d({\mathbf x}, {\mathbf z}) +
d({\mathbf z}, {\mathbf y})

1888. ⠸⠭ ⠨⠅ ⠷1101⠾

$\mathbf{x} = (1101)$

{\mathbf x} = (1101)

1889. ⠸⠽ ⠨⠅ ⠷1100⠾

$\mathbf{y} = (1100)$

{\mathbf y} = (1100)

1890. ⠷1101⠾

(1101)

(1101)

1891. ⠷1100⠾

(1100)

(1100)

1892. ⠙⠷⠸⠭⠠⠸⠽⠾ ⠨⠅ ⠂

$d(\mathbf{x}, \mathbf{y}) = 1$

d({\mathbf x}, {\mathbf y}) = 1

1893. ⠸⠭ ⠨⠅ ⠷1100⠾

$\mathbf{x} = (1100)$

{\mathbf x} = (1100)

1894. ⠸⠽ ⠨⠅ ⠷1010⠾

$\mathbf{y} = (1010)$

{\mathbf y} = (1010)

1895. ⠙⠷⠸⠭⠠⠸⠽⠾ ⠨⠅ ⠆

$d(\mathbf{x}, \mathbf{y}) = 2$

d({\mathbf x}, {\mathbf y}) = 2

1896. ⠼0000

0000

0000

1897. ⠼0011

0011

0011

1898. ⠼0101

0101

0101

1899. ⠼0110

0110

0110

1900. ⠼1001

1001

1001

1901. ⠼1010

1010

1010

1902. ⠼1100

1100

1100

1903. ⠼1111

1111

1111

1904. ⠙⠰⠍⠊⠝ ⠐ ⠨⠅ ⠆

$d_{\min} = 2$

d_{\min} = 2

1905. ⠙⠶⠸⠭⠠⠸⠵⠾ ⠨⠅ ⠙⠶⠸⠽⠠⠸⠵⠾ ⠨⠅ ⠂

$d(\mathbf{x}, \mathbf{z}) = d(\mathbf{y}, \mathbf{z}) = 1$

d({\mathbf x}, {\mathbf z}) = d({\mathbf y}, {\mathbf z}) = 1

1906. ⠙⠰⠍⠊⠝ ⠐ ⠨⠂⠱ ⠒

$d_{\min} \geq 3$

d_{\min} \geq 3

1907. ⠙⠶⠸⠵⠠⠸⠽⠾ ⠨⠂⠱ ⠆

$d(\mathbf{z}, \mathbf{y}) \geq 2$

d({\mathbf z}, {\mathbf y}) \geq 2

1908. ⠸⠵ ⠌⠨⠅ ⠸⠭

$\mathbf{z} \neq \mathbf{x}$

{\mathbf z} \neq {\mathbf x}

1909. ⠙⠰⠍⠊⠝ ⠐ ⠨⠅ ⠆⠝⠬⠂

$d_{\min} = 2n + 1$

d_{\min} = 2n + 1

1910. ⠙⠶⠸⠭⠠⠸⠽⠾ ⠐⠅⠱ ⠝

$d(\mathbf{x}, \mathbf{y}) \leq n$

d({\mathbf x}, {\mathbf y}) \leq n

1911. ⠆⠝⠬⠂ ⠐⠅⠱ ⠙⠶⠸⠭⠠⠸⠵⠾ ⠐⠅⠱ ⠙⠶⠸⠭⠠⠸⠽⠾⠬⠙⠶⠸⠽⠠⠸⠵⠾ ⠐⠅⠱ ⠝⠬⠙⠶⠸⠽⠠⠸⠵⠾

$2n + 1 \leq d(\mathbf{x}, \mathbf{z}) \leq d(\mathbf{x}, \mathbf{y}) + d(\mathbf{y}, \mathbf{z}) \leq n + d(\mathbf{y}, \mathbf{z})$

2n+1 \leq d({\mathbf x}, {\mathbf z}) \leq d({\mathbf x}, {\mathbf
y}) + d({\mathbf y}, {\mathbf z}) \leq n + d({\mathbf y}, {\mathbf
z})

1912. ⠙⠶⠸⠽⠠⠸⠵⠾ ⠨⠂⠱ ⠝⠬⠂

$d(\mathbf{y}, \mathbf{z}) \geq n + 1$

d({\mathbf y}, {\mathbf z}) \geq n+1

1913. ⠂ ⠐⠅⠱ ⠙⠶⠸⠭⠠⠸⠽⠾ ⠐⠅⠱ ⠆⠝

$1 \leq d(\mathbf{x}, \mathbf{y}) \leq 2n$

1 \leq d({\mathbf x}, {\mathbf y}) \leq 2n

1914. ⠆⠝⠬⠂

$2n + 1$

2n +1

1915. ⠸⠉⠂ ⠨⠅ ⠷00000⠾

${\mathbf c}_1 = (00000)$

{\mathbf c}_1 = (00000)

1916. ⠸⠉⠆ ⠨⠅ ⠷00111⠾

${\mathbf c}_2 = (00111)$

{\mathbf c}_2 = (00111)

1917. ⠸⠉⠒ ⠨⠅ ⠷11100⠾

${\mathbf c}_3 = (11100)$

{\mathbf c}_3 = (11100)

1918. ⠸⠉⠲ ⠨⠅ ⠷11011⠾

${\mathbf c}_4 = (11011)$

{\mathbf c}_4 = (11011)

1919. ⠼00000

00000

00000

1920. ⠼00111

00111

00111

1921. ⠼11100

11100

11100

1922. ⠼11011

11011

11011

1923. ⠷32⠠⠖⠾

$(32, 6)$

(32, 6)

1924. ⠸⠼⠴

${\mathbf 0}$

{\mathbf 0}

1925. ⠷11000101⠾⠬⠷11000101⠾ ⠨⠅ ⠷00000000⠾

$(11000101) + (11000101) = (00000000)$

(11000101) + (11000101) = (00000000)

1926. ⠈⠠⠵⠆⠘⠶

${\mathbb Z}_2^7$

{\mathbb Z}_2^7

1927. ⠙⠰⠍⠊⠝ ⠐ ⠨⠅ ⠒

$d_{\min} = 3$

d_{\min} = 3

1928. ⠺⠷⠸⠭⠬⠸⠽⠾ ⠨⠅ ⠙⠷⠸⠭⠠⠸⠽⠾

$w(\mathbf{x} + \mathbf{y}) = d(\mathbf{x}, \mathbf{y})$

w({\mathbf x} + {\mathbf y}) = d({\mathbf x}, {\mathbf y})

1929. ⠙⠰⠍⠊⠝ ⠐ ⠨⠅ ⠍⠊⠝ ⠨⠷⠺⠷⠸⠭⠾⠒ ww⠷⠸⠭⠾⠒ ⠌⠨⠅ ⠌⠨⠅ ⠸⠼⠴

$d_{\min} = \min\{w(\mathbf{x}) : \mathbf{x} \neq \mathbf{0}\}$

d_{\min} = \min\{ w({\mathbf x}) : { {\mathbf x} \neq {\mathbf 0}
} \}

1930. ⠸⠭⠡⠸⠽ ⠨⠅ ⠭⠂⠽⠂⠬⠄⠄⠄⠬⠭⠰⠝⠐⠽⠰⠝

$\mathbf{x} \cdot \mathbf{y} = x_1 y_1 + \cdots + x_n y_n$

{\mathbf x} \cdot {\mathbf y} = x_1 y_1 + \cdots + x_n y_n

1931. ⠸⠭ ⠨⠅ ⠷⠭⠂⠠⠭⠆⠠⠄⠄⠄⠠⠭⠰⠝⠐⠾⠘⠞

$\mathbf{x} = (x_1, x_2, \ldots, x_n)^{\mathsf{t}}$

{\mathbf x} = (x_1, x_2, \ldots, x_n)^\transpose

1932. ⠸⠽ ⠨⠅ ⠷⠽⠂⠠⠽⠆⠠⠄⠄⠄⠠⠽⠰⠝⠐⠾⠘⠞

$\mathbf{y} = (y_1, y_2, \ldots, y_n)^{\mathsf{t}}$

{\mathbf y} = (y_1, y_2, \ldots, y_n)^\transpose

1933. ⠸⠭ ⠨⠅ ⠷011001⠾⠘⠞

$\mathbf{x} = (011001)^{\mathsf{t}}$

{\mathbf x} = (011001)^\transpose

1934. ⠸⠽ ⠨⠅ ⠷110101⠾⠘⠞

$\mathbf{y} = (110101)^{\mathsf{t}}$

{\mathbf y} = (110101)^\transpose

1935. ⠸⠭⠡⠸⠽ ⠨⠅ ⠴

$\mathbf{x} \cdot \mathbf{y} = 0$

{\mathbf x} \cdot {\mathbf y} = 0

1936. ⠭⠂⠬⠭⠆⠬⠄⠄⠄⠬⠭⠰⠝⠐ ⠨⠅ ⠴

$x_1 + x_2 + \cdots + x_n = 0$

x_1 + x_2 + \cdots + x_n = 0

1937. ⠸⠭ ⠨⠅ ⠷⠭⠂⠠⠭⠆⠠⠭⠒⠠⠭⠲⠾⠘⠞

$\mathbf{x} = (x_1, x_2, x_3, x_4)^{\mathsf{t}}$

{\mathbf x} = (x_1, x_2, x_3, x_4)^\transpose

1938. ⠭⠂⠬⠭⠆⠬⠭⠒⠬⠭⠲ ⠨⠅ ⠴

$x_1 + x_2 + x_3 + x_4 = 0$

x_1+ x_2+ x_3+ x_4 = 0

1939. ⠸⠭⠡⠸⠼⠂ ⠨⠅ ⠸⠭⠘⠞⠐⠸⠼⠂ ⠨⠅ ⠠⠷⠭⠂ ⠭⠆ ⠭⠒ ⠭⠲⠠⠾⠂ ⠂ ⠂ ⠂ ⠨⠅ ⠴

$$\mathbf{x} \cdot \mathbf{1} = \mathbf{x}^{\mathsf{t}} \mathbf{1} = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = 0$$

{\mathbf x} \cdot {\mathbf 1} = {\mathbf x}^\transpose {\mathbf
1} = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \end{pmatrix} \begin{pmatrix}
1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = 0

1940. ⠈ ⠠⠍ ⠰⠍⠡⠠⠝ ⠐⠷ ⠈ ⠠⠵⠆ ⠾

$$\mathbb{M}_{m \times n}(\mathbb{Z}_2)$$

{\mathbb M}_{m \times n}({\mathbb Z}_2)

1941. ⠠⠓ ⠈⠑ ⠈ ⠠⠍ ⠰⠍⠡⠠⠝ ⠐⠷ ⠈ ⠠⠵⠆ ⠾

$$H \in \mathbb{M}_{m \times n}(\mathbb{Z}_2)$$

H \in {\mathbb M}_{m \times n}({\mathbb Z}_2)

1942. ⠠⠓⠸⠭ ⠨⠅ ⠸⠴

$$H\mathbf{x} = \mathbf{0}$$

H{\mathbf x} = {\mathbf 0}

1943. ⠠⠝⠥⠇ ⠇ ⠷ ⠠⠓⠾

$$\mathrm{Null}(H)$$

\Null(H)

1944. ⠈ ⠠⠵⠆

$$\mathbb{Z}_2$$

\mathbb Z_2

1945. ⠠⠓ ⠨⠅ ⠠⠷⠼⠴ ⠼⠂ ⠼⠴ ⠼⠂ ⠼⠴⠠⠾⠠⠷⠼⠂ ⠼⠂ ⠼⠂ ⠼⠂ ⠼⠴⠠⠾⠠⠷⠼⠴ ⠼⠴ ⠼⠂ ⠼⠂ ⠼⠂⠠⠾

$$H = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

H = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0
& 0 & 1 & 1 & 1 \end{pmatrix}

1946. ⠸⠭ ⠨⠅ ⠷⠭⠂ ⠠⠭⠆ ⠠⠭⠒ ⠠⠭⠲ ⠠⠭⠢⠾⠘⠞

$$\mathbf{x} = (x_1, x_2, x_3, x_4, x_5)^{\mathsf{t}}$$

{\mathbf x} = (x_1, x_2, x_3, x_4, x_5)^\transpose

1947. ⠷00000⠾⠷11110⠾⠷10101⠾⠷01011⠾

$$(00000) \qquad (11110) \qquad (10101) \qquad (01011)$$

(00000) \qquad (11110) \qquad (10101) \qquad (01011)

1948. ⠸⠭⠠⠸⠽ ⠈⠑ ⠠⠝⠥⠇ ⠇ ⠷ ⠠⠓⠾

$$\mathbf{x}, \mathbf{y} \in \mathrm{Null}(H)$$

{\mathbf x}, {\mathbf y} \in \Null(H)

1949. ⠠⠓⠸⠽ ⠨⠅ ⠸⠴

$$H\mathbf{y} = \mathbf{0}$$

H{\mathbf y} = {\mathbf 0}

1950. ⠠⠓⠷⠸⠭⠬⠸⠽⠾ ⠶ ⠠⠓⠸⠭⠬⠠⠓⠸⠽ ⠶ ⠸⠴⠬⠸⠴ ⠶ ⠸⠴

$H(\mathbf{x}+\mathbf{y}) = H\mathbf{x} + H\mathbf{y} = \mathbf{0} + \mathbf{0} = \mathbf{0}$

```
H({\mathbf x}+{\mathbf y}) = H{\mathbf x} + H{\mathbf y} = {\mathbf
0} + {\mathbf 0} = {\mathbf 0}
```

1951. ⠸⠭⠬⠸⠽

$\mathbf{x}+\mathbf{y}$

```
{\mathbf x} + {\mathbf y}
```

1952. ⠠⠓ ⠶ ⠠⠷⠼⠴ ⠼⠴ ⠼⠴ ⠼⠂ ⠼⠂ ⠼⠂ ⠠⠾ ⠠⠷⠼⠴ ⠼⠂ ⠼⠂ ⠼⠴ ⠼⠂ ⠼⠂ ⠠⠾
⠠⠷⠼⠂ ⠼⠴ ⠼⠂ ⠼⠴ ⠼⠴ ⠼⠂ ⠠⠾

$$H = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

```
H = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 &
1 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}
```

1953. ⠸⠭ ⠶ ⠷010011⠾ ⠘⠜

$\mathbf{x} = (010011)^{\mathsf{t}}$

```
{\mathbf x} = (010011)^\transpose
```

1954. ⠠⠓⠸⠭ ⠶ ⠴ ⠂ ⠂

$$H\mathbf{x} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

```
H{\mathbf x} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}
```

1955. ⠝ ⠖ ⠍

$n > m$

```
n \gt m
```

1956. ⠍⠡⠍

$m \times m$

```
m \times m
```

1957. ⠠⠊⠰⠍

I_m

```
I_m
```

1958. ⠠⠓ ⠶ ⠷⠠⠁⠳⠠⠊⠰⠍⠐⠾

$H = (A \mid I_m)$

```
H= (A \mid I_m)
```

1959. ⠍⠡⠷⠝⠤⠍⠾

$m \times (n - m)$

```
m \times (n-m)
```

1960. ⠠⠷⠁ 11 ⠁ 12 ⠄⠄⠄ ⠁ ⠰⠼⠂⠠⠝⠤⠍⠠⠾ ⠠⠷⠁ 21 ⠁ 22 ⠄⠄⠄ ⠁ ⠰⠼⠆⠠⠝⠤⠍⠠⠾
⠠⠷⠣⠄⠄⠄ ⠣⠄⠄⠄ ⠰⠄⠄⠄ ⠣⠄⠄⠄⠠⠾ ⠠⠷⠁ ⠰⠍Baseline⠂ ⠁ ⠰⠍Baseline⠆ ⠄⠄⠄ ⠁ ⠰⠍⠠⠝⠤⠍⠠⠾

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1,n-m} \\ a_{21} & a_{22} & \cdots & a_{2,n-m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{m,n-m} \end{pmatrix}$$

```
\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1,n-m} \\ a_{21} &
a_{22} & \cdots & a_{2,n-m} \\ \vdots & \vdots & \ddots & \vdots
\\ a_{m1} & a_{m2} & \cdots & a_{m,n-m} \end{pmatrix}
```

1961. ⠠⠷⠼⠂ ⠼⠴ ⠄⠄⠄ ⠼⠴⠠⠾ ⠠⠷⠼⠴ ⠼⠂ ⠄⠄⠄ ⠼⠴⠠⠾ ⠠⠷⠣⠄⠄⠄ ⠣⠄⠄⠄ ⠰⠄⠄⠄ ⠣⠄⠄⠄⠠⠾
⠠⠷⠼⠴ ⠼⠴ ⠄⠄⠄ ⠼⠂⠠⠾

$$\begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

```
\begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots
& \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}
```

1962. ⠝⠡⠷⠝⠤⠍⠾

$n \times (n-m)$

```
n \times (n-m)
```

1963. ⠠⠛ ⠶ ⠷⠹⠠⠊⠰⠝⠤⠍⠐⠌⠠⠁⠼⠾

$G = \left(\frac{I_{n-m}}{A} \right)$

```
G = \left( \frac{I_{n-m}}{A} \right)
```

1964. ⠠⠛ ⠸⠭ ⠶ ⠸⠽

$G {\mathbf x} = {\mathbf y}$

```
G {\mathbf x} = {\mathbf y}
```

1965. ⠷000⠾ ⠠⠷001⠾ ⠠⠷010⠾ ⠤ ⠠⠷111⠾

$(000), (001), (010), \ldots, (111)$

```
(000), (001), (010), \ldots, (111)
```

1966. ⠠⠁ ⠶ ⠠⠷⠼⠴ ⠼⠂ ⠼⠂⠠⠾ ⠠⠷⠼⠂ ⠼⠂ ⠼⠴⠠⠾ ⠠⠷⠼⠂ ⠼⠴ ⠼⠂⠠⠾

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

```
A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}
```

1967. ⠠⠛ ⠶ ⠠⠷⠼⠂ ⠼⠴ ⠼⠴⠠⠾ ⠠⠷⠼⠴ ⠼⠂ ⠼⠴⠠⠾ ⠠⠷⠼⠴ ⠼⠴ ⠼⠂⠠⠾
⠠⠷⠼⠴ ⠼⠂ ⠼⠂⠠⠾ ⠠⠷⠼⠂ ⠼⠂ ⠼⠴⠠⠾ ⠠⠷⠼⠂ ⠼⠴ ⠼⠂⠠⠾

$$G = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

```
G= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1
& 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}
```

1968. ⠠⠓ ⠨⠅ ⠠⠷⠼⠴ ⠼⠂ ⠼⠂ ⠼⠂ ⠼⠴ ⠼⠴⠠⠾⠠⠷⠼⠂ ⠼⠂ ⠼⠴ ⠼⠴ ⠼⠂ ⠼⠴⠠⠾
⠠⠷⠼⠂ ⠼⠴ ⠼⠂ ⠼⠴ ⠼⠴ ⠼⠂⠠⠾

$$H = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

```
H = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 &
0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}
```

1969. ⠸⠭ ⠨⠅ ⠷⠭⠂⠠⠭⠆⠠⠭⠒⠠⠭⠲⠠⠭⠢⠠⠭⠖⠾

$${\mathbf x} = (x_1, x_2, x_3, x_4, x_5, x_6)$$

```
{\mathbf x} = (x_1, x_2, x_3, x_4, x_5, x_6)
```

1970. ⠸⠴ ⠨⠅ ⠠⠓⠸⠭ ⠨⠅ ⠭⠆⠬⠭⠒⠬⠭⠲ ⠭⠂⠬⠭⠆⠬⠭⠢ ⠭⠂⠬⠭⠒⠬⠭⠖

$${\mathbf 0} = H{\mathbf x} = \begin{pmatrix} x_2 + x_3 + x_4 \\ x_1 + x_2 + x_5\\ x_1 + x_3 + x_6 \end{pmatrix}$$

```
{\mathbf 0} = H{\mathbf x} = \begin{pmatrix} x_2 + x_3 + x_4 \\
x_1 + x_2 + x_5\\ x_1 + x_3 + x_6 \end{pmatrix}
```

1971. ⠭⠲

x_4

```
x_4
```

1972. ⠭⠒

x_3

```
x_3
```

1973. ⠭⠢

x_5

```
x_5
```

1974. ⠭⠖

x_6

```
x_6
```

1975. StartLayout1stRow⠷000000⠾ ⠷001101⠾ ⠷010110⠾ ⠷011011⠾2ndRow⠷100011⠾ ⠷101

$$\begin{array}{cccc} (000000) & (001101) & (010110) & (011011) \\ (100011) & (101110) & (110101) & (111000). \end{array}$$

```
\begin{array}{cccc} (000000) & (001101) & (010110) & (011011) \\
(100011) & (101110) & (110101) & (111000). \end{array}
```

1976. ⠠⠛ ⠸⠭

$G \mathbf x$

```
G \mathbf x
```

1977. ⠼000000

000000

```
000000
```

1978. ⠼001101

001101

```
001101
```

1979. ⠼010110

010110

`010110`

1980. ⠼011011

011011

`011011`

1981. ⠼100011

100011

`100011`

1982. ⠼101110

101110

`101110`

1983. ⠼110101

110101

`110101`

1984. ⠼111000

111000

`111000`

1985. ⠸⠭ ⠈⠑ ⠈⠠⠵⠆ ⠘⠝

${\mathbf x} \in {\mathbb Z}_2^n$

`{\mathbf x} \in {\mathbb Z}_2^n`

1986. ⠝⠤⠍

$n-m$

`n-m`

1987. ⠷⠝⠠⠝⠤⠍⠾

$(n, n-m)$

`(n, n-m)`

1988. ⠝⠤⠍

$n - m$

`n - m`

1989. ⠝⠡⠅

$n \times k$

`n \times k`

1990. ⠠⠉ ⠨⠅ StartSet ⠸⠽⠱⠠⠛⠸⠭ ⠨⠅ ⠸⠽for ⠸⠭ ⠈⠑ ⠈⠠⠵⠆ ⠘⠅ ⠐EndSet

$C = \left\{{\mathbf y} : G{\mathbf x} ={\mathbf y}\text{ for }{\mathbf x}\in {\mathbb Z}_2^k\right\}$

`C = \left\{{\mathbf y} : G{\mathbf x} ={\mathbf y}\text{ for }{\mathbf x}\in {\mathbb Z}_2^k\right\}`

1991. ⠷⠝⠠⠅⠾

(n,k)

(n,k)

1992. ⠠⠛⠸⠭⠂ ⠨⠅ ⠸⠽⠂

$G {\mathbf x}_1 = {\mathbf y}_1$

G {\mathbf x}_1 = {\mathbf y}_1

1993. ⠠⠛⠸⠭⠆ ⠨⠅ ⠸⠽⠆

$G {\mathbf x}_2 ={\mathbf y}_2$

G {\mathbf x}_2 ={\mathbf y}_2

1994. ⠸⠽⠂⠬⠸⠽⠆

${\mathbf y}_1 + {\mathbf y}_2$

{\mathbf y}_1 + {\mathbf y}_2

1995. ⠠⠛⠷⠸⠭⠂⠬⠸⠭⠆⠾ ⠨⠅ ⠠⠛⠸⠭⠂⠬⠠⠛⠸⠭⠆ ⠨⠅ ⠸⠽⠂⠬⠸⠽⠆

$G({\mathbf x}_1 + {\mathbf x}_2) = G {\mathbf x}_1 + G {\mathbf x}_2 = {\mathbf y}_1 + {\mathbf y}_2$

G({\mathbf x}_1 + {\mathbf x}_2) = G {\mathbf x}_1 + G {\mathbf x}_2 = {\mathbf y}_1 + {\mathbf y}_2

1996. ⠠⠛⠸⠭ ⠨⠅ ⠠⠛⠸⠽

$G {\mathbf x} = G {\mathbf y}$

G {\mathbf x} = G {\mathbf y}

1997. ⠠⠛⠸⠭⠤⠠⠛⠸⠽ ⠨⠅ ⠠⠛⠷⠸⠭⠤⠸⠽⠾ ⠨⠅ ⠸⠴

$G {\mathbf x} - G {\mathbf y} = G({\mathbf x} - {\mathbf y}) = {\mathbf 0}$

G {\mathbf x} - G {\mathbf y} = G({\mathbf x} - {\mathbf y}) = {\mathbf 0}

1998. ⠠⠛⠷⠸⠭⠤⠸⠽⠾

$G({\mathbf x} - {\mathbf y})$

G({\mathbf x} - {\mathbf y})

1999. ⠭⠂⠤⠽⠂⠠⠄⠠⠭⠰⠅⠐⠤⠽⠰⠅⠐

$x_1 -y_1, \ldots, x_k - y_k$

x_1 -y_1, \ldots, x_k - y_k

2000. ⠠⠊⠰⠅

I_k

I_k

2001. ⠠⠛⠷⠸⠭⠤⠸⠽⠾ ⠨⠅ ⠸⠴

$G({\mathbf x} - {\mathbf y}) = {\mathbf 0}$

G({\mathbf x} - {\mathbf y}) = {\mathbf 0}

2002. ⠠⠓ ⠨⠅ ⠷⠠⠁⠳⠠⠊⠰⠍⠐⠾

$H = (A \mid I_m)$

H = (A \mid I_m)

2003. ⠠⠛⠀⠨⠅⠀⠷⠹⠠⠊⠰⠝⠤⠍⠐⠌⠠⠁⠼⠾

$$G = \left(\frac{I_{n-m}}{A} \right)$$

G = \left(\frac{I_{n-m} }{A} \right)

2004. ⠠⠓⠠⠛⠀⠨⠅⠀⠸⠴

$$HG = \mathbf{0}$$

HG = {\mathbf 0}

2005. ⠠⠉⠀⠨⠅⠀⠠⠓⠠⠛

$$C = HG$$

C = HG

2006. ⠊⠚

$$ij$$

ij

2007. ⠨⠙⠰⠊⠚⠐⠀⠨⠅⠀StartLayoutEnlarged ⠨⠾1stRow ⠨⠾⠼⠂⠀⠊⠀⠨⠅⠀⠚2ndRow ⠨⠾⠼⠴⠀⠊⠀⠌⠨⠅⠀⠚End

$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}

2008. ⠠⠓⠸⠽⠀⠨⠅⠀⠸⠴

$$H\mathbf{y} = \mathbf{0}$$

H {\mathbf y} = {\mathbf 0}

2009. ⠸⠽⠀⠈⠑⠀⠠⠉

$$\mathbf{y} \in C$$

{\mathbf y} \in C

2010. ⠸⠭⠀⠈⠑⠀⠈⠠⠵⠆⠘⠍

$$\mathbf{x} \in \mathbb{Z}_2^m$$

{\mathbf x} \in {\mathbb Z}_2^m

2011. ⠠⠓⠸⠽⠀⠨⠅⠀⠠⠓⠠⠛⠸⠭⠀⠨⠅⠀⠸⠴

$$H\mathbf{y} = HG\mathbf{x} = \mathbf{0}$$

H {\mathbf y} = HG {\mathbf x} = {\mathbf 0}

2012. ⠸⠽⠀⠨⠅⠀⠷⠽⠂⠠⠀⠄⠠⠀⠽⠰⠝⠐⠾⠘⠞

$$\mathbf{y} = (y_1, \ldots, y_n)^{\mathsf{t}}$$

{\mathbf y} = (y_1, \ldots, y_n)^\transpose

2013. ⠈⠠⠵⠆⠘⠝⠤⠍

$$\mathbb{Z}_2^{n-m}$$

{\mathbb Z}_2^{n-m}

2014. ⠠⠛⠸⠭⠘⠞⠐⠀⠨⠅⠀⠸⠽

$$G\mathbf{x}^{\mathsf{t}} = \mathbf{y}$$

G {\mathbf x}^\transpose = {\mathbf y}

2015. ⠽⠰⠝⠤⠍⠬⠂⠐ ⠠⠤ ⠠⠽⠰⠝⠐

$y_{n-m+1}, \ldots, y_n$

y_{n-m+1}, \ldots, y_n

2016. ⠽⠂ ⠤ ⠠⠽⠰⠝⠤⠍⠐

$y_1, \ldots, y_{n-m}$

y_1, \ldots, y_{n-m}

2017. ⠭⠰⠊⠐ ⠶ ⠽⠰⠊

$x_i = y_i$

x_i = y_i

2018. ⠊ ⠶ ⠂⠠⠤ ⠠⠝⠤⠍

$i = 1, \ldots, n - m$

i= 1, \ldots, n - m

2019. ⠠⠓⠸⠑⠰⠊

$H\mathbf{e}_i$

H{\mathbf e}_i

2020. ⠠⠷⠂⠂ ⠂⠂ ⠂⠂ ⠴⠴ ⠴⠴⠠⠾ ⠠⠷⠂⠂ ⠴⠴ ⠴⠴ ⠂⠂ ⠴⠴⠠⠾ ⠠⠷⠂⠂ ⠂⠂ ⠴⠴ ⠴⠴ ⠂⠂ ⠠⠾⠴ ⠐ ⠴⠴ ⠴⠴ ⠴ ⠶ ⠐ ⠴⠐

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

\begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}

2021. ⠸⠑⠰⠊

$\mathbf{e}_i$

{\mathbf e}_i

2022. ⠊ ⠶ ⠂⠠⠤ ⠠⠝

$i = 1, \ldots, n$

i = 1, \ldots, n

2023. ⠠⠓⠸⠑⠰⠊⠐ ⠌⠶ ⠸⠴

$H\mathbf{e}_i \neq \mathbf{0}$

H{\mathbf e}_i \neq {\mathbf 0}

2024. ⠠⠓⠆ ⠶ ⠠⠷⠂⠂ ⠂⠂ ⠂⠂ ⠴⠴ ⠴⠴⠠⠾ ⠠⠷⠂⠂ ⠴⠴ ⠴⠴ ⠂⠂ ⠴⠴⠠⠾ ⠠⠷⠂⠂ ⠂⠂ ⠴⠴ ⠴⠴ ⠂⠂ ⠠⠾

$$H_1 = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

H_1 = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix}

2025. ⠠⠓⠆ ⠨⠅ ⠠⠷⠼⠂ ⠼⠂ ⠼⠂ ⠼⠴ ⠼⠴⠠⠾ ⠠⠷⠼⠂ ⠼⠴ ⠼⠴ ⠼⠴ ⠼⠴⠠⠾
⠠⠷⠼⠂ ⠼⠂ ⠼⠴ ⠼⠴ ⠼⠂⠠⠾

$$H_2 = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

H_2 = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 1 \end{pmatrix}

2026. ⠠⠓⠂

H_1

H_1

2027. ⠠⠓⠆

H_2

H_2

2028. ⠠⠓ ⠨⠅ ⠠⠷⠼⠂ ⠼⠂ ⠼⠂ ⠼⠴⠠⠾ ⠠⠷⠼⠂ ⠼⠴ ⠼⠴ ⠼⠂⠠⠾ ⠠⠷⠼⠂ ⠼⠂ ⠼⠴ ⠼⠴⠠⠾

$$H = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{pmatrix}$$

H = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0
& 0 \end{pmatrix}

2029. ⠷1010⠾

(1010)

(1010)

2030. ⠷1001⠾

(1001)

(1001)

2031. ⠷0101⠾

(0101)

(0101)

2032. ⠷0011⠾

(0011)

(0011)

2033. ⠸⠑⠰⠊⠐⠬⠸⠑⠰⠚

$\mathbf{e}_i + \mathbf{e}_j$

{\mathbf e}_{i} +{\mathbf e}_{j}

2034. ⠺⠷⠸⠑⠰⠊⠐⠬⠸⠑⠰⠚⠐⠾ ⠨⠅ ⠆

$w(\mathbf{e}_i + \mathbf{e}_j) = 2$

w({\mathbf e}_{i} +{\mathbf e}_{j}) = 2

2035. ⠸⠴ ⠨⠅ ⠠⠓⠷⠸⠑⠰⠊⠐⠬⠸⠑⠰⠚⠐⠾ ⠨⠅ ⠠⠓⠸⠑⠰⠊⠐⠬⠠⠓⠸⠑⠰⠚

$\mathbf{0} = H(\mathbf{e}_i + \mathbf{e}_j) = H\mathbf{e}_i + H\mathbf{e}_j$

{\mathbf 0} = H({\mathbf e}_{i} +{\mathbf e}_{j}) = H{\mathbf e}_{i}
+ H{\mathbf e}_{j}

2036. ⠆⠘⠒⠐ ⠭ ⠦

$$2^3 = 8$$

```
2^3 = 8
```

2037. ⠴ ⠴ ⠴ ⠠⠂ ⠴ ⠴ ⠠⠴ ⠂ ⠴ ⠠⠴ ⠴ ⠂

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

```
\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\
0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix},
\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}
```

2038. ⠆⠘⠍

$$2^m$$

```
2^m
```

2039. ⠸⠼⠴⠠⠸⠑⠂ ⠠⠄ ⠠⠸⠑⠰⠍⠐

$$\mathbf{0}, \mathbf{e}_1, \ldots, \mathbf{e}_m$$

```
{\mathbf 0}, {\mathbf e}_1, \ldots, {\mathbf e}_m
```

2040. ⠆⠘⠍⠐⠤⠷⠂⠬⠍⠾

$$2^m - (1 + m)$$

```
2^m - (1 + m)
```

2041. ⠠⠓ ⠭ ⠠⠷⠼⠂ ⠼⠂ ⠼⠂ ⠼⠴ ⠼⠴⠠⠾⠠⠷⠼⠴ ⠼⠂ ⠼⠴ ⠼⠂ ⠼⠴⠠⠾⠠⠷⠼⠂ ⠼⠴ ⠼⠴ ⠼⠴ ⠼⠂⠠⠾

$$H = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
H = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1
& 0 & 0 & 0 & 1 \end{pmatrix}
```

2042. ⠸⠭ ⠭ ⠷11011⠾⠘⠞

$$\mathbf{x} = (11011)^{\mathsf{t}}$$

```
{\mathbf x} = (11011)^\transpose
```

2043. ⠸⠽ ⠭ ⠷01011⠾⠘⠞

$$\mathbf{y} = (01011)^{\mathsf{t}}$$

```
{\mathbf y} = (01011)^\transpose
```

2044. ⠠⠓⠸⠭ ⠭ ⠴ ⠴ ⠴and⠠⠓⠸⠽ ⠭ ⠂ ⠴ ⠂

$$H\mathbf{x} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad \text{and} \qquad H\mathbf{y} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

```
H{\mathbf x} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad
\text{and} \qquad H{\mathbf y} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}
```

2045. ⠠⠓⠸⠽

$$H\mathbf{y}$$

```
H{\mathbf y}
```

2046. ⠠⠓⠸⠭

$H\mathbf{x}$

H{\mathbf x}

2047. ⠸⠭ ⠨⠅ ⠸⠉⠬⠸⠑

$\mathbf{x} = \mathbf{c} + \mathbf{e}$

{\mathbf x} = {\mathbf c} +{\mathbf e}

2048. ⠸⠉

$\mathbf{c}$

{\mathbf c}

2049. ⠸⠑

$\mathbf{e}$

{\mathbf e}

2050. ⠠⠓⠸⠭ ⠨⠅ ⠠⠓⠷⠸⠉⠬⠸⠑⠾ ⠨⠅ ⠠⠓⠸⠉⠬⠠⠓⠸⠑ ⠨⠅ ⠸⠴⠬⠠⠓⠸⠑ ⠨⠅ ⠠⠓⠸⠑

$H\mathbf{x} = H(\mathbf{c} + \mathbf{e}) = H\mathbf{c} + H\mathbf{e} = \mathbf{0} + H\mathbf{e} = H\mathbf{e}$

H{\mathbf x} = H({\mathbf c} +{\mathbf e}) = H{\mathbf c} + H{\mathbf
e} = {\mathbf 0} + H{\mathbf e} = H{\mathbf e}

2051. ⠠⠓⠸⠑

$H\mathbf{e}$

H{\mathbf e}

2052. ⠠⠓⠈⠑⠈⠠⠍⠰⠍⠡⠝⠐⠷⠈⠠⠵⠆⠾

$H \in \mathbb{M}_{m \times n}(\mathbb{Z}_2)$

H \in {\mathbb M}_{ m \times n} ({\mathbb Z}_2)

2053. ⠸⠗

$\mathbf{r}$

{\mathbf r}

2054. ⠠⠓ ⠨⠅ ⠠⠷⠼⠂ ⠼⠴ ⠼⠂ ⠼⠂ ⠼⠴ ⠼⠴⠠⠾ ⠠⠷⠼⠴ ⠼⠂ ⠼⠂ ⠼⠴ ⠼⠂ ⠼⠴⠠⠾
⠠⠷⠼⠂ ⠼⠂ ⠼⠂ ⠼⠴ ⠼⠴ ⠼⠂⠠⠾

$$H = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

H = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 &
0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}

2055. ⠸⠭ ⠨⠅ ⠷111110⠾⠘⠞

$\mathbf{x} = (111110)^{\mathsf{t}}$

{\mathbf x} = (111110)^\transpose

2056. ⠸⠽ ⠨⠅ ⠷111111⠾⠘⠞

$\mathbf{y} = (111111)^{\mathsf{t}}$

{\mathbf y} = (111111)^\transpose

2057. ⠸⠵ ⠨⠅ ⠷010111⠾⠘⠞

$\mathbf{z} = (010111)^{\mathsf{t}}$

{\mathbf z} = (010111)^\transpose

2058. ⠠⠓⠸⠭ ⠨⠅ ⠂ ⠂ ⠂ ⠠⠠⠓⠸⠽ ⠨⠅ ⠂ ⠂ ⠴ ⠠⠠⠓⠸⠵ ⠨⠅ ⠂ ⠴ ⠴

$H\mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, H\mathbf{y} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, H\mathbf{z} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

H{\mathbf x} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, H{\mathbf y} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, H{\mathbf z} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}

2059. ⠷110110⠾

(110110)

(110110)

2060. ⠷010011⠾

(010011)

(010011)

2061. ⠷⠝⠠⠍⠾

(n,m)

(n,m)

2062. ⠸⠭⠬⠠⠉

$\mathbf{x} + C$

{\mathbf x} + C

2063. ⠆⠘⠝⠤⠍

2^{n-m}

2^{n-m}

2064. ⠷⠢⠠⠒⠾

$(5,3)$

(5,3)

2065. ⠠⠓ ⠨⠅ ⠠⠷⠼⠴ ⠼⠂ ⠼⠂ ⠼⠴ ⠼⠴⠠⠾⠠⠷⠼⠂ ⠼⠴ ⠼⠴ ⠼⠂ ⠼⠴⠠⠾⠠⠷⠼⠂ ⠼⠂ ⠼⠴ ⠼⠴ ⠼⠂⠠⠾

$H = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix}$

H = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix}

2066. ⠷00000⠾⠷01101⠾⠷10011⠾⠷11110⠾

$(00000) \quad (01101) \quad (10011) \quad (11110)$

(00000) \quad (01101) \quad (10011) \quad (11110)

2067. ⠆⠘⠢⠤⠆⠐ ⠨⠅ ⠆⠘⠒

$2^{5-2} = 2^3$

2^{5-2} = 2^3

2068. ⠈⠠⠵⠆⠘⠢

$\mathbb{Z}_2^5$

```
{\mathbb Z}_2^5
```

2069. ⠆⠘⠆⠐ ⠨⠅ ⠲

$2^2 = 4$

```
2^2 =4
```

2070. ⠷00000⠾⠷01101⠾⠷10011⠾⠷11110⠾

$(00000)(01101)(10011)(11110)$

```
(00000) (01101) (10011) (11110)
```

2071. ⠷10000⠾⠬ ⠠⠉

$(10000) + C$

```
(10000) + C
```

2072. ⠷10000⠾⠷11101⠾⠷00011⠾⠷01110⠾

$(10000)(11101)(00011)(01110)$

```
(10000) (11101) (00011) (01110)
```

2073. ⠷01000⠾⠬ ⠠⠉

$(01000) + C$

```
(01000) + C
```

2074. ⠷01000⠾⠷00101⠾⠷11011⠾⠷10110⠾

$(01000)(00101)(11011)(10110)$

```
(01000) (00101) (11011) (10110)
```

2075. ⠷00100⠾⠬ ⠠⠉

$(00100) + C$

```
(00100) + C
```

2076. ⠷00100⠾⠷01001⠾⠷10111⠾⠷11010⠾

$(00100)(01001)(10111)(11010)$

```
(00100) (01001) (10111) (11010)
```

2077. ⠷00010⠾⠬ ⠠⠉

$(00010) + C$

```
(00010) + C
```

2078. ⠷00010⠾⠷01111⠾⠷10001⠾⠷11100⠾

$(00010)(01111)(10001)(11100)$

```
(00010) (01111) (10001) (11100)
```

2079. ⠷00001⠾⠬ ⠠⠉

$(00001) + C$

```
(00001) + C
```

2080. ⠷00001⠾⠷01100⠾⠷10010⠾⠷11111⠾

$(00001)(01100)(10010)(11111)$

```
(00001) (01100) (10010) (11111)
```

2081. ⠷10100⠾⠬⠠⠉

$(10100) + C$

(10100) + C

2082. ⠷00111⠾⠷01010⠾⠷10100⠾⠷11001⠾

$(00111)(01010)(10100)(11001)$

(00111) (01010) (10100) (11001)

2083. ⠷00110⠾⠬⠠⠉

$(00110) + C$

(00110) + C

2084. ⠷00110⠾⠷01011⠾⠷10101⠾⠷11000⠾

$(00110)(01011)(10101)(11000)$

(00110) (01011) (10101) (11000)

2085. ⠸⠗ ⠨⠅ ⠸⠑⠬⠸⠭

$\mathbf{r} = \mathbf{e} + \mathbf{x}$

{\mathbf r} = {\mathbf e} + {\mathbf x}

2086. ⠸⠭ ⠨⠅ ⠸⠑⠬⠸⠗

$\mathbf{x} = \mathbf{e} + \mathbf{r}$

{\mathbf x} = {\mathbf e} + {\mathbf r}

2087. ⠸⠑⠬⠠⠉

$\mathbf{e} + C$

{\mathbf e} + C

2088. ⠸⠗⠬⠸⠑

$\mathbf{r} + \mathbf{e}$

{\mathbf r} + {\mathbf e}

2089. ⠸⠗ ⠨⠅ ⠷01111⠾

$\mathbf{r} = (01111)$

{\mathbf r} = (01111)

2090. ⠷01101⠾ ⠨⠅ ⠷01111⠾⠬⠷00010⠾

$(01101) = (01111) + (00010)$

(01101) = (01111) + (00010)

2091. ⠷00000⠾

(00000)

(00000)

2092. ⠷001⠾

(001)

(001)

2093. ⠷00001⠾

(00001)

(00001)

2094. ⠷010⠾

(010)

`(010)`

2095. ⠷00010⠾

(00010)

`(00010)`

2096. ⠷011⠾

(011)

`(011)`

2097. ⠷10000⠾

(10000)

`(10000)`

2098. ⠷100⠾

(100)

`(100)`

2099. ⠷00100⠾

(00100)

`(00100)`

2100. ⠷01000⠾

(01000)

`(01000)`

2101. ⠷110⠾

(110)

`(110)`

2102. ⠷00110⠾

(00110)

`(00110)`

2103. ⠷10100⠾

(10100)

`(10100)`

2104. ⠠⠓⠸⠭ ⠨⠅ ⠠⠓⠸⠽

$H{\mathbf x} = H{\mathbf y}$

`H{\mathbf x} = H{\mathbf y}`

2105. ⠸⠭⠤⠸⠽ ⠈⠑ ⠠⠉

${\mathbf x} - {\mathbf y} \in C$

`{\mathbf x} - {\mathbf y} \in C`

2106. ⠠⠓⠷⠸⠭⠤⠸⠽⠾ ⠨⠅ ⠴

$H({\mathbf x} - {\mathbf y}) = 0$

`H({\mathbf x} - {\mathbf y}) = 0`

2107. ⠠⠓⠸⠭ ⠨⠅ ⠠⠓⠸⠽

$H\mathbf{x} = H\mathbf{y}$

```
H {\mathbf x} = H{\mathbf y}
```

2108. ⠸⠭ ⠨⠅ ⠷01111⠾

$\mathbf{x} = (01111)$

```
{\mathbf x} = (01111)
```

2109. ⠠⠓⠸⠭ ⠨⠅ ⠴ ⠂ ⠂

$H\mathbf{x} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

```
H {\mathbf x} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}
```

2110. ⠆ ⠘⠝⠤⠅

2^{n-k}

```
2^{n - k}
```

2111. ⠷32 ⠠24⠾

$(32, 24)$

```
(32, 24)
```

2112. ⠆ ⠘24

2^{24}

```
2^{24}
```

2113. ⠆ ⠘32⠤24⠐ ⠨⠅ ⠆ ⠘⠦⠐ ⠨⠅ 256

$2^{32-24} = 2^8 = 256$

```
2^{32 - 24} = 2^{8} = 256
```

2114. ⠈ ⠠⠵⠆ ⠘⠲

$\mathbb{Z}_2^4$

```
{\mathbb Z}_2^4
```

2115. ⠷0110⠾⠷1001⠾⠷1010⠾⠷1100⠾

$(0110) \quad (1001) \quad (1010) \quad (1100)$

```
(0110) \quad (1001) \quad (1010) \quad (1100)
```

2116. ⠷0000⠾⠠⠈ ⠉⠐ ⠠⠉

$(0000) \notin C$

```
(0000) \notin C
```

2117. ⠷011010⠾ ⠠⠷011100⠾

$(011010), (011100)$

```
(011010), (011100)
```

2118. ⠷11110101⠾ ⠠⠷01010100⠾

$(11110101), (01010100)$

```
(11110101), (01010100)
```

2119. ⠗00110⠾ ⠠⠗01111⠾

$(00110), (01111)$

```
(00110), (01111)
```

2120. ⠗1001⠾ ⠠⠗0111⠾

$(1001), (0111)$

```
(1001), (0111)
```

2121. ⠗011010⠾

(011010)

```
(011010)
```

2122. ⠗11110101⠾

(11110101)

```
(11110101)
```

2123. ⠗01111⠾

(01111)

```
(01111)
```

2124. ⠗1011⠾

(1011)

```
(1011)
```

2125. ⠗011010⠾⠗011100⠾⠗110111⠾⠗110000⠾

$(011010) \; (011100) \; (110111) \; (110000)$

```
(011010) \; (011100) \; (110111) \; (110000)
```

2126. ⠗011100⠾⠗011011⠾⠗111011⠾⠗100011⠾⠗000000⠾⠗010101⠾⠗110100⠾⠗110011⠾

$$(011100) \; (011011) \; (111011) \; (100011) \\ (000000) \; (010101) \; (110100) \; (110011)$$

```
(011100) \; (011011) \; (111011) \; (100011) \\ (000000) \; (010101)
\; (110100) \; (110011)
```

2127. ⠗000000⠾⠗011100⠾⠗110101⠾⠗110001⠾

$(000000) \; (011100) \; (110101) \; (110001)$

```
(000000) \; (011100) \; (110101) \; (110001)
```

2128. ⠗0110110⠾⠗0111100⠾⠗1110000⠾⠗1111111⠾⠗1001001⠾⠗1000011⠾⠗0001111⠾⠗00000

$$(0110110) \; (0111100) \; (1110000) \; (1111111) \\ (1001001) \; (1000011) \; (0001111) \; (0000000)$$

```
(0110110) \; (0111100) \; (1110000) \; (1111111) \\ (1001001) \;
(1000011) \; (0001111) \; (0000000)
```

2129. ⠙⠰⠍⠊⠝ ⠂ ⠭ ⠂

$d_{\min} = 1$

```
d_{\min} = 1
```

2130. ⠠⠷⠼⠴ ⠼⠂ ⠼⠴ ⠼⠴ ⠼⠴⠠⠾ ⠠⠷⠼⠂ ⠼⠴ ⠼⠂ ⠼⠴ ⠼⠂⠠⠾ ⠠⠷⠼⠂ ⠼⠴ ⠼⠴ ⠼⠂ ⠼⠴⠠⠾

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

`\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix}`

2131. ⠠⠷⠼⠂ ⠼⠴ ⠼⠂ ⠼⠴ ⠼⠴ ⠼⠴⠠⠾ ⠠⠷⠼⠂ ⠼⠂ ⠼⠴ ⠼⠂ ⠼⠴ ⠼⠴⠠⠾ ⠠⠷⠼⠴ ⠼⠂ ⠼⠴ ⠼⠴ ⠼⠂ ⠼⠴
⠠⠷⠼⠂ ⠼⠂ ⠼⠴ ⠼⠴ ⠼⠴ ⠼⠂⠠⠾

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

`\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}`

2132. ⠠⠷⠼⠂ ⠼⠴ ⠼⠴ ⠼⠂ ⠼⠂⠠⠾ ⠠⠷⠼⠴ ⠼⠂ ⠼⠴ ⠼⠂ ⠼⠂⠠⠾

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

`\begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix}`

2133. ⠠⠷⠼⠴ ⠼⠴ ⠼⠴ ⠼⠂ ⠼⠂ ⠼⠂ ⠼⠂⠠⠾ ⠠⠷⠼⠴ ⠼⠂ ⠼⠂ ⠼⠴ ⠼⠴ ⠼⠂ ⠼⠂⠠⠾
⠠⠷⠼⠂ ⠼⠴ ⠼⠂ ⠼⠴ ⠼⠂ ⠼⠴ ⠼⠂⠠⠾ ⠠⠷⠼⠴ ⠼⠂ ⠼⠂ ⠼⠴ ⠼⠴ ⠼⠂ ⠼⠂⠠⠾

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$$

`\begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}`

2134. ⠷00000⠾ ⠠⠷00101⠾ ⠠⠷10011⠾ ⠠⠷10110⠾

$(00000), (00101), (10011), (10110)$

`(00000), (00101), (10011), (10110)`

2135. ⠠⠛ ⠨⠅ ⠠⠷⠼⠴ ⠼⠂⠠⠾ ⠠⠷⠼⠴ ⠼⠴⠠⠾ ⠠⠷⠼⠂ ⠼⠴⠠⠾ ⠠⠷⠼⠴ ⠼⠂⠠⠾
⠠⠷⠼⠂ ⠼⠂⠠⠾

$$G = \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$$

`G = \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}`

2136. ⠷000000⠾ ⠠⠷010111⠾ ⠠⠷101101⠾ ⠠⠷111010⠾

$(000000), (010111), (101101), (111010)$

`(000000), (010111), (101101), (111010)`

2137. ⠠⠛ ⠨⠅ ⠠⠷⠼⠂ ⠼⠴⠠⠾ ⠠⠷⠼⠴ ⠼⠂⠠⠾ ⠠⠷⠼⠂ ⠼⠴⠠⠾ ⠠⠷⠼⠂ ⠼⠂⠠⠾
⠠⠷⠼⠴ ⠼⠂⠠⠾

$$G = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$$

`G = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}`

2138. ⠷⠢⠠⠆⠾

$(5,2)$

`(5,2)`

2139. ⠠⠓ ⠨⠅ ⠠⠷⠼⠴ ⠼⠂ ⠼⠴ ⠼⠴ ⠼⠂⠠⠾⠠⠷⠼⠂ ⠼⠴ ⠼⠂ ⠼⠴ ⠼⠂⠠⠾⠠⠷⠼⠴ ⠼⠴ ⠼⠂ ⠼⠂ ⠼⠂⠠⠾

$$H = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

`H = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}`

2140. 01111101010111000011

$01111 \quad 10101 \quad 01110 \quad 00011$

`01111 \quad 10101 \quad 01110 \quad 00011`

2141. ⠏ ⠨⠅ 0.01

$p = 0.01$

`p = 0.01`

2142. ⠏ ⠨⠅ 0.0001

$p = 0.0001$

`p = 0.0001`

2143. ⠠⠷⠼⠂ ⠼⠂ ⠼⠴ ⠼⠴ ⠼⠴⠠⠾⠠⠷⠼⠴ ⠼⠴ ⠼⠂ ⠼⠴ ⠼⠴⠠⠾⠠⠷⠼⠴ ⠼⠴ ⠼⠴ ⠼⠂ ⠼⠴⠠⠾
⠠⠷⠼⠂ ⠼⠴ ⠼⠴ ⠼⠴ ⠼⠂⠠⠾

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

`\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{pmatrix}`

2144. ⠠⠷⠼⠴ ⠼⠂ ⠼⠂ ⠼⠴ ⠼⠴ ⠼⠴⠠⠾⠠⠷⠼⠂ ⠼⠂ ⠼⠴ ⠼⠂ ⠼⠴ ⠼⠴⠠⠾⠠⠷⠼⠴ ⠼⠂ ⠼⠴ ⠼⠴ ⠼⠂ ⠼⠴
⠠⠷⠼⠂ ⠼⠂ ⠼⠴ ⠼⠴ ⠼⠴ ⠼⠂⠠⠾

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

`\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}`

2145. ⠠⠷⠼⠂ ⠼⠂ ⠼⠂ ⠼⠴⠠⠾⠠⠷⠼⠂ ⠼⠴ ⠼⠴ ⠼⠂⠠⠾

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

`\begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}`

2146. ⠠⠷⠼⠴ ⠼⠴ ⠼⠴ ⠼⠂ ⠼⠴ ⠼⠴ ⠼⠴⠠⠾ ⠠⠷⠼⠴ ⠼⠂ ⠼⠂ ⠼⠴ ⠼⠂ ⠼⠴ ⠼⠴⠠⠾
⠠⠷⠼⠂ ⠼⠴ ⠼⠂ ⠼⠴ ⠼⠴ ⠼⠂ ⠼⠴⠠⠾ ⠠⠷⠼⠴ ⠼⠂ ⠼⠂ ⠼⠴ ⠼⠴ ⠼⠴ ⠼⠂⠠⠾

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 &
0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1
\end{pmatrix}

2147. ⠠⠛ ⠨⠅ ⠂⠂⠴⠴⠂

$$G = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

G = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}

2148. ⠠⠛ ⠨⠅ ⠠⠷⠼⠂ ⠼⠴⠠⠾ ⠠⠷⠼⠴ ⠼⠂⠠⠾ ⠠⠷⠼⠂ ⠼⠂⠠⠾ ⠠⠷⠼⠂ ⠼⠴⠠⠾

$$G = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{pmatrix}$$

G = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{pmatrix}

2149. ⠠⠓ ⠨⠅ ⠠⠷⠼⠴ ⠼⠂ ⠼⠂ ⠼⠂ ⠼⠂⠠⠾ ⠠⠷⠼⠴ ⠼⠴ ⠼⠴ ⠼⠂ ⠼⠂⠠⠾ ⠠⠷⠼⠂ ⠼⠴ ⠼⠂ ⠼⠴ ⠼⠂⠠⠾

$$H = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

H = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1
& 0 & 1 & 0 & 1 \end{pmatrix}

2150. ⠈⠠⠵⠆⠘⠒

$$\mathbb{Z}_2^3$$

{\mathbb Z}_2^3

2151. ⠠⠷⠼⠴ ⠼⠴ ⠼⠂ ⠼⠴ ⠼⠴⠠⠾ ⠠⠷⠼⠂ ⠼⠂ ⠼⠴ ⠼⠂ ⠼⠴⠠⠾ ⠠⠷⠼⠴ ⠼⠂ ⠼⠴ ⠼⠂ ⠼⠴⠠⠾
⠠⠷⠼⠂ ⠼⠂ ⠼⠴ ⠼⠴ ⠼⠂⠠⠾

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

\begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1
& 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix}

2152. ⠠⠷⠼⠂ ⠼⠴ ⠼⠴ ⠼⠂ ⠼⠂ ⠼⠂ ⠼⠂⠠⠾ ⠠⠷⠼⠂ ⠼⠂ ⠼⠂ ⠼⠴ ⠼⠴ ⠼⠂ ⠼⠂⠠⠾
⠠⠷⠼⠂ ⠼⠴ ⠼⠂ ⠼⠴ ⠼⠂ ⠼⠴ ⠼⠂⠠⠾ ⠠⠷⠼⠂ ⠼⠂ ⠼⠂ ⠼⠴ ⠼⠴ ⠼⠂ ⠼⠴⠠⠾

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 &
1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0
\end{pmatrix}

2153. ⠷11000⠾⠬ ⠠⠉

$(11000) + C$

(11000) + C

2154. ⠷01100⠾⠬ ⠠⠉

$(01100) + C$

(01100) + C

2155. ⠷01010⠾⠬ ⠠⠉

$(01010) + C$

(01010) + C

2156. ⠙⠷⠸⠭⠠⠸⠽⠾ ⠨⠅ ⠙⠷⠸⠭⠬⠸⠵⠠⠸⠽⠬⠸⠵⠾

$d(\mathbf{x}, \mathbf{y}) = d(\mathbf{x} + \mathbf{z}, \mathbf{y} + \mathbf{z})$

d({\mathbf x}, {\mathbf y}) = d({\mathbf x} + {\mathbf z}, {\mathbf y} + {\mathbf z})

2157. ⠙⠷⠸⠭⠠⠸⠽⠾ ⠨⠅ ⠺⠷⠸⠭⠤⠸⠽⠾

$d(\mathbf{x}, \mathbf{y}) = w(\mathbf{x} - \mathbf{y})$

d({\mathbf x}, {\mathbf y}) = w({\mathbf x}- {\mathbf y})

2158. ⠙⠱ ⠠⠭⠡⠠⠭ ⠫⠕ ⠈⠠⠗

$d: X \times X \rightarrow \mathbb{R}$

d: X \times X \rightarrow {\mathbb R}

2159. ⠸⠭⠠⠸⠽ ⠈⠑ ⠠⠭

$\mathbf{x}, \mathbf{y} \in X$

{\mathbf x}, {\mathbf y} \in X

2160. ⠸⠭ ⠈⠑ ⠠⠉

$\mathbf{x} \in C$

{\mathbf x} \in C

2161. ⠸⠽ ⠫⠳⠒⠒⠕ ⠸⠭⠬⠸⠽

$\mathbf{y} \mapsto \mathbf{x} + \mathbf{y}$

{\mathbf y} \mapsto {\mathbf x} + {\mathbf y}

2162. ⠼20

20

20

2163. ⠠⠉⠘⠏⠏ ⠐ ⠨⠅ StartSet⠸⠭ ⠈⠑⠈⠰⠵⠆ ⠫⠊ ⠐⠱⠸⠭⠡⠸⠽ ⠨⠅ ⠴forall⠸⠽ ⠈⠑ ⠠⠉EndSet

$C^\perp = \{\mathbf{x} \in \mathbb{Z}_2^n : \mathbf{x} \cdot \mathbf{y} = 0 \text{ for all } \mathbf{y} \in C\}$

C^\perp = \{ {\mathbf x} \in {\mathbb Z}_2^n : {\mathbf x} \cdot {\mathbf y} = 0 \text{ for all } {\mathbf y} \in C \}

2164. ⠠⠿⠠⠷⠂ ⠠⠷⠂ ⠠⠷⠂ ⠠⠷⠴ ⠠⠷⠴ ⠠⠾ ⠠⠿⠠⠷⠴ ⠠⠷⠴ ⠠⠷⠂ ⠠⠷⠴ ⠠⠷⠂ ⠠⠾ ⠠⠿⠠⠷⠂ ⠠⠷⠴ ⠠⠷⠴ ⠠⠷⠂ ⠠⠷⠴ ⠠⠾

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

\begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix}

2165. ⠠⠉⠘ ⠫⠏

$C^\perp$

```
C^\perp
```

2166. ⠷⠝⠠⠝⠤⠅⠾

$(n, n-k)$

```
(n, n-k)
```

2167. ⠠⠓ ⠶ ⠠⠷⠼⠴ ⠼⠴ ⠼⠴ ⠼⠂ ⠼⠂ ⠼⠂ ⠠⠾ ⠠⠷⠼⠴ ⠼⠂ ⠼⠂ ⠼⠴ ⠼⠴ ⠼⠂ ⠠⠾
⠠⠷⠼⠂ ⠼⠴ ⠼⠂ ⠼⠴ ⠼⠂ ⠼⠴ ⠠⠾

$$H = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

```
H = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 &
1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}
```

2168. ⠷101011⠾

(101011)

```
(101011)
```

2169. ⠷101101⠾

(101101)

```
(101101)
```

2170. ⠷001001⠾

(001001)

```
(001001)
```

2171. ⠷0010000101⠾

(0010000101)

```
(0010000101)
```

2172. ⠷0000101100⠾

(0000101100)

```
(0000101100)
```

2173. ⠷⠍⠠⠝⠾

(m,n)

```
(m,n)
```

2174. ⠷16⠠12⠾

$(16,12)$

```
(16, 12)
```

2175. ⠷⠶⠠⠲⠾

$(7,4)$

```
(7,4)
```

2176. ⠆⠘⠲⠐ ⠶ 16

$2^4 = 16$

```
2^4=16
```

2177. ⠙ ⠭ ⠒

$d = 3$

d=3

2178. ⠷⠙⠤⠂⠾⠌⠆

$(d-1)/2$

(d-1)/2

2179. ⠂ ⠬⠝ ⠂ ⠬⠝ ⠆ ⠬⠄⠄⠄ ⠬⠝ ⠗

$1 + {n\choose 1} + {n\choose 2} + \cdots + {n\choose r}$

1 + {n\choose 1} + {n\choose 2} + \cdots + {n\choose r}

2180. ⠆ ⠘⠅ ⠐⠷⠝ ⠴⠬⠝ ⠂ ⠬⠝ ⠆ ⠬⠄⠄⠄ ⠬⠝ ⠹⠙⠤⠂⠌⠆ ⠼⠾ ⠭ ⠆ ⠘⠝

$2^k\left({n\choose 0} + {n\choose 1} + {n\choose 2} + \cdots + {n\choose \frac{d-1}{2}}\right) = 2^n$

2^k\left({n\choose 0} + {n\choose 1} + {n\choose 2} + \cdots + {n\choose
\frac{d-1}{2}}\right) = 2^n

2181. ⠷ ⠠⠛ ⠠⠡⠾

$(G, \cdot)$

(G, \cdot)

2182. ⠷ ⠠⠓ ⠠ ⠘ ⠨⠡⠾

$(H, \circ)$

(H, \circ)

2183. ⠨⠋⠱ ⠠⠛ ⠫⠕ ⠠⠓

$\phi : G \rightarrow H$

\phi : G \rightarrow H

2184. ⠨⠋⠷⠁ ⠡⠃⠾ ⠭ ⠨⠋⠷⠁⠾ ⠘⠨⠡⠨⠋⠷⠃⠾

$\phi(a \cdot b) = \phi(a) \circ \phi(b)$

\phi(a \cdot b) = \phi(a) \circ \phi(b)

2185. ⠠⠛ ⠈⠱⠈⠱ ⠠⠓

$G \cong H$

G \cong H

2186. ⠈ ⠠⠵⠒ ⠈⠱⠈⠱ ⠠⡗⠭⠆ ⠶⠈⠢⠤ ⠌⠄ ⡗⠭⠆ ⠶⠈⠔⠄

${\mathbb Z}_4 \cong \langle i \rangle$

{\mathbb Z}_4 \cong \langle i \rangle

2187. ⠨⠋⠱⠈ ⠠⠵⠒ ⠫⠕ ⠠⡗⠭⠆ ⠶⠈⠢⠤ ⠌⠄ ⡗⠭⠆ ⠶⠈⠔⠄

$\phi: {\mathbb Z}_4 \rightarrow \langle i \rangle$

\phi: {\mathbb Z}_4 \rightarrow \langle i \rangle

2188. ⠨⠋⠷⠝⠾ ⠭ ⠊⠘⠝

$\phi(n) = i^n$

\phi(n) = i^n

2189. ⠨⠋⠷⠍⠬⠝⠾ ⠨⠅ ⠊⠘⠍⠬⠝⠐ ⠨⠅ ⠊⠘⠍⠐⠊⠘⠝⠐ ⠨⠅ ⠨⠋⠷⠍⠾⠨⠋⠷⠝⠾

$\phi(m+n) = i^{m+n} = i^m i^n = \phi(m)\phi(n)$

\phi(m + n) = i^{m+n} = i^m i^n = \phi(m) \phi(n)

2190. ⠷⠈⠠⠗⠠⠬⠾

$(\mathbb{R}, +)$

({\mathbb R}, +)

2191. ⠷⠈⠠⠗⠘⠬⠐⠠⠡⠾

$(\mathbb{R}^+, \cdot)$

({\mathbb R^+}, \cdot)

2192. ⠨⠋⠷⠭⠬⠽⠾ ⠨⠅ ⠑⠘⠭⠬⠽⠐ ⠨⠅ ⠑⠘⠭⠐⠑⠘⠽⠐ ⠨⠅ ⠨⠋⠷⠭⠾⠨⠋⠷⠽⠾

$\phi(x+y) = e^{x+y} = e^x e^y = \phi(x)\phi(y)$

\phi(x + y) = e^{x + y} = e^x e^y = \phi(x) \phi(y)

2193. ⠨⠋⠱⠈⠠⠵ ⠫⠕ ⠈⠠⠟⠘⠈⠼

$\phi : \mathbb{Z} \to \mathbb{Q}^*$

\phi: {\mathbb Z} \rightarrow {\mathbb Q}^\ast

2194. ⠨⠋⠷⠝⠾ ⠨⠅ ⠆⠘⠝

$\phi(n) = 2^n$

\phi(n) = 2^n

2195. ⠨⠋⠷⠍⠬⠝⠾ ⠨⠅ ⠆⠘⠍⠬⠝⠐ ⠨⠅ ⠆⠘⠍⠐⠆⠘⠝⠐ ⠨⠅ ⠨⠋⠷⠍⠾⠨⠋⠷⠝⠾

$\phi(m+n) = 2^{m+n} = 2^m 2^n = \phi(m)\phi(n)$

\phi(m + n) = 2^{m + n} = 2^m 2^n = \phi(m) \phi(n)

2196. StartSet: ⠘⠝⠐⠱⠝ ⠈⠑ ⠈⠠⠵EndSet

$\{2^n : n \in \mathbb{Z}\}$

\{2^n :n \in {\mathbb Z} \}

2197. ⠍ ⠌⠨⠅ ⠝

$m \neq n$

m \neq n

2198. ⠨⠋⠷⠍⠾ ⠌⠨⠅ ⠨⠋⠷⠝⠾

$\phi(m) \neq \phi(n)$

\phi(m) \neq \phi(n)

2199. ⠍ ⠨⠂ ⠝

$m > n$

m \gt n

2200. ⠨⠋⠷⠍⠾ ⠨⠅ ⠨⠋⠷⠝⠾

$\phi(m) = \phi(n)$

\phi(m) = \phi(n)

2201. ⠆⠘⠍⠐ ⠨⠅ ⠆⠘⠝

$2^m = 2^n$

2^m = 2^n

2202. ⠆ ⠘⠍⠤⠝⠐ ⠨⠅ ⠂

$2^{m-n} = 1$

2^{m - n} = 1

2203. ⠍⠤⠝ ⠨⠂ ⠴

$m - n > 0$

m - n \gt 0

2204. ⠠⠥⠷⠦⠾ ⠈⠱⠈⠱ ⠠⠥⠷12⠾

$U(8) \cong U(12)$

U(8) \cong U(12)

2205. ⠨⠋⠱⠠⠥⠷⠦⠾ ⠫⠕ ⠠⠥⠷12⠾

$\phi : U(8) \rightarrow U(12)$

\phi : U(8) \rightarrow U(12)

2206. ⠨⠓

ψ

\psi

2207. ⠨⠓⠷⠂⠾ ⠨⠅ ⠂

$\psi(1) = 1$

\psi(1) = 1

2208. ⠨⠓⠷⠒⠾ ⠨⠅ 11

$\psi(3) = 11$

\psi(3) = 11

2209. ⠨⠓⠷⠢⠾ ⠨⠅ ⠢

$\psi(5) = 5$

\psi(5) = 5

2210. ⠨⠓⠷⠶⠾ ⠨⠅ ⠶

$\psi(7) = 7$

\psi(7) = 7

2211. ⠨⠋⠱⠁⠠⠵⠖ ⠫⠕ ⠠⠎⠒

$\phi : \mathbb{Z}_6 \rightarrow S_3$

\phi : {\mathbb Z}_6 \rightarrow S_3

2212. ⠁ ⠠⠃ ⠈⠑⠠⠎⠒

$a, b \in S_3$

a , b \in S_3

2213. ⠁⠃ ⠌⠨⠅ ⠃⠁

$ab \neq ba$

ab \neq ba

2214. ⠨⠋⠷⠍⠾ ⠨⠅ ⠁ and ⠨⠋⠷⠝⠾ ⠨⠅ ⠃

$\phi(m) = a \quad \text{and} \quad \phi(n) = b$

\phi(m) = a \quad \text{and} \quad \phi(n) = b

2215. ⠁⠃ ⠶ ⠨⠋⠷⠍⠾⠨⠋⠷⠝⠾ ⠶ ⠨⠋⠷⠍⠬⠝⠾ ⠶ ⠨⠋⠷⠝⠬⠍⠾ ⠶ ⠨⠋⠷⠝⠾⠨⠋⠷⠍⠾ ⠶ ⠃⠁

$ab = \phi(m)\phi(n) = \phi(m+n) = \phi(n+m) = \phi(n)\phi(m) = ba$

```
ab = \phi(m ) \phi(n) = \phi(m + n) = \phi(n + m) = \phi(n ) \phi(m)
= ba
```

2216. ⠨⠋⠘⠤⠂ ⠐⠒⠠⠓ ⠫⠕ ⠠⠛

$\phi^{-1} : H \rightarrow G$

```
\phi^{-1} : H \rightarrow G
```

2217. ⠳⠠⠛⠳ ⠶ ⠳⠠⠓⠳

$|G| = |H|$

```
|G| = |H|
```

2218. ⠨⠋⠷⠛⠂⠾ ⠶ ⠓⠂

$\phi(g_1) = h_1$

```
\phi(g_1) = h_1
```

2219. ⠨⠋⠷⠛⠆⠾ ⠶ ⠓⠆

$\phi(g_2) = h_2$

```
\phi(g_2) = h_2
```

2220. ⠓⠂⠓⠆ ⠶ ⠨⠋⠷⠛⠂⠾⠨⠋⠷⠛⠆⠾ ⠶ ⠨⠋⠷⠛⠂⠛⠆⠾ ⠶ ⠨⠋⠷⠛⠆⠛⠂⠾ ⠶ ⠨⠋⠷⠛⠆⠾⠨⠋⠷⠛⠂⠾

$h_1 h_2 = \phi(g_1)\phi(g_2) = \phi(g_1 g_2) = \phi(g_2 g_1) = \phi(g_2)\phi(g_1) = h_2 h_1$

```
h_1 h_2 = \phi(g_1) \phi(g_2) = \phi(g_1 g_2) = \phi(g_2 g_1) =
\phi(g_2) \phi(g_1) = h_2 h_1
```

2221. ⠨⠋⠒⠈⠠⠵ ⠫⠕ ⠠⠛

$\phi : \mathbb{Z} \rightarrow G$

```
\phi : {\mathbb Z} \rightarrow G
```

2222. ⠨⠋⠒⠝ ⠫⠳⠒⠒⠕ ⠁⠘⠝

$\phi : n \mapsto a^n$

```
\phi : n \mapsto a^n
```

2223. ⠨⠋⠷⠍⠬⠝⠾ ⠶ ⠁⠘⠍⠬⠝⠐ ⠶ ⠁⠘⠍⠐⠁⠘⠝⠐ ⠶ ⠨⠋⠷⠍⠾⠨⠋⠷⠝⠾

$\phi(m+n) = a^{m+n} = a^m a^n = \phi(m)\phi(n)$

```
\phi( m+n ) = a^{m+n} = a^m a^n = \phi( m ) \phi( n )
```

2224. ⠁⠘⠍⠐ ⠌⠶ ⠁⠘⠝

$a^m \neq a^n$

```
a^m \neq a^n
```

2225. ⠁⠘⠍⠐ ⠶ ⠁⠘⠝

$a^m = a^n$

```
a^m = a^n
```

2226. ⠁⠘⠍⠤⠝⠐ ⠶ ⠑

$a^{m-n} = e$

```
a^{m - n} = e
```

2227. ⠨⠋⠷⠝⠾ ⠨⠅ ⠁⠘⠝

$\phi(n) = a^n$

\phi(n) = a^n

2228. ⠨⠋⠱⠈⠠⠵⠰⠝⠐ ⠫⠕ ⠠⠛

$\phi : {\mathbb Z}_n \rightarrow G$

\phi : {\mathbb Z}_n \rightarrow G

2229. ⠨⠋⠱⠅ ⠫⠳⠒⠒⠒⠕ ⠁⠘⠅

$\phi : k \mapsto a^k$

\phi : k \mapsto a^k

2230. ⠴ ⠅⠱ ⠅ ⠐⠅ ⠝

$0 \leq k < n$

0 \leq k \lt n

2231. ⠈⠠⠵⠒

${\mathbb Z}_3$

{\mathbb Z}_3

2232. StartLayout1stRow⠠⠆ ⠠⠼⠴ ⠠⠼⠂ ⠠⠼⠆ 2ndRow⠠⠼⠴ ⠠⠼⠴ ⠠⠼⠂ ⠠⠼⠆ 3rdRow⠠⠼⠂ ⠠⠼⠂ ⠠⠼⠆ ⠠⠼⠴4thRow⠠⠼

$$\begin{array}{c|ccc} + & 0 & 1 & 2 \\ \hline 0 & 0 & 1 & 2 \\ 1 & 1 & 2 & 0 \\ 2 & 2 & 0 & 1 \end{array}$$

\begin{array}{c|ccc} + & 0 & 1 & 2 \\ \hline 0 & 0 & 1 & 2 \\ 1
& 1 & 2 & 0 \\ 2 & 2 & 0 & 1 \end{array}

2233. ⠠⠛ ⠨⠅ StartSet⠷⠠⠴⠾ ⠠⠷⠠⠴⠂ ⠆ ⠾ ⠠⠷⠠⠴⠆ ⠂ ⠾EndSet

$G = \{ (0), (012), (021) \}$

G = \{ (0), (0 1 2), (0 2 1) \}

2234. ⠠⠛overbar

$\overline{G}$

\overline{G}

2235. ⠨⠇ ⠰⠛⠐⠷⠁⠾ ⠨⠅ ⠛⠁

$\lambda_g(a) = ga$

\lambda_g(a) = ga

2236. ⠨⠇ ⠰⠛⠐⠷⠁⠾ ⠨⠅ ⠨⠇ ⠰⠛⠐⠷⠃⠾

$\lambda_g(a) = \lambda_g(b)$

\lambda_g(a) = \lambda_g(b)

2237. ⠛⠁ ⠨⠅ ⠨⠇ ⠰⠛⠐⠷⠁⠾ ⠨⠅ ⠨⠇ ⠰⠛⠐⠷⠃⠾ ⠨⠅ ⠛⠃

$ga = \lambda_g(a) = \lambda_g(b) = gb$

ga =\lambda_g(a) = \lambda_g(b) = gb

2238. ⠁ ⠨⠅ ⠃

$a = b$

a = b

2239. ⠨⠇⠰⠛⠐⠷⠃⠾⠀⠨⠅⠀⠁

$\lambda_g(b) = a$

\lambda_g (b) = a

2240. ⠃⠀⠨⠅⠀⠛⠘⠤⠂⠐⠁

$b = g^{-1}a$

b = g^{-1} a

2241. ⠠⠛overbar ⠨⠅ StartSet⠨⠇⠰⠛⠀⠐⠂⠛⠀⠈⠑⠀⠠⠛EndSet

$\overline{G} = \{\lambda_g : g \in G\}$

\overline{G} = \{ \lambda_g : g \in G \}

2242. ⠷⠨⠇⠰⠛⠐⠨⠡⠨⠇⠰⠓⠐⠾⠷⠁⠾⠀⠨⠅⠀⠨⠇⠰⠛⠐⠷⠓⠁⠾⠀⠨⠅⠀⠛⠓⠁⠀⠨⠅⠀⠨⠇⠰⠛⠓⠐⠷⠁⠾

$(\lambda_g \circ \lambda_h)(a) = \lambda_g(ha) = gha = \lambda_{gh}(a)$

(\lambda_g \circ \lambda_h)(a) = \lambda_g(ha) = gha = \lambda_{gh}
(a)

2243. ⠨⠇⠰⠑⠐⠷⠁⠾⠀⠨⠅⠀⠑⠁⠀⠨⠅⠀⠁

$\lambda_e(a) = ea = a$

\lambda_e (a) = ea = a

2244. ⠷⠨⠇⠰⠛⠰⠘⠤⠂⠐⠨⠡⠨⠇⠰⠛⠐⠾⠷⠁⠾⠀⠨⠅⠀⠨⠇⠰⠛⠰⠘⠤⠂⠐⠷⠛⠁⠾⠀⠨⠅⠀⠛⠘⠤⠂⠐⠛⠁⠀⠨⠅⠀⠁⠀⠨⠅⠀⠨⠇

$(\lambda_{g^{-1}} \circ \lambda_g)(a) = \lambda_{g^{-1}}(ga) = g^{-1}ga = a = \lambda_e(a)$

(\lambda_{g^{-1}} \circ \lambda_g) (a) = \lambda_{g^{-1}} (ga) =
g^{-1} g a = a = \lambda_e (a)

2245. ⠨⠋⠸⠒⠛⠀⠫⠳⠒⠒⠕⠀⠨⠇⠰⠛

$\phi : g \mapsto \lambda_g$

\phi : g \mapsto \lambda_g

2246. ⠨⠋⠷⠛⠓⠾⠀⠨⠅⠀⠨⠇⠰⠛⠓⠐⠀⠨⠅⠀⠨⠇⠰⠛⠐⠨⠇⠰⠓⠐⠀⠨⠅⠀⠨⠋⠷⠛⠾⠨⠋⠷⠓⠾

$\phi(gh) = \lambda_{gh} = \lambda_g\lambda_h = \phi(g)\phi(h)$

\phi(gh) = \lambda_{gh} = \lambda_g \lambda_h = \phi(g) \phi(h)

2247. ⠨⠋⠷⠛⠾⠷⠁⠾⠀⠨⠅⠀⠨⠋⠷⠓⠾⠷⠁⠾

$\phi(g)(a) = \phi(h)(a)$

\phi(g)(a) = \phi(h)(a)

2248. ⠛⠁⠀⠨⠅⠀⠨⠇⠰⠛⠐⠁⠀⠨⠅⠀⠨⠇⠰⠓⠐⠁⠀⠨⠅⠀⠓⠁

$ga = \lambda_g a = \lambda_h a = ha$

ga = \lambda_g a = \lambda_h a= ha

2249. ⠛⠀⠨⠅⠀⠓

$g = h$

g = h

2250. ⠨⠋⠷⠛⠾⠀⠨⠅⠀⠨⠇⠰⠛

$\phi(g) = \lambda_g$

\phi(g) = \lambda_g

2251. ⠨⠇⠰⠛⠐⠈⠑⠠⠛overbar

$\lambda_g \in \overline{G}$

\lambda_g \in \overline{G}

2252. ⠛ ⠫⠳⠒⠒⠕ ⠨⠇⠰⠛

$g \mapsto \lambda_g$

g \mapsto \lambda_g

2253. ⠠⠛⠈⠡⠠⠓

$G \times H$

G \times H

2254. ⠷⠠⠛⠠⠡⠾

$(G,\cdot)$

(G,\cdot)

2255. ⠷⠛⠠⠓⠾ ⠈⠑ ⠠⠛⠈⠡⠠⠓

$(g, h) \in G \times H$

(g, h) \in G \times H

2256. ⠷⠛⠂⠠⠓⠂⠾⠷⠛⠆⠠⠓⠆⠾ ⠨⠅ ⠷⠛⠂⠡⠛⠆⠠⠓⠂⠨⠡⠓⠆⠾⠸⠆

$(g_1, h_1)(g_2, h_2) = (g_1 \cdot g_2, h_1 \circ h_2);$

(g_1, h_1)(g_2, h_2) = (g_1 \cdot g_2, h_1 \circ h_2);

2257. ⠡

$\cdot$

\cdot

2258. ⠷⠛⠂⠠⠓⠂⠾⠷⠛⠆⠠⠓⠆⠾ ⠨⠅ ⠷⠛⠂⠛⠆⠠⠓⠂⠓⠆⠾

$(g_1, h_1)(g_2, h_2) = (g_1 g_2, h_1 h_2)$

(g_1, h_1)(g_2, h_2) = (g_1 g_2, h_1 h_2)

2259. ⠑⠰⠠⠛

e_G

e_G

2260. ⠷⠑⠰⠠⠛⠐⠠⠑⠰⠠⠓⠐⠾

(e_G, e_H)

(e_G, e_H)

2261. ⠷⠛⠘⠤⠂⠐⠠⠓⠘⠤⠂⠐⠾

(g^{-1}, h^{-1})

(g^{-1}, h^{-1})

2262. ⠈⠠⠗⠈⠡⠈⠠⠗ ⠨⠅ ⠈⠠⠗⠘⠆

${\mathbb R} \times {\mathbb R} = {\mathbb R}^2$

{\mathbb R} \times {\mathbb R} = {\mathbb R}^2

2263. ⠷⠁⠠⠃⠾⠬⠷⠉⠠⠙⠾ ⠨⠅ ⠷⠁⠬⠉⠠⠃⠬⠙⠾

$(a, b) + (c, d) = (a + c, b + d)$

(a, b) + (c, d) = (a + c, b + d)

2264. ⠷⠴⠠⠴⠾

$(0, 0)$

(0,0)

2265. ⠷⠁⠠⠃⠾

(a, b)

(a, b)

2266. ⠷⠤⠁⠠⠤⠃⠾

$(-a, -b)$

(-a, -b)

2267. ⠈⠠⠵⠆⠡⠈⠠⠵⠆ ⠶ StartSet⠷⠴⠠⠴⠾⠠⠷⠴⠠⠂⠾⠠⠷⠂⠠⠴⠾⠠⠷⠂⠠⠂⠾EndSet

$\mathbb{Z}_2 \times \mathbb{Z}_2 = \{(0, 0), (0, 1), (1, 0), (1, 1)\}$

{\mathbb Z}_2 \times {\mathbb Z}_2 = \{ (0, 0), (0, 1), (1, 0),(1, 1) \}

2268. ⠷⠁⠠⠃⠾

(a, b)

(a,b)

2269. ⠷⠁⠠⠃⠾⠬⠷⠁⠠⠃⠾ ⠶ ⠷⠴⠠⠴⠾

$(a, b) + (a, b) = (0, 0)$

(a,b) + (a,b) = (0,0)

2270. ⠐⠄⡳⠭⠆⠆⠴⠋⠄⠩⠊ ⠶ ⠂⠣⠝⠻⠠⠛⠰⠊⠐ ⠶ ⠠⠛⠂⠡⠠⠛⠆⠡⠄⠄⠄⠡⠠⠛⠰⠝

$\prod_{i=1}^{n} G_i = G_1 \times G_2 \times \cdots \times G_n$

\prod_{i = 1}^n G_i = G_1 \times G_2 \times \cdots \times G_n

2271. ⠠⠛⠂⠠⠠⠛⠆⠠⠄⠠⠠⠛⠰⠝⠐

$G_1, G_2, \ldots, G_n$

G_1, G_2, \ldots, G_n

2272. ⠠⠛ ⠶ ⠠⠛⠂ ⠶ ⠠⠛⠆ ⠶ ⠄⠄⠄ ⠶ ⠠⠛⠰⠝

$G = G_1 = G_2 = \cdots = G_n$

G = G_1 = G_2 = \cdots = G_n

2273. ⠠⠛⠘⠝

G^n

G^n

2274. ⠠⠛⠂⠡⠠⠛⠆⠡⠄⠄⠄⠡⠠⠛⠰⠝

$G_1 \times G_2 \times \cdots \times G_n$

G_1 \times G_2 \times \cdots \times G_n

2275. ⠷01011101⠾⠬⠷01001011⠾ ⠶ ⠷00010110⠾

$(01011101) + (01001011) = (00010110)$

(01011101) + (01001011) = (00010110)

2276. ⠷⠛⠠⠓⠾

(g, h)

(g, h)

2277. ⠝ ⠨⠅ ⠳⠷⠛⠠⠓⠾⠳

$n = |(g,h)|$

n = |(g,h)|

2278. ⠝ ⠅⠱ ⠍

$n \leq m$

n \leq m

2279. ⠍ ⠅⠱ ⠝

$m \leq n$

m \leq n

2280. ⠷⠛⠂⠠⠄⠄⠠⠛⠰⠝⠐⠾ ⠈⠑⠄⡷⠭⠅⠅⠠⠣⠄⠠⠛⠰⠊

$(g_1, \ldots, g_n) \in \prod G_i$

(g_1, \ldots, g_n) \in \prod G_i

2281. ⠛⠰⠊

g_i

g_i

2282. ⠗⠰⠊

r_i

r_i

2283. ⠠⠛⠰⠊

G_i

G_i

2284. ⠷⠛⠂⠠⠄⠄⠠⠛⠰⠝⠐⠾

$(g_1, \ldots, g_n)$

(g_1, \ldots, g_n)

2285. ⠄⡷⠭⠅⠅⠠⠣⠄⠠⠛⠰⠊

$\prod G_i$

\prod G_i

2286. ⠗⠂⠠⠄⠄⠠⠗⠰⠝⠐

$r_1, \ldots, r_n$

r_1, \ldots, r_n

2287. ⠷⠦⠠56⠾ ⠈⠑ ⠈ ⠠⠵12⠡ ⠈ ⠠⠵60

$(8, 56) \in \mathbb{Z}_{12} \times \mathbb{Z}_{60}$

(8, 56) \in {\mathbb Z}_{12} \times {\mathbb Z}_{60}

2288. ⠛⠉⠙⠷⠦⠠12⠾ ⠨⠅ ⠲

$\gcd(8, 12) = 4$

\gcd(8,12) = 4

2289. 12⠌⠲ ⠨⠅ ⠒

$12/4 = 3$

12/4 = 3

2290. ⠼56

56

56

2291. ⠷⠦⠠56⠾

$(8, 56)$

(8, 56)

2292. ⠈⠠⠵12⠡⠈⠠⠵60

${\mathbb Z}_{12} \times {\mathbb Z}_{60}$

{\mathbb Z}_{12} \times {\mathbb Z}_{60}

2293. ⠈⠠⠵⠆⠡⠈⠠⠵⠒

${\mathbb Z}_2 \times {\mathbb Z}_3$

{\mathbb Z}_2 \times {\mathbb Z}_3

2294. ⠈⠠⠵⠆⠡⠈⠠⠵⠒ ⠈⠱⠈⠱ ⠈⠠⠵⠖

${\mathbb Z}_2 \times {\mathbb Z}_3 \cong {\mathbb Z}_6$

{\mathbb Z}_2 \times {\mathbb Z}_3 \cong {\mathbb Z}_6

2295. ⠷⠂⠠⠂⠾

$(1,1)$

(1,1)

2296. ⠈⠠⠵⠰⠍⠐⠡⠈⠠⠵⠰⠝

${\mathbb Z}_m \times {\mathbb Z}_n$

{\mathbb Z}_m \times {\mathbb Z}_n

2297. ⠈⠠⠵⠰⠍⠝

${\mathbb Z}_{mn}$

{\mathbb Z}_{mn}

2298. ⠛⠉⠙⠷⠍⠠⠝⠾ ⠨⠅ ⠂

$\gcd(m,n)=1$

\gcd(m,n)=1

2299. ⠈⠠⠵⠰⠍⠐⠡⠈⠠⠵⠰⠝⠐ ⠈⠱⠈⠱ ⠈⠠⠵⠰⠍⠝

${\mathbb Z}_m \times {\mathbb Z}_n \cong {\mathbb Z}_{mn}$

{\mathbb Z}_m \times {\mathbb Z}_n \cong {\mathbb Z}_{mn}

2300. ⠛⠉⠙⠷⠍⠠⠝⠾ ⠨⠅ ⠂

$\gcd(m, n) = 1$

\gcd(m, n) = 1

2301. ⠛⠉⠙⠷⠍⠠⠝⠾ ⠨⠅ ⠙ ⠨⠂ ⠂

$\gcd(m, n) = d \gt 1$

\gcd(m, n) = d \gt 1

2302. ⠍⠝⠌⠙

mn/d

mn/d

2303. ⠷⠁ ⠜⠾ ⠉⠑ ⠁ ⠨⠵ ⠰⠍⠉⠑ ⠁ ⠨⠵ ⠰⠝

$(a,b) \in \mathbb{Z}_m \times \mathbb{Z}_n$

(a,b) \in {\mathbb Z}_m \times {\mathbb Z}_n

2304. ⠐ModifyingBelow⠷⠁ ⠜⠾⠬⠷⠁ ⠜⠾⠬⠄⠄⠄⠬⠷⠁ ⠜⠾With⠸⠵⠍⠝ ⠤⠙⠗⠄ ⠍⠝⠌⠙times⠻ ⠶ ⠷⠴⠠⠴

$$\underbrace{(a,b) + (a,b) + \cdots + (a,b)}_{mn/d \text{ times}} = (0,0)$$

\underbrace{(a,b) + (a,b)+ \cdots + (a,b)}_{mn/d \; \text{times}}
= (0, 0)

2305. ⠇⠉⠍⠷⠍⠠⠝⠾ ⠶ ⠍⠝

$\operatorname{lcm}(m,n) = mn$

\lcm(m,n) = mn

2306. ⠝⠂ ⠄⠄ ⠠⠝⠰⠅ ⠂

$n_1, \ldots, n_k$

n_1, \ldots, n_k

2307. ⠐⠄⠸⠏⠗⠕⠙⠰⠊⠶⠂⠘⠅ ⠶ ⠐⠱⠵⠫⠁⠨⠵⠰⠝⠰⠊⠐ ⠉⠕⠝⠛ ⠁⠨⠵⠰⠝⠂⠄⠄⠄⠝⠰⠅

$\prod_{i=1}^k \mathbb{Z}_{n_i} \cong \mathbb{Z}_{n_1 \cdots n_k}$

\prod_{i=1}^k {\mathbb Z}_{n_i} \cong {\mathbb Z}_{n_1 \cdots n_k}

2308. ⠛⠉⠙⠷⠝⠰⠊⠠⠝⠰⠚⠐⠾ ⠶ ⠂

$\gcd(n_i, n_j) = 1$

\gcd(n_i, n_j) =1

2309. ⠍ ⠶ ⠏⠂ ⠑⠂⠂ ⠐⠄⠄⠄⠏⠰⠅ ⠑⠂⠰⠅

$m = p_1^{e_1} \cdots p_k^{e_k}$

m = p_1^{e_1} \cdots p_k^{e_k}

2310. ⠁⠨⠵⠰⠍⠂ ⠉⠕⠝⠛ ⠁⠨⠵⠰⠏⠂ ⠰⠑⠂⠂ ⠐⠡⠄⠄⠄⠡⠁⠨⠵⠰⠏⠰⠅ ⠰⠑⠰⠅

$\mathbb{Z}_m \cong \mathbb{Z}_{p_1^{e_1}} \times \cdots \times \mathbb{Z}_{p_k^{e_k}}$

{\mathbb Z}_m \cong {\mathbb Z}_{p_1^{e_1}} \times \cdots \times
{\mathbb Z}_{p_k^{e_k}}

2311. ⠏⠰⠊⠑⠂⠰⠊

$p_i^{e_i}$

p_i^{e_i}

2312. ⠏⠰⠚⠑⠂⠰⠚

$p_j^{e_j}$

p_j^{e_j}

2313. ⠁⠨⠵⠰⠏⠂ ⠰⠑⠂⠂ ⠐⠡⠄⠄⠄⠡⠁⠨⠵⠰⠏⠰⠅ ⠰⠑⠰⠅

$\mathbb{Z}_{p_1^{e_1}} \times \cdots \times \mathbb{Z}_{p_k^{e_k}}$

{\mathbb Z}_{p_1^{e_1}} \times \cdots \times {\mathbb Z}_{p_k^{e_k}}

2314. ⠠⠛ ⠶ ⠠⠓⠠⠅ ⠶ ⠨⠷⠓⠅⠱⠓⠈⠑⠠⠓⠠⠅⠈⠑⠠⠅⠨⠾

$G = HK = \{hk : h \in H, k \in K\}$

G = HK = \{ hk : h \in H, k \in K \}

2315. ⠠⠓⠨⠩⠠⠅ ⠶ StartSet⠑EndSet

$H \cap K = \{e\}$

H \cap K = \{ e \}

2316. ⠓⠅ ⠶ ⠅⠓

$hk = kh$

hk = kh

2317. ⠠⠓ ⠶ StartSet⠂⠠⠒EndSetand ⠠⠅ ⠶ StartSet⠂⠠⠢EndSet

$H = \{1, 3\} \quad \text{and} \quad K = \{1, 5\}$

H = \{1, 3 \} \quad \text{and} \quad K = \{1, 5 \}

2318. ⠠⠙⠖

D_6

D_6

2319. ⠠⠓ ⠶ StartSet⠊⠙⠠⠗⠘⠒⠐EndSetand ⠠⠅ ⠶ StartSet⠊⠙⠠⠗⠘⠆⠐⠠⠗⠘⠲⠐⠠⠎⠠⠗⠘⠆⠐⠎⠠⠗⠘⠲

$H = \{\mathrm{id}, r^3\} \quad \text{and} \quad K = \{\mathrm{id}, r^2, r^4, s, r^2 s, r^4 s\}$

H = \{\identity, r^3 \} \quad \text{and} \quad K = \{\identity,
r^2, r^4, s, r^2s, r^4 s \}

2320. ⠠⠅ ⠈⠱⠈⠱ ⠠⠎⠒

$K \cong S_3$

K \cong S_3

2321. ⠠⠙⠖ ⠈⠱⠈⠱ ⠈⠠⠵⠆⠡⠠⠎⠒

$D_6 \cong \mathbb{Z}_2 \times S_3$

D_6 \cong {\mathbb Z}_2 \times S_3

2322. StartSet⠷⠂ ⠾ ⠠⠷123⠾ ⠠⠷132⠾EndSet

$\{(1), (123), (132)\}$

\{ (1), (123), (132) \}

2323. ⠠⠓⠡⠠⠅

$H \times K$

H \times K

2324. ⠛ ⠶ ⠓⠅

$g = hk$

g =hk

2325. ⠨⠋⠒⠠⠛ ⠫⠕ ⠠⠓⠡⠠⠅

$\phi : G \rightarrow H \times K$

\phi : G \rightarrow H \times K

2326. ⠨⠋⠷⠛⠾ ⠨⠅ ⠷⠓⠠⠅⠾

$\phi(g) = (h,k)$

\phi(g) = (h,k)

2327. ⠛ ⠨⠅ ⠓⠅ ⠨⠅ ⠓⠄⠅⠄

$g = hk=h'k'$

g = hk=h'k'

2328. ⠓⠘⠤⠂⠐⠓⠘⠄⠐ ⠨⠅ ⠅⠷⠅⠘⠄⠐⠾⠘⠤⠂

$h^{-1} h'= k (k')^{-1}$

h^{-1} h'= k (k')^{-1}

2329. ⠓ ⠨⠅ ⠓⠄

$h = h'$

h = h'

2330. ⠅ ⠨⠅ ⠅⠄

$k = k'$

k = k'

2331. ⠛⠂ ⠨⠅ ⠓⠂⠅⠂

$g_1 = h_1 k_1$

g_1 = h_1 k_1

2332. ⠛⠆ ⠨⠅ ⠓⠆⠅⠆

$g_2 = h_2 k_2$

g_2 = h_2 k_2

2333. StartSet⠴ ⠠⠆ ⠠⠲EndSet⠡StartSet⠴ ⠠⠒EndSet

$\{ 0, 2, 4\} \times \{ 0, 3 \}$

\{ 0, 2, 4\} \times \{ 0, 3 \}

2334. ⠠⠓⠂ ⠠ ⠠⠓⠆ ⠤ ⠠ ⠠⠓⠰⠝⠐

$H_1, H_2, \ldots, H_n$

H_1, H_2, \ldots, H_n

2335. ⠠⠛ ⠨⠅ ⠠⠓⠂⠠⠓⠆⠄⠄⠄⠠⠓⠰⠝⠐ ⠨⠅ StartSet⠓⠂⠓⠆⠄⠄⠄⠓⠰⠝⠐⠈⠒⠓⠰⠊⠐ ⠈⠑ ⠠⠓⠰⠊⠐EndSet

$G = H_1 H_2 \cdots H_n = \{ h_1 h_2 \cdots h_n : h_i \in H_i \}$

G = H_1 H_2 \cdots H_n = \{ h_1 h_2 \cdots h_n : h_i \in H_i \}

2336. ⠠⠓⠰⠊⠐⠨⠩⠨⠨⠷⠨⠬⠰⠚⠌⠨⠅⠊⠐⠠⠓⠰⠚⠐⠨⠨⠾ ⠨⠅ StartSet⠑EndSet

$H_i \cap \langle \cup_{j \neq i} H_j \rangle = \{ e \}$

H_i \cap \langle \cup_{j \neq i} H_j \rangle = \{ e \}

2337. ⠓⠰⠊⠐⠓⠰⠚⠐ ⠨⠅ ⠓⠰⠚⠐⠓⠰⠊

$h_i h_j = h_j h_i$

h_i h_j = h_j h_i

2338. ⠓⠰⠊⠐⠈⠑⠠⠓⠰⠊

$h_i \in H_i$

h_i \in H_i

2339. ⠓⠰⠚⠐⠈⠑⠠⠓⠰⠚

$h_j \in H_j$

h_j \in H_j

2340. ⠠⠓⠰⠊

H_i

H_i

2341. ⠊⠀⠨⠅⠀⠂⠠⠀⠆⠠⠀⠄⠄⠄⠠⠀⠝

$i = 1, 2, \ldots, n$

i = 1, 2, \ldots, n

2342. ⠑⠄⠏⠗⠕⠙⠰⠊⠐⠎⠊⠩⠠⠓⠰⠊

$\prod_i H_i$

\prod_i H_i

2343. ⠈⠰⠵⠀⠈⠱⠈⠱⠀⠝⠈⠰⠵

$\mathbb{Z} \cong n\mathbb{Z}$

\mathbb Z \cong n \mathbb Z

2344. ⠝⠀⠌⠨⠅⠀⠴

$n \neq 0$

n \neq 0

2345. ⠠⠷⠁⠀⠃⠠⠾⠠⠷⠤⠃⠀⠁⠠⠾

$$\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$$

\begin{pmatrix} a & b \\ -b & a \end{pmatrix}

2346. ⠨⠋⠸⠒⠈⠠⠉⠘⠈⠼⠐⠀⠫⠕⠀⠠⠛⠠⠇⠰⠆⠷⠈⠠⠗⠾

$\phi: {\mathbb C}^* \rightarrow GL_2({\mathbb R})$

\phi: {\mathbb C}^* \rightarrow GL_2({\mathbb R})

2347. ⠨⠋⠷⠁⠬⠃⠊⠾⠀⠨⠅⠀⠠⠷⠁⠀⠃⠠⠾⠠⠷⠤⠃⠀⠁⠠⠾

$$\phi(a + bi) = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$$

\phi(a + bi) = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}

2348. ⠠⠥⠷⠦⠾⠀⠈⠱⠈⠱⠀⠈⠰⠵⠲

$U(8) \cong \mathbb{Z}_4$

U(8) \cong {\mathbb Z}_4

2349. ⠠⠷⠂⠀⠴⠠⠾⠠⠷⠴⠀⠂⠠⠾⠠⠠⠷⠂⠀⠴⠠⠾⠠⠷⠴⠀⠤⠂⠠⠾⠠⠠⠷⠤⠂⠀⠴⠠⠾
⠠⠷⠴⠀⠂⠠⠾⠠⠠⠷⠤⠂⠀⠴⠠⠾⠠⠷⠴⠀⠤⠂⠠⠾

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1
& 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix},
\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}

2350. ⠠⠥⠷⠢⠾

$U(5)$

```
U(5)
```

2351. ⠠⠥⠷10⠾

$U(10)$

```
U(10)
```

2352. ⠅ ⠫⠳⠒⠒⠕ ⠉⠊⠎⠷⠆⠅ ⠨⠏⠸⠌⠝⠾

$k \mapsto \operatorname{cis}(2k\pi / n)$

```
k \mapsto \cis(2k\pi / n)
```

2353. ⠠⠛ ⠨⠅ ⠈⠠⠗⠠⠸⠡⠐⠾ ⠨⠷⠤⠂ ⠨⠾

$G = \mathbb{R} \setminus \{-1\}$

```
G = {\mathbb R} \setminus \{ -1 \}
```

2354. ⠷⠠⠛⠠⠈⠼⠾

$(G, *)$

```
(G, *)
```

2355. ⠠⠙12

D_{12}

```
D_{12}
```

2356. ⠨⠺ ⠨⠅ ⠉⠊⠎⠷⠆⠨⠏⠸⠌⠝⠾

$\omega = \operatorname{cis}(2\pi/n)$

```
\omega = \cis(2 \pi /n)
```

2357. ⠠⠁ ⠨⠅ ⠠⠿⠨⠺ ⠴⠴⠠⠾ ⠠⠿⠴⠂ ⠨⠺⠘⠤⠂ ⠐⠠⠾and ⠠⠃ ⠨⠅ ⠠⠿⠴⠂ ⠴⠂ ⠠⠾
⠠⠿⠴⠂ ⠴⠴ ⠠⠾

$A = \begin{pmatrix} \omega & 0 \\ 0 & \omega^{-1} \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

```
A = \begin{pmatrix} \omega & 0 \\ 0 & \omega^{-1} \end{pmatrix}
\quad \text{and} \quad B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}
```

2358. ⠠⠿⠬⠂ ⠅ ⠠⠾ ⠠⠿⠴⠂ ⠴⠂ ⠠⠾

$\begin{pmatrix} \pm 1 & k \\ 0 & 1 \end{pmatrix}$

```
\begin{pmatrix} \pm 1 & k \\ 0 & 1 \end{pmatrix}
```

2359. ⠈⠠⠵⠲⠈⠡⠈⠠⠵⠆

$\mathbb{Z}_4 \times \mathbb{Z}_2$

```
{\mathbb Z}_4 \times {\mathbb Z}_2
```

2360. ⠷⠒⠠⠲⠾

$(3, 4)$

```
(3, 4)
```

2361. ⠈⠠⠵⠲⠈⠡⠈⠠⠵⠖

$\mathbb{Z}_4 \times \mathbb{Z}_6$

```
{\mathbb Z}_4 \times {\mathbb Z}_6
```

2362. ⠷⠖⠠15⠠⠲⠾

$(6, 15, 4)$

(6, 15, 4)

2363. ⠈⠠⠵30⠡⠈⠠⠵45⠡⠈⠠⠵24

${\mathbb Z}_{30} \times {\mathbb Z}_{45} \times {\mathbb Z}_{24}$

{\mathbb Z}_{30} \times {\mathbb Z}_{45} \times {\mathbb Z}_{24}

2364. ⠷⠢⠠10⠠15⠾

$(5, 10, 15)$

(5, 10, 15)

2365. ⠈⠠⠵25⠡⠈⠠⠵25⠡⠈⠠⠵25

${\mathbb Z}_{25} \times {\mathbb Z}_{25} \times {\mathbb Z}_{25}$

{\mathbb Z}_{25} \times {\mathbb Z}_{25} \times {\mathbb Z}_{25}

2366. ⠷⠦⠠⠦⠠⠦⠾

$(8, 8, 8)$

(8, 8, 8)

2367. ⠈⠠⠵10⠡⠈⠠⠵24⠡⠈⠠⠵80

${\mathbb Z}_{10} \times {\mathbb Z}_{24} \times {\mathbb Z}_{80}$

{\mathbb Z}_{10} \times {\mathbb Z}_{24} \times {\mathbb Z}_{80}

2368. ⠆⠘⠍⠐⠒⠘⠝

$2^m 3^n$

2^m 3^n

2369. ⠍⠠⠝⠈⠑⠈⠠⠵

$m,n \in {\mathbb Z}$

m,n \in {\mathbb Z}

2370. ⠈⠠⠵⠡⠈⠠⠵

${\mathbb Z} \times {\mathbb Z}$

{\mathbb Z} \times {\mathbb Z}

2371. ⠠⠎⠒⠡⠈⠠⠵⠆

$S_3 \times {\mathbb Z}_2$

S_3 \times {\mathbb Z}_2

2372. ⠠⠙⠰⠆⠝

D_{2n}

D_{2n}

2373. ⠠⠛⠡⠠⠅ ⠈⠱⠈⠱ ⠠⠓⠡⠠⠅

$G \times K \cong H \times K$

G \times K \cong H \times K

2374. ⠼51

51

51

2375. ⠼52

52

52

2376. ⠨⠋⠷⠭⠾ ⠨⠅ ⠑⠰⠠⠓

$\phi(x) = e_H$

\phi(x) = e_H

2377. ⠭ ⠨⠅ ⠑⠰⠠⠛

$x = e_G$

x=e_G

2378. ⠨⠋⠱⠠⠛ ⠫⠕ ⠠⠓

$\phi : G \rightarrow H$

\phi :G \rightarrow H

2379. ⠨⠋⠷⠁⠾

$\phi(a)$

\phi(a)

2380. ⠠⠁⠰⠝⠬⠆

A_{n+2}

A_{n+2}

2381. ⠨⠋⠱⠠⠛⠂ ⠫⠕ ⠠⠛⠆

$\phi : G_1 \rightarrow G_2$

\phi : G_1 \rightarrow G_2

2382. ⠨⠓⠱⠠⠛⠆ ⠫⠕ ⠠⠛⠒

$\psi : G_2 \rightarrow G_3$

\psi : G_2 \rightarrow G_3

2383. ⠨⠋⠘⠤⠂

ϕ^{-1}

\phi^{-1}

2384. ⠨⠓⠘⠨⠡⠨⠋

$\psi \circ \phi$

\psi \circ \phi

2385. ⠠⠥⠷⠢⠾ ⠈⠱⠈⠱ ⠈⠠⠵⠲

$U(5) \cong {\mathbb Z}_4$

U(5) \cong {\mathbb Z}_4

2386. ⠠⠥⠷⠏⠾

$U(p)$

U(p)

2387. ⠨⠋⠷⠁⠬⠃⠊⠾ ⠨⠅ ⠁⠤⠃⠊

$\phi(a + bi) = a - bi$

\phi(a + bi) = a - bi

2388. ⠈⠠⠉

$\mathbb{C}$

```
{\mathbb C}
```

2389. ⠁⠬⠊⠃ ⠫⠳⠒⠒⠕ ⠁⠤⠊⠃

$a + ib \mapsto a - ib$

```
a + ib \mapsto a - ib
```

2390. ⠠⠁ ⠫⠳⠒⠒⠕ ⠠⠃⠘⠤⠂⠐⠠⠁⠠⠃

$A \mapsto B^{-1}AB$

```
A \mapsto B^{-1}AB
```

2391. ⠠⠁⠥⠞⠷⠠⠛⠾

$\mathrm{Aut}(G)$

```
\aut(G)
```

2392. ⠠⠎⠰⠠⠛

S_G

```
S_G
```

2393. ⠠⠁⠥⠞⠷⠈⠠⠵⠖⠾

$\mathrm{Aut}(\mathbb{Z}_6)$

```
\aut( {\mathbb Z}_6)
```

2394. ⠠⠁⠥⠞⠷⠈⠠⠵⠾

$\mathrm{Aut}(\mathbb{Z})$

```
\aut( {\mathbb Z})
```

2395. ⠠⠁⠥⠞⠷⠠⠛⠾ ⠈⠱⠈⠱ ⠠⠁⠥⠞⠷⠠⠓⠾

$\mathrm{Aut}(G) \cong \mathrm{Aut}(H)$

```
\aut(G) \cong \aut(H)
```

2396. ⠊⠰⠛⠐⠱⠠⠛ ⠫⠕ ⠠⠛

$i_g : G \rightarrow G$

```
i_g : G \rightarrow G
```

2397. ⠊⠰⠛⠐⠷⠭⠾ ⠨⠅ ⠛⠭⠛⠘⠤⠂

$i_g(x) = gxg^{-1}$

```
i_g(x) = g x g^{-1}
```

2398. ⠊⠰⠛

i_g

```
i_g
```

2399. ⠠⠊⠝⠝⠷⠠⠛⠾

$\mathrm{Inn}(G)$

```
\inn(G)
```

2400. ⠊⠰⠛⠐⠷⠭⠾ ⠨⠅ ⠛⠭⠛⠘⠤⠂

$i_g(x) = gxg^{-1}$

```
i_g(x) = gxg^{-1}
```

2401. ⠠⠊⠝⠝⠷⠠⠛⠾ ⠨⠅ ⠠⠁⠥⠞⠷⠠⠛⠾

$\mathrm{Inn}(G) = \mathrm{Aut}(G)$

`\inn(G) = \aut(G)`

2402. ⠨⠇⠰⠛⠐⠱⠠⠛ ⠫⠕ ⠠⠛

$\lambda_g : G \rightarrow G$

`\lambda_g :G \rightarrow G`

2403. ⠨⠗⠰⠛⠐⠱⠠⠛ ⠫⠕ ⠠⠛

$\rho_g : G \rightarrow G$

`\rho_g :G \rightarrow G`

2404. ⠨⠇⠰⠛⠐⠷⠭⠾ ⠨⠅ ⠛⠭

$\lambda_g(x) = gx$

`\lambda_g(x) = gx`

2405. ⠨⠗⠰⠛⠐⠷⠭⠾ ⠨⠅ ⠭⠛⠘⠤⠂⠐

$\rho_g(x) = xg^{-1}$

`\rho_g(x) = xg^{-1}`

2406. ⠊⠰⠛⠐ ⠨⠅ ⠨⠗⠰⠛⠐⠘⠍⠄⠨⠇⠰⠛

$i_g = \rho_g \circ \lambda_g$

`i_g = \rho_g \circ \lambda_g`

2407. ⠛ ⠫⠳⠒⠒⠕ ⠨⠗⠰⠛

$g \mapsto \rho_g$

`g \mapsto \rho_g`

2408. ⠨⠋⠷⠛⠂⠾ ⠨⠅ ⠨⠋⠷⠛⠆⠾

$\phi(g_1) = \phi(g_2)$

`\phi(g_1) = \phi(g_2)`

2409. ⠠⠛ ⠈⠱⠈⠱ ⠠⠛overbar

$G \cong \overline{G}$

`G \cong \overline{G}`

2410. ⠠⠓ ⠈⠱⠈⠱ ⠠⠓overbar

$H \cong \overline{H}$

`H \cong \overline{H}`

2411. ⠠⠛⠈⠡⠠⠓ ⠈⠱⠈⠱ ⠠⠛overbar⠈⠡⠠⠓overbar

$G \times H \cong \overline{G} \times \overline{H}$

`G \times H \cong \overline{G} \times \overline{H}`

2412. ⠠⠓⠈⠡⠠⠛

$H \times G$

`H \times G`

2413. ⠠⠛⠂

G_1

`G_1`

2414. ⠠⠛⠆

G_2

G_2

2415. ⠠⠓⠂⠈⠡⠠⠓⠆

$H_1 \times H_2$

H_1 \times H_2

2416. ⠠⠛⠂⠈⠡⠠⠛⠆

$G_1 \times G_2$

G_1 \times G_2

2417. ⠄⡷⠭⠆⠀⠶⠑⠤⠄⠍⠠⠝⠄⡷⠭⠆⠀⠶⠑⠢⠄⠀⠀⠨⠅⠀⠀⠄⡷⠭⠆⠀⠶⠑⠤⠄⠙⠄⡷⠭⠆⠀⠶⠑⠢⠄

$\langle m,n \rangle = \langle d \rangle$

\langle m,n \rangle = \langle d \rangle

2418. ⠄⡷⠭⠆⠀⠶⠑⠤⠄⠍⠄⡷⠭⠆⠀⠶⠑⠢⠄⠀⠨⠩⠄⡷⠭⠆⠀⠶⠑⠤⠄⠝⠄⡷⠭⠆⠀⠶⠑⠢⠄⠀⠀⠨⠅⠀⠀⠄⡷⠭⠆⠀⠶⠑⠤⠄⠇⠄⡷⠭⠆⠀⠶⠑⠢⠄

$\langle m \rangle \cap \langle n \rangle = \langle l \rangle$

\langle m \rangle \cap \langle n \rangle = \langle l \rangle

2419. ⠇⠀⠀⠨⠅⠀⠀⠇⠉⠍⠷⠍⠠⠝⠾

$l = \mathrm{lcm}(m,n)$

l = \lcm(m,n)

2420. ⠆⠏

$2p$

2p

2421. ⠈⠠⠵⠰⠆⠏

$\mathbb{Z}_{2p}$

{\mathbb Z}_{2p}

2422. ⠽⠀⠈⠑⠀⠠⠛

$y \in G$

y \in G

2423. ⠽⠠⠏⠀⠀⠨⠅⠀⠀⠠⠏⠽

$yP = Py$

yP = Py

2424. ⠠⠏⠀⠀⠨⠅⠀⠀⠄⡷⠭⠆⠀⠶⠑⠤⠄⠵⠄⡷⠭⠆⠀⠶⠑⠢⠄

$P = \langle z \rangle$

P = \langle z \rangle

2425. ⠽⠵⠀⠀⠨⠅⠀⠀⠵⠘⠅⠐⠽

$yz = z^k y$

yz = z^ky

2426. ⠆⠀⠀⠐⠅⠱⠀⠅⠀⠀⠐⠅⠀⠏

$2 \leq k < p$

2 \leq k \lt p

2427. StartSet⠵⠘⠊⠐⠽⠘⠚⠐⠳⠴ ⠅⠱ ⠊ ⠐⠅ ⠏⠠⠴ ⠅⠱ ⠚ ⠐⠅ ⠆EndSet

$\{z^iy^j\mid 0\leq i \lt p, 0\leq j \lt 2\}$

`\{z^iy^j\mid 0\leq i \lt p, 0\leq j \lt 2\}`

2428. ⠷⠵⠘⠊⠐⠽⠘⠚⠐⠾⠷⠵⠘⠗⠐⠽⠘⠎⠐⠾

$(z^iy^j)(z^ry^s)$

`(z^iy^j)(z^ry^s)`

2429. ⠵⠘⠍⠐⠽⠘⠝

$z^m y^n$

`z^m y^n`

2430. ⠍ ⠠⠝

m, n

`m, n`

2431. ⠼⠂ ⠠⠼⠆ ⠠⠼⠒ ⠠⠼⠢ ⠠⠼⠶ ⠠⠼11

$1,2,3,5,7,11$

`1,2,3,5,7,11`

2432. ⠈ ⠠⠵⠆⠡⠈ ⠠⠵⠆

${\mathbb Z}_2\times{\mathbb Z}_2$

`{\mathbb Z}_2\times{\mathbb Z}_2`

2433. ⠈ ⠠⠵⠒⠡⠈ ⠠⠵⠆

${\mathbb Z}_3\times{\mathbb Z}_2$

`{\mathbb Z}_3\times{\mathbb Z}_2`

2434. ⠈ ⠠⠵⠖⠡⠈ ⠠⠵⠆

${\mathbb Z}_6\times{\mathbb Z}_2$

`{\mathbb Z}_6\times{\mathbb Z}_2`

2435. ⠈ ⠠⠵⠒⠡⠈ ⠠⠵⠆⠡⠈ ⠠⠵⠆

${\mathbb Z}_3\times{\mathbb Z}_2\times{\mathbb Z}_2$

`{\mathbb Z}_3\times{\mathbb Z}_2\times{\mathbb Z}_2`

2436. ⠬⠤⠂ ⠠⠬⠤⠠⠊ ⠠⠬⠤⠠⠚ ⠠⠬⠤⠠⠅

$\pm 1, \pm I, \pm J, \pm K$

`\pm 1, \pm I, \pm J, \pm K`

2437. ⠛ ⠈⠑ ⠠⠟

$g\in Q$

`g\in Q`

2438. ⠠⠞⠰⠛

T_g

`T_g`

2439. ⠠⠞⠰⠛⠐⠷⠭⠾ ⠨⠅ ⠭⠛

$T_g(x)=xg$

`T_g(x)=xg`

2440. StartSet⠈ ⠴ ⠤ ⠠⠲EndSet

$\{1,2,\dots,8\}$

```
\{1,2,\dots,8\}
```

2441. ⠈ ⠠⠵⠒ ⠑ ⠈ ⠠⠵⠖

${\mathbb Z}_2\times{\mathbb Z}_4$

```
{\mathbb Z}_2\times{\mathbb Z}_4
```

2442. ⠠⠥⠷24⠾

$U(24)$

```
U(24)
```

2443. ⠼180

180

```
180
```

2444. ⠼72

72

```
72
```

2445. ⠛⠓ ⠨⠅ ⠓⠛

$gh = hg$

```
gh = hg
```

2446. ⠷12⠾

(12)

```
(12)
```

2447. ⠷123⠾ ⠠⠓ ⠨⠅ StartSet⠷123⠾ ⠠⠷13⠾EndSetand ⠠⠓⠷123⠾ ⠨⠅ StartSet⠷123⠾ ⠠⠷23⠾EndS

$(123) H = \{ (123), (13) \} \quad \text{and} \quad H (123) = \{ (123), (23) \}$

```
(123) H = \{ (123), (13) \} \quad \text{and} \quad H (123) = \{
(123), (23) \}
```

2448. ⠷132⠾

(132)

```
(132)
```

2449. ⠛ ⠠⠝⠛ ⠘⠤⠂ ⠇⠐⠅ ⠠⠝

$gNg^{-1} \subset N$

```
gNg^{-1} \subset N
```

2450. ⠛ ⠠⠝⠛ ⠘⠤⠂ ⠐ ⠨⠅ ⠠⠝

$gNg^{-1} = N$

```
gNg^{-1} = N
```

2451. ⠫⠒⠒⠕⠕

$\Rightarrow$

```
\Rightarrow
```

2452. ⠛⠠⠝ ⠨⠅ ⠠⠝⠛

$gN = Ng$

gN = Ng

2453. ⠝ ⠈⠑ ⠠⠝

$n \in N$

n \in N

2454. ⠝⠄

n'

n'

2455. ⠛⠝ ⠨⠅ ⠝⠄⠛

$gn = n'g$

g n = n' g

2456. ⠛⠝⠛⠘⠤⠂⠐ ⠨⠅ ⠝⠄ ⠈⠑ ⠠⠝

$gng^{-1} = n' \in N$

gng^{-1} = n' \in N

2457. ⠠⠝⠸⠐⠅⠛⠠⠝⠛⠘⠤⠂

$N \subset gNg^{-1}$

N \subset gNg^{-1}

2458. ⠛⠘⠤⠂⠐⠝⠛ ⠨⠅ ⠛⠘⠤⠂⠐⠝⠷⠛⠘⠤⠂⠐⠾⠘⠤⠂⠐ ⠈⠑ ⠠⠝

$g^{-1}ng = g^{-1}n(g^{-1})^{-1} \in N$

g^{-1}ng=g^{-1}n(g^{-1})^{-1} \in N

2459. ⠛⠘⠤⠂⠐⠝⠛ ⠨⠅ ⠝⠄

$g^{-1}ng = n'$

g^{-1}ng = n'

2460. ⠝⠄ ⠈⠑ ⠠⠝

$n' \in N$

n' \in N

2461. ⠝ ⠨⠅ ⠛⠝⠄⠛⠘⠤⠂

$n = gn'g^{-1}$

n = g n' g^{-1}

2462. ⠛⠠⠝⠛⠘⠤⠂

gNg^{-1}

g N g^{-1}

2463. ⠛⠝⠛⠘⠤⠂⠐ ⠨⠅ ⠝⠄

$gng^{-1} = n'$

gng^{-1} = n'

2464. ⠛⠝ ⠨⠅ ⠝⠄⠛

$gn = n'g$

gn = n' g

2465. ⠛⠠⠝ ⠸⠐⠅ ⠠⠝⠛

$gN \subset Ng$

gN \subset Ng

2466. ⠠⠝⠛ ⠸⠐⠅ ⠛⠠⠝

$Ng \subset gN$

Ng \subset gN

2467. ⠠⠛⠌⠠⠝

G/N

G/N

2468. ⠷⠁⠠⠝⠾⠷⠃⠠⠝⠾ ⠨⠅ ⠁⠃⠠⠝

$(aN)(bN) = abN$

(aN) (bN) = abN

2469. ⠈⠷⠠⠛⠱⠠⠝⠈⠾

$[G:N]$

[G:N]

2470. ⠷⠁⠠⠝⠾⠷⠃⠠⠝⠾ ⠨⠅ ⠁⠃⠠⠝

$(aN)(bN) = abN$

(a N) (b N)= a b N

2471. ⠁⠠⠝ ⠨⠅ ⠃⠠⠝

$aN = bN$

aN = bN

2472. ⠉⠠⠝ ⠨⠅ ⠙⠠⠝

$cN = dN$

cN = dN

2473. ⠷⠁⠠⠝⠾⠷⠉⠠⠝⠾ ⠨⠅ ⠁⠉⠠⠝ ⠨⠅ ⠃⠙⠠⠝ ⠨⠅ ⠷⠃⠠⠝⠾⠷⠙⠠⠝⠾

$(aN)(cN) = acN = bdN = (bN)(dN)$

(aN) (cN) = acN = bd N = (b N)(d N)

2474. ⠁ ⠨⠅ ⠃⠝⠂

$a = bn_1$

a = b n_1

2475. ⠉ ⠨⠅ ⠙⠝⠆

$c = dn_2$

c = d n_2

2476. ⠝⠂

n_1

n_1

2477. ⠝⠆

n_2

n_2

2478. ⠑⠠⠝ ⠶ ⠠⠝

$eN = N$

eN = N

2479. ⠛⠘⠤⠂⠐⠠⠝

$g^{-1}N$

g^{-1} N

2480. ⠛⠠⠝

gN

gN

2481. ⠠⠝ ⠶ StartSet⠷⠂⠾ ⠠⠷123⠾ ⠠⠷132⠾EndSet

$N = \{(1), (123), (132)\}$

N = \{ (1), (123), (132) \}

2482. ⠷12⠾ ⠠⠝

$(12)N$

(12) N

2483. ⠠⠎⠒⠌⠠⠝

S_3/N

S_3 / N

2484. StartLayout1stRow ⠠⠝ ⠷12⠾ ⠠⠝2ndRow ⠠⠝ ⠠⠝ ⠷12⠾ ⠠⠝3rdRow⠷12⠾ ⠠⠝ ⠷12⠾ ⠠⠝ ⠠⠝E

$$\begin{array}{c|cc} & N & (12)N \\ \hline N & N & (12)N \\ (12)N & (12)N & N \end{array}$$

\begin{array}{c|cc} & N & (12) N \\ \hline N & N & (12) N \\ (12) N & (12) N & N \end{array}

2485. ⠠⠝ ⠶ ⠠⠁⠒

$N = A_3$

N = A_3

2486. ⠷12⠾ ⠠⠝ ⠶ StartSet⠷12⠾ ⠠⠷13⠾ ⠠⠷23⠾EndSet

$(12)N = \{(12), (13), (23)\}$

(12) N = \{ (12), (13), (23) \}

2487. ⠈⠠⠵⠌⠒⠈⠠⠵

$\mathbb{Z}/3\mathbb{Z}$

{\mathbb Z}/ 3 {\mathbb Z}

2488. StartLayout1stRow⠖ ⠴⠬⠒⠈⠠⠵ ⠂⠬⠒⠈⠠⠵ ⠆⠬⠒⠈⠠⠵2ndRow⠴⠬⠒⠈⠠⠵ ⠴⠬⠒⠈⠠⠵ ⠂⠬⠒⠈⠠⠵ ⠆

$$\begin{array}{c|ccc} + & 0+3\mathbb{Z} & 1+3\mathbb{Z} & 2+3\mathbb{Z} \\ \hline 0+3\mathbb{Z} & 0+3\mathbb{Z} & 1+3\mathbb{Z} & 2+3\mathbb{Z} \\ 1+3\mathbb{Z} & 1+3\mathbb{Z} & 2+3\mathbb{Z} & 0+3\mathbb{Z} \\ 2+3\mathbb{Z} & 2+3\mathbb{Z} & 0+3\mathbb{Z} & 1+3\mathbb{Z} \end{array}$$

\begin{array}{c|ccc} + & 0 + 3{\mathbb Z} & 1 + 3{\mathbb Z} & 2 + 3{\mathbb Z} \\\hline 0 + 3{\mathbb Z} & 0 + 3{\mathbb Z} & 1 + 3{\mathbb Z} & 2 + 3{\mathbb Z} \\ 1 + 3{\mathbb Z} & 1 + 3{\mathbb Z} & 2 + 3{\mathbb Z} & 0 + 3{\mathbb Z} \\ 2 + 3{\mathbb Z} & 2 + 3{\mathbb Z} & 0 + 3{\mathbb Z} & 1 + 3{\mathbb Z} \end{array}

2489. ⠈⠠⠵⠌⠝⠈⠠⠵

$\mathbb{Z}/n\mathbb{Z}$

{\mathbb Z } / n {\mathbb Z}

2490. ⠅⠬⠝⠈⠠⠵

$k + n\mathbb{Z}$

k + n{\mathbb Z}

2491. ⠇⠬⠝⠈⠠⠵

$l + n\mathbb{Z}$

l + n{\mathbb Z}

2492. ⠅⠬⠇⠬⠝⠈⠠⠵

$k + l + n\mathbb{Z}$

k+l + n{\mathbb Z}

2493. ⠠⠗⠰⠝

R_n

R_n

2494. ⠎⠗⠎⠘⠤⠂⠐ ⠶ ⠎⠗⠎ ⠶ ⠗⠘⠤⠂⠐⠈⠑⠠⠗⠰⠝

$srs^{-1} = srs = r^{-1} \in R_n$

srs^{-1} = srs = r^{-1} \in R_n

2495. ⠠⠙⠰⠝⠐⠌⠠⠗⠰⠝

D_n/R_n

D_n / R_n

2496. ⠝ ⠖⠱ ⠢

$n \geq 5$

n \geq 5

2497. ⠷⠁⠃⠾ ⠶ ⠷⠃⠁⠾

$(ab) = (ba)$

(a b) = (b a)

2498. ⠠⠝ ⠶ ⠠⠁⠰⠝

$N = A_n$

N = A_n

2499. ⠷⠊⠚⠅⠾

(ijk)

(ijk)

2500. StartSet⠂ ⠠⠆ ⠠⠄ ⠠⠝EndSet

$\{1, 2, \ldots, n\}$

\{ 1, 2, \ldots, n \}

2501. ⠷⠊⠚⠁⠾

(ija)

(i j a)

2502. ⠈⠷⠷⠊⠚⠾⠷⠁⠅⠾⠈⠾⠷⠊⠚⠁⠾⠘⠆⠐⠈⠷⠷⠊⠚⠾⠷⠁⠅⠾⠈⠾⠘⠤⠂⠀⠨⠅⠀⠷⠊⠚⠅⠾

$[(ij)(ak)](ija)^2[(ij)(ak)]^{-1} = (ijk)$

`[(i j)(a k)](i j a)^2 [(i j)(a k)]^{-1} = (i j k)`

2503. ⠷⠊⠚⠅⠾

(ijk)

`(i j k)`

2504. ⠨⠎⠀⠨⠅⠀⠨⠞⠷⠁⠂⠁⠆⠄⠄⠄⠁⠰⠗⠐⠾⠀⠈⠑⠀⠠⠝

$\sigma = \tau(a_1 a_2 \cdots a_r) \in N$

`\sigma = \tau(a_1 a_2 \cdots a_r) \in N`

2505. ⠗⠀⠨⠂⠀⠒

$r > 3$

`r \gt 3`

2506. ⠨⠎⠀⠨⠅⠀⠨⠞⠷⠁⠂⠁⠆⠁⠒⠾⠷⠁⠲⠁⠢⠁⠖⠾

$\sigma = \tau(a_1 a_2 a_3)(a_4 a_5 a_6)$

`\sigma = \tau(a_1 a_2 a_3)(a_4 a_5 a_6)`

2507. ⠨⠎⠀⠨⠅⠀⠨⠞⠷⠁⠂⠁⠆⠁⠒⠾

$\sigma = \tau(a_1 a_2 a_3)$

`\sigma = \tau(a_1 a_2 a_3)`

2508. ⠨⠎⠀⠨⠅⠀⠨⠞⠷⠁⠂⠁⠆⠾⠷⠁⠒⠁⠲⠾

$\sigma = \tau(a_1 a_2)(a_3 a_4)$

`\sigma = \tau (a_1 a_2) (a_3 a_4)`

2509. ⠨⠎⠀⠨⠅⠀⠨⠞⠷⠁⠂⠁⠆⠄⠄⠄⠁⠰⠗⠐⠾

$\sigma = \tau(a_1 a_2 \cdots a_r)$

`\sigma = \tau(a_1 a_2 \cdots a_r)`

2510. ⠷⠁⠂⠁⠆⠁⠒⠾⠨⠎⠷⠁⠂⠁⠆⠁⠒⠾⠘⠤⠂

$(a_1 a_2 a_3)\sigma(a_1 a_2 a_3)^{-1}$

`(a_1 a_2 a_3)\sigma(a_1 a_2 a_3)^{-1}`

2511. ⠨⠎⠘⠤⠂⠐⠷⠁⠂⠁⠆⠁⠒⠾⠨⠎⠷⠁⠂⠁⠆⠁⠒⠾⠘⠤⠂

$\sigma^{-1}(a_1 a_2 a_3)\sigma(a_1 a_2 a_3)^{-1}$

`\sigma^{-1}(a_1 a_2 a_3)\sigma(a_1 a_2 a_3)^{-1}`

2512. ⠨⠎⠘⠤⠂⠐⠷⠁⠂⠁⠆⠁⠲⠾⠨⠎⠷⠁⠂⠁⠆⠁⠲⠾⠘⠤⠂⠀⠈⠑⠀⠠⠝

$\sigma^{-1}(a_1 a_2 a_4)\sigma(a_1 a_2 a_4)^{-1} \in N$

`\sigma^{-1}(a_1 a_2 a_4)\sigma(a_1 a_2 a_4)^{-1} \in N`

2513. ⠷⠁⠂⠁⠆⠁⠲⠾⠨⠎⠷⠁⠂⠁⠆⠁⠲⠾⠘⠤⠂⠀⠈⠑⠀⠠⠝

$(a_1 a_2 a_4)\sigma(a_1 a_2 a_4)^{-1} \in N$

`(a_1 a_2 a_4)\sigma(a_1 a_2 a_4)^{-1} \in N`

2514. ⠨⠎⠀⠈⠑⠀⠠⠝

$\sigma \in N$

`\sigma \in N`

2515. ⠨⠎⠘⠆ ⠐⠈⠑ ⠠⠝

$\sigma^2 \in N$

`\sigma^2 \in N`

2516. ⠃ ⠈⠑⠐StartSet⠐ ⠠⠆ ⠄⠄ ⠠⠝EndSet

$b \in \{1, 2, \ldots, n\}$

`b \in \{1, 2, \ldots, n \}`

2517. ⠃ ⠌⠒ ⠈⠂ ⠠⠁⠆ ⠠⠁⠤ ⠠⠁⠲

$b \neq a_1, a_2, a_3, a_4$

`b \neq a_1, a_2, a_3, a_4`

2518. ⠨⠍ ⠒⠒ ⠷⠁⠐⠁⠤⠃⠾

$\mu = (a_1 a_3 b)$

`\mu = (a_1 a_3 b)`

2519. ⠨⠍⠘⠤⠂ ⠐⠷⠁⠐⠁⠤⠾⠷⠁⠆⠁⠲⠾⠨⠍⠷⠁⠐⠁⠤⠾⠷⠁⠆⠁⠲⠾ ⠈⠑ ⠠⠝

$\mu^{-1}(a_1 a_3)(a_2 a_4)\mu(a_1 a_3)(a_2 a_4) \in N$

`\mu^{-1} (a_1 a_3)(a_2 a_4) \mu (a_1 a_3)(a_2 a_4) \in N`

2520. 196,833⠈⠡196,833

$196{,}833 \times 196{,}833$

`196{,}833 \times 196{,}833`

2521. ⠠⠛⠸⠌⠠⠓

G/H

`G/H`

2522. ⠠⠛ ⠒⠒ ⠠⠎⠲

$G = S_4$

`G = S_4`

2523. ⠠⠓ ⠒⠒ ⠠⠁⠲

$H = A_4$

`H = A_4`

2524. ⠠⠛ ⠒⠒ ⠠⠁⠢

$G = A_5$

`G = A_5`

2525. ⠠⠓ ⠒⠒ ⠠⠙⠲

$H = D_4$

`H = D_4`

2526. ⠠⠛ ⠒⠒ ⠠⠟⠦

$G = Q_8$

`G = Q_8`

2527. ⠠⠓ ⠒⠒ StartSet⠐ ⠤⠤⠐ ⠠⠠⠊ ⠤⠤⠠⠊EndSet

$H = \{1, -1, I, -I\}$

`H = \{ 1, -1, I, -I \}`

2528. ⠠⠛ ⠨⠅ ⠈⠠⠵

$G = \mathbb{Z}$

```
G = {\mathbb Z}
```

2529. ⠠⠓ ⠨⠅ ⠢⠈⠠⠵

$H = 5\mathbb{Z}$

```
H = 5 {\mathbb Z}
```

2530. StartLayout1stRow ⠠⠁⠲ ⠷12⠾⠠⠁⠲ 2ndRow ⠠⠁⠲ ⠠⠁⠲ ⠷12⠾⠠⠁⠲ 3rdRow⠷12⠾⠠⠁⠲ ⠷12

$$\begin{array}{c|cc} & A_4 & (12)A_4 \\ \hline A_4 & A_4 & (12) A_4 \\ (12) A_4 & (12) A_4 & A_4 \end{array}$$

```
\begin{array}{c|cc} & A_4 & (12)A_4 \\ \hline A_4 & A_4 & (12) A_4
\\ (12) A_4 & (12) A_4 & A_4 \end{array}
```

2531. T

```
T
```

2532. ⠠⠷⠁ ⠃ ⠠⠾ ⠠⠷⠠⠴ ⠉ ⠠⠾

$$\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$$

```
\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}
```

2533. ⠉ ⠈⠑ ⠈⠠⠗

$c \in \mathbb{R}$

```
c \in {\mathbb R}
```

2534. ⠁⠉ ⠌⠨⠅ ⠴

$ac \neq 0$

```
ac \neq 0
```

2535. ⠠⠷⠠⠂ ⠭ ⠠⠾ ⠠⠷⠠⠴ ⠠⠂ ⠠⠾

$$\begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix}$$

```
\begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix}
```

2536. ⠭ ⠈⠑ ⠈⠠⠗

$x \in \mathbb{R}$

```
x \in {\mathbb R}
```

2537. ⠠⠞⠌⠠⠥

T/U

```
T/U
```

2538. ⠁ ⠠⠓

aH

```
aH
```

2539. ⠊⠰⠛ ⠐⠆ ⠠⠛ ⠫⠕ ⠠⠛

$i_g : G \to G$

```
i_g : G \to G
```

2540. ⠊⠰⠛⠐⠱⠭ ⠫⠳⠒⠒⠕ ⠛⠭⠛⠘⠤⠂

$i_g : x \mapsto gxg^{-1}$

i_g : x \mapsto gxg^{-1}

2541. ⠊⠰⠛⠐⠷⠠⠓⠾

$i_g(H)$

i_g(H)

2542. ⠠⠉⠷⠛⠾ ⠨⠅ StartSet⠭ ⠈⠑ ⠠⠛⠱⠭⠛ ⠨⠅ ⠛⠭EndSet

$C(g) = \{x \in G : xg = gx\}$

C(g) = \{ x \in G : xg = gx \}

2543. ⠠⠉⠷⠛⠾

$C(g)$

C(g)

2544. ⠭ ⠈⠑ ⠠⠉⠷⠛⠾

$x \in C(g)$

x \in C(g)

2545. ⠽⠭⠽⠘⠤⠂

yxy^{-1}

y x y^{-1}

2546. ⠷⠽⠭⠽⠘⠤⠂⠐⠾⠛ ⠨⠅ ⠛⠷⠽⠭⠽⠘⠤⠂⠐⠾

$(yxy^{-1})g = g(yxy^{-1})$

(y x y^{-1}) g = g (y x y^{-1})

2547. ⠠⠵⠷⠠⠛⠾ ⠨⠅ StartSet⠭ ⠈⠑ ⠠⠛⠱⠭⠛ ⠨⠅ ⠛⠭forall⠛ ⠈⠑ ⠠⠛EndSet

$Z(G) = \{x \in G : xg = gx \text{ for all } g \in G\}$

Z(G) = \{ x \in G : xg = gx \text{ for all } g \in G \}

2548. ⠠⠛⠠⠇⠆⠷⠈⠠⠗⠾

$GL_2(\mathbb{R})$

GL_2 ({\mathbb R})

2549. ⠠⠛⠌⠠⠵⠷⠠⠛⠾

$G/Z(G)$

G / Z(G)

2550. ⠠⠛⠄ ⠨⠅ ⠠⡷⠭⠆⠶⠑⠦⠄⠁⠃⠁⠘⠤⠂⠐⠃⠘⠤⠂⠐⠄⡷⠭⠆⠶⠑⠔⠄

$G' = \langle aba^{-1}b^{-1} \rangle$

G' = \langle aba^{- 1} b^{-1} \rangle

2551. ⠠⠛⠄

G'

G'

2552. ⠁⠃⠁⠘⠤⠂⠐⠃⠘⠤⠂

$aba^{-1}b^{-1}$

aba^{-1}b^{-1}

2553. ⠓ ⠈⠑ ⠠⠛⠄

$h \in G'$

h \in G'

2554. ⠓ ⠨⠅ ⠁⠃⠁⠘⠤⠂⠐⠃⠘⠤⠂

$h = aba^{-1}b^{-1}$

h = aba^{-1}b^{-1}

2555. ⠓ ⠨⠅ ⠓⠂ ⠄⠄⠄ ⠓⠰⠝

$h = h_1 \cdots h_n$

h = h_1 \cdots h_n

2556. ⠓⠰⠊⠐ ⠨⠅ ⠁⠰⠊⠐⠃⠰⠊⠐⠁⠰⠊⠘⠤⠂⠐⠃⠰⠊⠘⠤⠂

$h_i = a_i b_i a_i^{-1} b_i^{-1}$

h_i = a_i b_i a_i^{-1} b_i^{-1}

2557. ⠛⠓⠛⠘⠤⠂

ghg^{-1}

ghg^{-1}

2558. ⠛⠓⠛⠘⠤⠂⠐ ⠨⠅ ⠛⠓⠂ ⠄⠄⠄ ⠓⠰⠝⠐⠛⠘⠤⠂⠐ ⠨⠅ ⠷⠛⠓⠂⠛⠘⠤⠂⠐⠾⠷⠛⠓⠆⠛⠘⠤⠂⠐⠾ ⠄⠄⠄ ⠷⠛⠓⠰⠝⠐⠛⠘⠤

$ghg^{-1} = gh_1 \cdots h_n g^{-1} = (gh_1 g^{-1})(gh_2 g^{-1}) \cdots (gh_n g^{-1})$

ghg^{-1} = g h_1 \cdots h_n g^{-1} = (gh_1g^{-1})(gh_2g^{-1}) \cdots
(gh_ng^{-1})

2559. ⠠⠙⠦

D_8

D_{8}

2560. ⠠⠙⠰⠝

D_n

D_{n}

2561. ⠒ ⠅⠱ ⠝ ⠅⠱ 100

$3 \leq n \leq 100$

3\leq n \leq 100

2562. ⠠⠙470448

D_{470448}

D_{470448}

2563. ⠨⠋⠷⠛⠂⠡⠛⠆⠾ ⠨⠅ ⠨⠋⠷⠛⠂⠾⠨⠡⠨⠋⠷⠛⠆⠾

$\phi(g_1 \cdot g_2) = \phi(g_1) \circ \phi(g_2)$

\phi(g_1 \cdot g_2) = \phi(g_1) \circ \phi(g_2)

2564. StartLayout1stRow even odd2ndRoweven even odd3rdRowodd odd evenEndLayout

	even	odd
even	even	odd
odd	odd	even

\begin{array}{c|cc} & \text{even} & \text{odd} \\ \hline \text{even}
& \text{even} & \text{odd} \\ \text{odd} & \text{odd} & \text{even}
\end{array}

2565. ⠨⠋⠷⠝⠾ ⠨⠅ ⠛⠘⠝

$\phi(n) = g^n$

`\phi( n ) = g^n`

2566. ⠨⠋⠷⠍⠬⠝⠾ ⠨⠅ ⠛⠘⠍⠬⠝⠐ ⠨⠅ ⠛⠘⠍⠐⠛⠘⠝⠐ ⠨⠅ ⠨⠋⠷⠍⠾⠨⠋⠷⠝⠾

$\phi(m + n) = g^{m+n} = g^m g^n = \phi(m)\phi(n)$

`\phi( m + n ) = g^{ m + n} = g^m g^n = \phi( m ) \phi( n )`

2567. ⠠⠛ ⠨⠅ ⠠⠛⠠⠇⠆⠷⠈⠠⠗⠾

$G = GL_2(\mathbb{R})$

`G = GL_2( {\mathbb R })`

2568. ⠙⠑⠞⠷⠠⠁⠾ ⠨⠅ ⠁⠙⠤⠃⠉ ⠌⠨⠅ ⠴

$\det(A) = ad - bc \neq 0$

`\det(A) = ad - bc \neq 0`

2569. ⠨⠋⠸⠒ ⠠⠛⠠⠇⠆⠷⠈⠠⠗⠾ ⠫⠕ ⠈⠠⠗⠘⠈⠼

$\phi : GL_2(\mathbb{R}) \rightarrow \mathbb{R}^*$

`\phi : GL_2( {\mathbb R }) \rightarrow {\mathbb R}^\ast`

2570. ⠠⠁ ⠫⠳⠒⠒⠕ ⠙⠑⠞⠷⠠⠁⠾

$A \mapsto \det(A)$

`A \mapsto \det(A)`

2571. ⠈⠠⠞

$\mathbb{T}$

`{ \mathbb T}`

2572. ⠳⠵⠳ ⠨⠅ ⠂

$|z| = 1$

`|z|=1`

2573. ⠨⠋⠸⠒ ⠨⠹ ⠫⠳⠒⠒⠕ ⠉⠕⠎ ⠨⠹⠬⠊⠎⠊⠝ ⠨⠹

$\phi : \theta \mapsto \cos\theta + i\sin\theta$

`\phi : \theta \mapsto \cos \theta + i \sin \theta`

2574. ⠨⠋⠷⠑⠾

$\phi(e)$

`\phi( e)`

2575. ⠛ ⠈⠑ ⠠⠛⠂

$g \in G_1$

`g \in G_1`

2576. ⠨⠋⠷⠛⠘⠤⠂⠐⠾ ⠨⠅ ⠈⠷⠨⠋⠷⠛⠾⠈⠾⠘⠤⠂

$\phi(g^{-1}) = [\phi(g)]^{-1}$

`\phi( g^{-1}) = [\phi( g )]^{- 1}`

2577. ⠨⠋⠷⠠⠓⠂⠾

$\phi(H_1)$

`\phi( H_1 )`

2578. ⠨⠋⠘⠤⠂⠐⠷⠠⠓⠆⠾ ⠨⠅ StartSet⠛ ⠈⠑ ⠠⠛⠂ ⠱ ⠨⠋⠷⠛⠾ ⠈⠑ ⠠⠓⠆ EndSet

$\phi^{-1}(H_2) = \{g \in G_1 : \phi(g) \in H_2\}$

\phi^{-1}(H_2) = \{ g \in G _1: \phi(g) \in H_2 \}

2579. ⠨⠋⠘⠤⠂⠐⠷⠠⠓⠆⠾

$\phi^{-1}(H_2)$

\phi^{-1}(H_2)

2580. ⠑⠄ ⠨⠋⠷⠑⠾ ⠨⠅ ⠨⠋⠷⠑⠾ ⠨⠅ ⠨⠋⠷⠑⠑⠾ ⠨⠅ ⠨⠋⠷⠑⠾⠨⠋⠷⠑⠾

$e'\phi(e) = \phi(e) = \phi(ee) = \phi(e)\phi(e)$

e' \phi(e) = \phi(e) = \phi(e e) = \phi(e) \phi(e)

2581. ⠨⠋⠷⠑⠾ ⠨⠅ ⠑⠄

$\phi(e) = e'$

\phi(e) = e'

2582. ⠨⠋⠷⠛⠘⠤⠂⠐⠾⠨⠋⠷⠛⠾ ⠨⠅ ⠨⠋⠷⠛⠘⠤⠂⠐⠛⠾ ⠨⠅ ⠨⠋⠷⠑⠾ ⠨⠅ ⠑⠄

$\phi(g^{-1})\phi(g) = \phi(g^{-1}g) = \phi(e) = e'$

\phi(g^{-1}) \phi(g) = \phi(g^{-1} g) = \phi(e) = e'

2583. ⠨⠋⠷⠠⠓⠂⠾

$\phi(H_1)$

\phi(H_1)

2584. ⠁⠠⠃ ⠈⠑ ⠠⠓⠂

$a, b \in H_1$

a, b \in H_1

2585. ⠨⠋⠷⠁⠾ ⠨⠅ ⠭

$\phi(a) = x$

\phi(a) = x

2586. ⠨⠋⠷⠃⠾ ⠨⠅ ⠽

$\phi(b) = y$

\phi(b)=y

2587. ⠭⠽⠘⠤⠂⠐ ⠨⠅ ⠨⠋⠷⠁⠾⠈⠷⠨⠋⠷⠃⠾⠈⠾⠘⠤⠂⠐ ⠨⠅ ⠨⠋⠷⠁⠃⠘⠤⠂⠐⠾ ⠈⠑ ⠨⠋⠷⠠⠓⠂⠾

$xy^{-1} = \phi(a)[\phi(b)]^{-1} = \phi(ab^{-1}) \in \phi(H_1)$

xy^{-1} = \phi(a)[\phi(b)]^{-1} = \phi(a b^{-1}) \in \phi(H_1)

2588. ⠨⠋⠷⠛⠾ ⠈⠑ ⠠⠓⠆

$\phi(g) \in H_2$

\phi(g) \in H_2

2589. ⠨⠋⠷⠁⠃⠘⠤⠂⠐⠾ ⠨⠅ ⠨⠋⠷⠁⠾⠈⠷⠨⠋⠷⠃⠾⠈⠾⠘⠤⠂

$\phi(ab^{-1}) = \phi(a)[\phi(b)]^{-1}$

\phi(ab^{-1}) = \phi(a)[\phi(b)]^{-1}

2590. ⠁⠃⠘⠤⠂ ⠈⠑ ⠠⠓⠂

$ab^{-1} \in H_1$

ab^{-1} \in H_1

2591. ⠛⠘⠤⠂⠐⠓⠛ ⠈⠑ ⠠⠓⠂

$g^{-1}hg \in H_1$

g^{-1} h g \in H_1

2592. ⠓ ⠈⠑ ⠠⠓⠂

$h \in H_1$

h \in H_1

2593. ⠨⠋⠷⠛⠘⠤⠂⠐⠓⠛⠾ ⠨⠅ ⠈⠷⠨⠋⠷⠛⠾⠈⠾⠘⠤⠂⠐⠨⠋⠷⠓⠾⠨⠋⠷⠛⠾ ⠈⠑ ⠠⠓⠆

$\phi(g^{-1}hg) = [\phi(g)]^{-1}\phi(h)\phi(g) \in H_2$

\phi(g^{-1} h g) = [\phi(g)]^{-1} \phi(h) \phi(g) \in H_2

2594. ⠛⠘⠤⠂⠐⠓⠛ ⠈⠑ ⠠⠓⠂

$g^{-1}hg \in H_1$

g^{-1}hg \in H_1

2595. ⠨⠋⠘⠤⠂⠐⠷StartSet⠑EndSet⠾

$\phi^{-1}(\{e\})$

\phi^{-1} (\{ e \})

2596. ⠅⠑⠗ ⠨⠋

$\ker \phi$

\ker \phi

2597. ⠠⠁ ⠫⠳⠒⠒⠕ ⠙⠑⠞⠷⠠⠁⠾

$A \mapsto \det(A)$

A \mapsto \det(A)

2598. ⠅⠑⠗ ⠨⠋ ⠨⠅ ⠠⠎⠠⠇⠆⠷⠈⠠⠗⠾

$\ker \phi = SL_2(\mathbb{R})$

\ker \phi = SL_2({\mathbb R })

2599. ⠨⠋⠒ ⠈⠠⠗ ⠫⠕ ⠈⠠⠉⠘⠈⠼

$\phi : \mathbb{R} \rightarrow \mathbb{C}^\ast$

\phi : {\mathbb R} \rightarrow {\mathbb C}^\ast

2600. ⠨⠋⠷⠨⠹⠾ ⠨⠅ ⠉⠕⠎ ⠨⠹⠬⠊⠎⠊⠝ ⠨⠹

$\phi(\theta) = \cos \theta + i \sin \theta$

\phi(\theta) = \cos \theta + i \sin \theta

2601. StartSet⠆ ⠨⠏⠝⠒⠝ ⠈⠑ ⠈⠠⠵EndSet

$\{2\pi n : n \in \mathbb{Z}\}$

\{ 2 \pi n : n \in {\mathbb Z} \}

2602. ⠅⠑⠗ ⠨⠋ ⠈⠱⠈⠱ ⠈⠠⠵

$\ker \phi \cong \mathbb{Z}$

\ker \phi \cong {\mathbb Z}

2603. ⠈⠠⠵⠶

$\mathbb{Z}_7$

{\mathbb Z}_7

2604. ⠨⠋⠱⠠⠛ ⠫⠕ ⠠⠓

$\phi : G \rightarrow H$

\phi: G \rightarrow H

2605. ⠨⠋⠱⠠⠛ ⠫⠕ ⠠⠛⠌⠠⠓

$\phi : G \rightarrow G/H$

\phi : G \rightarrow G/H

2606. ⠨⠋⠷⠛⠾ ⠨⠅ ⠛⠠⠓

$\phi(g) = gH$

\phi(g) = gH

2607. ⠨⠋⠷⠛⠂⠛⠆⠾ ⠨⠅ ⠛⠂⠛⠆⠠⠓ ⠨⠅ ⠛⠂⠠⠓⠛⠆⠠⠓ ⠨⠅ ⠨⠋⠷⠛⠂⠾⠨⠋⠷⠛⠆⠾

$\phi(g_1 g_2) = g_1 g_2 H = g_1 H g_2 H = \phi(g_1)\phi(g_2)$

\phi(g_1 g_2) = g_1 g_2 H = g_1 H g_2 H = \phi(g_1) \phi(g_2
)

2608. ⠨⠓⠱⠠⠛ ⠫⠕ ⠠⠓

$\psi : G \rightarrow H$

\psi : G \rightarrow H

2609. ⠠⠅ ⠨⠅ ⠅⠑⠗⠨⠓

$K = \ker \psi$

K =\ker \psi

2610. ⠨⠋⠱⠠⠛ ⠫⠕ ⠠⠛⠌⠠⠅

$\phi : G \rightarrow G/K$

\phi: G \rightarrow G/K

2611. ⠨⠦⠱⠠⠛⠌⠠⠅ ⠫⠕ ⠨⠓⠷⠠⠛⠾

$\eta : G/K \rightarrow \psi(G)$

\eta: G/K \rightarrow \psi(G)

2612. ⠨⠓ ⠨⠅ ⠨⠦⠨⠋

$\psi = \eta\phi$

\psi = \eta \phi

2613. ⠨⠦⠷⠛⠠⠅⠾ ⠨⠅ ⠨⠓⠷⠛⠾

$\eta(gK) = \psi(g)$

\eta(gK) = \psi(g)

2614. ⠨⠦

η

\eta

2615. ⠛⠂⠠⠅ ⠨⠅ ⠛⠆⠠⠅

$g_1 K = g_2 K$

g_1 K =g_2 K

2616. ⠛⠂⠅ ⠨⠅ ⠛⠆

$g_1k = g_2$

g_1 k=g_2

2617. ⠨⠱⠷⠛⠂⠠⠅⠾ ⠨⠅ ⠨⠽⠷⠛⠂⠾ ⠨⠅ ⠨⠽⠷⠛⠂⠾⠨⠽⠷⠅⠾ ⠨⠅ ⠨⠽⠷⠛⠂⠅⠾ ⠨⠅ ⠨⠽⠷⠛⠆⠾ ⠨⠅ ⠨⠱⠷⠛

$\eta(g_1K) = \psi(g_1) = \psi(g_1)\psi(k) = \psi(g_1k) = \psi(g_2) = \eta(g_2K)$

\eta(g_1 K) = \psi(g_1) = \psi(g_1) \psi(k) = \psi(g_1k) = \psi(g_2)
= \eta(g_2 K)

2618. ⠨⠽⠷⠠⠛⠾

$\psi(G)$

\psi(G)

2619. ⠨⠱⠷⠛⠂⠠⠅⠾ ⠨⠅ ⠨⠱⠷⠛⠆⠠⠅⠾

$\eta(g_1K) = \eta(g_2K)$

\eta(g_1 K) = \eta(g_2 K)

2620. ⠨⠽⠷⠛⠂⠾ ⠨⠅ ⠨⠽⠷⠛⠆⠾

$\psi(g_1) = \psi(g_2)$

\psi(g_1) = \psi(g_2)

2621. ⠨⠽⠷⠛⠂⠘⠤⠂⠐⠛⠆⠾ ⠨⠅ ⠑

$\psi(g_1^{-1}g_2) = e$

\psi(g_1^{-1} g_2) = e

2622. ⠛⠂⠘⠤⠂⠐⠛⠆

$g_1^{-1}g_2$

g_1^{-1} g_2

2623. ⠛⠂⠘⠤⠂⠐⠛⠆⠠⠅ ⠨⠅ ⠠⠅

$g_1^{-1}g_2K = K$

g_1^{-1} g_2K = K

2624. ⠛⠂⠠⠅ ⠨⠅ ⠛⠆⠠⠅

$g_1K = g_2K$

g_1K =g_2K

2625. ⠝ ⠫⠳⠒⠒⠕ ⠛⠘⠝

$n \mapsto g^n$

n \mapsto g^n

2626. ⠨⠋⠷⠍⠬⠝⠾ ⠨⠅ ⠛⠘⠍⠬⠝⠐ ⠨⠅ ⠛⠘⠍⠐⠛⠘⠝⠐ ⠨⠅ ⠨⠋⠷⠍⠾⠨⠋⠷⠝⠾

$\phi(m + n) = g^{m+n} = g^m g^n = \phi(m)\phi(n)$

\phi(m + n) = g^{m+n} = g^m g^n = \phi(m) \phi(n)

2627. ⠳⠛⠳ ⠨⠅ ⠍

$|g| = m$

|g| = m

2628. ⠛⠘⠍⠐ ⠨⠅ ⠑

$g^m = e$

```
g^m = e
```

2629. ⠅⠑⠗ ⠨⠋ ⠨⠅ ⠍⠈⠠⠵

$\ker \phi = m\mathbb{Z}$

```
\ker \phi = m {\mathbb Z}
```

2630. ⠈⠠⠵⠌⠅⠑⠗ ⠨⠋ ⠨⠅ ⠈⠠⠵⠌⠍⠈⠠⠵ ⠈⠱⠈⠱ ⠠⠛

$\mathbb{Z}/\ker \phi = \mathbb{Z}/m\mathbb{Z} \cong G$

```
{\mathbb Z} / \ker \phi = {\mathbb Z} / m {\mathbb Z} \cong G
```

2631. ⠅⠑⠗ ⠨⠋ ⠨⠅ ⠴

$\ker \phi = 0$

```
\ker \phi = 0
```

2632. ⠠⠓⠠⠝

HN

```
HN
```

2633. ⠠⠓⠨⠩⠠⠝

$H \cap N$

```
H \cap N
```

2634. ⠠⠓⠌⠠⠓⠨⠩⠠⠝ ⠈⠱⠈⠱ ⠠⠓⠠⠝⠌⠠⠝

$H/H \cap N \cong HN/N$

```
H / H \cap N \cong HN /N
```

2635. ⠠⠓⠠⠝ ⠨⠅ ⠨⠷⠓⠝⠱⠓ ⠈⠑ ⠠⠓⠠⠝ ⠈⠑ ⠠⠝⠨⠾

$HN = \{hn : h \in H, n \in N\}$

```
HN = \{ hn : h \in H, n \in N \}
```

2636. ⠓⠂⠝⠂ ⠠⠓⠆⠝⠆ ⠈⠑ ⠠⠓⠠⠝

$h_1 n_1, h_2 n_2 \in HN$

```
h_1 n_1, h_2 n_2 \in HN
```

2637. ⠷⠓⠆⠾⠘⠤⠂⠐⠝⠂⠓⠆ ⠈⠑ ⠠⠝

$(h_2)^{-1} n_1 h_2 \in N$

```
(h_2)^{-1} n_1 h_2 \in N
```

2638. ⠷⠓⠂⠝⠂⠾⠷⠓⠆⠝⠆⠾ ⠨⠅ ⠓⠂⠓⠆⠷⠷⠓⠆⠾⠘⠤⠂⠐⠝⠂⠓⠆⠾⠝⠆

$(h_1 n_1)(h_2 n_2) = h_1 h_2 ((h_2)^{-1} n_1 h_2) n_2$

```
(h_1 n_1)(h_2 n_2) = h_1 h_2 ( (h_2)^{-1} n_1 h_2 )n_2
```

2639. ⠓⠝ ⠈⠑ ⠠⠓⠠⠝

$hn \in HN$

```
hn \in HN
```

2640. ⠷⠓⠝⠾⠘⠤⠂ ⠨⠅ ⠝⠘⠤⠂⠐⠓⠘⠤⠂ ⠨⠅ ⠓⠘⠤⠂⠐⠷⠓⠝⠘⠤⠂⠐⠓⠘⠤⠂⠐⠾

$(hn)^{-1} = n^{-1} h^{-1} = h^{-1}(h n^{-1} h^{-1})$

```
( hn )^{-1} = n^{-1 } h^{-1} = h^{-1} (h n^{-1} h^{-1} )
```

2641. ⠝ ⠈⠑ ⠠⠓ ⠨⠩ ⠠⠝

$n \in H \cap N$

n \in H \cap N

2642. ⠓⠘⠤⠂ ⠐⠝⠓ ⠈⠑ ⠠⠓

$h^{-1}nh \in H$

h^{-1} n h \in H

2643. ⠓⠘⠤⠂ ⠐⠝⠓ ⠈⠑ ⠠⠝

$h^{-1}nh \in N$

h^{-1} n h \in N

2644. ⠓⠘⠤⠂ ⠐⠝⠓ ⠈⠑ ⠠⠓ ⠨⠩ ⠠⠝

$h^{-1}nh \in H \cap N$

h^{-1} n h \in H \cap N

2645. ⠠⠓⠠⠝⠌⠠⠝

HN/N

HN / N

2646. ⠓ ⠫⠳⠒⠒⠕ ⠓⠠⠝

$h \mapsto hN$

h \mapsto h N

2647. ⠓⠝⠠⠝ ⠨⠅ ⠓⠠⠝

$hnN = hN$

h n N = h N

2648. ⠨⠋⠷⠓⠓⠄ ⠐⠾ ⠨⠅ ⠓⠓⠄ ⠠⠝ ⠨⠅ ⠓⠠⠝⠓⠄ ⠠⠝ ⠨⠅ ⠨⠋⠷⠓⠾⠨⠋⠷⠓⠄ ⠐⠾

$\phi(hh') = hh'N = hNh'N = \phi(h)\phi(h')$

\phi(h h') = h h' N = h N h' N = \phi(h) \phi(h')

2649. ⠠⠓⠌⠅⠑⠗ ⠨⠋

$H/\ker\phi$

H / \ker \phi

2650. ⠠⠓⠠⠝⠌⠠⠝ ⠨⠅ ⠨⠋⠷⠠⠓⠾ ⠈⠣⠈⠱ ⠠⠓⠌⠅⠑⠗ ⠨⠋

$HN/N = \phi(H) \cong H/\ker\phi$

HN/N = \phi(H) \cong H / \ker \phi

2651. ⠅⠑⠗ ⠨⠋ ⠨⠅ StartSet⠓ ⠈⠑ ⠠⠓⠸⠒⠓ ⠈⠑ ⠠⠝EndSet ⠨⠅ ⠠⠓ ⠨⠩ ⠠⠝

$\ker\phi = \{h \in H : h \in N\} = H \cap N$

\ker \phi = \{ h \in H : h \in N \} = H \cap N

2652. ⠠⠓⠠⠝⠌⠠⠝ ⠨⠅ ⠨⠋⠷⠠⠓⠾ ⠈⠣⠈⠱ ⠠⠓⠌⠠⠓ ⠨⠩ ⠠⠝

$HN/N = \phi(H) \cong H/H \cap N$

HN/N = \phi(H) \cong H / H \cap N

2653. ⠠⠓ ⠫⠳⠒⠒⠕ ⠠⠓⠌⠠⠝

$H \mapsto H/N$

H \mapsto H/N

2654. ⠠⠓⠌⠠⠝

H/N

H/N

2655. ⠁⠠⠝

aN

aN

2656. ⠃⠠⠝

bN

bN

2657. ⠷⠁⠠⠝⠾⠷⠃⠘⠤⠂⠐⠠⠝⠾ ⠨⠅ ⠁⠃⠘⠤⠂⠐⠠⠝⠈⠑⠠⠓⠌⠠⠝

$(aN)(b^{-1}N) = ab^{-1}N \in H/N$

(aN)(b^{-1} N)= ab^{-1}N \in H/N

2658. ⠠⠓ ⠨⠅ StartSet⠛ ⠈⠑⠠⠛⠱⠛⠠⠝⠈⠑⠠⠎EndSet

$H = \{g \in G : gN \in S\}$

H= \{ g \in G : gN \in S \}

2659. ⠷⠓⠂⠠⠝⠾⠷⠓⠆⠠⠝⠾ ⠨⠅ ⠓⠂⠓⠆⠠⠝⠈⠑⠠⠎

$(h_1N)(h_2N) = h_1h_2N \in S$

(h_1 N)(h_2 N)= h_1 h_2 N \in S

2660. ⠓⠂⠘⠤⠂⠐⠠⠝⠈⠑⠠⠎

$h_1^{-1}N \in S$

h_1^{-1} N \in S

2661. ⠠⠎ ⠨⠅ ⠠⠓⠌⠠⠝

$S = H/N$

S = H / N

2662. ⠠⠓⠂⠌⠠⠝ ⠨⠅ ⠠⠓⠆⠌⠠⠝

$H_1/N = H_2/N$

H_1/N = H_2/N

2663. ⠓⠂ ⠈⠑⠠⠓⠂

$h_1 \in H_1$

h_1 \in H_1

2664. ⠓⠂⠠⠝⠈⠑⠠⠓⠂⠌⠠⠝

$h_1N \in H_1/N$

h_1 N \in H_1/N

2665. ⠓⠂⠠⠝ ⠨⠅ ⠓⠆⠠⠝⠸⠐⠅⠠⠓⠆

$h_1N = h_2N \subset H_2$

h_1 N = h_2 N \subset H_2

2666. ⠓⠂ ⠈⠑⠠⠓⠆

$h_1 \in H_2$

h_1 \in H_2

2667. ⠠⠓⠂ ⠸⠐⠅ ⠠⠓⠆

$H_1 \subset H_2$

```
H_1 \subset H_2
```

2668. ⠠⠓⠆ ⠸⠐⠅ ⠠⠓⠂

$H_2 \subset H_1$

```
H_2 \subset H_1
```

2669. ⠠⠓⠂ ⠶ ⠠⠓⠆

$H_1 = H_2$

```
H_1 = H_2
```

2670. ⠠⠛⠌⠠⠝ ⠣⠕ ⠠⠛⠌⠠⠓

$G/N \rightarrow G/H$

```
G/N \rightarrow G/H
```

2671. ⠛⠠⠝ ⠣⠳⠒⠒⠕ ⠛⠠⠓

$gN \mapsto gH$

```
gN \mapsto gH
```

2672. ⠠⠛ ⠣⠕ ⠠⠛⠌⠠⠝ ⠣⠕ ⠹⠠⠛⠌⠠⠝⠌⠠⠓⠌⠠⠝⠼

$G \rightarrow G/N \rightarrow \frac{G/N}{H/N}$

```
G \rightarrow G/N \rightarrow \frac{G/N}{H/N}
```

2673. ⠠⠝⠸⠐⠅⠠⠓

$N \subset H$

```
N \subset H
```

2674. ⠠⠛⠌⠠⠓ ⠈⠱⠈⠱ ⠹⠠⠛⠌⠠⠝⠌⠠⠓⠌⠠⠝⠼

$G/H \cong \frac{G/N}{H/N}$

```
G/H \cong \frac{G/N}{H/N}
```

2675. ⠈⠠⠵⠌⠍⠈⠠⠵ ⠈⠱⠈⠱ ⠷⠈⠠⠵⠌⠍⠝⠈⠠⠵⠾⠌⠷⠍⠈⠠⠵⠌⠍⠝⠈⠠⠵⠾

$\mathbb{Z}/m\mathbb{Z} \cong (\mathbb{Z}/mn\mathbb{Z})/(m\mathbb{Z}/mn\mathbb{Z})$

```
{\mathbb Z} / m {\mathbb Z} \cong ({\mathbb Z}/ mn {\mathbb Z})/
(m {\mathbb Z}/ mn {\mathbb Z})
```

2676. ⠳⠈⠠⠵⠌⠍⠝⠈⠠⠵⠳ ⠶ ⠍⠝

$|\mathbb{Z}/mn\mathbb{Z}| = mn$

```
| {\mathbb Z} / mn {\mathbb Z} | = mn
```

2677. ⠳⠈⠠⠵⠌⠍⠈⠠⠵⠳ ⠶ ⠍

$|\mathbb{Z}/m\mathbb{Z}| = m$

```
|{\mathbb Z} / m{\mathbb Z}| = m
```

2678. ⠳⠍⠈⠠⠵⠌⠍⠝⠈⠠⠵⠳ ⠶ ⠝

$|m\mathbb{Z}/mn\mathbb{Z}| = n$

```
| m {\mathbb Z} / mn {\mathbb Z}| = n
```

2679. ⠙⠑⠞⠷⠠⠁⠠⠃⠾ ⠨⠅ ⠙⠑⠞⠷⠠⠁⠾⠙⠑⠞⠷⠠⠃⠾

$\det(AB) = \det(A)\det(B)$

`\det( AB) = \det(A) \det(B)`

2680. ⠠⠁⠠ ⠠⠃ ⠈⠑ ⠠⠛⠠⠇⠆⠷⠈⠠⠗⠾

$A, B \in GL_2(\mathbb{R})$

`A, B \in GL_2( {\mathbb R} )`

2681. ⠨⠋⠱⠈⠠⠗⠘⠈⠼⠐ ⠫⠕ ⠠⠛⠠⠇⠆⠷⠈⠠⠗⠾

$\phi : \mathbb{R}^\ast \rightarrow GL_2(\mathbb{R})$

`\phi : {\mathbb R}^\ast \rightarrow GL_2 ( {\mathbb R})`

2682. ⠨⠋⠷⠁⠾ ⠨⠅ ⠠⠷⠼⠂ ⠼⠴⠠⠾ ⠠⠷⠼⠴ ⠁⠠⠾

$\phi(a) = \begin{pmatrix} 1 & 0 \\ 0 & a \end{pmatrix}$

`\phi( a ) = \begin{pmatrix} 1 & 0 \\ 0 & a \end{pmatrix}`

2683. ⠨⠋⠱⠈⠠⠗ ⠫⠕ ⠠⠛⠠⠇⠆⠷⠈⠠⠗⠾

$\phi : \mathbb{R} \rightarrow GL_2(\mathbb{R})$

`\phi : {\mathbb R} \rightarrow GL_2 ( {\mathbb R})`

2684. ⠨⠋⠷⠁⠾ ⠨⠅ ⠠⠷⠼⠂ ⠼⠴⠠⠾ ⠠⠷⠁ ⠼⠂ ⠠⠾

$\phi(a) = \begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix}$

`\phi( a ) = \begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix}`

2685. ⠨⠋⠱⠠⠛⠠⠇⠆⠷⠈⠠⠗⠾ ⠫⠕ ⠈⠠⠗

$\phi : GL_2(\mathbb{R}) \rightarrow \mathbb{R}$

`\phi : GL_2 ({\mathbb R}) \rightarrow {\mathbb R}`

2686. ⠨⠋ ⠠⠷⠁ ⠃⠠⠾ ⠠⠷⠉ ⠙⠠⠾ ⠨⠅ ⠁⠬⠙

$\phi \left(\begin{pmatrix} a & b \\ c & d \end{pmatrix} \right) = a + d$

`\phi \left( \begin{pmatrix} a & b \\ c & d \end{pmatrix} \right)`
`= a + d`

2687. ⠨⠋⠱⠠⠛⠠⠇⠆⠷⠈⠠⠗⠾ ⠫⠕ ⠈⠠⠗⠘⠈⠼

$\phi : GL_2(\mathbb{R}) \rightarrow \mathbb{R}^\ast$

`\phi : GL_2 ( {\mathbb R}) \rightarrow {\mathbb R}^\ast`

2688. ⠨⠋ ⠠⠷⠁ ⠃⠠⠾ ⠠⠷⠉ ⠙⠠⠾ ⠨⠅ ⠁⠙⠤⠃⠉

$\phi \left(\begin{pmatrix} a & b \\ c & d \end{pmatrix} \right) = ad - bc$

`\phi \left( \begin{pmatrix} a & b \\ c & d \end{pmatrix} \right)`
`= ad - bc`

2689. ⠨⠋⠱⠈⠠⠍⠆⠷⠈⠠⠗⠾ ⠫⠕ ⠈⠠⠗

$\phi : \mathbb{M}_2(\mathbb{R}) \rightarrow \mathbb{R}$

`\phi : {\mathbb M}_2( {\mathbb R}) \rightarrow {\mathbb R}`

2690. ⠨⠋⠠⠷⠁ ⠃⠠⠾⠠⠷⠉ ⠙⠠⠾ ⠨⠅ ⠃

$$\phi \left(\begin{pmatrix} a & b \\ c & d \end{pmatrix} \right) = b$$

\phi \left(\begin{pmatrix} a & b \\ c & d \end{pmatrix} \right)
= b

2691. ⠈⠠⠍⠆⠷⠈⠠⠗⠾

$\mathbb{M}_2(\mathbb{R})$

{\mathbb M}_2({\mathbb R})

2692. StartSet⠂ EndSet

$\{1\}$

\{ 1 \}

2693. ⠭ ⠫⠳⠒⠒⠕ ⠠⠁⠭

$x \mapsto Ax$

x \mapsto Ax

2694. ⠨⠋⠱⠈⠠⠗⠘⠝⠐ ⠫⠕ ⠈⠠⠗⠘⠍

$\phi : \mathbb{R}^n \rightarrow \mathbb{R}^m$

\phi : {\mathbb R}^n \rightarrow {\mathbb R}^m

2695. ⠨⠋⠱⠈⠠⠵ ⠫⠕ ⠈⠠⠵

$\phi : \mathbb{Z} \rightarrow \mathbb{Z}$

\phi : {\mathbb Z} \rightarrow {\mathbb Z}

2696. ⠨⠋⠷⠝⠾ ⠨⠅ ⠶⠝

$\phi(n) = 7n$

\phi(n) = 7n

2697. ⠨⠋⠷⠍⠬⠝⠾ ⠨⠅ ⠶⠷⠍⠬⠝⠾ ⠨⠅ ⠶⠍⠬⠶⠝ ⠨⠅ ⠨⠋⠷⠍⠾⠬⠨⠋⠷⠝⠾

$\phi(m + n) = 7(m + n) = 7m + 7n = \phi(m) + \phi(n)$

\phi(m + n) = 7(m+n) = 7m + 7n = \phi(m) + \phi(n)

2698. ⠨⠋⠱⠈⠠⠵24 ⠫⠕ ⠈⠠⠵18

$\phi : \mathbb{Z}_{24} \rightarrow \mathbb{Z}_{18}$

\phi : {\mathbb Z}_{24} \rightarrow {\mathbb Z}_{18}

2699. ⠠⠓ ⠨⠅ ⠠⡗⠭⠆⠶⠑⠦⠄⠲⠄⡗⠭⠆⠶⠑⠠⠄

$H = \langle 4 \rangle$

H = \langle 4 \rangle

2700. ⠠⠝ ⠨⠅ ⠠⡗⠭⠆⠶⠑⠦⠄⠖⠄⡗⠭⠆⠶⠑⠠⠄

$N = \langle 6 \rangle$

N = \langle 6 \rangle

2701. ⠠⠓⠬⠠⠝

$H + N$

H + N

2702. ⠠⠓⠠⠝⠌⠠⠝

HN/N

HN/N

2703. ⠠⠓⠌⠷⠠⠓⠨⠩⠠⠝⠾

$H/(H \cap N)$

H/(H \cap N)

2704. ⠨⠋⠱⠠⠛⠀⠫⠕⠀⠠⠛

$\phi : G \rightarrow G$

\phi : G \rightarrow G

2705. ⠛⠀⠫⠳⠒⠒⠕⠀⠛⠘⠝

$g \mapsto g^n$

g \mapsto g^n

2706. ⠨⠋⠷⠠⠛⠾

$\phi(G)$

\phi(G)

2707. ⠨⠋⠷⠁⠾⠨⠋⠷⠃⠾⠀⠨⠅⠀⠨⠋⠷⠁⠃⠾⠀⠨⠅⠀⠨⠋⠷⠃⠁⠾⠀⠨⠅⠀⠨⠋⠷⠃⠾⠨⠋⠷⠁⠾

$\phi(a)\phi(b) = \phi(ab) = \phi(ba) = \phi(b)\phi(a)$

\phi(a) \phi(b) = \phi(ab) = \phi(ba) = \phi(b)\phi(a)

2708. ⠈⠠⠟⠌⠈⠠⠴⠀⠈⠱⠈⠱⠀⠈⠠⠟

$\mathbb{Q}/\mathbb{Z} \cong \mathbb{Q}$

{\mathbb Q} / {\mathbb Z} \cong {\mathbb Q}

2709. ⠨⠋⠘⠤⠂⠐⠷⠠⠓⠾

$\phi^{-1}(H)$

\phi^{-1}(H)

2710. ⠳⠠⠓⠳⠡⠳⠠⠝⠳

$|H| \cdot |N|$

|H| \cdot |N|

2711. ⠨⠋⠱⠠⠛⠀⠫⠕⠀⠠⠛⠌⠠⠝

$\phi : G \rightarrow G/N$

\phi : G \rightarrow G/N

2712. ⠨⠋overbar⠱⠷⠠⠛⠂⠌⠠⠓⠂⠾⠀⠫⠕⠀⠷⠠⠛⠆⠌⠠⠓⠆⠾

$\overline{\phi} : (G_1/H_1) \rightarrow (G_2/H_2)$

\overline{\phi} : (G_1/H_1) \rightarrow (G_2/H_2)

2713. ⠨⠋⠷⠠⠓⠂⠾⠀⠸⠪⠀⠠⠓⠆

$\phi(H_1) \subset H_2$

\phi(H_1) \subset H_2

2714. ⠠⠛⠌⠠⠓⠡⠠⠛⠌⠠⠅

$G/H \times G/K$

G/H \times G/K

2715. ⠨⠋⠷⠠⠓⠂⠾ ⠨⠅ ⠠⠓⠆

$\phi(H_1) = H_2$

`\phi(H_1) = H_2`

2716. ⠠⠛⠂⠌⠠⠓⠂ ⠈⠱⠈⠱ ⠠⠛⠆⠌⠠⠓⠆

$G_1/H_1 \cong G_2/H_2$

`G_1/H_1 \cong G_2/H_2`

2717. ⠨⠋⠘⠤⠂⠐⠷⠑⠾ ⠨⠅ StartSet⠑EndSet

$\phi^{-1}(e) = \{ e \}$

`\phi^{-1}(e) = \{ e \}`

2718. ⠨⠋⠷⠁⠾ ⠨⠅ ⠨⠋⠷⠃⠾

$\phi(a) = \phi(b)$

`\phi(a) = \phi(b)`

2719. ⠠⠁⠥⠞⠷⠠⠛⠾ ⠅⠱ ⠠⠎⠰⠠⠛

$\mathrm{Aut}(G) \leq S_G$

`\aut(G) \leq S_G`

2720. ⠊⠰⠛⠐⠈⠑⠠⠁⠥⠞⠷⠠⠛⠾

$i_g \in \mathrm{Aut}(G)$

`i_g \in \aut(G)`

2721. ⠠⠛ ⠫⠕ ⠠⠁⠥⠞⠷⠠⠛⠾

$G \rightarrow \mathrm{Aut}(G)$

`G \rightarrow \aut(G)`

2722. ⠛ ⠫⠳⠒⠒⠕ ⠊⠰⠛

$g \mapsto i_g$

`g \mapsto i_g`

2723. ⠠⠵⠷⠠⠛⠾

$Z(G)$

`Z(G)`

2724. ⠠⠛⠌⠠⠵⠷⠠⠛⠾ ⠈⠱⠈⠱ ⠠⠊⠝⠝⠷⠠⠛⠾

$G/Z(G) \cong \mathrm{Inn}(G)$

`G/Z(G) \cong \inn(G)`

2725. ⠠⠁⠥⠞⠷⠠⠎⠒⠾

$\mathrm{Aut}(S_3)$

`\aut(S_3)`

2726. ⠠⠊⠝⠝⠷⠠⠎⠒⠾

$\mathrm{Inn}(S_3)$

`\inn(S_3)`

2727. ⠠⠁⠥⠞⠷⠈⠠⠵⠾

$\mathrm{Aut}(\mathbb{Z})$

`\aut({\mathbb Z})`

2728. ⠠⠁⠥⠞⠷⠈⠠⠵⠦⠾ ⠈⠱⠈⠱ ⠠⠥⠷⠦⠾

$\mathrm{Aut}(\mathbb{Z}_8) \cong U(8)$

\aut({\mathbb Z}_8) \cong U(8)

2729. ⠨⠋⠰⠅ ⠐⠒ ⠈⠠⠵⠰⠝⠐ ⠫⠕ ⠈⠠⠵⠰⠝⠐

$\phi_k : \mathbb{Z}_n \rightarrow \mathbb{Z}_n$

\phi_k : {\mathbb Z}_n \rightarrow {\mathbb Z}_n

2730. ⠁ ⠫⠳⠒⠒⠕ ⠅⠁

$a \mapsto ka$

a \mapsto ka

2731. ⠨⠋⠰⠅

ϕ_k

\phi_k

2732. ⠨⠽⠐⠒ ⠠⠥⠷⠝⠾ ⠫⠕ ⠠⠁⠥⠞⠷⠈⠠⠵⠰⠝⠐⠾

$\psi : U(n) \rightarrow \mathrm{Aut}(\mathbb{Z}_n)$

\psi : U(n) \rightarrow \aut({\mathbb Z}_n)

2733. ⠨⠽⠐⠒⠅ ⠫⠳⠒⠒⠕ ⠨⠋⠰⠅⠐

$\psi : k \mapsto \phi_k$

\psi : k \mapsto \phi_k

2734. ⠠⠙20

D_{20}

D_{20}

2735. ⠠⠙⠢

D_5

D_{5}

2736. ⠠⠛⠈⠡⠠⠓

$G \times H$

G\times H

2737. ⠭ ⠫⠳⠒⠒⠕ ⠭

$x \mapsto x$

x\mapsto x

2738. ⠷⠂ ⠠⠆ ⠠⠒⠾

$(1, 2, 3)$

(1, 2, 3)

2739. ⠷⠲ ⠠⠢ ⠠⠖ ⠠⠶⠾

$(4, 5, 6, 7)$

(4, 5, 6, 7)

2740. ⠠⠎12

S_{12}

S_{12}

2741. ⠷⠂⠠⠆⠠⠒⠾⠷⠲⠠⠢⠠⠖⠾⠷⠶⠠⠦⠠⠔⠾⠷10⠠11⠠12⠾

$(1, 2, 3)(4, 5, 6)(7, 8, 9)(10, 11, 12)$

```
(1, 2, 3)(4, 5, 6)(7, 8, 9)(10, 11, 12)
```

2742. ⠷⠂ ⠠10⠠⠶⠠⠲⠾⠷⠆ ⠠11⠠⠦⠠⠢⠾⠷⠒ ⠠12⠠⠔⠠⠖⠾

$(1, 10, 7, 4)(2, 11, 8, 5)(3, 12, 9, 6)$

```
(1, 10, 7, 4)(2, 11, 8, 5)(3, 12, 9, 6)
```

2743. ⠠⠞⠒⠈⠠⠗⠘⠝⠐ ⠫⠕ ⠈⠠⠗⠘⠍

$T : \mathbb{R}^n \rightarrow \mathbb{R}^m$

```
T : {\mathbb R}^n \rightarrow {\mathbb R}^m
```

2744. ⠨⠁ ⠈⠑⠈⠠⠗

$\alpha \in \mathbb{R}$

```
\alpha \in {\mathbb R}
```

2745. ⠸⠭ ⠨⠅ ⠷⠭⠂⠠ ⠄⠄⠠⠭⠰⠝⠐⠾⠘⠞

$\mathbf{x} = (x_1, \ldots, x_n)^{\mathsf{t}}$

```
{\mathbf x} = (x_1, \ldots, x_n)^\transpose
```

2746. ⠠⠁ ⠨⠅ ⠠⠷⠁11 ⠁12 ⠄⠄⠄ ⠁⠰⠂⠝⠐⠠⠾ ⠠⠷⠁21 ⠁22 ⠄⠄⠄ ⠁⠰⠆⠝⠐⠠⠾
⠠⠷⠳⠄⠄⠄ ⠳⠄⠄⠄ ⠒⠄⠄⠄ ⠳⠄⠄⠄⠠⠾ ⠠⠷⠁ ⠰⠍Baseline⠂ ⠂ ⠁ ⠰⠍Baseline⠆ ⠂ ⠄⠄⠄ ⠁ ⠰⠍⠝⠠⠾

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

```
A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21}
& a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots
\\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}
```

2747. ⠠⠁⠷⠸⠭⠬⠸⠽⠾ ⠨⠅ ⠠⠁⠸⠭⠬⠠⠁⠸⠽and⠨⠁⠠⠁⠸⠭ ⠨⠅ ⠠⠁⠷⠨⠁⠸⠭⠾

$A(\mathbf{x} + \mathbf{y}) = A\mathbf{x} + A\mathbf{y} \qquad \text{and} \qquad \alpha A\mathbf{x} = A(\alpha\mathbf{x})$

```
A({\mathbf x} + {\mathbf y} ) = A {\mathbf x }+ A {\mathbf y} \qquad
\text{and} \qquad \alpha A {\mathbf x} = A ( \alpha {\mathbf x})
```

2748. ⠸⠭ ⠨⠅ ⠭⠂ ⠭⠆ ⠳⠄⠄⠄ ⠭⠝ ⠒⠟

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

```
{\mathbf x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}
```

2749. ⠷⠁ ⠰⠊⠚ ⠐⠾

(a_{ij})

```
(a_{ij})
```

2750. ⠭⠂ ⠸⠑⠂ ⠬⠭⠆ ⠸⠑⠆ ⠬⠄⠄⠄⠬⠭⠰⠝⠐⠸⠑⠰⠝

$x_1 \mathbf{e}_1 + x_2 \mathbf{e}_2 + \cdots + x_n \mathbf{e}_n$

```
x_1 {\mathbf e}_1 + x_2 {\mathbf e}_2 + \cdots + x_n {\mathbf e}_n
```

2751. ⠠⠞⠒⠈⠠⠗⠘⠆ ⠐ ⠫⠕ ⠈⠠⠗⠘⠆

$T : \mathbb{R}^2 \to \mathbb{R}^2$

```
T : {\mathbb R}^2 \rightarrow {\mathbb R}^2
```

2752. ⠠⠞⠷⠭⠂ ⠠⠭⠆⠾ ⠨⠅ ⠷⠆⠭⠂⠬⠢⠭⠆ ⠠⠤⠲⠭⠂⠬⠒⠭⠆⠾

$T(x_1, x_2) = (2x_1 + 5x_2, -4x_1 + 3x_2)$

```
T(x_1, x_2) = (2 x_1 + 5 x_2, - 4 x_1 + 3 x_2)
```

2753. ⠠⠞ ⠸⠑⠂ ⠨⠅ ⠷⠆ ⠠⠤⠲⠾⠘⠞

$T\mathbf{e}_1 = (2, -4)^{\mathsf{t}}$

```
T {\mathbf e}_1 = (2, -4)^\transpose
```

2754. ⠠⠞ ⠸⠑⠆ ⠨⠅ ⠷⠢ ⠠⠒⠾⠘⠞

$T\mathbf{e}_2 = (5, 3)^{\mathsf{t}}$

```
T {\mathbf e}_2 = (5,3)^\transpose
```

2755. ⠠⠁ ⠨⠅ ⠠⠷⠠⠷⠆ ⠠⠷⠢ ⠠⠾ ⠠⠷⠤⠲ ⠠⠷⠒ ⠠⠾

$$A = \begin{pmatrix} 2 & 5 \\ -4 & 3 \end{pmatrix}$$

```
A = \begin{pmatrix} 2 & 5 \\ -4 & 3 \end{pmatrix}
```

2756. ⠠⠊ ⠨⠅ ⠠⠷⠠⠷⠂ ⠠⠷⠴ ⠄⠄⠄ ⠠⠷⠴ ⠠⠾ ⠠⠷⠠⠷⠴ ⠠⠷⠂ ⠄⠄⠄ ⠠⠷⠴ ⠠⠾ ⠠⠷⠑⠄⠄⠄ ⠑⠄⠄⠄ ⠒⠄⠄⠄ ⠑⠄⠄⠄ ⠠⠾
⠠⠷⠠⠷⠴ ⠠⠷⠴ ⠄⠄⠄ ⠠⠷⠂ ⠠⠾

$$I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

```
I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}
```

2757. ⠠⠷⠠⠷⠆ ⠠⠷⠂ ⠠⠾ ⠠⠷⠠⠷⠢ ⠠⠷⠒ ⠠⠾

$$\begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}$$

```
\begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}
```

2758. ⠠⠁ ⠘⠤⠂ ⠐ ⠨⠅ ⠠⠷⠠⠷⠒ ⠤⠂ ⠠⠾ ⠠⠷⠤⠢ ⠠⠷⠆ ⠠⠾

$$A^{-1} = \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$$

```
A^{-1} = \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}
```

2759. ⠙⠑⠞⠷⠠⠁⠾ ⠨⠅ ⠆⠡⠒⠤⠢⠡⠂ ⠨⠅ ⠂

$\det(A) = 2 \cdot 3 - 5 \cdot 1 = 1$

```
\det(A) = 2 \cdot 3 - 5 \cdot 1 = 1
```

2760. ⠙⠑⠞⠷⠠⠁ ⠠⠃⠾ ⠨⠅ ⠷⠙⠑⠞⠠⠁⠾⠷⠙⠑⠞⠠⠃⠾

$\det(AB) = (\det A)(\det B)$

```
\det( A B) = (\det A )(\det B)
```

2761. ⠙⠑⠞⠷⠠⠁ ⠘⠤⠂ ⠐⠾ ⠨⠅ ⠂ ⠠⠌⠙⠑⠞⠠⠁

$\det(A^{-1}) = 1/\det A$

```
\det(A^{-1}) = 1 / \det A
```

2762. ⠠⠁⠀⠨⠅⠀⠷⠁⠰⠊⠚⠐⠾

$A = (a_{ij})$

`A = (a_{ij})`

2763. ⠠⠁⠘⠞⠐⠀⠨⠅⠀⠷⠁⠰⠚⠊⠐⠾

$A^{\mathsf{t}} = (a_{ji})$

`A^\transpose = (a_{ji})`

2764. ⠙⠑⠞⠷⠠⠁⠘⠞⠐⠾⠀⠨⠅⠀⠙⠑⠞⠠⠁

$\det(A^{\mathsf{t}}) = \det A$

`\det(A^\transpose) = \det A`

2765. ⠳⠙⠑⠞⠠⠁⠳

$|\det A|$

`|\det A|`

2766. ⠠⠛⠠⠇⠰⠝⠐⠷⠈⠠⠗⠾

$GL_n(\mathbb{R})$

`GL_n({\mathbb R})`

2767. ⠙⠑⠞⠷⠠⠁⠾⠀⠨⠅⠀⠂

$\det(A) = 1$

`\det(A) =1`

2768. ⠙⠑⠞⠷⠠⠃⠾⠀⠨⠅⠀⠂

$\det(B) = 1$

`\det(B) = 1`

2769. ⠙⠑⠞⠷⠠⠁⠠⠃⠾⠀⠨⠅⠀⠙⠑⠞⠷⠠⠁⠾⠙⠑⠞⠷⠠⠃⠾⠀⠨⠅⠀⠂

$\det(AB) = \det(A)\det(B) = 1$

`\det(AB) = \det(A) \det (B) = 1`

2770. ⠙⠑⠞⠷⠠⠁⠘⠤⠂⠐⠾⠀⠨⠅⠀⠂⠌⠙⠑⠞⠠⠁⠀⠨⠅⠀⠂

$\det(A^{-1}) = 1/\det A = 1$

`\det(A^{-1}) = 1 / \det A = 1`

2771. ⠠⠎⠠⠇⠰⠝⠐⠷⠈⠠⠗⠾

$SL_n(\mathbb{R})$

`SL_n({\mathbb R})`

2772. ⠁⠙⠤⠃⠉

$ad - bc$

`ad-bc`

2773. ⠁⠙⠤⠃⠉⠀⠌⠨⠅⠀⠴

$ad - bc \neq 0$

`ad-bc \neq 0`

2774. ⠠⠁⠀⠨⠅⠀⠠⠷⠼⠂⠀⠼⠂⠠⠾⠀⠠⠷⠼⠴⠀⠼⠂⠠⠾

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}

2775. ⠸⠭⠀⠨⠅⠀⠷⠂⠠⠴⠾⠘⠞

$${\mathbf x} = (1,0)^{\mathsf{t}}$$

{\mathbf x} = (1,0)^\transpose

2776. ⠸⠽⠀⠨⠅⠀⠷⠴⠠⠂⠾⠘⠞

$${\mathbf y} = (0,1)^{\mathsf{t}}$$

{\mathbf y} = (0,1)^\transpose

2777. ⠷⠂⠠⠴⠾⠘⠞

$$(1,0)^{\mathsf{t}}$$

(1,0)^\transpose

2778. ⠷⠂⠠⠂⠾⠘⠞

$$(1,1)^{\mathsf{t}}$$

(1, 1)^\transpose

2779. ⠠⠁⠸⠭⠀⠨⠅⠀⠷⠂⠠⠴⠾⠘⠞

$$A{\mathbf x} = (1,0)^{\mathsf{t}}$$

A {\mathbf x} = (1,0)^\transpose

2780. ⠠⠁⠸⠽⠀⠨⠅⠀⠷⠂⠠⠂⠾⠘⠞

$$A{\mathbf y} = (1,1)^{\mathsf{t}}$$

A {\mathbf y} = (1, 1)^\transpose

2781. ⠠⠎⠠⠇⠆⠷⠈⠠⠗⠾

$$SL_2(\mathbb R)$$

SL_2(\mathbb R)

2782. ⠠⠕⠷⠝⠾

$$O(n)$$

O(n)

2783. ⠠⠁⠘⠤⠂⠐⠀⠨⠅⠀⠠⠁⠘⠞

$$A^{-1} = A^{\mathsf{t}}$$

A^{-1} = A^\transpose

2784. ⠠⠛⠠⠇⠰⠝⠐⠷⠈⠠⠗⠾

$$GL_n({\mathbb R})$$

GL_n({\mathbb R})

2785. ⠠⠷⠒⠌⠢⠀⠤⠲⠌⠢⠠⠾⠀⠠⠷⠲⠌⠢⠀⠒⠌⠢⠠⠾⠠⠀⠠⠷⠂⠌⠆⠀⠤⠜⠒⠻⠌⠆⠠⠾⠀⠠⠷⠜⠒⠻⠌⠆⠀⠂⠌⠆⠠⠾⠠⠀⠠⠷⠤⠂⠌⠜⠆⠻⠀⠻
⠠⠷⠂⠌⠜⠖⠻⠀⠤⠆⠌⠜⠖⠻⠀⠂⠌⠜⠖⠻⠠⠾⠀⠠⠷⠂⠌⠜⠒⠻⠀⠂⠌⠜⠒⠻⠀⠂⠌⠜⠒⠻⠠⠾

$$\begin{pmatrix} 3/5 & -4/5 \\ 4/5 & 3/5 \end{pmatrix}, \quad \begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix}, \quad \begin{pmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{6} & -2/\sqrt{6} & 1/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix}$$

\begin{pmatrix} 3/5 & -4/5 \\ 4/5 & 3/5 \end{pmatrix}, \quad \begin{pmatrix}
1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix}, \quad \begin{pmatrix}
-1/\sqrt{2} & 0 & 1/ \sqrt{2} \\ 1/\sqrt{6} & -2/\sqrt{6} & 1/\sqrt{6}
\\ 1/ \sqrt{3} & 1/ \sqrt{3} & 1/ \sqrt{3} \end{pmatrix}

2786. ⠸⠭ ⠨⠅ ⠷⠭⠂⠠⠄⠠⠭⠰⠝⠐⠾⠘⠞

$\mathbf{x}=(x_1, \ldots, x_n)^{\mathsf{t}}$

{\mathbf x}=(x_1, \ldots, x_n)^\transpose

2787. ⠸⠽ ⠨⠅ ⠷⠽⠂⠠⠄⠠⠽⠰⠝⠐⠾⠘⠞

$\mathbf{y}=(y_1, \ldots, y_n)^{\mathsf{t}}$

{\mathbf y}=(y_1, \ldots, y_n)^\transpose

2788. ⠠⡷⠭⠆⠭⠈⠱⠄ ⠸⠭⠠⠸⠽⠠⡷⠭⠆⠭⠈⠣⠄ ⠨⠅ ⠸⠭⠘⠞⠐⠸⠽ ⠨⠅ ⠷⠭⠂⠠⠭⠆⠠⠄⠠⠭⠰⠝⠐⠾⠭⠂
⠽⠆ ⠑⠄⠄⠄ ⠽⠰⠝⠐⠾ ⠨⠅ ⠭⠂⠽⠂⠬⠄⠄⠄⠬⠭⠰⠝⠽⠰⠝

$$\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^{\mathsf{t}} \mathbf{y} = (x_1, x_2, \ldots, x_n) \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = x_1 y_1 + \cdots + x_n y_n$$

\langle {\mathbf x}, {\mathbf y} \rangle = {\mathbf x}^\transpose
{\mathbf y} = (x_1, x_2, \ldots, x_n) \begin{pmatrix} y_1 \\ y_2
\\ \vdots \\ y_n \end{pmatrix} = x_1 y_1 + \cdots + x_n y_n

2789. ⠳⠳⠸⠭⠳⠳ ⠨⠅ ⠜⠠⡷⠭⠆⠭⠈⠱⠄ ⠸⠭⠠⠸⠭⠠⡷⠭⠆⠭⠈⠣⠄⠻ ⠨⠅ ⠜⠭⠂⠘⠆⠐⠬⠄⠄⠄⠬⠭⠰⠝⠘⠆⠐⠻

$\|\mathbf{x}\| = \sqrt{\langle \mathbf{x}, \mathbf{x} \rangle} = \sqrt{x_1^2 + \cdots + x_n^2}$

\| {\mathbf x} \| = \sqrt{\langle {\mathbf x}, {\mathbf x} \rangle}
= \sqrt{x_1^2 + \cdots + x_n^2}

2790. ⠳⠳⠸⠭⠤⠸⠽⠳⠳

$\|\mathbf{x} - \mathbf{y}\|$

\| {\mathbf x}-{\mathbf y} \|

2791. ⠸⠺

$\mathbf{w}$

{\mathbf w}

2792. ⠠⡷⠭⠆⠭⠈⠱⠄ ⠸⠭⠠⠸⠽⠠⡷⠭⠆⠭⠈⠣⠄ ⠨⠅ ⠠⡷⠭⠆⠭⠈⠱⠄ ⠸⠽⠠⠸⠭⠠⡷⠭⠆⠭⠈⠣⠄

$\langle \mathbf{x}, \mathbf{y} \rangle = \langle \mathbf{y}, \mathbf{x} \rangle$

\langle {\mathbf x}, {\mathbf y} \rangle = \langle {\mathbf y},
{\mathbf x} \rangle

2793. ⠠⡷⠭⠆⠭⠈⠱⠄ ⠸⠭⠠⠸⠽⠬⠸⠺⠠⡷⠭⠆⠭⠈⠣⠄ ⠨⠅ ⠠⡷⠭⠆⠭⠈⠱⠄ ⠸⠭⠠⠸⠽⠠⡷⠭⠆⠭⠈⠣⠄⠬⠠⡷⠭⠆⠭⠈⠱⠄ ⠸⠭⠠⠸

$\langle \mathbf{x}, \mathbf{y} + \mathbf{w} \rangle = \langle \mathbf{x}, \mathbf{y} \rangle + \langle \mathbf{x}, \mathbf{w} \rangle$

\langle {\mathbf x}, {\mathbf y} + {\mathbf w} \rangle = \langle
{\mathbf x}, {\mathbf y} \rangle + \langle {\mathbf x}, {\mathbf
w} \rangle

2794. ⠠⡷⠭⠆⠭⠈⠱⠄ ⠨⠁⠸⠭⠠⠸⠽⠠⡷⠭⠆⠭⠈⠣⠄ ⠨⠅ ⠠⡷⠭⠆⠭⠈⠱⠄ ⠸⠭⠠⠨⠁⠸⠽⠠⡷⠭⠆⠭⠈⠣⠄ ⠨⠅ ⠨⠁⠠⡷⠭⠆⠭⠈

$\langle \alpha \mathbf{x}, \mathbf{y} \rangle = \langle \mathbf{x}, \alpha \mathbf{y} \rangle = \alpha \langle \mathbf{x}, \mathbf{y} \rangle$

\langle \alpha {\mathbf x}, {\mathbf y} \rangle = \langle {\mathbf
x}, \alpha {\mathbf y} \rangle = \alpha \langle {\mathbf x}, {\mathbf
y} \rangle

2795. ⠄⡷⠭⠆⠶⠑⠦⠄ ⠇⠭⠄⠇⠭⠄⡷⠭⠆⠶⠑⠔⠄ ⠕ ⠱ ⠴

$\langle \mathbf{x}, \mathbf{x} \rangle \geq 0$

\langle {\mathbf x}, {\mathbf x} \rangle \geq 0

2796. ⠇⠭ ⠶ ⠴

$\mathbf{x} = 0$

{\mathbf x} = 0

2797. ⠄⡷⠭⠆⠶⠑⠦⠄ ⠇⠭⠄⠇⠽⠄⡷⠭⠆⠶⠑⠔⠄ ⠶ ⠴

$\langle \mathbf{x}, \mathbf{y} \rangle = 0$

\langle {\mathbf x}, {\mathbf y} \rangle = 0

2798. ⠇⠽ ⠶ ⠴

$\mathbf{y} = 0$

{\mathbf y} = 0

2799. ⠇⠭ ⠶ ⠫⠒⠄⠒⠂⠻ ⠋⠞

$\mathbf{x} = (3, 4)^{\mathsf{t}}$

{\mathbf x} =(3,4)^\transpose

2800. ⠠⠇⠒⠅⠆ ⠂⠠⠇⠒⠅⠆ ⠂⠻ ⠶ ⠑

$\sqrt{3^2 + 4^2} = 5$

\sqrt{3^2 + 4^2} = 5

2801. ⠠⠁ ⠶ ⠠⠫⠒⠠⠑⠑ ⠤⠒⠠⠑⠑⠠⠻ ⠠⠫⠒⠠⠑⠑ ⠒⠠⠑⠑⠠⠻

$A = \begin{pmatrix} 3/5 & -4/5 \\ 4/5 & 3/5 \end{pmatrix}$

A= \begin{pmatrix} 3/5 & -4/5 \\ 4/5 & 3/5 \end{pmatrix}

2802. ⠠⠁ ⠇⠭ ⠶ ⠫⠤⠶⠠⠑⠑⠠24⠠⠑⠑⠠⠻ ⠋⠞

$A\mathbf{x} = (-7/5, 24/5)^{\mathsf{t}}$

A{\mathbf x} = (-7/5,24/5)^\transpose

2803. ⠙⠑⠞⠣⠠⠁ ⠠⠁ ⠋⠞ ⠂⠻ ⠶ ⠙⠑⠞⠣⠠⠊⠻ ⠶ ⠂

$\det(AA^{\mathsf{t}}) = \det(I) = 1$

\det(A A^\transpose) = \det(I) = 1

2804. ⠙⠑⠞⠣⠠⠁⠻ ⠶ ⠙⠑⠞⠣⠠⠁ ⠋⠞ ⠂⠻

$\det(A) = \det(A^{\mathsf{t}})$

\det(A) = \det(A^\transpose)

2805. ⠇⠁ ⠰⠚⠂ ⠶ ⠈ ⠰⠂⠉⠂⠈ ⠒⠉⠂⠱⠄⠄⠄ ⠈ ⠰⠝⠉

$$\mathbf{a}_j = \begin{pmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{nj} \end{pmatrix}$$

{\mathbf a}_j = \begin{pmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{nj}
\end{pmatrix}

2806. ⠠⠁⠀⠨⠅⠀⠷⠁⠰⠊⠚⠾

$A = (a_{ij})$

A= (a_{ij})

2807. ⠠⠁⠠⠁⠘⠞⠐⠀⠨⠅⠀⠠⠊

$AA^{\mathsf{t}} = I$

AA^\transpose = I

2808. ⠠⡷⠭⠆⠶⠑⠦⠄⠏⠀⠹⠂⠠⠏⠀⠭⠌⠂⠠⡷⠭⠆⠶⠑⠦⠄⠀⠨⠅⠀⠨⠙⠰⠗⠎

$\langle \mathbf{a}_r, \mathbf{a}_s \rangle = \delta_{rs}$

\langle {\mathbf a}_r, {\mathbf a}_s \rangle = \delta_{rs}

2809. ⠨⠙⠰⠗⠎⠐⠀⠨⠅⠀StartLayoutEnlarged⠸⠳1stRow⠸⠳⠼⠂⠀⠗⠀⠨⠅⠀⠎2ndRow⠸⠳⠼⠴⠀⠗⠀⠌⠨⠅⠀⠎Enc

$$\delta_{rs} = \begin{cases} 1 & r = s \\ 0 & r \neq s \end{cases}$$

\delta_{rs} = \begin{cases} 1 & r = s \\ 0 & r \neq s \end{cases}

2810. ⠳⠳⠠⠁⠸⠭⠤⠠⠁⠸⠽⠳⠳⠀⠨⠅⠀⠳⠳⠸⠭⠤⠸⠽⠳⠳

$\|A\mathbf{x} - A\mathbf{y}\| = \|\mathbf{x} - \mathbf{y}\|$

\| A{\mathbf x}- A{\mathbf y} \| =\| {\mathbf x}- {\mathbf y} \|

2811. ⠳⠳⠠⠁⠸⠭⠳⠳⠀⠨⠅⠀⠳⠳⠸⠭⠳⠳

$\|A\mathbf{x}\| = \|\mathbf{x}\|$

\| A{\mathbf x} \| =\| {\mathbf x} \|

2812. ⠠⡷⠭⠆⠶⠑⠦⠄⠠⠁⠸⠭⠠⠠⠁⠸⠽⠠⡷⠭⠆⠶⠑⠦⠄⠀⠨⠅⠀⠠⡷⠭⠆⠶⠑⠦⠄⠸⠭⠠⠸⠽⠠⡷⠭⠆⠶⠑⠦⠄

$\langle A\mathbf{x}, A\mathbf{y} \rangle = \langle \mathbf{x}, \mathbf{y} \rangle$

\langle A{\mathbf x}, A{\mathbf y} \rangle = \langle {\mathbf x},{\mathbf
y} \rangle

2813. ⠠⡷⠭⠆⠶⠑⠦⠄⠠⠁⠸⠭⠠⠠⠁⠸⠽⠠⡷⠭⠆⠶⠑⠦⠄⠀⠨⠅⠀⠠⡷⠭⠆⠶⠑⠦⠄⠸⠭⠠⠸⠽⠠⡷⠭⠆⠶⠑⠦⠄

$\langle A\mathbf{x}, A\mathbf{y} \rangle = \langle \mathbf{x}, \mathbf{y} \rangle$

\langle A{\mathbf x}, A {\mathbf y} \rangle = \langle {\mathbf x},
{\mathbf y} \rangle

2814. ⠳⠳⠠⠁⠸⠭⠤⠠⠁⠸⠽⠳⠳⠀⠨⠅⠀⠳⠳⠸⠭⠤⠸⠽⠳⠳

$\|A\mathbf{x} - A\mathbf{y}\| = \|\mathbf{x} - \mathbf{y}\|$

\| A{\mathbf x}- A{\mathbf y} \| = \| {\mathbf x}- {\mathbf y} \|

2815. ⠳⠳⠠⠁⠸⠭⠳⠳⠀⠨⠅⠀⠳⠳⠸⠭⠳⠳

$\|A\mathbf{x}\| = \|\mathbf{x}\|$

\| A{\mathbf x} \| = \| {\mathbf x}\|

2816. ⠷⠆⠾⠀⠫⠒⠒⠕⠕⠀⠷⠒⠾

$(2) \Rightarrow (3)$

(2) \Rightarrow (3)

2817. ⠷⠒⠾⠀⠫⠒⠒⠕⠕⠀⠷⠆⠾

$(3) \Rightarrow (2)$

(3) \Rightarrow (2)

2818. ⠄⡳⠭⠆ ⠶⠑⠤⠄ ⠸⠭⠠⠷⠠⠁⠘⠞⠐⠠⠁⠤⠠⠊⠾⠸⠭⠄⡳⠭⠆⠶⠑⠔⠄ ⠭ ⠴

$\langle \mathbf{x}, (A^{\mathsf{t}} A - I)\mathbf{x} \rangle = 0$

```
\langle {\mathbf x}, (A^\transpose A - I){\mathbf x} \rangle = 0
```

2819. ⠠⠁ ⠘⠞⠐⠠⠁⠤⠠⠊ ⠭ ⠴

$A^{\mathsf{t}} A - I = 0$

```
A^\transpose A -I = 0
```

2820. ⠷⠒⠾ ⠫⠒⠒⠕⠕ ⠷⠲⠾

$(3) \Rightarrow (4)$

```
(3) \Rightarrow (4)
```

2821. ⠷⠲⠾ ⠫⠒⠒⠕⠕ ⠷⠢⠾

$(4) \Rightarrow (5)$

```
(4) \Rightarrow (5)
```

2822. ⠳⠳⠠⠁⠸⠭⠳⠳ ⠭ ⠳⠳⠠⠁⠸⠭⠤⠠⠁⠸⠽⠳⠳ ⠭ ⠳⠳⠸⠭⠤⠸⠽⠳⠳ ⠭ ⠳⠳⠸⠭⠳⠳

$\|A\mathbf{x}\| = \|A\mathbf{x} - A\mathbf{y}\| = \|\mathbf{x} - \mathbf{y}\| = \|\mathbf{x}\|$

```
\| A{\mathbf x}\| = \| A{\mathbf x}- A{\mathbf y} \| = \| {\mathbf
x}- {\mathbf y} \| = \| {\mathbf x} \|
```

2823. ⠷⠢⠾ ⠫⠒⠒⠕⠕ ⠷⠒⠾

$(5) \Rightarrow (3)$

```
(5) \Rightarrow (3)
```

2824. ⠄⡳⠭⠆ ⠶⠑⠤⠄ ⠸⠭⠠⠸⠽⠄⡳⠭⠆⠶⠑⠔⠄ ⠭ one-half ⠈⠷⠳⠳⠸⠭⠬⠸⠽⠳⠳⠘⠆ ⠐⠤⠳⠳⠸⠭⠳⠳⠘⠆ ⠐⠤⠳⠳⠸⠽⠳⠳

$\langle \mathbf{x}, \mathbf{y} \rangle = \frac{1}{2} \left[\|\mathbf{x} + \mathbf{y}\|^2 - \|\mathbf{x}\|^2 - \|\mathbf{y}\|^2 \right]$

```
\langle {\mathbf x}, {\mathbf y} \rangle = \frac{1}{2} \left[ \|{\mathbf
x} +{\mathbf y}\|^2 - \|{\mathbf x}\|^2 - \|{\mathbf y}\|^2 \right]
```

2825. ⠠⠕⠷⠆⠾

$O(2)$

```
O(2)
```

2826. ⠈⠠⠗⠘⠆

$\mathbb{R}^2$

```
\mathbb R^2
```

2827. ⠠⠁ ⠈⠑ ⠠⠕⠷⠆⠾

$A \in O(2)$

```
A \in O(2)
```

2828. ⠸⠑⠂ ⠭ ⠷⠂⠠⠴⠾⠘⠞

$\mathbf{e}_1 = (1, 0)^{\mathsf{t}}$

```
{\mathbf e}_1 = (1, 0)^\transpose
```

2829. ⠸⠑⠆ ⠭ ⠷⠴⠠⠂⠾⠘⠞

$\mathbf{e}_2 = (0, 1)^{\mathsf{t}}$

```
{\mathbf e}_2 = (0, 1)^\transpose
```

2830. ⠠⠁⠸⠑⠂ ⠨⠅ ⠷⠁⠠⠃⠾⠘⠞

$A\mathbf{e}_1 = (a,b)^{\mathsf{t}}$

A{\mathbf e}_1 = (a,b)^\transpose

2831. ⠁⠘⠆⠐⠬⠃⠘⠆⠐ ⠨⠅ ⠂

$a^2 + b^2 = 1$

a^2 + b^2 = 1

2832. ⠠⠁⠸⠑⠆ ⠨⠅ ⠬⠤⠷⠤⠃⠠⠁⠾⠘⠞

$A\mathbf{e}_2 = \pm(-b,a)^{\mathsf{t}}$

A{\mathbf e}_2 = \pm(-b, a)^\transpose

2833. ⠠⠁⠸⠑⠆ ⠨⠅ ⠷⠤⠃⠠⠁⠾⠘⠞

$A\mathbf{e}_2 = (-b,a)^{\mathsf{t}}$

A{\mathbf e}_2 = (-b, a)^\transpose

2834. ⠠⠁ ⠨⠅ ⠠⠷⠁ ⠤⠃⠠⠾ ⠠⠷⠃ ⠁⠠⠾ ⠨⠅ ⠠⠷⠉⠕⠎ ⠨⠹ ⠤⠎⠊⠝ ⠨⠹⠠⠾
⠠⠷⠎⠊⠝ ⠨⠹ ⠉⠕⠎ ⠨⠹⠠⠾⠠

$A = \begin{pmatrix} a & -b \\ b & a \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix},$

A = \begin{pmatrix} a & -b \\ b & a \end{pmatrix} = \begin{pmatrix}
\cos \theta & - \sin \theta \\ \sin \theta & \cos \theta \end{pmatrix},

2835. ⠴ ⠅⠱ ⠨⠹ ⠐⠅ ⠆⠨⠏

$0 \leq \theta < 2\pi$

0 \leq \theta \lt 2 \pi

2836. ⠠⠁⠸⠑⠆ ⠨⠅ ⠷⠃⠠⠤⠁⠾⠘⠞

$A\mathbf{e}_2 = (b,-a)^{\mathsf{t}}$

A{\mathbf e}_2 = (b, -a)^\transpose

2837. ⠠⠃ ⠨⠅ ⠠⠷⠁ ⠃⠠⠾ ⠠⠷⠃ ⠤⠁⠠⠾ ⠨⠅ ⠠⠷⠉⠕⠎ ⠨⠹ ⠎⠊⠝ ⠨⠹⠠⠾ ⠠⠷⠎⠊⠝ ⠨⠹ ⠤⠉⠕⠎ ⠨⠹⠠⠾⠲

$B = \begin{pmatrix} a & b \\ b & -a \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}.$

B = \begin{pmatrix} a & b \\ b & -a \end{pmatrix} = \begin{pmatrix}
\cos \theta & \sin \theta \\ \sin \theta & -\cos \theta \end{pmatrix}.

2838. ⠙⠑⠞⠠⠃ ⠨⠅ ⠤⠂

$\det B = -1$

\det B = -1

2839. ⠠⠃⠘⠆⠐ ⠨⠅ ⠠⠷⠼⠂ ⠼⠴⠠⠾ ⠠⠷⠼⠴ ⠼⠂⠠⠾⠲

$B^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$

B^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.

2840. ⠠⠉ ⠨⠅ ⠠⠷⠼⠂ ⠼⠴⠠⠾ ⠠⠷⠼⠴ ⠤⠂⠠⠾⠠

$C = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$

C = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},

2841. ⠠⠃⠀⠨⠅⠀⠠⠁⠠⠉

$B = AC$

B = AC

2842. ⠈⠹

ℓ

\ell

2843. ⠠⠎⠠⠕⠷⠝⠾

$SO(n)$

SO(n)

2844. ⠠⠑⠷⠝⠾

$E(n)$

E(n)

2845. ⠷⠠⠁⠠⠸⠭⠾

$(A, \mathbf{x})$

(A, {\mathbf x})

2846. ⠷⠠⠁⠠⠸⠭⠾⠷⠠⠃⠠⠸⠽⠾⠀⠨⠅⠀⠷⠠⠁⠠⠃⠠⠠⠁⠸⠽⠬⠸⠭⠾

$(A, \mathbf{x})(B, \mathbf{y}) = (AB, A\mathbf{y} + \mathbf{x})$

(A, {\mathbf x}) (B, {\mathbf y}) = (AB, A {\mathbf y} +{\mathbf
x})

2847. ⠷⠠⠊⠠⠸⠴⠾

$(I, \mathbf{0})$

(I,{\mathbf 0})

2848. ⠷⠠⠁⠘⠤⠂⠐⠠⠤⠠⠁⠘⠤⠂⠐⠸⠭⠾

$(A^{-1}, -A^{-1}\mathbf{x})$

(A^{-1}, -A^{-1} {\mathbf x})

2849. ⠳⠳⠋⠷⠸⠭⠾⠤⠋⠷⠸⠽⠾⠳⠳⠀⠨⠅⠀⠳⠳⠸⠭⠤⠸⠽⠳⠳

$\|f(\mathbf{x}) - f(\mathbf{y})\| = \|\mathbf{x} - \mathbf{y}\|$

\| f({\mathbf x}) - f({\mathbf y}) \| =\|{\mathbf x} - {\mathbf
y} \|

2850. ⠸⠭⠠⠸⠽⠈⠑⠈⠠⠗⠘⠝

$\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$

{\mathbf x}, {\mathbf y} \in {\mathbb R}^n

2851. ⠠⠞⠰⠸⠽⠐⠷⠸⠭⠾⠀⠨⠅⠀⠸⠭⠬⠸⠽

$T_{\mathbf{y}}(\mathbf{x}) = \mathbf{x} + \mathbf{y}$

T_{\mathbf y}({\mathbf x}) = {\mathbf x} + {\mathbf y}

2852. ⠋⠷⠴⠾⠀⠨⠅⠀⠴

$f(0) = 0$

f(0) = 0

2853. ⠳⠳⠋⠷⠸⠭⠾⠳⠳ ⠨⠅ ⠳⠳⠸⠭⠳⠳

$\| f(\mathbf{x})\| = \| \mathbf{x} \|$

\| f({\mathbf x})\| = \| {\mathbf x} \|

2854. ⠈⠣⠋⠷⠸⠭⠾⠠ ⠋⠷⠸⠽⠾⠈⠜ ⠨⠅ ⠈⠣⠸⠭⠠ ⠸⠽⠈⠜

$\langle f(\mathbf{x}), f(\mathbf{y}) \rangle = \langle \mathbf{x}, \mathbf{y} \rangle$

\langle f({\mathbf x}), f({\mathbf y}) \rangle = \langle {\mathbf
x}, {\mathbf y} \rangle

2855. ⠸⠑⠂

$\mathbf{e}_1$

{\mathbf e}_1

2856. ⠸⠑⠆

$\mathbf{e}_2$

{\mathbf e_2}

2857. ⠷⠂ ⠠⠴⠾ ⠘⠞

$(1, 0)^{\mathsf{t}}$

(1, 0)^\transpose

2858. ⠷⠴⠠ ⠂⠾ ⠘⠞

$(0, 1)^{\mathsf{t}}$

(0, 1)^\transpose

2859. ⠸⠭ ⠨⠅ ⠷⠭⠂⠠ ⠭⠆⠾ ⠨⠅ ⠭⠂ ⠸⠑⠂ ⠬⠭⠆ ⠸⠑⠆

$\mathbf{x} = (x_1, x_2) = x_1 \mathbf{e}_1 + x_2 \mathbf{e}_2$

{\mathbf x} = (x_1, x_2) = x_1 {\mathbf e}_1 + x_2 {\mathbf e}_2

2860. ⠋⠷⠸⠭⠾ ⠨⠅ ⠈⠣⠋⠷⠸⠭⠾⠠ ⠋⠷⠸⠑⠂⠾⠈⠜⠋⠷⠸⠑⠂⠾⠬⠈⠣⠋⠷⠸⠭⠾⠠ ⠋⠷⠸⠑

$f(\mathbf{x}) = \langle f(\mathbf{x}), f(\mathbf{e}_1) \rangle f(\mathbf{e}_1) + \langle f(\mathbf{x}), f(\mathbf{e}_2) \rangle f(\mathbf{e}_2) = x_1 f(\mathbf{e}_1) + x_2 f(\mathbf{e}_2)$

f({\mathbf x}) = \langle f({\mathbf x}), f({\mathbf e}_1) \rangle
f({\mathbf e}_1) + \langle f({\mathbf x}), f({\mathbf e}_2) \rangle
f({\mathbf e}_2) = x_1 f({\mathbf e}_1)+x_2 f({\mathbf e}_2)

2861. ⠠⠞⠰⠸⠭⠐⠋

$T_{\mathbf{x}} f$

T_{\mathbf x} f

2862. ⠠⠞⠰⠸⠭⠐⠋⠷⠸⠽⠾ ⠨⠅ ⠠⠁⠸⠽

$T_{\mathbf{x}} f(\mathbf{y}) = A \mathbf{y}$

T_{\mathbf x} f({\mathbf y}) = A {\mathbf y}

2863. ⠋⠷⠸⠽⠾ ⠨⠅ ⠠⠁⠸⠽⠬⠸⠭

$f(\mathbf{y}) = A \mathbf{y} + \mathbf{x}$

f({\mathbf y}) = A {\mathbf y} + {\mathbf x}

2864. ⠋⠷⠛⠷⠸⠽⠾⠾ ⠨⠅ ⠋⠷⠠⠃⠸⠽⠬⠸⠭⠆⠾ ⠨⠅ ⠠⠁⠠⠃⠸⠽⠬⠠⠁⠸⠭⠆⠬⠸⠭⠂

$f(g(\mathbf{y})) = f(B \mathbf{y} + \mathbf{x}_2) = AB \mathbf{y} + A \mathbf{x}_2 + \mathbf{x}_1$

f(g({\mathbf y})) = f(B {\mathbf y} + {\mathbf x}_2) = AB {\mathbf
y} + A{\mathbf x}_2 + {\mathbf x}_1

2865. ⠠⠑⠷⠆⠾

$E(2)$

```
E(2)
```

2866. ⠠⠭ ⠸⠐⠅ ⠈⠠⠗⠘⠝

$X \subset {\mathbb R}^n$

```
X \subset {\mathbb R}^n
```

2867. ⠈⠠⠗⠘⠂

${\mathbb R}^1$

```
{\mathbb R}^1
```

2868. ⠠⠗⠰⠨⠹⠐ ⠨⠅ ⠠⠷⠉⠕⠎ ⠨⠹ ⠤⠎⠊⠝ ⠨⠹⠠⠾ ⠠⠷⠎⠊⠝ ⠨⠹ ⠉⠕⠎ ⠨⠹⠠⠾

$R_{\theta} = \begin{pmatrix} \cos \theta & - \sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$

```
R_{\theta} = \begin{pmatrix} \cos \theta & - \sin \theta \\ \sin
\theta & \cos \theta \end{pmatrix}
```

2869. ⠠⠞⠰⠨⠋⠐ ⠨⠅ ⠠⠷⠉⠕⠎ ⠨⠋ ⠤⠎⠊⠝ ⠨⠋⠠⠾ ⠠⠷⠎⠊⠝ ⠨⠋ ⠉⠕⠎ ⠨⠋⠠⠾⠠⠷⠂ ⠴⠠⠾
⠠⠷⠴ ⠤⠂ ⠠⠾ ⠨⠅ ⠠⠷⠉⠕⠎ ⠨⠋ ⠎⠊⠝ ⠨⠋⠠⠾ ⠠⠷⠎⠊⠝ ⠨⠋ ⠤⠉⠕⠎ ⠨⠋⠠⠾

$T_{\phi} = \begin{pmatrix} \cos \phi & - \sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} \cos \phi & \sin \phi \\ \sin \phi & - \cos \phi \end{pmatrix}$

```
T_{\phi} = \begin{pmatrix} \cos \phi & - \sin \phi \\ \sin \phi
& \cos \phi \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
= \begin{pmatrix} \cos \phi & \sin \phi \\ \sin \phi & - \cos \phi
\end{pmatrix}
```

2870. ⠙⠑⠞⠷⠠⠗⠰⠨⠹⠐⠾ ⠨⠅ ⠂

$\det(R_{\theta})=1$

```
\det(R_{\theta})=1
```

2871. ⠙⠑⠞⠷⠠⠞⠰⠨⠋⠐⠾ ⠨⠅ ⠤⠂

$\det(T_{\phi})=-1$

```
\det(T_{\phi})=-1
```

2872. ⠠⠞⠰⠨⠋⠘⠆⠐ ⠨⠅ ⠠⠊

$T_{\phi}^2=I$

```
T_{\phi}^2=I
```

2873. ⠨⠹⠴

θ_0

```
\theta_0
```

2874. ⠠⠗⠰⠨⠹⠰⠴

R_{θ_0}

```
R_{\theta_0}
```

2875. ⠨⠹⠂

θ_1

```
\theta_1
```

2876. ⠝ ⠨⠹⠴

$n\theta_0$

n \theta_0

2877. ⠷⠝⠬⠂ ⠾ ⠨⠹⠴

$(n+1)\theta_0$

(n+1) \theta_0

2878. ⠷⠝⠬⠂ ⠾ ⠨⠹⠴⠤ ⠨⠹⠂

$(n+1)\theta_0 - \theta_1$

(n+1) \theta_0 - \theta_1

2879. ⠨⠋⠒ ⠠⠛ ⠫⠕ StartSet⠤⠂ ⠠⠂ EndSet

$\phi : G \rightarrow \{-1, 1\}$

\phi : G \rightarrow \{-1, 1\}

2880. ⠳ ⠠⠛⠌⠅ ⠑⠗ ⠨⠋⠳ ⠨⠅ ⠆

$|G/ \ker \phi|=2$

|G/ \ker \phi|=2

2881. ⠠⠗ ⠰ ⠨⠹ ⠐ ⠤ ⠄ ⠠⠗ ⠰ ⠨⠹ ⠘⠝⠤⠂ ⠐ ⠄ ⠠⠞ ⠠⠗ ⠰ ⠨⠹ ⠐ ⠤ ⠄ ⠠⠞ ⠠⠗ ⠰ ⠨⠹ ⠘⠝⠤⠂

$R_{\theta}, \ldots, R_{\theta}^{n-1}, TR_{\theta}, \ldots, TR_{\theta}^{n-1}$

R_{\theta}, \ldots, R_{\theta}^{n-1}, TR_{\theta}, \ldots, TR_{\theta}^{n-1}

2882. ⠠⠞ ⠠⠗ ⠰ ⠨⠹ ⠐ ⠠⠞ ⠨⠅ ⠠⠗ ⠰ ⠨⠹ ⠘⠤⠂

$TR_{\theta}T = R_{\theta}^{-1}$

TR_{\theta}T = R_{\theta}^{-1}

2883. ⠈ ⠠⠗ ⠘⠒

$\mathbb R^3$

\mathbb R^3

2884. ⠍ ⠸⠭⠬⠝ ⠸⠽

$m {\mathbf x} + n {\mathbf y}$

m {\mathbf x} + n {\mathbf y}

2885. ⠷⠂ ⠠⠂ ⠾ ⠘⠞

$(1,1)^{\mathsf{t}}$

(1,1)^\transpose

2886. ⠷⠆ ⠠⠴⠾ ⠘⠞

$(2,0)^{\mathsf{t}}$

(2,0)^\transpose

2887. ⠷⠤⠂ ⠠⠂ ⠾ ⠘⠞

$(-1, 1)^{\mathsf{t}}$

(-1, 1)^\transpose

2888. ⠷⠤⠂ ⠠⠤⠂ ⠾ ⠘⠞

$(-1, -1)^{\mathsf{t}}$

(-1, -1)^\transpose

2889. StartSet ⠸⠭⠂ ⠠⠸⠭⠆ EndSet

$\{\mathbf{x}_1, \mathbf{x}_2\}$

\{ {\mathbf x}_1, {\mathbf x}_2 \}

2890. StartSet ⠸⠽⠂ ⠠⠸⠽⠆ EndSet

$\{\mathbf{y}_1, \mathbf{y}_2\}$

\{ {\mathbf y}_1, {\mathbf y}_2 \}

2891. ⠍⠐

α_1

\alpha_1

2892. ⠍⠰

α_2

\alpha_2

2893. ⠝⠐

β_1

\beta_1

2894. ⠝⠰

β_2

\beta_2

2895. ⠠⠥ ⠭ ⠠⠷⠍⠐ ⠍⠰ ⠠⠾ ⠠⠷⠝⠐ ⠝⠰ ⠠⠾

$U = \begin{pmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{pmatrix}$

U = \begin{pmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{pmatrix}

2896. ⠸⠭⠂

$\mathbf{x}_1$

{\mathbf x}_1

2897. ⠸⠭⠆

$\mathbf{x}_2$

{\mathbf x}_2

2898. ⠸⠽⠂

$\mathbf{y}_1$

{\mathbf y}_1

2899. ⠸⠽⠆

$\mathbf{y}_2$

{\mathbf y}_2

2900. ⠠⠥ ⠘⠤⠂

U^{-1}

U^{-1}

2901. ⠠⠥⠘⠤⠂⠐⠸⠽⠂ ⠸⠽⠆ ⠨⠅ ⠸⠭⠂ ⠸⠭⠆

$$U^{-1} \begin{pmatrix} {\mathbf y}_1 \\ {\mathbf y}_2 \end{pmatrix} = \begin{pmatrix} {\mathbf x}_1 \\ {\mathbf x}_2 \end{pmatrix}$$

```
U^{-1} \begin{pmatrix} {\mathbf y}_1 \\ {\mathbf y}_2 \end{pmatrix}
= \begin{pmatrix} {\mathbf x}_1 \\ {\mathbf x}_2 \end{pmatrix}
```

2902. ⠠⠥⠠⠥⠘⠤⠂⠐ ⠨⠅ ⠠⠊

$$UU^{-1} = I$$

```
U U^{-1} = I
```

2903. ⠙⠑⠞⠷⠠⠥⠠⠥⠘⠤⠂⠐⠾ ⠨⠅ ⠙⠑⠞⠷⠠⠥⠾⠙⠑⠞⠷⠠⠥⠘⠤⠂⠐⠾ ⠨⠅ ⠂⠸⠆

$$\det(UU^{-1}) = \det(U) \det(U^{-1}) = 1;$$

```
\det(U U^{-1}) =\det(U) \det( U^{-1}) = 1;
```

2904. ⠙⠑⠞⠷⠠⠥⠾ ⠨⠅ ⠬⠤⠂

$$\det(U) = \pm 1$$

```
\det(U) = \pm 1
```

2905. ⠬⠤⠂

$$\pm 1$$

```
\pm 1
```

2906. ⠠⠷⠼⠒ ⠼⠂ ⠠⠾ ⠠⠷⠼⠢ ⠼⠆ ⠠⠾

$$\begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$$

```
\begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}
```

2907. ⠠⠛ ⠸⠐⠅ ⠠⠑⠷⠆⠾

$$G \subset E(2)$$

```
G \subset E(2)
```

2908. StartSet⠷ ⠠⠊ ⠠⠞⠾⠈⠒⠞ ⠈⠑ ⠠⠇ EndSet

$$\{ (I, t) : t \in L \}$$

```
\{ (I, t) : t \in L \}
```

2909. ⠠⠛⠴ ⠨⠅ StartSet ⠠⠁ ⠈⠒⠷ ⠠⠁ ⠠⠃ ⠾ ⠈⠑ ⠠⠛forsome⠃ EndSet

$$G_0 = \{A : (A,b) \in G \text{ for some } b \}$$

```
G_0 = \{A : (A,b) \in G \text{ for some } b \}
```

2910. ⠠⠛⠴

$$G_0$$

```
G_0
```

2911. ⠷ ⠠⠁ ⠠ ⠸⠽⠾

$$(A, {\mathbf y})$$

```
(A, {\mathbf y})
```

2912. ⠷ ⠠⠊ ⠠ ⠠⠁ ⠸⠭⠾

$$(I, A {\mathbf x})$$

```
(I, A {\mathbf x})
```

2913. ⠠⠁ ⠸⠭

$A\mathbf{x}$

A {\mathbf x}

2914. ⠠⠛⠌⠠⠞ ⠈⠱⠈⠱ ⠠⠛⠴

$G/T \cong G_0$

G/T \cong G_0

2915. ⠝ ⠶ ⠂ ⠠⠼⠆ ⠠⠼⠒ ⠠⠼⠲ ⠠⠼⠖

$n = 1, 2, 3, 4, 6$

n = 1, 2, 3, 4, 6

2916. ⠈ ⠠⠵⠂

$\mathbb{Z}_1$

{\mathbb Z}_1

2917. ⠠⠙⠂

D_1

D_1

2918. ⠠⠙⠆

D_2

D_2

2919. ⠈ ⠠⠗⠘⠲

$\mathbb{R}^4$

{\mathbb R}^4

2920. ⠈ ⠠⠗⠘⠢

$\mathbb{R}^5$

{\mathbb R}^5

2921. ⠠⡷⠛⠅ ⠶⠈⠐⠤⠠ ⠇⠛ ⠄⠇⠻⠄ ⡷⠛⠅ ⠶⠈⠐⠔⠄ ⠒ one-half ⠈⠶⠳⠳ ⠇⠛⠤ ⠇⠻⠳⠳ ⠑ ⠐⠤⠳⠳ ⠇⠛⠳⠳ ⠑ ⠐⠤⠳⠳ ⠇⠻⠳⠳

$\langle \mathbf{x}, \mathbf{y} \rangle = \frac{1}{2} \left[\|\mathbf{x} + \mathbf{y}\|^2 - \|\mathbf{x}\|^2 - \| \mathbf{y}\|^2 \right]$

\langle {\mathbf x}, {\mathbf y} \rangle = \frac{1}{2} \left[\|{\mathbf x} + {\mathbf y}\|^2 - \|{\mathbf x}\|^2 - \| {\mathbf y}\|^2 \right]

2922. ⠠⠷⠂⠌⠜⠆⠻ ⠤⠂⠌⠜⠆⠻ ⠠⠾ ⠠⠷⠂⠌⠜⠆⠻ ⠂⠌⠜⠆⠻ ⠠⠾

$$\begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

\begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}

2923. ⠠⠷⠂⠌⠜⠢⠻ ⠆⠌⠜⠢⠻ ⠠⠾ ⠠⠷⠤⠆⠌⠜⠢⠻ ⠂⠌⠜⠢⠻ ⠠⠾

$$\begin{pmatrix} 1 / \sqrt{5} & 2 / \sqrt{5} \\ - 2 /\sqrt{5} & 1/ \sqrt{5} \end{pmatrix}$$

\begin{pmatrix} 1 / \sqrt{5} & 2 / \sqrt{5} \\ - 2 /\sqrt{5} & 1/ \sqrt{5} \end{pmatrix}

2924. ⠠⠷⠲⠌⠜⠢⠻ ⠼⠴ ⠒⠌⠜⠢⠻⠠⠾ ⠠⠷⠤⠒⠌⠜⠢⠻ ⠼⠴ ⠲⠌⠜⠢⠻⠠⠾ ⠠⠷⠼⠴ ⠤⠂ ⠼⠴⠠⠾

$$\begin{pmatrix} 4/\sqrt{5} & 0 & 3/\sqrt{5} \\ -3/\sqrt{5} & 0 & 4/\sqrt{5} \\ 0 & -1 & 0 \end{pmatrix}$$

\begin{pmatrix} 4/ \sqrt{5} & 0 & 3 / \sqrt{5} \\ -3 / \sqrt{5}
& 0 & 4 / \sqrt{5} \\ 0 & -1 & 0 \end{pmatrix}

2925. ⠠⠷⠂⠌⠒ ⠆⠌⠒ ⠤⠆⠌⠒⠠⠾ ⠠⠷⠤⠆⠌⠒ ⠆⠌⠒ ⠂⠌⠒⠠⠾ ⠠⠷⠤⠆⠌⠒ ⠂⠌⠒ ⠆⠌⠒⠠⠾

$$\begin{pmatrix} 1/3 & 2/3 & -2/3 \\ -2/3 & 2/3 & 1/3 \\ -2/3 & 1/3 & 2/3 \end{pmatrix}$$

\begin{pmatrix} 1/3 & 2/3 & - 2/3 \\ - 2/3 & 2/3 & 1/3 \\ -2/3 &
1/3 & 2/3 \end{pmatrix}

2926. ⠠⠎⠠⠕⠷⠆⠾

$SO(2)$

SO(2)

2927. ⠠⠕⠷⠒⠾

$O(3)$

O(3)

2928. ⠠⠑⠷⠝⠾ ⠨⠅ StartSet⠷⠠⠁⠠⠸⠭⠾⠱ ⠠⠁ ⠈⠑ ⠠⠕⠷⠝⠾and⠸⠭⠈⠑⠈⠠⠗⠘⠝⠐EndSet

$E(n) = \{(A, \mathbf{x}) : A \in O(n) \text{ and } \mathbf{x} \in \mathbb{R}^n\}$

E(n) = \{(A, {\mathbf x}) : A \in O(n) \text{ and } {\mathbf x}
\in {\mathbb R}^n \}

2929. StartSet⠷⠷⠆⠠⠂⠾⠠⠷⠂⠠⠂⠾EndSet

$\{(2, 1), (1, 1)\}$

\{ (2,1), (1,1) \}

2930. StartSet⠷12 ⠠⠢⠾⠠⠷⠶⠠⠒⠾EndSet

$\{(12, 5), (7, 3)\}$

\{ (12, 5), (7, 3) \}

2931. ⠠⠷⠢ ⠼⠆⠠⠾ ⠠⠷⠼⠆ ⠼⠂⠠⠾

$$\begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}$$

\begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}

2932. ⠠⠛⠌⠠⠞

G/T

G/T

2933. ⠠⠁ ⠈⠑ ⠠⠎⠠⠇⠆⠷⠈⠠⠗⠾

$A \in SL_2(\mathbb{R})$

A \in SL_2({\mathbb R})

2934. ⠠⠁ ⠸⠭

$A\mathbf{x}$

A{\mathbf x}

2935. ⠠⠁ ⠸⠽

$A\mathbf{y}$

A{\mathbf y}

2936. ⠙⠑⠞⠱⠠⠕⠷⠝⠾ ⠫⠕ ⠈⠠⠗⠘⠈⠼

$\det : O(n) \rightarrow \mathbb{R}^*$

\det : O(n) \rightarrow {\mathbb R}^*

2937. ⠸⠭ ⠌⠨⠅ ⠴

$\mathbf{x} \neq 0$

{\mathbf x} \neq 0

2938. ⠸⠭ ⠨⠅ ⠷⠭⠂⠠⠭⠆⠾

$\mathbf{x} = (x_1, x_2)$

{\mathbf x} = (x_1, x_2)

2939. ⠭⠂⠘⠆⠐⠬⠭⠆⠘⠆⠐ ⠨⠅ ⠂

$x_1^2 + x_2^2 = 1$

x_1^2 + x_2^2 = 1

2940. ⠠⠓⠨⠩⠠⠝ ⠨⠅ ⠨⠷⠊⠙⠨⠾

$H \cap N = \{\mathrm{id}\}$

H \cap N = \{ \identity \}

2941. ⠠⠓⠠⠝ ⠨⠅ ⠠⠛

$HN = G$

HN=G

2942. ⠠⠁⠒

A_3

A_3

2943. ⠠⠓ ⠨⠅ StartSet⠷⠂⠾⠠⠷12⠾EndSet

$H = \{(1), (12)\}$

H = \{(1), (12) \}

2944. ⠏Baseline⠖⠍

$p6m$

p6m

2945. ⠠⠛ ⠨⠅ ⠠⠓⠰⠝⠐⠸⠕ ⠠⠓⠰⠝⠤⠂⠐⠸⠕ ⠄⠄⠄ ⠸⠕ ⠠⠓⠂ ⠸⠕ ⠠⠓⠴ ⠨⠅ StartSet⠑EndSet

$G = H_n \supset H_{n-1} \supset \cdots \supset H_1 \supset H_0 = \{e\}$

G = H_n \supset H_{n - 1} \supset \cdots \supset H_1 \supset H_0
= \{ e \}

2946. ⠠⠓⠰⠊⠬⠂

H_{i+1}

H_{i+1}

2947. ⠠⠓⠰⠊⠬⠂⠐⠌⠠⠓⠰⠊

H_{i+1}/H_i

H_{i+1}/H_i

2948. ⠈⠠⠵⠰⠍⠝⠐ ⠈⠱⠈⠱ ⠈⠠⠵⠰⠍⠐⠡⠈⠠⠵⠰⠝

$\mathbb{Z}_{mn} \cong \mathbb{Z}_m \times \mathbb{Z}_n$

{\mathbb Z}_{mn} \cong {\mathbb Z}_m \times {\mathbb Z}_n

2949. ⠛⠉⠙⠷⠍⠠⠝⠾ ⠶ ⠂

$\gcd(m, n) = 1$

\gcd(m, n) =1

2950. ⠈⠠⠵⠰⠏⠂⠰⠘⠨⠁⠂⠐⠡⠄⠄⠄⠡⠈⠠⠵⠰⠏⠰⠰⠝⠰⠘⠨⠁⠰⠘⠰⠝

$\mathbb{Z}_{p_1^{\alpha_1}} \times \cdots \times \mathbb{Z}_{p_n^{\alpha_n}}$

{\mathbb Z}_{p_1^{\alpha_1}} \times \cdots \times {\mathbb Z}_{p_n^{\alpha_n}}

2951. ⠏⠰⠅

p_k

p_k

2952. ⠨⠷⠛⠰⠊⠐⠨⠾

$\{g_i\}$

\{ g_i\}

2953. StartSet⠛⠰⠊⠐⠱⠊⠈⠑⠠⠊EndSet

$\{g_i : i \in I\}$

\{g_i : i \in I \}

2954. StartSet⠛⠰⠊⠐⠱⠊⠈⠑⠠⠊EndSet

$\{g_i : i \in I\}$

\{ g_i : i \in I \}

2955. ⠈⠠⠵⠡⠈⠠⠵⠰⠝

$\mathbb{Z} \times \mathbb{Z}_n$

{\mathbb Z} \times {\mathbb Z}_n

2956. StartSet⠷⠂ ⠠⠴⠾⠠⠷⠴⠠⠂ ⠾EndSet

$\{(1,0),(0,1)\}$

\{ (1,0), (0,1) \}

2957. ⠏⠂⠌⠟⠂ ⠄⠄ ⠠⠏⠰⠝⠐⠌⠟⠰⠝⠐

$p_1/q_1, \ldots, p_n/q_n$

p_1/q_1, \ldots, p_n/q_n

2958. ⠏⠰⠊⠐⠌⠟⠰⠊

p_i/q_i

p_i/q_i

2959. ⠟⠂ ⠄⠄ ⠠⠟⠰⠝⠐

$q_1, \ldots, q_n$

q_1, \ldots, q_n

2960. ⠂⠌⠏

$1/p$

1/p

2961. ⠏⠰⠊⠌⠟⠰⠊⠐⠬⠏⠰⠚⠌⠟⠰⠚⠐ ⠶ ⠷⠏⠰⠊⠟⠰⠚⠐⠬⠏⠰⠚⠟⠰⠊⠐⠾⠌⠷⠟⠰⠊⠟⠰⠚⠐⠾

$p_i/q_i + p_j/q_j = (p_i q_j + p_j q_i)/(q_i q_j)$

p_i / q_i + p_j / q_j = (p_i q_j + p_j q_i)/(q_i q_j)

2962. StartSet⠛⠰⠊ ⠈⠑ ⠨⠛⠸⠒⠊ ⠈⠑ ⠠⠊EndSet

$\{g_i \in G : i \in I\}$

\{ g_i \in G : i \in I \}

2963. ⠓ ⠶ ⠛⠰⠊⠰⠰⠂⠘⠨⠁⠘⠰⠂⠐⠄⠄⠄⠛⠰⠊⠰⠰⠝⠘⠨⠁⠘⠰⠝

$h = g_{i_1}^{\alpha_1} \cdots g_{i_n}^{\alpha_n}$

h = g_{i_1}^{\alpha_1} \cdots g_{i_n}^{\alpha_n}

2964. ⠛⠰⠊⠰⠰⠅

g_{i_k}

g_{i_k}

2965. ⠛⠰⠊⠰⠰⠂⠘⠨⠁⠘⠰⠂⠐⠄⠄⠄⠛⠰⠊⠰⠰⠝⠘⠨⠁⠘⠰⠝

$g_{i_1}^{\alpha_1} \cdots g_{i_n}^{\alpha_n}$

g_{i_1}^{\alpha_1} \cdots g_{i_n}^{\alpha_n}

2966. ⠠⠅ ⠶ ⠠⠓

$K = H$

K=H

2967. ⠛⠰⠊⠘⠴ ⠶ ⠂

$g_i^0 = 1$

g_i^0 = 1

2968. ⠛ ⠶ ⠛⠰⠊⠰⠰⠂⠘⠅⠘⠰⠂⠐⠄⠄⠄⠛⠰⠊⠰⠰⠝⠘⠅⠘⠰⠝

$g = g_{i_1}^{k_1} \cdots g_{i_n}^{k_n}$

g =g_{i_1}^{k_1} \cdots g_{i_n}^{k_n}

2969. ⠛⠘⠤⠂ ⠶ ⠷⠛⠰⠊⠰⠰⠂⠘⠅⠘⠰⠂⠐⠄⠄⠄⠛⠰⠊⠰⠰⠝⠘⠅⠘⠰⠝⠐⠾⠘⠤⠂ ⠶ ⠷⠛⠰⠊⠰⠰⠝⠘⠤⠅⠘⠰⠝⠐⠄⠄⠄

$g^{-1} = (g_{i_1}^{k_1} \cdots g_{i_n}^{k_n})^{-1} = (g_{i_n}^{-k_n} \cdots g_{i_1}^{-k_1})$

g^{-1} = (g_{i_1}^{k_{1}} \cdots g_{i_n}^{k_n})^{-1} = (g_{i_n}^{-k_n}
\cdots g_{i_{1}}^{-k_{1}})

2970. ⠁⠘⠤⠒⠐⠃⠘⠢⠐⠁⠘⠶

$a^{-3}b^5a^7$

a^{-3} b^5 a^7

2971. ⠁⠘⠲⠐⠃⠘⠢

a^4b^5

a^4 b^5

2972. ⠈⠠⠵27

$\mathbb{Z}_{27}$

{\mathbb Z}_{27}

2973. ⠈⠠⠵⠰⠏⠰⠰⠘⠨⠁⠘⠰⠂ ⠈⠡⠈⠠⠵⠰⠏⠰⠰⠘⠨⠁⠘⠰⠆ ⠈⠡⠄⠄⠄⠈⠡⠈⠠⠵⠰⠏⠰⠰⠘⠨⠁⠘⠰⠝

$\mathbb{Z}_{p_1^{\alpha_1}} \times \mathbb{Z}_{p_2^{\alpha_2}} \times \cdots \times \mathbb{Z}_{p_n^{\alpha_n}}$

{\mathbb Z}_{p_1^{ \alpha_1 }} \times {\mathbb Z}_{p_2^{ \alpha_2
}} \times \cdots \times {\mathbb Z}_{p_n^{ \alpha_n }}

2974. 540 ⠨⠅ ⠆⠘⠆⠡⠒⠘⠒⠡⠢

$540=2^2 \cdot 3^3 \cdot 5$

540=2^2 \cdot 3^3 \cdot 5

2975. ⠈⠠⠵⠆⠡⠈⠠⠵⠆⠡⠈⠠⠵⠒⠡⠈⠠⠵⠒⠡⠈⠠⠵⠒⠡⠈⠠⠵⠢

$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_5$

{\mathbb Z}_2 \times {\mathbb Z}_2 \times {\mathbb Z}_3 \times {\mathbb
Z}_3 \times {\mathbb Z}_3 \times {\mathbb Z}_5

2976. ⠈⠠⠵⠆⠡⠈⠠⠵⠆⠡⠈⠠⠵⠒⠡⠈⠠⠵⠔⠡⠈⠠⠵⠢

$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_9 \times \mathbb{Z}_5$

{\mathbb Z}_2 \times {\mathbb Z}_2 \times {\mathbb Z}_3 \times {\mathbb
Z}_9 \times {\mathbb Z}_5

2977. ⠈⠠⠵⠆⠡⠈⠠⠵⠆⠡⠈⠠⠵27⠡⠈⠠⠵⠢

$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_{27} \times \mathbb{Z}_5$

{\mathbb Z}_2 \times {\mathbb Z}_2 \times {\mathbb Z}_{27} \times
{\mathbb Z}_5

2978. ⠈⠠⠵⠲⠡⠈⠠⠵⠒⠡⠈⠠⠵⠒⠡⠈⠠⠵⠒⠡⠈⠠⠵⠢

$\mathbb{Z}_4 \times \mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_5$

{\mathbb Z}_4 \times {\mathbb Z}_3 \times {\mathbb Z}_3 \times {\mathbb
Z}_3 \times {\mathbb Z}_5

2979. ⠈⠠⠵⠲⠡⠈⠠⠵⠒⠡⠈⠠⠵⠔⠡⠈⠠⠵⠢

$\mathbb{Z}_4 \times \mathbb{Z}_3 \times \mathbb{Z}_9 \times \mathbb{Z}_5$

{\mathbb Z}_4 \times {\mathbb Z}_3 \times {\mathbb Z}_9 \times {\mathbb
Z}_5

2980. ⠈⠠⠵⠲⠡⠈⠠⠵27⠡⠈⠠⠵⠢

$\mathbb{Z}_4 \times \mathbb{Z}_{27} \times \mathbb{Z}_5$

{\mathbb Z}_4 \times {\mathbb Z}_{27} \times {\mathbb Z}_5

2981. ⠅ ⠐⠅ ⠝

$k < n$

k \lt n

2982. ⠏ ⠨⠅ ⠝

$p = n$

p = n

2983. ⠏⠤⠂

$p - 1$

p - 1

2984. ⠂ ⠐⠅ ⠳⠠⠓⠳ ⠐⠅ ⠝

$1 < |H| < n$

1 \lt |H| \lt n

2985. ⠏⠳⠳⠠⠓⠳

$p \mid |H|$

p \mid |H|

2986. ⠳⠠⠛⠳ ⠨⠅ ⠳⠠⠓⠳⠡⠳⠠⠛⠌⠠⠓⠳

$|G| = |H| \cdot |G/H|$

|G| = |H| \cdot |G/H|

2987. ⠳⠠⠛⠌⠠⠓⠳

$|G/H|$

|G/H|

2988. ⠳⠠⠛⠌⠠⠓⠳ ⠐⠅ ⠳⠠⠛⠳ ⠨⠅ ⠝

$|G/H| < |G| = n$

|G/H| \lt |G| = n

2989. ⠠⠓ ⠨⠅ ⠷⠁⠠⠓⠾⠘⠏⠐ ⠨⠅ ⠁⠘⠏⠐⠠⠓

$H = (aH)^p = a^pH$

H = (aH)^p = a^pH

2990. ⠁⠘⠏⠐⠈⠑⠠⠓

$a^p \in H$

a^p \in H

2991. ⠁⠌⠈⠑⠠⠓

$a \notin H$

a \notin H

2992. ⠳⠠⠓⠳ ⠨⠅ ⠗

$|H| = r$

|H| = r

2993. ⠎⠏⠬⠞⠗ ⠨⠅ ⠂

$sp + tr = 1$

sp + tr = 1

2994. ⠁⠘⠏

a^p

a^p

2995. ⠷⠁⠘⠏⠐⠾⠘⠗⠐ ⠨⠅ ⠷⠁⠘⠗⠐⠾⠘⠏⠐ ⠨⠅ ⠂

$(a^p)^r = (a^r)^p = 1$

(a^p)^r = (a^r)^p = 1

2996. ⠁⠘⠗⠐⠀⠌⠨⠅⠀⠂

$a^r \neq 1$

a^r \neq 1

2997. ⠁⠘⠗⠐⠀⠨⠅⠀⠂

$a^r = 1$

a^r = 1

2998. ⠁⠀⠨⠅⠀⠷⠁⠘⠏⠐⠾⠘⠎⠐⠈⠑⠠⠓

$a = (a^p)^s \in H$

a= (a^p)^s \in H

2999. ⠳⠠⠛⠳⠀⠨⠅⠀⠏⠘⠝

$|G| = p^n$

|G| = p^n

3000. ⠏⠘⠝

p^n

p^n

3001. ⠳⠠⠛⠳

$|G|$

|G|

3002. ⠝⠀⠨⠅⠀⠏⠂⠘⠨⠁⠂⠐⠄⠄⠄⠏⠰⠅⠘⠨⠁⠘⠰⠅

$n = p_1^{\alpha_1} \cdots p_k^{\alpha_k}$

n = p_1^{\alpha_1} \cdots p_k^{\alpha_k}

3003. ⠨⠁⠂⠠⠀⠨⠁⠆⠠⠀⠄⠄⠄⠠⠀⠨⠁⠰⠅⠐

$\alpha_1, \alpha_2, \ldots, \alpha_k$

\alpha_1, \alpha_2, \ldots, \alpha_k

3004. ⠠⠛⠂⠠⠀⠠⠛⠆⠠⠀⠄⠄⠄⠠⠀⠠⠛⠰⠅⠐

$G_1, G_2, \ldots, G_k$

G_1, G_2, \ldots, G_k

3005. ⠏⠰⠊⠘⠗

p_i^r

p_i^r

3006. ⠏⠰⠊⠘⠴⠐⠀⠨⠅⠀⠂

$p_i^0 = 1$

p_i^0 = 1

3007. ⠂⠀⠈⠑⠀⠠⠛⠰⠊

$1 \in G_i$

1 \in G_i

3008. ⠛⠀⠈⠑⠀⠠⠛⠰⠊

$g \in G_i$

g \in G_i

3009. ⠓ ⠈⠑ ⠠⠛ ⠰⠊

$h \in G_i$

h \in G_i

3010. ⠏ ⠰⠊ ⠘⠎

p_i^s

p_i^s

3011. ⠷⠛⠓⠾⠘⠏⠘⠰⠊⠘⠘⠞⠐ ⠨⠅ ⠛⠘⠏⠘⠰⠊⠘⠘⠞⠐⠓⠘⠏⠘⠰⠊⠘⠘⠞⠐ ⠨⠅ ⠂⠡⠂ ⠨⠅ ⠂

$(gh)^{p_i^t} = g^{p_i^t} h^{p_i^t} = 1 \cdot 1 = 1$

(gh)^{p_i^t} = g^{p_i^t} h^{p_i^t} = 1 \cdot 1 = 1

3012. ⠠⠛ ⠨⠅ ⠠⠛⠂ ⠠⠛⠆⠄⠄⠄⠠⠛⠰⠅

$G = G_1 G_2 \cdots G_k$

G = G_1 G_2 \cdots G_k

3013. ⠠⠛⠰⠊⠐⠩⠠⠛⠰⠚⠐ ⠨⠅ StartSet⠂ EndSet

$G_i \cap G_j = \{1\}$

G_i \cap G_j = \{1 \}

3014. ⠛⠂ ⠈⠑ ⠠⠛⠂

$g_1 \in G_1$

g_1 \in G_1

3015. ⠠⠛⠆ ⠠⠠⠛⠒ ⠄⠄ ⠠⠠⠛⠰⠅ ⠂

$G_2, G_3, \ldots, G_k$

G_2, G_3, \ldots, G_k

3016. ⠛⠂ ⠨⠅ ⠛⠆⠛⠒⠄⠄⠄⠛⠰⠅

$g_1 = g_2 g_3 \cdots g_k$

g_1 = g_2 g_3 \cdots g_k

3017. ⠛⠰⠊⠐⠈⠑⠠⠛⠰⠊

$g_i \in G_i$

g_i \in G_i

3018. ⠏⠘⠨⠁ ⠘⠰⠊

p^{α_i}

p^{\alpha_i}

3019. ⠛⠰⠊⠘⠏⠘⠘⠨⠁ ⠘⠘⠰⠊⠐ ⠨⠅ ⠂

$g_i^{p^{\alpha_i}} = 1$

g_i^{p^{\alpha_i}} = 1

3020. ⠊ ⠨⠅ ⠆ ⠠⠠⠒⠤ ⠄⠄ ⠠⠰⠅

$i = 2, 3, \ldots, k$

i = 2, 3, \ldots, k

3021. ⠛⠂ ⠘⠏⠆ ⠘⠘⠨⠁ ⠆ ⠘⠄⠄⠄⠏⠘⠰⠅ ⠘⠘⠨⠁ ⠘⠘⠰⠅ ⠐ ⠨⠅ ⠂

$g_1^{p_2^{\alpha_2} \cdots p_k^{\alpha_k}} = 1$

g_1^{p_2^{\alpha_2} \cdots p_k^{\alpha_k}} = 1

3022. ⠛⠂

g_1

g_1

3023. ⠛⠉⠙⠷⠏⠂⠀⠠⠏⠆⠘⠨⠁⠘⠆⠀⠄⠄⠄⠀⠏⠰⠅⠘⠨⠁⠘⠰⠅⠐⠾⠀⠨⠅⠀⠂

$\gcd(p_1, p_2^{\alpha_2} \cdots p_k^{\alpha_k}) = 1$

\gcd(p_1, p_2^{\alpha_2} \cdots p_k^{\alpha_k}) = 1

3024. ⠛⠂⠀⠨⠅⠀⠂

$g_1 = 1$

g_1 = 1

3025. ⠛⠂⠀⠄⠄⠄⠀⠛⠰⠅

$g_1 \cdots g_k$

g_1 \cdots g_k

3026. ⠳⠛⠳⠀⠨⠅⠀⠏⠂⠘⠨⠃⠘⠂⠐⠏⠆⠘⠨⠃⠘⠆⠀⠄⠄⠄⠀⠏⠰⠅⠘⠨⠃⠘⠰⠅

$|g| = p_1^{\beta_1} p_2^{\beta_2} \cdots p_k^{\beta_k}$

|g| = p_1^{\beta_1} p_2^{\beta_2} \cdots p_k^{\beta_k}

3027. ⠨⠃⠂⠠⠀⠄⠄⠄⠠⠀⠨⠃⠰⠅

$\beta_1, \ldots, \beta_k$

\beta_1, \ldots, \beta_k

3028. ⠁⠰⠊⠀⠨⠅⠀⠳⠛⠳⠸⠌⠏⠰⠊⠘⠨⠃⠘⠰⠊

$a_i = |g|/p_i^{\beta_i}$

a_i = |g| / p_i^{\beta_i}

3029. ⠃⠂⠠⠀⠄⠄⠄⠠⠀⠃⠰⠅

$b_1, \ldots, b_k$

b_1, \ldots, b_k

3030. ⠁⠂⠃⠂⠬⠄⠄⠄⠬⠁⠰⠅⠃⠰⠅⠀⠨⠅⠀⠂

$a_1 b_1 + \cdots + a_k b_k = 1$

a_1 b_1 + \cdots + a_k b_k = 1

3031. ⠛⠀⠨⠅⠀⠛⠘⠁⠘⠂⠃⠘⠂⠬⠄⠄⠄⠬⠁⠘⠰⠅⠃⠘⠰⠅⠐⠀⠨⠅⠀⠛⠘⠁⠘⠂⠃⠘⠂⠐⠄⠄⠄⠛⠘⠁⠘⠰⠅⠃⠘⠰⠅

$g = g^{a_1 b_1 + \cdots + a_k b_k} = g^{a_1 b_1} \cdots g^{a_k b_k}$

g = g^{a_1 b_1 + \cdots + a_k b_k} = g^{a_1 b_1} \cdots g^{a_k b_k}

3032. ⠛⠘⠷⠁⠘⠰⠊⠃⠘⠰⠊⠾⠏⠘⠰⠊⠘⠘⠨⠃⠘⠘⠰⠊⠐⠀⠨⠅⠀⠛⠘⠃⠘⠰⠊⠳⠛⠳⠐⠀⠨⠅⠀⠑

$g^{(a_i b_i) p_i^{\beta_i}} = g^{b_i |g|} = e$

g^{(a_i b_i) p_i^{\beta_i}} = g^{b_i |g|} = e

3033. ⠛⠘⠁⠘⠰⠊⠃⠘⠰⠊

$g^{a_i b_i}$

g^{a_i b_i}

3034. ⠠⠛⠰⠊

G_{i}

G_{i}

3035. ⠛⠰⠊⠐ ⠨⠅ ⠛⠘⠁⠘⠰⠊⠃⠘⠰⠊

$g_i = g^{a_i b_i}$

g_i = g^{a_i b_i}

3036. ⠛ ⠨⠅ ⠛⠂⠄⠄⠄⠛⠰⠅ ⠐⠈⠑ ⠠⠛⠂⠠⠛⠆⠄⠄⠄⠠⠛⠰⠅

$g = g_1 \cdots g_k \in G_1 G_2 \cdots G_k$

g = g_1 \cdots g_k \in G_1 G_2 \cdots G_k

3037. ⠄⡳⠭⠆⠶⠑⠦⠄⠛⠄⡳⠭⠆⠶⠑⠔⠄⠡⠠⠓

$\langle g \rangle \times H$

\langle g \rangle \times H

3038. ⠝ ⠨⠅ ⠂

$n = 1$

n= 1

3039. ⠂ ⠅⠱ ⠅ ⠐⠅ ⠝

$1 \leq k < n$

1 \leq k \lt n

3040. ⠳⠛⠳ ⠨⠅ ⠏⠘⠍

$|g| = p^m$

|g| = p^{m}

3041. ⠁⠘⠏⠘⠘⠍⠐ ⠨⠅ ⠑

$a^{p^m} = e$

a^{p^m} = e

3042. ⠓⠌⠈⠑⠄⡳⠭⠆⠶⠑⠦⠄⠛⠄⡳⠭⠆⠶⠑⠔⠄

$h \notin \langle g \rangle$

h \notin \langle g \rangle

3043. ⠠⠛ ⠨⠅ ⠄⡳⠭⠆⠶⠑⠦⠄⠛⠄⡳⠭⠆⠶⠑⠔⠄

$G = \langle g \rangle$

G = \langle g \rangle

3044. ⠠⠓ ⠨⠅ ⠄⡳⠭⠆⠶⠑⠦⠄⠓⠄⡳⠭⠆⠶⠑⠔⠄

$H = \langle h \rangle$

H = \langle h \rangle

3045. ⠄⡳⠭⠆⠶⠑⠦⠄⠛⠄⡳⠭⠆⠶⠑⠔⠄ ⠨⠩⠠⠓ ⠨⠅ StartSet⠑EndSet

$\langle g \rangle \cap H = \{e\}$

\langle g \rangle \cap H = \{ e \}

3046. ⠳⠠⠓⠳ ⠨⠅ ⠏

$|H| = p$

|H|=p

3047. ⠳⠓⠘⠏⠐⠳ ⠨⠅ ⠳⠓⠳⠌⠏

$|h^p| = |h|/p$

|h^p| = |h| / p

3048. ⠓⠘⠏

h^p

h^p

3049. ⠓⠘⠏⠐ ⠨⠅ ⠛⠘⠗

$h^p = g^r$

h^p = g^r

3050. ⠷⠛⠘⠗⠐⠾⠘⠏⠘⠘⠍⠤⠂⠐ ⠨⠅ ⠷⠓⠘⠏⠐⠾⠘⠏⠘⠘⠍⠤⠂⠐ ⠨⠅ ⠓⠘⠏⠘⠘⠍⠐ ⠨⠅ ⠑

$(g^r)^{p^{m-1}} = (h^p)^{p^{m-1}} = h^{p^m} = e$

(g^r)^{p^{m - 1}} = (h^p)^{p^{m - 1}} = h^{p^{m}} = e

3051. ⠛⠘⠗

g^r

g^r

3052. ⠏⠘⠍⠤⠂

p^{m-1}

p^{m-1}

3053. ⠗ ⠨⠅ ⠏⠎

$r = ps$

r = ps

3054. ⠓⠘⠏⠐ ⠨⠅ ⠛⠘⠗⠐ ⠨⠅ ⠛⠘⠏⠎

$h^p = g^r = g^{ps}$

h^p = g^r = g^{ps}

3055. ⠛⠘⠤⠎⠐⠓

$g^{-s}h$

g^{-s}h

3056. ⠁⠘⠏⠐ ⠨⠅ ⠛⠘⠤⠎⠏⠐⠓⠘⠏⠐ ⠨⠅ ⠛⠘⠤⠗⠐⠓⠘⠏⠐ ⠨⠅ ⠓⠘⠤⠏⠐⠓⠘⠏⠐ ⠨⠅ ⠑

$a^p = g^{-sp}h^p = g^{-r}h^p = h^{-p}h^p = e$

a^p = g^{-sp} h^p = g^{-r} h^p = h^{-p} h^p = e

3057. ⠁ ⠌⠈⠑⠄⡷⠭⠆⠶⠑⠦⠄⠛⠄⡷⠭⠆⠶⠑⠔⠄

$a \notin \langle g \rangle$

a \notin \langle g \rangle

3058. ⠄⡷⠭⠆⠶⠑⠦⠄⠛⠄⡷⠭⠆⠶⠑⠔⠄

$\langle g \rangle$

\langle g\rangle

3059. ⠳⠠⠓⠳ ⠨⠅ ⠏

$|H| = p$

|H| = p

3060. ⠳⠛⠠⠓⠳ ⠐⠅ ⠳⠛⠳ ⠨⠅ ⠏⠘⠍

$|gH| < |g| = p^m$

|gH| \lt |g| = p^m

3061. ⠠⠓ ⠶ ⠷⠛⠠⠓⠾⠘⠏⠘⠘⠍⠤⠂⠘⠐ ⠶ ⠛⠘⠏⠘⠘⠍⠤⠂⠘⠐⠠⠓⠷

$H = (gH)^{p^{m-1}} = g^{p^{m-1}} H;$

H = (gH)^{p^{m-1}} = g^{p^{m-1}} H;

3062. ⠛⠘⠏⠘⠘⠍⠤⠂

$g^{p^{m-1}}$

g^{p^{m-1}}

3063. ⠏⠘⠍

p^m

p^m

3064. ⠠⠛⠌⠠⠓ ⠹⠹ ⠄⡗⠭⠅⠶⠑⠲⠄⠛⠠⠓⠄⡗⠭⠅⠶⠑⠢⠄ ⠁⠄⠅⠂⠌⠠⠓

$G/H \cong \langle gH \rangle \times K/H$

G/H \cong \langle gH \rangle \times K/H

3065. ⠄⡗⠭⠅⠶⠑⠲⠄⠛⠄⡗⠭⠅⠶⠑⠢⠄ ⠝⠅ ⠶ StartSet⠑EndSet

$\langle g \rangle \cap K = \{e\}$

\langle g \rangle \cap K = \{ e \}

3066. ⠃ ⠙⠄⡗⠭⠅⠶⠑⠲⠄⠛⠄⡗⠭⠅⠶⠑⠢⠄ ⠝⠅

$b \in \langle g \rangle \cap K$

b \in \langle g \rangle \cap K

3067. ⠃ ⠠⠓ ⠙⠄⡗⠭⠅⠶⠑⠲⠄⠛⠠⠓⠄⡗⠭⠅⠶⠑⠢⠄ ⠝⠅⠌⠠⠓ ⠶ StartSet⠠⠓EndSet

$bH \in \langle gH \rangle \cap K/H = \{H\}$

bH \in \langle gH \rangle \cap K/H = \{ H \}

3068. ⠃ ⠙⠄⡗⠭⠅⠶⠑⠲⠄⠛⠄⡗⠭⠅⠶⠑⠢⠄ ⠝⠠⠓ ⠶ StartSet⠑EndSet

$b \in \langle g \rangle \cap H = \{e\}$

b \in \langle g \rangle \cap H = \{ e \}

3069. ⠠⠛ ⠶ ⠄⡗⠭⠅⠶⠑⠲⠄⠛⠄⡗⠭⠅⠶⠑⠢⠄ ⠅

$G = \langle g \rangle K$

G = \langle g \rangle K

3070. ⠠⠛ ⠹⠹ ⠄⡗⠭⠅⠶⠑⠲⠄⠛⠄⡗⠭⠅⠶⠑⠢⠄⠁⠅

$G \cong \langle g \rangle \times K$

G \cong \langle g \rangle \times K

3071. ⠄⡗⠭⠅⠶⠑⠲⠄⠛⠄⡗⠭⠅⠶⠑⠢⠄ ⠶ ⠠⠛

$\langle g \rangle = G$

\langle g \rangle = G

3072. ⠠⠛ ⠹⠹ ⠈⠠⠵⠰⠳⠛⠳⠐⠁⠠⠓

$G \cong \mathbb{Z}_{|g|} \times H$

G \cong {\mathbb Z}_{|g|} \times H

3073. ⠳⠠⠓⠳ ⠐⠅ ⠳⠠⠛⠳

$|H| < |G|$

|H| \lt |G|

3074. ⠈⠰⠵⠰⠏⠂⠰⠘⠈⠁⠂⠈⠡⠈⠰⠵⠰⠏⠆⠰⠘⠈⠁⠆⠈⠡⠄⠄⠄⠈⠡⠈⠰⠵⠰⠏⠰⠝⠰⠘⠈⠁⠰⠝⠈⠡⠈⠰⠵⠈⠡⠄⠄⠄⠈⠡⠈⠰⠵

$\mathbb{Z}_{p_1^{\alpha_1}} \times \mathbb{Z}_{p_2^{\alpha_2}} \times \cdots \times \mathbb{Z}_{p_n^{\alpha_n}} \times \mathbb{Z} \times \cdots \times \mathbb{Z}$

{\mathbb Z}_{p_1^{ \alpha_1 }} \times {\mathbb Z}_{p_2^{ \alpha_2 }} \times \cdots \times {\mathbb Z}_{p_n^{ \alpha_n }} \times {\mathbb Z} \times \cdots \times {\mathbb Z}

3075. ⠠⠛ ⠶ ⠠⠓⠰⠝⠐⠨⠂⠠⠓⠰⠝⠤⠂⠐⠨⠂⠄⠄⠄⠐⠨⠂⠠⠓⠂⠐⠨⠂⠠⠓⠴ ⠶ StartSet⠑EndSet

$G = H_n \supset H_{n-1} \supset \cdots \supset H_1 \supset H_0 = \{e\}$

G = H_n \supset H_{n-1} \supset \cdots \supset H_1 \supset H_0 = \{ e \}

3076. ⠠⠙⠲⠐⠨⠂StartSet⠷⠂⠾ ⠠⠷12⠾⠷34⠾ ⠠⠷13⠾⠷24⠾ ⠠⠷14⠾⠷23⠾EndSet⠐⠨⠂StartSet⠷⠂⠾ ⠠⠷12⠾⠷

$D_4 \supset \{(1), (12)(34), (13)(24), (14)(23)\} \supset \{(1), (12)(34)\} \supset \{(1)\}$

D_4 \supset \{ (1), (12)(34), (13)(24), (14)(23) \} \supset \{ (1), (12)(34) \} \supset \{ (1) \}

3077. StartSet⠷⠂⠾ ⠠⠷12⠾⠷34⠾EndSet

$\{(1), (12)(34)\}$

\{ (1), (12)(34) \}

3078. ⠸⠷⠠⠅⠰⠚⠐⠸⠾

$\{K_j\}$

\{ K_j \}

3079. ⠸⠷⠠⠓⠰⠊⠐⠸⠾

$\{H_i\}$

\{ H_i \}

3080. ⠸⠷⠠⠓⠰⠊⠐⠸⠾⠸⠐⠅⠸⠷⠠⠅⠰⠚⠐⠸⠾

$\{H_i\} \subset \{K_j\}$

\{ H_i \} \subset \{ K_j \}

3081. ⠠⠅⠰⠚

K_j

K_j

3082. ⠈⠰⠵⠐⠨⠂⠒⠈⠰⠵⠐⠨⠂⠔⠈⠰⠵⠐⠨⠂45⠈⠰⠵⠐⠨⠂90⠈⠰⠵⠐⠨⠂180⠈⠰⠵⠐⠨⠂StartSet⠴EndSet

$\mathbb{Z} \supset 3\mathbb{Z} \supset 9\mathbb{Z} \supset 45\mathbb{Z} \supset 90\mathbb{Z} \supset 180\mathbb{Z} \supset \{0\}$

{\mathbb Z} \supset 3{\mathbb Z} \supset 9{\mathbb Z} \supset 45{\mathbb Z} \supset 90{\mathbb Z} \supset 180{\mathbb Z} \supset \{0\}

3083. ⠈⠰⠵⠐⠨⠂⠔⠈⠰⠵⠐⠨⠂45⠈⠰⠵⠐⠨⠂180⠈⠰⠵⠐⠨⠂StartSet⠴EndSet

$\mathbb{Z} \supset 9\mathbb{Z} \supset 45\mathbb{Z} \supset 180\mathbb{Z} \supset \{0\}$

{\mathbb Z} \supset 9{\mathbb Z} \supset 45{\mathbb Z} \supset 180{\mathbb Z} \supset \{0\}

3084. ⠸⠷⠠⠓⠰⠊⠐⠸⠾

$\{H_i\}$

\{H_i \}

3085. ⠨⠷⠠⠓⠰⠊⠬⠂⠐⠌⠠⠓⠰⠊⠐⠨⠾

$\{H_{i+1}/H_i\}$

\{H_{i+1}/H_i \}

3086. ⠨⠷⠠⠅⠰⠚⠬⠂⠐⠌⠠⠅⠰⠚⠐⠨⠾

$\{K_{j+1}/K_j\}$

\{ K_{j+1}/ K_j \}

3087. ⠈⠠⠵60⠸⠕⠄⡸⠭⠆ ⠶⠈⠢⠤⠄ ⠒⠄⡸⠭⠆ ⠶⠈⠢⠢ ⠸⠕⠄⡸⠭⠆ ⠶⠈⠢⠤⠄ 15⠄⡸⠭⠆ ⠶⠈⠢⠢ ⠸⠕⠄⡸⠭⠆ ⠶⠈⠢⠤⠄ 30⠄⡸⠭⠆

$\mathbb{Z}_{60} \supset \langle 3 \rangle \supset \langle 15 \rangle \supset \langle 30 \rangle \supset \{0\}$

{\mathbb Z}_{60} \supset \langle 3 \rangle \supset \langle 15 \rangle
\supset \langle 30 \rangle \supset \{ 0 \}

3088. ⠈⠠⠵60⠸⠕⠄⡸⠭⠆ ⠶⠈⠢⠤⠄ ⠆⠄⡸⠭⠆ ⠶⠈⠢⠢ ⠸⠕⠄⡸⠭⠆ ⠶⠈⠢⠤⠄ ⠲⠄⡸⠭⠆ ⠶⠈⠢⠢ ⠸⠕⠄⡸⠭⠆ ⠶⠈⠢⠤⠄ 20⠄⡸⠭⠆

$\mathbb{Z}_{60} \supset \langle 2 \rangle \supset \langle 4 \rangle \supset \langle 20 \rangle \supset \{0\}$

{\mathbb Z}_{60} \supset \langle 2 \rangle \supset \langle 4 \rangle
\supset \langle 20 \rangle \supset \{ 0 \}

3089. ⠠⠎⠰⠝⠐⠸⠕ ⠠⠁⠰⠝⠐⠸⠕ ⠨⠷⠷⠂⠾⠨⠾

$S_n \supset A_n \supset \{(1)\}$

S_n \supset A_n \supset \{ (1) \}

3090. ⠠⠎⠰⠝⠐⠌⠠⠁⠰⠝⠐ ⠈⠱⠈⠱ ⠈⠠⠵⠆

$S_n/A_n \cong \mathbb{Z}_2$

S_n / A_n \cong {\mathbb Z}_2

3091. StartSet⠴EndSet ⠶ ⠠⠓⠴⠸⠐⠅ ⠠⠓⠂⠸⠐⠅ ⠄⠄⠄ ⠸⠐⠅ ⠠⠓⠰⠝⠤⠂⠐⠸⠐⠅ ⠠⠓⠰⠝⠐ ⠶ ⠈⠠⠵

$\{0\} = H_0 \subset H_1 \subset \cdots \subset H_{n-1} \subset H_n = \mathbb{Z}$

\{ 0 \} = H_0 \subset H_1 \subset \cdots \subset H_{n-1} \subset
H_n = {\mathbb Z}

3092. ⠅ ⠈⠠⠵

$k\mathbb{Z}$

k {\mathbb Z}

3093. ⠅ ⠈⠑ ⠈⠠⠝

$k \in \mathbb{N}$

k \in {\mathbb N}

3094. ⠠⠓⠂⠐⠌⠠⠓⠴ ⠈⠱⠈⠱ ⠅ ⠈⠠⠵

$H_1/H_0 \cong k\mathbb{Z}$

H_1 / H_0 \cong k {\mathbb Z}

3095. ⠠⠓⠰⠊⠐⠨⠩⠠⠅⠰⠍⠤⠂

$H_i \cap K_{m-1}$

H_i \cap K_{m-1}

3096. ⠠⠓⠰⠊⠬⠂⠐⠨⠩⠠⠅⠰⠍⠤⠂

$H_{i+1} \cap K_{m-1}$

H_{i+1} \cap K_{m-1}

3097. ⠠⠅⠰⠚⠐⠨⠩⠠⠓⠰⠝⠤⠂

$K_j \cap H_{n-1}$

K_j \cap H_{n-1}

3098. ⠠⠅⠰⠚⠬⠂⠐⠨⠩⠠⠓⠰⠝⠤⠂

$K_{j+1} \cap H_{n-1}$

K_{j+1} \cap H_{n-1}

3099. ⠠⠓⠰⠊⠐⠷⠠⠓⠰⠊⠬⠂⠐⠨⠩⠠⠅⠰⠍⠤⠂⠐⠾

$H_i(H_{i+1} \cap K_{m-1})$

H_i (H_{i+1} \cap K_{m-1})

3100. ⠠⠓⠰⠊⠬⠂⠐⠌⠠⠓⠰⠊

H_{i+1}/H_i

H_{i+1} / H_i

3101. ⠠⠓⠰⠊⠐⠷⠠⠓⠰⠊⠬⠂⠐⠨⠩⠠⠅⠰⠍⠤⠂⠐⠾⠌⠠⠓⠰⠊

$H_i(H_{i+1} \cap K_{m-1})/H_i$

H_i (H_{i+1} \cap K_{m-1})/ H_i

3102. ⠠⠓⠰⠊⠐⠌⠠⠓⠰⠊

H_i/H_i

H_i/H_i

3103. ⠠⠓⠰⠝⠤⠂⠐⠸⠕⠠⠓⠰⠝⠤⠂⠐⠨⠩⠠⠅⠰⠍⠤⠂⠐⠸⠕⠄⠄⠄⠸⠕⠠⠓⠴⠨⠩⠠⠅⠰⠍⠤⠂⠐ ⠨⠅ StartSet⠑EndSe

$H_{n-1} \supset H_{n-1} \cap K_{m-1} \supset \cdots \supset H_0 \cap K_{m-1} = \{e\}$

H_{n-1} \supset H_{n-1} \cap K_{m-1} \supset \cdots \supset H_0
\cap K_{m-1} = \{ e \}

3104. ⠠⠓⠰⠝⠤⠂

H_{n-1}

H_{n-1}

3105. ⠠⠓⠰⠝⠤⠂⠐⠸⠕⠄⠄⠄⠸⠕⠠⠓⠂⠸⠕⠠⠓⠴ ⠨⠅ StartSet⠑EndSet

$H_{n-1} \supset \cdots \supset H_1 \supset H_0 = \{e\}$

H_{n-1} \supset \cdots \supset H_1 \supset H_0 = \{ e \}

3106. ⠠⠛ ⠨⠅ ⠠⠓⠰⠝⠐⠸⠕⠠⠓⠰⠝⠤⠂⠐⠸⠕⠠⠓⠰⠝⠤⠂⠐⠨⠩⠠⠅⠰⠍⠤⠂⠐⠸⠕⠄⠄⠄⠸⠕⠠⠓⠴⠨⠩⠠⠅⠰⠍⠤⠂⠐

$G = H_n \supset H_{n-1} \supset H_{n-1} \cap K_{m-1} \supset \cdots \supset H_0 \cap K_{m-1} = \{e\}$

G = H_n \supset H_{n-1} \supset H_{n-1} \cap K_{m-1} \supset \cdots
\supset H_0 \cap K_{m-1} = \{ e \}

3107. ⠠⠓⠰⠝⠤⠂⠐ ⠨⠅ ⠠⠅⠰⠍⠤⠂

$H_{n-1} = K_{m-1}$

H_{n-1} = K_{m-1}

3108. ⠠⠓⠰⠝⠤⠂⠐⠠⠅⠰⠍⠤⠂

$H_{n-1}K_{m-1}$

H_{n-1} K_{m-1}

3109. ⠠⠓⠰⠝⠤⠂⠐⠠⠅⠰⠍⠤⠂⠐ ⠨⠅ ⠠⠛

$H_{n-1}K_{m-1} = G$

```
H_{n-1} K_{m-1} = G
```

3110. ⠠⠅⠰⠍⠤⠂⠐⠸⠌⠷⠠⠅⠰⠍⠤⠂⠐⠨⠩⠠⠓⠰⠝⠤⠂⠐⠾ ⠈⠱⠈⠱ ⠷⠠⠓⠰⠝⠤⠂⠐⠠⠅⠰⠍⠤⠂⠐⠾⠸⠌⠠⠓⠰⠝⠤⠂⠐ ⠨⠅ ⠠⠛

$K_{m-1}/(K_{m-1} \cap H_{n-1}) \cong (H_{n-1}K_{m-1})/H_{n-1} = G/H_{n-1}$

```
K_{m-1} / (K_{m-1} \cap H_{n-1}) \cong (H_{n-1} K_{m-1}) / H_{n-1}
= G/H_{n-1}
```

3111. ⠠⠛ ⠨⠅ ⠠⠅⠰⠍⠐⠸⠕ ⠠⠅⠰⠍⠤⠂⠐⠸⠕ ⠠⠅⠰⠍⠤⠂⠐⠨⠩⠠⠓⠰⠝⠤⠂⠐⠸⠕⠄⠄⠄⠸⠕ ⠠⠅⠴⠨⠩⠠⠓⠰⠝⠤⠂⠐

$G = K_m \supset K_{m-1} \supset K_{m-1} \cap H_{n-1} \supset \cdots \supset K_0 \cap H_{n-1} = \{e\}$

```
G = K_m \supset K_{m-1} \supset K_{m-1} \cap H_{n-1} \supset \cdots
\supset K_0 \cap H_{n-1} = \{ e \}
```

3112. ⠠⠎⠲⠸⠕ ⠠⠁⠲⠸⠕ StartSet⠷⠂⠾ ⠠⠷12⠾⠷34⠾ ⠠⠷13⠾⠷24⠾ ⠠⠷14⠾⠷23⠾EndSet⠸⠕ ⠨⠷⠷⠂⠾ ⠾⠨⠾

$S_4 \supset A_4 \supset \{(1), (12)(34), (13)(24), (14)(23)\} \supset \{(1)\}$

```
S_4 \supset A_4 \supset \{ (1), (12)(34), (13)(24), (14)(23) \}
\supset \{ (1) \}
```

3113. ⠼200

200

```
200
```

3114. ⠼720

720

```
720
```

3115. ⠠⠎⠒⠈⠡⠈⠠⠵⠲

$S_3 \times \mathbb{Z}_4$

```
S_3 \times {\mathbb Z}_4
```

3116. StartSet⠼EndSet⠸⠐⠅ ⠨⠷⠼⠋⠨⠾ ⠸⠐⠅ ⠨⠷⠼⠒⠨⠾ ⠸⠐⠅ ⠈⠠⠵⠂⠆12

$\{0\} \subset \langle 6 \rangle \subset \langle 3 \rangle \subset \mathbb{Z}_{12}$

```
\{ 0 \} \subset \langle 6 \rangle \subset \langle 3 \rangle \subset
{\mathbb Z}_{12}
```

3117. ⠨⠷⠷⠂⠾⠨⠾⠈⠡StartSet⠼EndSet⠸⠐⠅ StartSet⠷⠂⠾ ⠠⠷123⠾ ⠠⠷132⠾EndSet⠈⠡StartSet⠼EndSet⠸⠐⠅

$\{(1)\} \times \{0\} \subset \{(1), (123), (132)\} \times \{0\} \subset S_3 \times \{0\} \subset S_3 \times \langle 2 \rangle \subset S_3 \times \mathbb{Z}_4$

```
\{ (1) \} \times \{ 0 \} \subset \{ (1), (123), (132) \} \times
\{ 0 \} \subset S_3 \times \{ 0 \} \subset S_3 \times \langle 2
\rangle\subset S_3 \times {\mathbb Z}_4
```

3118. ⠠⠛ ⠨⠅ ⠈⠠⠵⠆⠈⠡⠈⠠⠵⠆⠈⠡⠄⠄⠄

$G = \mathbb{Z}_2 \times \mathbb{Z}_2 \times \cdots$

```
G = {\mathbb Z}_2 \times {\mathbb Z}_2 \times \cdots
```

3119. ⠠⠛⠈⠡⠠⠓ ⠈⠱⠈⠱ ⠠⠛⠈⠡⠠⠅

$G \times H \cong G \times K$

```
G \times H \cong G \times K
```

3120. ⠠⠓ ⠈⠱⠈⠱ ⠠⠅

$H \cong K$

H \cong K

3121. ⠠⠛ ⠨⠅ ⠠⠏⠰⠝⠐⠸⠨⠂⠠⠏⠰⠝⠤⠂⠐⠸⠨⠂⠄⠄⠄⠸⠨⠂⠠⠏⠂⠸⠨⠂⠠⠏⠴ ⠨⠅ StartSet⠑EndSet

$G = P_n \supset P_{n-1} \supset \cdots \supset P_1 \supset P_0 = \{e\}$

G = P_n \supset P_{n - 1} \supset \cdots \supset P_1 \supset P_0
= \{ e \}

3122. ⠠⠏⠰⠊

P_i

P_i

3123. ⠠⠏⠰⠊⠬⠂

P_{i+1}

P_{i + 1}

3124. ⠠⠏⠰⠊⠬⠂⠐⠌⠠⠏⠰⠊

P_{i+1}/P_i

P_{i + 1} / P_i

3125. ⠁⠘⠤⠂⠐⠃⠘⠤⠂⠐⠁⠃

$a^{-1}b^{-1}ab$

a^{-1} b ^{-1} ab

3126. ⠠⠛⠘⠷⠴⠾⠐ ⠨⠅ ⠠⠛

$G^{(0)} = G$

G^{(0)} = G

3127. ⠠⠛⠘⠷⠂⠾⠐ ⠨⠅ ⠠⠛⠄

$G^{(1)} = G'$

G^{(1)} = G'

3128. ⠠⠛⠘⠷⠊⠬⠂⠾⠐ ⠨⠅ ⠷⠠⠛⠘⠷⠊⠾⠐⠾⠄

$G^{(i+1)} = (G^{(i)})'$

G^{(i + 1)} = (G^{(i)})'

3129. ⠠⠛⠘⠷⠊⠬⠂⠾

$G^{(i+1)}$

G^{(i+1)}

3130. ⠷⠠⠛⠘⠷⠊⠾⠐⠾⠄

$(G^{(i)})'$

(G^{(i)})'

3131. ⠠⠛⠘⠷⠴⠾⠐ ⠨⠅ ⠠⠛⠸⠨⠂⠠⠛⠘⠷⠂⠾⠐⠸⠨⠂⠠⠛⠘⠷⠆⠾⠐⠸⠨⠂⠄⠄⠄

$G^{(0)} = G \supset G^{(1)} \supset G^{(2)} \supset \cdots$

G^{(0)} = G \supset G^{(1)} \supset G^{(2)} \supset \cdots

3132. ⠠⠛⠘⠷⠝⠾⠐ ⠨⠅ StartSet⠑EndSet

$G^{(n)} = \{e\}$

G^{(n)} = \{ e \}

3133. ⠠⠛⠌⠠⠛⠄

G/G'

G/G'

3134. ⠠⠓⠘⠈⠼

H^*

H^*

3135. ⠠⠅⠘⠈⠼

K^*

K^*

3136. ⠠⠓⠘⠈⠼⠐⠷⠠⠓⠨⠩⠠⠅⠘⠈⠼⠐⠾

$H^*(H \cap K^*)$

H^* (H \cap K^*)

3137. ⠠⠓⠘⠈⠼⠐⠷⠠⠓⠨⠩⠠⠅⠾

$H^*(H \cap K)$

H^* (H \cap K)

3138. ⠠⠅⠘⠈⠼⠐⠷⠠⠓⠘⠈⠼⠐⠨⠩⠠⠅⠾

$K^*(H^* \cap K)$

K^* (H^* \cap K)

3139. ⠠⠅⠘⠈⠼⠐⠷⠠⠓⠨⠩⠠⠅⠾

$K^*(H \cap K)$

K^* (H \cap K)

3140. ⠠⠓⠘⠈⠼⠐⠷⠠⠓⠨⠩⠠⠅⠾⠌⠠⠓⠘⠈⠼⠐⠷⠠⠓⠨⠩⠠⠅⠘⠈⠼⠐⠾ ⠈⠱⠈⠱ ⠠⠅⠘⠈⠼⠐⠷⠠⠓⠨⠩⠠⠅⠾⠌⠠⠅⠘⠈⠼⠐⠷⠠

$H^*(H \cap K)/H^*(H \cap K^*) \cong K^*(H \cap K)/K^*(H^* \cap K) \cong (H \cap K)/(H^* \cap K)(H \cap K^*)$

H^* (H \cap K) / H^* (H \cap K^*) \cong K^* (H \cap K) / K^*
(H^* \cap K) \cong (H \cap K) / (H^* \cap K)(H \cap K^*)

3141. ⠝ ⠨⠅ ⠖⠠⠼10⠠⠼14

$n = 6,\, 10,\, 14$

n=6,\,10,\,14

3142. ⠝ ⠨⠅ ⠔

$n = 9$

n=9

3143. ⠏⠘⠆

p^2

p^2

3144. ⠈⠠⠵⠒⠡⠈⠠⠵⠒

$\mathbb{Z}_3 \times \mathbb{Z}_3$

`{\mathbb Z}_3\times{\mathbb Z}_3`

3145. ⠝ ⠨⠅ 15

$n = 15$

`n=15`

3146. ⠈⠠⠵⠒⠡⠈⠠⠵⠢ ⠈⠱⠈⠱ ⠈⠠⠵15

$\mathbb{Z}_3 \times \mathbb{Z}_5 \cong \mathbb{Z}_{15}$

`{\mathbb Z}_3\times{\mathbb Z}_5\cong{\mathbb Z}_{15}`

3147. ⠝ ⠨⠅ ⠦

$n = 8$

`n=8`

3148. ⠝ ⠨⠅ 12

$n = 12$

`n=12`

3149. ⠝ ⠨⠅ 16

$n = 16$

`n=16`

3150. ⠆⠘⠅

2^k

`2^k`

3151. ⠅ ⠨⠂ ⠆

$k > 2$

`k>2`

3152. ⠸⠼16

$\mathbf{16}$

`\mathbf{16}`

3153. ⠠⠵⠒⠄⡷⠭⠆⠆⠉⠁⠄⠠⠵⠲

$Z_3 \rtimes Z_4$

`Z_3\rtimes Z_4`

3154. ⠛⠭

gx

`gx`

3155. ⠠⠛⠡⠠⠭ ⠣⠕ ⠠⠭

$G \times X \rightarrow X$

`G \times X \rightarrow X`

3156. ⠷⠛⠠⠭⠾ ⠫⠳⠒⠒⠕ ⠛⠭

$(g, x) \mapsto gx$

`(g,x) \mapsto gx`

3157. ⠑⠭ ⠨⠅ ⠭

$ex = x$

ex = x

3158. ⠷⠛⠂⠛⠆⠾⠭ ⠨⠅ ⠛⠂⠷⠛⠆⠭⠾

$(g_1 g_2)x = g_1(g_2 x)$

(g_1 g_2)x = g_1(g_2 x)

3159. ⠷⠛⠠⠭⠾ ⠫⠳⠒⠒⠕ ⠭

$(g,x) \mapsto x$

(g,x) \mapsto x

3160. ⠠⠛ ⠨⠅ ⠠⠛⠠⠇⠆⠷⠈⠠⠗⠾

$G = GL_2(\mathbb{R})$

G = GL_2({\mathbb R})

3161. ⠠⠭ ⠨⠅ ⠈⠠⠗⠘⠆

$X = \mathbb{R}^2$

X = {\mathbb R}^2

3162. ⠧ ⠈⠑ ⠈⠠⠗⠘⠆

$v \in \mathbb{R}^2$

v \in {\mathbb R}^2

3163. ⠠⠊⠧ ⠨⠅ ⠧

$Iv = v$

Iv = v

3164. ⠷⠠⠁⠠⠃⠾⠧ ⠨⠅ ⠠⠁⠷⠠⠃⠧⠾

$(AB)v = A(Bv)$

(AB)v = A(Bv)

3165. ⠠⠛ ⠨⠅ ⠠⠙⠲

$G = D_4$

G = D_4

3166. ⠠⠭ ⠨⠅ StartSet⠂ ⠠⠆ ⠠⠒ ⠠⠲EndSet

$X = \{1, 2, 3, 4\}$

X = \{ 1, 2, 3, 4 \}

3167. StartSet⠷⠂ ⠾ ⠠⠷13⠾ ⠠⠷24⠾ ⠠⠷1432⠾ ⠠⠷1234⠾ ⠠⠷12⠾⠷34⠾ ⠠⠷14⠾⠷23⠾ ⠠⠷13⠾⠷24⠾EndSet

$\{(1), (13), (24), (1432), (1234), (12)(34), (14)(23), (13)(24)\}$

\{ (1), (13), (24), (1432), (1234), (12)(34), (14)(23), (13)(24)
\}

3168. ⠷13⠾⠷24⠾

$(13)(24)$

(13)(24)

3169. ⠷⠨⠎⠠⠭⠾ ⠫⠳⠒⠒⠕ ⠨⠎⠷⠭⠾

$(\sigma, x) \mapsto \sigma(x)$

(\sigma, x) \mapsto \sigma(x)

3170. ⠨⠎ ⠈⠑ ⠨⠛

$\sigma \in G$

\sigma \in G

3171. ⠠⠭ ⠨⠅ ⠠⠛

$X = G$

X = G

3172. ⠷⠛⠠⠭⠾ ⠫⠳⠒⠒⠕ ⠨⠇⠰⠛⠐⠷⠭⠾ ⠨⠅ ⠛⠭

$(g,x) \mapsto \lambda_g(x) = gx$

(g,x) \mapsto \lambda_g(x) = gx

3173. ⠠⠭ ⠨⠅ ⠠⠛

$X = G$

X=G

3174. ⠠⠓⠈⠡⠠⠛ ⠫⠕ ⠠⠛

$H \times G \rightarrow G$

H \times G \rightarrow G

3175. ⠷⠓⠠⠛⠾ ⠫⠳⠒⠒⠕ ⠓⠛⠓⠘⠤⠂

$(h,g) \mapsto hgh^{-1}$

(h,g) \mapsto hgh^{-1}

3176. ⠷⠛⠠⠭⠠⠓⠾ ⠫⠳⠒⠒⠕ ⠛⠭⠠⠓

$(g, xH) \mapsto gxH$

(g, xH) \mapsto gxH

3177. ⠷⠛⠛⠘⠄⠐⠾⠭⠠⠓ ⠨⠅ ⠛⠷⠛⠄⠭⠠⠓⠾

$(gg')xH = g(g'xH)$

(g g')xH = g(g'x H)

3178. ⠛⠭ ⠨⠅ ⠽

$gx = y$

gx =y

3179. ⠭⠄⡳⠭⠆⠆⠒⠉⠄⠰⠠⠛⠐⠽

$x \sim_G y$

x \sim_G y

3180. ⠛⠭ ⠨⠅ ⠽

$gx = y$

gx = y

3181. ⠛⠘⠤⠂⠐⠽ ⠨⠅ ⠭

$g^{-1}y = x$

g^{-1}y=x

3182. ⠽⠠ ⡷⠭⠆ ⠆ ⠒⠉⠄ ⠵

$y \sim z$

```
y \sim z
```

3183. ⠓⠽ ⠨⠅ ⠵

$hy = z$

```
hy= z
```

3184. ⠵ ⠨⠅ ⠓⠽ ⠨⠅ ⠷⠓⠛⠾⠭

$z = hy = (hg)x$

```
z = hy = (hg)x
```

3185. caligraphic ⠠⠕ ⠰⠭

$\mathcal{O}_x$

```
{\mathcal O}_x
```

3186. ⠠⠛ ⠨⠅ StartSet⠷⠂ ⠾ ⠠⠷⠂ ⠆ ⠒⠾ ⠠⠷⠂ ⠒⠆ ⠾ ⠠⠷⠲⠢⠠⠾ ⠠⠷⠂ ⠆ ⠒⠾⠷⠲⠢⠠⠾ ⠠⠷⠂ ⠒⠆ ⠾⠷⠲⠢⠠⠾EndSet

$G = \{(1), (123), (132), (45), (123)(45), (132)(45)\}$

```
G =\{(1), (1 2 3), (1 3 2), (4 5), (1 2 3)(4 5), (1 3 2)(4 5) \}
```

3187. ⠠⠭ ⠨⠅ StartSet⠂ ⠠⠆ ⠠⠒ ⠠⠲ ⠠⠢EndSet

$X = \{1, 2, 3, 4, 5\}$

```
X = \{ 1, 2, 3, 4, 5\}
```

3188. caligraphic ⠠⠕⠂ ⠨⠅ caligraphic ⠠⠕⠆ ⠨⠅ caligraphic ⠠⠕⠒ ⠨⠅ StartSet⠂ ⠠⠆ ⠠⠒EndSet

$\mathcal{O}_1 = \mathcal{O}_2 = \mathcal{O}_3 = \{1, 2, 3\}$

```
{\mathcal O}_1 = {\mathcal O}_2 = {\mathcal O}_3 =\{1, 2, 3\}
```

3189. caligraphic ⠠⠕⠲ ⠨⠅ caligraphic ⠠⠕⠢ ⠨⠅ StartSet⠲ ⠠⠢EndSet

$\mathcal{O}_4 = \mathcal{O}_5 = \{4, 5\}$

```
{\mathcal O}_4 = {\mathcal O}_5 = \{4, 5\}
```

3190. ⠠⠭ ⠰⠛

X_g

```
X_g
```

3191. ⠛⠭ ⠨⠅ ⠭

$gx = x$

```
gx = x
```

3192. ⠠⠛ ⠰⠭

G_x

```
G_x
```

3193. ⠠⠭ ⠰⠛⠐⠸⠐⠅ ⠠⠭

$X_g \subset X$

```
X_g \subset X
```

3194. ⠠⠛ ⠰⠭⠐⠸⠐⠅ ⠠⠛

$G_x \subset G$

```
G_x \subset G
```

3195. ⠠⠭ ⠨⠅ StartSet⠂⠠⠆⠠⠒⠠⠲⠠⠢⠠⠖EndSet

$X = \{1, 2, 3, 4, 5, 6\}$

X = \{1, 2, 3, 4, 5, 6\}

3196. StartSet⠷⠂⠾⠠⠷⠂⠆⠾⠷⠒⠲⠢⠖⠾⠠⠷⠒⠢⠾⠷⠲⠖⠾⠠⠷⠂⠆⠾⠷⠒⠖⠢⠲⠾EndSet

$\{(1), (12)(3456), (35)(46), (12)(3654)\}$

\{ (1), (1 2)(3 4 5 6), (3 5)(4 6), (1 2)(3 6 5 4) \}

3197. ⠑ ⠈⠑ ⠠⠛⠰⠭

$e \in G_x$

e \in G_x

3198. ⠛⠠⠓ ⠈⠑ ⠠⠛⠰⠭⠐

$g, h \in G_x$

g, h \in G_x

3199. ⠓⠭ ⠨⠅ ⠭

$hx = x$

hx = x

3200. ⠷⠛⠓⠾⠭ ⠨⠅ ⠛⠷⠓⠭⠾ ⠨⠅ ⠛⠭ ⠨⠅ ⠭

$(gh)x = g(hx) = gx = x$

(gh)x = g(hx) = gx = x

3201. ⠛ ⠈⠑ ⠠⠛⠰⠭

$g \in G_x$

g \in G_x

3202. ⠭ ⠨⠅ ⠑⠭ ⠨⠅ ⠷⠛⠘⠤⠂⠐⠛⠾⠭ ⠨⠅ ⠷⠛⠘⠤⠂⠐⠾⠛⠭ ⠨⠅ ⠛⠘⠤⠂⠐⠭

$x = ex = (g^{-1}g)x = (g^{-1})gx = g^{-1}x$

x = ex = (g^{-1}g)x = (g^{-1})gx = g^{-1} x

3203. ⠳⠠⠭⠰⠛⠐⠳

$|X_g|$

|X_g|

3204. ⠳caligraphic ⠠⠕⠰⠭⠐⠳

$|\mathcal{O}_x|$

|{\mathcal O}_x|

3205. ⠳caligraphic ⠠⠕⠰⠭⠐⠳ ⠨⠅ ⠈⠷⠠⠛⠸⠒⠠⠛⠰⠭⠐⠈⠾

$|\mathcal{O}_x| = [G : G_x]$

|{\mathcal O}_x| = [G:G_x]

3206. ⠳⠠⠛⠳⠸⠌⠳⠠⠛⠰⠭⠐⠳

$|G|/|G_x|$

|G|/|G_x|

3207. caligraphic ⠠⠇⠰⠠⠛⠰⠰⠭

$\mathcal{L}_{G_x}$

{\mathcal L}_{G_x}

3208. ⠽ ⠈⠑caligraphic ⠠⠕⠰⠭

$y \in \mathcal{O}_x$

```
y \in {\mathcal O}_x
```

3209. ⠛⠭ ⠨⠅ ⠽

$gx = y$

```
g x = y
```

3210. ⠨⠋⠷⠽⠾ ⠨⠅ ⠛⠠⠛⠰⠭

$\phi(y) = gG_x$

```
\phi( y ) = g G_x
```

3211. ⠨⠋⠷⠽⠂⠾ ⠨⠅ ⠨⠋⠷⠽⠆⠾

$\phi(y_1) = \phi(y_2)$

```
\phi(y_1) = \phi(y_2)
```

3212. ⠨⠋⠷⠽⠂⠾ ⠨⠅ ⠛⠂⠠⠛⠰⠭⠐ ⠨⠅ ⠛⠆⠠⠛⠰⠭⠐ ⠨⠅ ⠨⠋⠷⠽⠆⠾

$\phi(y_1) = g_1 G_x = g_2 G_x = \phi(y_2)$

```
\phi(y_1) = g_1 G_x = g_2 G_x = \phi(y_2)
```

3213. ⠛⠂⠭ ⠨⠅ ⠽⠂

$g_1 x = y_1$

```
g_1 x = y_1
```

3214. ⠛⠆⠭ ⠨⠅ ⠽⠆

$g_2 x = y_2$

```
g_2 x = y_2
```

3215. ⠛⠂⠠⠛⠰⠭⠐ ⠨⠅ ⠛⠆⠠⠛⠰⠭

$g_1 G_x = g_2 G_x$

```
g_1 G_x = g_2 G_x
```

3216. ⠛⠆ ⠨⠅ ⠛⠂⠛

$g_2 = g_1 g$

```
g_2 = g_1 g
```

3217. ⠽⠆ ⠨⠅ ⠛⠆⠭ ⠨⠅ ⠛⠂⠛⠭ ⠨⠅ ⠛⠂⠭ ⠨⠅ ⠽⠂⠸⠆

$y_2 = g_2 x = g_1 g x = g_1 x = y_1;$

```
y_2 = g_2 x = g_1 g x = g_1 x = y_1;
```

3218. ⠛⠠⠛⠰⠭

gG_x

```
g G_x
```

3219. ⠨⠋⠷⠽⠾ ⠨⠅ ⠛⠠⠛⠰⠭

$\phi(y) = gG_x$

```
\phi(y) = g G_x
```

3220. ⠠⠭⠰⠠⠛

X_G

```
X_G
```

3221. ⠠⠭⠰⠠⠛⠐ ⠨⠅ StartSet⠭ ⠈⠑ ⠠⠭⠸⠒⠛⠭ ⠨⠅ ⠭forall⠛ ⠈⠑ ⠠⠛EndSet

$X_G = \{x \in X : gx = x$ for all $g \in G\}$

X_G = \{ x \in X : gx = x \text{ for all } g \in G \}

3222. ⠳ ⠠⠭⠳ ⠨⠅ ⠳ ⠠⠭⠰⠠⠛⠐⠳⠬⠐⠅⠠⠎⠉⠐ ⠨⠅ ⠅⠳⠠⠏⠛⠳caligraphic ⠠⠕ ⠰⠭⠰⠰⠊⠰⠐⠳

$|X| = |X_G| + \sum_{i=k}^{n} |\mathcal{O}_{x_i}|$

|X| = |X_G| + \sum_{i = k}^n |{\mathcal O}_{x_i}|

3223. ⠭⠰⠅ ⠐ ⠄⠄ ⠠⠭⠰⠝⠐

$x_k, \ldots, x_n$

x_k, \ldots, x_n

3224. ⠷⠛ ⠠⠭⠾ ⠫⠳⠒⠒⠕ ⠛⠭⠛ ⠘⠤⠂⠐

$(g, x) \mapsto gxg^{-1}$

(g,x) \mapsto gxg^{-1}

3225. ⠠⠵⠷ ⠠⠛⠾ ⠨⠅ StartSet⠭⠸⠭⠛ ⠨⠅ ⠛⠭forall⠛ ⠈⠑ ⠠⠛EndSet

$Z(G) = \{x : xg = gx$ for all $g \in G\}$

Z(G) = \{x : xg = gx \text{ for all } g \in G \}

3226. ⠭⠂ ⠄⠄ ⠠⠭⠰⠅ ⠐

$x_1, \ldots, x_k$

x_1, \ldots, x_k

3227. ⠳caligraphic ⠠⠕ ⠰⠭⠂ ⠐⠳ ⠨⠅ ⠝⠂ ⠄⠄ ⠠⠳caligraphic ⠠⠕ ⠰⠭⠰⠰⠅ ⠰⠐⠳ ⠨⠅ ⠝⠰⠅ ⠐

$|\mathcal{O}_{x_1}| = n_1, \ldots, |\mathcal{O}_{x_k}| = n_k$

|{\mathcal O}_{x_1}| = n_1, \ldots, |{\mathcal O}_{x_k}| = n_k

3228. ⠳ ⠠⠛⠳ ⠨⠅ ⠳ ⠠⠵⠷ ⠠⠛⠾⠳⠬⠝⠂ ⠬⠄ ⠄ ⠄ ⠬⠝⠰⠅

$|G| = |Z(G)| + n_1 + \cdots + n_k$

|G| = |Z(G)| + n_1 + \cdots + n_k

3229. ⠭⠰⠊

x_i

x_i

3230. ⠠⠉⠷⠭⠰⠊⠐⠾ ⠨⠅ StartSet⠛ ⠈⠑ ⠠⠛⠸⠒⠛⠭⠰⠊⠐ ⠨⠅ ⠭⠰⠊⠐⠛EndSet

$C(x_i) = \{g \in G : gx_i = x_ig\}$

C(x_i) = \{ g \in G: g x_i = x_i g \}

3231. ⠳ ⠠⠛⠳ ⠨⠅ ⠳ ⠠⠵⠷ ⠠⠛⠾⠳⠬⠈⠷ ⠠⠛⠸⠒ ⠠⠉⠷⠭⠂⠾ ⠈⠾⠬⠄ ⠄ ⠄ ⠬⠈⠷ ⠠⠛⠸⠒ ⠠⠉⠷⠭⠰⠅ ⠐⠾ ⠈⠾

$|G| = |Z(G)| + [G : C(x_1)] + \cdots + [G : C(x_k)]$

|G| = |Z(G)| + [G: C(x_1)] + \cdots + [G: C(x_k)]

3232. ⠨⠷⠷⠂⠾ ⠾ ⠨⠾ ⠠StartSet⠷123⠾ ⠠⠷132⠾EndSet ⠠StartSet⠷12⠾ ⠠⠷13⠾ ⠠⠷23⠾EndSet

$\{(1)\}, \quad \{(123), (132)\}, \quad \{(12), (13), (23)\}$

\{ (1) \}, \quad \{ (123), (132) \}, \quad \{(12), (13), (23) \}

3233. ⠖ ⠨⠅ ⠐⠂⠬⠆⠬⠒

$6 = 1 + 2 + 3$

6 = 1+2+3

3234. StartSet⠷⠂ ⠾ ⠠⠷13⠾⠷24⠾EndSet

$\{(1),(13)(24)\}$

\{ (1), (13)(24) \}

3235. StartSet⠷13⠾ ⠠⠷24⠾EndSet ⠠StartSet⠷1432⠾ ⠠⠷1234⠾EndSet ⠠StartSet⠷12⠾⠷34⠾ ⠠⠷14⠾⠷

$\{(13),(24)\}, \quad \{(1432),(1234)\}, \quad \{(12)(34),(14)(23)\}$

\{ (13), (24) \}, \quad \{ (1432), (1234) \}, \quad \{ (12)(34), (14)(23) \}

3236. ⠦ ⠨⠅ ⠆⠬⠆⠬⠆⠬⠆

$8 = 2 + 2 + 2 + 2$

8 = 2 + 2 + 2 + 2

3237. ⠨⠎ ⠨⠅ ⠷⠁⠂⠠ ⠄⠄⠄⠠ ⠁⠰⠅⠐⠾

$\sigma = (a_1, \ldots, a_k)$

\sigma = (a_1, \ldots, a_k)

3238. ⠨⠞⠨⠎⠨⠞⠘⠤⠂⠐ ⠨⠅ ⠷⠨⠞⠷⠁⠂⠾⠠ ⠄⠄⠄⠠ ⠨⠞⠷⠁⠰⠅⠐⠾⠾

$\tau \sigma \tau^{-1} = (\tau(a_1), \ldots, \tau(a_k))$

\tau \sigma \tau^{-1} = (\tau(a_1), \ldots, \tau(a_k))

3239. ⠝⠰⠊⠐ ⠨⠂ ⠂

$n_i > 1$

n_i \gt 1

3240. ⠝⠰⠊⠐⠳⠳⠠⠛⠳

$n_i \mid |G|$

n_i \mid |G|

3241. ⠝⠰⠊

n_i

n_i

3242. ⠏⠳⠳⠠⠛⠳

$p \mid |G|$

p \mid |G|

3243. ⠳⠠⠵⠷⠠⠛⠾⠳

$|Z(G)|$

|Z(G)|

3244. ⠳⠠⠵⠷⠠⠛⠾⠳ ⠨⠂⠱ ⠂

$|Z(G)| \geq 1$

|Z(G)| \geq 1

3245. ⠳⠠⠵⠷⠠⠛⠾⠳ ⠨⠂⠱ ⠏

$|Z(G)| \geq p$

|Z(G)| \geq p

3246. ⠛ ⠈⠑ ⠠⠵⠷⠠⠛⠾

$g \in Z(G)$

g \in Z(G)

3247. ⠛ ⠌⠨⠅ ⠂

$g \neq 1$

g \neq 1

3248. ⠳⠠⠵⠷⠠⠛⠾⠳ ⠨⠅ ⠏

$|Z(G)| = p$

|Z(G)| = p

3249. ⠳⠠⠵⠷⠠⠛⠾⠳ ⠨⠅ ⠏⠘⠆

$|Z(G)| = p^2$

|Z(G)| = p^2

3250. ⠁⠠⠵⠷⠠⠛⠾

$aZ(G)$

aZ(G)

3251. ⠛⠠⠵⠷⠠⠛⠾

$gZ(G)$

gZ(G)

3252. ⠁⠘⠍⠐⠠⠵⠷⠠⠛⠾

$a^m Z(G)$

a^m Z(G)

3253. ⠛ ⠨⠅ ⠁⠘⠍⠐⠭

$g = a^m x$

g = a^m x

3254. ⠓⠠⠵⠷⠠⠛⠾ ⠈⠑ ⠠⠛⠌⠠⠵⠷⠠⠛⠾

$hZ(G) \in G/Z(G)$

hZ(G) \in G / Z(G)

3255. ⠓ ⠨⠅ ⠁⠘⠝⠐⠽

$h = a^n y$

h = a^n y

3256. ⠛⠓ ⠨⠅ ⠁⠘⠍⠐⠭⠁⠘⠝⠐⠽ ⠨⠅ ⠁⠘⠍⠬⠝⠐⠭⠽ ⠨⠅ ⠁⠘⠝⠐⠽⠁⠘⠍⠐⠭ ⠨⠅ ⠓⠛

$gh = a^m x a^n y = a^{m+n} x y = a^n y a^m x = hg$

gh = a^m x a^n y = a^{m+n} x y = a^n y a^m x = hg

3257. 90⠘⠘⠍⠄

90°

90^\circ

3258. ⠠⠛⠰⠽

G_y

G_y

3259. ⠳⠠⠛⠰⠭⠐⠳ ⠨⠅ ⠳⠠⠛⠰⠽⠐⠳

$|G_x| = |G_y|$

|G_x| = |G_y|

3260. ⠷⠛⠠⠭⠾ ⠫⠳⠒⠒⠕ ⠛⠡⠭

$(g,x) \mapsto g \cdot x$

(g,x) \mapsto g \cdot x

3261. ⠛⠡⠭ ⠨⠅ ⠽

$g \cdot x=y$

g \cdot x=y

3262. ⠁ ⠈⠑ ⠠⠛⠰⠭

$a \in G_x$

a \in G_x

3263. ⠛⠁⠛⠘⠤⠂⠐⠡⠽ ⠨⠅ ⠛⠁⠡⠛⠘⠤⠂⠐⠽ ⠨⠅ ⠛⠁⠡⠭ ⠨⠅ ⠛⠡⠭ ⠨⠅ ⠽

$gag^{-1} \cdot y = ga \cdot g^{-1}y = ga \cdot x = g \cdot x = y$

gag^{-1} \cdot y = ga \cdot g^{-1}y = ga \cdot x = g \cdot x = y

3264. ⠨⠋⠸⠒⠠⠛⠰⠭⠐ ⠫⠕ ⠠⠛⠰⠽⠐

$\phi: G_x \rightarrow G_y$

\phi: G_x \rightarrow G_y

3265. ⠨⠋⠷⠁⠾ ⠨⠅ ⠛⠁⠛⠘⠤⠂

$\phi(a) = gag^{-1}$

\phi(a) = gag^{-1}

3266. ⠨⠋⠷⠁⠃⠾ ⠨⠅ ⠛⠁⠃⠛⠘⠤⠂⠐ ⠨⠅ ⠛⠁⠛⠘⠤⠂⠐⠛⠃⠛⠘⠤⠂⠐ ⠨⠅ ⠨⠋⠷⠁⠾⠨⠋⠷⠃⠾

$\phi(ab) = gabg^{-1} = gag^{-1} gbg^{-1} = \phi(a) \phi(b)$

\phi(ab) = gabg^{-1} = gag^{-1} gbg^{-1} = \phi(a) \phi(b)

3267. ⠛⠁⠛⠘⠤⠂⠐ ⠨⠅ ⠛⠃⠛⠘⠤⠂

$gag^{-1}= gbg^{-1}$

gag^{-1}= gbg^{-1}

3268. ⠁ ⠨⠅ ⠃

$a=b$

a=b

3269. ⠛⠘⠤⠂⠐⠃⠛

$g^{-1}bg$

g^{-1}bg

3270. ⠛⠘⠤⠂⠐⠃⠛⠡⠭ ⠨⠅ ⠛⠘⠤⠂⠐⠃⠡⠛⠭ ⠨⠅ ⠛⠘⠤⠂⠐⠃⠡⠽ ⠨⠅ ⠛⠘⠤⠂⠐⠡⠽ ⠨⠅ ⠭⠆

$g^{-1}bg \cdot x = g^{-1}b \cdot gx = g^{-1}b \cdot y = g^{-1} \cdot y = x;$

g^{-1}bg \cdot x = g^{-1}b \cdot gx = g^{-1}b \cdot y = g^{-1} \cdot
y = x;

3271. ⠏⠷⠛⠃⠤⠠ ⠠⠃ ⠛⠾ ⠶ ⠃

$\phi(g^{-1}bg) = b$

\phi(g^{-1}bg) = b

3272. ⠅ ⠶ ⠹⠂ ⠠⠌⠳ ⠠⠛⠳⠠⠼ ⠂ ⠅ ⠠⠎⠉⠛ ⠉⠑ ⠠⠛⠳⠳ ⠠⠭ ⠰⠛ ⠐⠳

$k = \frac{1}{|G|} \sum_{g \in G} |X_g|$

k = \frac{1}{|G|} \sum_{g \in G} |X_g|

3273. ⠛⠭ ⠶ ⠭

$gx = x$

gx =x

3274. ⠐⠅⠠⠎⠉⠛ ⠉⠑ ⠠⠛⠳⠳ ⠠⠭ ⠰⠛ ⠐⠳

$\sum_{g \in G} |X_g|$

\sum_{g \in G} |X_g|

3275. ⠐⠅⠠⠎⠉⠭ ⠉⠑ ⠠⠭⠳⠳ ⠠⠛ ⠰⠭ ⠐⠳ ⠆

$\sum_{x \in X} |G_x|;$

\sum_{x \in X} |G_x|;

3276. ⠐⠅⠠⠎⠉⠛ ⠉⠑ ⠠⠛⠳⠳ ⠠⠭ ⠰⠛ ⠐⠳ ⠶ ⠐⠅⠠⠎⠉⠭ ⠉⠑ ⠠⠭⠳⠳ ⠠⠛ ⠰⠭ ⠐⠳

$\sum_{g \in G} |X_g| = \sum_{x \in X} |G_x|$

\sum_{g \in G} |X_g| = \sum_{x \in X} |G_x|

3277. ⠐⠅⠠⠎⠉⠽ ⠉⠑caligraphic ⠠⠕ ⠰⠭ ⠐⠳⠳ ⠠⠛ ⠰⠽ ⠐⠳ ⠶ ⠳caligraphic ⠠⠕ ⠰⠭ ⠐⠳⠡⠳ ⠠⠛ ⠰⠭ ⠐⠳

$\sum_{y \in \mathcal{O}_x} |G_y| = |\mathcal{O}_x| \cdot |G_x|$

\sum_{y \in {\mathcal O}_x} |G_y| = | {\mathcal O}_x| \cdot |G_x|

3278. ⠐⠅⠠⠎⠉⠛ ⠉⠑ ⠠⠛⠳⠳ ⠠⠭ ⠰⠛ ⠐⠳ ⠶ ⠐⠅⠠⠎⠉⠭ ⠉⠑ ⠠⠭⠳⠳ ⠠⠛ ⠰⠭ ⠐⠳ ⠶ ⠅ ⠡⠳ ⠠⠛⠳

$\sum_{g \in G} |X_g| = \sum_{x \in X} |G_x| = k \cdot |G|$

\sum_{g \in G} |X_g| = \sum_{x \in X} |G_x| = k \cdot |G|

3279. ⠠⠭ ⠶ StartSet⠂ ⠠⠆ ⠠⠒ ⠠⠲ ⠠⠢EndSet

$X = \{1, 2, 3, 4, 5\}$

X = \{1, 2, 3, 4, 5 \}

3280. ⠠⠛ ⠶ StartSet⠷⠂ ⠾ ⠠⠷⠂ ⠒⠾ ⠠⠷⠂ ⠒⠾⠷⠆ ⠢⠾ ⠠⠷⠆ ⠢⠾EndSet

$G = \{(1), (13), (13)(25), (25)\}$

G= \{(1), (1 3), (1 3)(2 5), (2 5) \}

3281. StartSet⠂ ⠠⠒EndSet

$\{1, 3\}$

\{1, 3\}

3282. StartSet⠆ ⠠⠢EndSet

$\{2, 5\}$

\{2, 5\}

3283. StartSet⠲EndSet

$\{4\}$

\{4\}

3284. ⠅ ⠨⠅ ⠹⠂⠌⠸⠠⠛⠸⠻ ⠐ ⠨⠠⠎⠩⠛⠈⠑⠠⠛⠻ ⠸⠠⠭⠰⠛⠸ ⠨⠅ one-fourth⠷⠢⠬⠒⠬⠂⠬⠒⠾ ⠨⠅ ⠒

$k = \frac{1}{|G|} \sum_{g \in G} |X_g| = \frac{1}{4}(5 + 3 + 1 + 3) = 3$

k = \frac{1}{|G|} \sum_{g \in G} |X_g| = \frac{1}{4}(5 + 3 + 1 +
3) = 3

3285. StartSet⠂ ⠠⠆ ⠠⠒ ⠠⠲EndSet

$\{ 1, 2, 3, 4\}$

\{ 1, 2, 3, 4\}

3286. ⠠⠽ ⠨⠅ StartSet ⠠⠃ ⠠ ⠠⠺EndSet

$Y = \{ B, W \}$

Y = \{ B, W \}

3287. W

W

3288. ⠋⠒ ⠠⠭ ⠫⠕ ⠠⠽

$f : X \rightarrow Y$

f : X \rightarrow Y

3289. ⠨⠎ ⠈⠑ ⠠⠙⠲

$\sigma \in D_4$

\sigma \in D_4

3290. ⠨⠎overTilde

$\widetilde{ \sigma }$

\widetilde{ \sigma }

3291. ModifyingAbove ⠨⠎ With ⠉⠒⠷⠋⠾ ⠨⠅ ⠋ ⠐ ⠨⠡ ⠨⠎

$\widetilde{\sigma}(f) = f \circ \sigma$

\widetilde{\sigma}(f) = f \circ \sigma

3292. ⠨⠎ ⠨⠅ ⠷⠂ ⠆⠾⠷⠒⠤⠲⠾

$\sigma = (1 2)(3 4)$

\sigma = (1 2)(3 4)

3293. ⠨⠎overTilde

$\widetilde{\sigma}$

\widetilde{\sigma}

3294. ⠠⠛overTilde

$\widetilde{G}$

\widetilde{G}

3295. ⠠⠭overTilde

$\widetilde{X}$

\widetilde{X}

3296. Y

Y

3297. ⠠⠭overTilde⠰⠷⠂⠾⠐ ⠨⠅ ⠠⠭overTilde

$\widetilde{X}_{(1)} = \widetilde{X}$

\widetilde{X}_{(1)} = \widetilde{X}

3298. ⠳⠠⠭overTilde⠳ ⠨⠅ ⠆⠘⠲⠐ ⠨⠅ 16

$|\widetilde{X}| = 2^4 =~16$

|\widetilde{X}| = 2^4 =~16

3299. ⠠⠭overTilde⠰⠷⠂⠆⠒⠲⠾

$\widetilde{X}_{(1 2 3 4)}$

\widetilde{X}_{(1 2 3 4)}

3300. ⠋ ⠈⠑ ⠠⠭overTilde

$f \in \widetilde{X}$

f \in \widetilde{X}

3301. ⠷⠂ 23⠲⠾

$(1 23 4)$

(1 23 4)

3302. ⠋⠷⠂⠾ ⠨⠅ ⠋⠷⠆⠾ ⠨⠅ ⠋⠷⠒⠾ ⠨⠅ ⠋⠷⠲⠾

$f(1) = f(2) = f(3) = f(4)$

f(1) = f(2) = f(3) = f(4)

3303. ⠋⠷⠭⠾ ⠨⠅ ⠠⠃

$f(x)= B$

f(x)= B

3304. ⠋⠷⠭⠾ ⠨⠅ ⠠⠺

$f(x)= W$

f(x)= W

3305. ⠳⠠⠭overTilde⠰⠷⠂⠆⠒⠲⠾⠐⠳ ⠨⠅ ⠆

$|\widetilde{X}_{(1 2 3 4)}| = 2$

|\widetilde{X}_{(1 2 3 4)}| = 2

3306. ⠳⠠⠭overTilde⠰⠷⠂⠲⠒⠆⠾⠐⠳ ⠨⠅ ⠆

$|\widetilde{X}_{(1 4 3 2)}| = 2$

|\widetilde{X}_{(1 4 3 2)}| = 2

3307. ⠠⠭overTilde⠰⠷⠂⠒⠾⠷⠆⠲⠾

$\widetilde{X}_{(1 3)(2 4)}$

\widetilde{X}_{(1 3)(2 4)}

3308. ⠋⠷⠂⠾ ⠨⠅ ⠋⠷⠒⠾

$f(1) = f(3)$

f(1) = f(3)

3309. ⠋⠷⠆⠾ ⠨⠅ ⠋⠷⠲⠾

$f(2) = f(4)$

f(2) = f(4)

3310. ⠳⠠⠭overTilde ⠰⠷13⠾⠷24⠾⠐⠳ ⠨⠅ ⠆⠘⠆⠐ ⠨⠅ ⠲

$|\widetilde{X}_{(13)(24)}| = 2^2 = 4$

`|\widetilde{X}_{(13)(24)}| = 2^2 = 4`

3311. ⠳⠠⠭overTilde ⠰⠷⠂ ⠆⠾⠷⠒⠲⠾⠐⠳ ⠨⠅ ⠲

$|\widetilde{X}_{(12)(34)}| = 4$

`|\widetilde{X}_{(1 2)(3 4)}| = 4`

3312. ⠳⠠⠭overTilde ⠰⠷⠂ ⠲⠾⠷⠆ ⠒⠾⠐⠳ ⠨⠅ ⠲

$|\widetilde{X}_{(14)(23)}| = 4$

`|\widetilde{X}_{(1 4)(2 3)}| = 4`

3313. ⠠⠭overTilde ⠰⠷⠂ ⠒⠾

$\widetilde{X}_{(13)}$

`\widetilde{X}_{(1 3 )}`

3314. ⠳⠠⠭overTilde ⠰⠷⠂ ⠒⠾⠐⠳ ⠨⠅ ⠆⠘⠒⠐ ⠨⠅ ⠦

$|\widetilde{X}_{(13)}| = 2^3 = 8$

`|\widetilde{X}_{(1 3)}| = 2^3 = 8`

3315. ⠳⠠⠭overTilde ⠰⠷⠆ ⠲⠾⠐⠳ ⠨⠅ ⠦

$|\widetilde{X}_{(24)}| = 8$

`|\widetilde{X}_{(2 4)}| = 8`

3316. one-eighth⠷⠆ ⠘⠲⠐⠬⠆ ⠘⠂⠐⠬⠆ ⠘⠆⠐⠬⠆ ⠘⠂⠐⠬⠆ ⠘⠆⠐⠬⠆ ⠘⠆⠐⠬⠆ ⠘⠒⠐⠬⠆ ⠘⠒⠐⠾ ⠨⠅ ⠖

$\frac{1}{8}(2^4 + 2^1 + 2^2 + 2^1 + 2^2 + 2^2 + 2^3 + 2^3) = 6$

`\frac{1}{8} ( 2^4 + 2^1 + 2^2 + 2^1 + 2^2 + 2^2 +2^3 + 2^3) = 6`

3317. ⠨⠎overTilde ⠈⠑ ⠠⠛overTilde

$\widetilde{\sigma} \in \widetilde{G}$

`\widetilde{\sigma} \in \widetilde{G}`

3318. ⠳⠠⠭overTilde ⠰⠨⠎⠐⠳ ⠨⠅ ⠳⠠⠽⠳⠘⠝

$|\widetilde{X}_{\sigma}| = |Y|^n$

`|\widetilde{X}_{\sigma}| = |Y|^n`

3319. ⠋ ⠘ ⠍⠄ ⠨⠎

$f \circ \sigma$

`f \circ \sigma`

3320. ModifyingAbove ⠨⠎With ⠈⠉⠷⠋⠾ ⠨⠅ ModifyingAbove ⠨⠎With ⠈⠉⠷⠛⠾

$\widetilde{\sigma}(f) = \widetilde{\sigma}(g)$

`\widetilde{\sigma}(f) = \widetilde{\sigma}(g)`

3321. ⠋⠷⠨⠎⠷⠭⠾⠾ ⠨⠅ ModifyingAbove ⠨⠎With ⠈⠉⠷⠋⠾⠷⠭⠾ ⠨⠅ ModifyingAbove ⠨⠎With ⠈⠉⠷⠛⠾⠷

$f(\sigma(x)) = \widetilde{\sigma}(f)(x) = \widetilde{\sigma}(g)(x) = g(\sigma(x))$

```
f( \sigma(x )) = \widetilde{\sigma}(f)(x) = \widetilde{\sigma}(g)(x)
= g( \sigma(x ))
```

3322. ⠋ ⠶ ⠛

$f = g$

f=g

3323. ⠨⠎ ⠫⠳⠒⠒⠕ ⠨⠎overTilde

$\sigma \mapsto \widetilde{\sigma}$

\sigma \mapsto \widetilde{\sigma}

3324. ⠨⠎ ⠶ ⠨⠎⠂⠨⠎⠆⠄⠄⠄⠨⠎⠰⠝

$\sigma = \sigma_1 \sigma_2 \cdots \sigma_n$

\sigma = \sigma_1 \sigma_2 \cdots \sigma_n

3325. ⠠⠭overTilde ⠰⠨⠎

${\widetilde{X}}_{\sigma}$

{\widetilde{X}}_{\sigma}

3326. ⠳⠠⠽⠳

$|Y|$

|Y|

3327. ⠳⠠⠭overTilde ⠰⠨⠎⠐⠳ ⠶ ⠳⠠⠽⠳⠘⠝

$|{\widetilde{X}}_{\sigma}| = |Y|^n$

|{\widetilde{X}}_{\sigma}| = |Y|^n

3328. ⠠⠭ ⠶ StartSet⠂ ⠠⠆ ⠄⠄ ⠠⠶EndSet

$X = \{1, 2, \ldots, 7\}$

X = \{1, 2, \ldots, 7\}

3329. ⠠⠽ ⠶ StartSet⠠⠁ ⠠⠠⠃ ⠠⠠⠉EndSet

$Y = \{ A, B, C \}$

Y = \{ A, B, C \}

3330. ⠷⠂⠒⠾⠷⠆⠲⠢⠾ ⠶ ⠷⠂⠒⠾⠷⠆⠲⠢⠾⠷⠖⠾⠷⠶⠾

$(1\ 3)(2\ 4\ 5) = (1\ 3)(2\ 4\ 5)(6)(7)$

(1 3)(2 4 5) = (1 3)(2 4 5)(6)(7)

3331. ⠝ ⠶ ⠲

$n = 4$

n = 4

3332. ⠋ ⠈⠑ ⠠⠭overTilde ⠰⠛

$f \in \widetilde{X}_g$

f \in \widetilde{X}_g

3333. ⠳⠠⠽⠳ ⠶ ⠒

$|Y|=3$

|Y|=3

3334. ⠳⠠⠭overTilde ⠰⠛⠐⠳ ⠶ ⠒⠘⠲⠐ ⠶ 81

$|\widetilde{X}_g| = 3^4 = 81$

|\widetilde{X}_g| = 3^4 = 81

3335. one-eighth⠷⠲⠘⠲⠐⠬⠲⠘⠂⠐⠬⠲⠘⠆⠐⠬⠲⠘⠂⠐⠬⠲⠘⠆⠐⠬⠲⠘⠆⠐⠬⠲⠘⠒⠐⠬⠲⠘⠒⠐⠾ ⠨⠅ 55

$\frac{1}{8}(4^4 + 4^1 + 4^2 + 4^1 + 4^2 + 4^2 + 4^3 + 4^3) = 55$

`\frac{1}{8} (4^4 + 4^1 + 4^2 + 4^1 + 4^2 + 4^ 2 + 4^3 + 4^3) = 55`

3336. ⠆⠘⠆⠘⠘⠝

2^{2^n}

`2^{2^n}`

3337. ⠛⠷⠁⠠⠃⠠⠉⠾ ⠨⠅ ⠋⠷⠃⠠⠉⠠⠁⠾

$g(a, b, c) = f(b, c, a)$

`g(a, b, c) = f(b, c, a)`

3338. ⠛⠠⡷⠭⠆⠆⠒⠉⠠⠋

$g \sim f$

`g \sim f`

3339. ⠷⠁⠉⠃⠾

(acb)

`(acb)`

3340. ⠷⠁⠃⠾

(ab)

`(ab)`

3341. ⠋⠴

f_0

`f_0`

3342. ⠋⠂

f_1

`f_1`

3343. ⠋⠆

f_2

`f_2`

3344. ⠋⠒

f_3

`f_3`

3345. ⠋⠲

f_4

`f_4`

3346. ⠋⠢

f_5

`f_5`

3347. ⠋⠖

f_6

`f_6`

3348. ⠋⠶

f_7

f_7

3349. ⠋⠦

f_8

f_8

3350. ⠋⠔

f_9

f_9

3351. ⠋10

f_{10}

f_{10}

3352. ⠋11

f_{11}

f_{11}

3353. ⠋12

f_{12}

f_{12}

3354. ⠋13

f_{13}

f_{13}

3355. ⠋14

f_{14}

f_{14}

3356. ⠋15

f_{15}

f_{15}

3357. ⠆ ⠘⠆ ⠘⠘⠒ ⠐ ⠨⠅ 256

$2^{2^3} = 256$

2^{2^3} = 256

3358. ⠆ ⠘⠆ ⠘⠘⠲ ⠐ ⠨⠅ 65,536

$2^{2^4} = 65{,}536$

2^{2^4} =65{,}536

3359. StartSet⠈ ⠄⠘ ⠄⠉EndSet

$\{a, b, c\}$

\{a, b, c\}

3360. ⠆ ⠘⠒

2^3

2^3

3361. ⠷⠁⠠⠃⠠⠉⠾

(a, b, c)

`(a, b, c)`

3362. ⠳⠠⠭⠳ ⠨⠅ ⠆⠘⠝

$|X| = 2^n$

`|X|=2^n`

3363. one-eighth⠷⠆⠘16 ⠐⠬⠆ ⠡⠆⠘12 ⠐⠬⠆ ⠡⠆⠘⠖ ⠐⠬⠒⠡⠆⠘10 ⠐⠾ ⠨⠅ 9616

$\frac{1}{8} (2^{16} + 2 \cdot 2^{12} + 2 \cdot 2^6 + 3 \cdot 2^{10}) = 9616$

```
\frac{1}{8} (2^{16} + 2 \cdot 2^{12} + 2 \cdot 2^6 + 3 \cdot 2^{10})
= 9616
```

3364. ⠷⠁⠾

(a)

`(a)`

3365. ⠷⠴⠾

(0)

`(0)`

3366. ⠷⠁⠉⠾

(ac)

`(a c)`

3367. ⠷⠆⠠⠦⠾⠷⠒⠠⠔⠾⠷⠖⠠12⠾⠷⠶⠠13⠾

$(2, 8)(3, 9)(6, 12)(7, 13)$

`(2,8)(3,9)(6,12)(7,13)`

3368. ⠷⠃⠙⠾

(bd)

`(b d)`

3369. ⠷⠂⠠⠲⠾⠷⠒⠠⠖⠾⠷⠔⠠12⠾⠷11⠠14⠾

$(1, 4)(3, 6)(9, 12)(11, 14)$

`(1,4)(3,6)(9,12)(11,14)`

3370. ⠷⠁⠙⠉⠃⠾

$(adcb)$

`(a d c b)`

3371. ⠷⠂⠠⠆⠠⠲⠠⠦⠾⠷⠒⠠6.12⠠⠔⠾⠷⠢⠠10⠾⠷⠶⠠14⠠13⠠11⠾

$(1, 2, 4, 8)(3, 6.12, 9)(5, 10)(7, 14, 13, 11)$

`(1,2,4,8)(3,6.12,9)(5,10)(7,14,13,11)`

3372. ⠷⠁⠃⠉⠙⠾

$(abcd)$

`(a b c d)`

3373. ⠷⠂ ⠠⠦ ⠠⠲ ⠠⠆ ⠾⠷⠒ ⠠⠔ ⠠12 ⠠⠖⠾⠷⠢ ⠠10⠾⠷⠶ ⠠11 ⠠13 ⠠14⠾

$(1, 8, 4, 2)(3, 9, 12, 6)(5, 10)(7, 11, 13, 14)$

(1,8,4,2)(3,9,12,6)(5,10)(7,11,13,14)

3374. ⠷⠁⠃⠾⠷⠉⠙⠾

$(ab)(cd)$

(a b)(c d)

3375. ⠷⠂ ⠠⠆⠾⠷⠲ ⠠⠦⠾⠷⠢ ⠠10⠾⠷⠖ ⠠⠔⠾⠷⠶ ⠠11⠾⠷13 ⠠14⠾

$(1, 2)(4, 8)(5, 10)(6, 9)(7, 11)(13, 14)$

(1,2)(4,8)(5,10)(6,9)(7,11)(13,14)

3376. ⠷⠁⠙⠾⠷⠃⠉⠾

$(ad)(bc)$

(a d)(b c)

3377. ⠷⠂ ⠠⠦⠾⠷⠆ ⠠⠲⠾⠷⠒ ⠠12⠾⠷⠢ ⠠10⠾⠷⠶ ⠠14⠾⠷11 ⠠13⠾

$(1, 8)(2, 4)(3, 12)(5, 10)(7, 14)(11, 13)$

(1,8)(2,4)(3,12)(5,10)(7,14)(11,13)

3378. ⠷⠁⠉⠾⠷⠃⠙⠾

$(ac)(bd)$

(a c)(b d)

3379. ⠷⠂ ⠠⠲⠾⠷⠆ ⠠⠦⠾⠷⠒ ⠠12⠾⠷⠖ ⠠⠔⠾⠷⠶ ⠠13⠾⠷11 ⠠14⠾

$(1, 4)(2, 8)(3, 12)(6, 9)(7, 13)(11, 14)$

(1,4)(2,8)(3,12)(6,9)(7,13)(11,14)

3380. ⠈ ⠠⠗ ⠘⠆ ⠐⠄ ⠸⠡⠩⠒ ⠒ ⠐ ⠖⠄ StartSet⠠⠴EndSet

$\mathbb{R}^2 \setminus \{0\}$

{\mathbb R}^2 \setminus \{ 0 \}

3381. ⠠⠭ ⠨⠅ StartSet⠂ ⠠⠆ ⠠⠒EndSet

$X = \{1, 2, 3\}$

X= \{1, 2, 3\}

3382. ⠠⠛ ⠨⠅ ⠠⠎⠢⠒ ⠨⠅ StartSet⠷⠂⠾ ⠠⠷12⠾ ⠠⠷13⠾ ⠠⠷23⠾ ⠠⠷123⠾ ⠠⠷132⠾EndSet

$G = S_3 = \{(1), (12), (13), (23), (123), (132)\}$

G=S_3=\{(1), (12), (13), (23), (123), (132) \}

3383. ⠠⠛ ⠨⠅ StartSet⠷⠂⠾ ⠠⠷12⠾ ⠠⠷345⠾ ⠠⠷354⠾ ⠠⠷12⠾⠷345⠾ ⠠⠷12⠾⠷354⠾EndSet

$G = \{(1), (12), (345), (354), (12)(345), (12)(354)\}$

G = \{(1), (12), (345), (354), (12)(345), (12)(354) \}

3384. ⠠⠭ ⠰⠷⠂⠾ ⠐ ⠨⠅ StartSet⠂ ⠠⠆ ⠠⠒EndSet

$X_{(1)} = \{1, 2, 3\}$

X_{(1)} = \{1, 2, 3 \}

3385. ⠠⠭ ⠰⠷12⠾ ⠐ ⠨⠅ StartSet⠒EndSet

$X_{(12)} = \{3\}$

X_{(12)} = \{3 \}

3386. ⠠⠭ ⠰⠷13⠾ ⠐ ⠨⠅ StartSet⠆ EndSet

$X_{(13)} = \{2\}$

X_{(13)} = \{ 2 \}

3387. ⠠⠭ ⠰⠷23⠾ ⠐ ⠨⠅ StartSet⠂ EndSet

$X_{(23)} = \{1\}$

X_{(23)} = \{1 \}

3388. ⠠⠭ ⠰⠷123⠾ ⠐ ⠨⠅ ⠠⠭ ⠰⠷132⠾ ⠐ ⠨⠅ ⠸⠴

$X_{(123)} = X_{(132)} = \emptyset$

X_{(123)} = X_{(132)} = \emptyset

3389. ⠠⠛⠂ ⠨⠅ StartSet⠷⠂⠾ ⠠⠷23⠾EndSet

$G_1 = \{(1), (23)\}$

G_1 = \{ (1), (23) \}

3390. ⠠⠛⠆ ⠨⠅ StartSet⠷⠂⠾ ⠠⠷13⠾EndSet

$G_2 = \{(1), (13)\}$

G_2 = \{(1), (13) \}

3391. ⠠⠛⠒ ⠨⠅ StartSet⠷⠂⠾ ⠠⠷12⠾EndSet

$G_3 = \{(1), (12)\}$

G_3 = \{ (1), (12)\}

3392. ⠳ ⠠⠛⠳ ⠨⠅ ⠳caligraphic ⠠⠕ ⠰⠭ ⠐⠳⠈⠡⠳ ⠠⠛ ⠰⠭ ⠐⠳

$|G| = |\mathcal{O}_x| \cdot |G_x|$

|G|=|{\mathcal O}_x| \cdot |G_x|

3393. caligraphic ⠠⠕⠂ ⠨⠅ caligraphic ⠠⠕⠆ ⠨⠅ caligraphic ⠠⠕⠒ ⠨⠅ StartSet⠂ ⠠⠆ ⠠⠒EndSet

$\mathcal{O}_1 = \mathcal{O}_2 = \mathcal{O}_3 = \{1, 2, 3\}$

{\mathcal O}_1 = {\mathcal O}_2 = {\mathcal O}_3 = \{ 1, 2, 3\}

3394. ⠨⠹ ⠈⠑ ⠠⠛

$\theta \in G$

\theta \in G

3395. ⠠⠛ ⠰ ⠠⠏

G_P

G_P

3396. ⠠⠛ ⠨⠅ ⠠⠁ ⠲

$G = A_4$

G = A_4

3397. ⠷⠛ ⠠⠓⠾ ⠫⠳⠒⠒⠕ ⠛⠓⠛ ⠘⠤⠂

$(g,h) \;\mapsto\; ghg^{-1}$

(g,h)~\mapsto~ghg^{-1}

3398. ⠂ ⠬⠒⠬⠖⠬⠖⠬⠦ ⠨⠅ 24

$1 + 3 + 6 + 6 + 8 = 24$

1 + 3 + 6 + 6 + 8 = 24

3399. ⠷⠒⠘⠲⠐⠬⠒⠘⠂⠐⠬⠒⠘⠆⠐⠬⠒⠘⠂⠐⠬⠒⠘⠆⠐⠬⠒⠘⠆⠐⠬⠒⠘⠒⠐⠬⠒⠘⠒⠐⠾⠸⠌⠦ ⠨⠅ 21

$(3^4 + 3^1 + 3^2 + 3^1 + 3^2 + 3^2 + 3^3 + 3^3)/8 = 21$

`(3^4 + 3^1 + 3^2 + 3^1 + 3^2 + 3^2 + 3^3 + 3^3)/8 = 21`

3400. ⠂⠠ ⠄⠄ ⠠⠖

$1, \ldots, 6$

`1, \ldots, 6`

3401. ⠷⠁⠃⠉⠙⠾

$(abcd)$

`(abcd)`

3402. ⠷⠁⠃⠾⠷⠉⠙⠾⠷⠑⠋⠾

$(ab)(cd)(ef)$

`(ab)(cd)(ef)`

3403. ⠷⠁⠃⠉⠾⠷⠙⠑⠋⠾

$(abc)(def)$

`(abc)(def)`

3404. ⠷⠂⠡⠆⠘⠖⠐⠬⠒⠡⠆⠘⠲⠐⠬⠲⠡⠆⠘⠒⠐⠬⠆⠡⠆⠘⠆⠐⠬⠆⠡⠆⠘⠂⠐⠾⠸⠌12 ⠨⠅ 13

$(1 \cdot 2^6 + 3 \cdot 2^4 + 4 \cdot 2^3 + 2 \cdot 2^2 + 2 \cdot 2^1)/12 = 13$

`(1 \cdot 2^6 + 3 \cdot 2^4 + 4 \cdot 2^3 + 2 \cdot 2^2 + 2 \cdot 2^1)/12 = 13`

3405. ⠠⠉⠠⠓⠒

CH_3

`CH_3`

3406. ⠷⠂⠡⠆⠘⠦⠐⠬⠒⠡⠆⠘⠖⠐⠬⠆⠡⠆⠘⠲⠐⠾⠸⠌⠖ ⠨⠅ 80

$(1 \cdot 2^8 + 3 \cdot 2^6 + 2 \cdot 2^4)/6 = 80$

`(1 \cdot 2^8 + 3 \cdot 2^6 + 2 \cdot 2^4)/6 = 80`

3407. ⠷⠭⠂⠭⠆⠭⠒⠭⠲⠾

$(x_1 x_2 x_3 x_4)$

`(x_1 x_2 x_3 x_4)`

3408. ⠛⠠⠉⠷⠁⠾⠛⠘⠤⠂⠐ ⠨⠅ ⠠⠉⠷⠛⠁⠛⠘⠤⠂⠐⠾

$gC(a)g^{-1} = C(gag^{-1})$

`gC(a) g^{-1} = C(gag^{-1})`

3409. ⠭ ⠈⠑ ⠛⠠⠉⠷⠁⠾⠛⠘⠤⠂⠐

$x \in gC(a)g^{-1}$

`x \in g C(a) g^{-1}`

3410. ⠛⠘⠤⠂⠐⠭⠛ ⠈⠑ ⠠⠉⠷⠁⠾

$g^{-1}xg \in C(a)$

`g^{-1}x g \in C(a)`

3411. ⠳⠠⠵⠷⠠⠛⠾⠳ ⠐⠅ ⠏⠘⠝⠤⠂

$|Z(G)| < p^{n-1}$

`|Z(G)| \lt p^{n - 1}`

3412. ⠠⠭⠰⠠⠛⠐ ⠨⠅ StartSet⠭ ⠈⠑ ⠠⠭⠱⠛⠭ ⠨⠅ ⠭forall⠛ ⠈⠑ ⠠⠛EndSet

$X_G = \{ x \in X : gx = x \text{ for all } g \in G \}$

`X_G = \{ x \in X : gx = x \text{ for all }g \in G \}`

3413. ⠳⠠⠭⠳ ⠿ ⠳⠠⠭⠰⠠⠛⠐⠳⠷⠍⠕⠙⠏⠾

$|X| \equiv |X_G| \pmod{p}$

`|X| \equiv |X_G| \pmod{ p}`

3414. ⠛⠭⠛⠘⠤⠂

gxg^{-1}

`gxg^{-1}`

3415. ⠛⠘⠤⠂⠐⠭⠛

$g^{-1}xg$

`g^{-1}xg`

3416. ⠼48

48

`48`

3417. ⠼000⠠⠼010⠠⠼110⠠⠼100

$000, 010, 110, 100$

`000, 010, 110, 100`

3418. ⠼001⠠⠼011⠠⠼111⠠⠼101

$001, 011, 111, 101$

`001, 011, 111, 101`

3419. ⠼⠒⠯ ⠨⠅ ⠖

$3! = 6$

`3!=6`

3420. ⠠⠙⠶

D_7

`D_{7}`

3421. ⠠⠟⠲

Q_4

`Q_4`

3422. ⠠⠵⠷⠠⠛⠾ ⠨⠅ StartSet⠛ ⠈⠑ ⠠⠛⠱⠛⠭ ⠨⠅ ⠭⠛forall⠭ ⠈⠑ ⠠⠛EndSet

$Z(G) = \{g \in G : gx = xg \text{ for all } x \in G\}$

`Z(G) = \{g \in G : gx = xg \text{ for all } x \in G\}`

3423. ⠠⠉⠷⠭⠰⠊⠐⠾ ⠨⠅ StartSet⠛ ⠈⠑ ⠠⠛⠱⠛⠭⠰⠊⠐ ⠨⠅ ⠭⠰⠊⠐⠛EndSet

$C(x_i) = \{ g \in G : g x_i = x_i g \}$

`C(x_i) = \{ g \in G : g x_i = x_i g \}`

3424. ⠳⠠⠛⠳ ⠶ ⠏

$|G| = p$

|G|=p

3425. ⠏ ⠅⠱ ⠅ ⠪ ⠝

$p \leq k < n$

p \leq k \lt n

3426. ⠳⠠⠛⠳ ⠶ ⠝

$|G| = n$

|G|= n

3427. ⠏⠳⠝

$p \mid n$

p \mid n

3428. ⠠⠉⠷⠭ ⠰⠊ ⠐⠾

$C(x_i)$

C(x_i)

3429. ⠳⠠⠉⠷⠭ ⠰⠊ ⠐⠾⠳

$|C(x_i)|$

|C(x_i)|

3430. ⠈⠷⠠⠛⠐⠂⠠⠉⠷⠭ ⠰⠊ ⠐⠾ ⠈⠾

$[G : C(x_i)]$

[G:C(x_i)]

3431. ⠳⠠⠁⠢⠳ ⠶ 60 ⠶ ⠆⠘⠆⠐⠡⠒⠡⠢

$|A_5| = 60 = 2^2 \cdot 3 \cdot 5$

|A_5| = 60 = 2^2 \cdot 3 \cdot 5

3432. ⠏⠘⠗

p^r

p^r

3433. ⠝ ⠨⠂ ⠏

$n > p$

n \gt p

3434. ⠏⠘⠗ ⠐⠳⠳⠠⠉⠷⠭ ⠰⠊ ⠐⠾⠳

$p^r \mid |C(x_i)|$

p^r \mid |C(x_i)|

3435. ⠳⠠⠛⠳ ⠶ ⠳⠠⠉⠷⠭ ⠰⠊ ⠐⠾⠳⠡⠈⠷⠠⠛⠐⠂⠠⠉⠷⠭ ⠰⠊ ⠐⠾ ⠈⠾

$|G| = |C(x_i)| \cdot [G : C(x_i)]$

|G| = |C(x_i)| \cdot [G:C(x_i)]

3436. ⠳⠠⠛⠳⠠⠈⠏

$|G|/p$

|G|/p

3437. ⠏⠘⠗⠤⠂

p^{r-1}

p^{r- 1}

3438. caligraphic ⠠⠎

$\mathcal{S}$

{\mathcal S}

3439. ⠠⠓⠈⠡caligraphic ⠠⠎ ⠫⠕ caligraphic ⠠⠎

$H \times \mathcal{S} \rightarrow \mathcal{S}$

H \times {\mathcal S} \rightarrow {\mathcal S}

3440. ⠓⠡⠠⠅ ⠫⠳⠒⠒⠕ ⠓⠠⠅⠓⠘⠤⠂

$h \cdot K \mapsto hKh^{-1}$

h \cdot K \mapsto hKh^{-1}

3441. ⠠⠝⠷⠠⠓⠾ ⠨⠅ StartSet⠛ ⠈⠑ ⠠⠛⠒⠛ ⠠⠓⠛⠘⠤⠂ ⠐ ⠨⠅ ⠠⠓EndSet

$N(H) = \{g \in G : gHg^{-1} = H\}$

N(H) = \{ g \in G : g H g^{-1} = H\}

3442. ⠠⠝⠷⠠⠓⠾

$N(H)$

N(H)

3443. ⠭⠘⠤⠂⠐⠠⠏⠭ ⠨⠅ ⠠⠏

$x^{-1}Px = P$

x^{-1} P x = P

3444. ⠭ ⠈⠑ ⠠⠏

$x \in P$

x \in P

3445. ⠭ ⠈⠑ ⠠⠝⠷⠠⠏⠾

$x \in N(P)$

x \in N(P)

3446. ⠠⠳⠉⠅⠭⠈⠹⠭⠠⠏⠠⠳⠉⠅⠭⠈⠔⠠ ⠸⠐⠅ ⠠⠝⠷⠠⠏⠾⠸⠌⠠⠏

$\langle xP \rangle \subset N(P)/P$

\langle xP \rangle \subset N(P)/P

3447. ⠠⠝⠷⠠⠏⠾

$N(P)$

N(P)

3448. ⠠⠓⠌⠠⠏ ⠨⠅ ⠠⠳⠉⠅⠭⠈⠹⠭⠠⠏⠠⠳⠉⠅⠭⠈⠔⠠

$H/P = \langle xP \rangle$

H/P = \langle xP \rangle

3449. ⠳⠠⠓⠳ ⠨⠅ ⠳⠠⠏⠳⠈⠡⠳⠠⠳⠉⠅⠭⠈⠹⠭⠠⠏⠠⠳⠉⠅⠭⠈⠔⠠⠳

$|H| = |P| \cdot |\langle xP \rangle|$

|H| = |P| \cdot |\langle xP \rangle|

3450. ⠠⠓ ⠨⠅ ⠠⠏

$H = P$

`H=P`

3451. ⠠⠓⠌⠠⠏

H/P

`H/P`

3452. ⠭⠠⠏ ⠨⠅ ⠠⠏

$xP = P$

`xP = P`

3453. ⠈⠷⠠⠓⠱⠠⠝⠷⠠⠅⠾⠨⠩⠠⠓⠈⠾

$[H : N(K) \cap H]$

`[H:N(K) \cap H]`

3454. ⠠⠝⠷⠠⠅⠾⠨⠩⠠⠓

$N(K) \cap H$

`N(K) \cap H`

3455. ⠓⠘⠤⠂⠐⠠⠅⠓ ⠫⠳⠒⠒⠕ ⠷⠠⠝⠷⠠⠅⠾⠨⠩⠠⠓⠾⠓

$h^{-1}Kh \mapsto (N(K) \cap H)h$

`h^{-1}Kh \mapsto (N(K) \cap H)h`

3456. ⠷⠠⠝⠷⠠⠅⠾⠨⠩⠠⠓⠾⠓⠂ ⠨⠅ ⠷⠠⠝⠷⠠⠅⠾⠨⠩⠠⠓⠾⠓⠆

$(N(K) \cap H)h_1 = (N(K) \cap H)h_2$

`(N(K) \cap H)h_1 = (N(K) \cap H)h_2`

3457. ⠓⠆⠓⠂⠘⠤⠂ ⠈⠑ ⠠⠝⠷⠠⠅⠾

$h_2 h_1^{-1} \in N(K)$

`h_2 h_1^{-1} \in N(K)`

3458. ⠠⠅ ⠨⠅ ⠓⠆⠓⠂⠘⠤⠂⠐⠠⠅⠓⠂⠓⠆⠘⠤⠂

$K = h_2 h_1^{-1} K h_1 h_2^{-1}$

`K = h_2 h_1^{-1} K h_1 h_2^{-1}`

3459. ⠓⠂⠘⠤⠂⠐⠠⠅⠓⠂ ⠨⠅ ⠓⠆⠘⠤⠂⠐⠠⠅⠓⠆

$h_1^{-1} K h_1 = h_2^{-1} K h_2$

`h_1^{-1} K h_1 = h_2^{-1} K h_2`

3460. ⠠⠏⠂

P_1

`P_1`

3461. ⠠⠏⠆

P_2

`P_2`

3462. ⠛⠠⠏⠂⠛⠘⠤⠂⠐ ⠨⠅ ⠠⠏⠆

$g P_1 g^{-1} = P_2$

`g P_1 g^{-1} = P_2`

3463. ⠳⠠⠛⠳ ⠨⠅ ⠏⠘⠗⠐⠍

$|G| = p^r m$

|G|=p^r m

3464. ⠳⠠⠏⠳ ⠨⠅ ⠏⠘⠗

$|P| = p^r$

|P|=p^r

3465. caligraphic ⠠⠎ ⠨⠅ StartSet ⠠⠏ ⠨⠅ ⠠⠏⠂ ⠠⠠⠏⠆ ⠠⠄⠄⠄ ⠠⠠⠏⠰⠅ ⠐EndSet

$\mathcal{S} = \{P = P_1, P_2, \ldots, P_k\}$

{\mathcal S} = \{ P = P_1, P_2, \ldots, P_k \}

3466. ⠅ ⠨⠅ ⠈⠷⠠⠛⠱⠠⠝⠷⠠⠏⠾⠈⠾

$k = [G : N(P)]$

k = [G: N(P)]

3467. ⠳⠠⠛⠳ ⠨⠅ ⠏⠘⠗⠐⠍ ⠨⠅ ⠳⠠⠝⠷⠠⠏⠾⠳⠡⠈⠷⠠⠛⠱⠠⠝⠷⠠⠏⠾⠈⠾ ⠨⠅ ⠳⠠⠝⠷⠠⠏⠾⠳⠡⠅

$|G| = p^r m = |N(P)| \cdot [G : N(P)] = |N(P)| \cdot k$

|G|= p^r m = |N(P)| \cdot [G: N(P)]= |N(P)| \cdot k

3468. ⠳⠠⠝⠷⠠⠏⠾⠳

$|N(P)|$

|N(P)|

3469. ⠠⠟ ⠈⠑caligraphic ⠠⠎

$Q \in \mathcal{S}$

Q \in {\mathcal S}

3470. ⠈⠷⠠⠟⠱⠠⠝⠷⠠⠏⠰⠊⠐⠾⠨⠩⠠⠟⠈⠾

$[Q : N(P_i) \cap Q]$

[Q :N(P_i) \cap Q]

3471. ⠳⠠⠟⠳ ⠨⠅ ⠈⠷⠠⠟⠱⠠⠝⠷⠠⠏⠰⠊⠐⠾⠨⠩⠠⠟⠈⠾⠳⠠⠝⠷⠠⠏⠰⠊⠐⠾⠨⠩⠠⠟⠳

$|Q| = [Q : N(P_i) \cap Q]|N(P_i) \cap Q|$

|Q| = [Q :N(P_i) \cap Q] |N(P_i) \cap Q|

3472. ⠳⠠⠟⠳ ⠨⠅ ⠏⠘⠗

$|Q| = p^r$

|Q|= p^r

3473. ⠠⠏⠰⠚

P_j

P_j

3474. ⠭⠘⠤⠂⠐⠠⠏⠰⠚⠐⠭ ⠨⠅ ⠠⠏⠰⠚

$x^{-1} P_j x = P_j$

x^{-1} P_j x = P_j

3475. ⠭ ⠈⠑ ⠠⠟

$x \in Q$

x \in Q

3476. ⠠⠏⠰⠚⠐⠀⠨⠅⠀⠠⠟

$P_j = Q$

`P_j = Q`

3477. ⠂⠷⠍⠕⠙⠏⠾

$1 \pmod{p}$

`1 \pmod{p}`

3478. ⠳caligraphic ⠠⠎⠳

$|{\mathcal S}|$

`|{\mathcal S}|`

3479. StartSet ⠠⠏EndSet

$\{ P \}$

`\{ P \}`

3480. ⠳caligraphic ⠠⠎⠳⠀⠸⠇⠀⠂⠷⠍⠕⠙⠏⠾

$|{\mathcal S}| \equiv 1 \pmod{p}$

`|{\mathcal S}| \equiv 1 \pmod{p}`

3481. ⠠⠏⠀⠈⠑caligraphic ⠠⠎

$P \in {\mathcal S}$

`P \in {\mathcal S}`

3482. ⠳caligraphic ⠠⠎⠳⠀⠨⠅⠀⠳orbitof ⠠⠏⠳⠀⠨⠅⠀⠈⠷⠠⠛⠱⠠⠝⠷⠠⠏⠾⠈⠾

$|{\mathcal S}| = |\text{orbit of }P| = [G : N(P)]$

`|{\mathcal S}| = |\text{orbit of }P| = [G : N(P)]`

3483. ⠈⠷⠠⠛⠱⠠⠝⠷⠠⠏⠾⠈⠾

$[G : N(P)]$

`[G : N(P)]`

3484. ⠂⠷⠍⠕⠙⠢⠾

$1 \pmod{5}$

`1 \pmod{5}`

3485. ⠏⠀⠐⠅⠀⠟

$p < q$

`p \lt q`

3486. ⠟⠄⡷⠭⠆⠆⠖⠆⠄⠂⠷⠍⠕⠙⠏⠾

$q \not\equiv 1 \pmod{p}$

`q \not\equiv 1 \pmod{p}`

3487. ⠂⠬⠅⠟

$1 + kq$

`1 + kq`

3488. ⠅⠀⠨⠅⠀⠴⠠⠼⠂⠠⠄

$k = 0, 1, \ldots$

`k = 0, 1, \ldots`

3489. ⠂⠬⠟

$1 + q$

1 + q

3490. ⠂⠬⠅⠏

$1 + kp$

1 + kp

3491. ⠂⠬⠅⠏ ⠨⠅ ⠟

$1 + kp = q$

1 + kp = q

3492. ⠂⠬⠅⠏ ⠨⠅ ⠂

$1 + kp = 1$

1 + kp = 1

3493. ⠈⠠⠵⠰⠟

${\mathbb Z}_q$

{\mathbb Z}_q

3494. ⠠⠛ ⠈⠱⠈⠱ ⠈⠠⠵⠰⠏⠐⠡⠈⠠⠵⠰⠟⠐ ⠈⠱⠈⠱ ⠈⠠⠵⠰⠏⠟

$G \cong {\mathbb Z}_p \times {\mathbb Z}_q \cong {\mathbb Z}_{pq}$

G \cong {\mathbb Z}_p \times {\mathbb Z}_q \cong {\mathbb Z}_{pq}

3495. 15 ⠨⠅ ⠢⠡⠒

$15 = 5 \cdot 3$

15 = 5 \cdot 3

3496. ⠢⠄⡷⠭⠆⠆⠖⠆⠄⠂⠷⠍⠕⠙⠒⠾

$5 \not\equiv 1 \pmod{3}$

5 \not\equiv 1 \pmod{3}

3497. 99 ⠨⠅ ⠒⠘⠆⠐⠡11

$99 = 3^2 \cdot 11$

99 = 3^2 \cdot 11

3498. ⠂⠬⠒⠅

$1 + 3k$

1 + 3k

3499. ⠅ ⠨⠅ ⠴⠠ ⠂⠠ ⠆⠠ ⠄⠄⠄

$k = 0, 1, 2, \ldots$

k = 0, 1, 2, \ldots

3500. ⠂⠬11⠅

$1 + 11k$

1 +11k

3501. ⠈⠠⠵11

${\mathbb Z}_{11}$

{\mathbb Z}_{11}

3502. ⠈⠠⠵⠒⠡⠈⠠⠵⠒⠡⠈⠠⠵11

$\mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_{11}$

{\mathbb Z}_3 \times {\mathbb Z}_3 \times {\mathbb Z}_{11}

3503. ⠈⠠⠵⠔⠡⠈⠠⠵11

$\mathbb{Z}_9 \times \mathbb{Z}_{11}$

{\mathbb Z}_9 \times {\mathbb Z}_{11}

3504. ⠢⠡⠶⠡47 ⠨⠅ 1645

$5 \cdot 7 \cdot 47 = 1645$

5 \cdot 7 \cdot 47 = 1645

3505. ⠠⠛⠄ ⠨⠅ ⠠⡷⠭⠆⠶⠑⠦⠄ ⠈⠃⠈ ⠃⠤⠂ ⠐⠃ ⠃⠤⠂ ⠐⠱⠈ ⠠⠃ ⠉⠂⠠⠛⠄⡷⠭⠆⠶⠑⠔⠄

$G' = \langle aba^{-1}b^{-1} : a, b \in G \rangle$

G' = \langle a b a^{-1} b^{-1} : a, b \in G \rangle

3506. ⠼47

47

47

3507. ⠠⠛⠌⠠⠓⠂

G/H_1

G/H_1

3508. ⠳⠠⠛⠣⠐⠳

$|G'|$

|G'|

3509. ⠳⠠⠛⠣⠐⠳ ⠨⠅ ⠂

$|G'| = 1$

|G'|=1

3510. ⠳⠠⠛⠣⠐⠳ ⠨⠅ 47

$|G'| = 47$

|G'|=47

3511. ⠠⠓⠒

H_3

H_3

3512. ⠳⠠⠓⠆⠳ ⠨⠅ ⠢

$|H_2| = 5$

|H_2| = 5

3513. ⠳⠠⠓⠒⠳ ⠨⠅ ⠶

$|H_3| = 7$

|H_3| = 7

3514. ⠊ ⠨⠅ ⠂⠠⠼⠆

$i = 1, 2$

i= 1, 2

3515. ⠳⠠⠛⠘⠄⠐⠳ ∷ ⠂

$|G'| = 1$

|G'| =1

3516. ⠼27

27

27

3517. ⠼49

49

49

3518. ⠼64

64

64

3519. ⠼81

81

81

3520. 56 ∷ ⠆ ⠘⠒ ⠐⠡⠶

$56 = 2^3 \cdot 7$

56= 2^3 \cdot 7

3521. ⠦⠡⠖ ∷ 48

$8 \cdot 6 = 48$

8 \cdot 6 = 48

3522. ⠳⠠⠓⠠⠅⠳ ∷ ⠹⠳⠠⠓⠳⠡⠳⠠⠅⠳⠌⠳⠠⠓⠨⠩⠠⠅⠳⠼

$|HK| = \frac{|H| \cdot |K|}{|H \cap K|}$

|HK| = \frac{|H| \cdot |K|}{|H \cap K|}

3523. ⠠⠓⠠⠅ ∷ ⠨⠷⠓⠅⠈⠆⠓ ⠈⠑ ⠠⠓⠠⠅ ⠈⠑ ⠠⠅ ⠨⠾

$HK = \{hk : h \in H, k \in K\}$

HK = \{ hk : h \in H, k \in K \}

3524. ⠳⠠⠓⠠⠅⠳ ⠰⠱ ⠳⠠⠓⠳⠡⠳⠠⠅⠳

$|HK| \leq |H| \cdot |K|$

|HK| \leq |H| \cdot |K|

3525. ⠠⠓⠠⠅

HK

HK

3526. ⠓⠂⠅⠂ ∷ ⠓⠆⠅⠆

$h_1 k_1 = h_2 k_2$

h_1 k_1 = h_2 k_2

3527. ⠅⠂⠠⠅⠆ ⠈⠑⠠⠅

$k_1, k_2 \in K$

k_1, k_2 \in K

3528. ⠁ ⠨⠅ ⠷⠓⠂⠾⠘⠤⠂⠐⠓⠆ ⠨⠅ ⠅⠂⠷⠅⠆⠾⠘⠤⠂

$a = (h_1)^{-1}h_2 = k_1(k_2)^{-1}$

a = (h_1)^{-1} h_2 = k_1 (k_2)^{-1}

3529. ⠁ ⠈⠑ ⠠⠓ ⠨⠩ ⠠⠅

$a \in H \cap K$

a \in H \cap K

3530. ⠷⠓⠂⠾⠘⠤⠂⠐⠓⠆

$(h_1)^{-1}h_2$

(h_1)^{-1} h_2

3531. ⠅⠆⠷⠅⠂⠾⠘⠤⠂

$k_2(k_1)^{-1}$

k_2 (k_1)^{-1}

3532. ⠓ ⠨⠅ ⠓⠂⠃⠘⠤⠂

$h = h_1b^{-1}$

h = h_1 b^{-1}

3533. ⠅ ⠨⠅ ⠃⠅⠂

$k = bk_1$

k = b k_1

3534. ⠃ ⠈⠑ ⠠⠓ ⠨⠩ ⠠⠅

$b \in H \cap K$

b \in H \cap K

3535. ⠓⠅ ⠨⠅ ⠓⠂⠅⠂

$hk = h_1k_1$

h k = h_1 k_1

3536. ⠓⠅ ⠈⠑ ⠠⠓⠠⠅

$hk \in HK$

hk \in HK

3537. ⠓⠰⠊⠐⠅⠰⠊

h_ik_i

h_i k_i

3538. ⠓⠰⠊⠐ ⠈⠑ ⠠⠓

$h_i \in H$

h_i \in H

3539. ⠅⠰⠊⠐ ⠈⠑ ⠠⠅

$k_i \in K$

k_i \in K

3540. ⠳⠠⠓ ⠨⠩ ⠠⠅⠳

$|H \cap K|$

|H \cap K|

3541. ⠳⠠⠓⠠⠅⠳ ⠨⠅ ⠷⠳⠠⠓⠳⠡⠳⠠⠅⠳⠾⠸⠌⠳⠠⠓⠨⠩⠠⠅⠳

$|HK| = (|H| \cdot |K|)/|H \cap K|$

|HK| = (|H| \cdot |K|)/|H \cap K|

3542. ⠳⠠⠓⠨⠩⠠⠅⠳ ⠨⠅ ⠦

$|H \cap K| = 8$

|H \cap K| = 8

3543. ⠳⠠⠓⠨⠩⠠⠅⠳ ⠐⠅⠱ ⠲

$|H \cap K| \leq 4$

|H \cap K| \leq 4

3544. ⠳⠠⠓⠠⠅⠳ ⠨⠅ ⠹16⠡16⠌⠲⠼ ⠨⠅ 64

$|HK| = \frac{16 \cdot 16}{4} = 64$

|HK| = \frac{16 \cdot 16}{4} =64

3545. ⠠⠝⠷⠠⠓⠨⠩⠠⠅⠾

$N(H \cap K)$

N(H \cap K)

3546. ⠳⠠⠝⠷⠠⠓⠨⠩⠠⠅⠾⠳

$|N(H \cap K)|$

|N(H \cap K)|

3547. ⠳⠠⠝⠷⠠⠓⠨⠩⠠⠅⠾⠳ ⠨⠅ 48

$|N(H \cap K)| = 48$

|N(H \cap K)|= 48

3548. ⠠⠝⠷⠠⠓⠨⠩⠠⠅⠾ ⠨⠅ ⠠⠛

$N(H \cap K) = G$

N(H \cap K) = G

3549. ⠼18

18

18

3550. ⠼54

54

54

3551. ⠼80

80

80

3552. ⠳⠠⠛⠳ ⠨⠅ 18 ⠨⠅ ⠆⠡⠒⠘⠆

$|G| = 18 = 2 \cdot 3^2$

|G| = 18 = 2 \cdot 3^2

3553. ⠠⠏⠂ ⠨⠅ StartSet⠷⠂⠾ ⠠⠷123⠾ ⠠⠷132⠾EndSet

$P_1 = \{(1), (123), (132)\}$

P_1 = \{ (1), (123), (132) \}

3554. ⠠⠏⠆ ⠀⠨⠅⠀ StartSet⠷⠂ ⠾ ⠠⠷124⠾ ⠠⠷142⠾EndSet

$P_2 = \{(1), (124), (142)\}$

P_2 = \{ (1), (124), (142) \}

3555. ⠠⠏⠒ ⠀⠨⠅⠀ StartSet⠷⠂ ⠾ ⠠⠷134⠾ ⠠⠷143⠾EndSet

$P_3 = \{(1), (134), (143)\}$

P_3 = \{ (1), (134), (143) \}

3556. ⠠⠏⠲ ⠀⠨⠅⠀ StartSet⠷⠂ ⠾ ⠠⠷234⠾ ⠠⠷243⠾EndSet

$P_4 = \{(1), (234), (243)\}$

P_4 = \{ (1), (234), (243) \}

3557. ⠼45

45

45

3558. ⠼96

96

96

3559. ⠳⠠⠛⠳ ⠀⠨⠅⠀ 96 ⠀⠨⠅⠀ ⠆ ⠘⠢ ⠐⠡⠒

$|G| = 96 = 2^5 \cdot 3$

|G| = 96 = 2^5 \cdot 3

3560. ⠳⠠⠓ ⠨⠩ ⠠⠅⠳ ⠨⠂ ⠱ 16

$|H \cap K| \geq 16$

|H \cap K| \geq 16

3561. ⠷32⠐⠡32⠾⠌⠦ ⠀⠨⠅⠀ 128

$(32 \cdot 32)/8 = 128$

(32 \cdot 32)/8 = 128

3562. ⠼160

160

160

3563. ⠳⠠⠓⠳ ⠀⠨⠅⠀ ⠏⠘⠅

$|H| = p^k$

|H| = p^k

3564. ⠏⠘⠆⠐⠟⠘⠆

p^2q^2

p^2 q^2

3565. ⠟⠌⠳⠏⠘⠆⠐⠤⠂

$q \nmid p^2 - 1$

q \nmid p^2 - 1

3566. ⠏⠌⠳⠟⠘⠆⠐⠤⠂

$p \nmid q^2 - 1$

p \nmid q^2 - 1

3567. ⠟⠘⠆

q^2

q^2

3568. ⠼33

33

33

3569. ⠏⠘⠗⠤⠂

p^{r-1}

p^{r-1}

3570. ⠏⠘⠝⠐⠅

$p^n k$

p^n k

3571. ⠅ ⠐⠅ ⠏

$k < p$

k \lt p

3572. ⠛⠠⠝⠷⠠⠓⠾⠛⠘⠤⠂⠐ ⠨⠅ ⠠⠝⠷⠛⠠⠓⠛⠘⠤⠂⠐⠾

$gN(H)g^{-1} = N(gHg^{-1})$

g N(H) g^{-1} = N(gHg^{-1})

3573. ⠼108

108

108

3574. ⠼175

175

175

3575. ⠼255

255

255

3576. ⠳⠠⠛⠳ ⠨⠅ ⠒⠡⠢⠡17

$|G| = 3 \cdot 5 \cdot 17$

|G| = 3 \cdot 5 \cdot 17

3577. ⠏⠂⠘⠑⠂ ⠐⠄⠄⠄⠏⠰⠝⠘⠑⠘⠰⠝

$p_1^{e_1} \cdots p_n^{e_n}$

p_1^{e_1} \cdots p_n^{e_n}

3578. ⠠⠏⠂⠠⠄⠠⠠⠏⠰⠝⠐

$P_1, \ldots, P_n$

P_1, \ldots, P_n

3579. ⠳⠠⠏⠰⠊⠐⠳ ⠨⠅ ⠏⠰⠊⠘⠑⠘⠰⠊

$|P_i| = p_i^{e_i}$

|P_i| = p_i^{e_i}

3580. ⠠⠏⠂⠈⠡⠄⠄⠄⠈⠡⠠⠏⠰⠝

$P_1 \times \cdots \times P_n$

P_1 \times \cdots \times P_n

3581. ⠛⠠⠏⠛⠘⠤⠂⠐ ⠨⠅ ⠓⠠⠏⠓⠘⠤⠂⠐

$gPg^{-1} = hPh^{-1}$

gPg^{-1} = hPh^{-1}

3582. ⠠⠛ ⠨⠅ ⠠⠓⠠⠝

$G = HN$

G= HN

3583. ⠏⠘⠝⠐⠟

$p^n q$

p^nq

3584. ⠏ ⠨⠂ ⠟

$p > q$

p>q

3585. ⠈⠷⠠⠛⠈⠒⠠⠝⠷⠠⠓⠾⠈⠾

$[G : N(H)]$

[G : N(H)]

3586. ⠠⠝⠷⠠⠓⠾⠛ ⠫⠳⠒⠒⠕ ⠛⠘⠤⠂⠐⠠⠓⠛

$N(H)g \mapsto g^{-1}Hg$

N(H) g \mapsto g^{-1} H g

3587. ⠏⠄⠈⠳⠏⠘⠅ ⠐⠍ ⠏⠘⠅

$p \nmid \binom{p^k m}{p^k}$

p \nmid \binom{p^k m}{p^k}

3588. ⠏⠘⠅

p^k

p^k

3589. ⠁⠠⠞ ⠨⠅ StartSet⠁⠞⠒⠞ ⠈⠑ ⠠⠞EndSet

$aT = \{at : t \in T\}$

aT = \{ at : t \in T \}

3590. ⠠⠞ ⠈⠑caligraphic ⠠⠎

$T \in \mathcal{S}$

T \in {\mathcal S}

3591. ⠏⠌⠳⠳caligraphic ⠠⠕⠰⠠⠞⠐⠳

$p \nmid |\mathcal{O}_T|$

p \nmid | {\mathcal O}_T|

3592. StartSet ⠠⠞⠂ ⠠⠄ ⠄⠠⠞⠰⠥ ⠐EndSet

$\{T_1, \ldots, T_u\}$

\{ T_1, \ldots, T_u \}

3593. ⠏⠌⠳⠥

$p \nmid u$

p \nmid u

3594. ⠠⠓ ⠶ StartSet⠛ ⠈⠑ ⠠⠛⠱⠛ ⠠⠞⠂ ⠶ ⠠⠞⠂ EndSet

$H = \{g \in G : gT_1 = T_1\}$

H = \{ g \in G : gT_1 = T_1 \}

3595. ⠳⠠⠛⠳ ⠶ ⠥⠳⠠⠓⠳

$|G| = u|H|$

|G| = u |H|

3596. ⠏⠘⠅ ⠐ ⠅⠱ ⠳⠠⠓⠳

$p^k \leq |H|$

p^k \leq |H|

3597. ⠳⠠⠓⠳ ⠶ ⠳caligraphic ⠠⠕ ⠰ ⠠⠞ ⠐⠳ ⠅⠱ ⠏⠘⠅

$|H| = |\mathcal{O}_T| \leq p^k$

|H| = |{\mathcal O}_T| \leq p^k

3598. ⠏⠘⠅ ⠐ ⠶ ⠳⠠⠓⠳

$p^k = |H|$

p^k = |H|

3599. ⠨⠷⠁⠃⠁ ⠘⠤⠂ ⠐⠃ ⠘⠤⠂ ⠐⠒⠁ ⠠⠃ ⠈⠑ ⠠⠛⠨⠾

$\{aba^{-1}b^{-1} : a, b \in G\}$

\{ a b a^{-1} b^{-1} : a, b \in G \}

3600. ⠁ ⠠⠛⠣ ⠐ ⠠⠃ ⠠⠛⠣ ⠐ ⠈⠑ ⠠⠛⠌⠠⠛⠣

$aG', bG' \in G/G'$

a G', b G' \in G/G'

3601. ⠷⠁ ⠠⠛⠣ ⠐⠾⠷⠃ ⠠⠛⠣ ⠐⠾ ⠶ ⠁⠃ ⠠⠛⠣ ⠐ ⠶ ⠁⠃⠷⠃ ⠘⠤⠂ ⠐⠁ ⠘⠤⠂ ⠐⠃⠁⠾ ⠠⠛⠣ ⠐ ⠶ ⠷⠁⠃⠃ ⠘⠤

$(aG')(bG') = abG' = ab(b^{-1}a^{-1}ba)G' = (abb^{-1}a^{-1})baG' = baG'$

(a G')(b G') = ab G' = ab(b^{-1}a^{-1}ba) G' = (abb^{-1}a^{-1})ba
G' = ba G'

3602. ⠳⠠⠛⠳ ⠅⠱ 60

$|G| \leq 60$

|G| \leq 60

3603. ⠼19

19

19

3604. ⠼34

34

34

3605. ⠨35

35

35

3606. ⠨50

50

50

3607. ⠨36

36

36

3608. ⠨22

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22

3609. ⠨37

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37

3610. ⠨23

23

23

3611. ⠨38

38

38

3612. ⠨53

53

53

3613. ⠨55

55

55

3614. ⠨41

41

41

3615. ⠨42

42

42

3616. ⠨57

57

57

3617. ⠨43

43

43

3618. ⠠⠼58

58

58

3619. ⠠⠼29

29

29

3620. ⠠⠼44

44

44

3621. ⠠⠼59

59

59

3622. ⠝ ⠒⠒ ⠂ ⠄⠄ ⠄⠠⠼60

$n = 1, \ldots, 60$

n = 1, \ldots, 60

3623. ⠏ ⠘⠴

p^0

p^0

3624. ⠠⠙18

D_{18}

D_{18}

3625. 36 ⠒⠒ ⠆ ⠘⠆ ⠡⠲⠒ ⠘⠆

$36 = 2^2 \cdot 3^2$

36=2^2\cdot 3^2

3626. ⠏ ⠒⠒ ⠆

$p = 2$

p=2

3627. ⠠⠼⠂ ⠄⠠⠼⠒

$1, 3$

1, 3

3628. ⠏ ⠒⠒ ⠒

$p = 3$

p=3

3629. ⠖ ⠒⠒ ⠆⠑⠒

$6 = 2 \cdot 3$

6=2\cdot 3

3630. ⠠⠓⠠⠎

HS

HS

3631. 44352000

$44\,352\,000$

`44\,352\,000`

3632. ⠠⠙36

D_{36}

`D_{36}`

3633. ⠲⠝

$4n$

`4n`

3634. ⠝ ⠶ ⠆

$n=2$

`n=2`

3635. ⠁⠬⠃ ⠶ ⠃⠬⠁

$a + b = b + a$

`a + b = b + a`

3636. ⠁⠠⠃⠈⠑⠠⠗

$a, b \in R$

`a, b \in R`

3637. ⠷⠁⠬⠃⠾⠬⠉ ⠶ ⠁⠬⠷⠃⠬⠉⠾

$(a + b) + c = a + (b + c)$

`(a + b) + c = a + ( b + c)`

3638. ⠁⠠⠃⠠⠉⠈⠑⠠⠗

$a, b, c \in R$

`a, b, c \in R`

3639. ⠁⠬⠴ ⠶ ⠁

$a + 0 = a$

`a + 0 = a`

3640. ⠁⠈⠑⠠⠗

$a \in R$

`a \in R`

3641. ⠁⠬⠷⠤⠁⠾ ⠶ ⠴

$a + (-a) = 0$

`a + (-a) = 0`

3642. ⠷⠁⠃⠾⠉ ⠶ ⠁⠷⠃⠉⠾

$(ab) c = a (b c)$

`(ab) c = a ( b c)`

3643. ⠷⠠⠗⠠⠬⠾

$(R, +)$

`(R, +)`

3644. ⠂ ⠈⠑ ⠠⠗

$1 \in R$

```
1 \in R
```

3645. ⠂ ⠌⠨⠅ ⠴

$1 \neq 0$

```
1 \neq 0
```

3646. ⠂⠁ ⠨⠅ ⠁Baseline⠂ ⠨⠅ ⠁

$1a = a1 = a$

```
1a = a1 = a
```

3647. ⠁ ⠠⠃

a, b

```
a, b
```

3648. ⠁⠃ ⠨⠅ ⠴

$ab = 0$

```
ab = 0
```

3649. ⠁ ⠨⠅ ⠴

$a = 0$

```
a = 0
```

3650. ⠃ ⠨⠅ ⠴

$b = 0$

```
b = 0
```

3651. ⠁⠘⠤⠂⠐⠁ ⠨⠅ ⠁⠁⠘⠤⠂⠐ ⠨⠅ ⠂

$a^{-1}a = aa^{-1} = 1$

```
a^{-1} a = a a^{-1} = 1
```

3652. ⠁⠃ ⠨⠅ ⠴

$ab = 0$

```
a b = 0
```

3653. ⠁ ⠨⠅ ⠴

$a = 0$

```
a=0
```

3654. ⠃ ⠨⠅ ⠴

$b = 0$

```
b=0
```

3655. ⠂⠌⠆

$1/2$

```
1/2
```

3656. ⠁⠃⠷⠍⠕⠙⠝⠾

$ab \pmod{n}$

```
ab \pmod{n}
```

3657. ⠢⠈⠡⠶ ⠸⠇ 11⠷⠍⠕⠙⠘12⠾

$5 \cdot 7 \equiv 11 \pmod{12}$

```
5 \cdot 7 \equiv 11 \pmod{12}
```

3658. ⠒⠈⠡⠲ ⠸⠇ ⠴⠷⠍⠕⠙⠘12⠾

$3 \cdot 4 \equiv 0 \pmod{12}$

```
3 \cdot 4 \equiv 0 \pmod{12}
```

3659. ⠈⠷⠁ ⠠⠃ ⠈⠾

$[a,b]$

```
[a,b]
```

3660. ⠛⠷⠭⠾ ⠭ ⠉⠕⠎ ⠭

$g(x) = \cos x$

```
g(x) = \cos x
```

3661. ⠷⠋⠬⠛⠾⠷⠭⠾ ⠭ ⠋⠷⠭⠾⠬⠛⠷⠭⠾ ⠭ ⠭⠘⠆ ⠐⠬⠉⠕⠎ ⠭

$(f+g)(x) = f(x) + g(x) = x^2 + \cos x$

```
(f+g)(x) = f(x) + g(x) = x^2 + \cos x
```

3662. ⠷⠋⠛⠾⠷⠭⠾ ⠭ ⠋⠷⠭⠾⠛⠷⠭⠾ ⠭ ⠭⠘⠆ ⠐⠉⠕⠎ ⠭

$(fg)(x) = f(x)g(x) = x^2 \cos x$

```
(fg)(x) = f(x) g(x) = x^2 \cos x
```

3663. ⠠⠁⠠⠃ ⠭ ⠴

$AB = 0$

```
AB = 0
```

3664. ⠂ ⠭ ⠠⠷⠂⠐ ⠴⠂⠴ ⠠⠾ ⠠⠷⠴⠂ ⠴⠐ ⠠⠾⠠⠸⠊ ⠭ ⠠⠷⠴⠂ ⠴⠐ ⠠⠾⠤⠂ ⠴⠂⠴ ⠠⠾⠠⠸⠚ ⠭ ⠠⠷⠴⠂ ⠊
⠠⠷⠊ ⠴⠂⠴ ⠠⠾⠠⠸⠅ ⠭ ⠠⠷⠊ ⠴⠂⠴ ⠠⠷⠴⠂ ⠤⠊ ⠠⠾

$$1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad {\mathbf i} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad {\mathbf j} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \quad {\mathbf k} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

```
1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad {\mathbf
i} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad {\mathbf
j} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \quad {\mathbf
k} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}
```

3665. ⠈⠠⠓

$\mathbb{H}$

```
{\mathbb H}
```

3666. ⠁ ⠬⠃ ⠸⠊⠬⠉ ⠸⠚⠬⠙ ⠸⠅

$a + b\mathbf{i} + c\mathbf{j} + d\mathbf{k}$

```
a + b {\mathbf i} + c {\mathbf j} +d {\mathbf k}
```

3667. ⠁ ⠠⠃ ⠠⠉ ⠠⠙

a, b, c, d

```
a, b , c, d
```

3668. ⠠⠷⠨⠁ ⠨⠃ ⠠⠾⠠⠷⠤⠨⠃ overbar ⠨⠁ overbar ⠠⠾

$$\begin{pmatrix} \alpha & \beta \\ -\overline{\beta} & \overline{\alpha} \end{pmatrix}$$

\begin{pmatrix} \alpha & \beta \\ -\overline{\beta} & \overline{\alpha
} \end{pmatrix}

3669. ⠨⠁ ⠨⠅ ⠁⠬⠙⠊

$$\alpha = a + di$$

\alpha = a + di

3670. ⠨⠃ ⠨⠅ ⠃⠬⠉⠊

$$\beta = b + ci$$

\beta = b + ci

3671. ⠸⠊

$$\mathbf{i}$$

{\mathbf i}

3672. ⠸⠚

$$\mathbf{j}$$

{\mathbf j}

3673. ⠸⠅

$$\mathbf{k}$$

{\mathbf k}

3674. ⠷⠁⠂⠬⠃⠂⠸⠊⠬⠉⠂⠸⠚⠬⠙⠂⠸⠅⠾⠷⠁⠆⠬⠃⠆⠸⠊⠬⠉⠆⠸⠚⠬⠙⠆⠸⠅⠾ ⠨⠅ ⠨⠁⠬⠨⠃⠸⠊⠬⠨⠛⠸⠚⠬⠨⠙⠸⠅

$$(a_1 + b_1 \mathbf{i} + c_1 \mathbf{j} + d_1 \mathbf{k})(a_2 + b_2 \mathbf{i} + c_2 \mathbf{j} + d_2 \mathbf{k}) = \alpha + \beta \mathbf{i} + \gamma \mathbf{j} + \delta \mathbf{k}$$

(a_1 + b_1 {\mathbf i} + c_1 {\mathbf j} +d_1 {\mathbf k}) (a_2
+ b_2 {\mathbf i} + c_2 {\mathbf j} +d_2 {\mathbf k}) = \alpha
+ \beta {\mathbf i} + \gamma {\mathbf j} + \delta {\mathbf k}

3675. ⠷⠁⠬⠃⠸⠊⠬⠉⠸⠚⠬⠙⠸⠅⠾⠷⠁⠤⠃⠸⠊⠤⠉⠸⠚⠤⠙⠸⠅⠾ ⠨⠅ ⠁⠘⠆⠐⠬⠃⠘⠆⠐⠬⠉⠘⠆⠐⠬⠙⠘⠆

$$(a + b\mathbf{i} + c\mathbf{j} + d\mathbf{k})(a - b\mathbf{i} - c\mathbf{j} - d\mathbf{k}) = a^2 + b^2 + c^2 + d^2$$

(a + b {\mathbf i} + c {\mathbf j} + d {\mathbf k})(a - b {\mathbf
i} - c {\mathbf j} - d {\mathbf k}) = a^2 + b^2 + c^2 + d^2

3676. ⠁⠬⠃⠸⠊⠬⠉⠸⠚⠬⠙⠸⠅ ⠌⠨⠅ ⠴

$$a + b\mathbf{i} + c\mathbf{j} + d\mathbf{k} \neq 0$$

a + b {\mathbf i} + c {\mathbf j} +d {\mathbf k} \neq 0

3677. ⠷⠁⠬⠃⠸⠊⠬⠉⠸⠚⠬⠙⠸⠅⠾⠷⠹⠁⠤⠃⠸⠊⠤⠉⠸⠚⠤⠙⠸⠅⠌⠁⠘⠆⠐⠬⠃⠘⠆⠐⠬⠉⠘⠆⠐⠬⠙⠘⠆⠐⠼⠾ ⠨⠅ ⠂

$$(a + b\mathbf{i} + c\mathbf{j} + d\mathbf{k})\left(\frac{a - b\mathbf{i} - c\mathbf{j} - d\mathbf{k}}{a^2 + b^2 + c^2 + d^2} \right) = 1$$

(a + b {\mathbf i} + c {\mathbf j} + d {\mathbf k})\left(\frac{a
- b {\mathbf i} - c {\mathbf j} - d {\mathbf k} }{a^2 + b^2 + c^2
+ d^2} \right) = 1

3678. ⠁ Baseline⠴ ⠨⠅ ⠴⠁ ⠨⠅ ⠴

$$a0 = 0a = 0$$

a0 = 0a = 0

3679. ⠁⠷⠤⠃⠾ ⠭ ⠷⠤⠁⠾⠃ ⠭ ⠤⠁⠃

$a(-b) = (-a)b = -ab$

a(-b) = (-a)b = -ab

3680. ⠷⠤⠁⠾⠷⠤⠃⠾ ⠭ ⠁⠃

$(-a)(-b) = ab$

(-a)(-b) =ab

3681. ⠁ Baseline⠴ ⠭ ⠁⠷⠴⠬⠴⠾ ⠭ ⠁ Baseline⠴⠬ Baseline⠴ ⠷

$a0 = a(0 + 0) = a0 + a0;$

a0 = a(0+0)= a0+ a0;

3682. ⠁ Baseline⠴ ⠭ ⠴

$a0 = 0$

a0=0

3683. ⠴⠁ ⠭ ⠴

$0a = 0$

0a = 0

3684. ⠁⠃⠬⠁⠷⠤⠃⠾ ⠭ ⠁⠷⠃⠤⠃⠾ ⠭ ⠁ Baseline⠴ ⠭ ⠴

$ab + a(-b) = a(b - b) = a0 = 0$

ab + a(-b) = a(b-b) = a0 = 0

3685. ⠤⠁⠃ ⠭ ⠁⠷⠤⠃⠾

$-ab = a(-b)$

-ab = a(-b)

3686. ⠤⠁⠃ ⠭ ⠷⠤⠁⠾⠃

$-ab = (-a)b$

-ab = (-a)b

3687. ⠷⠤⠁⠾⠷⠤⠃⠾ ⠭ ⠤⠷⠁⠷⠤⠃⠾⠾ ⠭ ⠤⠷⠤⠁⠃⠾ ⠭ ⠁⠃

$(-a)(-b) = -(a(-b)) = -(-ab) = ab$

(-a)(-b) = -(a(- b)) = -(-ab) = ab

3688. ⠈⠠⠵⠇⠪⠈⠠⠟⠇⠪⠈⠠⠗⠇⠪⠈⠠⠉

$\mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$

{\mathbb Z} \subset {\mathbb Q} \subset {\mathbb R} \subset {\mathbb C}

3689. ⠗⠎⠈⠑⠠⠎

$rs \in S$

rs \in S

3690. ⠗ ⠠⠎⠈⠑⠠⠎

$r, s \in S$

r, s \in S

3691. ⠗⠤⠎⠀⠈⠑⠀⠠⠎

$r - s \in S$

r-s \in S

3692. ⠠⠗⠀⠨⠅⠀⠈⠠⠍⠆⠷⠈⠠⠗⠾

$R = \mathbb{M}_2(\mathbb{R})$

R ={\mathbb M}_2({\mathbb R})

3693. ⠠⠞⠀⠨⠅⠀⠨⠷⠠⠷⠁⠀⠃⠠⠾⠀⠠⠷⠼⠴⠀⠉⠠⠾⠱⠁⠠⠃⠠⠉⠈⠑⠈⠠⠗⠨⠾

$$T = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} : a, b, c \in \mathbb{R} \right\}$$

T = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} : a, b,
c \in {\mathbb R} \right\}

3694. ⠠⠁⠀⠨⠅⠀⠠⠷⠁⠀⠃⠠⠾⠀⠠⠷⠼⠴⠀⠉⠠⠾and⠠⠃⠀⠨⠅⠀⠠⠷⠁⠘⠄⠐⠀⠃⠘⠄⠐⠠⠾
⠠⠷⠼⠴⠀⠉⠄⠠⠾

$$A = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} a' & b' \\ 0 & c' \end{pmatrix}$$

A = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \quad \text{and}
\quad B = \begin{pmatrix} a' & b' \\ 0 & c' \end{pmatrix}

3695. ⠠⠁⠤⠠⠃

$A - B$

A-B

3696. ⠠⠁⠠⠃⠀⠨⠅⠀⠠⠷⠁⠁⠘⠄⠐⠀⠁⠃⠘⠄⠐⠬⠃⠉⠘⠄⠐⠠⠾⠀⠠⠷⠼⠴⠀⠉⠉⠄⠠⠾

$$AB = \begin{pmatrix} aa' & ab' + bc' \\ 0 & cc' \end{pmatrix}$$

AB = \begin{pmatrix} a a' & ab' + bc' \\ 0 & cc' \end{pmatrix}

3697. ⠎⠀⠈⠑⠀⠠⠗

$s \in R$

s \in R

3698. ⠗⠎⠀⠨⠅⠀⠴

$rs = 0$

rs = 0

3699. ⠈⠠⠵⠈⠷⠊⠈⠾⠀⠨⠅⠀⠨⠷⠍⠬⠝⠊⠱⠍⠠⠝⠈⠑⠈⠠⠵⠨⠾

$\mathbb{Z}[i] = \{m + ni : m, n \in \mathbb{Z}\}$

{\mathbb Z}[i] = \{ m + ni : m, n \in {\mathbb Z} \}

3700. ⠨⠁⠀⠨⠅⠀⠁⠬⠃⠊

$\alpha = a + bi$

\alpha = a + bi

3701. ⠈⠠⠵⠈⠷⠊⠈⠾

$\mathbb{Z}[i]$

{\mathbb Z}[i]

3702. ⠍ overbar ⠭ ⠁⠤⠃⠊

$\overline{\alpha} = a - bi$

\overline{\alpha} = a - bi

3703. ⠍ ⠝ ⠭ ⠂

$\alpha\beta = 1$

\alpha \beta = 1

3704. ⠍ overbar ⠝ overbar ⠭ ⠂

$\overline{\alpha}\overline{\beta} = 1$

\overline{\alpha} \overline{\beta} = 1

3705. ⠝ ⠭ ⠉⠖⠙⠊

$\beta = c + di$

\beta = c + di

3706. ⠂ ⠭ ⠍ ⠝ ⠍ overbar ⠝ overbar ⠭ ⠷⠁ ⠘⠆ ⠖⠃ ⠘⠆ ⠾⠷⠉ ⠘⠆ ⠖⠙ ⠘⠆ ⠾

$1 = \alpha\beta\overline{\alpha}\overline{\beta} = (a^2 + b^2)(c^2 + d^2)$

1 = \alpha \beta \overline{\alpha} \overline{\beta} = (a^2 + b^2
)(c^2 + d^2)

3707. ⠁ ⠘⠆ ⠖⠃ ⠘⠆

$a^2 + b^2$

a^2 + b^2

3708. ⠁ ⠖⠃⠊ ⠭ ⠬⠤⠂

$a + bi = \pm 1$

a + bi = \pm 1

3709. ⠁ ⠖⠃⠊ ⠭ ⠬⠤⠊

$a + bi = \pm i$

a + bi = \pm i

3710. ⠬⠤⠊

$\pm i$

\pm i

3711. ⠠⠋ ⠭ StartSet ⠠⠷⠂⠴ ⠂⠴ ⠠⠷⠂⠂ ⠂⠴ ⠠⠾⠂ ⠠⠷⠂⠂ ⠂⠴ ⠠⠾⠂ ⠠⠷⠂⠴ ⠂⠂ ⠠⠾⠂ ⠠⠷⠴⠂ ⠂⠂ ⠠⠾⠂ ⠠⠷⠴⠴ ⠴⠴ ⠠⠾EndSet

$$F = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \right\}$$

F = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix}
1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix},
\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \right\}

3712. ⠁ ⠠⠟⠷⠜⠆⠻⠾ ⠭ ⠸⠷⠁ ⠖⠃⠜⠆⠻ ⠐⠂ ⠁⠠⠂ ⠃ ⠈⠑ ⠠⠟⠸⠾

$\mathbb{Q}(\sqrt{2}\,) = \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$

{\mathbb Q}(\sqrt{2}\,) = \{ a + b \sqrt{2} : a, b \in {\mathbb
Q} \}

3713. ⠁ ⠤⠃ ⠜⠆ ⠻

$a + b\sqrt{2}$

a + b \sqrt{2}

3714. ⠈ ⠠⠟⠷⠜⠆ ⠻⠾

$\mathbb{Q}(\sqrt{2}\,)$

{\mathbb Q}(\sqrt{2}\,)

3715. ⠹⠁ ⠠⠉ ⠣⠆ ⠐⠤⠆⠃ ⠣⠆ ⠐⠰⠆⠤⠹⠤⠃ ⠠⠉ ⠣⠆ ⠐⠤⠆⠃ ⠣⠆ ⠐⠰⠆⠜⠆ ⠻

$\frac{a}{a^2-2b^2} + \frac{-b}{a^2-2b^2}\sqrt{2}$

\frac{a}{a^2 - 2 b^2} +\frac{- b}{ a^2 - 2 b^2} \sqrt{2}

3716. ⠁ ⠈⠑ ⠠⠙

$a \in D$

a \in D

3717. ⠃ ⠶ ⠉

$b = c$

b=c

3718. ⠁⠷⠃⠤⠉⠾ ⠶ ⠴

$a(b-c) = 0$

a(b - c) =0

3719. ⠃⠤⠉ ⠶ ⠴

$b - c = 0$

b - c = 0

3720. ⠁⠃ ⠶ ⠁ Baseline⠴

$ab = a0$

ab = a 0

3721. ⠠⠙⠘⠈⠼

D^*

D^\ast

3722. ⠠⠙⠘⠈⠼

D^*

D^*

3723. ⠁ ⠈⠑ ⠠⠙⠘⠈⠼

$a \in D^*$

a \in D^\ast

3724. ⠨⠇ ⠰⠁ ⠐⠱ ⠠⠙⠘⠈⠼⠐ ⠫⠕ ⠠⠙⠘⠈⠼

$\lambda_a : D^* \rightarrow D^*$

\lambda_a : D^\ast \rightarrow D^\ast

3725. ⠨⠇ ⠰⠁ ⠐⠷⠙⠾ ⠶ ⠁⠙

$\lambda_a(d) = ad$

\lambda_a(d) = ad

3726. ⠙ ⠌⠨⠅ ⠴

$d \neq 0$

d \neq 0

3727. ⠁⠙ ⠌⠨⠅ ⠴

$ad \neq 0$

ad \neq 0

3728. ⠨⠇⠰⠁

λ_a

\lambda_a

3729. ⠙⠂⠠⠙⠆⠈⠑⠠⠙⠘⠈⠼

$d_1, d_2 \in D^*$

d_1, d_2 \in D^*

3730. ⠁⠙⠂ ⠨⠅ ⠨⠇⠰⠁⠐⠷⠙⠂⠾ ⠨⠅ ⠨⠇⠰⠁⠐⠷⠙⠆⠾ ⠨⠅ ⠁⠙⠆

$ad_1 = \lambda_a(d_1) = \lambda_a(d_2) = ad_2$

ad_1 = \lambda_a(d_1) = \lambda_a(d_2) = ad_2

3731. ⠙⠂ ⠨⠅ ⠙⠆

$d_1 = d_2$

d_1 = d_2

3732. ⠙⠈⠑⠠⠙⠘⠈⠼

$d \in D^\ast$

d \in D^\ast

3733. ⠨⠇⠰⠁⠐⠷⠙⠾ ⠨⠅ ⠁⠙ ⠨⠅ ⠂

$\lambda_a(d) = ad = 1$

\lambda_a(d) = ad = 1

3734. ⠗⠬⠄⠄⠄⠬⠗

$r + \cdots + r$

r + \cdots + r

3735. ⠝⠗

nr

nr

3736. ⠝⠗ ⠨⠅ ⠴

$nr = 0$

nr = 0

3737. ⠗⠈⠑⠠⠗

$r \in R$

r \in R

3738. ⠉⠓⠁⠗ ⠠⠗

$\operatorname{char} R$

\chr R

3739. ⠏⠁ ⠨⠅ ⠴

$pa = 0$

pa =0

3740. ⠝Baseline⠐ ⠨⠅ ⠴

$n1 = 0$

n 1 = 0

3741. ⠝⠗ ⠨⠅ ⠝⠷⠂⠗⠾ ⠨⠅ ⠷⠝Baseline⠐⠾⠗ ⠨⠅ ⠴⠗ ⠨⠅ ⠴

$nr = n(1r) = (n1)r = 0r = 0$

nr = n(1r) = (n 1) r = 0r = 0

3742. ⠝Baseline⠐ ⠨⠅ ⠴

$n1 = 0$

n1 = 0

3743. ⠝ ⠨⠅ ⠁⠃

$n = ab$

n = ab

3744. ⠂ ⠐⠅ ⠁ ⠐⠅ ⠝

$1 < a < n$

1 \lt a \lt n

3745. ⠂ ⠐⠅ ⠃ ⠐⠅ ⠝

$1 < b < n$

1 \lt b \lt n

3746. ⠴ ⠨⠅ ⠝Baseline⠐ ⠨⠅ ⠷⠁⠃⠾⠂ ⠨⠅ ⠷⠁ Baseline⠐⠾⠷⠃ Baseline⠐⠾

$0 = n1 = (ab)1 = (a1)(b1)$

0 = n 1 = (ab)1 = (a1)(b1)

3747. ⠁ Baseline⠐ ⠨⠅ ⠴

$a1 = 0$

a1 =0

3748. ⠃ Baseline⠐ ⠨⠅ ⠴

$b1 = 0$

b1=0

3749. ⠨⠋⠈⠆ ⠠⠗ ⠫⠕ ⠠⠎

$\phi : R \rightarrow S$

\phi : R \rightarrow S

3750. ⠅⠑⠗ ⠨⠋ ⠨⠅ StartSet⠗ ⠈⠑ ⠠⠗⠈⠆ ⠨⠋⠷⠗⠾ ⠨⠅ ⠴EndSet

$\ker \phi = \{r \in R : \phi(r) = 0\}$

\ker \phi = \{ r \in R : \phi(r) = 0 \}

3751. ⠨⠋⠈⠆ ⠈⠰⠵ ⠫⠕ ⠈⠰⠵⠰⠝⠐

$\phi : \mathbb{Z} \rightarrow \mathbb{Z}_n$

\phi : {\mathbb Z} \rightarrow {\mathbb Z}_n

3752. ⠁ ⠫⠳⠒⠒⠕ ⠁⠷⠍⠕⠙⠝⠾

$a \mapsto a \pmod{n}$

a \mapsto a \pmod{n}

3753. ⠠⠉⠈⠷⠁⠠⠃⠈⠾

$C[a, b]$

C[a, b]

3754. ⠨⠁⠈⠑⠈⠷⠁⠠⠃⠈⠾

$\alpha \in [a, b]$

\alpha \in [a, b]

3755. ⠨⠋⠰⠨⠁⠐⠱⠠⠉⠈⠷⠁⠠⠃⠈⠾ ⠫⠕ ⠈⠠⠗

$\phi_{\alpha} : C[a, b] \rightarrow {\mathbb R}$

\phi_{\alpha} : C[a, b] \rightarrow {\mathbb R}

3756. ⠨⠋⠰⠨⠁⠐⠷⠋⠾ ⠰⠆ ⠋⠷⠨⠁⠾

$\phi_{\alpha} (f) = f(\alpha)$

\phi_{\alpha} (f) = f(\alpha)

3757. ⠨⠋⠰⠨⠁

ϕ_{α}

\phi_{\alpha}

3758. ⠨⠋⠷⠠⠗⠾

$\phi(R)$

\phi(R)

3759. ⠨⠋⠷⠴⠾ ⠰⠆ ⠴

$\phi(0) = 0$

\phi(0) = 0

3760. ⠂⠰⠠⠗

1_R

1_R

3761. ⠂⠰⠠⠎

1_S

1_S

3762. ⠨⠋⠷⠂⠰⠠⠗⠐⠾ ⠰⠆ ⠂⠰⠠⠎

$\phi(1_R) = 1_S$

\phi(1_R) = 1_S

3763. ⠨⠋⠷⠠⠗⠾ ⠌⠰⠆ StartSet⠴EndSet

$\phi(R) \neq \{ 0 \}$

\phi(R) \neq \{ 0 \}

3764. ⠁⠗

ar

ar

3765. ⠗⠁

ra

ra

3766. ⠗ ⠠⠊ ⠸⠐⠅ ⠠⠊

$rI \subset I$

rI \subset I

3767. ⠠⠊⠗ ⠸⠐⠅ ⠠⠊

$Ir \subset I$

Ir \subset I

3768. ⠗Baseline⠂ ⠨⠅ ⠗ ⠈⠑ ⠠⠊

$r1 = r \in I$

r1 = r \in I

3769. ⠠⠊ ⠨⠅ ⠠⠗

$I = R$

I = R

3770. ⠄⡷⠛⠅ ⠿⠑⠒⠄ ⠁ ⠄⡷⠛⠅ ⠿⠑⠢⠄ ⠨⠅ StartSet⠁ ⠗⠈⠒⠗ ⠈⠑ ⠠⠗EndSet

$\langle a \rangle = \{ar : r \in R\}$

\langle a \rangle = \{ ar : r \in R \}

3771. ⠴ ⠨⠅ ⠁Baseline⠴

$0 = a0$

0 = a0

3772. ⠁ ⠨⠅ ⠁Baseline⠂

$a = a1$

a = a1

3773. ⠁ ⠗⠬⠁ ⠗ ⠄⠠ ⠐ ⠨⠅ ⠁ ⠷⠗⠬⠗ ⠄⠠ ⠐⠾

$ar + ar' = a(r + r')$

ar + ar' = a(r + r')

3774. ⠤⠁ ⠗ ⠨⠅ ⠁ ⠷⠤⠗⠾ ⠈⠑ ⠄⡷⠛⠅ ⠿⠑⠒⠄ ⠁ ⠄⡷⠛⠅ ⠿⠑⠢⠄

$-ar = a(-r) \in \langle a \rangle$

-ar = a (-r) \in \langle a \rangle

3775. ⠁ ⠗ ⠈⠑ ⠄⡷⠛⠅ ⠿⠑⠒⠄ ⠁ ⠄⡷⠛⠅ ⠿⠑⠢⠄

$ar \in \langle a \rangle$

ar \in \langle a \rangle

3776. ⠎⠷⠁ ⠗⠾ ⠨⠅ ⠁ ⠷⠎⠗⠾

$s(ar) = a(sr)$

s(ar) = a(sr)

3777. ⠄⡷⠛⠅ ⠿⠑⠒⠄ ⠴ ⠄⡷⠛⠅ ⠿⠑⠢⠄ ⠨⠅ StartSet⠴EndSet

$\langle 0 \rangle = \{0\}$

\langle 0 \rangle = \{ 0 \}

3778. ⠁⠀⠨⠅⠀⠝⠟⠬⠗

$a = nq + r$

a = nq + r

3779. ⠗⠀⠨⠅⠀⠁⠤⠝⠟⠀⠈⠑⠀⠠⠊

$r = a - nq \in I$

r = a - nq \in I

3780. ⠁⠀⠨⠅⠀⠝⠟

$a = nq$

a = nq

3781. ⠠⠊⠀⠨⠅⠀⠠⡷⠭⠆⠶⠑⠦⠄⠝⠄⡷⠭⠆⠶⠑⠔⠄

$I = \langle n \rangle$

I = \langle n \rangle

3782. ⠝⠁

na

na

3783. ⠝⠁⠃

nab

nab

3784. ⠁⠀⠈⠑⠅⠑⠗⠀⠨⠋

$a \in \ker \phi$

a \in \ker \phi

3785. ⠨⠋⠷⠁⠗⠾⠀⠨⠅⠀⠨⠋⠷⠁⠾⠨⠋⠷⠗⠾⠀⠨⠅⠀⠴⠨⠋⠷⠗⠾⠀⠨⠅⠀⠴

$\phi(ar) = \phi(a)\phi(r) = 0\phi(r) = 0$

\phi(ar) = \phi(a) \phi(r) = 0 \phi(r) = 0

3786. ⠨⠋⠷⠗⠁⠾⠀⠨⠅⠀⠨⠋⠷⠗⠾⠨⠋⠷⠁⠾⠀⠨⠅⠀⠨⠋⠷⠗⠾⠴⠀⠨⠅⠀⠴

$\phi(ra) = \phi(r)\phi(a) = \phi(r)0 = 0$

\phi(ra) = \phi(r) \phi(a) = \phi(r)0 = 0

3787. ⠠⠗⠌⠠⠊

R/I

R/I

3788. ⠷⠗⠬⠠⠊⠾⠷⠎⠬⠠⠊⠾⠀⠨⠅⠀⠗⠎⠬⠠⠊

$(r + I)(s + I) = rs + I$

(r + I)(s + I) = rs + I

3789. ⠗⠬⠠⠊

$r + I$

r+I

3790. ⠎⠬⠠⠊

$s + I$

s +I

3791. ⠗⠄ ⠈⠑⠗⠬⠠⠊

$r' \in r + I$

r' \in r+I

3792. ⠎⠄ ⠈⠑⠎⠬⠠⠊

$s' \in s + I$

s' \in s+I

3793. ⠗⠄⠎⠄

$r's'$

r's'

3794. ⠗⠎⠬⠠⠊

$rs + I$

rs+I

3795. ⠗⠄ ⠨⠅ ⠗⠬⠁

$r' = r + a$

r' = r + a

3796. ⠃ ⠈⠑⠠⠊

$b \in I$

b \in I

3797. ⠎⠄ ⠨⠅ ⠎⠬⠃

$s' = s + b$

s' = s + b

3798. ⠗⠄⠎⠘⠄ ⠐ ⠨⠅ ⠷⠗⠬⠁⠾⠷⠎⠬⠃⠾ ⠨⠅ ⠗⠎⠬⠁⠎⠬⠗⠃⠬⠁⠃

$r's' = (r + a)(s + b) = rs + as + rb + ab$

r' s' = (r+a)(s+b) = rs + as + rb + ab

3799. ⠁⠎⠬⠗⠃⠬⠁⠃ ⠈⠑⠠⠊

$as + rb + ab \in I$

as + rb + ab \in I

3800. ⠗⠄⠎⠄ ⠈⠑⠗⠎⠬⠠⠊

$r's' \in rs + I$

r' s' \in rs + I

3801. ⠨⠋⠒⠠⠗ ⠫⠕ ⠠⠗⠈⠠⠊

$\phi : R \rightarrow R/I$

\phi : R \rightarrow R/I

3802. ⠨⠋⠷⠗⠾ ⠨⠅ ⠗⠬⠠⠊

$\phi(r) = r + I$

\phi(r) = r + I

3803. ⠨⠋⠷⠗⠾⠨⠋⠷⠎⠾ ⠨⠅ ⠷⠗⠬⠠⠊⠾⠷⠎⠬⠠⠊⠾ ⠨⠅ ⠗⠎⠬⠠⠊ ⠨⠅ ⠨⠋⠷⠗⠎⠾

$\phi(r)\phi(s) = (r + I)(s + I) = rs + I = \phi(rs)$

\phi(r) \phi(s) = (r + I)(s+I) = rs + I = \phi(rs)

3804. ⠨⠓⠱⠠⠗⠀⠫⠕⠀⠠⠎

$\psi : R \rightarrow S$

```
\psi : R \rightarrow S
```

3805. ⠅⠑⠗⠨⠓

$\ker \psi$

```
\ker \psi
```

3806. ⠨⠋⠱⠠⠗⠀⠫⠕⠀⠠⠗⠌⠅⠑⠗⠨⠓

$\phi : R \rightarrow R/\ker \psi$

```
\phi : R \rightarrow R/\ker \psi
```

3807. ⠨⠦⠱⠠⠗⠌⠅⠑⠗⠨⠓⠀⠫⠕⠀⠨⠓⠷⠠⠗⠾

$\eta: R/\ker \psi \rightarrow \psi(R)$

```
\eta: R/\ker \psi \rightarrow \psi(R)
```

3808. ⠠⠅⠀⠨⠅⠀⠅⠑⠗⠨⠓

$K = \ker \psi$

```
K = \ker \psi
```

3809. ⠨⠦⠱⠠⠗⠌⠠⠅⠀⠫⠕⠀⠨⠓⠷⠠⠗⠾

$\eta: R/K \rightarrow \psi(R)$

```
\eta: R/K \rightarrow \psi(R)
```

3810. ⠨⠦⠷⠗⠬⠠⠅⠾⠀⠨⠅⠀⠨⠓⠷⠗⠾

$\eta(r + K) = \psi(r)$

```
\eta(r + K) = \psi(r)
```

3811. ⠠⠗⠌⠠⠅

R/K

```
R/K
```

3812. ⠨⠦⠷⠷⠗⠬⠠⠅⠾⠷⠎⠬⠠⠅⠾⠾⠀⠨⠅⠀⠨⠦⠷⠗⠬⠠⠅⠾⠨⠦⠷⠎⠬⠠⠅⠾

$\eta((r + K)(s + K)) = \eta(r + K) \eta(s + K)$

```
\eta( (r + K)(s + K) ) = \eta(r + K) \eta( s + K)
```

3813. ⠠⠊⠨⠩⠠⠚

$I \cap J$

```
I \cap J
```

3814. ⠠⠊⠌⠠⠊⠨⠩⠠⠚⠀⠈⠱⠈⠱⠀⠷⠠⠊⠬⠠⠚⠾⠌⠠⠚

$I / I \cap J \cong (I+ J) /J$

```
I / I \cap J \cong (I+ J) /J
```

3815. ⠠⠚⠀⠸⠐⠅⠀⠠⠊

$J \subset I$

```
J \subset I
```

3816. ⠠⠗⠌⠠⠊⠀⠈⠱⠈⠱⠀⠹⠠⠗⠌⠠⠚⠌⠠⠊⠌⠠⠚⠼

$R/I \cong \frac{R/J}{I/J}$

```
R/I \cong \frac{R/J}{I/J}
```

3817. ⠠⠎ ⠫⠳⠒⠒⠕ ⠠⠎⠌⠠⠊

$S \mapsto S/I$

S \mapsto S/I

3818. M

M

3819. ⠠⠗⠌⠠⠍

R/M

R/M

3820. ⠂ ⠬⠠⠍

$1 + M$

1 + M

3821. ⠁ ⠬⠠⠍

$a + M$

a + M

3822. ⠁ ⠌⠈⠑ ⠠⠍

$a \notin M$

a \notin M

3823. StartSet⠗⠁ ⠬⠍⠱⠗ ⠈⠑ ⠠⠗and⠍ ⠈⠑ ⠠⠍EndSet

$\{ ra + m : r \in R \text{ and } m \in M \}$

\{ ra + m : r \in R \text{ and } m \in M \}

3824. ⠴⠁⠬⠴ ⠨⠅ ⠴

$0a + 0 = 0$

0a+0=0

3825. ⠗⠂⠁⠬⠍⠂

$r_1 a + m_1$

r_1 a + m_1

3826. ⠗⠆⠁⠬⠍⠆

$r_2 a + m_2$

r_2 a + m_2

3827. ⠷⠗⠂⠁⠬⠍⠂⠾⠤⠷⠗⠆⠁⠬⠍⠆⠾ ⠨⠅ ⠷⠗⠂⠤⠗⠆⠾⠁⠬⠷⠍⠂⠤⠍⠆⠾

$(r_1 a + m_1) - (r_2 a + m_2) = (r_1 - r_2)a + (m_1 - m_2)$

(r_1 a + m_1) - (r_2 a + m_2) = (r_1 - r_2)a + (m_1 - m_2)

3828. ⠠⠊ ⠨⠅ ⠠⠗

$I = R$

I=R

3829. ⠂ ⠨⠅ ⠁⠃⠬⠍

$1 = ab + m$

1=ab+m

3830. ⠂⠬⠠⠍ ⠨⠅ ⠁⠃⠬⠠⠍ ⠨⠅ ⠃⠁⠬⠠⠍ ⠨⠅ ⠷⠁⠬⠠⠍⠾⠷⠃⠬⠠⠍⠾

$1 + M = ab + M = ba + M = (a + M)(b + M)$

`1 + M = ab + M = ba + M = (a+M)(b+M)`

3831. ⠴⠬⠠⠍ ⠨⠅ ⠠⠍

$0 + M = M$

`0 + M = M`

3832. ⠁⠬⠠⠍

$a + M$

`a+ M`

3833. ⠃⠬⠠⠍

$b + M$

`b +M`

3834. ⠷⠁⠬⠠⠍⠾⠷⠃⠬⠠⠍⠾ ⠨⠅ ⠁⠃⠬⠠⠍ ⠨⠅ ⠂⠬⠠⠍

$(a + M)(b + M) = ab + M = 1 + M$

`(a+M)(b+M) = ab + M = 1+M`

3835. ⠍ ⠈⠑ ⠠⠍

$m \in M$

`m \in M`

3836. ⠁⠃⠬⠍ ⠨⠅ ⠂

$ab + m = 1$

`ab + m = 1`

3837. ⠗Baseline⠂ ⠨⠅ ⠗ ⠈⠑ ⠠⠊

$r1 = r \in I$

`r1 =r \in I`

3838. ⠏⠈⠠⠵

$p\mathbb{Z}$

`p{\mathbb Z}`

3839. ⠈⠠⠵⠌⠏⠈⠠⠵ ⠈⠱⠈⠱ ⠈⠠⠵⠰⠏

$\mathbb{Z}/p\mathbb{Z} \cong \mathbb{Z}_p$

`{\mathbb Z}/ p {\mathbb Z} \cong {\mathbb Z}_p`

3840. ⠁⠃ ⠈⠑ ⠠⠏

$ab \in P$

`ab \in P`

3841. ⠁ ⠈⠑ ⠠⠏

$a \in P$

`a \in P`

3842. ⠃ ⠈⠑ ⠠⠏

$b \in P$

`b \in P`

3843. ⠠⠏ ⠨⠅ StartSet⠴ ⠠⠆ ⠠⠲ ⠠⠖ ⠠⠦ ⠠10EndSet

$P = \{0, 2, 4, 6, 8, 10\}$

P = \{ 0, 2, 4, 6, 8, 10 \}

3844. ⠠⠗⠌⠠⠏

R/P

R/P

3845. ⠁ ⠬⠠⠏

$a + P$

a + P

3846. ⠃ ⠬⠠⠏

$b + P$

b + P

3847. ⠷⠁ ⠬⠠⠏⠾⠷⠃ ⠬⠠⠏⠾ ⠨⠅ ⠴⠬⠠⠏ ⠨⠅ ⠠⠏

$(a + P)(b + P) = 0 + P = P$

(a + P)(b + P) = 0 + P = P

3848. ⠁ ⠬⠠⠏ ⠨⠅ ⠠⠏

$a + P = P$

a + P = P

3849. ⠃ ⠬⠠⠏ ⠨⠅ ⠠⠏

$b + P = P$

b + P = P

3850. ⠷⠁ ⠬⠠⠏⠾⠷⠃ ⠬⠠⠏⠾ ⠨⠅ ⠁⠃ ⠬⠠⠏ ⠨⠅ ⠴⠬⠠⠏ ⠨⠅ ⠠⠏

$(a + P)(b + P) = ab + P = 0 + P = P$

(a + P)(b + P) = ab + P = 0 + P = P

3851. ⠁ ⠌⠈⠑ ⠠⠏

$a \notin P$

a \notin P

3852. ⠃ ⠬⠠⠏ ⠨⠅ ⠴⠬⠠⠏

$b + P = 0 + P$

b + P = 0 + P

3853. ⠈⠠⠵⠌⠝⠈⠠⠵ ⠈⠱⠈⠱ ⠈⠠⠵⠰⠝

$\mathbb{Z}/n\mathbb{Z} \cong \mathbb{Z}_n$

{\mathbb Z} / n{\mathbb Z} \cong {\mathbb Z}_n

3854. ⠛⠉⠙⠷⠍⠠⠝⠾ ⠨⠅ ⠂

$\gcd(m, n) = 1$

\gcd(m, n) = 1

3855. ⠁ ⠠⠃ ⠈⠑ ⠈⠠⠵

$a, b \in \mathbb{Z}$

a, b \in {\mathbb Z}

3856. ⠭⠂ ⠸⠇ ⠭⠆ ⠷⠍⠕⠙⠍⠝⠾

$x_1 \equiv x_2 \pmod{mn}$

`x_1 \equiv x_2 \pmod{mn}`

3857. ⠭ ⠸⠇ ⠁ ⠷⠍⠕⠙⠍⠾

$x \equiv a \pmod{m}$

`x \equiv a \pmod{m}`

3858. ⠁ ⠬⠅⠍

$a + km$

`a + km`

3859. ⠅⠂

k_1

`k_1`

3860. ⠁ ⠬⠅⠂⠍ ⠸⠇ ⠃ ⠷⠍⠕⠙⠝⠾

$a + k_1 m \equiv b \pmod{n}$

`a + k_1 m \equiv b \pmod{n}`

3861. ⠅⠂⠍ ⠸⠇ ⠷⠃⠤⠁⠾⠷⠍⠕⠙⠝⠾

$k_1 m \equiv (b-a) \pmod{n}$

`k_1 m \equiv (b-a) \pmod{n}`

3862. ⠍⠎⠬⠝⠞ ⠨⠅ ⠂

$ms + nt = 1$

`ms + nt = 1`

3863. ⠷⠃⠤⠁⠾⠍⠎ ⠨⠅ ⠷⠃⠤⠁⠾⠤⠷⠃⠤⠁⠾⠝⠞

$(b-a)ms = (b-a) - (b-a)nt$

`(b-a) ms = (b-a) -(b-a) nt`

3864. ⠈⠷⠷⠃⠤⠁⠾⠎⠈⠾⠍ ⠸⠇ ⠷⠃⠤⠁⠾⠷⠍⠕⠙⠝⠾

$[(b-a)s]m \equiv (b-a) \pmod{n}$

`[(b-a)s]m \equiv (b-a) \pmod{n}`

3865. ⠅⠂ ⠨⠅ ⠷⠃⠤⠁⠾⠎

$k_1 = (b-a)s$

`k_1 = (b-a)s`

3866. ⠍⠝

mn

`mn`

3867. ⠊ ⠨⠅ ⠂ ⠠⠼⠆

$i = 1, 2$

`i = 1, 2`

3868. ⠉⠂⠤⠉⠆

$c_1 - c_2$

`c_1 - c_2`

3869. ⠉⠆ ⠸⠇ ⠉⠂ ⠷⠍⠕⠙⠍⠝⠾

$c_2 \equiv c_1 \pmod{mn}$

`c_2 \equiv c_1 \pmod{mn}`

3870. ⠲⠎⠬⠢⠞ ⠨⠅ ⠂

$4s + 5t = 1$

`4s + 5t = 1`

3871. ⠎ ⠨⠅ ⠲

$s = 4$

`s = 4`

3872. ⠞ ⠨⠅ ⠤⠒

$t = -3$

`t = -3`

3873. ⠭ ⠨⠅ ⠁⠬⠅⠂⠍ ⠨⠅ ⠒⠬⠲⠅⠂ ⠨⠅ ⠒⠬⠲⠈⠷⠷⠢⠤⠲⠾⠲⠈⠾ ⠨⠅ 19

$x = a + k_1 m = 3 + 4k_1 = 3 + 4[(5 - 4)4] = 19$

`x = a + k_1 m = 3 + 4k_1 = 3 + 4[(5 - 4)4] = 19`

3874. ⠝⠂ ⠠⠝⠆ ⠄⠄ ⠠⠝⠰⠅ ⠂

$n_1, n_2, \ldots, n_k$

`n_1, n_2, \ldots, n_k`

3875. ⠛⠉⠙⠷⠝⠰⠊⠂⠠⠝⠰⠚⠂⠾ ⠨⠅ ⠂

$\gcd(n_i, n_j) = 1$

`\gcd(n_i, n_j) = 1`

3876. ⠁⠂ ⠄⠄ ⠠⠁⠰⠅ ⠂

$a_1, \ldots, a_k$

`a_1, \ldots, a_k`

3877. ⠝⠂⠝⠆⠄⠄⠄⠝⠰⠅

$n_1 n_2 \cdots n_k$

`n_1 n_2 \cdots n_k`

3878. ⠅ ⠨⠅ ⠆

$k = 2$

`k= 2`

3879. ⠝⠂⠄⠄⠄⠝⠰⠅

$n_1 \cdots n_k$

`n_1 \cdots n_k`

3880. ⠝⠰⠅⠬⠂

n_{k+1}

`n_{k+1}`

3881. ⠝⠂⠄⠝⠰⠅⠬⠂

$n_1 \ldots n_{k+1}$

`n_1 \ldots n_{k+1}`

3882. 19⠷⠍⠕⠙20⠾

$19 \pmod{20}$

```
19 \pmod{20}
```

3883. ⠼1260

1260

```
1260
```

3884. ⠆⠘63⠐⠤⠂ ⠭ 9,223,372,036,854,775,807

$2^{63} - 1 = 9{,}223{,}372{,}036{,}854{,}775{,}807$

```
2^{63} - 1 = 9{,}223{,}372{,}036{,}854{,}775{,}807
```

3885. ⠆⠘511⠐⠤⠂

$2^{511} - 1$

```
2^{511} - 1
```

3886. ⠼2134

2134

```
2134
```

3887. ⠼1531

1531

```
1531
```

3888. ⠼95

95

```
95
```

3889. ⠼97

97

```
97
```

3890. ⠼98

98

```
98
```

3891. 2134⠡1531

$2134 \cdot 1531$

```
2134 \cdot 1531
```

3892. 95⠡97⠡98⠡99 ⠭ 89,403,930

$95 \cdot 97 \cdot 98 \cdot 99 = 89{,}403{,}930$

```
95 \cdot 97 \cdot 98 \cdot 99 = 89{,}403{,}930
```

3893. 2134⠡1531 ⠭ 3,267,154

$2134 \cdot 1531 = 3{,}267{,}154$

```
2134 \cdot 1531 = 3{,}267{,}154
```

3894. ⠶⠈⠠⠵

$7 {\mathbb Z}$

```
7 {\mathbb Z}
```

3895. ⠈⠠⠟⠷⠜⠆⠻⠾ ⠨⠅ ⠨⠷⠁⠬⠃⠜⠆⠻⠱⠁⠠⠃⠈⠑⠈⠠⠟⠨⠾

$\mathbb{Q}(\sqrt{2}\,) = \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$

```
{\mathbb Q} ( \sqrt{2}\, ) = \{a + b \sqrt{2} : a, b \in {\mathbb
Q}\}
```

3896. ⠈⠠⠟⠷⠜⠆⠻⠠⠜⠒⠻⠾ ⠨⠅ ⠨⠷⠁⠬⠃⠜⠆⠻⠬⠉⠜⠒⠻⠬⠙⠜⠖⠻⠱⠁⠠⠃⠠⠉⠠⠙⠈⠑⠈⠠⠟⠨⠾

$\mathbb{Q}(\sqrt{2}, \sqrt{3}\,) = \{a + b\sqrt{2} + c\sqrt{3} + d\sqrt{6} : a, b, c, d \in \mathbb{Q}\}$

```
{\mathbb Q} ( \sqrt{2}, \sqrt{3}\, ) = \{a + b \sqrt{2} + c \sqrt{3}
+ d \sqrt{6} : a, b, c, d \in {\mathbb Q}\}
```

3897. ⠈⠠⠵⠈⠷⠜⠒⠻⠈⠾ ⠨⠅ ⠨⠷⠁⠬⠃⠜⠒⠻⠱⠁⠠⠃⠈⠑⠈⠠⠵⠨⠾

$\mathbb{Z}[\sqrt{3}\,] = \{ a + b \sqrt{3} : a, b \in \mathbb{Z} \}$

```
{\mathbb Z}[\sqrt{3}\, ] = \{ a + b \sqrt{3} : a, b \in {\mathbb
Z} \}
```

3898. ⠠⠗ ⠨⠅ ⠨⠷⠁⠬⠃⠣⠒⠜⠒⠻⠱⠁⠠⠃⠈⠑⠈⠠⠟⠨⠾

$R = \{a + b \sqrt[3]{3} : a, b \in \mathbb{Q} \}$

```
R = \{a + b \sqrt[3]{3} : a, b \in {\mathbb Q} \}
```

3899. ⠈⠠⠵⠈⠷⠊⠈⠾ ⠨⠅ ⠨⠷⠁⠬⠃⠊⠱⠁⠠⠃⠈⠑⠈⠠⠵and⠊⠘⠆⠐ ⠨⠅ ⠤⠂⠨⠾

$\mathbb{Z}[i] = \{ a + b i : a, b \in \mathbb{Z} \text{ and } i^2 = -1 \}$

```
{\mathbb Z}[ i ] = \{ a + b i : a, b \in {\mathbb Z} \text{ and
} i^2 = -1 \}
```

3900. ⠈⠠⠟⠷⠣⠒⠜⠒⠻⠾ ⠨⠅ ⠨⠷⠁⠬⠃⠣⠒⠜⠒⠻⠬⠉⠣⠒⠜⠔⠻⠱⠁⠠⠃⠠⠉⠈⠑⠈⠠⠟⠨⠾

$\mathbb{Q}(\sqrt[3]{3}\,) = \{ a + b \sqrt[3]{3} + c \sqrt[3]{9} : a, b, c \in \mathbb{Q} \}$

```
{\mathbb Q}( \sqrt[3]{3}\, ) = \{ a + b \sqrt[3]{3} + c \sqrt[3]{9}
: a, b, c \in {\mathbb Q} \}
```

3901. ⠈⠠⠟⠷⠜⠆⠻⠾

$\mathbb{Q}(\sqrt{2}\,)$

```
{\mathbb Q}(\sqrt{2}\, )
```

3902. ⠠⠷⠁ ⠃⠠⠾ ⠠⠷⠼⠴ ⠼⠴⠠⠾

$$\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix}$$

```
\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix}
```

3903. ⠁⠠⠃⠈⠑⠈⠠⠗

$a, b \in \mathbb{R}$

```
a, b \in {\mathbb R}
```

3904. ⠈⠠⠵10

$\mathbb{Z}_{10}$

```
{\mathbb Z}_{10}
```

3905. ⠈⠠⠵⠶

$\mathbb{Z}_{7}$

```
{\mathbb Z}_{7}
```

3906. ⠈⠠⠍⠆⠷⠈⠠⠵⠾

$\mathbb{M}_2(\mathbb{Z})$

{\mathbb M}_2({\mathbb Z})

3907. ⠈⠠⠍⠆⠷⠈⠠⠵⠆ ⠾

$\mathbb{M}_2(\mathbb{Z}_2)$

{\mathbb M}_2({\mathbb Z}_2)

3908. StartSet⠂ ⠠⠒ ⠠⠶ ⠠⠔EndSet

$\{1, 3, 7, 9\}$

\{1, 3, 7, 9 \}

3909. StartSet⠂ ⠠⠆ ⠠⠒ ⠠⠲ ⠠⠢ ⠠⠖EndSet

$\{1, 2, 3, 4, 5, 6\}$

\{ 1, 2, 3, 4, 5, 6 \}

3910. StartSet ⠠⠷⠼⠂ ⠼⠴ ⠠⠾ ⠠⠷⠼⠴ ⠼⠂ ⠠⠾ ⠠ ⠠⠷⠼⠂ ⠼⠂ ⠠⠾ ⠠⠷⠼⠴ ⠼⠂ ⠠⠾ ⠠ ⠠⠷⠼⠂ ⠼⠴ ⠠⠾ ⠠⠷⠼⠂ ⠼⠂ ⠠⠾ ⠠ ⠠⠷⠼⠴ ⠼⠂ ⠠⠾ ⠠⠷⠼⠂ ⠼⠴ ⠠⠾ ⠠ ⠠⠷⠼⠂ ⠼⠂ ⠠⠾ ⠠⠷⠼⠂ ⠼⠴ ⠠⠾ ⠠ ⠠⠷⠼⠴ ⠼⠂ ⠠⠾ ⠠⠷⠼⠂ ⠼⠂ ⠠⠾ ⠠EndSet

$$\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \right\}$$

\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix}
1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix},
\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1
& 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix},
\right\}

3911. ⠈⠠⠵25

$\mathbb{Z}_{25}$

{\mathbb Z}_{25}

3912. ⠈⠠⠍⠆⠷⠈⠠⠗⠾

$\mathbb{M}_2(\mathbb{R})$

{\mathbb M}_2({\mathbb R})

3913. StartSet⠴EndSet

$\{0\}$

\{0 \}

3914. StartSet⠴ ⠠⠔EndSet

$\{0, 9\}$

\{0, 9 \}

3915. StartSet⠴ ⠠⠖ ⠠12EndSet

$\{0, 6, 12\}$

\{0, 6, 12 \}

3916. StartSet⠴ ⠠⠒ ⠠⠖ ⠠⠔ ⠠12 ⠠15EndSet

$\{0, 3, 6, 9, 12, 15\}$

\{0, 3, 6, 9, 12, 15 \}

3917. StartSet⠴⠠⠆⠠⠲⠠⠖⠠⠦⠠10⠠12⠠14⠠16EndSet

$\{0, 2, 4, 6, 8, 10, 12, 14, 16\}$

```
\{0, 2, 4, 6, 8, 10, 12, 14, 16 \}
```

3918. ⠠⠗ ⠨⠅ ⠈⠠⠵

$R = \mathbb{Z}$

```
R = {\mathbb Z}
```

3919. ⠠⠊ ⠨⠅ ⠖⠈⠠⠵

$I = 6\mathbb{Z}$

```
I = 6 {\mathbb Z}
```

3920. ⠠⠗ ⠨⠅ ⠈⠠⠵12

$R = \mathbb{Z}_{12}$

```
R = {\mathbb Z}_{12}
```

3921. ⠠⠊ ⠨⠅ StartSet⠴⠠⠒⠠⠖⠠⠔EndSet

$I = \{0, 3, 6, 9\}$

```
I = \{ 0, 3, 6, 9 \}
```

3922. ⠨⠋⠱⠈⠠⠵⠌⠖⠈⠠⠵ ⠫⠕ ⠈⠠⠵⠌15⠈⠠⠵

$\phi : \mathbb{Z}/6\mathbb{Z} \rightarrow \mathbb{Z}/15\mathbb{Z}$

```
\phi : {\mathbb Z} / 6 {\mathbb Z} \rightarrow {\mathbb Z} / 15
{\mathbb Z}
```

3923. ⠨⠋⠱⠈⠠⠉ ⠫⠕ ⠈⠠⠗

$\phi : \mathbb{C} \rightarrow \mathbb{R}$

```
\phi: {\mathbb C} \rightarrow {\mathbb R}
```

3924. ⠨⠋⠷⠊⠾ ⠨⠅ ⠁

$\phi(i) = a$

```
\phi(i) = a
```

3925. ⠈⠠⠟⠷⠜⠒⠻⠾ ⠨⠅ ⠨⠷⠁⠬⠃⠜⠒⠻⠱⠁⠠⠃ ⠈⠑ ⠈⠠⠟ ⠨⠾

$\mathbb{Q}(\sqrt{3}\,) = \{a + b\sqrt{3} : a, b \in \mathbb{Q}\}$

```
{\mathbb Q}( \sqrt{3}\, ) = \{a + b \sqrt{3} : a, b \in {\mathbb
Q} \}
```

3926. ⠨⠋⠱⠈⠠⠟⠷⠜⠆⠻⠾ ⠫⠕ ⠈⠠⠟⠷⠜⠒⠻⠾

$\phi : \mathbb{Q}(\sqrt{2}\,) \rightarrow \mathbb{Q}(\sqrt{3}\,)$

```
\phi: {\mathbb Q}(\sqrt{2}\, ) \rightarrow {\mathbb Q}(\sqrt{3}\,
)
```

3927. ⠨⠋⠷⠜⠆⠻⠾ ⠨⠅ ⠁

$\phi(\sqrt{2}\,) = a$

```
\phi(\sqrt{2}\, ) = a
```

3928. ⠨⠋⠱⠈⠠⠉ ⠫⠕ ⠈⠠⠍⠆⠷⠈⠠⠗⠾

$\phi : \mathbb{C} \rightarrow \mathbb{M}_2(\mathbb{R})$

```
\phi : {\mathbb C} \rightarrow {\mathbb M}_2 ({\mathbb R})
```

3929. ⠨⠋⠷⠁⠬⠃⠊⠾ ⠨⠅ ⠠⠷⠁ ⠃ ⠠⠾ ⠠⠷⠤⠃ ⠁ ⠠⠾

$\phi(a+bi) = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$

`\phi( a + bi) = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}`

3930. ⠈⠠⠍⠆⠷⠈⠠⠗⠾

$\mathbb{M}_2(\mathbb{R})$

`{\mathbb M}_2 ({\mathbb R})`

3931. ⠈⠠⠵⠈⠷⠊⠈⠾

$\mathbb{Z}[i]$

`{\mathbb Z}[i ]`

3932. ⠈⠠⠵⠈⠷⠜⠒⠻⠊⠈⠾ ⠨⠅ ⠸⠷⠁⠬⠃⠜⠒⠻⠊⠸⠒ ⠁⠠ ⠃ ⠈⠑ ⠈⠠⠵⠸⠾

$\mathbb{Z}[\sqrt{3}\, i] = \{a + b\sqrt{3}\, i : a, b \in \mathbb{Z}\}$

`{\mathbb Z}[ \sqrt{3}\, i ] = \{ a + b \sqrt{3}\, i : a, b \in {\mathbb Z} \}`

3933. ⠶ ⠿ 17⠷⠍⠕⠙55⠾

$x \equiv 17 \pmod{55}$

`x \equiv 17 \pmod{55}`

3934. ⠶ ⠿ 214⠷⠍⠕⠙2772⠾

$x \equiv 214 \pmod{2772}$

`x \equiv 214 \pmod{2772}`

3935. 2234⠬4121

$2234 + 4121$

`2234 + 4121`

3936. ⠠⠊ ⠌⠨⠅ StartSet⠴EndSet

$I \neq \{0\}$

`I \neq \{ 0 \}`

3937. ⠂ ⠈⠑ ⠠⠊

$1 \in I$

`1 \in I`

3938. ⠷⠤⠂⠾⠁ ⠨⠅ ⠤⠁

$(-1)a = -a$

`(-1)a = -a`

3939. ⠨⠋⠷⠠⠗⠾ ⠌⠨⠅ ⠴

$\phi(R) \neq 0$

`\phi(R) \neq 0`

3940. ⠨⠋⠷⠁⠾⠨⠋⠷⠃⠾ ⠨⠅ ⠨⠋⠷⠁⠃⠾ ⠨⠅ ⠨⠋⠷⠃⠁⠾ ⠨⠅ ⠨⠋⠷⠃⠾⠨⠋⠷⠁⠾

$\phi(a)\phi(b) = \phi(ab) = \phi(ba) = \phi(b)\phi(a)$

`\phi(a) \phi(b) = \phi(ab) = \phi(ba) = \phi(b) \phi(a)`

3941. ⠠⠊⠌⠠⠊⠨⠩⠠⠚ ⠈⠱⠈⠱ ⠠⠊⠬⠠⠚⠌⠠⠚

$I/I \cap J \cong I + J/J$

I / I \cap J \cong I + J /J

3942. ⠠⠎ ⠫⠕ ⠠⠎⠌⠠⠊

$S \rightarrow S/I$

S \rightarrow S/I

3943. ⠗⠤⠎⠈⠑⠠⠎

$r - s \in S$

r - s \in S

3944. ⠨⠷⠠⠗⠰⠨⠁⠐⠨⠾

$\{R_\alpha\}$

\{ R_{\alpha} \}

3945. ⠄⡷⠭⠆⠆⠉⠆⠄⠠⠗⠰⠨⠁

$\bigcap R_\alpha$

\bigcap R_{\alpha}

3946. ⠨⠷⠠⠊⠰⠨⠁⠐⠨⠾⠰⠨⠁⠈⠑⠠⠁

$\{I_\alpha\}_{\alpha \in A}$

\{ I_{\alpha} \}_{\alpha \in A}

3947. ⠈⠂⡷⠭⠆⠆⠉⠆⠄⠩⠨⠁⠈⠑⠠⠁⠻⠠⠊⠰⠨⠁

$\bigcap_{\alpha \in A} I_\alpha$

\bigcap_{\alpha \in A} I_{\alpha}

3948. ⠠⠊⠂

I_1

I_1

3949. ⠠⠊⠆

I_2

I_2

3950. ⠠⠊⠂⠨⠬⠠⠊⠆

$I_1 \cup I_2$

I_1 \cup I_2

3951. ⠃⠈⠑⠠⠗

$b \in R$

b \in R

3952. ⠁⠃ ⠨⠅ ⠂

$ab = 1$

ab =1

3953. ⠁⠘⠝⠐ ⠨⠅ ⠴

$a^n = 0$

a^n = 0

3954. ⠁⠘⠆⠐⠀⠨⠅⠀⠁

$a^2 = a$

a^2 = a

3955. ⠷⠁⠬⠃⠾⠘⠆

$(a+b)^2$

(a+b)^2

3956. ⠷⠤⠁⠃⠾⠘⠆

$(-ab)^2$

(-ab)^2

3957. ⠁⠘⠒⠐⠀⠨⠅⠀⠁

$a^3 = a$

a^3 =a

3958. ⠂⠰⠠⠗⠐⠀⠨⠅⠀⠂⠰⠠⠎

$1_R = 1_S$

1_R = 1_S

3959. ⠂⠀⠨⠅⠀⠴

$1 = 0$

1 = 0

3960. ⠠⠗⠀⠨⠅⠀StartSet⠴EndSet

$R = \{0\}$

R = \{ 0 \}

3961. ⠠⠗⠄

R'

R'

3962. ⠠⠵⠷⠠⠗⠾⠀⠨⠅⠀StartSet⠁⠀⠈⠑⠀⠠⠗⠈⠒⠁⠗⠀⠨⠅⠀⠗⠁forall⠗⠀⠈⠑⠀⠠⠗EndSet

$Z(R) = \{a \in R : ar = ra$ for all $r \in R\}$

Z(R) = \{ a \in R : ar = ra \text{ for all } r \in R \}

3963. ⠠⠵⠷⠠⠗⠾

$Z(R)$

Z(R)

3964. ⠈⠠⠵⠰⠷⠏⠾⠐⠀⠨⠅⠀⠸⠷⠁⠠⠌⠃⠀⠠⠒⠀⠁⠠⠀⠃⠀⠈⠑⠀⠈⠠⠵and⠛⠉⠙⠷⠃⠠⠏⠾⠀⠨⠅⠀⠂⠀⠸⠾

$\mathbb{Z}_{(p)} = \{a/b : a, b \in \mathbb{Z}$ and $\gcd(b,p) = 1\}$

{\mathbb Z}_{(p)} = \{ a / b : a, b \in {\mathbb Z} \text{ and }
\gcd(b,p) = 1 \}

3965. ⠈⠠⠵⠰⠷⠏⠾

$\mathbb{Z}_{(p)}$

{\mathbb Z}_{(p)}

3966. ⠁⠌⠃⠠⠉⠌⠙⠈⠑⠈⠠⠵⠰⠷⠏⠾⠐

$a/b, c/d \in \mathbb{Z}_{(p)}$

a/b, c/d \in {\mathbb Z}_{(p)}

3967. ⠁⠌⠃⠬⠉⠌⠙ ⠨⠅ ⠷⠁⠙⠬⠃⠉⠾⠌⠃⠙

$a/b + c/d = (ad + bc)/bd$

a/b + c/d = (ad + bc)/bd

3968. ⠷⠁⠌⠃⠾⠡⠷⠉⠌⠙⠾ ⠨⠅ ⠷⠁⠉⠾⠌⠷⠃⠙⠾

$(a/b) \cdot (c/d) = (ac)/(bd)$

(a/b) \cdot (c/d) = (ac)/(bd)

3969. ⠛⠉⠙⠷⠃⠙⠠⠏⠾ ⠨⠅ ⠂

$\gcd(bd, p) = 1$

\gcd(bd,p) = 1

3970. ⠊⠰⠥⠐⠱⠠⠗ ⠫⠕ ⠠⠗

$i_u : R \rightarrow R$

i_u : R \rightarrow R

3971. ⠗ ⠫⠳⠒⠒⠕ ⠥⠗⠥⠘⠤⠂

$r \mapsto uru^{-1}$

r \mapsto uru^{-1}

3972. ⠊⠰⠥

i_u

i_u

3973. ⠠⠊⠝⠝⠷⠠⠗⠾

$\mathrm{Inn}(R)$

\inn(R)

3974. ⠠⠁⠥⠞⠷⠠⠗⠾

$\mathrm{Aut}(R)$

\aut(R)

3975. ⠠⠥⠷⠠⠗⠾

$U(R)$

U(R)

3976. ⠨⠋⠐⠱⠠⠥⠷⠠⠗⠾ ⠫⠕ ⠠⠊⠝⠝⠷⠠⠗⠾

$\phi : U(R) \rightarrow \mathrm{Inn}(R)$

\phi : U(R) \rightarrow \inn(R)

3977. ⠥ ⠫⠳⠒⠒⠕ ⠊⠰⠥

$u \mapsto i_u$

u \mapsto i_u

3978. ⠠⠊⠝⠝⠷⠈⠠⠵⠾

$\mathrm{Inn}(\mathbb{Z})$

\inn({\mathbb Z})

3979. ⠠⠥⠷⠈⠠⠵⠾

$U(\mathbb{Z})$

```
U( {\mathbb Z})
```

3980. ⠠⠗⠈⠡⠠⠎

$R \times S$

```
R \times S
```

3981. ⠷⠗⠠⠎⠾⠬⠷⠗⠄⠠⠎⠘⠄⠐⠾ ⠨⠅ ⠷⠗⠬⠗⠘⠄⠐⠠⠎⠬⠎⠘⠄⠐⠾

$(r, s) + (r', s') = (r + r', s + s')$

```
(r, s) + (r', s') = ( r + r', s + s')
```

3982. ⠷⠗⠠⠎⠾⠷⠗⠄⠠⠎⠘⠄⠐⠾ ⠨⠅ ⠷⠗⠗⠘⠄⠐⠠⠎⠎⠘⠄⠐⠾

$(r, s)(r', s') = (rr', ss')$

```
(r, s)(r', s') = ( rr', ss')
```

3983. ⠭⠘⠆⠐ ⠨⠅ ⠭

$x^2 = x$

```
x^2 = x
```

3984. ⠭ ⠌⠨⠅ ⠴

$x \neq 0$

```
x \neq 0
```

3985. ⠭ ⠨⠅ ⠂

$x = 1$

```
x = 1
```

3986. ⠈⠠⠍⠆⠷⠈⠠⠗⠾

$\mathbb{M}_2(\mathbb{R})$

```
{\mathbb M}_2({\mathbb R})
```

3987. ⠛⠉⠙⠷⠁⠠⠝⠾ ⠨⠅ ⠙

$\gcd(a, n) = d$

```
\gcd(a, n) = d
```

3988. ⠛⠉⠙⠷⠃⠠⠙⠾ ⠌⠨⠅ ⠂

$\gcd(b, d) \neq 1$

```
\gcd(b, d) \neq 1
```

3989. ⠁⠭ ⠸⠇ ⠃ ⠷⠍⠕⠙⠝⠾

$ax \equiv b \pmod{n}$

```
ax \equiv b \pmod{n}
```

3990. ⠠⠊⠬⠠⠚ ⠨⠅ ⠠⠗

$I + J = R$

```
I+J = R
```

3991. ⠠⠊⠬⠠⠚ ⠨⠅ ⠠⠗

$I + J = R$

```
I + J = R
```

3992. ⠠⠗⠌⠷⠠⠊⠨⠩⠠⠚⠾ ⠈⠱⠈⠱ ⠠⠗⠌⠠⠊⠡⠠⠗⠌⠠⠚

$R/(I \cap J) \cong R/I \times R/J$

`R/(I \cap J) \cong R/I \times R/J`

3993. ⠈⠠⠵⠰⠝

${\mathbb Z_n}$

`{\mathbb Z_n}`

3994. ⠭⠘⠆⠐⠤⠝ ⠨⠅ ⠴

$x^2-n=0$

`x^2-n=0`

3995. ⠈⠠⠟⠈⠷⠜⠝⠻⠈⠾

${\mathbb Q}[\sqrt{n}]$

`{\mathbb Q}[\sqrt{n}]`

3996. ⠭⠘⠝⠐⠤⠂ ⠨⠅ ⠴

$x^n-1=0$

`x^n-1=0`

3997. ⠈⠠⠟overbar

$\overline{{\mathbb Q}}$

`\overline{{\mathbb Q}}`

3998. ⠈⠠⠵⠰⠏

${\mathbb Z_p}$

`{\mathbb Z_p}`

3999. ⠜⠶⠻

$\sqrt{7}$

`\sqrt{7}`

4000. ⠭⠘⠆⠐⠤⠶

x^2-7

`x^2-7`

4001. ⠤⠜⠶⠻

$-\sqrt{7}$

`-\sqrt{7}`

4002. ⠗⠘⠆⠐ ⠨⠅ ⠝

$r^2=n$

`r^2=n`

4003. ⠎⠘⠆⠐ ⠨⠅ ⠍

$s^2=m$

`s^2=m`

4004. ⠞ ⠨⠅ ⠗⠎ ⠨⠅ ⠤⠎⠗

$t = rs = -sr$

`t = rs = -sr`

4005. ⠬overbar

$\overline{y}$

\overline{y}

4006. ⠄⡸⠩⠒⠆ ⠿⠁⠒⠤⠄ ⠐⠒⠄ ⡸⠩⠒⠆ ⠿⠁⠢⠄⠄

$\langle 4\rangle$

\langle 4\rangle

4007. StartSet⠈ ⠁⠄⠤⠠⠻ ⠁⠄⠢⠹⠈ ⠠⠆ ⠉⠂ ⠈ ⠠⠹EndSet

$\{a\cdot 3+ b\cdot 5\mid a,b\in{\mathbb Z}\}$

\{a\cdot 3+ b\cdot 5\mid a,b\in{\mathbb Z}\}

4008. F

F

4009. ⠵ ⠘⠆ ⠐⠖⠵⠖⠒

z^2+z+3

z^2+z+3

4010. ⠏⠷⠭⠾⠖⠟⠷⠭⠾

$p(x) + q(x)$

p(x) + q(x)

4011. ⠏⠷⠭⠾⠟⠷⠭⠾

$p(x) q(x)$

p(x) q(x)

4012. ⠋⠷⠭⠾ ⠶ ⠐⠅⠠⠊⠉⠊⠁ ⠶ ⠰⠳⠝⠟⠁ ⠪⠁⠐⠭⠳⠁⠐ ⠶ ⠁⠰⠴⠁⠐⠭⠴⠁⠆⠭⠣ ⠐⠖⠄⠄⠄⠰⠁ ⠪⠝⠐⠭⠳⠝

$f(x) = \sum^{n}_{i=0} a_i x^i = a_0 + a_1 x +a_2 x^2 + \cdots + a_n x^n$

f(x) = \sum^{n}_{i=0} a_i x^i = a_0 + a_1 x +a_2 x^2 + \cdots +
a_n x^n

4013. ⠁ ⠣⠂⠐ ⠈⠑ ⠠⠗

$a_i \in R$

a_i \in R

4014. ⠁ ⠣⠝⠐ ⠌⠶ ⠴

$a_n \neq 0$

a_n \neq 0

4015. ⠁ ⠴ ⠠⠁ ⠐ ⠤ ⠠⠁ ⠣⠝ ⠐

$a_0, a_1, \ldots, a_n$

a_0, a_1, \ldots, a_n

4016. ⠙⠑⠛⠋⠷⠭⠾ ⠶ ⠝

$\deg f(x) = n$

\deg f(x) = n

4017. ⠋ ⠶ ⠴

$f = 0$

f=0

4018. ⠤⠠⠿

$-\infty$

-\infty

4019. ⠠⠗⠈⠷⠭⠈⠾

$R[x]$

R[x]

4020. ⠏⠷⠭⠾ ⠨⠅ ⠟⠷⠭⠾

$p(x) = q(x)$

p(x) = q(x)

4021. ⠁⠰⠊⠐ ⠨⠅ ⠃⠰⠊

$a_i = b_i$

a_i = b_i

4022. ⠊ ⠨⠂⠱ ⠴

$i \geq 0$

i \geq 0

4023. ⠏⠷⠭⠾

$p(x)$

p(x)

4024. ⠟⠷⠭⠾

$q(x)$

q(x)

4025. ⠏⠷⠭⠾⠬⠟⠷⠭⠾ ⠨⠅ ⠉⠴⠬⠉⠂⠭⠬⠄⠄⠄⠬⠉⠰⠅⠐⠭⠘⠅

$p(x) + q(x) = c_0 + c_1 x + \cdots + c_k x^k$

p(x) + q(x) = c_0 + c_1 x + \cdots + c_k x^k

4026. ⠉⠰⠊⠐ ⠨⠅ ⠁⠰⠊⠐⠬⠃⠰⠊

$c_i = a_i + b_i$

c_i = a_i + b_i

4027. ⠏⠷⠭⠾⠟⠷⠭⠾ ⠨⠅ ⠉⠴⠬⠉⠂⠭⠬⠄⠄⠄⠬⠉⠰⠍⠬⠝⠐⠭⠘⠍⠬⠝

$p(x)q(x) = c_0 + c_1 x + \cdots + c_{m+n} x^{m+n}$

p(x) q(x) = c_0 + c_1 x + \cdots + c_{m + n} x^{m + n}

4028. ⠉⠰⠊⠐ ⠨⠅ ⠐⠨⠠⠎⠩⠅ ⠨⠅ ⠴⠣⠊⠻⠁⠰⠅⠐⠃⠰⠊⠤⠅⠐ ⠨⠅ ⠁⠴⠃⠰⠊⠐⠬⠁⠂⠃⠰⠊⠤⠂⠐⠬⠄⠄⠄⠬⠁⠰⠊⠤⠂⠐

$c_i = \sum_{k=0}^i a_k b_{i-k} = a_0 b_i + a_1 b_{i-1} + \cdots + a_{i-1} b_1 + a_i b_0$

c_i = \sum_{k = 0}^i a_k b_{i - k} = a_0 b_i + a_1 b_{i -1} + \cdots
+ a_{i -1} b _1 + a_i b_0

4029. ⠏⠷⠭⠾ ⠨⠅ ⠒⠬⠴⠭⠬⠴⠭⠘⠆⠐⠬⠆⠭⠘⠒⠐⠬⠴⠭⠘⠲

$p(x) = 3 + 0x + 0x^2 + 2x^3 + 0x^4$

p(x) = 3 + 0 x + 0 x^2 + 2 x^3 + 0 x^4

4030. ⠟⠷⠭⠾⠀⠨⠅⠀⠆⠬⠴⠭⠤⠭⠘⠆⠐⠬⠴⠭⠘⠒⠐⠬⠲⠭⠘⠲

$q(x) = 2 + 0x - x^2 + 0x^3 + 4x^4$

```
q(x) = 2 + 0 x - x^2 + 0 x^3 + 4 x^4
```

4031. ⠈⠠⠵⠈⠷⠭⠈⠾

${\mathbb Z}[x]$

```
{\mathbb Z}[x]
```

4032. ⠏⠷⠭⠾⠀⠨⠅⠀⠒⠬⠆⠭⠘⠒

$p(x) = 3 + 2x^3$

```
p(x) = 3 + 2 x^3
```

4033. ⠟⠷⠭⠾⠀⠨⠅⠀⠆⠤⠭⠘⠆⠐⠬⠲⠭⠘⠲

$q(x) = 2 - x^2 + 4x^4$

```
q(x) = 2 - x^2 + 4 x^4
```

4034. ⠏⠷⠭⠾⠬⠟⠷⠭⠾⠀⠨⠅⠀⠢⠤⠭⠘⠆⠐⠬⠆⠭⠘⠒⠐⠬⠲⠭⠘⠲

$p(x) + q(x) = 5 - x^2 + 2x^3 + 4x^4$

```
p(x) + q(x)= 5 - x^2 + 2 x^3 + 4 x^4
```

4035. ⠏⠷⠭⠾⠟⠷⠭⠾⠀⠨⠅⠀⠷⠒⠬⠆⠭⠘⠒⠐⠾⠷⠆⠤⠭⠘⠆⠐⠬⠲⠭⠘⠲⠐⠾⠀⠨⠅⠀⠖⠤⠒⠭⠘⠆⠐⠬⠲⠭⠘⠒⠐⠬12⠭⠘⠲⠐⠤⠆⠭

$p(x)q(x) = (3 + 2x^3)(2 - x^2 + 4x^4) = 6 - 3x^2 + 4x^3 + 12x^4 - 2x^5 + 8x^7$

```
p(x) q(x) = (3 + 2 x^3)( 2 - x^2 + 4 x^4 ) = 6 - 3x^2 + 4 x^3 +
12 x^4 - 2 x^5 + 8 x^7
```

4036. ⠉⠰⠊

c_i

```
c_i
```

4037. ⠏⠷⠭⠾⠀⠨⠅⠀⠒⠬⠒⠭⠘⠒⠐and⠏⠷⠭⠾⠀⠨⠅⠀⠲⠬⠲⠭⠘⠆⠐⠬⠲⠭⠘⠲

$p(x) = 3 + 3x^3 \qquad \text{and} \qquad q(x) = 4 + 4x^2 + 4x^4$

```
p(x) = 3 + 3 x^3 \qquad \text{and} \qquad q(x) = 4 + 4 x^2 + 4 x^4
```

4038. ⠈⠠⠵12⠈⠷⠭⠈⠾

${\mathbb Z}_{12}[x]$

```
{\mathbb Z}_{12}[x]
```

4039. ⠶⠬⠲⠭⠘⠆⠐⠬⠒⠭⠘⠒⠐⠬⠲⠭⠘⠲

$7 + 4x^2 + 3x^3 + 4x^4$

```
7 + 4 x^2 + 3 x^3 + 4 x^4
```

4040. ⠋⠷⠭⠾⠀⠨⠅⠀⠴

$f(x) = 0$

```
f(x) = 0
```

4041. ⠏⠷⠭⠾⠀⠨⠅⠀⠐⠨⠠⠎⠰⠊⠀⠨⠅⠀⠴⠘⠝⠐⠁⠰⠊⠭⠘⠊

$p(x) = \sum_{i=0}^{n} a_i x^i$

```
p(x) = \sum_{i = 0}^{n} a_i x^i
```

4042. ⠤⠏⠷⠭⠾⠀⠨⠅⠀⠐⠨⠠⠎⠩⠊⠀⠨⠅⠀⠴⠣⠝⠻⠷⠤⠁⠰⠊⠐⠾⠭⠘⠊⠐⠀⠨⠅⠀⠤⠐⠨⠠⠎⠩⠊⠀⠨⠅⠀⠴⠣⠝⠻⠁⠰⠊⠐⠭⠘⠊

$-p(x) = \sum_{i=0}^{n}(-a_i)x^i = -\sum_{i=0}^{n} a_i x^i$

-p(x) = \sum_{i = 0}^{n} (-a_i) x^i = -\sum_{i = 0}^{n} a_i x^i

4043. ⠙⠑⠛⠏⠷⠭⠾⠬⠙⠑⠛⠟⠷⠭⠾⠀⠨⠅⠀⠙⠑⠛⠷⠏⠷⠭⠾⠟⠷⠭⠾⠾

$\deg p(x) + \deg q(x) = \deg(p(x)q(x))$

\deg p(x) + \deg q(x) = \deg(p(x) q(x))

4044. ⠏⠷⠭⠾⠀⠨⠅⠀⠁⠰⠍⠐⠭⠘⠍⠐⠬⠄⠄⠄⠬⠁⠂⠭⠬⠁⠴

$p(x) = a_m x^m + \cdots + a_1 x + a_0$

p(x) = a_m x^m + \cdots + a_1 x + a_0

4045. ⠟⠷⠭⠾⠀⠨⠅⠀⠃⠰⠝⠐⠭⠘⠝⠐⠬⠄⠄⠄⠬⠃⠂⠭⠬⠃⠴

$q(x) = b_n x^n + \cdots + b_1 x + b_0$

q(x) = b_n x^n + \cdots + b_1 x + b_0

4046. ⠁⠰⠍⠐⠀⠌⠨⠅⠀⠴

$a_m \neq 0$

a_m \neq 0

4047. ⠃⠰⠝⠐⠀⠌⠨⠅⠀⠴

$b_n \neq 0$

b_n \neq 0

4048. ⠁⠰⠍⠐⠃⠰⠝⠐⠭⠘⠍⠬⠝

$a_m b_n x^{m+n}$

a_m b_n x^{m + n}

4049. ⠏⠷⠭⠾⠟⠷⠭⠾⠀⠌⠨⠅⠀⠴

$p(x)q(x) \neq 0$

p(x)q(x) \neq 0

4050. ⠏⠷⠭⠾⠀⠌⠨⠅⠀⠴

$p(x) \neq 0$

p(x) \neq 0

4051. ⠟⠷⠭⠾⠀⠌⠨⠅⠀⠴

$q(x) \neq 0$

q(x) \neq 0

4052. ⠭⠘⠆⠐⠤⠒⠭⠽⠬⠆⠽⠘⠒

$x^2 - 3xy + 2y^3$

x^2 - 3 x y + 2 y^3

4053. ⠷⠠⠗⠈⠷⠭⠈⠾⠾⠈⠷⠽⠈⠾

$(R[x])[y]$

(R[x])[y]

4054. ⠷⠠⠗⠈⠷⠭⠈⠾⠾⠈⠷⠽⠈⠾⠀⠈⠱⠈⠱⠀⠠⠗⠷⠈⠷⠽⠈⠾⠾⠈⠷⠭⠈⠾

$(R[x])[y] \cong R([y])[x]$

(R[x])[y] \cong R([y])[x]

4055. ⠠⠗⠈⠷⠭⠠⠽⠈⠾

$R[x,y]$

R[x,y]

4056. ⠠⠗⠈⠷⠭⠠⠽⠈⠾

$R[x,y]$

R[x, y]

4057. ⠠⠗⠈⠷⠭⠂⠠⠭⠆⠠⠄⠠⠭⠰⠝⠐⠈⠾

$R[x_1, x_2, \ldots, x_n]$

R[x_1, x_2, \ldots, x_n]

4058. ⠨⠁⠈⠑⠠⠗

$\alpha \in R$

\alpha \in R

4059. ⠨⠋⠰⠨⠁⠐⠱⠠⠗⠈⠷⠭⠈⠾ ⠫⠕ ⠠⠗

$\phi_{\alpha} : R[x] \rightarrow R$

\phi_{\alpha} : R[x] \rightarrow R

4060. ⠨⠋⠰⠨⠁⠐⠷⠏⠷⠭⠾⠾ ⠨⠅ ⠏⠷⠨⠁⠾ ⠨⠅ ⠁⠰⠝⠐⠨⠁⠘⠝⠐⠬⠄⠄⠄⠬⠁⠂⠨⠁⠬⠁⠴

$\phi_{\alpha}(p(x)) = p(\alpha) = a_n \alpha^n + \cdots + a_1 \alpha + a_0$

\phi_{\alpha} (p(x)) = p(\alpha) = a_n \alpha^n + \cdots + a_1
\alpha + a_0

4061. ⠏⠷⠭⠾ ⠨⠅ ⠁⠰⠝⠐⠭⠘⠝⠐⠬⠄⠄⠄⠬⠁⠂⠭⠬⠁⠴

$p(x) = a_n x^n + \cdots + a_1 x + a_0$

p(x) = a_n x^n + \cdots + a_1 x + a_0

4062. ⠏⠷⠭⠾ ⠨⠅ ⠐⠨⠠⠎⠩⠊ ⠨⠅ ⠼⠚⠣⠝⠻ ⠁⠰⠊⠐⠭⠘⠊

$p(x) = \sum_{i=0}^{n} a_i x^i$

p(x) = \sum_{i = 0}^n a_i x^i

4063. ⠟⠷⠭⠾ ⠨⠅ ⠐⠨⠠⠎⠩⠊ ⠨⠅ ⠼⠚⠣⠍⠻ ⠃⠰⠊⠐⠭⠘⠊

$q(x) = \sum_{i=0}^{m} b_i x^i$

q(x) = \sum_{i = 0}^m b_i x^i

4064. ⠨⠋⠰⠨⠁⠐⠷⠏⠷⠭⠾⠬⠟⠷⠭⠾⠾ ⠨⠅ ⠨⠋⠰⠨⠁⠐⠷⠏⠷⠭⠾⠾⠬⠨⠋⠰⠨⠁⠐⠷⠟⠷⠭⠾⠾

$\phi_{\alpha}(p(x) + q(x)) = \phi_{\alpha}(p(x)) + \phi_{\alpha}(q(x))$

\phi_{\alpha}(p(x) + q(x)) = \phi_{\alpha}(p(x)) + \phi_{\alpha}(q(x))

4065. ⠠⠋⠈⠷⠭⠈⠾

$F[x]$

F[x]

4066. ⠟⠷⠭⠾⠠⠗⠷⠭⠾⠈⠑⠠⠋⠈⠷⠭⠈⠾

$q(x), r(x) \in F[x]$

q(x), r(x) \in F[x]

4067. ⠋⠷⠭⠾ ⠨⠅ ⠛⠷⠭⠾⠟⠷⠭⠾⠬⠗⠷⠭⠾

$f(x) = g(x)q(x) + r(x)$

`f(x) = g(x) q(x) + r(x)`

4068. ⠙⠑⠛⠗⠷⠭⠾ ⠐⠅ ⠙⠑⠛⠛⠷⠭⠾

$\deg r(x) < \deg g(x)$

`\deg r(x) \lt \deg g(x)`

4069. ⠗⠷⠭⠾

$r(x)$

`r(x)`

4070. ⠴ ⠨⠅ ⠴⠡⠛⠷⠭⠾⠬⠴ ⠸⠆

$0 = 0 \cdot g(x) + 0;$

`0 = 0 \cdot g(x) + 0;`

4071. ⠙⠑⠛⠛⠷⠭⠾ ⠨⠅ ⠍

$\deg g(x) = m$

`\deg g(x) = m`

4072. ⠟⠷⠭⠾ ⠨⠅ ⠴

$q(x) = 0$

`q(x) = 0`

4073. ⠗⠷⠭⠾ ⠨⠅ ⠋⠷⠭⠾

$r(x) = f(x)$

`r(x) = f(x)`

4074. ⠋⠄⠷⠭⠾ ⠨⠅ ⠋⠷⠭⠾⠤⠹⠁⠰⠝⠐⠌⠃⠰⠍⠐⠼⠭⠘⠝⠤⠍⠐⠛⠷⠭⠾

$f'(x) = f(x) - \frac{a_n}{b_m} x^{n - m} g(x)$

`f'(x) = f(x) - \frac{a_n}{b_m} x^{n - m} g(x)`

4075. ⠟⠄⠷⠭⠾

$q'(x)$

`q'(x)`

4076. ⠋⠄⠷⠭⠾ ⠨⠅ ⠟⠄⠷⠭⠾⠛⠷⠭⠾⠬⠗⠷⠭⠾

$f'(x) = q'(x)g(x) + r(x)$

`f'(x) = q'(x) g(x) + r(x)`

4077. ⠗⠷⠭⠾ ⠨⠅ ⠴

$r(x) = 0$

`r(x) = 0`

4078. ⠟⠷⠭⠾ ⠨⠅ ⠟⠄⠷⠭⠾⠬⠹⠁⠰⠝⠐⠌⠃⠰⠍⠐⠼⠭⠘⠝⠤⠍

$q(x) = q'(x) + \frac{a_n}{b_m} x^{n - m}$

`q(x) = q'(x) + \frac{a_n}{b_m} x^{n - m}`

4079. ⠟⠂⠷⠭⠾

$q_1(x)$

`q_1(x)`

4080. ⠗⠂⠷⠭⠾

$r_1(x)$

r_1(x)

4081. ⠋⠷⠭⠾ ⠨⠅ ⠛⠷⠭⠾⠟⠂⠷⠭⠾⠬⠗⠂⠷⠭⠾

$f(x) = g(x)q_1(x) + r_1(x)$

f(x) = g(x) q_1(x) + r_1(x)

4082. ⠙⠑⠛⠗⠂⠷⠭⠾ ⠐⠅ ⠙⠑⠛⠛⠷⠭⠾

$\deg r_1(x) < \deg g(x)$

\deg r_1(x) \lt \deg g(x)

4083. ⠗⠂⠷⠭⠾ ⠨⠅ ⠴

$r_1(x) = 0$

r_1(x) = 0

4084. ⠋⠷⠭⠾ ⠨⠅ ⠛⠷⠭⠾⠟⠷⠭⠾⠬⠗⠷⠭⠾ ⠨⠅ ⠛⠷⠭⠾⠟⠂⠷⠭⠾⠬⠗⠂⠷⠭⠾

$f(x) = g(x)q(x) + r(x) = g(x)q_1(x) + r_1(x)$

f(x) = g(x) q(x) + r(x) = g(x) q_1(x) + r_1(x)

4085. ⠛⠷⠭⠾⠈⠷⠟⠷⠭⠾⠤⠟⠂⠷⠭⠾⠈⠾ ⠨⠅ ⠗⠂⠷⠭⠾⠤⠗⠷⠭⠾

$g(x)[q(x) - q_1(x)] = r_1(x) - r(x)$

g(x) [q(x) - q_1(x)] = r_1(x) - r(x)

4086. ⠟⠷⠭⠾⠤⠟⠂⠷⠭⠾

$q(x) - q_1(x)$

q(x) - q_1(x)

4087. ⠙⠑⠛⠷⠛⠷⠭⠾⠈⠷⠟⠷⠭⠾⠤⠟⠂⠷⠭⠾⠈⠾⠾ ⠨⠅ ⠙⠑⠛⠷⠗⠂⠷⠭⠾⠤⠗⠷⠭⠾⠾ ⠨⠂⠱ ⠙⠑⠛⠛⠷⠭⠾

$\deg(g(x)[q(x) - q_1(x)]) = \deg(r_1(x) - r(x)) \geq \deg g(x)$

\deg(g(x) [q(x) - q_1(x)])= \deg(r_1(x) - r(x)) \geq \deg g(x)

4088. ⠗⠷⠭⠾ ⠨⠅ ⠗⠂⠷⠭⠾

$r(x) = r_1(x)$

r(x) = r_1(x)

4089. ⠟⠷⠭⠾ ⠨⠅ ⠟⠂⠷⠭⠾

$q(x) = q_1(x)$

q(x) = q_1(x)

4090. ⠭⠘⠒⠐⠤⠭⠘⠆⠐⠬⠆⠭⠤⠒

$x^3 - x^2 + 2x - 3$

x^3 - x^2 + 2 x - 3

4091. ⠭⠤⠆

$x - 2$

x - 2

4092. ⠭⠘⠆

x^2

x^2

4093. ⠤

$-$

-

4094. ⠭ ⠘⠒

x^3

x^3

4095. ⠆ ⠭

$2x$

2x

4096. ⠆ ⠭ ⠘⠆

$2x^2$

2x^2

4097. ⠲⠭

$4x$

4x

4098. ⠭ ⠘⠒ ⠐⠤⠭⠘⠆ ⠐⠬⠆⠭ ⠐⠤⠒ ⠭ ⠷⠭⠤⠆⠾⠷⠭⠘⠆ ⠐⠬⠭⠬⠲⠾⠬⠢

$x^3 - x^2 + 2x - 3 = (x-2)(x^2+x+4)+5$

x^3 - x^2 + 2 x - 3 = (x - 2) (x^2 + x + 4) + 5

4099. ⠨⠁ ⠈⠑ ⠠⠋

$\alpha \in F$

\alpha \in F

4100. ⠏⠷⠨⠁⠾ ⠭ ⠴

$p(\alpha) = 0$

p(\alpha) = 0

4101. ⠏⠷⠭⠾ ⠈⠑ ⠠⠋ ⠈⠷⠭ ⠈⠾

$p(x) \in F[x]$

p(x) \in F[x]

4102. ⠭⠤ ⠨⠁

$x - \alpha$

x - \alpha

4103. ⠏⠷⠨⠁⠾ ⠭ ⠴

$p(\alpha) = 0$

p(\alpha) = 0

4104. ⠏⠷⠭⠾ ⠭ ⠷⠭⠤ ⠨⠁⠾⠟⠷⠭⠾⠬⠗⠷⠭⠾

$p(x) = (x-\alpha)q(x) + r(x)$

p(x) = (x -\alpha) q(x) + r(x)

4105. ⠭⠤ ⠨⠁

$x - \alpha$

x -\alpha

4106. ⠗⠷⠭⠾ ⠨⠅ ⠁

$r(x) = a$

r(x) = a

4107. ⠁ ⠈⠑ ⠠⠋

$a \in F$

a \in F

4108. ⠏⠷⠭⠾ ⠨⠅ ⠷⠭⠤⠨⠁⠾⠟⠷⠭⠾⠬⠁

$p(x) = (x - \alpha)q(x) + a$

p(x) = (x -\alpha) q(x) + a

4109. ⠴ ⠨⠅ ⠏⠷⠨⠁⠾ ⠨⠅ ⠴⠡⠟⠷⠨⠁⠾⠬⠁ ⠨⠅ ⠁⠸⠆

$0 = p(\alpha) = 0 \cdot q(\alpha) + a = a;$

0 = p(\alpha) = 0 \cdot q(\alpha) + a = a;

4110. ⠏⠷⠭⠾ ⠨⠅ ⠷⠭⠤⠨⠁⠾⠟⠷⠭⠾

$p(x) = (x - \alpha)q(x)$

p(x) = (x - \alpha) q(x)

4111. ⠏⠷⠨⠁⠾ ⠨⠅ ⠴⠡⠟⠷⠨⠁⠾ ⠨⠅ ⠴

$p(\alpha) = 0 \cdot q(\alpha) = 0$

p(\alpha) = 0 \cdot q(\alpha) = 0

4112. ⠙⠑⠛⠏⠷⠭⠾ ⠨⠅ ⠴

$\deg p(x) = 0$

\deg p(x) = 0

4113. ⠙⠑⠛⠏⠷⠭⠾ ⠨⠅ ⠂

$\deg p(x) = 1$

\deg p(x) = 1

4114. ⠏⠷⠭⠾ ⠨⠅ ⠁⠭⠬⠃

$p(x) = ax + b$

p(x) = ax + b

4115. ⠁⠨⠁⠂⠬⠃ ⠨⠅ ⠁⠨⠁⠆⠬⠃

$a\alpha_1 + b = a\alpha_2 + b$

a\alpha_1 + b = a\alpha_2 +b

4116. ⠨⠁⠂ ⠨⠅ ⠨⠁⠆

$\alpha_1 = \alpha_2$

\alpha_1 = \alpha_2

4117. ⠙⠑⠛⠏⠷⠭⠾ ⠨⠂ ⠂

$\deg p(x) > 1$

\deg p(x) \gt 1

4118. ⠏⠷⠭⠾ ⠨⠅ ⠷⠭⠤⠨⠁⠾⠟⠷⠭⠾

$p(x) = (x - \alpha)q(x)$

p(x) = (x - \alpha) q(x)

4119. ⠟⠷⠭⠾ ⠈⠑ ⠠⠋⠈⠷⠭⠈⠾

$q(x) \in F[x]$

q(x) \in F[x]

4120. ⠏⠷⠨⠃⠾ ⠨⠅ ⠷⠨⠃⠤⠨⠁⠾⠟⠷⠨⠃⠾ ⠨⠅ ⠴

$p(\beta) = (\beta - \alpha)q(\beta) = 0$

p(\beta) = (\beta - \alpha) q(\beta) = 0

4121. ⠨⠁ ⠌⠨⠅ ⠨⠃

$\alpha \neq \beta$

\alpha \neq \beta

4122. ⠟⠷⠨⠃⠾ ⠨⠅ ⠴

$q(\beta) = 0$

q(\beta) = 0

4123. ⠙⠷⠭⠾

$d(x)$

d(x)

4124. ⠏⠷⠭⠾⠠⠟⠷⠭⠾ ⠈⠑ ⠠⠋⠈⠷⠭⠈⠾

$p(x), q(x) \in F[x]$

p(x), q(x) \in F[x]

4125. ⠙⠄⠷⠭⠾

$d'(x)$

d'(x)

4126. ⠙⠄⠷⠭⠾⠳⠙⠷⠭⠾

$d'(x) \mid d(x)$

d'(x) \mid d(x)

4127. ⠙⠷⠭⠾ ⠨⠅ ⠛⠉⠙⠷⠏⠷⠭⠾⠠⠟⠷⠭⠾⠾

$d(x) = \gcd(p(x), q(x))$

d(x) = \gcd(p(x), q(x))

4128. ⠛⠉⠙⠷⠏⠷⠭⠾⠠⠟⠷⠭⠾⠾ ⠨⠅ ⠂

$\gcd(p(x), q(x)) = 1$

\gcd(p(x), q(x)) = 1

4129. ⠎⠷⠭⠾

$s(x)$

s(x)

4130. ⠙⠷⠭⠾ ⠨⠅ ⠗⠷⠭⠾⠏⠷⠭⠾⠬⠎⠷⠭⠾⠟⠷⠭⠾

$d(x) = r(x)p(x) + s(x)q(x)$

d(x) = r(x) p(x) + s(x) q(x)

4131. ⠠⠎ ⠨⠅ ⠨⠷⠋⠷⠭⠾⠏⠷⠭⠾⠬⠛⠷⠭⠾⠟⠷⠭⠾⠸⠒⠋⠷⠭⠾⠠⠛⠷⠭⠾ ⠈⠑ ⠠⠋⠈⠷⠭⠈⠾⠨⠾

$S = \{f(x)p(x) + g(x)q(x) : f(x), g(x) \in F[x]\}$

S = \{ f(x) p(x) + g(x) q(x) : f(x), g(x) \in F[x] \}

4132. ⠁⠷⠭⠾

$a(x)$

`a(x)`

4133. ⠃⠷⠭⠾

$b(x)$

`b(x)`

4134. ⠏⠷⠭⠾ ⠨⠅ ⠁⠷⠭⠾⠙⠷⠭⠾⠬⠃⠷⠭⠾

$p(x) = a(x)d(x) + b(x)$

`p(x) = a(x) d(x) + b(x)`

4135. ⠙⠑⠛⠃⠷⠭⠾ ⠐⠅ ⠙⠑⠛⠙⠷⠭⠾

$\deg b(x) < \deg d(x)$

`\deg b(x) \lt \deg d(x)`

4136. ⠥⠷⠭⠾

$u(x)$

`u(x)`

4137. ⠧⠷⠭⠾

$v(x)$

`v(x)`

4138. ⠏⠷⠭⠾ ⠨⠅ ⠥⠷⠭⠾⠙⠄⠷⠭⠾

$p(x) = u(x)d'(x)$

`p(x) = u(x) d'(x)`

4139. ⠟⠷⠭⠾ ⠨⠅ ⠧⠷⠭⠾⠙⠄⠷⠭⠾

$q(x) = v(x)d'(x)$

`q(x) = v(x) d'(x)`

4140. ⠙⠷⠭⠾ ⠨⠅ ⠙⠄⠷⠭⠾⠈⠷⠗⠷⠭⠾⠥⠷⠭⠾⠬⠎⠷⠭⠾⠧⠷⠭⠾⠈⠾

$d(x) = d'(x)[r(x)u(x) + s(x)v(x)]$

`d(x) = d'(x)[r(x) u(x) + s(x) v(x)]`

4141. ⠙⠑⠛⠙⠷⠭⠾ ⠨⠅ ⠙⠑⠛⠙⠄⠷⠭⠾⠬⠙⠑⠛⠈⠷⠗⠷⠭⠾⠥⠷⠭⠾⠬⠎⠷⠭⠾⠧⠷⠭⠾⠈⠾

$\deg d(x) = \deg d'(x) + \deg[r(x)u(x) + s(x)v(x)]$

`\deg d(x) = \deg d'(x) + \deg[r(x) u(x) + s(x) v(x)]`

4142. ⠙⠑⠛⠙⠷⠭⠾ ⠨⠅ ⠙⠑⠛⠙⠄⠷⠭⠾

$\deg d(x) = \deg d'(x)$

`\deg d(x) = \deg d'(x)`

4143. ⠙⠷⠭⠾ ⠨⠅ ⠙⠄⠷⠭⠾

$d(x) = d'(x)$

`d(x) = d'(x)`

4144. ⠋⠷⠭⠾⠈⠑⠠⠋⠈⠷⠭⠈⠾

$f(x) \in F[x]$

`f(x) \in F[x]`

4145. ⠓⠷⠭⠾

$h(x)$

h(x)

4146. ⠭⠘⠆⠐⠤⠆ ⠈⠑ ⠈⠠⠟⠈⠷⠭⠈⠾

$x^2 - 2 \in {\mathbb Q}[x]$

x^2 - 2 \in {\mathbb Q}[x]

4147. ⠭⠘⠆⠐⠬⠂

$x^2 + 1$

x^2 + 1

4148. ⠏⠷⠭⠾ ⠨⠅ ⠭⠘⠒⠐⠬⠭⠘⠆⠐⠬⠆

$p(x) = x^3 + x^2 + 2$

p(x) = x^3 + x^2 + 2

4149. ⠈⠠⠵⠒⠈⠷⠭⠈⠾

${\mathbb Z}_3[x]$

{\mathbb Z}_3[x]

4150. ⠭⠤⠁

$x - a$

x - a

4151. ⠏⠷⠁⠾ ⠨⠅ ⠴

$p(a) = 0$

p(a) = 0

4152. ⠏⠷⠭⠾ ⠈⠑ ⠈⠠⠟⠈⠷⠭⠈⠾

$p(x) \in {\mathbb Q}[x]$

p(x) \in {\mathbb Q}[x]

4153. ⠏⠷⠭⠾ ⠨⠅ ⠹⠗⠌⠎⠼⠷⠁⠴⠬⠁⠂⠭⠬⠄⠄⠄⠬⠁⠰⠝⠐⠭⠘⠝⠐⠾

$p(x) = \frac{r}{s}(a_0 + a_1 x + \cdots + a_n x^n)$

p(x) = \frac{r}{s}(a_0 + a_1 x + \cdots + a_n x^n)

4154. ⠗⠠⠎⠠⠁⠴⠠⠄⠠⠁⠰⠝⠐

$r, s, a_0, \ldots, a_n$

r, s, a_0, \ldots, a_n

4155. ⠏⠷⠭⠾ ⠨⠅ ⠹⠃⠴⠌⠉⠴⠼⠬⠹⠃⠂⠌⠉⠂⠼⠭⠬⠄⠄⠄⠬⠹⠃⠰⠝⠐⠌⠉⠰⠝⠐⠼⠭⠘⠝

$p(x) = \frac{b_0}{c_0} + \frac{b_1}{c_1} x + \cdots + \frac{b_n}{c_n} x^n$

p(x) = \frac{b_0}{c_0} + \frac{b_1}{c_1} x + \cdots + \frac{b_n}{c_n}
x^n

4156. ⠃⠰⠊

b_i

b_i

4157. ⠏⠷⠭⠾⠀⠨⠅⠀⠹⠂⠌⠉⠴⠄⠄⠄⠉⠰⠝⠐⠼⠷⠙⠴⠬⠙⠂⠭⠬⠄⠄⠄⠬⠙⠰⠝⠐⠭⠘⠝⠐⠾

$p(x) = \frac{1}{c_0 \cdots c_n} (d_0 + d_1 x + \cdots + d_n x^n)$

p(x) = \frac{1}{c_0 \cdots c_n} (d_0 + d_1 x + \cdots + d_n x^n)

4158. ⠙⠴⠠⠀⠄⠄⠄⠠⠀⠙⠰⠝

$d_0, \ldots, d_n$

d_0, \ldots, d_n

4159. ⠏⠷⠭⠾⠀⠨⠅⠀⠹⠙⠌⠉⠴⠄⠄⠄⠉⠰⠝⠐⠼⠷⠁⠴⠬⠁⠂⠭⠬⠄⠄⠄⠬⠁⠰⠝⠐⠭⠘⠝⠐⠾

$p(x) = \frac{d}{c_0 \cdots c_n} (a_0 + a_1 x + \cdots + a_n x^n)$

p(x) = \frac{d}{c_0 \cdots c_n} (a_0 + a_1 x + \cdots + a_n x^n)

4160. ⠙⠰⠊⠀⠨⠅⠀⠙⠁⠰⠊

$d_i = d a_i$

d_i = d a_i

4161. ⠙⠸⠌⠷⠉⠴⠄⠄⠄⠉⠰⠝⠐⠾

$d /(c_0 \cdots c_n)$

d /(c_0 \cdots c_n)

4162. ⠛⠉⠙⠷⠗⠠⠎⠾⠀⠨⠅⠀⠂

$\gcd(r,s) = 1$

\gcd(r,s) = 1

4163. ⠏⠷⠭⠾⠀⠈⠑⠀⠈⠠⠵⠈⠷⠭⠈⠾

$p(x) \in {\mathbb Z}[x]$

p(x) \in {\mathbb Z}[x]

4164. ⠨⠁⠷⠭⠾

$\alpha(x)$

\alpha(x)

4165. ⠨⠃⠷⠭⠾

$\beta(x)$

\beta(x)

4166. ⠈⠠⠟⠈⠷⠭⠈⠾

${\mathbb Q}[x]$

{\mathbb Q}[x]

4167. ⠏⠷⠭⠾⠀⠨⠅⠀⠁⠷⠭⠾⠃⠷⠭⠾

$p(x) = a(x) b(x)$

p(x) = a(x) b(x)

4168. ⠙⠑⠛⠀⠨⠁⠷⠭⠾⠀⠨⠅⠀⠙⠑⠛⠀⠁⠷⠭⠾

$\deg \alpha(x) = \deg a(x)$

\deg \alpha(x) = \deg a(x)

4169. ⠙⠑⠛⠀⠨⠃⠷⠭⠾⠀⠨⠅⠀⠙⠑⠛⠀⠃⠷⠭⠾

$\deg \beta(x) = \deg b(x)$

\deg \beta(x) = \deg b(x)

4170. ⠏⠷⠭⠾ ⠨⠅ ⠨⠁⠷⠭⠾⠨⠃⠷⠭⠾ ⠨⠅ ⠹⠉⠂⠉⠆⠌⠙⠂⠙⠆⠼⠨⠁⠂⠷⠭⠾⠨⠃⠂⠷⠭⠾ ⠨⠅ ⠹⠉⠌⠙⠼⠨⠁⠂⠷⠭⠾⠨⠃

$p(x) = \alpha(x)\beta(x) = \frac{c_1 c_2}{d_1 d_2}\alpha_1(x)\beta_1(x) = \frac{c}{d}\alpha_1(x)\beta_1(x)$

p(x) = \alpha(x) \beta(x) = \frac{c_1 c_2}{d_1 d_2} \alpha_1(x)
\beta_1(x) = \frac{c}{d} \alpha_1(x) \beta_1(x)

4171. ⠉⠌⠙

c/d

c/d

4172. ⠉⠂⠌⠙⠂

c_1/d_1

c_1/d_1

4173. ⠉⠆⠌⠙⠆

c_2/d_2

c_2/d_2

4174. ⠙⠏⠷⠭⠾ ⠨⠅ ⠉⠨⠁⠂⠷⠭⠾⠨⠃⠂⠷⠭⠾

$dp(x) = c\alpha_1(x)\beta_1(x)$

d p(x) = c \alpha_1(x) \beta_1(x)

4175. ⠉⠁⠰⠍⠐⠃⠰⠝⠐ ⠨⠅ ⠂

$ca_m b_n = 1$

c a_m b_n = 1

4176. ⠉ ⠨⠅ ⠂

$c = 1$

c=1

4177. ⠉ ⠨⠅ ⠤⠂

$c = -1$

c = -1

4178. ⠉ ⠨⠅ ⠂

$c = 1$

c = 1

4179. ⠁⠰⠍⠐ ⠨⠅ ⠃⠰⠝⠐ ⠨⠅ ⠂

$a_m = b_n = 1$

a_m = b_n = 1

4180. ⠁⠰⠍⠐ ⠨⠅ ⠃⠰⠝⠐ ⠨⠅ ⠤⠂

$a_m = b_n = -1$

a_m = b_n = -1

4181. ⠏⠷⠭⠾ ⠨⠅ ⠨⠁⠂⠷⠭⠾⠨⠃⠂⠷⠭⠾

$p(x) = \alpha_1(x)\beta_1(x)$

p(x) = \alpha_1(x) \beta_1(x)

4182. ⠨⠁⠂⠷⠭⠾

$\alpha_1(x)$

\alpha_1(x)

4183. ⠨⠃⠂⠷⠭⠾

$\beta_1(x)$

\beta_1(x)

4184. ⠙⠑⠛⠨⠁⠷⠭⠾⠀⠨⠅⠀⠙⠑⠛⠨⠁⠂⠷⠭⠾

$\deg \alpha(x) = \deg \alpha_1(x)$

\deg \alpha(x) = \deg \alpha_1(x)

4185. ⠙⠑⠛⠨⠃⠷⠭⠾⠀⠨⠅⠀⠙⠑⠛⠨⠃⠂⠷⠭⠾

$\deg \beta(x) = \deg \beta_1(x)$

\deg \beta(x) = \deg \beta_1(x)

4186. ⠁⠷⠭⠾⠀⠨⠅⠀⠤⠨⠁⠂⠷⠭⠾

$a(x) = -\alpha_1(x)$

a(x) = -\alpha_1(x)

4187. ⠃⠷⠭⠾⠀⠨⠅⠀⠤⠨⠃⠂⠷⠭⠾

$b(x) = -\beta_1(x)$

b(x) = -\beta_1(x)

4188. ⠏⠷⠭⠾⠀⠨⠅⠀⠷⠤⠨⠁⠂⠷⠭⠾⠾⠷⠤⠨⠃⠂⠷⠭⠾⠾⠀⠨⠅⠀⠁⠷⠭⠾⠃⠷⠭⠾

$p(x) = (-\alpha_1(x))(-\beta_1(x)) = a(x)b(x)$

p(x) = (-\alpha_1(x))(- \beta_1(x)) = a(x) b(x)

4189. ⠙⠀⠌⠨⠅⠀⠂

$d \neq 1$

d \neq 1

4190. ⠛⠉⠙⠷⠉⠠⠙⠾⠀⠨⠅⠀⠂

$\gcd(c, d) = 1$

\gcd(c, d) = 1

4191. ⠏⠱⠙

$p \mid d$

p \mid d

4192. ⠏⠌⠱⠉

$p \nmid c$

p \notdivide c

4193. ⠏⠌⠱⠁⠰⠊

$p \nmid a_i$

p \notdivide a_i

4194. ⠃⠰⠚

b_j

b_j

4195. ⠏⠌⠳⠃⠰⠚

$p \nmid b_j$

p \notdivide b_j

4196. ⠨⠁⠄⠂⠷⠭⠾

$\alpha_1'(x)$

\alpha_1'(x)

4197. ⠨⠃⠄⠂⠷⠭⠾

$\beta_1'(x)$

\beta_1'(x)

4198. ⠈⠠⠵⠰⠏⠐⠈⠷⠭⠈⠾

${\mathbb Z}_p[x]$

{\mathbb Z}_p[x]

4199. ⠨⠁⠄⠂⠷⠭⠾⠨⠃⠄⠂⠷⠭⠾⠀⠨⠅⠀⠴

$\alpha_1'(x) \beta_1'(x) = 0$

\alpha_1'(x) \beta_1'(x) = 0

4200. ⠙⠀⠨⠅⠀⠂

$d=1$

d=1

4201. ⠏⠷⠭⠾⠀⠨⠅⠀⠭⠘⠝⠐⠬⠁⠰⠝⠤⠂⠐⠭⠘⠝⠤⠂⠐⠬⠄⠄⠄⠬⠁⠴

$p(x) = x^n + a_{n - 1} x^{n - 1} + \cdots + a_0$

p(x) = x^n + a_{n - 1} x^{n - 1} + \cdots + a_0

4202. ⠁⠴⠀⠌⠨⠅⠀⠴

$a_0 \neq 0$

a_0 \neq 0

4203. ⠁⠴

a_0

a_0

4204. ⠁⠀⠈⠑⠀⠈⠠⠟

$a \in {\mathbb Q}$

a \in {\mathbb Q}

4205. ⠨⠁⠀⠈⠑⠀⠈⠠⠵

$\alpha \in {\mathbb Z}$

\alpha \in {\mathbb Z}

4206. ⠏⠷⠭⠾⠀⠨⠅⠀⠷⠭⠤⠨⠁⠾⠷⠭⠘⠝⠤⠂⠐⠬⠄⠄⠄⠤⠁⠴⠌⠨⠁⠾

$p(x) = (x - \alpha)(x^{n - 1} + \cdots - a_0 / \alpha)$

p(x) = (x - \alpha)(x^{n - 1} + \cdots - a_0 / \alpha)

4207. ⠁⠴⠌⠨⠁⠀⠈⠑⠀⠈⠠⠵

$a_0 /\alpha \in {\mathbb Z}$

a_0 /\alpha \in {\mathbb Z}

4208. ⠨⠁⠳⠁⠴

$\alpha \mid a_0$

\alpha \mid a_0

4209. ⠏⠷⠭⠾⠀⠨⠅⠀⠭⠘⠲⠐⠤⠆⠭⠘⠒⠐⠬⠭⠬⠂

$p(x) = x^4 - 2x^3 + x + 1$

p(x) = x^4 - 2 x^3 + x + 1

4210. ⠏⠷⠂⠾⠀⠨⠅⠀⠂

$p(1) = 1$

p(1) = 1

4211. ⠏⠷⠤⠂⠾⠀⠨⠅⠀⠒

$p(-1) = 3$

p(-1)= 3

4212. ⠃⠙⠀⠨⠅⠀⠂

$bd = 1$

bd = 1

4213. ⠃⠀⠨⠅⠀⠙⠀⠨⠅⠀⠂

$b = d = 1$

b = d = 1

4214. ⠃⠀⠨⠅⠀⠙⠀⠨⠅⠀⠤⠂

$b = d = -1$

b = d = -1

4215. ⠃⠀⠨⠅⠀⠙

$b = d$

b = d

4216. ⠁⠙⠬⠃⠉⠀⠨⠅⠀⠃⠷⠁⠬⠉⠾⠀⠨⠅⠀⠂

$ad + bc = b(a + c) = 1$

ad + bc = b(a + c) = 1

4217. ⠁⠬⠉⠀⠨⠅⠀⠤⠆

$a + c = -2$

a + c = -2

4218. ⠤⠆⠃⠀⠨⠅⠀⠂

$-2b = 1$

-2b = 1

4219. ⠋⠷⠭⠾⠀⠨⠅⠀⠁⠰⠝⠐⠭⠘⠝⠐⠬⠄⠄⠄⠬⠁⠴⠈⠑⠈⠠⠵⠈⠷⠭⠈⠾

$f(x) = a_n x^n + \cdots + a_0 \in \mathbb{Z}[x]$

f(x) = a_n x^n + \cdots + a_0 \in {\mathbb Z}[x]

4220. ⠏⠳⠁⠰⠊

$p \mid a_i$

p \mid a_i

4221. ⠊ ⠨⠅ ⠴⠠⠼⠂⠠⠄⠠⠝⠤⠂

$i = 0, 1, \ldots, n-1$

```
i = 0, 1, \ldots, n-1
```

4222. ⠏⠌⠳⠁⠰⠝

$p \nmid a_n$

```
p \notdivide a_n
```

4223. ⠏⠘⠆⠐⠌⠳⠁⠴

$p^2 \nmid a_0$

```
p^2 \notdivide a_0
```

4224. ⠋⠷⠭⠾ ⠨⠅ ⠷⠃⠰⠗⠐⠭⠘⠗⠐⠬⠄⠄⠄⠬⠃⠴⠾⠷⠉⠰⠎⠐⠭⠘⠎⠐⠬⠄⠄⠄⠬⠉⠴⠾

$f(x) = (b_rx^r + \cdots + b_0)(c_s x^s + \cdots + c_0)$

```
f(x) = (b_rx^r + \cdots + b_0)(c_s x^s + \cdots + c_0 )
```

4225. ⠃⠰⠗

b_r

```
b_r
```

4226. ⠉⠰⠎

c_s

```
c_s
```

4227. ⠗⠠⠎ ⠐⠅ ⠝

$r, s < n$

```
r, s \lt n
```

4228. ⠁⠴ ⠨⠅ ⠃⠴⠉⠴

$a_0 = b_0 c_0$

```
a_0 = b_0 c_0
```

4229. ⠃⠴

b_0

```
b_0
```

4230. ⠉⠴

c_0

```
c_0
```

4231. ⠏⠌⠳⠃⠴

$p \nmid b_0$

```
p \notdivide b_0
```

4232. ⠏⠳⠉⠴

$p \mid c_0$

```
p \mid c_0
```

4233. ⠁⠰⠝⠐ ⠨⠅ ⠃⠰⠗⠐⠉⠰⠎

$a_n = b_r c_s$

```
a_n = b_r c_s
```

4234. ⠏⠌⠳⠉⠰⠅

$p \nmid c_k$

p \notdivide c_k

4235. ⠁ ⠰⠍⠐ ⠨⠅ ⠃⠴⠉⠰⠍⠐⠬⠃⠂⠉⠰⠍⠤⠂⠐⠬ ⠄⠄⠄⠐⠬⠃⠰⠍⠐⠉⠴

$a_m = b_0 c_m + b_1 c_{m-1} + \cdots + b_m c_0$

a_m = b_0 c_m + b_1 c_{m - 1} + \cdots + b_m c_0

4236. ⠃⠴⠉⠰⠍

$b_0 c_m$

b_0 c_m

4237. ⠍ ⠨⠅ ⠝

$m = n$

m = n

4238. ⠍ ⠐⠅ ⠝

$m < n$

m \lt n

4239. ⠋⠷⠭⠾ ⠨⠅ 16⠭⠘⠢⠐⠤⠔⠭⠘⠲⠐⠬⠒⠭⠘⠆⠐⠬⠖⠭⠐⠤21

$f(x) = 16x^5 - 9x^4 + 3x^2 + 6x - 21$

f(x) = 16 x^5 - 9 x^4 + 3x^2 + 6 x - 21

4240. ⠏ ⠨⠅ ⠒

$p = 3$

p = 3

4241. ⠠⠋⠈⠷⠭⠈⠾

$F[x]$

F\lbrack x \rbrack

4242. ⠄⡗⠛⠆ ⠶⠁⠢⠄⠄⠏⠷⠭⠾⠄⡗⠛⠆ ⠶⠁⠂⠄

$\langle p(x) \rangle$

\langle p(x) \rangle

4243. ⠄⡗⠛⠆ ⠶⠁⠢⠄⠄⠏⠷⠭⠾⠄⡗⠛⠆ ⠶⠁⠂⠄ ⠨⠅ StartSet⠏⠷⠭⠾⠟⠷⠭⠾⠈⠒⠟⠷⠭⠾ ⠈⠑ ⠠⠋ ⠈⠷⠭ ⠈⠾EndSet

$\langle p(x) \rangle = \{p(x)q(x) : q(x) \in F[x]\}$

\langle p(x) \rangle = \{ p(x) q(x) : q(x) \in F[x] \}

4244. ⠄⡗⠛⠆ ⠶⠁⠢⠄⠄⠭ ⠘⠆ ⠐⠄⡗⠛⠆ ⠶⠁⠂⠄

$\langle x^2 \rangle$

\langle x^2 \rangle

4245. ⠏⠷⠭⠾ ⠈⠑ ⠠⠊

$p(x) \in I$

p(x) \in I

4246. ⠙⠑⠛⠏⠷⠭⠾ ⠨⠅ ⠴

$\deg p(x) = 0$

\deg p(x)= 0

4247. ⠄⡷⠭⠆⠶⠑⠦⠄⠂⠄⡷⠭⠆⠶⠑⠔⠄ ⠨⠅ ⠠⠊ ⠨⠅ ⠠⠋⠈⠷⠭⠈⠾

$\langle 1 \rangle = I = F[x]$

\langle 1 \rangle = I = F[x]

4248. ⠙⠑⠛⠏⠷⠭⠾ ⠨⠂⠱ ⠂

$\deg p(x) \geq 1$

\deg p(x) \geq 1

4249. ⠋⠷⠭⠾ ⠨⠅ ⠏⠷⠭⠾⠟⠷⠭⠾⠬⠗⠷⠭⠾

$f(x) = p(x) q(x) + r(x)$

f(x) = p(x) q(x) + r(x)

4250. ⠙⠑⠛⠗⠷⠭⠾ ⠐⠅ ⠙⠑⠛⠏⠷⠭⠾

$\deg r(x) \lt \deg p(x)$

\deg r(x) \lt \deg p(x)

4251. ⠋⠷⠭⠾⠠⠏⠷⠭⠾ ⠈⠑ ⠠⠊

$f(x), p(x) \in I$

f(x), p(x) \in I

4252. ⠗⠷⠭⠾ ⠨⠅ ⠋⠷⠭⠾⠤⠏⠷⠭⠾⠟⠷⠭⠾

$r(x) = f(x) - p(x) q(x)$

r(x) = f(x) - p(x) q(x)

4253. ⠠⠊ ⠨⠅ ⠄⡷⠭⠆⠶⠑⠦⠄⠏⠷⠭⠾⠄⡷⠭⠆⠶⠑⠔⠄

$I = \langle p(x) \rangle$

I = \langle p(x) \rangle

4254. ⠠⠋⠈⠷⠭⠠⠽⠈⠾

$F[x,y]$

F[x,y]

4255. ⠠⠋⠈⠷⠭⠠⠽⠈⠾

$F[x, y]$

F[x, y]

4256. ⠏⠷⠭⠾ ⠨⠅ ⠋⠷⠭⠾⠛⠷⠭⠾

$p(x) = f(x) g(x)$

p(x) = f(x) g(x)

4257. ⠄⡷⠭⠆⠶⠑⠦⠄⠏⠷⠭⠾⠄⡷⠭⠆⠶⠑⠔⠄ ⠸⠐⠅ ⠄⡷⠭⠆⠶⠑⠦⠄⠋⠷⠭⠾⠄⡷⠭⠆⠶⠑⠔⠄

$\langle p(x) \rangle \subset \langle f(x) \rangle$

\langle p(x) \rangle \subset \langle f(x) \rangle

4258. ⠠⠊ ⠨⠅ ⠄⡷⠭⠆⠶⠑⠦⠄⠋⠷⠭⠾⠄⡷⠭⠆⠶⠑⠔⠄

$I = \langle f(x) \rangle$

I = \langle f(x) \rangle

4259. ⠛⠷⠭⠾ ⠈⠑ ⠠⠋⠈⠷⠭⠈⠾

$g(x) \in F[x]$

g(x) \in F[x]

4260. ⠠⠊ ⠨⠅ ⠠⠋⠈⠷⠭⠈⠾

$I = F[x]$

I = F[x]

4261. ⠄⡷⠭⠆⠶⠑⠦⠄⠏⠷⠭⠾⠄⡷⠭⠆⠶⠑⠔⠄

$\langle p(x)\rangle$

\langle p(x)\rangle

4262. ⠁⠭⠘⠒⠐⠬⠃⠭⠘⠆⠐⠬⠉⠭⠐⠬⠙ ⠨⠅ ⠴

$ax^3 + bx^2 + cx + d = 0$

ax^3 + bx^2 + cx + d = 0

4263. ⠁⠭⠘⠒⠐⠬⠉⠭⠐⠬⠙ ⠨⠅ ⠴

$ax^3 + cx + d = 0$

ax^3 + cx + d = 0

4264. ⠁⠭⠘⠒⠐⠬⠃⠭⠘⠆⠐⠬⠉⠭⠐⠬⠙ ⠨⠅ ⠴

$ax^3 + bx^2 + cx + d = 0$

a x^3 + bx^2 + cx + d = 0

4265. ⠁⠭⠘⠲⠐⠬⠃⠭⠘⠒⠐⠬⠉⠭⠘⠆⠐⠬⠙⠭⠐⠬⠑ ⠨⠅ ⠴

$ax^4 + bx^3 + cx^2 + dx + e = 0$

a x^4 + b x^3 + cx^2 + d x + e = 0

4266. ⠈⠠⠵⠆⠈⠷⠭⠈⠾

$\mathbb{Z}_2[x]$

{\mathbb Z}_2[x]

4267. ⠷⠢⠭⠘⠆⠐⠬⠒⠭⠤⠲⠾⠬⠷⠲⠭⠘⠆⠐⠤⠭⠬⠔⠾

$(5x^2 + 3x - 4) + (4x^2 - x + 9)$

(5x^2 + 3x - 4) + (4x^2 - x + 9)

4268. ⠷⠢⠭⠘⠆⠐⠬⠒⠭⠤⠲⠾⠷⠲⠭⠘⠆⠐⠤⠭⠬⠔⠾

$(5x^2 + 3x - 4)(4x^2 - x + 9)$

(5x^2 + 3x - 4) (4x^2 - x + 9)

4269. ⠷⠶⠭⠘⠒⠐⠬⠒⠭⠘⠆⠐⠤⠭⠾⠬⠷⠖⠭⠘⠆⠐⠤⠦⠭⠬⠲⠾

$(7x^3 + 3x^2 - x) + (6x^2 - 8x + 4)$

(7x^3 + 3x^2 - x) + (6x^2 - 8x + 4)

4270. ⠷⠒⠭⠘⠆⠐⠬⠆⠭⠤⠲⠾⠬⠷⠲⠭⠘⠆⠐⠬⠆⠾

$(3x^2 + 2x - 4) + (4x^2 + 2)$

(3x^2 + 2x - 4) + (4x^2 + 2)

4271. ⠷⠒⠭⠘⠆⠐⠬⠆⠭⠤⠲⠾⠷⠲⠭⠘⠆⠐⠬⠆⠾

$(3x^2 + 2x - 4)(4x^2 + 2)$

(3x^2 + 2x - 4) (4x^2 + 2)

4272. ⠷⠢⠭⠘⠆⠐⠬⠒⠭⠤⠆⠾⠘⠆

$(5x^2 + 3x - 2)^2$

(5x^2 + 3x - 2)^2

4273. ⠔⠭⠘⠆⠬⠆⠭⠬⠢

$9x^2 + 2x + 5$

9x^2 + 2x + 5

4274. ⠦⠭⠘⠲⠬⠶⠭⠘⠒⠬⠆⠭⠘⠆⠬⠶⠭

$8x^4 + 7x^3 + 2x^2 + 7x$

8x^4 + 7x^3 + 2x^2 + 7x

4275. ⠁⠷⠭⠾ ⠨⠅ ⠟⠷⠭⠾⠃⠷⠭⠾⠬⠗⠷⠭⠾

$a(x) = q(x)b(x) + r(x)$

a(x) = q(x) b(x) + r(x)

4276. ⠙⠑⠛⠗⠷⠭⠾ ⠐⠅ ⠙⠑⠛⠃⠷⠭⠾

$\deg r(x) < \deg b(x)$

\deg r(x) \lt \deg b(x)

4277. ⠁⠷⠭⠾ ⠨⠅ ⠢⠭⠘⠒⠐⠬⠖⠭⠘⠆⠐⠤⠒⠭⠬⠲

$a(x) = 5x^3 + 6x^2 - 3x + 4$

a(x) = 5 x^3 + 6x^2 - 3 x + 4

4278. ⠃⠷⠭⠾ ⠨⠅ ⠭⠤⠆

$b(x) = x - 2$

b(x) = x - 2

4279. ⠈⠠⠵⠶⠈⠷⠭⠈⠾

$\mathbb{Z}_7[x]$

{\mathbb Z}_7[x]

4280. ⠁⠷⠭⠾ ⠨⠅ ⠖⠭⠘⠲⠐⠤⠆⠭⠘⠒⠐⠬⠭⠘⠆⠐⠤⠒⠭⠬⠂

$a(x) = 6x^4 - 2x^3 + x^2 - 3x + 1$

a(x) = 6 x^4 - 2 x^3 + x^2 - 3 x + 1

4281. ⠃⠷⠭⠾ ⠨⠅ ⠭⠘⠆⠐⠬⠭⠤⠆

$b(x) = x^2 + x - 2$

b(x) = x^2 + x - 2

4282. ⠁⠷⠭⠾ ⠨⠅ ⠲⠭⠘⠢⠐⠤⠭⠘⠒⠐⠬⠭⠘⠆⠐⠬⠲

$a(x) = 4x^5 - x^3 + x^2 + 4$

a(x) = 4 x^5 - x^3 + x^2 + 4

4283. ⠃⠷⠭⠾ ⠨⠅ ⠭⠘⠒⠐⠤⠆

$b(x) = x^3 - 2$

b(x) = x^3 - 2

4284. ⠈⠠⠵⠢⠈⠷⠭⠈⠾

$\mathbb{Z}_5[x]$

{\mathbb Z}_5[x]

4285. ⠁⠷⠭⠾ ⠨⠅ ⠭⠘⠢⠐⠬⠭⠘⠒⠐⠤⠭⠘⠆⠐⠤⠭

$a(x) = x^5 + x^3 - x^2 - x$

a(x) = x^5 + x^3 -x^2 - x

4286. ⠃⠷⠭⠾ ⠭ ⠭⠘⠒⠐⠬⠭

$b(x) = x^3 + x$

b(x) = x^3 + x

4287. ⠢⠭⠘⠒⠐⠬⠖⠭⠘⠆⠐⠤⠒⠭⠬⠲ ⠭ ⠷⠢⠭⠘⠆⠐⠬⠆⠭⠬⠂⠾⠷⠭⠤⠆⠾⠬⠖

$5x^3 + 6x^2 - 3x + 4 = (5x^2 + 2x + 1)(x - 2) + 6$

5 x^3 + 6 x^2 - 3 x + 4 = (5 x^2 + 2x + 1)(x -2) + 6

4288. ⠲⠭⠘⠢⠐⠤⠭⠘⠒⠐⠬⠭⠘⠆⠐⠬⠲ ⠭ ⠷⠲⠭⠘⠆⠐⠬⠲⠾⠷⠭⠘⠒⠐⠬⠒⠾⠬⠲⠭⠘⠆⠐⠬⠆

$4x^5 - x^3 + x^2 + 4 = (4x^2 + 4)(x^3 + 3) + 4x^2 + 2$

4x^5 - x^3 + x^2 + 4 = (4x^2 + 4)(x^3 + 3) + 4x^2 + 2

4289. ⠙⠷⠭⠾ ⠭ ⠛⠉⠙⠷⠏⠷⠭⠾⠠⠟⠷⠭⠾⠾

$d(x) = \gcd(p(x), q(x))$

d(x) = \gcd(p(x), q(x))

4290. ⠁⠷⠭⠾⠏⠷⠭⠾⠬⠃⠷⠭⠾⠟⠷⠭⠾ ⠭ ⠙⠷⠭⠾

$a(x)p(x) + b(x)q(x) = d(x)$

a(x) p(x) + b(x) q(x) = d(x)

4291. ⠏⠷⠭⠾ ⠭ ⠭⠘⠒⠐⠤⠖⠭⠘⠆⠐⠬14⠭⠤15

$p(x) = x^3 - 6x^2 + 14x - 15$

p(x) = x^3 - 6x^2 + 14x - 15

4292. ⠟⠷⠭⠾ ⠭ ⠭⠘⠒⠐⠤⠦⠭⠘⠆⠐⠬21⠭⠤18

$q(x) = x^3 - 8x^2 + 21x - 18$

q(x) = x^3 - 8x^2 + 21x - 18

4293. ⠏⠷⠭⠾⠠⠟⠷⠭⠾⠈⠑⠈⠠⠟⠈⠷⠭⠈⠾

$p(x), q(x) \in \mathbb{Q}[x]$

p(x), q(x) \in {\mathbb Q}[x]

4294. ⠏⠷⠭⠾ ⠭ ⠭⠘⠒⠐⠬⠭⠘⠆⠐⠤⠭⠬⠂

$p(x) = x^3 + x^2 - x + 1$

p(x) = x^3 + x^2 - x + 1

4295. ⠟⠷⠭⠾ ⠭ ⠭⠘⠒⠐⠬⠭⠤⠂

$q(x) = x^3 + x - 1$

q(x) = x^3 + x - 1

4296. ⠏⠷⠭⠾⠠⠟⠷⠭⠾⠈⠑⠈⠠⠵⠆⠈⠷⠭⠈⠾

$p(x), q(x) \in \mathbb{Z}_2[x]$

p(x), q(x) \in {\mathbb Z}_2[x]

4297. ⠏⠷⠭⠾ ⠭ ⠭⠘⠒⠐⠬⠭⠘⠆⠐⠤⠲⠭⠬⠲

$p(x) = x^3 + x^2 - 4x + 4$

p(x) = x^3 + x^2 - 4x + 4

4298. ⠟⠷⠭⠾ ⠭ ⠭⠘⠒⠐⠬⠒⠭⠤⠆

$q(x) = x^3 + 3x - 2$

q(x) = x^3 + 3 x -2

4299. ⠏⠷⠭⠾⠠⠟⠷⠭⠾⠈⠑⠈⠠⠵⠢⠈⠷⠭⠈⠾

$p(x), q(x) \in \mathbb{Z}_5[x]$

`p(x), q(x) \in {\mathbb Z}_5[x]`

4300. ⠏⠷⠭⠾ ⠶ ⠭⠘⠒⠐⠤⠆⠭⠬⠲

$p(x) = x^3 - 2x + 4$

`p(x) = x^3 - 2 x + 4`

4301. ⠟⠷⠭⠾ ⠶ ⠲⠭⠘⠒⠐⠬⠭⠬⠒

$q(x) = 4x^3 + x + 3$

`q(x) = 4 x^3 + x + 3`

4302. ⠢⠭⠘⠒⠬⠲⠭⠘⠆⠤⠭⠬⠔

$5x^3 + 4x^2 - x + 9$

`5x^3 + 4x^2 - x + 9`

4303. ⠒⠭⠘⠒⠤⠲⠭⠘⠆⠤⠭⠬⠲

$3x^3 - 4x^2 - x + 4$

`3x^3 - 4x^2 - x + 4`

4304. ⠈⠠⠵⠢

$\mathbb{Z}_5$

`{\mathbb Z}_{5}`

4305. ⠢⠭⠘⠲⠬⠆⠭⠘⠆⠤⠒

$5x^4 + 2x^2 - 3$

`5x^4 + 2x^2 - 3`

4306. ⠭⠘⠒⠐⠬⠭⠬⠂

$x^3 + x + 1$

`x^3 + x + 1`

4307. ⠈⠠⠵⠲⠈⠷⠭⠈⠾

$\mathbb{Z}_4[x]$

`{\mathbb Z}_4[x]`

4308. ⠷⠆⠭⠬⠂⠾

$(2x + 1)$

`(2x + 1)`

4309. ⠭⠘⠲⠐⠤⠆⠭⠘⠒⠬⠆⠭⠘⠆⠬⠭⠬⠲

$x^4 - 2x^3 + 2x^2 + x + 4$

`x^4 - 2x^3 + 2x^2 + x + 4`

4310. ⠭⠘⠲⠐⠤⠢⠭⠘⠒⠬⠒⠭⠤⠆

$x^4 - 5x^3 + 3x - 2$

`x^4 - 5x^3 + 3x - 2`

4311. ⠒⠭⠘⠢⠤⠲⠭⠘⠒⠤⠖⠭⠘⠆⠬⠖

$3x^5 - 4x^3 - 6x^2 + 6$

`3x^5 - 4x^3 - 6x^2 + 6`

4312. ⠢⠭ ⠘⠢⠤⠖⠭ ⠘⠲⠤⠒⠭ ⠘⠆ ⠬⠔⠭⠤15

$5x^5 - 6x^4 - 3x^2 + 9x - 15$

`5x^5 - 6x^4 - 3x^2 + 9 x - 15`

4313. ⠭ ⠘⠆ ⠐⠬⠭⠬⠦

$x^2 + x + 8$

`x^2 + x + 8`

4314. ⠈ ⠠⠵10⠈⠷⠭ ⠈⠾

${\mathbb Z}_{10}[x]$

`{\mathbb Z}_{10}[x]`

4315. ⠭ ⠘⠆ ⠐⠬⠭⠬⠦ ⠨⠅ ⠷⠭⠬⠆⠾⠷⠭⠬⠔⠾

$x^2 + x + 8 = (x + 2)(x + 9)$

`x^2 + x + 8 = (x + 2)(x + 9)`

4316. ⠈ ⠠⠵⠖ ⠈⠷⠭ ⠈⠾

${\mathbb Z}_6[x]$

`{\mathbb Z}_6[x]`

4317. ⠠⠋ ⠈⠷⠭⠂ ⠄⠄ ⠠⠭ ⠰⠝ ⠂ ⠈⠾

$F[x_1, \ldots, x_n]$

`F[x_1, \ldots, x_n]`

4318. ⠭ ⠘⠏ ⠐⠬⠁

$x^p + a$

`x^p + a`

4319. ⠁ ⠈⠑ ⠈ ⠠⠵ ⠰⠏

$a \in {\mathbb Z}_p$

`a \in {\mathbb Z}_p`

4320. ⠋⠷⠭⠾⠳⠏⠷⠭⠾⠟⠷⠭⠾

$f(x) \mid p(x)q(x)$

`f(x) \mid p(x)q(x)`

4321. ⠋⠷⠭⠾⠳⠏⠷⠭⠾

$f(x) \mid p(x)$

`f(x) \mid p(x)`

4322. ⠋⠷⠭⠾⠳⠟⠷⠭⠾

$f(x) \mid q(x)$

`f(x) \mid q(x)`

4323. ⠠⠗ ⠈⠷⠭ ⠈⠾ ⠈⠱ ⠈⠱ ⠠⠎ ⠈⠷⠭ ⠈⠾

$R[x] \cong S[x]$

`R[x] \cong S[x]`

4324. ⠨⠋overbar⠱ ⠠⠗ ⠈⠷⠭ ⠈⠾ ⠫⠕ ⠠⠎ ⠈⠷⠭ ⠈⠾

$\overline{\phi} : R[x] \rightarrow S[x]$

`\overline{\phi} : R[x] \rightarrow S[x]`

4325.

$\overline{\phi}(a_0 + a_1 x + \cdots + a_n x^n) = \phi(a_0) + \phi(a_1) x + \cdots + \phi(a_n) x^n$

\overline{\phi}(a_0 + a_1 x + \cdots + a_n x^n) = \phi(a_0) + \phi(a_1) x + \cdots + \phi(a_n) x^n

4326.

$p(a)$

p(a)

4327.

$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0 \in \mathbb Z[x]$

p(x) = a_n x^n + a_{n - 1}x^{n - 1} + \cdots + a_0 \in \mathbb Z[x]

4328.

$p(r/s) = 0$

p(r/s) = 0

4329.

$\gcd(r, s) = 1$

\gcd(r, s) = 1

4330.

$r \mid a_0$

r \mid a_0

4331.

$s \mid a_n$

s \mid a_n

4332.

$({\mathbb Z}[x], +)$

({\mathbb Z}[x], +)

4333.

$\Phi_n(x) = \frac{x^n - 1}{x - 1} = x^{n - 1} + x^{n - 2} + \cdots + x + 1$

\Phi_n(x) = \frac{x^n - 1}{x - 1} = x^{n - 1} + x^{n - 2} + \cdots + x + 1

4334.

$\Phi_p(x)$

\Phi_p(x)

4335.

$x^p - x$

x^p - x

4336.

$x^p - x = x(x - 1)(x - 2) \cdots (x - (p - 1))$

x^p - x = x(x - 1)(x - 2) \cdots (x - (p - 1))

4337. ⠋⠷⠭⠾⠀⠨⠅⠀⠁⠴⠬⠁⠂⠭⠬⠄⠄⠄⠬⠁⠰⠝⠐⠭⠘⠝

$f(x) = a_0 + a_1x + \cdots + a_nx^n$

```
f(x) = a_0 + a_1 x + \cdots + a_n x^n
```

4338. ⠋⠄⠷⠭⠾⠀⠨⠅⠀⠁⠂⠬⠆⠁⠆⠭⠬⠄⠄⠄⠬⠝⠁⠰⠝⠐⠭⠘⠝⠤⠂

$f'(x) = a_1 + 2a_2x + \cdots + na_nx^{n-1}$

```
f'(x) = a_1 + 2 a_2 x + \cdots + n a_n x^{n - 1}
```

4339. ⠷⠋⠬⠛⠾⠄⠷⠭⠾⠀⠨⠅⠀⠋⠄⠷⠭⠾⠬⠛⠄⠷⠭⠾

$(f+g)'(x) = f'(x) + g'(x)$

```
(f + g)'(x) = f'(x) + g'(x)
```

4340. ⠠⠙⠱⠠⠋⠈⠷⠭⠈⠾⠀⠫⠕⠀⠠⠋⠈⠷⠭⠈⠾

$D : F[x] \to F[x]$

```
D : F[x] \rightarrow F[x]
```

4341. ⠠⠙⠷⠋⠷⠭⠾⠾⠀⠨⠅⠀⠋⠄⠷⠭⠾

$D(f(x)) = f'(x)$

```
D(f(x)) = f'(x)
```

4342. ⠉⠓⠁⠗⠠⠋⠀⠨⠅⠀⠴

$\operatorname{char} F = 0$

```
\chr F = 0
```

4343. ⠉⠓⠁⠗⠠⠋⠀⠨⠅⠀⠏

$\operatorname{char} F = p$

```
\chr F = p
```

4344. ⠷⠋⠛⠾⠄⠷⠭⠾⠀⠨⠅⠀⠋⠄⠷⠭⠾⠛⠷⠭⠾⠬⠋⠷⠭⠾⠛⠄⠷⠭⠾

$(fg)'(x) = f'(x)g(x) + f(x)g'(x)$

```
(fg)'(x) = f'(x)g(x) + f(x) g'(x)
```

4345. ⠋⠷⠭⠾⠀⠨⠅⠀⠁⠷⠭⠤⠁⠂⠾⠷⠭⠤⠁⠆⠾⠄⠄⠄⠷⠭⠤⠁⠰⠝⠐⠾

$f(x) = a(x-a_1)(x-a_2)\cdots(x-a_n)$

```
f(x) = a(x - a_1) (x - a_2) \cdots ( x - a_n)
```

4346. ⠋⠄⠷⠭⠾

$f'(x)$

```
f'(x)
```

4347. ⠠⠗⠈⠷⠭⠂⠠⠄⠠⠭⠰⠝⠐⠈⠾

$R[x_1, \ldots, x_n]$

```
R[x_1, \ldots, x_n]
```

4348. ⠙⠑⠛⠷⠏⠷⠭⠾⠬⠟⠷⠭⠾⠾⠀⠅⠱⠀⠍⠁⠭⠀⠷⠙⠑⠛⠏⠷⠭⠾⠠⠙⠑⠛⠟⠷⠭⠾⠾

$\deg(p(x) + q(x)) \leq \max(\deg p(x), \deg q(x))$

```
\deg( p(x) + q(x) ) \leq \max( \deg p(x), \deg q(x) )
```

4349. ⠨⠠⠙⠀⠨⠅⠀⠃⠘⠆⠐⠤⠲⠁⠉

$\Delta = b^2 - 4ac$

```
\Delta = b^2 - 4ac
```

4350. ⠨⠠⠙ ⠨⠂ ⠴

$\Delta > 0$

```
\Delta \gt 0
```

4351. ⠨⠠⠙ ⠨⠅ ⠴

$\Delta = 0$

```
\Delta = 0
```

4352. ⠨⠠⠙ ⠐⠅ ⠴

$\Delta < 0$

```
\Delta \lt 0
```

4353. ⠭⠘⠒⠐⠬⠃⠭⠘⠆⠐⠬⠉⠭⠐⠬⠙ ⠨⠅ ⠴

$x^3 + bx^2 + cx + d = 0$

```
x^3 + bx^2 + cx + d = 0
```

4354. ⠽⠘⠒⠐⠬⠏⠽⠐⠬⠟ ⠨⠅ ⠴

$y^3 + py + q = 0$

```
y^3 + py + q = 0
```

4355. ⠭ ⠨⠅ ⠽⠤⠃⠸⠌⠒

$x = y - b/3$

```
x = y - b/3
```

4356. ⠽ ⠨⠅ ⠵⠤⠹⠏⠌⠒⠵⠼

$y = z - \frac{p}{3z}$

```
y = z - \frac{p}{3 z}
```

4357. ⠵⠘⠒

z^3

```
z^3
```

4358. ⠤⠏⠘⠒⠌27

$-p^3/27$

```
-p^3/27
```

4359. ⠣⠒⠜⠠⠁⠠⠃⠻ ⠨⠅ ⠤⠏⠸⠌⠒

$\sqrt[3]{AB} = -p/3$

```
\sqrt[3]{A B} = -p/3
```

4360. ⠣⠒⠜⠠⠁⠻⠠⠨⠺⠣⠒⠜⠠⠁⠻⠠⠨⠺⠘⠆ ⠐⠣⠒⠜⠠⠁⠻⠠⠣⠒⠜⠠⠃⠻⠠⠨⠺⠣⠒⠜⠠⠃⠻⠠⠨⠺⠘⠆ ⠐⠣⠒⠜⠠⠃⠻

$\sqrt[3]{A}, \quad \omega \sqrt[3]{A}, \quad \omega^2 \sqrt[3]{A}, \quad \sqrt[3]{B}, \quad \omega \sqrt[3]{B}, \quad \omega^2 \sqrt[3]{B}$

```
\sqrt[3]{A}, \quad \omega \sqrt[3]{A}, \quad \omega^2 \sqrt[3]{A},
\quad \sqrt[3]{B}, \quad \omega \sqrt[3]{B}, \quad \omega^2 \sqrt[3]{B}
```

4361. ⠨⠺⠘⠊ ⠐⠣⠒⠜⠤⠹⠟⠌⠆⠼⠬⠜⠹⠏⠘⠒⠐⠌27⠼⠬⠹⠟⠘⠆⠐⠌⠲⠼⠻⠐⠬⠨⠺⠘⠆⠊ ⠐⠣⠒⠜⠤⠹⠟⠌⠆⠼⠤⠜⠹⠏⠘⠒ ⠐

$\omega^i \sqrt[3]{-\frac{q}{2}+ \sqrt{\ \frac{p^3}{27} + \frac{q^2}{4}} } + \omega^{2i} \sqrt[3]{-\frac{q}{2}- \sqrt{\ \frac{p^3}{27} + \frac{q^2}{4}} }$

```
\omega^i \sqrt[3]{-\frac{q}{2}+ \sqrt{\ \frac{p^3}{27} + \frac{q^2}{4}}
} + \omega^{2i} \sqrt[3]{-\frac{q}{2}- \sqrt{\ \frac{p^3}{27} +
\frac{q^2}{4}} }
```

4362. ⠊⠀⠨⠅⠀⠴⠠⠼⠂⠠⠼⠆

$i = 0, 1, 2$

i = 0, 1, 2

4363. ⠨⠠⠙⠀⠨⠅⠀⠹⠏⠘⠒⠐⠌27⠼⠬⠹⠟⠘⠆⠐⠌⠲⠼

$\Delta = \frac{p^3}{27} + \frac{q^2}{4}$

\Delta = \frac{p^3}{27} + \frac{q^2}{4}

4364. ⠽⠘⠒⠐⠬⠏⠽⠬⠟⠀⠨⠅⠀⠴

$y^3 + py + q = 0$

y^3 + py + q=0

4365. ⠭⠘⠒⠐⠤⠲⠭⠘⠆⠐⠬11⠭⠬30⠀⠨⠅⠀⠴

$x^3 - 4x^2 + 11x + 30 = 0$

x^3 - 4x^2 + 11 x + 30 = 0

4366. ⠭⠘⠒⠐⠤⠒⠭⠬⠢⠀⠨⠅⠀⠴

$x^3 - 3x + 5 = 0$

x^3 - 3x +5 = 0

4367. ⠭⠘⠒⠐⠤⠒⠭⠬⠆⠀⠨⠅⠀⠴

$x^3 - 3x + 2 = 0$

x^3 - 3x +2 = 0

4368. ⠭⠘⠒⠐⠬⠭⠬⠒⠀⠨⠅⠀⠴

$x^3 + x + 3 = 0$

x^3 + x + 3 = 0

4369. ⠭⠘⠲⠐⠬⠁⠭⠘⠒⠐⠬⠃⠭⠘⠆⠐⠬⠉⠭⠬⠙⠀⠨⠅⠀⠴

$x^4 + ax^3 + bx^2 + cx + d = 0$

x^4 + ax^3 + bx^2 + cx + d = 0

4370. ⠽⠘⠲⠐⠬⠏⠽⠘⠆⠐⠬⠟⠽⠬⠗⠀⠨⠅⠀⠴

$y^4 + py^2 + qy + r = 0$

y^4 + py^2 + qy + r = 0

4371. ⠭⠀⠨⠅⠀⠽⠤⠁⠌⠲

$x = y - a/4$

x = y - a/4

4372. ⠷⠽⠘⠆⠐⠬one-half⠵⠾⠘⠆⠐⠀⠨⠅⠀⠷⠵⠤⠏⠾⠽⠘⠆⠐⠤⠟⠽⠬⠷one-fourth⠵⠘⠆⠐⠤⠗⠾

$\left(y^2 + \frac{1}{2} z \right)^2 = (z - p)y^2 - qy + \left(\frac{1}{4} z^2 - r \right)$

\left(y^2 + \frac{1}{2} z \right)^2 = (z - p)y^2 - qy + \left(
\frac{1}{4} z^2 - r \right)

4373. ⠷⠍⠽⠬⠅⠾⠘⠆

$(my + k)^2$

(my + k)^2

4374. ⠟⠘⠆⠀⠐⠤⠲⠷⠵⠐⠤⠏⠾⠷⠷one-fourth⠵⠘⠆⠀⠐⠤⠗⠾⠀⠀⠨⠅⠀⠀⠴

$q^2 - 4(z-p)\left(\frac{1}{4}z^2 - r\right) = 0$

q^2 - 4(z - p)\left(\frac{1}{4} z^2 - r \right) = 0

4375. ⠵⠘⠒⠀⠐⠤⠏⠵⠘⠆⠀⠐⠤⠲⠗⠵⠬⠷⠲⠏⠗⠐⠤⠟⠘⠆⠀⠐⠾⠀⠀⠨⠅⠀⠀⠴

$z^3 - pz^2 - 4rz + (4pr - q^2) = 0$

z^3 - pz^2 - 4rz + (4pr - q^2) = 0

4376. ⠷⠽⠘⠆⠀⠐⠬one-half⠵⠾⠘⠆⠐⠀⠨⠅⠀⠷⠍⠽⠬⠅⠀⠾⠘⠆

$\left(y^2 + \frac{1}{2}z\right)^2 = (my+k)^2$

\left(y^2 + \frac{1}{2} z \right)^2 = (my + k)^2

4377. ⠭⠘⠲⠐⠤⠭⠘⠆⠀⠐⠤⠒⠭⠬⠆⠀⠀⠨⠅⠀⠀⠴

$x^4 - x^2 - 3x + 2 = 0$

x^4 - x^2 - 3x + 2 = 0

4378. ⠭⠘⠲⠐⠬⠭⠘⠒⠀⠐⠤⠶⠭⠘⠆⠀⠐⠤⠭⠬⠖⠀⠀⠨⠅⠀⠀⠴

$x^4 + x^3 - 7x^2 - x + 6 = 0$

x^4 + x^3 - 7 x^2 - x + 6 = 0

4379. ⠭⠘⠲⠐⠤⠆⠭⠘⠆⠀⠐⠬⠲⠭⠤⠒⠀⠀⠨⠅⠀⠀⠴

$x^4 - 2x^2 + 4x - 3 = 0$

x^4 -2 x^2 + 4 x -3 = 0

4380. ⠭⠘⠲⠐⠤⠲⠭⠘⠒⠀⠐⠬⠒⠭⠘⠆⠀⠐⠤⠢⠭⠬⠆⠀⠀⠨⠅⠀⠀⠴

$x^4 - 4x^3 + 3x^2 - 5x + 2 = 0$

x^4 - 4 x^3 + 3x^2 - 5x +2 = 0

4381. ⠁⠘⠆⠐⠬⠲⠁⠬⠆⠀⠀⠨⠅⠀⠀⠴

$a^2 + 4a + 2 = 0$

a^2+4a+2=0

4382. ⠁⠘⠆⠐⠀⠨⠅⠀⠤⠲⠁⠤⠒⠀⠨⠅⠀⠁⠬⠆

$a^2 = -4a - 3 = a + 2$

a^2=-4a-3=a+2

4383. ⠁⠘⠆

a^2

a^2

4384. ⠁⠬⠆

$a + 2$

a+2

4385. ⠶⠘⠢⠐⠀⠨⠅⠀16807

$7^5 = 16\,807$

7^5=16\,807

4386. ⠭⠬⠨⠷⠭⠘⠢⠐⠬⠭⠬⠲⠨⠾⠀⠭⠘⠢⠐⠬⠭⠬⠲⠨⠾

$x + \langle x^5 + x + 4\rangle$

x + \langle x^5+ x + 4\rangle

4387. ⠭⠘⠒⠐⠤⠒⠭⠬⠲

$x^3 - 3x + 4$

x^3-3x+4

4388. ⠏ ⠶ ⠭⠘⠲⠐⠬⠲⠭⠘⠆⠐⠬⠲⠭⠬⠆

$p = x^{4} + 4x^{2} + 4x + 2$

p = x^{4} + 4x^{2} + 4x + 2

4389. ⠼729

729

729

4390. ⠏ ⠶ ⠭⠘⠒⠐⠬⠆⠭⠘⠆⠐⠬⠆⠭⠬⠲

$p = x^3 + 2x^2 + 2x + 4$

p = x^3 + 2x^2 + 2x + 4

4391. ⠟ ⠶ ⠭⠘⠲⠐⠬⠆⠭⠘⠆

$q = x^4 + 2x^2$

q = x^4 + 2x^2

4392. ⠗⠷⠭⠾⠏⠷⠭⠾⠬⠎⠷⠭⠾⠟⠷⠭⠾

$r(x)p(x)+s(x)q(x)$

r(x)p(x)+s(x)q(x)

4393. ⠏⠌⠟ ⠈⠑ ⠈⠠⠟

$p/q \in {\mathbb Q}$

p/q \in {\mathbb Q}

4394. ⠂⠌⠆ ⠶ ⠆⠌⠲ ⠶ ⠒⠌⠖

$1/2 = 2/4 = 3/6$

1/2 = 2/4 = 3/6

4395. ⠹⠁⠌⠃⠼ ⠶ ⠹⠉⠌⠙⠼

$\frac{a}{b} = \frac{c}{d}$

\frac{a}{b} = \frac{c}{d}

4396. ⠁⠙ ⠶ ⠃⠉

$ad = bc$

ad = bc

4397. ⠷⠏⠠⠟⠾

(p, q)

(p, q)

4398. ⠷⠒⠠⠶⠾

$(3, 7)$

(3, 7)

4399. ⠒⠌⠶

$3/7$

3/7

4400. ⠢⠌⠴

$5/0$

5/0

4401. ⠷⠢⠠⠴⠾

$(5, 0)$

(5,0)

4402. ⠷⠒⠠⠖⠾

$(3, 6)$

(3,6)

4403. ⠷⠆⠠⠲⠾

$(2, 4)$

(2,4)

4404. ⠷⠉⠠⠙⠾

(c, d)

(c, d)

4405. ⠠⠎⠀⠨⠅⠀⠨⠷⠷⠁⠠⠃⠾⠱⠁⠠⠃⠈⠑⠠⠙and⠃⠀⠌⠨⠅⠀⠴⠨⠾

$S = \{(a, b) : a, b \in D$ and $b \neq 0\}$

S = \{ (a, b) : a, b \in D \text{ and } b \neq 0 \}

4406. ⠷⠁⠠⠃⠾⠄⡷⠽⠆⠆⠒⠉⠄⠷⠉⠠⠙⠾

$(a, b) \sim (c, d)$

(a,b) \sim (c,d)

4407. ⠁⠙⠀⠨⠅⠀⠃⠉

$ad = bc$

ad=bc

4408. ⠉⠃⠀⠨⠅⠀⠙⠁

$cb = da$

cb = da

4409. ⠷⠉⠠⠙⠾⠄⡷⠽⠆⠆⠒⠉⠄⠷⠁⠠⠃⠾

$(c, d) \sim (a, b)$

(c,d) \sim (a, b)

4410. ⠷⠉⠠⠙⠾⠄⡷⠽⠆⠆⠒⠉⠄⠷⠑⠠⠋⠾

$(c, d) \sim (e, f)$

(c, d) \sim (e,f)

4411. ⠉⠋⠀⠨⠅⠀⠙⠑

$cf = de$

cf = de

4412. ⠁⠋⠙⠀⠨⠅⠀⠁⠙⠋⠀⠨⠅⠀⠃⠉⠋⠀⠨⠅⠀⠃⠙⠑⠀⠨⠅⠀⠃⠑⠙

$afd = adf = bcf = bde = bed$

a f d = a d f = b c f = b d e = bed

4413. ⠁⠋ ⠶ ⠃⠑

$af = be$

af = be

4414. ⠷⠁ ⠠⠃⠾⠄⡷⠭⠆ ⠆ ⠒⠉⠄ ⠷⠑ ⠠⠋⠾

$(a, b) \sim (e, f)$

(a,b) \sim (e, f)

4415. ⠠⠋⠰⠠⠙

F_D

F_D

4416. ⠷⠁ ⠠⠃⠾ ⠈⠑ ⠠⠎

$(a, b) \in S$

(a, b) \in S

4417. ⠈⠷⠁ ⠠⠃ ⠈⠾

$[a, b]$

[a, b]

4418. ⠈⠷⠁ ⠠⠃ ⠈⠾⠬ ⠈⠷⠉ ⠠⠙ ⠈⠾ ⠶ ⠈⠷⠁ ⠙⠬⠃ ⠉ ⠠⠃ ⠙ ⠈⠾

$[a, b] + [c, d] = [ad + bc, bd]$

[a, b] + [c, d] = [ad + b c,b d]

4419. ⠈⠷⠁ ⠠⠃ ⠈⠾⠡ ⠈⠷⠉ ⠠⠙ ⠈⠾ ⠶ ⠈⠷⠁ ⠉ ⠠⠃ ⠙ ⠈⠾

$[a, b] \cdot [c, d] = [ac, bd]$

[a, b] \cdot [c, d] = [ac, b d]

4420. ⠈⠷⠁⠂ ⠠⠃⠂ ⠈⠾ ⠶ ⠈⠷⠁⠆ ⠠⠃⠆ ⠈⠾

$[a_1, b_1] = [a_2, b_2]$

[a_1, b_1] = [a_2, b_2]

4421. ⠈⠷⠉⠂ ⠠⠙⠂ ⠈⠾ ⠶ ⠈⠷⠉⠆ ⠠⠙⠆ ⠈⠾

$[c_1, d_1] = [c_2, d_2]$

[c_1, d_1] =[c_2, d_2]

4422. ⠈⠷⠁⠂⠙⠂⠬⠃⠂⠉⠂ ⠠⠃⠂⠙⠂ ⠈⠾ ⠶ ⠈⠷⠁⠆⠙⠆⠬⠃⠆⠉⠆ ⠠⠃⠆⠙⠆ ⠈⠾

$[a_1 d_1 + b_1 c_1, b_1 d_1] = [a_2 d_2 + b_2 c_2, b_2 d_2]$

[a_1 d_1 + b_1 c_1,b_1 d_1] = [a_2 d_2 + b_2 c_2,b_2 d_2]

4423. ⠷⠁⠂⠙⠂⠬⠃⠂⠉⠂⠾⠷⠃⠆⠙⠆⠾ ⠶ ⠷⠃⠂⠙⠂⠾⠷⠁⠆⠙⠆⠬⠃⠆⠉⠆⠾

$(a_1 d_1 + b_1 c_1)(b_2 d_2) = (b_1 d_1)(a_2 d_2 + b_2 c_2)$

(a_1 d_1 + b_1 c_1)(b_2 d_2) = (b_1 d_1) (a_2 d_2 + b_2 c_2)

4424. ⠁⠂⠃⠆ ⠶ ⠃⠂⠁⠆

$a_1 b_2 = b_1 a_2$

a_1 b_2 = b_1 a_2

4425. ⠉⠂⠙⠆ ⠶ ⠙⠂⠉⠆

$c_1 d_2 = d_1 c_2$

c_1 d_2 = d_1 c_2

4426. ⠈⠷⠴⠠⠂⠈⠾

$[0, 1]$

[0,1]

4427. ⠈⠷⠂⠠⠂⠈⠾

$[1, 1]$

[1,1]

4428. ⠈⠷⠁⠠⠃⠈⠾⠬⠈⠷⠴⠠⠂⠈⠾ ⠨⠅ ⠈⠷⠁ Baseline⠂ ⠬⠃ Baseline⠴ ⠠⠃ Baseline⠂ ⠈⠾ ⠨⠅ ⠈⠷⠁⠠⠃⠈⠾

$[a, b] + [0, 1] = [a1 + b0, b1] = [a, b]$

[a, b] + [0, 1] = [a 1 + b 0, b 1] = [a,b]

4429. ⠈⠷⠂⠠⠂⠈⠾

$[1, 1]$

[1, 1]

4430. ⠈⠷⠁⠠⠃⠈⠾⠈⠑⠠⠋⠰⠠⠙

$[a, b] \in F_D$

[a, b] \in F_D

4431. ⠈⠷⠃⠠⠁⠈⠾

$[b, a]$

[b, a]

4432. ⠈⠷⠁⠠⠃⠈⠾⠡⠈⠷⠃⠠⠁⠈⠾ ⠨⠅ ⠈⠷⠂⠠⠂⠈⠾

$[a, b] \cdot [b, a] = [1, 1]$

[a,b] \cdot [b, a] = [1,1]

4433. ⠈⠷⠤⠁⠠⠃⠈⠾

$[-a, b]$

[-a,b]

4434. ⠨⠽⠒⠠⠋⠰⠠⠙ ⠫⠕ ⠠⠑

$\psi : F_D \rightarrow E$

\psi : F_D \rightarrow E

4435. ⠨⠽⠷⠁⠾ ⠨⠅ ⠁

$\psi(a) = a$

\psi(a) = a

4436. ⠨⠋⠒⠠⠙ ⠫⠕ ⠠⠋⠰⠠⠙

$\phi : D \rightarrow F_D$

\phi : D \rightarrow F_D

4437. ⠨⠋⠷⠁⠾ ⠨⠅ ⠈⠷⠁⠠⠂⠈⠾

$\phi(a) = [a, 1]$

\phi(a) = [a, 1]

4438. ⠨⠋⠷⠁⠬⠃⠾ ⠨⠅ ⠈⠷⠁⠬⠃⠠⠂⠈⠾ ⠨⠅ ⠈⠷⠁⠠⠂⠈⠾⠬⠈⠷⠃⠠⠂⠈⠾ ⠨⠅ ⠨⠋⠷⠁⠾⠬⠨⠋⠷⠃⠾

$\phi(a + b) = [a + b, 1] = [a, 1] + [b, 1] = \phi(a) + \phi(b)$

\phi(a + b) = [a+b, 1] = [a, 1] + [b, 1] = \phi(a) + \phi(b)

4439.

$\phi(ab) = [ab, 1] = [a, 1][b, 1] = \phi(a)\phi(b);$

\phi(a b) = [a b, 1] = [a, 1] [b, 1] = \phi(a) \phi(b);

4440.

$\phi(a) = \phi(b)$

\phi(a) = \phi(b)

4441.

$[a, 1] = [b, 1]$

[a, 1] = [b, 1]

4442. Baseline

$a = a1 = 1b = b$

a = a1 = 1b = b

4443.

$\phi(a)[\phi(b)]^{-1} = [a, 1][b, 1]^{-1} = [a, 1] \cdot [1, b] = [a, b]$

\phi(a) [\phi(b)]^{-1} = [a, 1] [b, 1]^{-1} = [a, 1] \cdot [1, b]
= [a, b]

4444.

$\psi : F_D \rightarrow E$

\psi :F_D \rightarrow E

4445.

$\psi([a, b]) = ab^{-1}$

\psi([a, b]) = a b^{-1}

4446.

$a_1 b_1^{-1} = a_2 b_2^{-1}$

a_1 b_1^{-1} = a_2 b_2^{-1}

4447.

$\psi([a_1, b_1]) = \psi([a_2, b_2])$

\psi([a_1, b_1]) = \psi([a_2, b_2])

4448.

$[a, b]$

[a, b]

4449.

$[c, d]$

[c, d]

4450.

$\psi([a, b]) = ab^{-1} = 0$

\psi([a, b]) = ab^{-1} = 0

4451. ⠁ ⠨⠅ ⠴⠃ ⠨⠅ ⠴

$a = 0b = 0$

a = 0b = 0

4452. ⠈⠷⠁⠠⠃⠈⠾ ⠨⠅ ⠈⠷⠴⠠⠃⠈⠾

$[a, b] = [0, b]$

[a, b] = [0, b]

4453. ⠈⠷⠴⠠⠃⠈⠾

$[0, b]$

[0, b]

4454. ⠏⠷⠭⠾⠌⠟⠷⠭⠾

$p(x)/q(x)$

p(x)/q(x)

4455. ⠈⠠⠟⠷⠭⠾

$\mathbb{Q}(x)$

{\mathbb Q}(x)

4456. ⠈⠠⠟

$\mathbb{Q}$

\mathbb Q

4457. ⠉⠈⠑⠠⠗

$c \in R$

c \in R

4458. ⠃ ⠨⠅ ⠁⠉

$b = ac$

b = ac

4459. ⠁ ⠨⠅ ⠥⠃

$a = ub$

a = ub

4460. ⠏⠈⠑⠠⠙

$p \in D$

p \in D

4461. ⠏ ⠨⠅ ⠁⠃

$p = ab$

p = ab

4462. ⠈⠠⠟⠈⠷⠭⠠⠽⠈⠾

$\mathbb{Q}[x, y]$

{\mathbb Q}[x, y]

4463. ⠽⠘⠆

y^2

y^2

4464. ⠭⠘⠆ ⠐⠽⠘⠆

x^2y^2

x^2 y^2

4465. ⠏⠂⠄⠄⠄⠏⠰⠅

$p_1 \cdots p_k$

p_1 \cdots p_k

4466. ⠁ ⠨⠅ ⠏⠂⠄⠄⠄⠏⠰⠗⠐ ⠨⠅ ⠟⠂⠄⠄⠄⠟⠰⠎

$a = p_1 \cdots p_r = q_1 \cdots q_s$

a = p_1 \cdots p_r = q_1 \cdots q_s

4467. ⠗ ⠨⠅ ⠎

$r = s$

r=s

4468. ⠨⠏ ⠈⠑ ⠠⠎⠰⠗

$\pi \in S_r$

\pi \in S_r

4469. ⠟⠰⠨⠏⠷⠚⠾

$q_{\pi(j)}$

q_{\pi(j)}

4470. ⠚ ⠨⠅ ⠂ ⠤ ⠠⠗

$j = 1, \ldots, r$

j = 1, \ldots, r

4471. ⠈⠠⠵⠈⠷⠜⠒⠻⠊⠈⠾ ⠨⠅ ⠸⠷⠁⠬⠃⠜⠒⠻⠊⠸⠾

$\mathbb{Z}[\sqrt{3}\, i] = \{a + b\sqrt{3}\, i\}$

{\mathbb Z}[\sqrt{3}\, i] = \{ a + b \sqrt{3}\, i\}

4472. ⠵ ⠨⠅ ⠁⠬⠃⠜⠒⠻⠊

$z = a + b\sqrt{3}\, i$

z = a + b \sqrt{3}\, i

4473. ⠨⠝⠐⠂ ⠈⠠⠵⠈⠷⠜⠒⠻⠊⠈⠾ ⠫⠕ ⠈⠠⠝ ⠨⠬StartSet⠴EndSet

$\nu : \mathbb{Z}[\sqrt{3}\, i] \rightarrow \mathbb{N} \cup \{0\}$

\nu : {\mathbb Z}[\sqrt{3}\, i] \rightarrow {\mathbb N} \cup \{
0 \}

4474. ⠨⠝⠷⠵⠾ ⠨⠅ ⠳⠵⠳⠘⠆⠐ ⠨⠅ ⠁⠘⠆ ⠐⠬⠒⠃⠘⠆

$\nu(z) = |z|^2 = a^2 + 3b^2$

\nu(z) = |z|^2 = a^2 + 3 b^2

4475. ⠨⠝⠷⠵⠾ ⠨⠂⠱ ⠴

$\nu(z) \geq 0$

\nu(z) \geq 0

4476. ⠵ ⠨⠅ ⠴

$z = 0$

`z = 0`

4477. ⠨⠝⠷⠵⠺⠾ ⠨⠅ ⠨⠝⠷⠵⠾⠨⠝⠷⠺⠾

$\nu(zw) = \nu(z)\nu(w)$

`\nu(z w) = \nu(z) \nu(w)`

4478. ⠨⠝⠷⠵⠾ ⠨⠅ ⠂

$\nu(z) = 1$

`\nu(z) = 1`

4479. ⠈⠠⠵⠈⠷⠜⠒⠻⠊⠈⠾

$\mathbb{Z}[\sqrt{3}\, i]$

`{\mathbb Z}[ \sqrt{3}\, i ]`

4480. ⠲ ⠨⠅ ⠆⠡⠆ ⠨⠅ ⠷⠂⠤⠜⠒⠻⠊⠾⠷⠂⠬⠜⠒⠻⠊⠾

$4 = 2 \cdot 2 = (1 - \sqrt{3}\, i)(1 + \sqrt{3}\, i)$

`4 = 2 \cdot 2 = (1 - \sqrt{3}\, i) (1 + \sqrt{3}\, i)`

4481. ⠆ ⠨⠅ ⠵⠺

$2 = zw$

`2 = z w`

4482. ⠵⠠⠺

z, w

`z, w`

4483. ⠨⠝⠷⠵⠾ ⠨⠅ ⠨⠝⠷⠺⠾ ⠨⠅ ⠆

$\nu(z) = \nu(w) = 2$

`\nu( z) = \nu(w) = 2`

4484. ⠈⠠⠵⠈⠷⠜⠒⠻⠊⠈⠾

$\mathbb{Z}[\sqrt{3}\, i]$

`{\mathbb Z}[\sqrt{3}\, i ]`

4485. ⠨⠝⠷⠵⠾ ⠨⠅ ⠆

$\nu(z) = 2$

`\nu(z) = 2`

4486. ⠁⠘⠆⠐⠬⠒⠃⠘⠆⠐ ⠨⠅ ⠆

$a^2 + 3b^2 = 2$

`a^2 + 3 b^2 = 2`

4487. ⠂⠤⠜⠒⠻⠊

$1 - \sqrt{3}\, i$

`1 - \sqrt{3}\, i`

4488. ⠂⠬⠜⠒⠻⠊

$1 + \sqrt{3}\, i$

`1 + \sqrt{3}\, i`

4489. ⠄⡷⠭⠆⠶⠑⠦⠄⠁⠄⡷⠭⠆⠶⠑⠔⠄ ⠨⠅ StartSet⠗⠁⠱⠗⠈⠑⠠⠗EndSet

$\langle a \rangle = \{ ra : r \in R \}$

\langle a \rangle = \{ ra : r \in R \}

4490. ⠁⠠⠃⠈⠑⠠⠙

$a, b \in D$

a, b \in D

4491. ⠄⡷⠭⠆⠶⠑⠦⠄⠃⠄⡷⠭⠆⠶⠑⠔⠄⠸⠐⠅⠄⡷⠭⠆⠶⠑⠦⠄⠁⠄⡷⠭⠆⠶⠑⠔⠄

$\langle b \rangle \subset \langle a \rangle$

\langle b \rangle \subset \langle a \rangle

4492. ⠄⡷⠭⠆⠶⠑⠦⠄⠃⠄⡷⠭⠆⠶⠑⠔⠄ ⠨⠅ ⠄⡷⠭⠆⠶⠑⠦⠄⠁⠄⡷⠭⠆⠶⠑⠔⠄

$\langle b \rangle = \langle a \rangle$

\langle b \rangle = \langle a \rangle

4493. ⠄⡷⠭⠆⠶⠑⠦⠄⠁⠄⡷⠭⠆⠶⠑⠔⠄ ⠨⠅ ⠠⠙

$\langle a \rangle = D$

\langle a \rangle = D

4494. ⠃ ⠨⠅ ⠁⠭

$b = ax$

b = ax

4495. ⠭⠈⠑⠠⠙

$x \in D$

x \in D

4496. ⠃⠗ ⠨⠅ ⠷⠁⠭⠾⠗ ⠨⠅ ⠁⠷⠭⠗⠾

$br = (ax)r = a(xr)$

br =(ax)r = a(xr)

4497. ⠃⠈⠑⠄⡷⠭⠆⠶⠑⠦⠄⠁⠄⡷⠭⠆⠶⠑⠔⠄

$b \in \langle a \rangle$

b \in \langle a \rangle

4498. ⠃ ⠨⠅ ⠁⠭

$b = ax$

b =a x

4499. ⠁ ⠨⠅ ⠥⠃

$a = ub$

a = u b

4500. ⠃⠳⠁

$b \mid a$

b \mid a

4501. ⠄⡷⠭⠆⠶⠑⠦⠄⠁⠄⡷⠭⠆⠶⠑⠔⠄⠸⠐⠅⠄⡷⠭⠆⠶⠑⠦⠄⠃⠄⡷⠭⠆⠶⠑⠔⠄

$\langle a \rangle \subset \langle b \rangle$

\langle a \rangle \subset \langle b \rangle

4502. ⠄⡳⠭⠆⠶⠑⠦⠄⠁⠄⡳⠭⠆⠶⠑⠔⠄ ⠭ ⠄⡳⠭⠆⠶⠑⠦⠄⠃⠄⡳⠭⠆⠶⠑⠔⠄

$\langle a \rangle = \langle b \rangle$

\langle a \rangle = \langle b \rangle

4503. ⠁ ⠭ ⠃⠭

$a = bx$

a = bx

4504. ⠃ ⠭ ⠁⠽

$b = ay$

b = ay

4505. ⠭⠠⠽⠈⠑⠠⠙

$x, y \in D$

x, y \in D

4506. ⠁ ⠭ ⠃⠭ ⠭ ⠁⠽⠭

$a = bx = ayx$

a = bx = ayx

4507. ⠭⠽ ⠭ ⠂

$xy = 1$

x y = 1

4508. ⠄⡳⠭⠆⠶⠑⠦⠄⠁⠄⡳⠭⠆⠶⠑⠔⠄ ⠭ ⠄⡳⠭⠆⠶⠑⠦⠄⠂⠄⡳⠭⠆⠶⠑⠔⠄ ⠭ ⠠⠙

$\langle a \rangle = \langle 1 \rangle = D$

\langle a \rangle = \langle 1 \rangle = D

4509. ⠄⡳⠭⠆⠶⠑⠦⠄⠏⠄⡳⠭⠆⠶⠑⠔⠄

$\langle p \rangle$

\langle p \rangle

4510. ⠄⡳⠭⠆⠶⠑⠦⠄⠏⠄⡳⠭⠆⠶⠑⠔⠄ ⠇⠪ ⠄⡳⠭⠆⠶⠑⠦⠄⠁⠄⡳⠭⠆⠶⠑⠔⠄

$\langle p \rangle \subset \langle a \rangle$

\langle p \rangle \subset \langle a \rangle

4511. ⠠⠙ ⠭ ⠄⡳⠭⠆⠶⠑⠦⠄⠁⠄⡳⠭⠆⠶⠑⠔⠄

$D = \langle a \rangle$

D = \langle a \rangle

4512. ⠄⡳⠭⠆⠶⠑⠦⠄⠏⠄⡳⠭⠆⠶⠑⠔⠄ ⠭ ⠄⡳⠭⠆⠶⠑⠦⠄⠁⠄⡳⠭⠆⠶⠑⠔⠄

$\langle p \rangle = \langle a \rangle$

\langle p \rangle = \langle a \rangle

4513. ⠄⡳⠭⠆⠶⠑⠦⠄⠏⠄⡳⠭⠆⠶⠑⠔⠄ ⠇⠪ ⠄⡳⠭⠆⠶⠑⠦⠄⠁⠄⡳⠭⠆⠶⠑⠔⠄ ⠇⠪ ⠠⠙

$\langle p \rangle \subset \langle a \rangle \subset D$

\langle p \rangle \subset \langle a \rangle \subset D

4514. ⠁⠳⠏

$a \mid p$

a \mid p

4515. ⠠⡷⠭⠆ ⠿⠑⠤⠄ ⠈⠃⠄ ⡷⠭⠆ ⠿⠑⠢⠄ ⠇⠐⠅ ⡷⠭⠆ ⠿⠑⠤⠄ ⠏⠄ ⡷⠭⠆ ⠿⠑⠢⠄

$\langle ab \rangle \subset \langle p \rangle$

\langle ab \rangle \subset \langle p \rangle

4516. ⠁ ⠈⠑⠄ ⡷⠭⠆ ⠶⠑⠤⠄ ⠏⠄ ⡷⠭⠆ ⠶⠑⠢⠄

$a \in \langle p \rangle$

a \in \langle p \rangle

4517. ⠃ ⠈⠑⠄ ⡷⠭⠆ ⠶⠑⠤⠄ ⠏⠄ ⡷⠭⠆ ⠶⠑⠢⠄

$b \in \langle p \rangle$

b \in \langle p \rangle

4518. ⠠⠊⠂ ⠄⠠⠊⠆ ⠄⠄

$I_1, I_2, \ldots$

I_1, I_2, \ldots

4519. ⠠⠊⠂ ⠇⠐⠅ ⠠⠊⠆ ⠇⠐⠅ ⠄⠄⠄

$I_1 \subset I_2 \subset \cdots$

I_1 \subset I_2 \subset \cdots

4520. ⠠⠊⠰⠝⠂ ⠒ ⠠⠊⠰⠠⠝

$I_n = I_N$

I_n = I_N

4521. ⠝ ⠨⠂⠱ ⠠⠝

$n \geq N$

n \geq N

4522. ⠠⠊ ⠒ ⠐⠄⡷⠭⠆⠘⠉⠤⠄⠙⠊ ⠒ ⠂⠑⠄⠿⠻⠠⠊⠰⠊

$I = \bigcup_{i=1}^{\infty} I_i$

I= \bigcup_{i = 1}^\infty I_i

4523. ⠠⠊⠂ ⠇⠐⠅ ⠠⠊

$I_1 \subset I$

I_1 \subset I

4524. ⠴ ⠈⠑⠄⠠⠊

$0 \in I$

0 \in I

4525. ⠁ ⠠⠃ ⠈⠑⠄⠠⠊

$a, b \in I$

a, b \in I

4526. ⠁ ⠈⠑⠄⠠⠊⠰⠊

$a \in I_i$

a \in I_i

4527. ⠃ ⠈⠑⠄⠠⠊⠰⠚

$b \in I_j$

b \in I_j

4528. ⠊ ⠅⠱ ⠚

$i \leq j$

i \leq j

4529. ⠠⠊⠰⠚

I_j

I_j

4530. ⠗ ⠈⠑ ⠠⠙

$r \in D$

r \in D

4531. ⠁ ⠈⠑ ⠠⠊

$a \in I$

a \in I

4532. ⠠⠊⠰⠊

I_i

I_i

4533. ⠗⠁ ⠈⠑ ⠠⠊⠰⠊

$ra \in I_i$

ra \in I_i

4534. ⠁ overbar ⠈⠑ ⠠⠙

$\overline{a} \in D$

\overline{a} \in D

4535. ⠁ overbar

$\overline{a}$

\overline{a}

4536. ⠠⠊⠰⠠⠝

I_N

I_N

4537. ⠠⠝ ⠈⠑ ⠈⠠⠝

$N \in {\mathbb N}$

N \in {\mathbb N}

4538. ⠠⠊⠰⠠⠝⠐ ⠶ ⠠⠊ ⠶ ⠄⡷⠭⠆⠶⠑⠦⠄⠁ overbar⠄⡷⠭⠆⠶⠑⠔⠄

$I_N = I = \langle \overline{a} \rangle$

I_N = I = \langle \overline{a} \rangle

4539. ⠁ ⠶ ⠁⠂⠃⠂

$a = a_1 b_1$

a = a_1 b_1

4540. ⠃⠂

b_1

b_1

4541. ⠄⡳⠭⠆ ⠶⠑⠦⠄⠁⠄⡳⠭⠆ ⠶⠑⠄⠄ ⠇⠐⠅⠄⡳⠭⠆ ⠶⠑⠦⠄⠁⠂⠄⡳⠭⠆ ⠶⠑⠄⠄

$\langle a \rangle \subset \langle a_1 \rangle$

\langle a \rangle \subset \langle a_1 \rangle

4542. ⠄⡳⠭⠆ ⠶⠑⠦⠄⠁⠄⡳⠭⠆ ⠶⠑⠄⠄ ⠌⠶ ⠄⡳⠭⠆ ⠶⠑⠦⠄⠁⠂⠄⡳⠭⠆ ⠶⠑⠄⠄

$\langle a \rangle \neq \langle a_1 \rangle$

\langle a \rangle \neq \langle a_1 \rangle

4543. ⠁⠂ ⠶ ⠁⠆⠃⠆

$a_1 = a_2 b_2$

a_1 = a_2 b_2

4544. ⠁⠆

a_2

a_2

4545. ⠃⠆

b_2

b_2

4546. ⠄⡳⠭⠆ ⠶⠑⠦⠄⠁⠂⠄⡳⠭⠆ ⠶⠑⠄⠄ ⠇⠐⠅⠄⡳⠭⠆ ⠶⠑⠦⠄⠁⠆⠄⡳⠭⠆ ⠶⠑⠄⠄

$\langle a_1 \rangle \subset \langle a_2 \rangle$

\langle a_1 \rangle \subset \langle a_2 \rangle

4547. ⠄⡳⠭⠆ ⠶⠑⠦⠄⠁⠄⡳⠭⠆ ⠶⠑⠄⠄ ⠇⠐⠅⠄⡳⠭⠆ ⠶⠑⠦⠄⠁⠂⠄⡳⠭⠆ ⠶⠑⠄⠄ ⠇⠐⠅⠄⡳⠭⠆ ⠶⠑⠦⠄⠁⠆⠄⡳⠭⠆ ⠶⠑⠄⠄ ⠇⠐⠅

$\langle a \rangle \subset \langle a_1 \rangle \subset \langle a_2 \rangle \subset \cdots$

\langle a \rangle \subset \langle a_1 \rangle \subset \langle a_2
\rangle \subset \cdots

4548. ⠄⡳⠭⠆ ⠶⠑⠦⠄⠁ ⠆⠝⠂⠄⡳⠭⠆ ⠶⠑⠄⠄ ⠶ ⠄⡳⠭⠆ ⠶⠑⠦⠄⠁ ⠆⠠⠝⠂⠄⡳⠭⠆ ⠶⠑⠄⠄

$\langle a_n \rangle = \langle a_N \rangle$

\langle a_n \rangle = \langle a_N \rangle

4549. ⠁ ⠆⠠⠝

a_N

a_N

4550. ⠁ ⠶ ⠉⠂⠏⠂

$a = c_1 p_1$

a = c_1 p_1

4551. ⠄⡳⠭⠆ ⠶⠑⠦⠄⠁⠄⡳⠭⠆ ⠶⠑⠄⠄ ⠇⠐⠅⠄⡳⠭⠆ ⠶⠑⠦⠄⠉⠂⠄⡳⠭⠆ ⠶⠑⠄⠄

$\langle a \rangle \subset \langle c_1 \rangle$

\langle a \rangle \subset \langle c_1 \rangle

4552. ⠉⠂ ⠶ ⠉⠆⠏⠆

$c_1 = c_2 p_2$

c_1 = c_2 p_2

4553. ⠄⡷⠭⠆ ⠶⠑⠤⠄ ⠁ ⠄⡷⠭⠆ ⠶⠑⠔⠄ ⠸⠐⠅ ⠄⡷⠭⠆ ⠶⠑⠤⠄ ⠉⠂ ⠄⡷⠭⠆ ⠶⠑⠔⠄ ⠸⠐⠅ ⠄⡷⠭⠆ ⠶⠑⠤⠄ ⠉⠆ ⠄⡷⠭⠆ ⠶⠑⠔⠄ ⠸⠐⠅

$\langle a \rangle \subset \langle c_1 \rangle \subset \langle c_2 \rangle \subset \cdots$

\langle a \rangle \subset \langle c_1 \rangle \subset \langle c_2
\rangle \subset \cdots

4554. ⠁ ⠨⠅ ⠏⠂⠏⠆⠄⠄⠄⠏⠰⠗

$a = p_1 p_2 \cdots p_r$

a = p_1 p_2 \cdots p_r

4555. ⠏⠂⠠ ⠄⠄⠄⠠ ⠏⠰⠗⠐

$p_1, \ldots, p_r$

p_1, \ldots, p_r

4556. ⠁ ⠨⠅ ⠏⠂⠏⠆⠄⠄⠄⠏⠰⠗⠐ ⠨⠅ ⠟⠂⠟⠆⠄⠄⠄⠟⠰⠎

$a = p_1 p_2 \cdots p_r = q_1 q_2 \cdots q_s$

a = p_1 p_2 \cdots p_r = q_1 q_2 \cdots q_s

4557. ⠗ ⠐⠅ ⠎

$r < s$

r \lt s

4558. ⠟⠂⠟⠆⠄⠄⠄⠟⠰⠎

$q_1 q_2 \cdots q_s$

q_1 q_2 \cdots q_s

4559. ⠏⠂⠳⠟⠂

$p_1 \mid q_1$

p_1 \mid q_1

4560. ⠟⠂ ⠨⠅ ⠥⠂⠏⠂

$q_1 = u_1 p_1$

q_1 = u_1 p_1

4561. ⠥⠂

u_1

u_1

4562. ⠁ ⠨⠅ ⠏⠂⠏⠆⠄⠄⠄⠏⠰⠗⠐ ⠨⠅ ⠥⠂⠏⠂⠟⠆⠄⠄⠄⠟⠰⠎

$a = p_1 p_2 \cdots p_r = u_1 p_1 q_2 \cdots q_s$

a = p_1 p_2 \cdots p_r = u_1 p_1 q_2 \cdots q_s

4563. ⠏⠆⠄⠄⠄⠏⠰⠗⠐ ⠨⠅ ⠥⠂⠟⠆⠄⠄⠄⠟⠰⠎

$p_2 \cdots p_r = u_1 q_2 \cdots q_s$

p_2 \cdots p_r = u_1 q_2 \cdots q_s

4564. ⠏⠆ ⠨⠅ ⠟⠆⠠ ⠏⠒ ⠨⠅ ⠟⠒⠠ ⠄⠄⠄⠠ ⠏⠰⠗⠐ ⠨⠅ ⠟⠰⠗⠐

$p_2 = q_2, p_3 = q_3, \ldots, p_r = q_r$

p_2 = q_2, p_3 = q_3, \ldots, p_r = q_r

4565. ⠥⠂⠥⠆⠄⠄⠄⠥⠰⠗⠐⠟⠰⠗⠬⠂⠐⠄⠄⠄⠟⠰⠎⠐ ⠀⠨⠅⠀ ⠂

$u_1 u_2 \cdots u_r q_{r+1} \cdots q_s = 1$

```
u_1 u_2 \cdots u_r q_{r + 1} \cdots q_s = 1
```

4566. ⠟⠰⠗⠬⠂⠐⠄⠄⠄⠟⠰⠎

$q_{r+1} \cdots q_s$

```
q_{r + 1} \cdots q_s
```

4567. ⠟⠰⠗⠬⠂⠠⠀⠄⠄⠄⠠⠀⠟⠰⠎⠐

$q_{r+1}, \ldots, q_s$

```
q_{r + 1}, \ldots, q_s
```

4568. ⠠⠊⠀⠨⠅⠀⠨⠷⠢⠋⠷⠭⠾⠬⠭⠛⠷⠭⠾⠸⠒⠋⠷⠭⠾⠠⠛⠷⠭⠾⠀⠈⠑⠀⠠⠵⠈⠷⠭⠈⠾⠨⠾

$I = \{5f(x) + xg(x) : f(x), g(x) \in \mathbb{Z}[x]\}$

```
I = \{ 5 f(x) + x g(x) : f(x), g(x) \in {\mathbb Z}[x] \}
```

4569. ⠢⠀⠈⠑⠀⠠⠊

$5 \in I$

```
5 \in I
```

4570. ⠢⠀⠨⠅⠀⠋⠷⠭⠾⠏⠷⠭⠾

$5 = f(x)p(x)$

```
5 = f(x) p(x)
```

4571. ⠏⠷⠭⠾⠀⠨⠅⠀⠏

$p(x) = p$

```
p(x) = p
```

4572. ⠭⠀⠈⠑⠀⠠⠊

$x \in I$

```
x \in I
```

4573. ⠭⠀⠨⠅⠀⠏⠛⠷⠭⠾

$x = pg(x)$

```
x = p g(x)
```

4574. ⠏⠀⠨⠅⠀⠬⠤⠂

$p = \pm 1$

```
p = \pm 1
```

4575. ⠨⠨⠈⠣⠏⠷⠭⠾⠨⠨⠈⠜⠀⠨⠅⠀⠠⠵⠈⠷⠭⠈⠾

$\langle p(x) \rangle = \mathbb{Z}[x]$

```
\langle p(x) \rangle = {\mathbb Z}[x]
```

4576. ⠒⠀⠨⠅⠀⠢⠋⠷⠭⠾⠬⠭⠛⠷⠭⠾

$3 = 5f(x) + xg(x)$

```
3 = 5 f(x) + x g(x)
```

4577. ⠒⠀⠨⠅⠀⠢⠋⠷⠭⠾

$3 = 5f(x)$

```
3 = 5 f(x)
```

4578. ⠨⠝⠷⠁⠾

$\nu(a)$

```
\nu(a)
```

4579. ⠨⠝⠷⠁⠾⠀⠀⠅⠱⠀⠨⠝⠷⠁⠃⠾

$\nu(a) \leq \nu(ab)$

```
\nu(a) \leq \nu(ab)
```

4580. ⠃⠀⠌⠨⠅⠀⠴

$b \neq 0$

```
b \neq 0
```

4581. ⠟⠠⠗⠈⠑⠠⠙

$q, r \in D$

```
q, r \in D
```

4582. ⠨⠝⠷⠗⠾⠀⠐⠅⠀⠨⠝⠷⠃⠾

$\nu(r) < \nu(b)$

```
\nu(r) \lt \nu(b)
```

4583. ⠨⠝

ν

```
\nu
```

4584. ⠈⠠⠵⠈⠷⠊⠈⠾⠀⠨⠅⠀⠨⠷⠁⠬⠃⠊⠱⠁⠠⠃⠈⠑⠈⠠⠵⠨⠾

$\mathbb{Z}[i] = \{a + bi : a, b \in \mathbb{Z}\}$

```
{\mathbb Z}[i] = \{ a + b i : a, b \in {\mathbb Z} \}
```

4585. ⠳⠁⠬⠃⠊⠳⠀⠨⠅⠀⠜⠁⠘⠆⠐⠬⠃⠘⠆⠐⠻

$|a + bi| = \sqrt{a^2 + b^2}$

```
|a + bi| = \sqrt{ a^2 + b^2}
```

4586. ⠜⠁⠘⠆⠐⠬⠃⠘⠆⠐⠻

$\sqrt{a^2 + b^2}$

```
\sqrt{a^2 + b^2}
```

4587. ⠨⠝⠷⠁⠬⠃⠊⠾⠀⠨⠅⠀⠁⠘⠆⠐⠬⠃⠘⠆

$\nu(a + bi) = a^2 + b^2$

```
\nu(a + bi) = a^2 + b^2
```

4588. ⠨⠝⠷⠁⠬⠃⠊⠾⠀⠨⠅⠀⠁⠘⠆⠐⠬⠃⠘⠆

$\nu(a + bi) = a^2 + b^2$

```
\nu( a+ bi) = a^2 + b^2
```

4589. ⠈⠠⠵⠈⠷⠊⠈⠾

$\mathbb{Z}[i]$

```
{\mathbb Z}[i]
```

4590. ⠵⠠⠺⠈⠑⠈⠠⠵⠈⠷⠊⠈⠾

$z, w \in \mathbb{Z}[i]$

```
z, w \in {\mathbb Z}[i]
```

4591. ⠨⠝⠷⠵⠺⠾ ⠨⠅ ⠳⠵⠺⠳⠘⠆⠐ ⠨⠅ ⠳⠵⠳⠘⠆⠐⠳⠺⠳⠘⠆⠐ ⠨⠅ ⠨⠝⠷⠵⠾⠨⠝⠷⠺⠾

$\nu(zw) = |zw|^2 = |z|^2|w|^2 = \nu(z)\nu(w)$

\nu(zw) = |zw|^2 = |z|^2 |w|^2 = \nu(z) \nu(w)

4592. ⠨⠝⠷⠵⠾ ⠨⠂⠱ ⠂

$\nu(z) \geq 1$

\nu(z) \geq 1

4593. ⠵ ⠈⠑ ⠈⠠⠵⠈⠷⠊⠈⠾

$z \in \mathbb{Z}[i]$

z \in {\mathbb Z}[i]

4594. ⠨⠝⠷⠵⠾ ⠐⠅⠱ ⠨⠝⠷⠵⠾⠨⠝⠷⠺⠾

$\nu(z) \leq \nu(z)\nu(w)$

\nu(z) \leq \nu(z) \nu(w)

4595. ⠵ ⠨⠅ ⠁⠬⠃⠊

$z = a + bi$

z= a+bi

4596. ⠺ ⠨⠅ ⠉⠬⠙⠊

$w = c + di$

w = c+di

4597. ⠺ ⠌⠨⠅ ⠴

$w \neq 0$

w \neq 0

4598. ⠵ ⠨⠅ ⠟⠺⠬⠗

$z = qw + r$

z = qw + r

4599. ⠨⠝⠷⠗⠾ ⠐⠅ ⠨⠝⠷⠺⠾

$\nu(r) < \nu(w)$

\nu(r) \lt \nu(w)

4600. ⠈⠠⠟⠷⠊⠾ ⠨⠅ ⠨⠷⠏⠬⠟⠊⠒⠏⠠⠟ ⠈⠑ ⠈⠠⠟⠨⠾

$\mathbb{Q}(i) = \{p + qi : p, q \in \mathbb{Q}\}$

{\mathbb Q}(i) = \{ p + qi : p, q \in {\mathbb Q} \}

4601. ⠈⠠⠟⠷⠊⠾

$\mathbb{Q}(i)$

{\mathbb Q}(i)

4602. ⠍⠰⠊

m_i

m_i

4603. ⠳⠝⠰⠊⠸⠌⠷⠁⠘⠆⠐⠬⠃⠘⠆⠐⠾⠳ ⠐⠅⠱ ⠂⠌⠆

$|n_i/(a^2 + b^2)| \leq 1/2$

|n_i / (a^2 + b^2)| \leq 1/2

4604. ⠵⠺⠘⠤⠂⠐⠀⠨⠅⠀⠷⠍⠂⠬⠍⠆⠊⠾⠬⠷⠎⠬⠞⠊⠾

$zw^{-1} = (m_1 + m_2 i) + (s + ti)$

```
z w^{-1} = (m_1 + m_2 i) + (s + ti)
```

4605. ⠎⠘⠆⠐⠬⠞⠘⠆⠐⠀⠅⠱⠀⠂⠌⠲⠬⠂⠌⠲⠀⠨⠅⠀⠂⠌⠆

$s^2 + t^2 \leq 1/4 + 1/4 = 1/2$

```
s^2 + t^2 \leq 1/4 + 1/4 = 1/2
```

4606. ⠵⠀⠨⠅⠀⠵⠺⠘⠤⠂⠐⠺⠀⠨⠅⠀⠺⠷⠍⠂⠬⠍⠆⠊⠾⠬⠺⠷⠎⠬⠞⠊⠾⠀⠨⠅⠀⠟⠺⠬⠗

$z = zw^{-1}w = w(m_1 + m_2 i) + w(s + ti) = qw + r$

```
z = z w^{-1} w = w (m_1 + m_2 i) + w (s + ti) = q w + r
```

4607. ⠟⠀⠨⠅⠀⠍⠂⠬⠍⠆⠊

$q = m_1 + m_2 i$

```
q = m_1 + m_2 i
```

4608. ⠗⠀⠨⠅⠀⠺⠷⠎⠬⠞⠊⠾

$r = w(s + ti)$

```
r = w (s + ti)
```

4609. ⠟⠺

qw

```
qw
```

4610. ⠨⠝⠷⠗⠾⠀⠨⠅⠀⠨⠝⠷⠺⠾⠨⠝⠷⠎⠬⠞⠊⠾⠀⠅⠱⠀one-half⠨⠝⠷⠺⠾⠀⠐⠅⠀⠨⠝⠷⠺⠾

$\nu(r) = \nu(w)\nu(s + ti) \leq \frac{1}{2}\nu(w) < \nu(w)$

```
\nu(r) = \nu(w) \nu(s + ti) \leq \frac{1}{2} \nu(w) \lt \nu(w)
```

4611. ⠨⠝⠷⠃⠾

$\nu(b)$

```
\nu(b)
```

4612. ⠁⠀⠨⠅⠀⠃⠟

$a = bq$

```
a = bq
```

4613. ⠠⠊⠀⠨⠅⠀⠄⡷⠭⠆⠶⠑⠦⠄⠃⠄⡷⠭⠆⠶⠑⠔⠄

$I = \langle b \rangle$

```
I = \langle b \rangle
```

4614. ⠠⠙⠈⠷⠭⠈⠾

$D[x]$

```
D\lbrack x \rbrack
```

4615. ⠏⠷⠭⠾⠀⠨⠅⠀⠁⠰⠝⠐⠭⠘⠝⠐⠬⠄⠄⠄⠬⠁⠂⠭⠬⠁⠴

$p(x) = a_n x^n + \cdots + a_1 x + a_0$

```
p(x) = a_n x^n + \cdots + a_1 x + a_0
```

4616. ⠠⠙⠈⠷⠭⠈⠾

$D[x]$

```
D[x]
```

4617. ⠁⠴⠠⠀⠄⠄⠄⠠⠀⠁⠰⠝⠐

$a_0, \ldots, a_n$

a_0, \ldots, a_n

4618. ⠛⠉⠙⠷⠁⠴⠠⠀⠄⠄⠄⠠⠀⠁⠰⠝⠐⠾⠀⠨⠅⠀⠂

$\gcd(a_0, \ldots, a_n) = 1$

\gcd(a_0, \ldots, a_n) = 1

4619. ⠏⠷⠭⠾⠀⠨⠅⠀⠼⠢⠭⠘⠲⠐⠤⠒⠭⠘⠒⠐⠬⠭⠤⠲

$p(x) = 5x^4 - 3x^3 + x - 4$

p(x)= 5 x^4 - 3 x^3 + x -4

4620. ⠟⠷⠭⠾⠀⠨⠅⠀⠼⠲⠭⠘⠆⠐⠤⠖⠭⠬⠦

$q(x) = 4x^2 - 6x + 8$

q(x) = 4 x^2 - 6 x + 8

4621. ⠋⠷⠭⠾⠛⠷⠭⠾

$f(x)g(x)$

f(x) g(x)

4622. ⠋⠷⠭⠾⠀⠨⠅⠀⠐⠎⠩⠊⠀⠨⠅⠀⠴⠣⠍⠻⠁⠰⠊⠐⠭⠘⠊

$f(x) = \sum_{i=0}^{m} a_i x^i$

f(x) = \sum_{i=0}^{m} a_i x^i

4623. ⠛⠷⠭⠾⠀⠨⠅⠀⠐⠎⠩⠊⠀⠨⠅⠀⠴⠣⠝⠻⠃⠰⠊⠐⠭⠘⠊

$g(x) = \sum_{i=0}^{n} b_i x^i$

g(x) = \sum_{i=0}^{n} b_i x^i

4624. ⠏⠌⠳⠀⠁⠰⠗

$p \nmid a_r$

p \notdivide a_r

4625. ⠏⠌⠳⠀⠃⠰⠎

$p \nmid b_s$

p \notdivide b_s

4626. ⠭⠘⠗⠬⠎

x^{r+s}

x^{r+s}

4627. ⠉⠰⠗⠬⠎⠐⠀⠨⠅⠀⠁⠴⠃⠰⠗⠬⠎⠐⠬⠁⠂⠃⠰⠗⠬⠎⠤⠂⠐⠬⠄⠄⠄⠬⠁⠰⠗⠬⠎⠤⠂⠐⠃⠂⠬⠁⠰⠗⠬⠎⠐⠃⠴

$c_{r+s} = a_0 b_{r+s} + a_1 b_{r+s-1} + \cdots + a_{r+s-1} b_1 + a_{r+s} b_0$

c_{r + s} = a_0 b_{r + s} + a_1 b_{r + s - 1} + \cdots + a_{r +
s - 1} b_1 + a_{r + s} b_0

4628. ⠁⠴⠠⠀⠄⠄⠄⠠⠀⠁⠰⠗⠤⠂⠐

$a_0, \ldots, a_{r-1}$

a_0, \ldots, a_{r-1}

4629. ⠃⠴⠠ ⠄⠄⠄⠠ ⠃⠰⠎⠤⠂ ⠐

$b_0, \ldots, b_{s-1}$

b_0, \ldots, b_{s-1}

4630. ⠉ ⠰⠗⠬⠎

c_{r+s}

c_{r+s}

4631. ⠁ ⠰⠗⠐⠃ ⠰⠎

$a_r b_s$

a_r b_s

4632. ⠏⠳⠉ ⠰⠗⠬⠎

$p \mid c_{r+s}$

p \mid c_{r+s}

4633. ⠁ ⠰⠗

a_r

a_r

4634. ⠃ ⠰⠎

b_s

b_s

4635. ⠏⠷⠭⠾ ⠨⠅ ⠉⠏⠂⠷⠭⠾

$p(x) = cp_1(x)$

p(x) = c p_1(x)

4636. ⠟⠷⠭⠾ ⠨⠅ ⠙⠟⠂⠷⠭⠾

$q(x) = dq_1(x)$

q(x) = d q_1(x)

4637. ⠏⠂⠷⠭⠾

$p_1(x)$

p_1(x)

4638. ⠏⠷⠭⠾⠟⠷⠭⠾ ⠨⠅ ⠉⠙⠏⠂⠷⠭⠾⠟⠂⠷⠭⠾

$p(x)q(x) = cdp_1(x)q_1(x)$

p(x) q(x) = c d p_1(x) q_1(x)

4639. ⠏⠂⠷⠭⠾⠟⠂⠷⠭⠾

$p_1(x)q_1(x)$

p_1(x) q_1(x)

4640. ⠉⠙

cd

cd

4641. ⠏⠷⠭⠾ ⠈⠑ ⠠⠙⠈⠷⠭⠈⠾

$p(x) \in D[x]$

p(x) \in D[x]

4642. ⠏⠷⠭⠾⠀⠭⠀⠋⠂⠷⠭⠾⠛⠂⠷⠭⠾

$p(x) = f_1(x)g_1(x)$

p(x) = f_1(x) g_1(x)

4643. ⠋⠂⠷⠭⠾

$f_1(x)$

f_1(x)

4644. ⠛⠂⠷⠭⠾

$g_1(x)$

g_1(x)

4645. ⠙⠑⠛⠋⠷⠭⠾⠀⠭⠀⠙⠑⠛⠋⠂⠷⠭⠾

$\deg f(x) = \deg f_1(x)$

\deg f(x) = \deg f_1(x)

4646. ⠙⠑⠛⠛⠷⠭⠾⠀⠭⠀⠙⠑⠛⠛⠂⠷⠭⠾

$\deg g(x) = \deg g_1(x)$

\deg g(x) = \deg g_1(x)

4647. ⠁⠋⠷⠭⠾⠠⠃⠛⠷⠭⠾

$af(x), bg(x)$

a f(x), b g(x)

4648. ⠁⠂⠠⠃⠆⠈⠑⠠⠙

$a_1, b_2 \in D$

a_1, b_2 \in D

4649. ⠁⠋⠷⠭⠾⠀⠭⠀⠁⠂⠋⠂⠷⠭⠾

$af(x) = a_1 f_1(x)$

a f(x) = a_1 f_1(x)

4650. ⠃⠛⠷⠭⠾⠀⠭⠀⠃⠂⠛⠂⠷⠭⠾

$bg(x) = b_1 g_1(x)$

b g(x) = b_1 g_1(x)

4651. ⠁⠃⠏⠷⠭⠾⠀⠭⠀⠷⠁⠂⠋⠂⠷⠭⠾⠾⠷⠃⠂⠛⠂⠷⠭⠾⠾

$abp(x) = (a_1 f_1(x))(b_1 g_1(x))$

a b p(x) = (a_1 f_1(x))(b_1 g_1(x))

4652. ⠁⠃⠳⠁⠂⠃⠂

$ab \mid a_1 b_1$

ab \mid a_1 b_1

4653. ⠉⠈⠑⠠⠙

$c \in D$

c \in D

4654. ⠏⠷⠭⠾⠀⠭⠀⠉⠋⠂⠷⠭⠾⠛⠂⠷⠭⠾

$p(x) = c f_1(x) g_1(x)$

p(x) = c f_1(x) g_1(x)

4655. ⠏⠷⠭⠾ ⠨⠅ ⠋⠂⠷⠭⠾⠋⠆⠷⠭⠾⠄⠄⠄⠋⠰⠝⠐⠷⠭⠾

$p(x) = f_1(x) f_2(x) \cdots f_n(x)$

```
p(x) = f_1(x) f_2(x) \cdots f_n(x)
```

4656. ⠋⠰⠊⠐⠷⠭⠾

$f_i(x)$

```
f_i(x)
```

4657. ⠁⠰⠊⠐⠈⠑⠠⠙

$a_i \in D$

```
a_i \in D
```

4658. ⠁⠰⠊⠐⠋⠰⠊⠐⠷⠭⠾

$a_i f_i(x)$

```
a_i f_i(x)
```

4659. ⠃⠂⠠⠄⠠⠃⠰⠝⠐⠈⠑⠠⠙

$b_1, \ldots, b_n \in D$

```
b_1, \ldots, b_n \in D
```

4660. ⠁⠰⠊⠐⠋⠰⠊⠐⠷⠭⠾ ⠨⠅ ⠃⠰⠊⠐⠛⠰⠊⠐⠷⠭⠾

$a_i f_i(x) = b_i g_i(x)$

```
a_i f_i(x) = b_i g_i(x)
```

4661. ⠛⠰⠊⠐⠷⠭⠾

$g_i(x)$

```
g_i(x)
```

4662. ⠁⠂⠄⠄⠄⠁⠰⠝⠐⠏⠷⠭⠾ ⠨⠅ ⠃⠂⠄⠄⠄⠃⠰⠝⠐⠛⠂⠷⠭⠾⠄⠄⠄⠛⠰⠝⠐⠷⠭⠾

$a_1 \cdots a_n p(x) = b_1 \cdots b_n g_1(x) \cdots g_n(x)$

```
a_1 \cdots a_n p(x) = b_1 \cdots b_n g_1(x) \cdots g_n(x)
```

4663. ⠃ ⠨⠅ ⠃⠂⠄⠄⠄⠃⠰⠝

$b = b_1 \cdots b_n$

```
b = b_1 \cdots b_n
```

4664. ⠛⠂⠷⠭⠾⠄⠄⠄⠛⠰⠝⠐⠷⠭⠾

$g_1(x) \cdots g_n(x)$

```
g_1(x) \cdots g_n(x)
```

4665. ⠁⠂⠄⠄⠄⠁⠰⠝

$a_1 \cdots a_n$

```
a_1 \cdots a_n
```

4666. ⠏⠷⠭⠾ ⠨⠅ ⠁⠛⠂⠷⠭⠾⠄⠄⠄⠛⠰⠝⠐⠷⠭⠾

$p(x) = a g_1(x) \cdots g_n(x)$

```
p(x) = a g_1(x) \cdots g_n(x)
```

4667. ⠥⠉⠂⠄⠄⠄⠉⠰⠅

$u c_1 \cdots c_k$

```
u c_1 \cdots c_k
```

4668. ⠏⠷⠭⠾ ⠨⠅ ⠁⠂⠄⠄⠄⠁⠰⠍⠐⠋⠂⠷⠭⠾⠄⠄⠄⠋⠰⠝⠐⠷⠭⠾ ⠨⠅ ⠃⠂⠄⠄⠄⠃⠰⠗⠐⠛⠂⠷⠭⠾⠄⠄⠄⠛⠰⠎⠐⠷⠭⠾

$p(x) = a_1 \cdots a_m f_1(x) \cdots f_n(x) = b_1 \cdots b_r g_1(x) \cdots g_s(x)$

p(x) = a_1 \cdots a_m f_1(x) \cdots f_n(x) = b_1 \cdots b_r g_1(x)
\cdots g_s(x)

4669. ⠋⠰⠊

f_i

f_i

4670. ⠝ ⠨⠅ ⠎

$n = s$

n=s

4671. ⠉⠂ ⠄⠄ ⠠⠉⠰⠝⠐

$c_1, \ldots, c_n$

c_1, \ldots, c_n

4672. ⠙⠂ ⠄⠄ ⠠⠙⠰⠝⠐

$d_1, \ldots, d_n$

d_1, \ldots, d_n

4673. ⠷⠉⠰⠊⠐⠌⠙⠰⠊⠐⠾⠋⠰⠊⠐⠷⠭⠾ ⠨⠅ ⠛⠰⠊⠐⠷⠭⠾

$(c_i/d_i)f_i(x) = g_i(x)$

(c_i / d_i) f_i(x) = g_i(x)

4674. ⠉⠰⠊⠐⠋⠰⠊⠐⠷⠭⠾ ⠨⠅ ⠙⠰⠊⠐⠛⠰⠊⠐⠷⠭⠾

$c_i f_i(x) = d_i g_i(x)$

c_i f_i(x) = d_i g_i(x)

4675. ⠁⠂⠄⠄⠄⠁⠰⠍⠐ ⠨⠅ ⠥⠃⠂⠄⠄⠄⠃⠰⠗

$a_1 \cdots a_m = ub_1 \cdots b_r$

a_1 \cdots a_m = u b_1 \cdots b_r

4676. ⠍ ⠨⠅ ⠎

$m = s$

m = s

4677. ⠠⠙⠈⠷⠭⠂⠠ ⠄⠄ ⠠⠭⠰⠝⠐⠈⠾

$D[x_1, \ldots, x_n]$

D[x_1, \ldots, x_n]

4678. ⠠⠝ ⠨⠅ ⠆⠘⠆⠘⠘⠝⠐⠬⠂

$N = 2^{2^n} + 1$

N= 2^{2^n} + 1

4679. ⠜⠤⠂⠻

$\sqrt{-1}$

\sqrt{-1}

4680. ⠈⠠⠵⠈⠷⠜⠒⠻⠊⠈⠾

$\mathbb{Z}[\sqrt{3}\, i]$

{\mathbb Z}[\sqrt{3}\, i]

4681. ⠁⠘⠆⠐⠬⠒⠃⠘⠆⠐⠀⠨⠅⠀⠂

$a^2 + 3b^2 = 1$

a^2 + 3 b^2 = 1

4682. ⠵⠘⠤⠂⠐⠀⠨⠅⠀⠂⠌⠷⠁⠬⠃⠜⠒⠻⠊⠾⠀⠨⠅⠀⠷⠁⠤⠃⠜⠒⠻⠊⠾⠌⠷⠁⠘⠆⠐⠬⠒⠃⠘⠆⠐⠾

$z^{-1} = 1/(a + b\sqrt{3}\, i) = (a - b\sqrt{3}\, i)/(a^2 + 3b^2)$

z^{-1} = 1/(a + b\sqrt{3}\, i) = (a -b \sqrt{3}\, i)/(a^2 + 3b^2)

4683. ⠈⠠⠵⠈⠷⠜⠒⠻⠊⠈⠾

$\mathbb{Z}[\sqrt{3}\, i]$

{\mathbb Z}[\sqrt{3}\, i]

4684. ⠁⠀⠨⠅⠀⠬⠤⠂⠠⠃⠀⠨⠅⠀⠴

$a = \pm 1, b = 0$

a = \pm 1, b = 0

4685. ⠂⠬⠒⠊

$1 + 3i$

1 + 3i

4686. ⠖⠬⠦⠊

$6 + 8i$

6 + 8i

4687. ⠢⠀⠨⠅⠀⠤⠊⠷⠂⠬⠆⠊⠾⠷⠆⠬⠊⠾

$5 = -i(1 + 2i)(2 + i)$

5 = -i(1 + 2i)(2 + i)

4688. ⠖⠬⠦⠊⠀⠨⠅⠀⠤⠊⠷⠂⠬⠊⠾⠘⠆⠐⠷⠆⠬⠊⠾⠘⠆

$6 + 8i = -i(1 + i)^2(2 + i)^2$

6 + 8i = -i(1 + i)^2(2 + i)^2

4689. ⠠⠋⠷⠭⠾

$F(x)$

F(x)

4690. ⠏⠷⠭⠾⠌⠟⠷⠭⠾

$p(x)/q(x)$

p(x) / q(x)

4691. ⠏⠷⠭⠂⠀⠤⠀⠠⠭⠰⠝⠐⠾

$p(x_1, \ldots, x_n)$

p(x_1, \ldots, x_n)

4692. ⠟⠷⠭⠂⠀⠤⠀⠠⠭⠰⠝⠐⠾

$q(x_1, \ldots, x_n)$

q(x_1, \ldots, x_n)

4693. ⠏⠷⠭⠂⠠⠀⠄⠄⠄⠠⠀⠭⠰⠝⠐⠾⠸⠌⠟⠷⠭⠂⠠⠀⠄⠄⠄⠠⠀⠭⠰⠝⠐⠾

$p(x_1, \ldots, x_n)/q(x_1, \ldots, x_n)$

p(x_1, \ldots, x_n) / q(x_1, \ldots, x_n)

4694. ⠠⠋⠷⠭⠂⠠⠀⠄⠄⠄⠠⠀⠭⠰⠝⠐⠾

$F(x_1, \ldots, x_n)$

F(x_1, \ldots, x_n)

4695. ⠭⠂⠠⠀⠄⠄⠄⠠⠀⠭⠰⠝

$x_1, \ldots, x_n$

x_1, \ldots, x_n

4696. ⠈⠠⠵⠰⠏⠐⠷⠭⠾

$\mathbb{Z}_p(x)$

{\mathbb Z}_p(x)

4697. ⠈⠠⠟⠷⠊⠾⠀⠨⠅⠀⠨⠷⠏⠬⠟⠊⠸⠒⠀⠏⠠⠀⠟⠀⠈⠑⠀⠈⠠⠟⠨⠾

$\mathbb{Q}(i) = \{p + qi : p, q \in \mathbb{Q}\}$

{\mathbb Q}(i) = \{ p + q i : p, q \in {\mathbb Q} \}

4698. ⠺⠀⠨⠅⠀⠉⠬⠙⠊⠀⠌⠨⠅⠀⠴

$w = c + di \neq 0$

w = c + di \neq 0

4699. ⠵⠸⠌⠺⠀⠈⠑⠀⠈⠠⠟⠷⠊⠾

$z/w \in \mathbb{Q}(i)$

z/w \in {\mathbb Q}(i)

4700. ⠈⠠⠵⠈⠷⠜⠆⠻⠈⠾⠀⠨⠅⠀⠨⠷⠁⠬⠃⠜⠆⠻⠸⠒⠀⠁⠠⠀⠃⠀⠈⠑⠀⠈⠠⠵⠨⠾

$\mathbb{Z}[\sqrt{2}\,] = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\}$

{\mathbb Z}[\sqrt{2}\,] = \{ a + b \sqrt{2} : a, b \in {\mathbb Z} \}

4701. ⠈⠠⠵⠈⠷⠜⠆⠻⠈⠾

$\mathbb{Z}[\sqrt{2}\,]$

{\mathbb Z}[\sqrt{2}\,]

4702. ⠈⠠⠵⠈⠷⠜⠆⠻⠈⠾

$\mathbb{Z}[\sqrt{2}\,]$

{\mathbb Z}[\sqrt{2}\,]

4703. ⠈⠠⠵⠈⠷⠜⠆⠻⠊⠈⠾

$\mathbb{Z}[\sqrt{2} i]$

{\mathbb Z}[\sqrt{2} i]

4704. ⠨⠝⠷⠁⠬⠃⠜⠆⠻⠊⠾⠀⠨⠅⠀⠁⠘⠆⠐⠬⠆⠃⠘⠆

$\nu(a + b\sqrt{2}\, i) = a^2 + 2b^2$

\nu(a + b \sqrt{2}\, i) = a^2 + 2b^2

4705. ⠙⠈⠑⠠⠙

$d \in D$

d \in D

4706. ⠛⠉⠙⠷⠁⠠⠃⠾

$\gcd(a, b)$

\gcd(a, b)

4707. ⠛⠉⠙⠷⠁⠠⠃⠾ ⠨⠅ ⠁⠎⠬⠃⠞

$\gcd(a, b) = as + bt$

\gcd(a, b) = as + bt

4708. ⠨⠝⠷⠥⠾ ⠨⠅ ⠨⠝⠷⠂⠾

$\nu(u) = \nu(1)$

\nu(u) = \nu(1)

4709. ⠨⠝⠷⠁⠾ ⠨⠅ ⠨⠝⠷⠃⠾

$\nu(a) = \nu(b)$

\nu(a) = \nu(b)

4710. ⠨⠝⠷⠃⠾ ⠐⠅⠱ ⠨⠝⠷⠥⠃⠾ ⠐⠅⠱ ⠨⠝⠷⠁⠾

$\nu(b) \leq \nu(ub) \leq \nu(a)$

\nu(b) \leq \nu(ub) \leq \nu(a)

4711. ⠨⠝⠷⠁⠾ ⠐⠅⠱ ⠨⠝⠷⠃⠾

$\nu(a) \leq \nu(b)$

\nu(a) \leq \nu(b)

4712. ⠈⠠⠵⠈⠷⠜⠢⠻⠊⠈⠾

${\mathbb Z}[\sqrt{5}\, i]$

{\mathbb Z}[\sqrt{5}\, i]

4713. ⠁⠂⠠⠄⠠⠁⠰⠝⠐

$a_1, \ldots, a_n$

a_1, \ldots, a_n

4714. ⠁⠂⠗⠂⠬⠄⠄⠄⠬⠁⠰⠝⠐⠗⠰⠝

$a_1 r_1 + \cdots + a_n r_n$

a_1 r_1 + \cdots + a_n r_n

4715. ⠠⠊⠂⠸⠨⠂⠠⠊⠆⠸⠨⠂⠠⠊⠒⠸⠨⠂⠄⠄⠄

$I_1 \supset I_2 \supset I_3 \supset \cdots$

I_1 \supset I_2 \supset I_3 \supset \cdots

4716. ⠠⠊⠰⠅⠐ ⠨⠅ ⠠⠊⠰⠠⠝

$I_k = I_N$

I_k = I_N

4717. ⠅ ⠨⠂⠱ ⠠⠝

$k \geq N$

k \geq N

4718. ⠁⠃⠈⠑⠠⠎

$ab \in S$

ab \in S

4719. ⠁⠠⠃⠈⠑⠠⠎

$a, b \in S$

a, b \in S

4720. ⠷⠁⠠⠎⠾⠄⡷⠭⠆⠆⠒⠉⠄⠷⠁⠄⠠⠎⠘⠄⠐⠾

$(a, s) \sim (a', s')$

(a, s) \sim (a', s')

4721. ⠎⠘⠈⠼⠐⠈⠑⠠⠎

$s^\ast \in S$

s^\ast \in S

4722. ⠎⠘⠈⠼⠐⠷⠎⠄⠁⠤⠎⠁⠘⠄⠐⠾ ⠨⠅ ⠴

$s^\ast(s' a -s a') = 0$

s^\ast(s' a -s a') = 0

4723. ⠁⠌⠎

a/s

a/s

4724. ⠷⠁⠠⠎⠾⠈⠑⠠⠗⠡⠠⠎

$(a,s) \in R \times S$

(a,s) \in R \times S

4725. ⠠⠎⠘⠤⠂⠐⠠⠗

$S^{-1}R$

S^{-1}R

4726. ⠠⠎⠘⠤⠂⠐⠠⠗

$S^{-1} R$

S^{-1} R

4727. ⠨⠓⠱⠠⠗ ⠫⠕ ⠠⠎⠘⠤⠂⠐⠠⠗

$\psi : R \rightarrow S^{-1}R$

\psi : R \rightarrow S^{-1}R

4728. ⠨⠓⠷⠁⠾ ⠨⠅ ⠁⠌⠂

$\psi(a) = a/1$

\psi(a) = a/1

4729. ⠠⠎ ⠨⠅ ⠠⠗⠄⡷⠭⠆⠆⠂⠖⠄⠠⠏

$S = R \setminus P$

S = R \setminus P

4730. ⠈⠠⠵⠈⠷⠜⠒⠻⠊⠈⠾

${\mathbb Z}[\sqrt{3}i]$

{\mathbb Z}[\sqrt{3}i]

4731. ⠠⠭⠈⠡⠠⠭

$X \times X$

X \times X

4732. ⠷⠁⠠⠁⠾⠈⠑⠠⠏

$(a, a) \in P$

(a, a) \in P

4733. ⠁⠈⠑⠠⠭

$a \in X$

a \in X

4734. ⠷⠁⠠⠃⠾⠈⠑⠠⠏

$(a, b) \in P$

(a,b) \in P

4735. ⠷⠃⠠⠁⠾⠈⠑⠠⠏

$(b, a) \in P$

(b,a) \in P

4736. ⠷⠁⠠⠃⠾⠈⠑⠠⠏

$(a, b) \in P$

(a, b) \in P

4737. ⠷⠃⠠⠉⠾⠈⠑⠠⠏

$(b, c) \in P$

(b, c) \in P

4738. ⠷⠁⠠⠉⠾⠈⠑⠠⠏

$(a, c) \in P$

(a, c) \in P

4739. ⠁⠠⡷⠭⠆⠈⠈⠋⠠⠃

$a \preceq b$

a \preceq b

4740. ⠁   ⠅⠱ ⠃

$a \leq b$

a \leq b

4741. ⠠⡷⠭⠆⠈⠈⠋⠠

$\preceq$

\preceq

4742. ⠠⠭ ⠨⠅ StartSet⠁⠠⠃⠠⠉EndSet

$X = \{a, b, c\}$

X = \{ a, b, c \}

4743. StartSet⠁⠠⠃⠠⠉EndSet

$\{a, b, c\}$

\{ a, b, c \}

4744. ⠇ ⠪

$\subset$

\subset

4745. caligraphic ⠠⠏⠷StartSet⠁ ⠠⠃ ⠠⠉EndSet⠾

$\mathcal P(\{a, b, c\})$

\mathcal P(\{ a, b, c \})

4746. ⠁ ⠳⠁

$a \mid a$

a \mid a

4747. ⠁ ⠈⠑ ⠈ ⠠⠝

$a \in {\mathbb N}$

a \in {\mathbb N}

4748. ⠍⠳⠝

$m \mid n$

m \mid n

4749. ⠝⠳⠍

$n \mid m$

n \mid m

4750. ⠝⠳⠏

$n \mid p$

n \mid p

4751. ⠍⠳⠏

$m \mid p$

m \mid p

4752. ⠠⠭ ⠶ StartSet⠂ ⠠⠆ ⠠⠒ ⠠⠲ ⠠⠖ ⠠⠦ ⠠12 ⠠24EndSet

$X = \{1, 2, 3, 4, 6, 8, 12, 24\}$

X = \{ 1, 2, 3, 4, 6, 8, 12, 24 \}

4753. ⠁ ⠄⡷⠭⠆ ⠁ ⠁ ⠋⠄ ⠥

$a \preceq u$

a \preceq u

4754. ⠁ ⠈⠑ ⠠⠽

$a \in Y$

a \in Y

4755. ⠥⠄ ⡷⠭⠆ ⠁ ⠁ ⠋⠄ ⠧

$u \preceq v$

u \preceq v

4756. v

v

4757. ⠇⠄⡳⠭⠆⠁⠁⠋⠄⠁

$l \preceq a$

l \preceq a

4758. ⠅⠄⡳⠭⠆⠁⠁⠋⠄⠇

$k \preceq l$

k \preceq l

4759. ⠠⠽ ⠭ StartSet: ⠠⠒ ⠠⠲ ⠠⠖EndSet

$Y = \{2, 3, 4, 6\}$

Y = \{ 2, 3, 4, 6 \}

4760. ⠥⠆

u_2

u_2

4761. ⠥⠂⠄⡳⠭⠆⠁⠁⠋⠄⠥

$u_1 \preceq u$

u_1 \preceq u

4762. ⠥⠂⠄⡳⠭⠆⠁⠁⠋⠄⠥⠆

$u_1 \preceq u_2$

u_1 \preceq u_2

4763. ⠥⠆⠄⡳⠭⠆⠁⠁⠋⠄⠥⠂

$u_2 \preceq u_1$

u_2 \preceq u_1

4764. ⠥⠂ ⠭ ⠥⠆

$u_1 = u_2$

u_1 = u_2

4765. ⠁ ⠠⠃ ⠈⠑ ⠠⠇

$a, b \in L$

a, b \in L

4766. ⠁⠄⡳⠭⠆⠆⠆⠦⠄⠃

$a \vee b$

a \vee b

4767. ⠁⠄⡳⠭⠆⠆⠆⠶⠄⠃

$a \wedge b$

a \wedge b

4768. ⠠⠁ ⠇⠐⠅ ⠠⠁ ⠅⠬ ⠠⠃

$A \subset A \cup B$

A \subset A \cup B

4769. ⠠⠃ ⠇⠐⠅ ⠠⠁ ⠅⠬ ⠠⠃

$B \subset A \cup B$

B \subset A \cup B

4770. ⠷⠠⠁⠨⠬⠠⠃⠾⠄

$(A \cup B)'$

(A \cup B)'

4771. ⠠⠁⠄⠨⠩⠠⠃⠄

$A' \cap B'$

A' \cap B'

4772. ⠄⡳⠭⠆⠁⠃⠴⠄

$\succeq$

\succeq

4773. ⠄⡳⠭⠆⠆⠆⠦⠄

$\vee$

\vee

4774. ⠄⡳⠭⠆⠆⠆⠶⠄

$\wedge$

\wedge

4775. ⠁⠠⠃⠠⠉⠈⠑⠠⠇

$a, b, c \in L$

a, b, c \in L

4776. ⠁⠄⡳⠭⠆⠆⠆⠦⠄⠃⠀⠰⠆⠀⠃⠄⡳⠭⠆⠆⠆⠦⠄⠁

$a \vee b = b \vee a$

a \vee b = b \vee a

4777. ⠁⠄⡳⠭⠆⠆⠆⠶⠄⠃⠀⠰⠆⠀⠃⠄⡳⠭⠆⠆⠆⠶⠄⠁

$a \wedge b = b \wedge a$

a \wedge b = b \wedge a

4778. ⠁⠄⡳⠭⠆⠆⠆⠦⠄⠷⠃⠄⡳⠭⠆⠆⠆⠦⠄⠉⠾⠀⠰⠆⠀⠷⠁⠄⡳⠭⠆⠆⠆⠦⠄⠃⠾⠄⡳⠭⠆⠆⠆⠦⠄⠉

$a \vee (b \vee c) = (a \vee b) \vee c$

a \vee (b \vee c) = (a \vee b) \vee c

4779. ⠁⠄⡳⠭⠆⠆⠆⠶⠄⠷⠃⠄⡳⠭⠆⠆⠆⠶⠄⠉⠾⠀⠰⠆⠀⠷⠁⠄⡳⠭⠆⠆⠆⠶⠄⠃⠾⠄⡳⠭⠆⠆⠆⠶⠄⠉

$a \wedge (b \wedge c) = (a \wedge b) \wedge c$

a \wedge (b \wedge c) = (a \wedge b) \wedge c

4780. ⠁⠄⡳⠭⠆⠆⠆⠦⠄⠁⠀⠰⠆⠀⠁

$a \vee a = a$

a \vee a = a

4781. ⠁⠄⡳⠭⠆⠆⠆⠶⠄⠁⠀⠰⠆⠀⠁

$a \wedge a = a$

a \wedge a = a

4782. ⠁⠄⡳⠭⠆⠆⠆⠦⠄⠷⠁⠄⡳⠭⠆⠆⠆⠶⠄⠃⠾⠀⠰⠆⠀⠁

$a \vee (a \wedge b) = a$

a \vee (a \wedge b) = a

4783. ⠁⠄⡳⠭⠆⠆⠆⠶⠄⠷⠁⠄⡳⠭⠆⠆⠆⠦⠄⠃⠾⠀⠨⠅⠀⠁

$a \wedge (a \vee b) = a$

a \wedge (a \vee b) =a

4784. StartSet⠁ ⠠⠃ EndSet

$\{a, b\}$

\{ a, b\}

4785. ⠃⠄⡳⠭⠆⠆⠆⠦⠄⠁

$b \vee a$

b \vee a

4786. StartSet⠃ ⠠⠁ EndSet

$\{b, a\}$

\{ b, a \}

4787. StartSet⠁ ⠠⠃ EndSet ⠨⠅ StartSet⠃ ⠠⠁ EndSet

$\{a, b\} = \{b, a\}$

\{ a, b\} = \{ b, a \}

4788. ⠁⠄⡳⠭⠆⠆⠆⠦⠄⠷⠃⠄⡳⠭⠆⠆⠆⠦⠄⠉⠾

$a \vee (b \vee c)$

a \vee (b \vee c)

4789. ⠷⠁⠄⡳⠭⠆⠆⠆⠦⠄⠃⠾⠄⡳⠭⠆⠆⠆⠦⠄⠉

$(a \vee b) \vee c$

(a \vee b) \vee c

4790. ⠙⠀⠨⠅⠀⠁⠄⡳⠭⠆⠆⠆⠦⠄⠃

$d = a \vee b$

d = a \vee b

4791. ⠉⠄⡳⠭⠆⠁⠁⠋⠄⠙⠄⡳⠭⠆⠆⠆⠦⠄⠉⠀⠨⠅⠀⠷⠁⠄⡳⠭⠆⠆⠆⠦⠄⠃⠾⠄⡳⠭⠆⠆⠆⠦⠄⠉

$c \preceq d \vee c = (a \vee b) \vee c$

c \preceq d \vee c = (a \vee b) \vee c

4792. ⠁⠄⡳⠭⠆⠁⠁⠋⠄⠁⠄⡳⠭⠆⠆⠆⠦⠄⠃⠀⠨⠅⠀⠙⠄⡳⠭⠆⠁⠁⠋⠄⠙⠄⡳⠭⠆⠆⠆⠦⠄⠉⠀⠨⠅⠀⠷⠁⠄⡳⠭⠆⠆⠆⠦⠄⠃⠾⠄⡳⠭⠆

$a \preceq a \vee b = d \preceq d \vee c = (a \vee b) \vee c$

a \preceq a \vee b =d \preceq d \vee c = (a \vee b) \vee c

4793. ⠃⠄⡳⠭⠆⠁⠁⠋⠄⠷⠁⠄⡳⠭⠆⠆⠆⠦⠄⠃⠾⠄⡳⠭⠆⠆⠆⠦⠄⠉

$b \preceq (a \vee b) \vee c$

b \preceq (a \vee b) \vee c

4794. StartSet⠁ ⠠⠃ ⠠⠉EndSet

$\{a, b, c\}$

\{ a, b, c\}

4795. ⠃⠄⡳⠭⠆⠁⠁⠋⠄⠥

$b \preceq u$

b \preceq u

4796. ⠙ ⠭ ⠁⠄⡳⠭⠆⠆⠆⠦⠄⠃⠄⡳⠭⠆⠁⠁⠋⠄⠥

$d = a \vee b \preceq u$

d = a \vee b \preceq u

4797. ⠉⠄⡳⠭⠆⠁⠁⠋⠄⠥

$c \preceq u$

c \preceq u

4798. ⠷⠁⠄⡳⠭⠆⠆⠆⠦⠄⠃⠾⠄⡳⠭⠆⠆⠆⠦⠄⠉ ⠭ ⠙⠄⡳⠭⠆⠆⠆⠦⠄⠉⠄⡳⠭⠆⠁⠁⠋⠄⠥

$(a \vee b) \vee c = d \vee c \preceq u$

(a \vee b) \vee c = d \vee c \preceq u

4799. StartSet⠁ EndSet

$\{a\}$

\{ a \}

4800. ⠙ ⠭ ⠁⠄⡳⠭⠆⠆⠆⠶⠄⠃

$d = a \wedge b$

d = a \wedge b

4801. ⠁⠄⡳⠭⠆⠁⠁⠋⠄⠁⠄⡳⠭⠆⠆⠆⠦⠄⠙

$a \preceq a \vee d$

a \preceq a \vee d

4802. ⠙ ⠭ ⠁⠄⡳⠭⠆⠆⠆⠶⠄⠃⠄⡳⠭⠆⠁⠁⠋⠄⠁

$d = a \wedge b \preceq a$

d = a \wedge b \preceq a

4803. ⠁⠄⡳⠭⠆⠆⠆⠦⠄⠙⠄⡳⠭⠆⠁⠁⠋⠄⠁

$a \vee d \preceq a$

a \vee d \preceq a

4804. ⠁⠄⡳⠭⠆⠆⠆⠦⠄⠷⠁⠄⡳⠭⠆⠆⠆⠶⠄⠃⠾ ⠭ ⠁

$a \vee (a \wedge b) = a$

a \vee (a \wedge b) = a

4805. ⠁⠄⡳⠭⠆⠆⠆⠦⠄⠃ ⠭ ⠃

$a \vee b = b$

a \vee b = b

4806. ⠁⠄⡳⠭⠆⠁⠁⠋⠄⠁

$a \preceq a$

a \preceq a

4807. ⠃⠄⡳⠭⠆⠁⠁⠋⠄⠁

$b \preceq a$

b \preceq a

4808. ⠃⠄⡳⠭⠆⠆⠆⠦⠄⠁ ⠭ ⠁

$b \vee a = a$

b \vee a = a

4809. ⠃ ⠨⠅ ⠁⠄⡳⠭⠆⠆⠆⠦⠄⠃ ⠨⠅ ⠃⠄⡳⠭⠆⠆⠆⠦⠄⠁ ⠨⠅ ⠁

$b = a \vee b = b \vee a = a$

b = a \vee b = b \vee a = a

4810. ⠃⠄⡳⠭⠆⠁⠁⠋⠄⠉

$b \preceq c$

b \preceq c

4811. ⠃⠄⡳⠭⠆⠆⠆⠦⠄⠉ ⠨⠅ ⠉

$b \vee c = c$

b \vee c = c

4812. ⠁⠄⡳⠭⠆⠆⠆⠦⠄⠉ ⠨⠅ ⠁⠄⡳⠭⠆⠆⠆⠦⠄⠷⠃⠄⡳⠭⠆⠆⠆⠦⠄⠉⠾ ⠨⠅ ⠷⠁⠄⡳⠭⠆⠆⠆⠦⠄⠃⠾⠄⡳⠭⠆⠆⠆⠦⠄⠉ ⠨⠅ ⠃⠄⡳⠭⠆⠆⠆⠦⠄⠉ ⠨

$a \vee c = a \vee (b \vee c) = (a \vee b) \vee c = b \vee c = c$

a \vee c = a \vee (b \vee c) = (a \vee b) \vee c = b \vee c =
c

4813. ⠁⠄⡳⠭⠆⠁⠁⠋⠄⠉

$a \preceq c$

a \preceq c

4814. ⠁ ⠨⠅ ⠷⠁⠄⡳⠭⠆⠆⠆⠦⠄⠃⠾⠄⡳⠭⠆⠆⠆⠶⠄⠁ ⠨⠅ ⠁⠄⡳⠭⠆⠆⠆⠶⠄⠷⠁⠄⡳⠭⠆⠆⠆⠦⠄⠃⠾

$a = (a \vee b) \wedge a = a \wedge (a \vee b)$

a=(a \vee b) \wedge a = a \wedge (a \vee b)

4815. ⠁⠄⡳⠭⠆⠁⠁⠋⠄⠁⠄⡳⠭⠆⠆⠆⠦⠄⠃

$a \preceq a \vee b$

a \preceq a \vee b

4816. ⠃⠄⡳⠭⠆⠁⠁⠋⠄⠁⠄⡳⠭⠆⠆⠆⠦⠄⠃

$b \preceq a \vee b$

b \preceq a \vee b

4817. ⠁⠄⡳⠭⠆⠆⠆⠦⠄⠃⠄⡳⠭⠆⠁⠁⠋⠄⠥

$a \vee b \preceq u$

a \vee b \preceq u

4818. ⠷⠁⠄⡳⠭⠆⠆⠆⠦⠄⠃⠾⠄⡳⠭⠆⠆⠆⠦⠄⠥ ⠨⠅ ⠁⠄⡳⠭⠆⠆⠆⠦⠄⠷⠃⠄⡳⠭⠆⠆⠆⠦⠄⠥⠾ ⠨⠅ ⠁⠄⡳⠭⠆⠆⠆⠦⠄⠥ ⠨

$(a \vee b) \vee u = a \vee (b \vee u) = a \vee u = u$

(a \vee b) \vee u = a \vee (b \vee u) = a \vee u = u

4819. ⠠⠁ ⠨⠩⠠⠭ ⠨⠅ ⠠⠁

$A \cap X = A$

A \cap X = A

4820. ⠁⠄⡳⠭⠆⠁⠁⠋⠄⠠⠊

$a \preceq I$

a \preceq I

4821. ⠠⠕⠄⡷⠭⠆⠁⠁⠋⠄⠁

$O \preceq a$

O \preceq a

4822. ⠠⠁⠄⠀⠨⠅⠀⠠⠭⠄⡷⠭⠆⠆⠂⠖⠄⠠⠁⠀⠨⠅⠀StartSet⠭⠱⠭⠈⠑⠠⠭and⠭⠌⠈⠑⠠⠁EndSet

$A' = X \setminus A = \{x : x \in X \text{ and } x \notin A\}$

A' = X \setminus A = \{ x : x \in X \text{ and } x \notin A \}

4823. ⠠⠁⠨⠬⠠⠁⠘⠄⠐⠀⠨⠅⠀⠠⠭

$A \cup A' = X$

A \cup A' = X

4824. ⠠⠁⠨⠩⠠⠁⠘⠄⠐⠀⠨⠅⠀⠸⠴

$A \cap A' = \emptyset$

A \cap A' = \emptyset

4825. ⠁⠈⠑⠠⠇

$a \in L$

a \in L

4826. ⠁⠄

a'

a'

4827. ⠁⠄⡷⠭⠆⠆⠆⠦⠄⠁⠘⠄⠐⠀⠨⠅⠀⠠⠊

$a \vee a' = I$

a \vee a' = I

4828. ⠁⠄⡷⠭⠆⠆⠆⠶⠄⠁⠘⠄⠐⠀⠨⠅⠀⠠⠕

$a \wedge a' = O$

a \wedge a' = O

4829. ⠁⠄⡷⠭⠆⠆⠆⠶⠄⠷⠃⠄⡷⠭⠆⠆⠆⠦⠄⠉⠾⠀⠨⠅⠀⠷⠁⠄⡷⠭⠆⠆⠆⠶⠄⠃⠾⠄⡷⠭⠆⠆⠆⠦⠄⠷⠁⠄⡷⠭⠆⠆⠆⠶⠄⠉⠾⠸⠆

$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c);$

a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c);

4830. ⠠⠁⠨⠩⠷⠠⠃⠨⠬⠠⠉⠾⠀⠨⠅⠀⠷⠠⠁⠨⠩⠠⠃⠾⠨⠬⠷⠠⠁⠨⠩⠠⠉⠾

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

A \cap (B \cup C) = (A \cap B) \cup (A \cap C)

4831. ⠠⠁⠠⠠⠃⠠⠠⠉⠈⠑caligraphic⠠⠏⠷⠠⠭⠾

$A, B, C \in \mathcal{P}(X)$

A, B, C \in {\mathcal P}(X)

4832. ⠁⠄⡷⠭⠆⠆⠆⠶⠄⠷⠃⠄⡷⠭⠆⠆⠆⠦⠄⠉⠾⠀⠨⠅⠀⠷⠁⠄⡷⠭⠆⠆⠆⠶⠄⠃⠾⠄⡷⠭⠆⠆⠆⠦⠄⠷⠁⠄⡷⠭⠆⠆⠆⠶⠄⠉⠾

$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)

4833. ⠁⠄⡷⠭⠆⠆⠆⠦⠄⠷⠃⠄⡷⠭⠆⠆⠆⠶⠄⠉⠾⠀⠨⠅⠀⠷⠁⠄⡷⠭⠆⠆⠆⠦⠄⠃⠾⠄⡷⠭⠆⠆⠆⠶⠄⠷⠁⠄⡷⠭⠆⠆⠆⠦⠄⠉⠾

$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)

4834. ⠁⠠⠃ ⠈⠑⠠⠃

$a, b \in B$

a, b \in B

4835. ⠁⠄⡳⠭⠆⠆⠆⠶⠄⠷⠃⠄⡳⠭⠆⠆⠆⠶⠄⠉⠾ ⠶ ⠷⠁⠄⡳⠭⠆⠆⠆⠶⠄⠃⠾⠄⡳⠭⠆⠆⠆⠶⠄⠉

$a \wedge (b \wedge c) = (a \wedge b) \wedge c$

a \wedge (b \wedge c) = (a \wedge b) \wedge c

4836. ⠁⠠⠃⠠⠉ ⠈⠑⠠⠃

$a, b, c \in B$

a, b, c \in B

4837. ⠁⠄⡳⠭⠆⠆⠆⠦⠄⠷⠃⠄⡳⠭⠆⠆⠆⠶⠄⠉⠾ ⠶ ⠷⠁⠄⡳⠭⠆⠆⠆⠦⠄⠃⠾⠄⡳⠭⠆⠆⠆⠶⠄⠷⠁⠄⡳⠭⠆⠆⠆⠦⠄⠉⠾

$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)

4838. ⠁⠄⡳⠭⠆⠆⠆⠦⠄⠠⠕ ⠶ ⠁

$a \vee O = a$

a \vee 0 = a

4839. ⠁⠄⡳⠭⠆⠆⠆⠶⠄⠠⠊ ⠶ ⠁

$a \wedge I = a$

a \wedge I = a

4840. ⠁ ⠈⠑⠠⠃

$a \in B$

a \in B

4841. ⠁⠄ ⠈⠑⠠⠃

$a' \in B$

a' \in B

4842. ⠠⠊⠄⡳⠭⠆⠆⠆⠦⠄⠃ ⠶ ⠷⠃⠄⡳⠭⠆⠆⠆⠦⠄⠃ ⠃⠄⠐⠾⠄⡳⠭⠆⠆⠆⠦⠄⠃ ⠶ ⠷⠃⠄⠄⡳⠭⠆⠆⠆⠦⠄⠃⠾⠄⡳⠭⠆⠆

$I \vee b = (b \vee b') \vee b = (b' \vee b) \vee b = b' \vee (b \vee b) = b' \vee b = I$

I \vee b = (b \vee b') \vee b = (b' \vee b) \vee b = b' \vee (b
\vee b) = b' \vee b = I

4843. ⠠⠕⠄⡳⠭⠆⠆⠆⠦⠄⠁ ⠶ ⠁

$O \vee a = a$

0 \vee a = a

4844. ⠁⠄⡳⠭⠆⠆⠆⠶⠄⠃ ⠶ ⠁

$a \wedge b = a$

a \wedge b = a

4845. ⠁⠄⡳⠭⠆⠆⠆⠦⠄⠠⠊ ⠶ ⠁

$a \vee I = a$

a \vee I = a

4846. ⠁⠄⡳⠭⠆⠆⠆⠦⠄⠠⠊ ⠨⠅ ⠷⠁⠄⡳⠭⠆⠆⠆⠶⠄⠠⠊⠾⠄⡳⠭⠆⠆⠆⠦⠄⠠⠊ ⠨⠅ ⠠⠊⠄⡳⠭⠆⠆⠆⠦⠄⠷⠠⠊⠄⡳⠭⠆⠆⠆⠶

$a \vee I = (a \wedge I) \vee I = I \vee (I \wedge a) = I$

a \vee I =(a \wedge I) \vee I = I \vee (I \wedge a) = I

4847. ⠁⠄⡳⠭⠆⠆⠆⠦⠄⠠⠊ ⠨⠅ ⠠⠊

$a \vee I = I$

a \vee I = I

4848. ⠁⠄⡳⠭⠆⠆⠆⠶⠄⠠⠕ ⠨⠅ ⠠⠕

$a \wedge O = O$

a \wedge 0 = 0

4849. ⠁⠄⡳⠭⠆⠆⠆⠦⠄⠃ ⠨⠅ ⠁⠄⡳⠭⠆⠆⠆⠦⠄⠉

$a \vee b = a \vee c$

a \vee b = a \vee c

4850. ⠁⠄⡳⠭⠆⠆⠆⠶⠄⠃ ⠨⠅ ⠁⠄⡳⠭⠆⠆⠆⠶⠄⠉

$a \wedge b = a \wedge c$

a \wedge b = a \wedge c

4851. ⠁⠄⡳⠭⠆⠆⠆⠦⠄⠃ ⠨⠅ ⠠⠊

$a \vee b = I$

a \vee b = I

4852. ⠁⠄⡳⠭⠆⠆⠆⠶⠄⠃ ⠨⠅ ⠠⠕

$a \wedge b = O$

a \wedge b = 0

4853. ⠃ ⠨⠅ ⠁⠄

$b = a'$

b = a'

4854. ⠷⠁⠘⠄⠐⠾⠄ ⠨⠅ ⠁

$(a')' = a$

(a')'=a

4855. ⠠⠊⠄ ⠨⠅ ⠠⠕

$I' = O$

I' = 0

4856. ⠠⠕⠄ ⠨⠅ ⠠⠊

$O' = I$

0' = I

4857. ⠷⠁⠄⡳⠭⠆⠆⠆⠦⠄⠃⠾⠄ ⠨⠅ ⠁⠄⠄⡳⠭⠆⠆⠆⠶⠄⠃⠄

$(a \vee b)' = a' \wedge b'$

(a \vee b)' = a' \wedge b'

4858. ⠷⠁⠄⡳⠭⠆⠆⠆⠶⠄⠃⠾⠄ ⠨⠅ ⠁⠄⠄⡳⠭⠆⠆⠆⠦⠄⠃⠄

$(a \wedge b)' = a' \vee b'$

(a \wedge b)' = a' \vee b'

4859. ⠨⠋⠱⠠⠃ ⠫⠕ ⠠⠉

$\phi : B \rightarrow C$

\phi : B \rightarrow C

4860. ⠁ ⠌⠭ ⠠⠕

$a \neq O$

a \neq O

4861. ⠠⠕⠄⡷⠭⠆⠁⠁⠋⠄⠃⠄⡷⠭⠆⠁⠁⠋⠄⠁

$O \preceq b \preceq a$

O \preceq b \preceq a

4862. ⠁ ⠭ ⠃

$a = b$

a =b

4863. ⠃⠂⠄⡷⠭⠆⠁⠁⠋⠄⠃

$b_1 \preceq b$

b_1 \preceq b

4864. ⠃⠆⠄⡷⠭⠆⠁⠁⠋⠄⠃⠂

$b_2 \preceq b_1$

b_2 \preceq b_1

4865. ⠁ ⠭ ⠃⠆

$a = b_2$

a = b_2

4866. ⠠⠕⠄⡷⠭⠆⠁⠁⠋⠄⠄⠄⠄⠄⡷⠭⠆⠁⠁⠋⠄⠃⠒⠄⡷⠭⠆⠁⠁⠋⠄⠃⠆⠄⡷⠭⠆⠁⠁⠋⠄⠃⠂⠄⡷⠭⠆⠁⠁⠋⠄⠃

$O \preceq \cdots \preceq b_3 \preceq b_2 \preceq b_1 \preceq b$

O \preceq \cdots \preceq b_3 \preceq b_2 \preceq b_1 \preceq b

4867. ⠃⠮

b_k

b_k

4868. ⠁ ⠭ ⠃⠮

$a = b_k$

a = b_k

4869. ⠁ ⠌⠭ ⠃

$a \neq b$

a \neq b

4870. ⠁⠄⡷⠭⠆⠆⠆⠶⠄⠃⠄⡷⠭⠆⠁⠁⠋⠄⠁

$a \wedge b \preceq a$

a \wedge b \preceq a

4871. ⠁ ⠭ ⠠⠕

$a = O$

a = O

4872. ⠁⠄⡷⠭⠆⠆⠆⠶⠄⠃⠘⠄⠐ ⠨⠅ ⠠⠕

$a \wedge b' = O$

a \wedge b' = 0

4873. ⠁⠄⠄⡷⠭⠆⠆⠆⠤⠄⠃ ⠨⠅ ⠠⠊

$a' \vee b = I$

a' \vee b = I

4874. ⠁⠄⠄⡷⠭⠆⠆⠆⠤⠄⠃ ⠨⠅ ⠷⠁⠄⡷⠭⠆⠆⠆⠶⠄⠃⠘⠄⠐⠾⠄ ⠨⠅ ⠠⠕⠄ ⠨⠅ ⠠⠊

$a' \vee b = (a \wedge b')' = O' = I$

a' \vee b = (a \wedge b')' = O' = I

4875. ⠃⠄⡷⠭⠆⠈⠈⠻⠌⠉

$b \npreceq c$

b \not\preceq c

4876. ⠁⠄⡷⠭⠆⠈⠈⠻⠌⠉

$a \npreceq c$

a \not\preceq c

4877. ⠃⠄⡷⠭⠆⠆⠆⠶⠄⠉⠘⠄⠐⠌⠨⠅ ⠠⠕

$b \wedge c' \neq O$

b \wedge c' \neq 0

4878. ⠁⠄⡷⠭⠆⠈⠈⠻⠄⠃⠄⡷⠭⠆⠆⠆⠶⠄⠉⠄

$a \preceq b \wedge c'$

a \preceq b \wedge c'

4879. ⠁ ⠰⠊⠄⡷⠭⠆⠈⠈⠻⠄⠃

$a_i \preceq b$

a_i \preceq b

4880. ⠃ ⠨⠅ ⠁⠂⠄⡷⠭⠆⠆⠆⠤⠄⠄⠄⠄⠄⡷⠭⠆⠆⠆⠤⠄⠁⠰⠝

$b = a_1 \vee \cdots \vee a_n$

b = a_1 \vee \cdots \vee a_n

4881. ⠁ ⠠⠁⠂ ⠠⠠ ⠠⠁ ⠰⠝⠂

$a, a_1, \ldots, a_n$

a, a_1, \ldots, a_n

4882. ⠃ ⠨⠅ ⠁⠄⡷⠭⠆⠆⠆⠤⠄⠁⠂⠄⡷⠭⠆⠆⠆⠤⠄⠄⠄⠄⠄⡷⠭⠆⠆⠆⠤⠄⠁⠰⠝

$b = a \vee a_1 \vee \cdots \vee a_n$

b = a \vee a_1 \vee \cdots \vee a_n

4883. ⠁ ⠨⠅ ⠁⠰⠊

$a = a_i$

a = a_i

4884. ⠃⠂ ⠨⠅ ⠁⠂⠄⡷⠭⠆⠆⠆⠤⠄⠄⠄⠄⠄⡷⠭⠆⠆⠆⠤⠄⠁⠰⠝

$b_1 = a_1 \vee \cdots \vee a_n$

b_1 = a_1 \vee \cdots \vee a_n

4885. ⠃ ⠄ ⡸⠭⠆ ⠁ ⠁ ⠋⠄⠃⠂

$b \preceq b_1$

b \preceq b_1

4886. ⠃ ⠄ ⡸⠭⠆ ⠁ ⠁ ⠋/⠃⠂

$b \not\preceq b_1$

b \not\preceq b_1

4887. ⠁ ⠄ ⡸⠭⠆ ⠁ ⠁ ⠋/⠃⠂

$a \not\preceq b_1$

a \not\preceq b_1

4888. ⠁ ⠄ ⡸⠭⠆ ⠁ ⠁ ⠋⠄⠃⠂

$a \preceq b_1$

a \preceq b_1

4889. ⠁ ⠶ ⠁ ⠄ ⡸⠭⠆ ⠆ ⠆ ⠿⠄⠃ ⠶ ⠁ ⠄ ⡸⠭⠆ ⠆ ⠆ ⠿⠄ ⠷⠁⠂⠄ ⡸⠭⠆ ⠆ ⠆ ⠵⠄ ⠄ ⠄ ⠄ ⠄ ⡸⠭⠆ ⠆ ⠆ ⠵⠄ ⠁ ⠰⠝ ⠂⠾ ⠶ ⠷⠁ ⠄

$a = a \wedge b = a \wedge (a_1 \vee \cdots \vee a_n) = (a \wedge a_1) \vee \cdots \vee (a \wedge a_n)$

a = a \wedge b = a \wedge(a_1 \vee \cdots \vee a_n) = (a \wedge
a_1) \vee \cdots \vee (a \wedge a_n)

4890. ⠁ ⠄ ⡸⠭⠆ ⠆ ⠆ ⠿⠄ ⠁ ⠰⠊

$a \wedge a_i$

a \wedge a_i

4891. ⠁ ⠶ ⠁⠂⠄ ⡸⠭⠆ ⠆ ⠆ ⠵⠄ ⠄ ⠄ ⠄ ⠄ ⡸⠭⠆ ⠆ ⠆ ⠵⠄ ⠁ ⠰⠝

$a = a_1 \vee \cdots \vee a_n$

a = a_1 \vee \cdots \vee a_n

4892. ⠁⠂ ⠤ ⠠⠁ ⠰⠝ ⠂ ⠈⠑ ⠠⠭

$a_1, \ldots, a_n \in X$

a_1, \ldots, a_n \in X

4893. ⠸⠋⠒ ⠠⠆ ⠫⠕ caligraphic ⠠⠏⠳ ⠠⠭⠾

$\phi : B \rightarrow \mathcal{P}(X)$

\phi : B \rightarrow {\mathcal P}(X)

4894. ⠸⠋⠳⠁⠾ ⠶ ⠸⠋⠳⠁⠂⠄ ⡸⠭⠆ ⠆ ⠆ ⠵⠄ ⠄ ⠄ ⠄ ⠄ ⡸⠭⠆ ⠆ ⠆ ⠵⠄ ⠁ ⠰⠝ ⠂⠾ ⠶ StartSet⠁ ⠂ ⠤ ⠠⠁ ⠰⠝ ⠂EndSet

$\phi(a) = \phi(a_1 \vee \cdots \vee a_n) = \{a_1, \ldots, a_n\}$

\phi(a) = \phi(a_1 \vee \cdots \vee a_n) = \{a_1, \ldots, a_n
\}

4895. ⠃ ⠶ ⠃⠂⠄ ⡸⠭⠆ ⠆ ⠆ ⠵⠄ ⠄ ⠄ ⠄ ⠄ ⡸⠭⠆ ⠆ ⠆ ⠵⠄ ⠃ ⠰⠍

$b = b_1 \vee \cdots \vee b_m$

b = b_1 \vee \cdots \vee b_m

4896. StartSet⠁ ⠂ ⠤ ⠠⠁ ⠰⠝ ⠂EndSet ⠶ StartSet⠃ ⠂ ⠤ ⠠⠃ ⠰⠍ ⠂EndSet

$\{a_1, \ldots, a_n\} = \{b_1, \ldots, b_m\}$

\{a_1, \ldots, a_n \} = \{b_1, \ldots, b_m \}

4897. ⠨⠋⠷⠁⠠⡸⠛⠒⠒⠱⠠⠃⠾ ⠶ ⠨⠋⠷⠁⠾⠨⠩⠨⠋⠷⠃⠾

$\phi(a \wedge b) = \phi(a) \cap \phi(b)$

`\phi( a \wedge b ) = \phi(a) \cap \phi(b)`

4898. ⠃⠠⡸⠛⠒⠒⠱⠠⠁

$b \wedge a$

`b \wedge a`

4899. ⠷⠁⠠⡸⠛⠒⠒⠒⠡⠠⠃⠾⠠⡸⠛⠒⠒⠱⠠⠷⠁⠠⡸⠛⠒⠒⠒⠡⠠⠃⠄⠠⠐⠾⠠⡸⠛⠒⠒⠱⠠⠷⠁⠠⡸⠛⠒⠒⠒⠡⠠⠃⠾

$(a \vee b) \wedge (a \vee b') \wedge (a \vee b)$

`(a \vee b) \wedge (a \vee b') \wedge (a \vee b)`

4900. ⠠⠭ ⠶ StartSet⠁⠠⠃⠠⠉⠠⠙EndSet

$X = \{a, b, c, d\}$

`X = \{ a, b, c, d \}`

4901. ⠷⠁⠠⡸⠛⠒⠒⠒⠡⠠⠃⠠⡸⠛⠒⠒⠒⠡⠠⠁⠄⠠⠐⠾⠠⡸⠛⠒⠒⠱⠠⠁

$(a \vee b \vee a') \wedge a$

`(a \vee b \vee a') \wedge a`

4902. ⠷⠁⠠⡸⠛⠒⠒⠒⠡⠠⠃⠾⠠⠠⡸⠛⠒⠒⠱⠠⠷⠁⠠⡸⠛⠒⠒⠒⠡⠠⠃⠾

$(a \vee b)' \wedge (a \vee b)$

`(a \vee b)' \wedge (a \vee b)`

4903. ⠁⠠⡸⠛⠒⠒⠒⠡⠠⠷⠁⠠⡸⠛⠒⠒⠱⠠⠃⠾

$a \vee (a \wedge b)$

`a \vee (a \wedge b)`

4904. ⠷⠉⠠⡸⠛⠒⠒⠒⠡⠠⠁⠠⡸⠛⠒⠒⠒⠡⠠⠃⠾⠠⡸⠛⠒⠒⠱⠠⠉⠠⠠⡸⠛⠒⠒⠱⠠⠷⠁⠠⡸⠛⠒⠒⠒⠡⠠⠃⠾⠠

$(c \vee a \vee b) \wedge c' \wedge (a \vee b)'$

`(c \vee a \vee b) \wedge c' \wedge (a \vee b)'`

4905. caligraphic ⠠⠏⠷⠠⠭⠾ ⠶ ⠆⠘⠝

$\mathcal{P}(X) = 2^n$

`{\cal P}(X) = 2^n`

4906. ⠁⠠⠠⡸⠛⠒⠒⠱⠠⠈⠷⠷⠁⠠⡸⠛⠒⠒⠱⠠⠃⠄⠠⠐⠾⠠⡸⠛⠒⠒⠒⠡⠠⠃⠈⠾ ⠶ ⠁⠠⡸⠛⠒⠒⠱⠠⠷⠁⠠⡸⠛⠒⠒⠒⠡⠠⠃

$a' \wedge [(a \wedge b') \vee b] = a \wedge (a \vee b)$

`a' \wedge [(a \wedge b') \vee b] = a \wedge (a \vee b)`

4907. ⠠⠊⠬⠠⠚

$I + J$

`I + J`

4908. ⠠⠊⠠⠠⠚

I, J

`I, J`

4909. ⠠⠊⠬⠠⠚ ⠶ StartSet⠗⠬⠎⠒⠗ ⠈⠑ ⠠⠊and⠎ ⠈⠑ ⠠⠚EndSet

$I + J = \{r + s : r \in I \text{ and } s \in J\}$

`I + J = \{ r + s : r \in I \text{ and } s \in J \}`

4910. ⠗⠂ ⠠⠗⠆ ⠈⠑ ⠠⠊

$r_1, r_2 \in I$

```
r_1, r_2 \in I
```

4911. ⠎⠂ ⠠⠎⠆ ⠈⠑ ⠠⠚

$s_1, s_2 \in J$

```
s_1, s_2 \in J
```

4912. ⠷⠗⠂ ⠬⠎⠂ ⠾⠬⠷⠗⠆ ⠬⠎⠆ ⠾ ⠨⠅ ⠷⠗⠂ ⠬⠗⠆ ⠾⠬⠷⠎⠂ ⠬⠎⠆ ⠾

$(r_1 + s_1) + (r_2 + s_2) = (r_1 + r_2) + (s_1 + s_2)$

```
(r_1 + s_1) + (r_2 + s_2) = (r_1 + r_2) +(s_1 + s_2)
```

4913. ⠁ ⠷⠗⠂ ⠬⠎⠂ ⠾ ⠨⠅ ⠁ ⠗⠂ ⠬⠁ ⠎⠂ ⠈⠑ ⠠⠊⠬ ⠠⠚

$a(r_1 + s_1) = ar_1 + as_1 \in I + J$

```
a(r_1 + s_1) = ar_1 + as_1 \in I + J
```

4914. ⠅⠱

$\leq$

```
\leq
```

4915. ⠨⠋⠱⠠⠭ ⠫⠕ ⠠⠽

$\phi : X \rightarrow Y$

```
\phi : X \rightarrow Y
```

4916. ⠨⠋⠷⠁ ⠾⠠⡷⠭⠆ ⠁ ⠁ ⠉⠄ ⠨⠋⠷⠘ ⠾

$\phi(a) \preceq \phi(b)$

```
\phi(a) \preceq \phi(b)
```

4917. ⠨⠓⠱⠠⠸ ⠫⠕ ⠠⠍

$\psi : L \rightarrow M$

```
\psi: L \rightarrow M
```

4918. ⠨⠓⠷⠁ ⠠⡷⠭⠆⠆⠆⠦⠄ ⠘⠾ ⠨⠅ ⠨⠓⠷⠁ ⠾⠠⡷⠭⠆⠆⠆⠦⠄ ⠨⠓⠷⠘ ⠾

$\psi(a \vee b) = \psi(a) \vee \psi(b)$

```
\psi( a \vee b ) = \psi(a) \vee \psi(b)
```

4919. ⠨⠓⠷⠁ ⠠⡷⠭⠆⠆⠆⠿⠄ ⠘⠾ ⠨⠅ ⠨⠓⠷⠁ ⠾⠠⡷⠭⠆⠆⠆⠿⠄ ⠨⠓⠷⠘ ⠾

$\psi(a \wedge b) = \psi(a) \wedge \psi(b)$

```
\psi( a \wedge b ) = \psi(a) \wedge \psi(b)
```

4920. ⠷⠁ ⠠⡷⠭⠆⠆⠆⠿⠄ ⠘ ⠑⠄ ⠐⠾⠠⡷⠭⠆⠆⠆⠦⠄ ⠷⠁ ⠄⠠⡷⠭⠆⠆⠆⠿⠄ ⠘⠾ ⠨⠅ ⠠⠕

$(a \wedge b') \vee (a' \wedge b) = O$

```
(a \wedge b') \vee ( a' \wedge b) = 0
```

4921. ⠷ ⠫⠒⠒⠕⠕ ⠾

$(\Rightarrow)$

```
( \Rightarrow)
```

4922. ⠁ ⠶ ⠃ ⠹⠒⠒⠕⠕ ⠷⠁⠄⡸⠭⠆⠆⠆⠿⠄⠃⠋⠄ ⠐⠿⠄⡸⠭⠆⠆⠆⠦⠄⠷⠁⠄⠄⡸⠭⠆⠆⠆⠿⠄⠃⠾ ⠶ ⠷⠁⠄⡸⠭⠆⠆⠆⠿

$a = b \Rightarrow (a \wedge b') \vee (a' \wedge b) = (a \wedge a') \vee (a' \wedge a) = O \vee O = O$

```
a = b \Rightarrow (a \wedge b') \vee (a' \wedge b) = (a \wedge a')
\vee (a' \wedge a) = O \vee O = O
```

4923. ⠷ ⠹⠪⠪⠒⠒ ⠾

$(\Leftarrow)$

```
( \Leftarrow)
```

4924. ⠷⠁⠄⡸⠭⠆⠆⠆⠿⠄⠃⠋⠄ ⠐⠿⠄⡸⠭⠆⠆⠆⠦⠄⠷⠁⠄⠄⡸⠭⠆⠆⠆⠿⠄⠃⠾ ⠶ ⠠⠕ ⠹⠒⠒⠕⠕ ⠁⠄⡸⠭⠆⠆⠆⠦⠄⠃ ⠶

$(a \wedge b') \vee (a' \wedge b) = O \Rightarrow a \vee b = (a \vee a) \vee b = a \vee (a \vee b) = a \vee [I \wedge (a \vee b)] =$
$a \vee [(a \vee a') \wedge (a \vee b)] = [a \vee (a \wedge b')] \vee [a \vee (a' \wedge b)] = a \vee [(a \wedge b') \vee (a' \wedge b)] =$
$a \vee 0 = a$

```
( a \wedge b') \vee (a' \wedge b) = O \Rightarrow a \vee b = (a \vee
a) \vee b = a \vee (a \vee b) = a \vee [I \wedge (a \vee b)] = a
\vee [(a \vee a') \wedge (a \vee b)] = [a \vee (a \wedge b')] \vee
[a \vee (a' \wedge b)] = a \vee [(a \wedge b') \vee (a' \wedge b)]
= a \vee 0 = a
```

4925. ⠷⠁⠄⡸⠭⠆⠆⠆⠿⠄⠃⠋⠄ ⠐⠿⠄⡸⠭⠆⠆⠆⠦⠄⠷⠁⠄⠄⡸⠭⠆⠆⠆⠿⠄⠃⠾ ⠶ ⠃

$(a \wedge b') \vee (a' \wedge b) = b$

```
(a \wedge b') \vee ( a' \wedge b) = b
```

4926. ⠠⠇⠑⠠⠍

$L \times M$

```
L \times M
```

4927. ⠷⠁ ⠠⠃⠾⠄⡸⠭⠆⠈⠈⠋⠄⠷⠉ ⠠⠙⠾

$(a, b) \preceq (c, d)$

```
( a, b) \preceq (c, d)
```

4928. ⠃⠄⡸⠭⠆⠈⠈⠋⠄⠙

$b \preceq d$

```
b \preceq d
```

4929. ⠋⠒ ⠨⠷ ⠠⠕ ⠠⠄⠊ ⠨⠾⠘⠝⠐ ⠫⠕ StartSet⠼ ⠠⠄⠊EndSet

$f : \{O, I\}^n \rightarrow \{0, I\}$

```
f : \{O, I\}^n \rightarrow \{ 0, I\}
```

4930. ⠄

$'$

```
'
```

4931. ⠭⠄⡸⠭⠆⠆⠆⠦⠄⠽

$x \vee y$

```
x \vee y
```

4932. ⠭⠄⡸⠭⠆⠆⠆⠿⠄⠽

$x \wedge y$

```
x \wedge y
```

4933. ⠠⠉⠂

C_1

`C_1`

4934. ⠠⠉⠆

C_2

`C_2`

4935. ⠠⠉⠂⠀⠨⠅⠀⠈⠷⠆⠠⠂⠠⠆⠈⠾⠄⡳⠭⠆⠁⠃⠴⠄⠈⠷⠒⠠⠆⠈⠾⠀⠨⠅⠀⠠⠉⠆

$C_1 = [2, 1, 2] \succeq [3, 2] = C_2$

`C_1 = [2, 1, 2] \succeq [3, 2] = C_2`

4936. 72⠀⠨⠅⠀⠆⠘⠒⠐⠡⠒⠘⠆

$72=2^3\cdot 3^2$

`72=2^3\cdot 3^2`

4937. V

`V`

4938. ⠨⠁⠡⠧

$\alpha \cdot v$

`\alpha \cdot v`

4939. ⠨⠁⠧

αv

`\alpha v`

4940. ⠧⠀⠈⠑⠀⠠⠧

$v \in V$

`v \in V`

4941. ⠨⠁⠷⠨⠃⠧⠾⠀⠨⠅⠀⠷⠨⠁⠨⠃⠾⠧

$\alpha(\beta v) =(\alpha \beta)v$

`\alpha(\beta v) =(\alpha \beta)v`

4942. ⠷⠨⠁⠬⠨⠃⠾⠧⠀⠨⠅⠀⠨⠁⠧⠬⠨⠃⠧

$(\alpha + \beta)v =\alpha v + \beta v$

`(\alpha + \beta)v =\alpha v + \beta v`

4943. ⠨⠁⠷⠥⠬⠧⠾⠀⠨⠅⠀⠨⠁⠥⠬⠨⠁⠧

$\alpha(u + v) = \alpha u + \alpha v$

`\alpha(u + v) = \alpha u + \alpha v`

4944. ⠂⠧⠀⠨⠅⠀⠧

$1v=v$

`1v=v`

4945. ⠨⠁⠠⠨⠃⠀⠈⠑⠀⠠⠋

$\alpha, \beta \in F$

`\alpha, \beta \in F`

4946. ⠥ ⠠⠧ ⠈⠑ ⠠⠧

$u, v \in V$

u, v \in V

4947. ⠥ ⠶ ⠷⠥⠂ ⠤ ⠠⠥ ⠰⠝ ⠐⠾

$u = (u_1, \ldots, u_n)$

u = (u_1, \ldots, u_n)

4948. ⠧ ⠶ ⠷⠧⠂ ⠤ ⠠⠧ ⠰⠝ ⠐⠾

$v = (v_1, \ldots, v_n)$

v = (v_1, \ldots, v_n)

4949. ⠥⠬⠧ ⠶ ⠷⠥⠂ ⠤ ⠠⠥ ⠰⠝ ⠐⠾⠬⠷⠧⠂ ⠤ ⠠⠧ ⠰⠝ ⠐⠾ ⠶ ⠷⠥⠂⠬⠧⠂ ⠤ ⠠⠥ ⠰⠝ ⠐⠬⠧ ⠰⠝ ⠐⠾

$u + v = (u_1, \ldots, u_n) + (v_1, \ldots, v_n) = (u_1 + v_1, \ldots, u_n + v_n)$

u + v = (u_1, \ldots, u_n) + (v_1, \ldots, v_n) = (u_1 + v_1, \ldots,
u_n + v_n)

4950. ⠨⠁⠥ ⠶ ⠨⠁⠷⠥⠂ ⠤ ⠠⠥ ⠰⠝ ⠐⠾ ⠶ ⠷⠨⠁⠥⠂ ⠤ ⠠⠨⠁⠥ ⠰⠝ ⠐⠾

$\alpha u = \alpha(u_1, \ldots, u_n) = (\alpha u_1, \ldots, \alpha u_n)$

\alpha u = \alpha(u_1, \ldots, u_n)= (\alpha u_1, \ldots, \alpha
u_n)

4951. ⠨⠁⠏⠷⠭⠾

$\alpha p(x)$

\alpha p(x)

4952. ⠷⠋⠬⠛⠾⠷⠭⠾

$(f + g)(x)$

(f+g)(x)

4953. ⠋⠷⠭⠾⠬⠛⠷⠭⠾

$f(x) + g(x)$

f(x) + g(x)

4954. ⠷⠨⠁⠋⠾⠷⠭⠾ ⠶ ⠨⠁⠋⠷⠭⠾

$(\alpha f)(x) = \alpha f(x)$

(\alpha f)(x) = \alpha f(x)

4955. ⠛⠷⠭⠾ ⠶ ⠭⠘⠆

$g(x) = x^2$

g(x)= x^2

4956. ⠷⠆⠋⠬⠢⠛⠾⠷⠭⠾ ⠶ ⠆⠎⠊⠝ ⠭⠬⠢⠭⠘⠆

$(2f + 5g)(x) = 2 \sin x + 5x^2$

(2f + 5g)(x) =2 \sin x + 5 x^2

4957. ⠠⠧ ⠶ ⠈⠠⠟⠷⠜⠆⠻⠾ ⠶ ⠸⠷⠁⠬⠃⠜⠆⠻ ⠒ ⠁⠠ ⠃ ⠈⠑ ⠈⠠⠟⠸⠾

$V = \mathbb{Q}(\sqrt{2}\,) = \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$

V = {\mathbb Q}(\sqrt{2}\,) = \{ a + b \sqrt{2} : a, b \in {\mathbb
Q } \}

4958. ⠥ ⠶ ⠁⠬⠃⠜⠆⠻

$u = a + b\sqrt{2}$

u = a + b \sqrt{2}

4959. ⠧ ⠶ ⠉⠬⠙⠜⠆⠻

$v = c + d\sqrt{2}$

v = c + d \sqrt{2}

4960. ⠥⠬⠧ ⠶ ⠷⠁⠬⠉⠾⠬⠷⠃⠬⠙⠾⠜⠆⠻

$u + v = (a + c) + (b + d)\sqrt{2}$

u + v = (a + c) + (b + d) \sqrt{2}

4961. ⠨⠁ ⠈⠑ ⠈⠠⠟

$\alpha \in \mathbb{Q}$

\alpha \in {\mathbb Q}

4962. ⠴⠧ ⠶ ⠸⠴

$0v = \mathbf{0}$

0v ={\mathbf 0}

4963. ⠨⠁ Baselinebold⠴ ⠶ ⠸⠴

$\alpha\mathbf{0} = \mathbf{0}$

\alpha {\mathbf 0} = {\mathbf 0}

4964. ⠨⠁⠧ ⠶ ⠸⠴

$\alpha v = \mathbf{0}$

\alpha v = {\mathbf 0}

4965. ⠨⠁ ⠶ ⠴

$\alpha = 0$

\alpha = 0

4966. ⠧ ⠶ ⠸⠴

$v = \mathbf{0}$

v = {\mathbf 0}

4967. ⠷⠤⠂⠾⠧ ⠶ ⠤⠧

$(-1)v = -v$

(-1) v = -v

4968. ⠤⠷⠨⠁⠧⠾ ⠶ ⠷⠤⠨⠁⠾⠧ ⠶ ⠨⠁⠷⠤⠧⠾

$-(\alpha v) = (-\alpha)v = \alpha(-v)$

-(\alpha v) = (-\alpha)v = \alpha(-v)

4969. ⠴⠧ ⠶ ⠷⠴⠬⠴⠾⠧ ⠶ ⠴⠧⠬⠴⠧⠸⠆

$0v = (0 + 0)v = 0v + 0v;$

0 v = (0 + 0)v = 0v + 0v;

4970. ⠸⠴⠬⠴⠧ ⠶ ⠴⠧⠬⠴⠧

$\mathbf{0} + 0v = 0v + 0v$

{\mathbf 0} + 0 v = 0v + 0v

4971. ⠸⠴ ⠨⠅ ⠴⠧

$\mathbf{0} = 0v$

{\mathbf 0} = 0v

4972. ⠨⠁ ⠌⠨⠅ ⠴

$\alpha \neq 0$

\alpha \neq 0

4973. ⠂⠌⠨⠁

$1/\alpha$

1/ \alpha

4974. ⠧⠬⠷⠤⠂⠾⠧ ⠨⠅ ⠂⠧⠬⠷⠤⠂⠾⠧ ⠨⠅ ⠷⠂⠤⠂⠾⠧ ⠨⠅ ⠴⠧ ⠨⠅ ⠸⠴

$v + (-1)v = 1v + (-1)v = (1-1)v = 0v = \mathbf{0}$

v + (-1)v = 1v + (-1)v = (1-1)v = 0v = {\mathbf 0}

4975. ⠤⠧ ⠨⠅ ⠷⠤⠂⠾⠧

$-v = (-1)v$

-v = (-1)v

4976. ⠥ ⠠⠧ ⠈⠑ ⠠⠺

$u, v \in W$

u, v \in W

4977. ⠥⠬⠧

$u + v$

u + v

4978. ⠠⠺ ⠨⠅ ⠸⠷⠷⠭⠂⠠ ⠆⠭⠂⠬⠭⠆⠠ ⠭⠂⠤⠭⠆⠾ ⠸⠒ ⠭⠂⠠ ⠭⠆ ⠈⠑ ⠠⠨⠗ ⠸⠾

$W = \{(x_1, 2x_1 + x_2, x_1 - x_2) : x_1, x_2 \in \mathbb{R}\}$

W = \{ (x_1, 2 x_1 + x_2, x_1 - x_2) : x_1, x_2 \in {\mathbb R}
\}

4979. ⠥ ⠨⠅ ⠷⠭⠂⠠ ⠆⠭⠂⠬⠭⠆⠠ ⠭⠂⠤⠭⠆⠾

$u = (x_1, 2x_1 + x_2, x_1 - x_2)$

u = (x_1, 2 x_1 + x_2, x_1 - x_2)

4980. ⠧ ⠨⠅ ⠷⠽⠂⠠ ⠆⠽⠂⠬⠽⠆⠠ ⠽⠂⠤⠽⠆⠾

$v = (y_1, 2y_1 + y_2, y_1 - y_2)$

v = (y_1, 2 y_1 + y_2, y_1 - y_2)

4981. ⠥⠬⠧ ⠨⠅ ⠷⠭⠂⠬⠽⠂⠠ ⠆⠷⠭⠂⠬⠽⠂⠾⠬⠷⠭⠆⠬⠽⠆⠾⠠ ⠷⠭⠂⠬⠽⠂⠾⠤⠷⠭⠆⠬⠽⠆⠾⠾

$u + v = (x_1 + y_1, 2(x_1 + y_1) + (x_2 + y_2), (x_1 + y_1) - (x_2 + y_2))$

u + v = (x_1 + y_1, 2(x_1 + y_1) +(x_2 + y_2), (x_1 + y_1) - (x_2+
y_2))

4982. ⠨⠁⠏⠷⠭⠾ ⠈⠑ ⠠⠺

$\alpha p(x) \in W$

\alpha p(x) \in W

4983. ⠏⠷⠭⠾ ⠈⠑ ⠠⠺

$p(x) \in W$

p(x) \in W

4984. ⠧⠂⠠ ⠧⠆⠠ ⠄⠄⠄⠠ ⠧⠰⠝⠐

$v_1, v_2, \ldots, v_n$

v_1, v_2, \ldots, v_n

4985. ⠨⠁⠂⠠ ⠨⠁⠆⠠ ⠄⠄⠄⠠ ⠨⠁⠰⠝⠐

$\alpha_1, \alpha_2, \ldots, \alpha_n$

\alpha_1, \alpha_2, \ldots, \alpha_n

4986. ⠺ ⠨⠅ ⠐⠨⠠⠎⠰⠊⠨⠅⠂⠘⠝⠐⠨⠁⠰⠊⠐⠧⠰⠊ ⠨⠅ ⠨⠁⠂⠧⠂⠬⠨⠁⠆⠧⠆⠬⠄⠄⠄⠬⠨⠁⠰⠝⠐⠧⠰⠝

$w = \sum_{i=1}^n \alpha_i v_i = \alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_n v_n$

w = \sum_{i=1}^n \alpha_i v_i = \alpha_1 v_1 + \alpha_2 v_2 + \cdots
+ \alpha_n v_n

4987. ⠠⠎ ⠨⠅ StartSet⠧⠂⠠ ⠧⠆⠠ ⠄⠄⠄⠠ ⠧⠰⠝⠐EndSet

$S = \{v_1, v_2, \ldots, v_n\}$

S= \{v_1, v_2, \ldots, v_n \}

4988. ⠧⠰⠊

v_i

v_i

4989. ⠥⠬⠧ ⠨⠅ ⠷⠨⠁⠂⠬⠨⠃⠂⠾⠧⠂⠬⠷⠨⠁⠆⠬⠨⠃⠆⠾⠧⠆⠬⠄⠄⠄⠬⠷⠨⠁⠰⠝⠐⠬⠨⠃⠰⠝⠐⠾⠧⠰⠝

$u + v = (\alpha_1 + \beta_1)v_1 + (\alpha_2 + \beta_2)v_2 + \cdots + (\alpha_n + \beta_n)v_n$

u + v =(\alpha_1 + \beta_1) v_1 + (\alpha_2+ \beta_2) v_2 + \cdots
+ (\alpha_n + \beta_n) v_n

4990. ⠨⠁⠥ ⠨⠅ ⠷⠨⠁⠨⠁⠂⠾⠧⠂⠬⠷⠨⠁⠨⠁⠆⠾⠧⠆⠬⠄⠄⠄⠬⠷⠨⠁⠨⠁⠰⠝⠐⠾⠧⠰⠝

$\alpha u = (\alpha\alpha_1)v_1 + (\alpha\alpha_2)v_2 + \cdots + (\alpha\alpha_n)v_n$

\alpha u = (\alpha \alpha_1) v_1 + (\alpha \alpha_2) v_2 + \cdots
+ (\alpha \alpha_n) v_n

4991. ⠠⠎ ⠨⠅ StartSet⠧⠂⠠ ⠧⠆⠠ ⠄⠄⠄⠠ ⠧⠰⠝⠐EndSet

$S = \{v_1, v_2, \ldots, v_n\}$

S = \{v_1, v_2, \ldots, v_n\}

4992. ⠨⠁⠂⠠ ⠨⠁⠆⠄⠄⠄⠨⠁⠰⠝ ⠈⠑ ⠠⠋

$\alpha_1, \alpha_2 \ldots \alpha_n \in F$

\alpha_1, \alpha_2 \ldots \alpha_n \in F

4993. ⠨⠁⠰⠊

α_i

\alpha_i

4994. ⠨⠁⠂⠧⠂⠬⠨⠁⠆⠧⠆⠬⠄⠄⠄⠬⠨⠁⠰⠝⠐⠧⠰⠝ ⠨⠅ ⠸⠼⠚

$\alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_n v_n = \mathbf{0}$

\alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_n v_n = {\mathbf 0
}

4995. ⠨⠁⠂⠀⠨⠅⠀⠨⠁⠆⠀⠨⠅⠀⠄⠄⠄⠀⠨⠅⠀⠨⠁⠰⠝⠐⠀⠨⠅⠀⠴

$\alpha_1 = \alpha_2 = \cdots = \alpha_n = 0$

\alpha_1 = \alpha_2 = \cdots = \alpha_n = 0

4996. StartSet ⠨⠁⠂⠠⠀⠨⠁⠆⠄⠀⠨⠁⠰⠝⠐EndSet

$\{\alpha_1, \alpha_2 \ldots \alpha_n\}$

\{ \alpha_1, \alpha_2 \ldots \alpha_n \}

4997. StartSet⠧⠂⠠⠀⠧⠆⠠⠀⠄⠄⠠⠀⠧⠰⠝⠐EndSet

$\{v_1, v_2, \ldots, v_n\}$

\{ v_1, v_2, \ldots, v_n \}

4998. ⠧⠀⠨⠅⠀⠨⠁⠂⠧⠂⠬⠨⠁⠆⠧⠆⠬⠄⠄⠄⠬⠨⠁⠰⠝⠐⠧⠰⠝⠀⠨⠅⠀⠨⠃⠂⠧⠂⠬⠨⠃⠆⠧⠆⠬⠄⠄⠄⠬⠨⠃⠰⠝⠐⠧⠰⠝

$v = \alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_n v_n = \beta_1 v_1 + \beta_2 v_2 + \cdots + \beta_n v_n$

v = \alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_n v_n = \beta_1
v_1 + \beta_2 v_2 + \cdots + \beta_n v_n

4999. ⠨⠁⠂⠀⠨⠅⠀⠨⠃⠂⠠⠀⠨⠁⠆⠀⠨⠅⠀⠨⠃⠆⠠⠀⠄⠄⠠⠀⠨⠁⠰⠝⠐⠀⠨⠅⠀⠨⠃⠰⠝⠐

$\alpha_1 = \beta_1, \alpha_2 = \beta_2, \ldots, \alpha_n = \beta_n$

\alpha_1 = \beta_1, \alpha_2 = \beta_2, \ldots, \alpha_n = \beta_n

5000. ⠷⠨⠁⠂⠤⠨⠃⠂⠾⠧⠂⠬⠷⠨⠁⠆⠤⠨⠃⠆⠾⠧⠆⠬⠄⠄⠄⠬⠷⠨⠁⠰⠝⠐⠤⠨⠃⠰⠝⠐⠾⠧⠰⠝⠐⠀⠨⠅⠀⠸⠴

$(\alpha_1 - \beta_1)v_1 + (\alpha_2 - \beta_2)v_2 + \cdots + (\alpha_n - \beta_n)v_n = \mathbf{0}$

(\alpha_1 - \beta_1) v_1 + (\alpha_2 - \beta_2) v_2 + \cdots + (\alpha_n
- \beta_n) v_n = {\mathbf 0}

5001. ⠧⠂⠠⠀⠄⠄⠠⠀⠧⠰⠝⠐

$v_1, \ldots, v_n$

v_1, \ldots, v_n

5002. ⠨⠁⠰⠊⠐⠤⠨⠃⠰⠊⠐⠀⠨⠅⠀⠴

$\alpha_i - \beta_i = 0$

\alpha_i - \beta_i = 0

5003. StartSet⠧⠂⠠⠀⠧⠆⠠⠀⠄⠄⠠⠀⠧⠰⠝⠐EndSet

$\{v_1, v_2, \ldots, v_n\}$

\{ v_1, v_2, \dots, v_n \}

5004. ⠨⠁⠂⠠⠀⠄⠄⠠⠀⠨⠁⠰⠝⠐

$\alpha_1, \ldots, \alpha_n$

\alpha_1, \ldots, \alpha_n

5005. ⠨⠁⠰⠅⠐⠀⠌⠨⠅⠀⠴

$\alpha_k \neq 0$

\alpha_k \neq 0

5006. ⠧⠰⠅⠐⠀⠨⠅⠀⠤⠹⠨⠁⠂⠌⠨⠁⠰⠅⠐⠼⠧⠂⠤⠄⠄⠄⠤⠹⠨⠁⠰⠅⠤⠂⠐⠌⠨⠁⠰⠅⠐⠼⠧⠰⠅⠤⠂⠐⠤⠹⠨⠁⠰⠅⠬⠂⠐⠌⠨⠁⠰⠅⠐⠼⠧⠰⠅⠬⠂⠐⠤⠄⠄⠄⠤⠹⠨⠁⠰⠝⠐⠌⠨⠁⠰⠅

$v_k = -\frac{\alpha_1}{\alpha_k} v_1 - \cdots - \frac{\alpha_{k-1}}{\alpha_k} v_{k-1} - \frac{\alpha_{k+1}}{\alpha_k} v_{k+1} - \cdots - \frac{\alpha_n}{\alpha_k} v_n$

v_k = - \frac{\alpha_1}{\alpha_k} v_1 - \cdots - \frac{\alpha_{k
- 1}}{\alpha_k} v_{k-1} - \frac{\alpha_{k + 1}}{\alpha_k} v_{k +
1} - \cdots - \frac{\alpha_n}{\alpha_k} v_n

5007. ⠧⠰⠅⠀⠨⠅⠀⠨⠃⠂⠧⠂⠬⠄⠄⠄⠬⠨⠃⠰⠅⠤⠂⠧⠰⠅⠤⠂⠐⠬⠨⠃⠰⠅⠬⠂⠧⠰⠅⠬⠂⠐⠬⠄⠄⠄⠬⠨⠃⠰⠝⠧⠰⠝

$v_k = \beta_1 v_1 + \cdots + \beta_{k-1} v_{k-1} + \beta_{k+1} v_{k+1} + \cdots + \beta_n v_n$

```
v_k = \beta_1 v_1 + \cdots + \beta_{k - 1} v_{k - 1} + \beta_{k
+ 1} v_{k + 1} + \cdots + \beta_n v_n
```

5008. ⠨⠃⠂⠧⠂⠬⠄⠄⠄⠬⠨⠃⠰⠅⠤⠂⠧⠰⠅⠤⠂⠐⠤⠧⠰⠅⠐⠬⠨⠃⠰⠅⠬⠂⠧⠰⠅⠬⠂⠐⠬⠄⠄⠄⠬⠨⠃⠰⠝⠧⠰⠝⠀⠨⠅⠀⠸⠰

$\beta_1 v_1 + \cdots + \beta_{k-1} v_{k-1} - v_k + \beta_{k+1} v_{k+1} + \cdots + \beta_n v_n = \mathbf{0}$

```
\beta_1 v_1 + \cdots + \beta_{k - 1} v_{k - 1} - v_k + \beta_{k
+ 1} v_{k + 1} + \cdots + \beta_n v_n = {\mathbf 0}
```

5009. StartSet⠑⠂⠠⠀⠑⠆⠠⠀⠄⠄⠄⠠⠀⠑⠰⠝⠐EndSet

$\{e_1, e_2, \ldots, e_n\}$

```
\{ e_1, e_2, \ldots, e_n \}
```

5010. ⠑⠂⠀⠨⠅⠀⠷⠂⠠⠀⠴⠠⠀⠴⠾

$e_1 = (1, 0, 0)$

```
e_1 = (1, 0, 0)
```

5011. ⠑⠆⠀⠨⠅⠀⠷⠴⠠⠀⠂⠠⠀⠴⠾

$e_2 = (0, 1, 0)$

```
e_2 = (0, 1, 0)
```

5012. ⠑⠒⠀⠨⠅⠀⠷⠴⠠⠀⠴⠠⠀⠂⠾

$e_3 = (0, 0, 1)$

```
e_3 =(0, 0, 1)
```

5013. ⠷⠭⠂⠠⠀⠭⠆⠠⠀⠭⠒⠾

(x_1, x_2, x_3)

```
(x_1, x_2, x_3)
```

5014. ⠭⠂⠑⠂⠬⠭⠆⠑⠆⠬⠭⠒⠑⠒

$x_1 e_1 + x_2 e_2 + x_3 e_3$

```
x_1 e_1 + x_2 e_2 + x_3 e_3
```

5015. ⠑⠂⠠⠀⠑⠆⠠⠀⠑⠒

e_1, e_2, e_3

```
e_1, e_2, e_3
```

5016. StartSet⠷⠒⠠⠀⠆⠠⠀⠂⠾⠠⠀⠷⠒⠠⠀⠆⠠⠀⠴⠾⠠⠀⠷⠂⠠⠀⠂⠠⠀⠂⠾EndSet

$\{(3, 2, 1), (3, 2, 0), (1, 1, 1)\}$

```
\{ (3, 2, 1), (3, 2, 0), (1, 1, 1) \}
```

5017. StartSet⠂⠠⠀⠜⠆⠻EndSet

$\{1, \sqrt{2}\, \}$

```
\{1, \sqrt{2}\, \}
```

5018. StartSet⠂⠬⠜⠆⠻⠠⠀⠂⠤⠜⠆⠻EndSet

$\{1 + \sqrt{2}, 1 - \sqrt{2}\, \}$

```
\{1 + \sqrt{2}, 1 - \sqrt{2}\, \}
```

5019. StartSet⠑⠂ ⠠⠑⠆ ⠄⠄ ⠠⠑ ⠰⠍ ⠐EndSet

$\{e_1, e_2, \ldots, e_m\}$

\{ e_1, e_2, \ldots, e_m \}

5020. StartSet⠋⠂ ⠠⠋⠆ ⠄⠄ ⠠⠋ ⠰⠝ ⠐EndSet

$\{f_1, f_2, \ldots, f_n\}$

\{ f_1, f_2, \ldots, f_n \}

5021. ⠙⠊⠍ ⠠⠧ ⠶ ⠝

$\dim V = n$

\dim V =n

5022. ⠠⠎ ⠶ StartSet⠧⠂ ⠄⠄ ⠠⠧ ⠰⠝ ⠐EndSet

$S = \{v_1, \ldots, v_n\}$

S = \{v_1, \ldots, v_n \}

5023. ⠠⠎ ⠶ StartSet⠧⠂ ⠄⠄ ⠠⠧ ⠰⠅ ⠐EndSet

$S = \{v_1, \ldots, v_k\}$

S = \{v_1, \ldots, v_k \}

5024. ⠧ ⠰⠅ ⠬⠂ ⠐⠄⠄ ⠠⠧ ⠰⠝ ⠐

$v_{k+1}, \ldots, v_n$

v_{k + 1}, \ldots, v_n

5025. StartSet⠧⠂ ⠄⠄ ⠠⠧ ⠰⠅ ⠐⠠⠧ ⠰⠅ ⠬⠂ ⠐⠄⠄ ⠠⠧ ⠰⠝ ⠐EndSet

$\{v_1, \ldots, v_k, v_{k+1}, \ldots, v_n\}$

\{v_1, \ldots, v_k, v_{k + 1}, \ldots, v_n \}

5026. ⠈ ⠠⠟⠷⠠⠜⠆ ⠻⠾

$\mathbb{Q}(\sqrt{2}\,)$

{\mathbb Q }(\sqrt{2}\,)

5027. ⠈ ⠠⠟⠷⠠⠜⠆ ⠻ ⠠⠠⠜⠒⠻⠾

$\mathbb{Q}(\sqrt{2}, \sqrt{3}\,)$

{\mathbb Q }(\sqrt{2}, \sqrt{3}\,)

5028. ⠁ ⠬⠃ ⠠⠜⠆ ⠻⠬⠉⠠⠜⠒⠻⠬⠙⠠⠜⠖⠻

$a + b\sqrt{2} + c\sqrt{3} + d\sqrt{6}$

a + b \sqrt{2} + c \sqrt{3} + d \sqrt{6}

5029. ⠁ ⠠⠃ ⠠⠉ ⠠⠙

a, b, c, d

a, b, c, d

5030. ⠈ ⠠⠟⠷⠠⠜⠆ ⠻ ⠠⠠⠜⠒⠻⠾

$\mathbb{Q}(\sqrt{2}, \sqrt{3}\,)$

{\mathbb Q}(\sqrt{2}, \sqrt{3}\,)

5031. StartSet⠂ ⠠⠠⠜⠆ ⠻ ⠠⠠⠜⠒⠻ ⠠⠠⠜⠖⠻EndSet

$\{1, \sqrt{2}, \sqrt{3}, \sqrt{6}\,\}$

\{ 1, \sqrt{2}, \sqrt{3}, \sqrt{6}\, \}

5032. ⠠⠏ ⠰⠝

P_n

P_n

5033. StartSet⠂ ⠠⠭ ⠠⠭ ⠘⠆ ⠂ ⠄⠄ ⠠⠭ ⠘⠝⠤⠂ ⠂EndSet

$\{1, x, x^2, \ldots, x^{n-1}\}$

\{ 1, x, x^2, \ldots, x^{n-1} \}

5034. ⠠⠋ ⠘⠝

F^n

F^n

5035. StartSet⠷⠭⠂ ⠠⠭⠆ ⠠⠭⠒⠾⠒⠭⠂ ⠤⠆ ⠭⠆ ⠤⠭⠒ ⠶ ⠴EndSet

$\{(x_1, x_2, x_3) : 3x_1 - 2x_2 + x_3 = 0\}$

\{ (x_1, x_2, x_3) : 3 x_1 - 2 x_2 + x_3 = 0 \}

5036. ⠨⠷⠷⠭⠂ ⠠⠭⠆ ⠠⠭⠒⠾⠒⠭⠂ ⠼⠒⠭⠒ ⠶ ⠴ ⠠⠆ ⠭⠂ ⠤⠭⠆ ⠤⠭⠒ ⠶ ⠴ ⠸⠾

$\{(x_1, x_2, x_3) : 3x_1 + 4x_3 = 0, 2x_1 - x_2 + x_3 = 0\}$

\{ (x_1, x_2, x_3) : 3 x_1 + 4 x_3 = 0, 2 x_1 - x_2 + x_3 = 0 \}

5037. StartSet⠷⠭⠂ ⠠⠭⠆ ⠠⠭⠒⠾⠭⠂ ⠤⠆ ⠭⠆ ⠬⠆ ⠭⠒ ⠶ ⠆EndSet

$\{(x_1, x_2, x_3) : x_1 - 2x_2 + 2x_3 = 2\}$

\{ (x_1, x_2, x_3) : x_1 - 2 x_2 + 2 x_3 = 2 \}

5038. StartSet⠷⠭⠂ ⠠⠭⠆ ⠠⠭⠒⠾⠒⠭⠂ ⠤⠆ ⠭⠆ ⠘⠆ ⠐ ⠶ ⠴EndSet

$\{(x_1, x_2, x_3) : 3x_1 - 2x_2^2 = 0\}$

\{ (x_1, x_2, x_3) : 3 x_1 - 2 x_2^2 = 0 \}

5039. StartSet⠷⠂ ⠠⠆ ⠤⠤⠒⠾ ⠠⠷⠴⠆ ⠠⠂ ⠠⠆ ⠾EndSet

$\{(1, 0, -3), (0, 1, 2)\}$

\{(1, 0, -3), (0, 1, 2) \}

5040. ⠷⠭ ⠠⠽ ⠠⠵⠾ ⠈⠑ ⠈ ⠠⠗ ⠘⠒

$(x, y, z) \in \mathbb{R}^3$

(x, y, z) \in {\mathbb R}^3

5041. ⠈⠷⠴ ⠠⠂ ⠈⠾

$[0, 1]$

[0, 1]

5042. ⠠⠉ ⠈⠷⠴ ⠠⠂ ⠈⠾

$C[0, 1]$

C[0, 1]

5043. ⠴ ⠶ ⠨⠁ Baseline⠴ ⠶ ⠨⠁ ⠷⠤⠧⠬⠧⠾ ⠶ ⠨⠁ ⠷⠤⠧⠾⠬ ⠨⠁ ⠧

$0 = \alpha 0 = \alpha(-v + v) = \alpha(-v) + \alpha v$

0 = \alpha 0 = \alpha(-v + v) = \alpha(-v) + \alpha v

5044. ⠤ ⠨⠁ ⠧ ⠶ ⠨⠁ ⠷⠤⠧⠾

$-\alpha v = \alpha(-v)$

- \alpha v = \alpha(-v)

5045. ⠧⠴ ⠶ ⠴⠠⠧⠂ ⠤ ⠠⠧⠝⠂⠉⠑⠠⠧

$v_0 = 0, v_1, \ldots, v_n \in V$

```
v_0 = 0, v_1, \ldots, v_n \in V
```

5046. ⠋⠴ ⠊⠶ ⠴⠠⠋⠂ ⠤ ⠠⠋ ⠫⠂⠉⠑⠠⠋

$\alpha_0 \neq 0, \alpha_1, \ldots, \alpha_n \in F$

```
\alpha_0 \neq 0, \alpha_1, \ldots, \alpha_n \in F
```

5047. ⠋⠴⠧⠴⠲⠄⠄⠤⠋ ⠫⠂⠧⠫⠂ ⠶ ⠴

$\alpha_0 v_0 + \cdots + \alpha_n v_n = 0$

```
\alpha_0 v_0 + \cdots + \alpha_n v_n = 0
```

5048. StartSet ⠇⠴EndSet

$\{\mathbf{0}\}$

```
\{ {\mathbf 0} \}
```

5049. ⠠⠞⠒⠠⠧ ⠳⠕ ⠠⠺

$T : V \to W$

```
T: V \rightarrow W
```

5050. ⠅⠑⠗⠷⠠⠞⠾ ⠶ StartSet⠧ ⠉⠑⠠⠧⠒⠠⠞⠷⠧⠾ ⠶ ⠇⠴EndSet

$\ker(T) = \{v \in V : T(v) = \mathbf{0}\}$

```
\ker(T) = \{ v \in V : T(v) = {\mathbf 0} \}
```

5051. ⠠⠗⠷⠠⠧⠾ ⠶ StartSet⠺ ⠉⠑⠠⠺⠒⠠⠞⠷⠧⠾ ⠶ ⠺forsome⠧ ⠉⠑⠠⠧EndSet

$R(V) = \{w \in W : T(v) = w \text{ for some } v \in V\}$

```
R(V) = \{ w \in W : T(v) = w \text{ for some } v \in V \}
```

5052. ⠠⠞⠒⠠⠧ ⠳⠕ ⠠⠺

$T : V \to W$

```
T : V \rightarrow W
```

5053. ⠅⠑⠗⠷⠠⠞⠾ ⠶ StartSet ⠇⠴EndSet

$\ker(T) = \{\mathbf{0}\}$

```
\ker(T) = \{ \mathbf 0 \}
```

5054. StartSet⠧⠂ ⠤ ⠠⠧⠅ ⠂EndSet

$\{v_1, \ldots, v_k\}$

```
\{ v_1, \ldots, v_k \}
```

5055. StartSet⠧⠂ ⠤ ⠠⠧⠅ ⠂⠠⠧⠅⠢⠂ ⠂⠤ ⠠⠧⠍ ⠂EndSet

$\{v_1, \ldots, v_k, v_{k+1}, \ldots, v_m\}$

```
\{ v_1, \ldots, v_k, v_{k + 1}, \ldots, v_m\}
```

5056. StartSet ⠠⠞⠷⠧⠅⠢⠂ ⠐⠾⠤ ⠠⠠⠞⠷⠧⠍ ⠐⠾EndSet

$\{T(v_{k+1}), \ldots, T(v_m)\}$

```
\{ T(v_{k + 1}), \ldots, T(v_m) \}
```

5057. ⠍⠤⠅

$m - k$

```
m - k
```

5058. ⠙⠊⠍⠀⠠⠧ ⠨⠅ ⠙⠊⠍⠀⠠⠺

$\dim V = \dim W$

`\dim V = \dim W`

5059. ⠥⠠⠧⠈⠑⠅⠑⠗⠷⠠⠞⠾

$u, v \in \ker(T)$

`u, v \in \ker(T)`

5060. ⠥⠬⠧⠠⠨⠁⠧⠈⠑⠅⠑⠗⠷⠠⠞⠾

$u + v, \alpha v \in \ker(T)$

`u + v, \alpha v \in \ker(T)`

5061. ⠅⠑⠗⠷⠠⠞⠾

$\ker(T)$

`\ker(T)`

5062. ⠠⠞⠷⠥⠾ ⠨⠅ ⠠⠞⠷⠧⠾

$T(u) = T(v)$

`T(u) = T(v)`

5063. ⠠⠞⠷⠥⠤⠧⠾ ⠨⠅ ⠠⠞⠷⠥⠾⠤⠠⠞⠷⠧⠾ ⠨⠅ ⠴

$T(u-v) = T(u) - T(v) = 0$

`T(u-v) = T(u) - T(v) = 0`

5064. ⠥⠤⠧ ⠨⠅ ⠴

$u-v = 0$

`u-v = 0`

5065. ⠥ ⠨⠅ ⠧

$u = v$

`u = v`

5066. StartSet⠧⠂⠠⠄⠠⠧⠰⠝⠐EndSet

$\{ v_1, \ldots, v_n \}$

`\{ v_1, \ldots, v_n \}`

5067. StartSet ⠠⠞⠷⠧⠂⠾⠠⠄⠠⠠⠞⠷⠧⠰⠝⠐⠾EndSet

$\{ T(v_1), \ldots, T(v_n) \}$

`\{ T(v_1), \ldots, T(v_n) \}`

5068. ⠠⠥⠬⠠⠧

$U + V$

`U + V`

5069. ⠥⠈⠑⠠⠥

$u \in U$

`u \in U`

5070. ⠠⠥⠨⠩⠠⠧

$U \cap V$

`U \cap V`

5071. ⠠⠥⠬⠠⠧ ⠨⠅ ⠠⠺

$U + V = W$

U + V = W

5072. ⠠⠥⠨⠩⠠⠧ ⠨⠅ ⠸⠴

$U \cap V = \mathbf{0}$

U \cap V = {\mathbf 0}

5073. ⠠⠺ ⠨⠅ ⠠⠥⠫⠉⠸⠫⠬⠻⠠⠧

$W = U \oplus V$

W = U \oplus V

5074. ⠺⠈⠑⠠⠺

$w \in W$

w \in W

5075. ⠺ ⠨⠅ ⠥⠬⠧

$w = u + v$

w = u + v

5076. ⠙⠊⠍⠷⠠⠥⠬⠠⠧⠾ ⠨⠅ ⠙⠊⠍ ⠠⠥⠬⠙⠊⠍ ⠠⠧⠤⠙⠊⠍⠷⠠⠥⠨⠩⠠⠧⠾

$\dim(U + V) = \dim U + \dim V - \dim(U \cap V)$

\dim(U + V) = \dim U + \dim V - \dim(U \cap V)

5077. ⠥⠠⠥⠄⠈⠑⠠⠥

$u, u' \in U$

u, u' \in U

5078. ⠧⠠⠧⠄⠈⠑⠠⠧

$v, v' \in V$

v, v' \in V

5079. ⠠⠓⠕⠍⠷⠠⠧⠠⠠⠺⠾

$\mathrm{Hom}(V, W)$

\Hom(V, W)

5080. ⠠⠎⠠⠠⠞⠈⠑⠠⠓⠕⠍⠷⠠⠧⠠⠠⠺⠾

$S, T \in \mathrm{Hom}(V, W)$

S, T \in \Hom(V, W)

5081. ⠠⠧⠘⠈⠼⠐ ⠨⠅ ⠠⠓⠕⠍⠷⠠⠧⠠⠠⠋⠾

$V^* = \mathrm{Hom}(V, F)$

V^* = \Hom(V, F)

5082. ⠧ ⠨⠅ ⠨⠁⠂⠧⠂⠬⠄⠄⠄⠬⠨⠁⠰⠝⠐⠧⠰⠝

$v = \alpha_1 v_1 + \cdots + \alpha_n v_n$

v = \alpha_1 v_1 + \cdots + \alpha_n v_n

5083. ⠨⠋⠰⠊⠐⠱⠠⠧ ⠫⠕ ⠠⠋

$\phi_i : V \rightarrow F$

\phi_i : V \rightarrow F

5084. ⠨⠋⠰⠊⠐⠷⠧⠾ ⠨⠅ ⠨⠁⠰⠊

$\phi_i (v) = \alpha_i$

\phi_i (v) = \alpha_i

5085. ⠨⠋⠰⠊

ϕ_i

\phi_i

5086. ⠠⠧⠘⠈⠼

V^*

V^*

5087. StartSet⠷⠒⠠⠂⠾⠠⠷⠆⠠⠤⠆⠾EndSet

$\{ (3, 1), (2, -2) \}$

\{ (3, 1), (2, -2) \}

5088. ⠷⠈⠠⠗⠘⠆⠐⠾⠘⠈⠼

$({\mathbb R}^2)^*$

({\mathbb R}^2)^*

5089. ⠠⠧⠘⠈⠼⠈⠼

$V^{* *}$

V^{* *}

5090. ⠨⠇⠰⠧

λ_v

\lambda_v

5091. ⠠⠧⠘⠈⠼⠈⠼

V^{**}

V^{**}

5092. ⠧ ⠫⠳⠒⠒⠕ ⠨⠇⠰⠧

$v \mapsto \lambda_v$

v \mapsto \lambda_v

5093. ⠶⠘⠖⠐ ⠨⠅ 117649

$7^6=117\,649$

7^6=117\,649

5094. StartSet⠂⠠ ⠁⠠ ⠁⠘⠆⠐⠠ ⠁⠘⠒⠐⠠ ⠁⠘⠲⠐⠠ ⠁⠘⠢⠐EndSet

$\{1,\,a,\,a^2,\,a^3,\,a^4,\,a^5\}$

\{1,\,a,\,a^2,\,a^3,\,a^4,\,a^5\}

5095. ⠷⠈⠠⠵⠰⠶⠐⠾⠘⠖

$({\mathbb Z}_7)^6$

({\mathbb Z}_7)^6

5096. ⠠⠥⠬⠠⠺

$U+W$

U+W

5097. ⠠⠥⠬⠠⠺ ⠨⠅ StartSet⠥⠬⠺⠳⠥⠈⠑⠠⠥⠠⠺⠈⠑⠠⠺EndSet

$U+W=\{u+w\mid u\in U,\ w\in W\}$

```
U+W=\{u+w\mid u\in U,\ w\in W\}
```

5098. ⠠⠥⠨⠩⠠⠺

$U\cap W$

```
U\cap W
```

5099. ⠈⠠⠟⠈⠷⠣⠲⠜⠆⠻⠈⠾

${\mathbb Q}[\sqrt[4]{2}]$

```
{\mathbb Q}[\sqrt[4]{2}]
```

5100. ⠉ ⠨⠅ ⠣⠲⠜⠆⠻

$c = \sqrt[4]{2}$

```
c = \sqrt[4]{2}
```

5101. ⠍⠡⠍

$m\times m$

```
m\times m
```

5102. ⠆⠡⠆

2×2

```
2\times 2
```

5103. ⠒⠡⠒

3×3

```
3\times 3
```

5104. ⠼⠆⠠⠼⠒⠠⠼⠲⠠⠼⠢

$2,3,4,5$

```
2,3,4,5
```

5105. ⠠⠋⠘⠍

F^m

```
F^m
```

5106. ⠢⠘⠒

5^3

```
5^3
```

5107. ⠭⠈⠑⠠⠋

$x\in F$

```
x\in F
```

5108. StartSet⠂⠠⠁⠠⠁⠘⠆⠐EndSet

$\{1,a,a^2\}$

```
\{1,a,a^2\}
```

5109. ⠁ ⠫⠳⠒⠒⠕ ⠁⠘⠢

$a\mapsto a^5$

```
a\mapsto a^5
```

5110. ⠠⠑⠈⠷⠭⠈⠾

$E[x]$

E[x]

5111. ⠏⠷⠭⠾ ⠨⠅ ⠭⠘⠲⠐⠤⠢⠭⠘⠆⠐⠬⠖

$p(x) = x^4 - 5x^2 + 6$

p(x) = x^4 -5 x^2 + 6

5112. ⠷⠭⠘⠆⠐⠤⠆⠾⠷⠭⠘⠆⠐⠤⠒⠾

$(x^2 - 2)(x^2 - 3)$

(x^2 - 2)(x^2 - 3)

5113. ⠏⠷⠭⠾ ⠨⠅ ⠷⠭⠤⠜⠆⠻⠾⠷⠭⠬⠜⠆⠻⠾⠷⠭⠤⠜⠒⠻⠾⠷⠭⠬⠜⠒⠻⠾

$p(x) = (x - \sqrt{2})(x + \sqrt{2})(x - \sqrt{3})(x + \sqrt{3})$

p(x) = (x - \sqrt{2}) (x + \sqrt{2})(x - \sqrt{3})(x + \sqrt{3})

5114. ⠈⠠⠟⠷⠜⠆⠻⠾ ⠨⠅ ⠨⠷⠁⠬⠃⠜⠆⠻⠱⠁⠠⠃⠈⠑⠈⠠⠟⠨⠾

$\mathbb{Q}(\sqrt{2}) = \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$

{\mathbb Q }(\sqrt{2}) = \{ a + b \sqrt{2} : a, b \in {\mathbb
Q} \}

5115. ⠠⠋ ⠸⠐⠅ ⠠⠑

$F \subset E$

F \subset E

5116. ⠠⠋ ⠨⠅ ⠈⠠⠟⠷⠜⠆⠻⠾ ⠨⠅ ⠨⠷⠁⠬⠃⠜⠆⠻⠱⠁⠠⠃⠈⠑⠈⠠⠟⠨⠾

$F = \mathbb{Q}(\sqrt{2}\,) = \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$

F = {\mathbb Q}(\sqrt{2}\,) = \{ a + b \sqrt{2} : a, b \in {\mathbb
Q} \}

5117. ⠠⠑ ⠨⠅ ⠈⠠⠟⠷⠜⠆⠻⠬⠜⠒⠻⠾

$E = \mathbb{Q}(\sqrt{2} + \sqrt{3}\,)$

E = {\mathbb Q }(\sqrt{2} + \sqrt{3}\,)

5118. ⠜⠆⠻⠬⠜⠒⠻

$\sqrt{2} + \sqrt{3}$

\sqrt{2} + \sqrt{3}

5119. ⠂⠌⠷⠜⠆⠻⠬⠜⠒⠻⠾ ⠨⠅ ⠜⠒⠻⠤⠜⠆⠻

$1/(\sqrt{2} + \sqrt{3}\,) = \sqrt{3} - \sqrt{2}$

1 / (\sqrt{2} + \sqrt{3}\,) = \sqrt{3} - \sqrt{2}

5120. ⠜⠒⠻⠤⠜⠆⠻

$\sqrt{3} - \sqrt{2}$

\sqrt{3} - \sqrt{2}

5121. ⠜⠒⠻

$\sqrt{3}$

\sqrt{3}

5122. ⠏⠷⠭⠾ ⠨⠅ ⠭⠘⠆⠐⠬⠭⠬⠂⠈⠑⠈⠠⠵⠆⠈⠷⠭⠈⠾

$p(x) = x^2 + x + 1 \in \mathbb{Z}_2[x]$

```
p(x) = x^2 + x + 1 \in {\mathbb Z}_2[x]
```

5123. ⠈⠠⠵⠆⠈⠷⠭⠈⠾⠌⠄⡷⠭⠆⠶⠑⠦⠄⠏⠷⠭⠾⠄⡷⠭⠆⠶⠑⠔⠄

$\mathbb{Z}_2[x]/\langle p(x)\rangle$

```
{\mathbb Z}_2[x] / \langle p(x) \rangle
```

5124. ⠋⠷⠭⠾⠬⠄⡷⠭⠆⠶⠑⠦⠄⠏⠷⠭⠾⠄⡷⠭⠆⠶⠑⠔⠄

$f(x) + \langle p(x)\rangle$

```
f(x) + \langle p(x) \rangle
```

5125. ⠋⠷⠭⠾ ⠨⠅ ⠷⠭⠘⠆⠐⠬⠭⠬⠂⠾⠟⠷⠭⠾⠬⠗⠷⠭⠾

$f(x) = (x^2 + x + 1)q(x) + r(x)$

```
f(x) = (x^2 + x + 1) q(x) + r(x)
```

5126. ⠭⠘⠆⠐⠬⠭⠬⠂

$x^2 + x + 1$

```
x^2 + x + 1
```

5127. ⠋⠷⠭⠾⠬⠄⡷⠭⠆⠶⠑⠦⠄⠭⠘⠆⠐⠬⠭⠬⠂⠄⡷⠭⠆⠶⠑⠔⠄ ⠨⠅ ⠗⠷⠭⠾⠬⠄⡷⠭⠆⠶⠑⠦⠄⠭⠘⠆⠐⠬⠭⠬⠂⠄⡷⠭⠆⠶⠑⠔⠄

$f(x) + \langle x^2 + x + 1\rangle = r(x) + \langle x^2 + x + 1\rangle$

```
f(x) + \langle x^2 + x + 1 \rangle = r(x) + \langle x^2 + x + 1
\rangle
```

5128. ⠂⠬⠭

$1 + x$

```
1 + x
```

5129. ⠠⠑ ⠨⠅ ⠈⠠⠵⠆⠈⠷⠭⠈⠾⠌⠄⡷⠭⠆⠶⠑⠦⠄⠭⠘⠆⠐⠬⠭⠬⠂⠄⡷⠭⠆⠶⠑⠔⠄

$E = \mathbb{Z}_2[x]/\langle x^2 + x + 1\rangle$

```
E = {\mathbb Z}_2[x] / \langle x^2 + x + 1 \rangle
```

5130. ⠈⠠⠵⠆⠷⠨⠁⠾

$\mathbb{Z}_2(\alpha)$

```
{\mathbb Z}_2( \alpha)
```

5131. ⠨⠁⠘⠆⠐⠬⠨⠁⠬⠂ ⠨⠅ ⠴

$\alpha^2 + \alpha + 1 = 0$

```
{\alpha}^2 + {\alpha} + 1 = 0
```

5132. ⠷⠂⠬⠨⠁⠾⠘⠆

$(1 + \alpha)^2$

```
(1 + \alpha)^2
```

5133. ⠷⠂⠬⠨⠁⠾⠷⠂⠬⠨⠁⠾ ⠨⠅ ⠂⠬⠨⠁⠬⠨⠁⠬⠷⠨⠁⠾⠘⠆⠐ ⠨⠅ ⠨⠁

$(1 + \alpha)(1 + \alpha) = 1 + \alpha + \alpha + (\alpha)^2 = \alpha$

```
(1 + \alpha)(1 + \alpha)= 1 + \alpha + \alpha + (\alpha)^2 = \alpha
```

5134. ⠈⠠⠵⠆⠷⠨⠁⠾

$\mathbb{Z}_2(\alpha)$

`{\mathbb Z}_2(\alpha)`

5135. StartLayout1stRow⠬ ⠼⠴⠆ ⠼⠂ ⠨⠁ ⠂⠬⠨⠁ 2ndRow⠼⠴⠆ ⠼⠴⠆ ⠼⠂ ⠨⠁ ⠂⠬⠨⠁ 3rdRow⠼⠂ ⠼⠂ ⠼

$$\begin{array}{c|cccc} + & 0 & 1 & \alpha & 1 + \alpha \\ \hline 0 & 0 & 1 & \alpha & 1 + \alpha \\ 1 & 1 & 0 & 1 + \alpha & \alpha \\ \alpha & \alpha & 1 + \alpha & 0 & 1 \\ 1 + \alpha & 1 + \alpha & \alpha & 1 & 0 \end{array}$$

```
\begin{array}{c|cccc} + & 0 & 1 & \alpha & 1 + \alpha \\ \hline
0 & 0 & 1 & \alpha & 1 + \alpha \\ 1 & 1 & 0 & 1 + \alpha & \alpha
\\ \alpha & \alpha & 1 + \alpha & 0 & 1 \\ 1 + \alpha & 1 + \alpha
& \alpha & 1 & 0 \end{array}
```

5136. StartLayout1stRow⠡ ⠼⠴⠆ ⠼⠂ ⠨⠁ ⠂⠬⠨⠁ 2ndRow⠼⠴⠆ ⠼⠴⠆ ⠼⠴⠆ ⠼⠴⠆ ⠼⠴⠆3rdRow⠼⠂ ⠼⠴⠆ ⠼⠂

$$\begin{array}{c|cccc} \cdot & 0 & 1 & \alpha & 1 + \alpha \\ \hline 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & \alpha & 1 + \alpha \\ \alpha & 0 & \alpha & 1 + \alpha & 1 \\ 1 + \alpha & 0 & 1 + \alpha & 1 & \alpha \end{array}$$

```
\begin{array}{c|cccc} \cdot & 0 & 1 & \alpha & 1 + \alpha \\ \hline
0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & \alpha & 1 + \alpha \\ \alpha &
0 & \alpha & 1 + \alpha & 1 \\ 1 + \alpha & 0 & 1 + \alpha & 1 &
\alpha \end{array}
```

5137. ⠨⠁ ⠈⠑ ⠠⠑

$\alpha \in E$

`\alpha \in E`

5138. ⠠⠋ ⠈⠷⠭ ⠈⠾⠠⠂ ⠳⠭⠆ ⠿⠈⠱⠄ ⠏⠷⠭⠾⠄ ⠳⠭⠆ ⠿⠈⠢⠂

$F[x]/\langle p(x) \rangle$

`F[x]/\langle p(x) \rangle`

5139. ⠠⠑ ⠨⠅ ⠠⠋ ⠈⠷⠭ ⠈⠾⠠⠂ ⠳⠭⠆ ⠿⠈⠱⠄ ⠏⠷⠭⠾⠄ ⠳⠭⠆ ⠿⠈⠢⠂

$E = F[x]/\langle p(x) \rangle$

`E = F[x]/\langle p(x) \rangle`

5140. ⠨⠽⠒⠠⠋ ⠫⠕ ⠠⠋ ⠈⠷⠭ ⠈⠾⠠⠂ ⠳⠭⠆ ⠿⠈⠱⠄ ⠏⠷⠭⠾⠄ ⠳⠭⠆ ⠿⠈⠢⠂

$\psi : F \rightarrow F[x]/\langle p(x) \rangle$

`\psi:F \rightarrow F[x]/\langle p(x) \rangle`

5141. ⠨⠽⠷⠁⠾ ⠨⠅ ⠁⠬⠄ ⠳⠭⠆ ⠿⠈⠱⠄ ⠏⠷⠭⠾⠄ ⠳⠭⠆ ⠿⠈⠢⠂

$\psi(a) = a + \langle p(x) \rangle$

`\psi(a) = a + \langle p(x)\rangle`

5142. ⠨⠽⠷⠁⠾⠬⠨⠽⠷⠃⠾ ⠨⠅ ⠷⠁⠬⠄ ⠳⠭⠆ ⠿⠈⠱⠄ ⠏⠷⠭⠾⠄ ⠳⠭⠆ ⠿⠈⠢⠂ ⠾⠬⠷⠃⠬⠄ ⠳⠭⠆ ⠿⠈⠱⠄ ⠏⠷⠭⠾⠄ ⠳⠭⠆ ⠿⠈⠢

$\psi(a) + \psi(b) = (a + \langle p(x) \rangle) + (b + \langle p(x) \rangle) = (a + b) + \langle p(x) \rangle = \psi(a + b)$

```
\psi( a ) + \psi( b ) = (a + \langle p(x) \rangle) + (b + \langle
p(x) \rangle) = (a + b) + \langle p(x) \rangle = \psi( a + b )
```

5143. ⠨⠽⠷⠁⠾⠨⠽⠷⠃⠾⠀⠨⠅⠀⠷⠁⠬⠨⠷⠏⠷⠭⠾⠨⠾⠾⠷⠃⠬⠨⠷⠏⠷⠭⠾⠨⠾⠾⠀⠨⠅⠀⠁⠃⠬⠨⠷⠏⠷⠭⠾⠨⠾⠀⠨⠅⠀⠨⠽⠷⠁⠃⠾

$\psi(a)\psi(b) = (a + \langle p(x)\rangle)(b + \langle p(x)\rangle) = ab + \langle p(x)\rangle = \psi(ab)$

\psi(a) \psi(b) = (a + \langle p(x) \rangle) (b + \langle p(x)
\rangle) = ab + \langle p(x) \rangle = \psi(ab)

5144. ⠁⠬⠨⠷⠏⠷⠭⠾⠨⠾⠀⠨⠅⠀⠨⠽⠷⠁⠾⠀⠨⠅⠀⠨⠽⠷⠃⠾⠀⠨⠅⠀⠃⠬⠨⠷⠏⠷⠭⠾⠨⠾

$a + \langle p(x)\rangle = \psi(a) = \psi(b) = b + \langle p(x)\rangle$

a + \langle p(x) \rangle = \psi(a) = \psi(b) = b + \langle p(x)
\rangle

5145. ⠁⠤⠃⠀⠨⠅⠀⠼⠴

$a - b = 0$

a - b = 0

5146. StartSet⠁⠬⠨⠷⠏⠷⠭⠾⠨⠾⠀⠸⠒⠀⠁⠀⠈⠑⠀⠠⠋EndSet

$\{a + \langle p(x)\rangle : a \in F\}$

\{ a + \langle p(x) \rangle : a \in F \}

5147. ⠨⠁⠀⠨⠅⠀⠭⠬⠨⠷⠏⠷⠭⠾⠨⠾

$\alpha = x + \langle p(x)\rangle$

\alpha = x + \langle p(x) \rangle

5148. ⠏⠷⠭⠾⠀⠨⠅⠀⠁⠴⠬⠁⠂⠭⠬⠄⠄⠄⠬⠁⠰⠝⠐⠭⠘⠝

$p(x) = a_0 + a_1 x + \cdots + a_n x^n$

p(x) = a_0 + a_1 x + \cdots + a_n x^n

5149. ⠨⠁⠀⠈⠑⠀⠠⠑⠀⠨⠅⠀⠠⠋⠈⠷⠭⠈⠾⠸⠌⠨⠷⠏⠷⠭⠾⠨⠾

$\alpha \in E = F[x]/\langle p(x)\rangle$

\alpha \in E = F[x]/\langle p(x) \rangle

5150. ⠏⠷⠭⠾⠀⠨⠅⠀⠭⠘⠢⠐⠬⠭⠘⠲⠐⠬⠂⠀⠈⠑⠀⠨⠰⠠⠵⠆⠈⠷⠭⠈⠾

$p(x) = x^5 + x^4 + 1 \in \mathbb{Z}_2[x]$

p(x) = x^5 + x^4 + 1 \in {\mathbb Z}_2[x]

5151. ⠨⠰⠠⠵⠆⠈⠷⠭⠈⠾⠸⠌⠨⠷⠭⠘⠆⠐⠬⠭⠬⠂⠨⠾

$\mathbb{Z}_2[x]/\langle x^2 + x + 1\rangle$

{\mathbb Z}_2[x] / \langle x^2 + x + 1 \rangle

5152. ⠨⠰⠠⠵⠆⠈⠷⠭⠈⠾⠸⠌⠨⠷⠭⠘⠒⠐⠬⠭⠬⠂⠨⠾

$\mathbb{Z}_2[x]/\langle x^3 + x + 1\rangle$

{\mathbb Z}_2[x] / \langle x^3 + x + 1 \rangle

5153. ⠋⠷⠨⠁⠾⠀⠨⠅⠀⠼⠴

$f(\alpha) = 0$

f(\alpha)=0

5154. ⠠⠋⠷⠨⠁⠂⠠⠀⠄⠄⠄⠠⠀⠨⠁⠰⠝⠐⠾

$F(\alpha_1, \ldots, \alpha_n)$

F(\alpha_1, \ldots, \alpha_n)

5155. ⠠⠑⠀⠨⠅⠀⠠⠋⠷⠨⠁⠾

$E = F(\alpha)$

E = F(\alpha)

5156. ⠭⠘⠆⠐⠤⠆

$x^2 - 2$

x^2 -2

5157. ⠨⠏⠬⠑

$\pi + e$

\pi + e

5158. ⠜⠆⠬⠨⠜⠒⠨⠻⠻

$\sqrt{2 + \sqrt{3}}$

\sqrt{2 + \sqrt{3} }

5159. ⠨⠁⠀⠨⠅⠀⠜⠆⠬⠨⠜⠒⠨⠻⠻

$\alpha = \sqrt{2 + \sqrt{3}}$

\alpha = \sqrt{2 + \sqrt{3} }

5160. ⠨⠁⠘⠆⠐⠀⠨⠅⠀⠆⠬⠜⠒⠻

$\alpha^2 = 2 + \sqrt{3}$

\alpha^2 = 2 + \sqrt{3}

5161. ⠨⠁⠘⠆⠐⠤⠆⠀⠨⠅⠀⠜⠒⠻

$\alpha^2 - 2 = \sqrt{3}$

\alpha^2 - 2 = \sqrt{3}

5162. ⠷⠨⠁⠘⠆⠐⠤⠆⠾⠘⠆⠐⠀⠨⠅⠀⠒

$(\alpha^2 - 2)^2 = 3$

(\alpha^2 - 2)^2 = 3

5163. ⠨⠁⠘⠲⠐⠤⠲⠨⠁⠘⠆⠐⠬⠂⠀⠨⠅⠀⠴

$\alpha^4 - 4\alpha^2 + 1 = 0$

\alpha^4 - 4 \alpha^2 + 1 = 0

5164. ⠭⠘⠲⠐⠤⠲⠭⠘⠆⠬⠂⠈⠑⠈⠠⠟⠈⠷⠭⠈⠾

$x^4 - 4x^2 + 1 \in \mathbb{Q}[x]$

x^4 - 4 x^2 + 1 \in {\mathbb Q}[x]

5165. ⠠⠋⠷⠨⠁⠾

$F(\alpha)$

F(\alpha)

5166. ⠨⠋⠰⠨⠁⠐⠱⠠⠋⠈⠷⠭⠈⠾⠀⠫⠕⠀⠠⠑

$\phi_\alpha : F[x] \to E$

\phi_{\alpha} : F[x] \rightarrow E

5167. ⠨⠋⠰⠨⠁⠐⠷⠏⠷⠭⠾⠾⠀⠨⠅⠀⠏⠷⠨⠁⠾⠀⠌⠨⠅⠀⠴

$\phi_\alpha(p(x)) = p(\alpha) \neq 0$

\phi_{\alpha} (p(x)) = p(\alpha) \neq 0

5168. ⠅⠑⠗ ⠨⠋⠰⠨⠁ ⠐ ⠨⠅ StartSet⠴EndSet

$\ker \phi_{\alpha} = \{0\}$

\ker \phi_{\alpha} = \{ 0 \}

5169. ⠋⠷⠨⠁⠾ ⠨⠅ ⠴

$f(\alpha) = 0$

f(\alpha) = 0

5170. ⠋⠷⠨⠁⠾ ⠨⠅ ⠴

$f(\alpha) = 0$

f(\alpha) = 0

5171. ⠋⠷⠭⠾ ⠈⠑⠨⠷⠏⠷⠭⠾⠨⠾ ⠋⠷⠭⠾⠨⠷⠏⠷⠭⠾⠨⠾

$f(x) \in \langle p(x) \rangle$

f(x) \in \langle p(x) \rangle

5172. ⠨⠃⠏⠷⠭⠾

$\beta p(x)$

\beta p(x)

5173. ⠨⠃ ⠈⠑⠠⠋

$\beta \in F$

\beta \in F

5174. ⠏⠷⠭⠾ ⠨⠅ ⠗⠷⠭⠾⠎⠷⠭⠾

$p(x) = r(x)s(x)$

p(x) = r(x) s(x)

5175. ⠗⠷⠨⠁⠾⠎⠷⠨⠁⠾ ⠨⠅ ⠴

$r(\alpha)s(\alpha) = 0$

r(\alpha) s(\alpha) = 0

5176. ⠗⠷⠨⠁⠾ ⠨⠅ ⠴

$r(\alpha) = 0$

r(\alpha)=0

5177. ⠎⠷⠨⠁⠾ ⠨⠅ ⠴

$s(\alpha) = 0$

s(\alpha) = 0

5178. ⠋⠷⠭⠾ ⠨⠅ ⠭⠘⠆⠐⠤⠆

$f(x) = x^2 - 2$

f(x) = x^2 - 2

5179. ⠛⠷⠭⠾ ⠨⠅ ⠭⠘⠲⠐⠤⠲⠭⠘⠆⠐⠬⠂

$g(x) = x^4 - 4x^2 + 1$

g(x) = x^4 - 4 x^2 + 1

5180. ⠠⠋⠷⠨⠁⠾ ⠈⠱⠈⠱ ⠠⠋⠈⠷⠭⠈⠾⠸⠌⠨⠷⠏⠷⠭⠾⠨⠾

$F(\alpha) \cong F[x]/\langle p(x) \rangle$

F(\alpha) \cong F[x] / \langle p(x) \rangle

5181. ⠨⠃⠈⠑⠠⠑

$\beta \in E$

\beta \in E

5182. ⠨⠃ ⠨⠅ ⠃⠴⠬⠃⠂⠨⠁⠬⠄⠄⠄⠬⠃⠰⠝⠤⠂⠐⠨⠁⠘⠝⠤⠂

$\beta = b_0 + b_1 \alpha + \cdots + b_{n-1} \alpha^{n-1}$

\beta = b_0 + b_1 \alpha + \cdots + b_{n-1} \alpha^{n - 1}

5183. ⠃⠰⠊⠐⠈⠑⠠⠋

$b_i \in F$

b_i \in F

5184. ⠨⠋⠰⠨⠁⠐⠷⠠⠋⠈⠷⠭⠈⠾⠾ ⠈⠱⠈⠱ ⠠⠋⠷⠨⠁⠾

$\phi_{\alpha}(F[x]) \cong F(\alpha)$

\phi_{\alpha} (F[x]) \cong F(\alpha)

5185. ⠨⠋⠰⠨⠁⠐⠷⠋⠷⠭⠾⠾ ⠨⠅ ⠋⠷⠨⠁⠾

$\phi_{\alpha}(f(x)) = f(\alpha)$

\phi_{\alpha} (f(x)) = f(\alpha)

5186. ⠋⠷⠨⠁⠾

$f(\alpha)$

f(\alpha)

5187. ⠨⠁⠘⠝⠐ ⠨⠅ ⠤⠁⠰⠝⠤⠂⠐⠨⠁⠘⠝⠤⠂⠐⠤⠄⠄⠄⠤⠁⠴

$\alpha^n = -a_{n-1} \alpha^{n-1} - \cdots - a_0$

{\alpha}^n = - a_{n - 1} {\alpha}^{n - 1} - \cdots - a_0

5188. ⠨⠁⠘⠍

α^m

{\alpha}^m

5189. ⠍ ⠨⠂⠱ ⠝

$m \geq n$

m \geq n

5190. ⠨⠁

α

{\alpha}

5191. ⠨⠃⠈⠑⠠⠋⠷⠨⠁⠾

$\beta \in F(\alpha)$

\beta \in F(\alpha)

5192. ⠨⠃ ⠨⠅ ⠃⠴⠬⠃⠂⠨⠁⠬⠄⠄⠄⠬⠃⠰⠝⠤⠂⠐⠨⠁⠘⠝⠤⠂

$\beta = b_0 + b_1 \alpha + \cdots + b_{n-1} \alpha^{n-1}$

\beta = b_0 + b_1 \alpha + \cdots + b_{n - 1} \alpha^{n - 1}

5193. ⠨⠃ ⠶ ⠃⠴⠬⠃⠂⠨⠁⠬⠄⠄⠄⠬⠃⠆⠝⠤⠂⠐⠨⠁⠘⠝⠤⠂⠐ ⠶ ⠉⠴⠬⠉⠂⠨⠁⠬⠄⠄⠄⠬⠉⠆⠝⠤⠂⠐⠨⠁⠘⠝⠤⠂

$\beta = b_0 + b_1 \alpha + \cdots + b_{n-1} \alpha^{n-1} = c_0 + c_1 \alpha + \cdots + c_{n - 1} \alpha^{n - 1}$

\beta = b_0 + b_1 \alpha + \cdots + b_{n-1} \alpha^{n-1} = c_0 +
c_1 \alpha + \cdots + c_{n - 1} \alpha^{n - 1}

5194. ⠛⠷⠭⠾ ⠶ ⠷⠃⠴⠤⠉⠴⠾⠬⠷⠃⠂⠤⠉⠂⠾⠭⠬⠄⠄⠄⠬⠷⠃⠆⠝⠤⠂⠐⠤⠉⠆⠝⠤⠂⠐⠾⠭⠘⠝⠤⠂

$g(x) = (b_0 - c_0) + (b_1 - c_1) x + \cdots + (b_{n - 1} - c_{n - 1})x^{n - 1}$

g(x) = (b_0 - c_0) + (b_1 - c_1) x + \cdots + (b_{n - 1} - c_{n
- 1})x^{n - 1}

5195. ⠛⠷⠨⠁⠾ ⠶ ⠴

$g(\alpha) = 0$

g(\alpha) = 0

5196. ⠏⠷⠭⠾

$p(x)$

p(x)

5197. ⠃⠴⠤⠉⠴ ⠶ ⠃⠂⠤⠉⠂ ⠶ ⠄⠄⠄ ⠶ ⠃⠆⠝⠤⠂⠐⠤⠉⠆⠝⠤⠂⠐ ⠶ ⠴

$b_0 - c_0 = b_1 - c_1 = \cdots = b_{n - 1} - c_{n - 1} = 0$

b_0 - c_0 = b_1 - c_1 = \cdots = b_{n - 1} - c_{n - 1} = 0

5198. ⠃⠰⠊⠐ ⠶ ⠉⠰⠊

$b_i = c_i$

b_i = c_i

5199. ⠨⠷⠭⠘⠆⠐⠬⠂⠨⠾

$\langle x^2 + 1 \rangle$

\langle x^2 + 1 \rangle

5200. ⠈⠠⠗⠈⠷⠭⠈⠾

${\mathbb R}[x]$

{\mathbb R}[x]

5201. ⠠⠑ ⠶ ⠈⠠⠗⠈⠷⠭⠈⠾⠸⠌⠨⠷⠭⠘⠆⠐⠬⠂⠨⠾

$E = {\mathbb R}[x]/\langle x^2 + 1 \rangle$

E = {\mathbb R}[x]/\langle x^2 + 1 \rangle

5202. ⠨⠁ ⠶ ⠭⠬⠨⠷⠭⠘⠆⠐⠬⠂⠨⠾

$\alpha = x + \langle x^2 + 1 \rangle$

\alpha = x + \langle x^2 + 1 \rangle

5203. ⠈⠠⠗⠷⠨⠁⠾ ⠶ ⠸⠷⠁⠬⠃⠨⠁ ⠐⠁⠠⠃ ⠈⠑⠈⠠⠗⠸⠾

${\mathbb R}(\alpha) = \{ a + b \alpha : a, b \in {\mathbb R} \}$

{\mathbb R}(\alpha) = \{ a + b \alpha : a, b \in {\mathbb R} \}

5204. ⠨⠁⠘⠆ ⠶ ⠤⠂

$\alpha^2 = -1$

\alpha^2 = -1

5205. ⠈⠠⠗⠷⠨⠁⠾

$\mathbb{R}(\alpha)$

{\mathbb R}(\alpha)

5206. ⠁⠬⠃⠨⠁

$a + b\alpha$

a + b \alpha

5207. ⠠⠑⠀⠨⠅⠀⠠⠋⠷⠨⠁⠾

$E = F(\alpha)$

E = F(\alpha)

5208. StartSet⠂⠠⠨⠁⠠⠨⠁⠘⠆⠐⠠⠄⠄⠄⠠⠨⠁⠘⠝⠤⠂⠐EndSet

$\{1, \alpha, \alpha^2, \ldots, \alpha^{n-1}\}$

\{ 1, \alpha, {\alpha}^2, \ldots, {\alpha}^{n - 1} \}

5209. ⠈⠷⠠⠑⠒⠠⠋⠈⠾⠀⠨⠅⠀⠝

$[E : F] = n$

[E:F]= n

5210. ⠈⠷⠠⠑⠒⠠⠋⠈⠾⠀⠨⠅⠀⠝

$[E : F] = n$

[E:F] = n

5211. ⠼⠂⠠⠨⠁⠠⠄⠄⠄⠠⠨⠁⠘⠝

$1, \alpha, \ldots, \alpha^n$

1, \alpha, \ldots, {\alpha}^n

5212. ⠁⠰⠊⠐⠈⠑⠠⠋

$a_i \in F$

a_i \in F

5213. ⠁⠰⠝⠐⠨⠁⠘⠝⠐⠬⠁⠰⠝⠤⠂⠐⠨⠁⠘⠝⠤⠂⠐⠬⠄⠄⠄⠬⠁⠰⠂⠐⠨⠁⠬⠁⠰⠴⠀⠨⠅⠀⠼⠴

$a_n\alpha^n + a_{n-1}\alpha^{n-1} + \cdots + a_1\alpha + a_0 = 0$

a_n {\alpha}^n + a_{n - 1} {\alpha}^{n - 1} + \cdots + a_1 \alpha
+ a_0 = 0

5214. ⠏⠷⠭⠾⠀⠨⠅⠀⠁⠰⠝⠐⠭⠘⠝⠐⠬⠄⠄⠄⠬⠁⠰⠴⠐⠈⠑⠠⠋⠈⠷⠭⠈⠾

$p(x) = a_n x^n + \cdots + a_0 \in F[x]$

p(x) = a_n x^n + \cdots + a_0 \in F[x]

5215. ⠈⠷⠠⠅⠒⠠⠋⠈⠾⠀⠨⠅⠀⠈⠷⠠⠅⠒⠠⠑⠈⠾⠈⠷⠠⠑⠒⠠⠋⠈⠾

$[K : F] = [K : E][E : F]$

[K:F]= [K:E] [E:F]

5216. StartSet⠨⠁⠂⠠⠄⠄⠄⠠⠨⠁⠰⠝⠐EndSet

$\{\alpha_1, \ldots, \alpha_n\}$

\{ \alpha_1, \ldots, \alpha_n \}

5217. StartSet ⠨⠃⠂⠠ ⠄⠄⠄⠠ ⠨⠃⠰⠍⠐EndSet

$\{\beta_1, \ldots, \beta_m\}$

\{ \beta_1, \ldots, \beta_m \}

5218. ⠨⠷⠨⠁⠰⠊⠐⠨⠃⠰⠚⠐⠨⠾

$\{\alpha_i \beta_j\}$

\{ \alpha_i \beta_j \}

5219. ⠥ ⠈⠑ ⠠⠅

$u \in K$

u \in K

5220. ⠥ ⠨⠅ ⠐⠨⠠⠎⠩⠚ ⠨⠅ ⠂⠣⠍⠻⠃⠰⠚⠐⠨⠃⠰⠚

$u = \sum_{j=1}^{m} b_j \beta_j$

u = \sum_{j = 1}^{m} b_j \beta_j

5221. ⠃⠰⠚ ⠨⠅ ⠐⠨⠠⠎⠩⠊ ⠨⠅ ⠂⠣⠝⠻⠁⠰⠊⠚⠐⠨⠁⠰⠊

$b_j = \sum_{i=1}^{n} a_{ij} \alpha_i$

b_j = \sum_{i = 1}^{n} a_{ij} \alpha_i

5222. ⠃⠰⠚⠐⠈⠑⠠⠑

$b_j \in E$

b_j \in E

5223. ⠁⠰⠊⠚⠐⠈⠑⠠⠋

$a_{ij} \in F$

a_{ij} \in F

5224. ⠥ ⠨⠅ ⠐⠨⠠⠎⠩⠚ ⠨⠅ ⠂⠣⠍⠻⠷⠐⠨⠠⠎⠩⠊ ⠨⠅ ⠂⠣⠝⠻⠁⠰⠊⠚⠐⠨⠁⠰⠊⠾⠨⠃⠰⠚ ⠨⠅ ⠐⠨⠠⠎⠩⠊⠠⠚

$u = \sum_{j=1}^{m} \left(\sum_{i=1}^{n} a_{ij} \alpha_i \right) \beta_j = \sum_{i,j} a_{ij} (\alpha_i \beta_j)$

u = \sum_{j = 1}^{m} \left(\sum_{i = 1}^{n} a_{ij} \alpha_i \right)
\beta_j = \sum_{i,j} a_{ij} (\alpha_i \beta_j)

5225. ⠨⠁⠰⠊⠐⠨⠃⠰⠚

$\alpha_i \beta_j$

\alpha_i \beta_j

5226. ⠉⠂⠧⠂⠬⠉⠆⠧⠆⠬⠄⠄⠄⠬⠉⠰⠝⠐⠧⠰⠝ ⠨⠅ ⠴

$c_1 v_1 + c_2 v_2 + \cdots + c_n v_n = 0$

c_1 v_1 + c_2 v_2 + \cdots + c_n v_n = 0

5227. ⠉⠂ ⠨⠅ ⠉⠆ ⠨⠅ ⠄⠄⠄ ⠨⠅ ⠉⠰⠝ ⠨⠅ ⠴

$c_1 = c_2 = \cdots = c_n = 0$

c_1 = c_2 = \cdots = c_n = 0

5228. ⠥ ⠨⠅ ⠐⠨⠠⠎⠩⠊⠪⠚⠐⠉⠰⠊⠚⠐⠷⠨⠁⠰⠊⠐⠨⠃⠰⠚⠐⠾ ⠨⠅ ⠴

$u = \sum_{i,j} c_{ij} (\alpha_i \beta_j) = 0$

u = \sum_{i,j} c_{ij} (\alpha_i \beta_j) = 0

5229. ⠉⠰⠊⠚⠐⠈⠑⠠⠋

$c_{ij} \in F$

c_{ij} \in F

5230. ⠉⠰⠊⠚

c_{ij}

c_{ij}

5231. ⠐⠨⠠⠎⠩⠚⠀⠨⠅⠀⠂⠣⠍⠻⠷⠐⠨⠠⠎⠩⠊⠀⠨⠅⠀⠂⠣⠝⠻⠉⠰⠊⠚⠐⠨⠁⠰⠊⠐⠾⠨⠃⠰⠚⠐⠀⠨⠅⠀⠴

$\sum_{j=1}^{m} \left(\sum_{i=1}^{n} c_{ij} \alpha_i \right) \beta_j = 0$

\sum_{j = 1}^{m} \left(\sum_{i = 1}^{n} c_{ij} \alpha_i \right)
\beta_j = 0

5232. ⠐⠨⠠⠎⠩⠊⠻⠉⠰⠊⠚⠐⠨⠁⠰⠊⠈⠑⠠⠑

$\sum_i c_{ij} \alpha_i \in E$

\sum_i c_{ij} \alpha_i \in E

5233. ⠨⠃⠰⠚

β_j

\beta_j

5234. ⠐⠨⠠⠎⠩⠊⠀⠨⠅⠀⠂⠣⠝⠻⠉⠰⠊⠚⠐⠨⠁⠰⠊⠐⠀⠨⠅⠀⠴

$\sum_{i=1}^n c_{ij} \alpha_i = 0$

\sum_{i = 1}^n c_{ij} \alpha_i = 0

5235. ⠨⠁⠰⠚

α_j

\alpha_j

5236. ⠉⠰⠊⠚⠐⠀⠨⠅⠀⠴

$c_{ij} = 0$

c_{ij} = 0

5237. ⠠⠋⠰⠊

F_i

F_i

5238. ⠊⠀⠨⠅⠀⠂⠠⠄⠠⠅

$i = 1, \dots, k$

i = 1, \dots, k

5239. ⠠⠋⠰⠊⠬⠂

F_{i+1}

F_{i+1}

5240. ⠠⠋⠰⠅

F_k

F_k

5241. ⠠⠋⠂

F_1

F_1

5242. ⠈⠷ ⠠⠋ ⠰⠅ ⠐⠆ ⠠⠋⠂ ⠈⠾ ⠨⠅ ⠈⠷ ⠠⠋ ⠰⠅ ⠐⠆ ⠠⠋ ⠰⠅ ⠤⠂ ⠐⠈⠾⠄ ⠄ ⠄ ⠈⠷ ⠠⠋⠆ ⠐⠆ ⠠⠋⠂ ⠈⠾

$[F_k : F_1] = [F_k : F_{k-1}] \cdots [F_2 : F_1]$

[F_k : F_1] = [F_k : F_{k-1}] \cdots [F_2 : F_1]

5243. ⠙⠑⠛⠟⠷⠭⠾

$\deg q(x)$

\deg q(x)

5244. ⠙⠑⠛⠏⠷⠭⠾

$\deg p(x)$

\deg p(x)

5245. ⠙⠑⠛⠏⠷⠭⠾ ⠨⠅ ⠈⠷ ⠠⠋⠷ ⠨⠁ ⠾⠐⠆ ⠠⠋ ⠈⠾

$\deg p(x) = [F(\alpha) : F]$

\deg p(x) = [F(\alpha) : F]

5246. ⠙⠑⠛⠟⠷⠭⠾ ⠨⠅ ⠈⠷ ⠠⠋⠷ ⠨⠃ ⠾⠐⠆ ⠠⠋ ⠈⠾

$\deg q(x) = [F(\beta) : F]$

\deg q(x) = [F(\beta) : F]

5247. ⠠⠋ ⠸⠐⠅ ⠠⠋⠷ ⠨⠃ ⠾ ⠸⠐⠅ ⠠⠋⠷ ⠨⠁ ⠾

$F \subset F(\beta) \subset F(\alpha)$

F \subset F(\beta) \subset F(\alpha)

5248. ⠈⠷ ⠠⠋⠷ ⠨⠁ ⠾⠐⠆ ⠠⠋ ⠈⠾ ⠨⠅ ⠈⠷ ⠠⠋⠷ ⠨⠁ ⠾⠐⠆ ⠠⠋⠷ ⠨⠃ ⠾ ⠈⠾ ⠈⠷ ⠠⠋⠷ ⠨⠃ ⠾⠐⠆ ⠠⠋ ⠈⠾

$[F(\alpha) : F] = [F(\alpha) : F(\beta)][F(\beta) : F]$

[F(\alpha) : F]= [F(\alpha) : F(\beta)] [F(\beta) :
F]

5249. ⠜⠒⠻⠬⠜⠢⠻

$\sqrt{3} + \sqrt{5}$

\sqrt{3} + \sqrt{5}

5250. ⠭ ⠘⠲ ⠐⠤16⠭ ⠘⠆ ⠐⠬⠲

$x^4 - 16x^2 + 4$

x^4 - 16 x^2 + 4

5251. ⠈⠷ ⠈ ⠠⠟⠷⠜⠒⠻⠬⠜⠢⠻⠾⠐⠆ ⠈ ⠠⠟ ⠈⠾ ⠨⠅ ⠲

$[\mathbb{Q}(\sqrt{3} + \sqrt{5}\,) : \mathbb{Q}] = 4$

[{\mathbb Q}(\sqrt{3} + \sqrt{5}\,) : {\mathbb Q}] = 4

5252. StartSet⠂ ⠠⠠⠜⠒⠻EndSet

$\{1, \sqrt{3}\,\}$

\{ 1, \sqrt{3}\, \}

5253. ⠈⠠⠟⠷⠜⠒⠻⠾

$\mathbb{Q}(\sqrt{3}\,)$

{\mathbb Q}(\sqrt{3}\,)

5254. ⠜⠢⠻

$\sqrt{5}$

\sqrt{5}

5255. StartSet⠂ ⠠⠜⠢⠻EndSet

$\{1, \sqrt{5}\,\}$

\{ 1, \sqrt{5}\, \}

5256. ⠈⠠⠟⠷⠜⠒⠻⠠⠜⠢⠻⠾ ⠨⠅ ⠷⠈⠠⠟⠷⠜⠒⠻⠾⠾⠷⠜⠢⠻⠾

$\mathbb{Q}(\sqrt{3}, \sqrt{5}\,) = (\mathbb{Q}(\sqrt{3}\,))(\sqrt{5}\,)$

{\mathbb Q}(\sqrt{3}, \sqrt{5}\,) = ({\mathbb Q}(\sqrt{3}\,))(
\sqrt{5}\,)

5257. StartSet⠂ ⠠⠜⠒⠻ ⠠⠜⠢⠻ ⠠⠜⠒⠻⠜⠢⠻ ⠨⠅ ⠜15⠻EndSet

$\{1, \sqrt{3}, \sqrt{5}, \sqrt{3}\sqrt{5} = \sqrt{15}\,\}$

\{ 1, \sqrt{3}, \sqrt{5}, \sqrt{3} \sqrt{5} = \sqrt{15}\, \}

5258. ⠈⠠⠟⠷⠜⠒⠻⠠⠜⠢⠻⠾ ⠨⠅ ⠈⠠⠟⠷⠜⠒⠻⠬⠜⠢⠻⠾

$\mathbb{Q}(\sqrt{3}, \sqrt{5}\,) = \mathbb{Q}(\sqrt{3} + \sqrt{5}\,)$

{\mathbb Q}(\sqrt{3}, \sqrt{5}\,) = {\mathbb Q}(\sqrt{3} + \sqrt{5}\,
)

5259. ⠠⠋⠷⠨⠁⠂ ⠠⠄ ⠠⠨⠁ ⠰⠝⠐⠾

$F(\alpha_1, \ldots, \alpha_n)$

F(\alpha_1, \ldots, \alpha_n)

5260. ⠈⠠⠟⠷⠣⠒⠜⠢⠻⠠⠜⠢⠻⠊⠾

$\mathbb{Q}(\sqrt[3]{5}, \sqrt{5}\, i)$

{\mathbb Q}(\sqrt[3]{5}, \sqrt{5} \, i)

5261. ⠣⠒⠜⠢⠻

$\sqrt[3]{5}$

\sqrt[3]{5}

5262. ⠜⠢⠻⠊⠌⠈⠑ ⠈⠠⠟⠷⠣⠒⠜⠢⠻⠾

$\sqrt{5}\, i \notin \mathbb{Q}(\sqrt[3]{5}\,)$

\sqrt{5} \, i \notin {\mathbb Q}(\sqrt[3]{5}\,)

5263. ⠈⠷ ⠈⠠⠟⠷⠣⠒⠜⠢⠻⠠⠜⠢⠻⠊⠾⠈⠆⠈⠠⠟⠷⠣⠒⠜⠢⠻⠾⠈⠾ ⠨⠅ ⠆

$[\mathbb{Q}(\sqrt[3]{5}, \sqrt{5}\, i) : \mathbb{Q}(\sqrt[3]{5}\,)] = 2$

[{\mathbb Q}(\sqrt[3]{5}, \sqrt{5}\, i) : {\mathbb Q}(\sqrt[3]{5}\,
)] = 2

5264. StartSet⠂ ⠠⠜⠢⠻⠊EndSet

$\{1, \sqrt{5}i\,\}$

\{ 1, \sqrt{5}i\, \}

5265. ⠈⠠⠟⠷⠣⠒⠜⠢⠻⠠⠜⠢⠻⠊⠾

$\mathbb{Q}(\sqrt[3]{5}, \sqrt{5}\, i)$

```
{\mathbb Q}( \sqrt[3]{5}, \sqrt{5}\, i )
```

5266. ⠈⠠⠟⠷⠣⠒⠜⠢⠻⠾

$\mathbb{Q}(\sqrt[3]{5}\,)$

```
{\mathbb Q}( \sqrt[3]{5}\, )
```

5267. StartSet⠂⠠⠣⠒⠜⠢⠻⠠⠷⠣⠒⠜⠢⠻⠾⠘⠆⠐EndSet

$\{1, \sqrt[3]{5}, (\sqrt[3]{5}\,)^2\}$

```
\{ 1, \sqrt[3]{5}, (\sqrt[3]{5}\, )^2 \}
```

5268. ⠈⠠⠟⠷⠣⠒⠜⠢⠻⠾

$\mathbb{Q}(\sqrt[3]{5}\,)$

```
{\mathbb Q}(\sqrt[3]{5}\, )
```

5269. ⠈⠠⠟⠷⠣⠒⠜⠢⠻⠠⠜⠢⠻⠊⠾

$\mathbb{Q}(\sqrt[3]{5}, \sqrt{5}\, i)$

```
{\mathbb Q}(\sqrt[3]{5}, \sqrt{5}\, i )
```

5270. StartSet⠂⠠⠜⠢⠻⠊⠠⠣⠒⠜⠢⠻⠠⠷⠣⠒⠜⠢⠻⠾⠘⠆⠐⠠⠷⠣⠖⠜⠢⠻⠾⠘⠢⠐⠊⠠⠷⠣⠖⠜⠢⠻⠾⠘⠶⠐⠊ ⠨⠅ ⠢⠣⠖⠜

$\{1, \sqrt{5}\, i, \sqrt[3]{5}, (\sqrt[3]{5}\,)^2, (\sqrt[6]{5}\,)^5 i, (\sqrt[6]{5}\,)^7 i = 5 \sqrt[6]{5}\, i$ or $\sqrt[6]{5}\, i\}$

```
\{ 1, \sqrt{5}\, i, \sqrt[3]{5}, (\sqrt[3]{5}\, )^2, (\sqrt[6]{5}\,
)^5 i, (\sqrt[6]{5}\, )^7 i = 5 \sqrt[6]{5}\, i \text{ or } \sqrt[6]{5}\,
i \}
```

5271. ⠣⠖⠜⠢⠻⠊

$\sqrt[6]{5}\, i$

```
\sqrt[6]{5}\, i
```

5272. ⠭⠘⠖⠐⠬⠢

$x^6 + 5$

```
x^6 + 5
```

5273. ⠏ ⠨⠅ ⠢

$p = 5$

```
p = 5
```

5274. ⠈⠠⠟⠸⠐⠅⠈⠠⠟⠷⠣⠖⠜⠢⠻⠊⠾⠸⠐⠅⠈⠠⠟⠷⠣⠒⠜⠢⠻⠠⠜⠢⠻⠊⠾

$\mathbb{Q} \subset \mathbb{Q}(\sqrt[6]{5}\, i) \subset \mathbb{Q}(\sqrt[3]{5}, \sqrt{5}\, i)$

```
{\mathbb Q} \subset {\mathbb Q}( \sqrt[6]{5}\, i) \subset {\mathbb
Q}( \sqrt[3]{5}, \sqrt{5}\, i )
```

5275. ⠈⠠⠟⠷⠣⠖⠜⠢⠻⠊⠾ ⠨⠅ ⠈⠠⠟⠷⠣⠒⠜⠢⠻⠠⠜⠢⠻⠊⠾

$\mathbb{Q}(\sqrt[6]{5}\, i) = \mathbb{Q}(\sqrt[3]{5}, \sqrt{5}\, i)$

```
{\mathbb Q}( \sqrt[6]{5}\, i) = {\mathbb Q}( \sqrt[3]{5}, \sqrt{5}\,
i )
```

5276. ⠨⠁⠂⠠⠄⠠⠨⠁⠰⠝⠐⠈⠑⠠⠑

$\alpha_1, \ldots, \alpha_n \in E$

```
\alpha_1, \ldots, \alpha_n \in E
```

5277.

$E = F(\alpha_1, \ldots, \alpha_n)$

```
E = F(\alpha_1, \ldots, \alpha_n)
```

5278.

$E = F(\alpha_1, \ldots, \alpha_n) \supset F(\alpha_1, \ldots, \alpha_{n-1}) \supset \cdots \supset F(\alpha_1) \supset F$

```
E = F(\alpha_1, \ldots, \alpha_n) \supset F(\alpha_1, \ldots, \alpha_{n-1}
) \supset \cdots \supset F( \alpha_1 ) \supset F
```

5279.

$F(\alpha_1, \ldots, \alpha_i)$

```
F(\alpha_1, \ldots, \alpha_i)
```

5280.

$F(\alpha_1, \ldots, \alpha_{i-1})$

```
F(\alpha_1, \ldots, \alpha_{i-1})
```

5281.

$E = F(\alpha_1, \ldots, \alpha_n) \supset F(\alpha_1, \ldots, \alpha_{n-1}) \supset \cdots \supset F(\alpha_1) \supset F$

```
E = F(\alpha_1, \ldots, \alpha_n) \supset F(\alpha_1, \ldots, \alpha_{n
- 1} ) \supset \cdots \supset F( \alpha_1 ) \supset F
```

5282.

$F(\alpha_1, \ldots, \alpha_{i-1})$

```
F(\alpha_1, \ldots, \alpha_{i - 1})
```

5283.

$F(\alpha_1, \ldots, \alpha_i) = F(\alpha_1, \ldots, \alpha_{i-1})(\alpha_i)$

```
F(\alpha_1, \ldots, \alpha_i) = F(\alpha_1, \ldots, \alpha_{i -
1} )(\alpha_i)
```

5284.

$[F(\alpha_1, \ldots, \alpha_i) : F(\alpha_1, \ldots, \alpha_{i-1})]$

```
[ F(\alpha_1, \ldots, \alpha_i) : F(\alpha_1, \ldots, \alpha_{i
- 1} )]
```

5285.

$[E : F]$

```
[E : F]
```

5286.

$\alpha, \beta \in E$

```
\alpha, \beta \in E
```

5287.

$F(\alpha, \beta)$

```
F( \alpha, \beta )
```

5288.

$\alpha \pm \beta$

```
\alpha \pm \beta
```

5289. ⠍ ⠝

$\alpha\beta$

\alpha \beta

5290. ⠍⠌⠝

α/β

\alpha / \beta

5291. ⠝ ⠌⠭ ⠴

$\beta \neq 0$

\beta \neq 0

5292. ⠭⠤⠍

$x-\alpha$

x-\alpha

5293. ⠏⠷⠭⠾ ⠭ ⠷⠭⠤⠍⠾⠟⠂⠷⠭⠾

$p(x) = (x-\alpha)q_1(x)$

p(x) = (x - \alpha) q_1(x)

5294. ⠙⠑⠛⠟⠂⠷⠭⠾ ⠭ ⠙⠑⠛⠏⠷⠭⠾⠤⠂

$\deg q_1(x) = \deg p(x) - 1$

\deg q_1(x) = \deg p(x) - 1

5295. ⠏⠷⠭⠾ ⠭ ⠷⠭⠤⠍⠾⠷⠭⠤⠝⠾⠟⠆⠷⠭⠾

$p(x) = (x-\alpha)(x-\beta)q_2(x)$

p(x) = (x - \alpha)(x - \beta)q_2(x)

5296. ⠙⠑⠛⠟⠆⠷⠭⠾ ⠭ ⠙⠑⠛⠏⠷⠭⠾⠤⠆

$\deg q_2(x) = \deg p(x) - 2$

\deg q_2(x) = \deg p(x) -2

5297. ⠁⠭⠤⠃

$ax-b$

ax - b

5298. ⠏⠷⠃⠌⠁⠾ ⠭ ⠴

$p(b/a) = 0$

p(b/a) = 0

5299. ⠠⠋ ⠭ ⠠⠑

$F = E$

F = E

5300. ⠠⠑ ⠭ ⠠⠋⠷⠍⠂ ⠤ ⠠⠍ ⠰⠝⠐⠾

$E = F(\alpha_1, \ldots, \alpha_n)$

E = F(\alpha_1, \ldots, \alpha_n)

5301. ⠏⠷⠭⠾ ⠭ ⠷⠭⠤⠍⠂⠾⠷⠭⠤⠍⠆⠾⠄⠄⠄⠷⠭⠤⠍ ⠰⠝⠐⠾

$p(x) = (x-\alpha_1)(x-\alpha_2)\cdots(x-\alpha_n)$

p(x) = (x - \alpha_1)(x - \alpha_2) \cdots (x - \alpha_n)

5302. ⠏⠷⠭⠾ ⠨⠅ ⠭⠘⠲⠐⠬⠆⠭⠘⠆⠐⠤⠦

$p(x) = x^4 + 2x^2 - 8$

`p(x) = x^4 + 2x^2 - 8`

5303. ⠭⠘⠆⠐⠬⠲

$x^2 + 4$

`x^2 + 4`

5304. ⠈⠀⠠⠟⠷⠜⠆⠻⠠⠀⠊⠾

${\mathbb Q}(\sqrt{2}, i)$

`{\mathbb Q}( \sqrt{2}, i )`

5305. ⠏⠷⠭⠾ ⠨⠅ ⠭⠘⠒⠐⠤⠒

$p(x) = x^3 - 3$

`p(x) = x^3 - 3`

5306. ⠈⠀⠠⠟⠷⠣⠒⠜⠒⠻⠾

${\mathbb Q}(\sqrt[3]{3}\,)$

`{\mathbb Q}( \sqrt[3]{3}\, )`

5307. ⠹⠤⠣⠒⠜⠒⠻⠬⠤⠷⠣⠖⠜⠒⠻⠾⠘⠢⠐⠊⠌⠆⠼

$\frac{ -\sqrt[3]{3} \pm (\sqrt[6]{3}\,)^5 i }{2}$

`\frac{ -\sqrt[3]{3} \pm (\sqrt[6]{3}\, )^5 i }{2}`

5308. ⠠⠑ ⠨⠅ ⠠⠋

$E = F$

`E = F`

5309. ⠙⠑⠛⠏⠷⠭⠾ ⠨⠅ ⠝

$\deg p(x) = n$

`\deg p(x) = n`

5310. ⠏⠷⠭⠾ ⠨⠅ ⠷⠭⠤⠨⠁⠂⠾⠟⠷⠭⠾

$p(x) = (x - \alpha_1)q(x)$

`p(x) = (x - \alpha_1)q(x)`

5311. ⠟⠷⠭⠾ ⠈⠑ ⠠⠅⠈⠷⠭⠈⠾

$q(x) \in K[x]$

`q(x) \in K[x]`

5312. ⠙⠑⠛⠟⠷⠭⠾ ⠨⠅ ⠝⠤⠂

$\deg q(x) = n -1$

`\deg q(x) = n -1`

5313. ⠠⠑ ⠸⠨⠂ ⠠⠅

$E \supset K$

`E \supset K`

5314. ⠨⠁⠆⠠⠀⠄⠄⠄⠠⠀⠨⠁⠰⠝

$\alpha_2, \ldots, \alpha_n$

`\alpha_2, \ldots, \alpha_n`

5315. ⠠⠑ ⠨⠅ ⠠⠅⠷⠨⠁⠆⠠⠄⠠⠨⠁⠰⠝⠐⠾ ⠨⠅ ⠠⠋⠷⠨⠁⠂⠠⠄⠠⠨⠁⠰⠝⠐⠾

$E = K(\alpha_2, \ldots, \alpha_n) = F(\alpha_1, \ldots, \alpha_n)$

`E = K(\alpha_2, \ldots, \alpha_n) = F(\alpha_1, \ldots, \alpha_n)`

5316. ⠨⠋⠱⠠⠅ ⠫⠕ ⠠⠇

$\phi : K \rightarrow L$

`\phi : K \rightarrow L`

5317. ⠨⠋⠱⠠⠑ ⠫⠕ ⠠⠋

$\phi : E \rightarrow F$

`\phi : E \rightarrow F`

5318. ⠨⠁⠈⠑⠠⠅

$\alpha \in K$

`\alpha \in K`

5319. ⠨⠋overbar⠱⠠⠑⠷⠨⠁⠾ ⠫⠕ ⠠⠋⠷⠨⠃⠾

$\overline{\phi} : E(\alpha) \rightarrow F(\beta)$

`\overline{\phi} : E( \alpha ) \rightarrow F( \beta )`

5320. ModifyingAbove ⠨⠋With ⠷⠨⠁⠾ ⠨⠅ ⠨⠃

$\overline{\phi}(\alpha) = \beta$

`\overline{\phi}( \alpha ) = \beta`

5321. ⠨⠋overbar

$\overline{\phi}$

`\overline{\phi}`

5322. ⠠⠑⠷⠨⠁⠾

$E(\alpha)$

`E( \alpha )`

5323. ⠼⠂⠠⠨⠁⠠⠄⠠⠨⠁⠘⠝⠤⠂

$1, \alpha, \ldots, \alpha^{n - 1}$

`1, \alpha, \ldots, \alpha^{n - 1}`

5324. ⠨⠋overbar⠷⠁⠴⠬⠁⠂⠨⠁⠬⠄⠄⠄⠬⠁⠰⠝⠤⠂⠐⠨⠁⠘⠝⠤⠂⠐⠾ ⠨⠅ ⠨⠋⠷⠁⠴⠾⠬⠨⠋⠷⠁⠂⠾⠨⠃⠬⠄⠄⠄⠬⠨⠋⠷⠁

$\overline{\phi}(a_0 + a_1 \alpha + \cdots + a_{n - 1} \alpha^{n - 1}) = \phi(a_0) + \phi(a_1) \beta + \cdots + \phi(a_{n - 1}) \beta^{n - 1}$

```
\overline{\phi}( a_0 + a_1 \alpha + \cdots + a_{n - 1} \alpha^{n
- 1}) = \phi(a_0) + \phi(a_1) \beta + \cdots + \phi(a_{n - 1}) \beta^{n
- 1}
```

5325. ⠁⠴⠬⠁⠂⠨⠁⠬⠄⠄⠄⠬⠁⠰⠝⠤⠂⠐⠨⠁⠘⠝⠤⠂

$a_0 + a_1 \alpha + \cdots + a_{n - 1} \alpha^{n - 1}$

`a_0 + a_1 \alpha + \cdots + a_{n - 1} \alpha^{n - 1}`

5326. ⠠⠑⠷⠨⠁⠾

$E(\alpha)$

`E(\alpha)`

5327. ⠨⠋⠷⠁⠴⠬⠁⠂⠭⠬⠄⠄⠄⠬⠁⠰⠝⠭⠘⠝⠐⠾ ⠨⠅ ⠨⠋⠷⠁⠴⠾⠬⠨⠋⠷⠁⠂⠾⠭⠬⠄⠄⠄⠬⠨⠋⠷⠁⠰⠝⠐⠾⠭⠘⠝

$\phi(a_0 + a_1 x + \cdots + a_n x^n) = \phi(a_0) + \phi(a_1)x + \cdots + \phi(a_n)x^n$

```
\phi( a_0 + a_1 x + \cdots + a_n x^n ) = \phi( a_0 ) + \phi(a_1)
x + \cdots + \phi(a_n) x^n
```

5328. ⠨⠋⠷⠏⠷⠭⠾⠾ ⠨⠅ ⠟⠷⠭⠾

$\phi(p(x)) = q(x)$

```
\phi(p(x)) = q(x)
```

5329. ⠄⡗⠉⠇ ⠭⠈⠢⠤ ⠟⠷⠭⠾⠄⡗⠉⠇ ⠭⠈⠢⠠⠄

$\langle q(x) \rangle$

```
\langle q(x) \rangle
```

5330. ⠨⠽⠒⠠⠑⠈⠷⠭⠈⠾⠸⠌⠄⡗⠉⠇ ⠭⠈⠢⠤ ⠏⠷⠭⠾⠄⡗⠉⠇ ⠭⠈⠢⠠⠄ ⠫⠕ ⠠⠋⠈⠷⠭⠈⠾⠸⠌⠄⡗⠉⠇ ⠭⠈⠢⠤ ⠟⠷⠭⠾⠄⡗⠉⠇ ⠭⠈⠢⠠⠄

$\psi : E[x]/\langle p(x) \rangle \rightarrow F[x]/\langle q(x) \rangle$

```
\psi : E[x] / \langle p(x) \rangle \rightarrow F[x]/\langle q(x)
\rangle
```

5331. ⠨⠎⠒⠠⠑⠈⠷⠭⠈⠾⠸⠌⠄⡗⠉⠇ ⠭⠈⠢⠤ ⠏⠷⠭⠾⠄⡗⠉⠇ ⠭⠈⠢⠠⠄ ⠫⠕ ⠠⠑⠷⠨⠁⠾

$\sigma : E[x]/\langle p(x) \rangle \rightarrow E(\alpha)$

```
\sigma: E[x]/\langle p(x) \rangle \rightarrow E(\alpha)
```

5332. ⠨⠞⠒⠠⠋⠈⠷⠭⠈⠾⠸⠌⠄⡗⠉⠇ ⠭⠈⠢⠤ ⠟⠷⠭⠾⠄⡗⠉⠇ ⠭⠈⠢⠠⠄ ⠫⠕ ⠠⠋⠷⠨⠃⠾

$\tau : F[x]/\langle q(x) \rangle \rightarrow F(\beta)$

```
\tau : F[x]/\langle q(x) \rangle \rightarrow F( \beta )
```

5333. ⠨⠋overbar ⠨⠅ ⠨⠞⠨⠽⠨⠎⠘⠤⠂

$\overline{\phi} = \tau\psi\sigma^{-1}$

```
\overline{\phi} = \tau \psi \sigma^{-1}
```

5334. ⠨⠽⠒⠠⠅ ⠫⠕ ⠠⠇

$\psi : K \rightarrow L$

```
\psi : K \rightarrow L
```

5335. ⠠⠅ ⠨⠅ ⠠⠑

$K = E$

```
K = E
```

5336. ⠠⠇ ⠨⠅ ⠠⠋

$L = F$

```
L = F
```

5337. ⠠⠑ ⠸⠐⠅ ⠠⠑⠷⠨⠁⠾ ⠸⠐⠅ ⠠⠅

$E \subset E(\alpha) \subset K$

```
E \subset E( \alpha ) \subset K
```

5338. ⠠⠋ ⠸⠐⠅ ⠠⠋⠷⠨⠃⠾ ⠸⠐⠅ ⠠⠇

$F \subset F(\beta) \subset L$

```
F \subset F( \beta) \subset L
```

5339. ⠨⠋overbar⠱⠠⠑⠷⠨⠁⠾ ⠫⠕ ⠠⠋⠷⠨⠃⠾

$\overline{\phi} : E(\alpha) \rightarrow F(\beta)$

\overline{\phi} : E(\alpha) \rightarrow F(\beta)

5340. ⠏⠷⠭⠾ ⠨⠅ ⠷⠭⠤⠨⠁⠾⠋⠷⠭⠾

$p(x) = (x - \alpha) f(x)$

p(x) = (x - \alpha) f(x)

5341. ⠟⠷⠭⠾ ⠨⠅ ⠷⠭⠤⠨⠃⠾⠛⠷⠭⠾

$q(x) = (x - \beta) g(x)$

q(x) = (x - \beta) g(x)

5342. ⠠⠑⠷⠨⠁⠾

$E(\alpha)$

E(\alpha)

5343. ⠠⠋⠷⠨⠃⠾

$F(\beta)$

F(\beta)

5344. 30⠘⠘⠨⠡

30°

30^\circ

5345. 20⠘⠘⠨⠡

20°

20^\circ

5346. 60⠘⠘⠨⠡

60°

60^\circ

5347. ⠳⠨⠁⠳

$|\alpha|$

| \alpha |

5348. ⠨⠁⠬⠨⠃

$\alpha + \beta$

\alpha + \beta

5349. ⠨⠁⠤⠨⠃

$\alpha - \beta$

\alpha - \beta

5350. ⠨⠁ ⠨⠂ ⠨⠃

$\alpha > \beta$

\alpha \gt \beta

5351. ⠨⠃ ⠨⠂ ⠂

$\beta > 1$

\beta \gt 1

5352. ⠫⠞⠠⠁⠠⠃⠠⠉

$\triangle ABC$

\triangle ABC

5353. ⠫⠞⠠⠁⠠⠙⠠⠑

$\triangle ADE$

\triangle ADE

5354. ⠨⠁⠌⠂⠀⠨⠅⠀⠭⠌⠨⠃

$\alpha/1 = x/\beta$

\alpha / 1 = x / \beta

5355. ⠨⠃⠀⠐⠅⠀⠂

$\beta < 1$

\beta \lt 1

5356. ⠜⠨⠁⠻

$\sqrt{\alpha}$

\sqrt{\alpha}

5357. ⠫⠞⠠⠁⠠⠃⠠⠙

$\triangle ABD$

\triangle ABD

5358. ⠫⠞⠠⠃⠠⠉⠠⠙

$\triangle BCD$

\triangle BCD

5359. ⠂⠌⠭⠀⠨⠅⠀⠭⠌⠨⠁

$1/x = x/\alpha$

1 /x = x / \alpha

5360. ⠭⠘⠆⠐⠀⠨⠅⠀⠨⠁

$x^2 = \alpha$

x^2 = \alpha

5361. ⠠⠏⠀⠨⠅⠀⠷⠏⠠⠟⠾

$P = (p, q)$

P =(p, q)

5362. ⠁⠭⠬⠃⠽⠬⠉⠀⠨⠅⠀⠴

$ax + by + c = 0$

a x + by + c = 0

5363. ⠭⠘⠆⠐⠬⠽⠘⠆⠐⠬⠙⠭⠬⠑⠽⠬⠋⠀⠨⠅⠀⠴

$x^2 + y^2 + dx + ey + f = 0$

x^2 + y^2 + d x + e y + f = 0

5364. ⠷⠭⠂⠠⠽⠂⠾

(x_1, y_1)

(x_1, y_1)

5365. ⠭⠂⠀⠨⠅⠀⠭⠆

$x_1 = x_2$

`x_1 = x_2`

5366. ⠭⠤⠭⠂⠀⠨⠅⠀⠴

$x - x_1 = 0$

`x - x_1 = 0`

5367. ⠭⠂⠀⠌⠨⠅⠀⠭⠆

$x_1 \neq x_2$

`x_1 \neq x_2`

5368. ⠽⠤⠽⠂⠀⠨⠅⠀⠷⠹⠽⠆⠤⠽⠂⠌⠭⠆⠤⠭⠂⠼⠾⠷⠭⠤⠭⠂⠾

$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1} \right) (x - x_1)$

`y - y_1 = \left( \frac{y_2 - y_1}{x_2 - x_1} \right) (x - x_1)`

5369. ⠷⠭⠤⠭⠂⠾⠘⠆⠐⠬⠷⠽⠤⠽⠂⠾⠘⠆⠐⠤⠗⠘⠆⠐⠀⠨⠅⠀⠴

$(x - x_1)^2 + (y - y_1)^2 - r^2 = 0$

`(x - x_1)^2 + (y - y_1)^2 - r^2 = 0`

5370. ⠑⠰⠊

e_i

`e_i`

5371. ⠭⠘⠆⠐⠬⠽⠘⠆⠐⠬⠙⠂⠭⠬⠑⠂⠭⠬⠋⠂⠀⠨⠅⠀⠴

$x^2 + y^2 + d_1 x + e_1 x + f_1 = 0$

`x^2 + y^2 + d_1 x +e_1 x + f_1 = 0`

5372. ⠷⠙⠂⠤⠙⠆⠾⠭⠬⠃⠷⠑⠆⠤⠑⠂⠾⠽⠬⠷⠋⠆⠤⠋⠂⠾⠀⠨⠅⠀⠴

$(d_1 - d_2) x + b(e_2 - e_1)y + (f_2 - f_1) = 0$

`(d_1 - d_2) x + b(e_2 - e_1)y + (f_2 - f_1) = 0`

5373. ⠠⠁⠭⠘⠆⠐⠬⠠⠃⠭⠬⠠⠉⠀⠨⠅⠀⠴

$Ax^2 + B x + C = 0$

`Ax^2 + B x + C = 0`

5374. ⠭⠀⠨⠅⠀⠹⠤⠠⠃⠬⠤⠜⠠⠃⠘⠆⠐⠤⠲⠠⠁⠠⠉⠻⠌⠆⠠⠁⠼

$x = \frac{- B \pm \sqrt{B^2 - 4 A C} }{2 A}$

`x = \frac{- B \pm \sqrt{B^2 - 4 A C} }{2 A}`

5375. ⠠⠋⠷⠜⠨⠁⠻⠾

$F(\sqrt{\alpha}\,)$

`F( \sqrt{\alpha}\, )`

5376. ⠨⠁⠀⠨⠅⠀⠠⠃⠘⠆⠐⠤⠲⠠⠁⠠⠉⠀⠨⠂⠀⠴

$\alpha = B^2 - 4 A C > 0$

`\alpha = B^2 - 4 A C \gt 0`

5377. ⠈⠠⠟⠀⠨⠅⠀⠠⠋⠴⠀⠸⠐⠅⠀⠠⠋⠂⠀⠸⠐⠅⠀⠄⠄⠄⠀⠸⠐⠅⠀⠠⠋⠰⠅

${\mathbb Q} = F_0 \subset F_1 \subset \cdots \subset F_k$

`{\mathbb Q} = F_0 \subset F_1 \subset \cdots \subset F_k`

5378. ⠠⠋⠰⠊⠐ ⠨⠅ ⠠⠋⠰⠊⠤⠂⠐⠷⠜⠨⠁⠰⠊⠐⠻⠾

$F_i = F_{i-1}(\sqrt{ \alpha_i}\,)$

F_i = F_{i-1}(\sqrt{ \alpha_i}\,)

5379. ⠨⠁⠰⠊⠐⠈⠑⠠⠋⠰⠊

$\alpha_i \in F_i$

\alpha_i \in F_i

5380. ⠨⠁⠈⠑⠠⠋⠰⠅

$\alpha \in F_k$

\alpha \in F_k

5381. ⠅ ⠨⠂ ⠴

$k \gt 0$

k \gt 0

5382. ⠈⠷⠈⠠⠟⠷⠨⠁⠾⠱⠈⠠⠟⠈⠾ ⠨⠅ ⠆⠘⠅

$[{\mathbb Q}(\alpha) : {\mathbb Q}] = 2^k$

[{\mathbb Q}(\alpha) : {\mathbb Q}] = 2^k

5383. ⠈⠷⠠⠋⠰⠅⠐⠱⠈⠠⠟⠈⠾ ⠨⠅ ⠈⠷⠠⠋⠰⠅⠐⠱⠠⠋⠰⠅⠤⠂⠐⠈⠾⠈⠷⠠⠋⠰⠅⠤⠂⠐⠱⠠⠋⠰⠅⠤⠆⠐⠈⠾⠄⠄⠄⠈⠷⠠⠋⠂⠱

$[F_k: {\mathbb Q}] = [F_k : F_{k - 1}][F_{k - 1} : F_{k - 2}] \cdots [F_1: {\mathbb Q}] = 2^k$

[F_k: {\mathbb Q}] = [F_k : F_{k - 1}][F_{k - 1} : F_{k - 2}] \cdots
[F_1: {\mathbb Q}] = 2^k

5384. ⠣⠒⠜⠆⠻

$\sqrt[3]{2}$

\sqrt[3]{2}

5385. ⠭⠘⠒ ⠐⠤⠆

$x^3 -2$

x^3 -2

5386. ⠈⠷⠈⠠⠟⠷⠣⠒⠜⠆⠻⠾⠱⠈⠠⠟⠈⠾ ⠨⠅ ⠒

$[{\mathbb Q}(\sqrt[3]{2}\,) : {\mathbb Q}] = 3$

[{\mathbb Q}(\sqrt[3]{2}\,) : {\mathbb Q}] = 3

5387. ⠜⠨⠏⠻

$\sqrt{\pi}$

\sqrt{\pi}

5388. 60⠘⠘⠨⠡

60°

60^{\circ}

5389. ⠨⠁ ⠨⠅ ⠉⠕⠎ ⠨⠹

$\alpha = \cos \theta$

\alpha = \cos \theta

5390. ⠨⠹ ⠨⠅ 20⠘⠘⠨⠡

$\theta = 20^{\circ}$

```
\theta = 20^{\circ}
```

5391. ⠉⠕⠎ ⠒⠨⠹ ⠨⠅ ⠉⠕⠎ 60⠘⠘⠨⠡⠐ ⠨⠅ ⠂⠌⠆

$\cos 3 \theta = \cos 60^\circ = 1/2$

```
\cos 3 \theta = \cos 60^\circ = 1/2
```

5392. ⠲⠨⠁ ⠘⠒ ⠐⠤⠒⠨⠁ ⠨⠅ one-half

$4 \alpha^3 - 3 \alpha = \frac{1}{2}$

```
4 \alpha^3 - 3 \alpha = \frac{1}{2}
```

5393. ⠦⠭⠘⠒⠤⠖⠭⠤⠂

$8 x^3 - 6 x -1$

```
8 x^3 - 6 x -1
```

5394. ⠈⠷⠈⠠⠟⠷⠨⠁⠾⠱⠈⠠⠟⠈⠾ ⠨⠅ ⠒

$[{\mathbb Q}(\alpha) : {\mathbb Q }] = 3$

```
[{\mathbb Q}( \alpha ) : {\mathbb Q }] = 3
```

5395. ⠭⠘⠆⠐⠬⠽⠘⠆⠐ ⠨⠅ ⠵⠘⠆

$x^2 + y^2 = z^2$

```
x^2 + y^2 = z^2
```

5396. ⠭⠘⠝⠐⠬⠽⠘⠝⠐ ⠨⠅ ⠵⠘⠝

$x^n + y^n = z^n$

```
x^n + y^n = z^n
```

5397. ⠏⠷⠭⠠⠽⠾

$p(x, y)$

```
p(x, y)
```

5398. ⠈⠠⠵⠈⠷⠭⠠⠽⠈⠾

${\mathbb Z}[x,y]$

```
{\mathbb Z}[x,y]
```

5399. ⠜⠂⠌⠒⠬⠨⠜⠶⠨⠻⠻

$\sqrt{ 1/3 + \sqrt{7} }$

```
\sqrt{ 1/3 + \sqrt{7} }
```

5400. ⠜⠒⠻⠬⠣⠒⠜⠢⠻

$\sqrt{ 3} + \sqrt[3]{5}$

```
\sqrt{ 3} + \sqrt[3]{5}
```

5401. ⠜⠒⠻⠬⠜⠆ ⠻⠊

$\sqrt{3} + \sqrt{2}\, i$

```
\sqrt{3} + \sqrt{2}\, i
```

5402. ⠨⠹ ⠨⠅ ⠆⠨⠏⠌⠝

$\theta = 2 \pi /n$

```
\theta = 2 \pi /n
```

5403. ⠜⠨⠣⠒⠜⠆⠨⠻⠤⠊⠻

$\sqrt{\sqrt[3]{2} - i}$

\sqrt{ \sqrt[3]{2} - i }

5404. ⠭⠘⠲⠐⠤⠷⠆⠌⠒⠾⠭⠘⠆⠤62⠌⠔

$x^4 - (2/3)x^2 - 62/9$

x^4 - (2/3) x^2 - 62/9

5405. ⠭⠘⠲⠐⠤⠆⠭⠘⠆⠬25

$x^4 - 2x^2 + 25$

x^4 - 2 x^2 + 25

5406. ⠈⠠⠟⠷⠜⠒⠻⠠⠜⠖⠻⠾

$\mathbb{Q}(\sqrt{3}, \sqrt{6}\,)$

{\mathbb Q}(\sqrt{3}, \sqrt{6}\,)

5407. ⠈⠠⠟⠷⠣⠒⠜⠆⠻⠠⠣⠒⠜⠒⠻⠾

$\mathbb{Q}(\sqrt[3]{2}, \sqrt[3]{3}\,)$

{\mathbb Q}(\sqrt[3]{2}, \sqrt[3]{3}\,)

5408. ⠈⠠⠟⠷⠜⠆⠻⠠⠊⠾

$\mathbb{Q}(\sqrt{2}, i)$

{\mathbb Q}(\sqrt{2}, i)

5409. ⠈⠠⠟⠷⠜⠒⠻⠠⠜⠢⠻⠠⠜⠶⠻⠾

$\mathbb{Q}(\sqrt{3}, \sqrt{5}, \sqrt{7}\,)$

{\mathbb Q}(\sqrt{3}, \sqrt{5}, \sqrt{7}\,)

5410. ⠈⠠⠟⠷⠜⠆⠻⠠⠣⠒⠜⠆⠻⠾

$\mathbb{Q}(\sqrt{2}, \sqrt[3]{2}\,)$

{\mathbb Q}(\sqrt{2}, \root 3 \of{2}\,)

5411. ⠈⠠⠟⠷⠜⠦⠻⠾

$\mathbb{Q}(\sqrt{8}\,)$

{\mathbb Q}(\sqrt{8}\,)

5412. ⠈⠠⠟⠷⠊⠠⠜⠆⠻⠬⠊⠠⠜⠒⠻⠬⠊⠾

$\mathbb{Q}(i, \sqrt{2} + i, \sqrt{3} + i)$

{\mathbb Q}(i, \sqrt{2} +i, \sqrt{3} + i)

5413. ⠈⠠⠟⠷⠜⠆⠻⠬⠜⠢⠻⠾

$\mathbb{Q}(\sqrt{2} + \sqrt{5}\,)$

{\mathbb Q}(\sqrt{2} + \sqrt{5}\,)

5414. ⠈⠠⠟⠷⠜⠢⠻⠾

$\mathbb{Q}(\sqrt{5}\,)$

{\mathbb Q} (\sqrt{5}\,)

5415. ⠈⠠⠟⠷⠜⠆⠻⠠⠜⠖⠻⠬⠜10⠻⠾

$\mathbb{Q}(\sqrt{2}, \sqrt{6} + \sqrt{10}\,)$

{\mathbb Q}(\sqrt{2}, \sqrt{6} + \sqrt{10}\,)

5416. ⠈⠰⠟⠷⠜⠒⠻⠬⠜⠢⠻⠾

$\mathbb{Q}(\sqrt{3} + \sqrt{5}\,)$

```
{\mathbb Q} ( \sqrt{3} + \sqrt{5}\, )
```

5417. StartSet⠂ ⠠⠊ ⠠⠜⠆ ⠻ ⠠⠜⠆ ⠻⠊EndSet

$\{1, i, \sqrt{2}, \sqrt{2}\, i\}$

```
\{ 1, i, \sqrt{2}, \sqrt{2}\, i \}
```

5418. StartSet⠂ ⠠⠆ ⠘⠂⠌⠖ ⠐⠠⠆ ⠘⠂⠌⠒ ⠐⠠⠆ ⠘⠂⠌⠆ ⠐⠠⠆ ⠘⠆⠌⠒ ⠐⠠⠆ ⠘⠢⠌⠖ ⠐EndSet

$\{1, 2^{1/6}, 2^{1/3}, 2^{1/2}, 2^{2/3}, 2^{5/6}\}$

```
\{1, 2^{1/6}, 2^{1/3}, 2^{1/2}, 2^{2/3}, 2^{5/6} \}
```

5419. ⠭ ⠘⠲ ⠐⠤10⠭ ⠘⠆ ⠐⠬21

$x^4 - 10x^2 + 21$

```
x^4 - 10 x^2 + 21
```

5420. ⠭ ⠘⠲ ⠐⠬⠂

$x^4 + 1$

```
x^4 + 1
```

5421. ⠭ ⠘⠒ ⠐⠬⠆ ⠭⠬⠆

$x^3 + 2x + 2$

```
x^3 + 2x + 2
```

5422. ⠭ ⠘⠒ ⠐⠤⠒

$x^3 - 3$

```
x^3 - 3
```

5423. ⠈⠰⠟⠷⠜⠒⠻ ⠠⠜⠶⠻⠾

$\mathbb{Q}(\sqrt{3}, \sqrt{7}\,)$

```
{\mathbb Q}(\sqrt{3}, \sqrt{7}\, )
```

5424. ⠈⠰⠟⠷⠣⠲⠜⠒⠻ ⠠⠊⠾

$\mathbb{Q}(\sqrt[4]{3}, i)$

```
{\mathbb Q}( \sqrt[4]{3}, i )
```

5425. ⠈⠷ ⠈⠰⠟⠷⠣⠲⠜⠒⠻ ⠠⠊⠾⠸⠒ ⠈⠰⠟ ⠈⠾ ⠨⠅ ⠦

$[\mathbb{Q}(\sqrt[4]{3}, i) : \mathbb{Q}] = 8$

```
[{\mathbb Q}( \sqrt[4]{3}, i ): \mathbb Q] = 8
```

5426. ⠈⠷ ⠠⠋⠸⠒ ⠈⠰⠟ ⠈⠾ ⠨⠅ ⠆

$[F : \mathbb{Q}] = 2$

```
[F:\mathbb Q] = 2
```

5427. ⠈⠷ ⠠⠋⠸⠒ ⠈⠰⠟ ⠈⠾ ⠨⠅ ⠲

$[F : \mathbb{Q}] = 4$

```
[F:\mathbb Q] = 4
```

5428. ⠈⠰⠵⠰⠆ ⠈⠷⠭ ⠈⠾⠸⠌ ⠨⠣⠭ ⠘⠒ ⠐⠬⠭⠬⠂ ⠨⠜ ⠸⠌⠠⠄

$\mathbb{Z}_2[x]/\langle x^3 + x + 1\rangle$

```
{\mathbb Z}_2[x]/ \langle x^3 + x + 1 \rangle
```

5429. ⠂⠬⠨⠁

$1 + \alpha$

1 + \alpha

5430. ⠨⠁⠘⠆

α^2

\alpha^2

5431. ⠂⠬⠨⠁⠘⠆

$1 + \alpha^2$

1 + \alpha^2

5432. ⠨⠁⠬⠨⠁⠘⠆

$\alpha + \alpha^2$

\alpha + \alpha^2

5433. ⠂⠬⠨⠁⠬⠨⠁⠘⠆

$1 + \alpha + \alpha^2$

1 + \alpha + \alpha^2

5434. ⠨⠁⠘⠒⠐⠬⠨⠁⠬⠂ ⠨⠅ ⠴

$\alpha^3 + \alpha + 1 = 0$

\alpha^3 + \alpha + 1 = 0

5435. ⠉⠕⠎ ⠂⠘⠘⠨⠡

$\cos 1^\circ$

\cos 1^\circ

5436. ⠈⠠⠟⠷⠜⠒⠻⠠⠣⠲⠜⠒⠻⠠⠣⠦⠜⠒⠻⠠⠄⠾

${\mathbb Q}(\sqrt{3}, \sqrt[4]{3}, \sqrt[8]{3}, \ldots)$

{\mathbb Q}(\sqrt{3}, \sqrt[4]{3}, \sqrt[8]{3}, \ldots)

5437. ⠈⠠⠟⠷⠨⠏⠘⠒⠐⠾

${\mathbb Q}(\pi^3)$

{\mathbb Q}(\pi^3)

5438. ⠈⠷⠠⠑⠱⠠⠋⠈⠾ ⠅⠱ ⠝⠯

$[E : F] \leq n!$

[E : F] \leq n!

5439. ⠈⠠⠟⠷⠜⠆⠻⠾ ⠈⠱⠈⠱ ⠈⠠⠟⠷⠜⠒⠻⠾

${\mathbb Q}(\sqrt{2}\,) \cong {\mathbb Q}(\sqrt{3}\,)$

{\mathbb Q}(\sqrt{2}\,) \cong {\mathbb Q}(\sqrt{3}\,)

5440. ⠈⠠⠟⠷⠣⠲⠜⠒⠻⠾

${\mathbb Q}(\sqrt[4]{3}\,)$

{\mathbb Q}(\sqrt[4]{3}\,)

5441. ⠈⠠⠟⠷⠣⠲⠜⠒⠻⠊⠾

${\mathbb Q}(\sqrt[4]{3}\, i)$

{\mathbb Q}(\sqrt[4]{3}\, i)

5442. ⠏⠷⠭⠾⠀⠨⠅⠀⠨⠃⠴⠬⠨⠃⠂⠭⠬⠄⠄⠄⠬⠨⠃⠰⠝⠐⠭⠘⠝

$p(x) = \beta_0 + \beta_1 x + \cdots + \beta_n x^n$

p(x) = \beta_0 + \beta_1 x + \cdots + \beta_n x^n

5443. ⠠⠋⠷⠨⠃⠴⠠⠀⠄⠄⠄⠠⠀⠨⠃⠰⠝⠐⠾

$F(\beta_0, \ldots, \beta_n)$

F(\beta_0, \ldots, \beta_n)

5444. ⠈⠠⠵⠈⠷⠭⠈⠾⠸⠌⠨⠨⠷⠭⠘⠒⠐⠤⠆⠨⠨⠾

$\mathbb{Z}[x]/\langle x^3 - 2\rangle$

{\mathbb Z}[x] / \langle x^3 -2 \rangle

5445. ⠏⠷⠭⠾⠀⠨⠅⠀⠭⠘⠏⠐⠤⠁

$p(x) = x^p - a$

p(x) = x^p - a

5446. ⠈⠠⠟⠷⠜⠒⠻⠠⠀⠜⠶⠻⠾⠀⠨⠅⠀⠈⠠⠟⠷⠜⠒⠻⠬⠜⠶⠻⠾

$\mathbb{Q}(\sqrt{3}, \sqrt{7}\,) = \mathbb{Q}(\sqrt{3} + \sqrt{7}\,)$

{\mathbb Q}(\sqrt{3}, \sqrt{7}\,) = {\mathbb Q}(\sqrt{3} + \sqrt{7}\,
)

5447. ⠈⠠⠟⠷⠜⠁⠻⠠⠀⠜⠃⠻⠾⠀⠨⠅⠀⠈⠠⠟⠷⠜⠁⠻⠬⠜⠃⠻⠾

$\mathbb{Q}(\sqrt{a}, \sqrt{b}\,) = \mathbb{Q}(\sqrt{a} + \sqrt{b}\,)$

{\mathbb Q}(\sqrt{a}, \sqrt{b}\,) = {\mathbb Q}(\sqrt{a} + \sqrt{b}\,
)

5448. ⠛⠉⠙⠷⠁⠠⠀⠃⠾⠀⠨⠅⠀⠂

$\gcd(a, b) = 1$

\gcd(a, b) = 1

5449. StartSet⠂⠠⠀⠜⠒⠻⠠⠀⠜⠶⠻⠠⠀⠜21⠻EndSet

$\{1, \sqrt{3}, \sqrt{7}, \sqrt{21}\,\}$

\{ 1, \sqrt{3}, \sqrt{7}, \sqrt{21}\, \}

5450. ⠈⠠⠟⠷⠜⠒⠻⠠⠀⠜⠶⠻⠾

$\mathbb{Q}(\sqrt{3}, \sqrt{7}\,)$

{\mathbb Q}(\sqrt{3}, \sqrt{7}\,)

5451. ⠈⠠⠟⠷⠜⠒⠻⠠⠀⠜⠶⠻⠾⠀⠸⠐⠕⠀⠈⠠⠟⠷⠜⠒⠻⠬⠜⠶⠻⠾

$\mathbb{Q}(\sqrt{3}, \sqrt{7}\,) \supset \mathbb{Q}(\sqrt{3} + \sqrt{7}\,)$

{\mathbb Q}(\sqrt{3}, \sqrt{7}\,) \supset {\mathbb Q}(\sqrt{3}
+\sqrt{7}\,)

5452. ⠈⠷⠈⠠⠟⠷⠜⠒⠻⠠⠀⠜⠶⠻⠾⠸⠒⠈⠠⠟⠈⠾⠀⠨⠅⠀⠲

$[\mathbb{Q}(\sqrt{3}, \sqrt{7}\,) : \mathbb{Q}] = 4$

[{\mathbb Q}(\sqrt{3}, \sqrt{7}\,) : {\mathbb Q}] = 4

5453. ⠈⠷⠈⠠⠟⠷⠜⠒⠻⠬⠜⠶⠻⠾⠸⠒⠈⠠⠟⠈⠾⠀⠨⠅⠀⠆

$[\mathbb{Q}(\sqrt{3} + \sqrt{7}\,) : \mathbb{Q}] = 2$

[{\mathbb Q}(\sqrt{3} + \sqrt{7}\,) : {\mathbb Q}] = 2

5454. ⠜⠒⠻⠬⠜⠶⠻

$\sqrt{3} +\sqrt{7}$

\sqrt{3} +\sqrt{7}

5455. ⠈⠠⠏⠷⠜⠒⠻⠠⠜⠶⠻⠾ ⠨⠅ ⠈⠠⠏⠷⠜⠒⠻⠬⠜⠶⠻⠾

$\mathbb{Q}(\sqrt{3}, \sqrt{7}\,) = \mathbb{Q}(\sqrt{3} +\sqrt{7}\,)$

{\mathbb Q}(\sqrt{3}, \sqrt{7}\,) = {\mathbb Q}(\sqrt{3} +\sqrt{7}\,
)

5456. ⠈⠷⠠⠑⠱⠠⠋⠈⠾ ⠨⠅ ⠆

$[E:F] = 2$

[E:F] = 2

5457. ⠈⠷⠠⠋⠷⠨⠁⠾⠱⠠⠋⠷⠨⠁⠘⠒⠐⠾⠈⠾

$[F(\alpha): F(\alpha^3)]$

[F(\alpha): F(\alpha^3)]

5458. ⠨⠁⠠⠨⠃

α, β

\alpha, \beta

5459. ⠠⠋⠷⠨⠁⠾

$F(\alpha)$

F(\alpha)

5460. ⠨⠃ ⠈⠑ ⠠⠋⠷⠨⠁⠾

$\beta \in F(\alpha)$

\beta \in F(\alpha)

5461. ⠨⠃ ⠨⠅ ⠏⠷⠨⠁⠾⠸⠌⠟⠷⠨⠁⠾

$\beta = p(\alpha)/q(\alpha)$

\beta = p(\alpha)/q(\alpha)

5462. ⠟⠷⠨⠁⠾ ⠌⠨⠅ ⠴

$q(\alpha) \neq 0$

q(\alpha) \neq 0

5463. ⠋⠷⠨⠃⠾ ⠨⠅ ⠴

$f(\beta) = 0$

f(\beta) = 0

5464. ⠴ ⠨⠅ ⠋⠷⠨⠃⠾ ⠨⠅ ⠋⠷⠹⠏⠷⠨⠁⠾⠌⠟⠷⠨⠁⠾⠼⠾ ⠨⠅ ⠁⠴⠬⠁⠂⠷⠹⠏⠷⠨⠁⠾⠌⠟⠷⠨⠁⠾⠼⠾⠬⠄⠄⠄⠬⠁

$$0 = f(\beta) = f\left(\frac{p(\alpha)}{q(\alpha)} \right) = a_0 + a_1 \left(\frac{p(\alpha)}{q(\alpha)} \right) + \cdots + a_n \left(\frac{p(\alpha)}{q(\alpha)} \right)^n$$

0 = f(\beta) = f\left(\frac{p(\alpha)}{q(\alpha)} \right) = a_0
+ a_1 \left(\frac{p(\alpha)}{q(\alpha)} \right) + \cdots + a_n
\left(\frac{p(\alpha)}{q(\alpha)} \right)^n

5465. ⠟⠷⠨⠁⠾⠘⠝

$q(\alpha)^n$

q(\alpha)^n

5466. ⠙⠑⠛⠏ ⠨⠅ ⠝

$\deg p = n$

\deg p = n

5467. ⠈⠷⠠⠋⠷⠨⠁⠾⠱⠠⠋⠈⠾ ⠨⠅ ⠝

$[F(\alpha) : F] = n$

[F(\alpha) : F] = n

5468. ⠈⠠⠟ ⠸⠐⠅ ⠈⠠⠟⠈⠷⠜⠒⠻⠈⠾ ⠸⠐⠅ ⠈⠠⠟⠈⠷⠜⠒⠻⠠⠜⠆⠻⠈⠾

${\mathbb Q}\subset{\mathbb Q}[\sqrt{3}]\subset{\mathbb Q}[\sqrt{3},\sqrt{2}]$

{\mathbb Q}\subset{\mathbb Q}[\sqrt{3}]\subset{\mathbb Q}[\sqrt{3},\sqrt{2}]

5469. ⠜⠆⠻⠤⠜⠒⠻

$\sqrt{2}-\sqrt{3}$

\sqrt{2}-\sqrt{3}

5470. ⠏⠷⠭⠾ ⠨⠅ ⠭⠘⠲⠐⠬⠭⠘⠆⠐⠤⠂

$p(x)=x^4+x^2-1$

p(x)=x^4+x^2-1

5471. ⠁⠘⠆⠐⠬⠂

a^2+1

a^2+1

5472. ⠷⠺⠤⠗⠾

$(w-r)$

(w-r)

5473. ⠏⠷⠭⠾ ⠨⠅ ⠭⠘⠢⠐⠬⠆⠭⠘⠲⠐⠬⠂

$p(x)=x^5+2x^4+1$

p(x)=x^5+2x^4+1

5474. ⠒⠘⠢

3^5

3^5

5475. ⠒⠘⠢⠐ ⠨⠅ 243

$3^5 = 243$

3^5 = 243

5476. ⠗⠷⠭⠾ ⠨⠅ ⠭⠘⠲⠐⠬⠆⠭⠬⠆

$r(x)=x^4+2x+2$

r(x)=x^4+2x+2

5477. ⠎⠷⠭⠾ ⠨⠅ ⠭⠘⠲⠐⠬⠭⠘⠆⠐⠬⠂

$s(x)=x^4+x^2+1$

s(x)=x^4+x^2+1

5478. ⠟⠷⠭⠾ ⠨⠅ ⠭⠘⠒⠐⠬⠒⠭⠘⠆⠐⠬⠒⠭⠤⠆

$q(x)=x^3+3x^2+3x-2$

q(x)=x^3+3x^2+3x-2

5479. ⠏⠨⠁ ⠨⠅ ⠴

$p\alpha = 0$

p \alpha = 0

5480. ⠝⠨⠁ ⠨⠅ ⠴

$n\alpha = 0$

n \alpha = 0

5481. ⠨⠋⠱⠈⠠⠵ ⠫⠕ ⠠⠋

$\phi : \mathbb{Z} \rightarrow F$

\phi : {\mathbb Z} \rightarrow F

5482. ⠨⠋⠷⠝⠾ ⠨⠅ ⠝⠡⠂

$\phi(n) = n \cdot 1$

\phi(n) = n \cdot 1

5483. ⠏⠈⠠⠵

$p\mathbb{Z}$

p {\mathbb Z}

5484. ⠈⠷⠠⠋⠱⠠⠅⠈⠾ ⠨⠅ ⠝

$[F : K] = n$

[F : K] = n

5485. ⠨⠁⠂ ⠄⠄ ⠠⠨⠁⠰⠝⠐⠈⠑⠠⠋

$\alpha_1, \ldots, \alpha_n \in F$

\alpha_1, \ldots, \alpha_n \in F

5486. ⠨⠁ ⠨⠅ ⠁⠂⠨⠁⠂⠬⠄⠄⠄⠬⠁⠰⠝⠐⠨⠁⠰⠝

$\alpha = a_1\alpha_1 + \cdots + a_n\alpha_n$

\alpha = a_1 \alpha_1 + \cdots + a_n \alpha_n

5487. ⠁⠘⠏⠘⠘⠝⠐⠬⠃⠘⠏⠘⠘⠝⠐ ⠨⠅ ⠷⠁⠬⠃⠾⠘⠏⠘⠘⠝

$a^{p^n} + b^{p^n} = (a + b)^{p^n}$

a^{p^n} + b^{p^n} = (a + b)^{p^n}

5488. ⠷⠁⠬⠃⠾⠘⠏⠐ ⠨⠅ ⠐⠅⠠⠎⠈⠉⠅ ⠨⠅ ⠴⠘⠏⠐⠷⠏⠩⠅⠾⠁⠘⠅⠐⠃⠘⠏⠤⠅

$(a + b)^p = \sum_{k=0}^{p} \binom{p}{k} a^k b^{p-k}$

(a + b)^p = \sum_{k = 0}^{p} \binom{p}{k} a^k b^{p - k}

5489. ⠴ ⠐⠅ ⠅ ⠐⠅ ⠏

$0 < k < p$

0 \lt k \lt p

5490. ⠏ ⠅ ⠨⠅ ⠹⠏⠖⠌⠅⠖⠷⠏⠤⠅⠾⠖⠼

$\binom{p}{k} = \frac{p!}{k!(p-k)!}$

\binom{p}{k} = \frac{p!}{k!(p - k)!}

5491. ⠅⠖⠷⠏⠤⠅⠾⠖

$k!(p - k)!$

k!(p - k)!

5492. ⠷⠁⠬⠃⠾⠘⠏⠐ ⠨⠅ ⠁⠘⠏⠐⠬⠃⠘⠏

$(a+b)^p = a^p + b^p$

```
(a + b)^p = a^p + b^p
```

5493. ⠷⠁⠬⠃⠾⠘⠏⠘⠘⠝⠘⠘⠬⠂⠐ ⠨⠅ ⠷⠷⠁⠬⠃⠾⠘⠏⠐⠾⠘⠏⠘⠘⠝⠐ ⠨⠅ ⠷⠁⠘⠏⠐⠬⠃⠘⠏⠐⠾⠘⠏⠘⠘⠝⠐ ⠨⠅ ⠷⠁⠘⠏

$(a+b)^{p^{n+1}} = ((a+b)^p)^{p^n} = (a^p+b^p)^{p^n} = (a^p)^{p^n} + (b^p)^{p^n} = a^{p^{n+1}} + b^{p^{n+1}}$

```
(a + b)^{p^{n + 1}} = ((a + b)^p)^{p^{n}} = (a^p + b^p)^{p^{n}}
= (a^p)^{p^{n}} + (b^p)^{p^{n}} = a^{p^{n + 1}} + b^{p^{n + 1}}
```

5494. ⠭⠘⠆⠐⠤⠆

$x^2 - 2$

```
x^2 - 2
```

5495. ⠷⠭⠤⠜⠆⠻⠾⠷⠭⠬⠜⠆⠻⠾

$(x-\sqrt{2}\,)(x+\sqrt{2}\,)$

```
(x - \sqrt{2}\, )(x + \sqrt{2}\, )
```

5496. ⠨⠁ ⠨⠅ ⠁⠬⠃⠜⠆⠻

$\alpha = a + b\sqrt{2}$

```
\alpha = a + b \sqrt{2}
```

5497. ⠭⠘⠆⠐⠤⠆⠁⠭⠬⠁⠘⠆⠐⠤⠆⠃⠘⠆ ⠨⠅ ⠷⠭⠤⠷⠁⠬⠃⠜⠆⠻⠾⠾⠷⠭⠤⠷⠁⠤⠃⠜⠆⠻⠾⠾

$x^2 - 2ax + a^2 - 2b^2 = (x-(a+b\sqrt{2}\,))(x-(a-b\sqrt{2}\,))$

```
x^2 - 2 a x + a^2 - 2 b^2 = (x - (a + b \sqrt{2}\, ))(x - (a - b
\sqrt{2}\, ))
```

5498. ⠋⠷⠭⠾ ⠨⠅ ⠷⠭⠤⠨⠁⠂⠾⠷⠭⠤⠨⠁⠆⠾⠄⠄⠄⠷⠭⠤⠨⠁⠰⠝⠐⠾

$f(x) = (x-\alpha_1)(x-\alpha_2)\cdots(x-\alpha_n)$

```
f(x) = (x - \alpha_1) (x - \alpha_2) \cdots (x - \alpha_n)
```

5499. ⠨⠁⠰⠊ ⠌⠨⠅ ⠨⠁⠰⠚

$\alpha_i \neq \alpha_j$

```
\alpha_i \neq \alpha_j
```

5500. ⠋⠷⠭⠾ ⠨⠅ ⠷⠭⠤⠨⠁⠾⠘⠅⠐⠛⠷⠭⠾

$f(x) = (x-\alpha)^k g(x)$

```
f(x) = (x - \alpha)^k g(x)
```

5501. ⠅ ⠨⠂ ⠂

$k > 1$

```
k \gt 1
```

5502. ⠋⠄⠷⠭⠾ ⠨⠅ ⠅⠷⠭⠤⠨⠁⠾⠘⠅⠤⠂⠐⠛⠷⠭⠾⠬⠷⠭⠤⠨⠁⠾⠘⠅⠐⠛⠄⠷⠭⠾

$f'(x) = k(x-\alpha)^{k-1}g(x) + (x-\alpha)^k g'(x)$

```
f'(x) = k ( x - \alpha)^{k-1} g(x) + (x- \alpha)^k g'(x)
```

5503. ⠭⠘⠏⠘⠘⠝⠐⠤⠭

$x^{p^n} - x$

```
x^{p^n} -x
```

5504. ⠋⠷⠭⠾ ⠨⠅ ⠭⠘⠏⠘⠘⠝⠐⠤⠭

$f(x) = x^{p^n} - x$

f(x) = x^{p^n} - x

5505. ⠋⠄⠷⠭⠾ ⠨⠅ ⠏⠘⠝⠐⠭⠘⠏⠘⠘⠝⠘⠤⠂⠐⠤⠂ ⠨⠅ ⠤⠂

$f'(x) = p^n x^{p^n - 1} - 1 = -1$

f'(x) = p^n x^{p^n - 1} - 1 = -1

5506. ⠨⠁⠘⠏⠘⠘⠝⠐⠬⠨⠃⠘⠏⠘⠘⠝⠐ ⠨⠅ ⠷⠨⠁⠬⠨⠃⠾⠘⠏⠘⠘⠝

$\alpha^{p^n} + \beta^{p^n} = (\alpha + \beta)^{p^n}$

\alpha^{p^n} + \beta^{p^n} = (\alpha + \beta)^{p^n}

5507. ⠨⠁⠘⠏⠘⠘⠝⠐⠨⠃⠘⠏⠘⠘⠝⠐ ⠨⠅ ⠷⠨⠁⠨⠃⠾⠘⠏⠘⠘⠝

$\alpha^{p^n} \beta^{p^n} = (\alpha\beta)^{p^n}$

\alpha^{p^n} \beta^{p^n} = (\alpha \beta)^{p^n}

5508. ⠤⠨⠁

$-\alpha$

-\alpha

5509. ⠋⠷⠤⠨⠁⠾ ⠨⠅ ⠷⠤⠨⠁⠾⠘⠏⠘⠘⠝⠐⠤⠷⠤⠨⠁⠾ ⠨⠅ ⠤⠨⠁⠘⠏⠘⠘⠝⠐⠬⠨⠁ ⠨⠅ ⠤⠷⠨⠁⠘⠏⠘⠘⠝⠐⠤⠨⠁⠾

$f(-\alpha) = (-\alpha)^{p^n} - (-\alpha) = -\alpha^{p^n} + \alpha = -(\alpha^{p^n} - \alpha) = 0$

f(-\alpha) = (-\alpha)^{p^n} - (-\alpha) = -\alpha^{p^n} + \alpha
= -(\alpha^{p^n} - \alpha) = 0

5510. ⠏ ⠨⠅ ⠆

$p = 2$

p = 2

5511. ⠋⠷⠤⠨⠁⠾ ⠨⠅ ⠷⠤⠨⠁⠾⠘⠆⠘⠘⠝⠐⠤⠷⠤⠨⠁⠾ ⠨⠅ ⠨⠁⠬⠨⠁ ⠨⠅ ⠴

$f(-\alpha) = (-\alpha)^{2^n} - (-\alpha) = \alpha + \alpha = 0$

f(-\alpha) = (-\alpha)^{2^n} - (-\alpha) = \alpha + \alpha = 0

5512. ⠷⠨⠁⠘⠤⠂⠐⠾⠘⠏⠘⠘⠝⠐ ⠨⠅ ⠷⠨⠁⠘⠏⠘⠘⠝⠐⠾⠘⠤⠂⠐ ⠨⠅ ⠨⠁⠘⠤⠂

$(\alpha^{-1})^{p^n} = (\alpha^{p^n})^{-1} = \alpha^{-1}$

(\alpha^{-1})^{p^n} = (\alpha^{p^n})^{-1} = \alpha^{-1}

5513. ⠏⠘⠝⠐⠤⠂

$p^n - 1$

p^n-1

5514. ⠨⠁⠘⠏⠘⠘⠝⠘⠤⠂⠐ ⠨⠅ ⠂

$\alpha^{p^n - 1} = 1$

\alpha^{p^n-1} =1

5515. ⠨⠁⠘⠏⠘⠘⠝⠐⠤⠨⠁ ⠨⠅ ⠴

$\alpha^{p^n} - \alpha = 0$

\alpha^{p^n} -\alpha = 0

5516. ⠠⠛⠠⠋⠷⠏⠘⠝⠐⠾

$\mathrm{GF}(p^n)$

\gf(p^n)

5517. ⠍ ⠨⠂ ⠴

$m > 0$

m \gt 0

5518. ⠠⠛⠠⠋⠷⠏⠘⠍⠐⠾

$\mathrm{GF}(p^m)$

\gf(p^m)

5519. ⠠⠑ ⠨⠅ ⠠⠛⠠⠋⠷⠏⠘⠝⠐⠾

$E = \mathrm{GF}(p^n)$

E = \gf(p^n)

5520. ⠈⠷⠠⠑⠱⠠⠅⠈⠾ ⠨⠅ ⠈⠷⠠⠑⠱⠠⠋⠈⠾⠈⠷⠠⠋⠱⠠⠅⠈⠾

$[E : K] = [E : F][F : K]$

[E:K] = [E:F][F:K]

5521. ⠏⠘⠍⠐⠤⠂

$p^m - 1$

p^m -1

5522. ⠏⠘⠝⠐⠤⠂

$p^n - 1$

p^n -1

5523. ⠭⠘⠏⠘⠘⠍⠘⠤⠂ ⠐⠤⠂

$x^{p^m - 1} - 1$

x^{p^m -1} - 1

5524. ⠭⠘⠏⠘⠘⠝⠘⠤⠂ ⠐⠤⠂

$x^{p^n - 1} - 1$

x^{p^n -1} -1

5525. ⠭⠘⠏⠘⠘⠍⠐⠤⠭

$x^{p^m} - x$

x^{p^m} - x

5526. ⠭⠘⠏⠘⠘⠝⠐⠤⠭

$x^{p^n} - x$

x^{p^n} - x

5527. ⠠⠛⠠⠋⠷⠏⠘24⠐⠾

$\mathrm{GF}(p^{24})$

\gf(p^{24})

5528. ⠠⠋⠘⠈⠼

F^*

F^*

5529. ⠠⠋⠘⠈⠼

F^*

`F^\ast`

5530. ⠠⠛⠀⠈⠱⠈⠱⠀⠈⠠⠵⠰⠏⠂⠰⠘⠑⠂⠐⠡⠄⠄⠄⠈⠡⠈⠠⠵⠰⠏⠰⠰⠅⠰⠘⠑⠰⠘⠰⠅

$G \cong \mathbb{Z}_{p_1^{e_1}} \times \cdots \times \mathbb{Z}_{p_k^{e_k}}$

`G \cong {\mathbb Z}_{p_1^{e_1}} \times \cdots \times {\mathbb Z}_{p_k^{e_k}}`

5531. ⠝⠀⠶⠀⠏⠂⠘⠑⠂⠀⠐⠄⠄⠄⠏⠰⠅⠀⠘⠑⠘⠰⠅

$n = p_1^{e_1} \cdots p_k^{e_k}$

`n = p_1^{e_1} \cdots p_k^{e_k}`

5532. ⠏⠂⠀⠘⠑⠂⠀⠐⠄⠄⠀⠠⠏⠰⠅⠀⠘⠑⠘⠰⠅⠀⠘

$p_1^{e_1}, \ldots, p_k^{e_k}$

`p_1^{e_1}, \ldots, p_k^{e_k}`

5533. ⠭⠘⠗⠐⠤⠂

$x^r - 1$

`x^r - 1`

5534. ⠭⠘⠍⠐⠤⠂

$x^m - 1$

`x^m - 1`

5535. ⠭⠘⠍⠐⠤⠂

$x^m - 1$

`x^m -1`

5536. ⠍⠀⠀⠅⠱⠀⠳⠠⠛⠳

$m \leq |G|$

`m \leq |G|`

5537. ⠠⠑⠘⠈⠼

E^*

`E^{\ast}`

5538. ⠠⠛⠠⠋⠷⠆⠘⠲⠐⠾

$\mathrm{GF}(2^4)$

`\gf(2^4)`

5539. ⠈⠠⠵⠆⠠⠌⠄⡗⠭⠆⠶⠈⠢⠤⠄⠐⠬⠭⠬⠭⠘⠲⠐⠠⡗⠭⠆⠶⠈⠢⠠⠄

$\mathbb{Z}_2/\langle 1 + x + x^4 \rangle$

`{\mathbb Z}_2/ \langle 1 + x + x^4 \rangle`

5540. StartSet⠈ ⠤⠵⠤⠋⠐ ⠏⠤⠋⠆ ⠏⠲ ⠐⠤⠋⠒ ⠏⠒⠋ ⠓⠤⠐⠟⠁ ⠆⠠⠁⠐ ⠉⠢ ⠈ ⠠⠵⠆ and⠐ ⠤⠅⠋ ⠤⠅⠋ ⠓⠆⠐ ⠶ ⠠⠼EndSet

$\{a_0 + a_1\alpha + a_2\alpha^2 + a_3\alpha^3 : a_i \in \mathbb{Z}_2 \text{ and } 1 + \alpha + \alpha^4 = 0\}$

`\{ a_0 + a_1 \alpha + a_2 \alpha^2 + a_3 \alpha^3 : a_i \in {\mathbb Z}_2 \text{ and } 1 + \alpha + \alpha^4 = 0 \}`

5541. ⠂⠬⠨⠁⠬⠨⠁ ⠘⠲⠐ ⠨⠅ ⠴

$1 + \alpha + \alpha^4 = 0$

1 + \alpha +\alpha^4 = 0

5542. ⠈⠠⠵15

$\mathbb{Z}_{15}$

{\mathbb Z}_{15}

5543. ⠷⠝ ⠠⠅ ⠾

(n, k)

(n, k)

5544. ⠠⠑⠱⠈⠠⠵⠆⠘⠅⠐ ⠫⠕ ⠈⠠⠵⠆⠘⠝

$E : \mathbb{Z}_2^k \rightarrow \mathbb{Z}_2^n$

E:{\mathbb Z}^{k}_{2} \rightarrow {\mathbb Z}^{n}_{2}

5545. ⠠⠙⠱⠈⠠⠵⠆⠘⠝⠐ ⠫⠕ ⠈⠠⠵⠆⠘⠅

$D : \mathbb{Z}_2^n \rightarrow \mathbb{Z}_2^k$

D:{\mathbb Z}^{n}_{2} \rightarrow {\mathbb Z}^{k}_{2}

5546. ⠠⠓ ⠈⠑ ⠈⠠⠍⠰⠅⠈⠡⠝⠐⠷⠈⠠⠵⠆⠾

$H \in \mathbb{M}_{k \times n}(\mathbb{Z}_2)$

H \in {\mathbb M}_{k \times n}({\mathbb Z}_2)

5547. ⠨⠋⠱⠈⠠⠵⠆⠘⠅⠐ ⠫⠕ ⠈⠠⠵⠆⠘⠝

$\phi : \mathbb{Z}_2^k \rightarrow \mathbb{Z}_2^n$

\phi : {\mathbb Z}_2^k \rightarrow {\mathbb Z}_2^n

5548. ⠷⠁ ⠂ ⠠⠁⠆ ⠄⠄ ⠠⠁ ⠰⠝ ⠐⠾

$(a_1, a_2, \ldots, a_n)$

(a_1, a_2, \ldots, a_n)

5549. ⠷⠁ ⠰⠝⠂⠠⠁⠂ ⠠⠁⠆ ⠄⠄ ⠠⠁ ⠰⠝⠤⠂ ⠐⠾

$(a_n, a_1, a_2, \ldots, a_{n-1})$

(a_n, a_1, a_2, \ldots, a_{n - 1})

5550. ⠷⠖ ⠠⠒⠾

$(6, 3)$

(6,3)

5551. ⠠⠛⠂ ⠨⠅ ⠠⠷⠼⠂ ⠼⠴ ⠼⠴ ⠼⠴⠠⠾ ⠠⠷⠼⠴ ⠼⠂ ⠼⠴ ⠼⠴⠠⠾ ⠠⠷⠼⠴ ⠼⠴ ⠼⠂ ⠠⠾
⠠⠷⠼⠂ ⠼⠴ ⠼⠴ ⠼⠴⠠⠾ ⠠⠷⠼⠴ ⠼⠂ ⠼⠴ ⠼⠴⠠⠾ ⠠⠷⠼⠴ ⠼⠴ ⠼⠂ ⠠⠾and ⠠⠛⠆ ⠨⠅ ⠠⠷⠼⠂ ⠼⠴ ⠼⠴ ⠼⠴⠠⠾
⠠⠷⠼⠂ ⠼⠂ ⠼⠴ ⠼⠴⠠⠾ ⠠⠷⠼⠂ ⠼⠂ ⠼⠂ ⠠⠾ ⠠⠷⠼⠂ ⠼⠂ ⠼⠂ ⠠⠾ ⠠⠷⠼⠴ ⠼⠂ ⠼⠂ ⠠⠾
⠠⠷⠼⠴ ⠼⠴ ⠼⠂ ⠠⠾

$$G_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad G_2 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

G_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad G_2 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}

5552. StartLayout1stRow⠷000⠾ ⠳⠳⠒⠒⠕ ⠷000000⠾ ⠷100⠾ ⠳⠳⠒⠒⠕ ⠷100100⠾2nd

$$\begin{array}{rclccrcl} (000) & \mapsto & (000000) & & & (100) & \mapsto & (100100) \\ (001) & \mapsto & (001001) & & & (101) & \mapsto & (101101) \\ (010) & \mapsto & (010010) & & & (110) & \mapsto & (110110) \\ (011) & \mapsto & (011011) & & & (111) & \mapsto & (111111). \end{array}$$

\begin{array}{rclccrcl} (000) & \mapsto & (000000) & & & (100) & \mapsto & (100100) \\ (001) & \mapsto & (001001) & & & (101) & \mapsto & (101101) \\ (010) & \mapsto & (010010) & & & (110) & \mapsto & (110110) \\ (011) & \mapsto & (011011) & & & (111) & \mapsto & (111111). \end{array}

5553. StartLayout1stRow⠷000⠾ ⠳⠳⠒⠒⠕ ⠷000000⠾ ⠷100⠾ ⠳⠳⠒⠒⠕ ⠷111100⠾2nd

$$\begin{array}{rclccrcl} (000) & \mapsto & (000000) & & & (100) & \mapsto & (111100) \\ (001) & \mapsto & (001111) & & & (101) & \mapsto & (110011) \\ (010) & \mapsto & (011110) & & & (110) & \mapsto & (100010) \\ (011) & \mapsto & (010001) & & & (111) & \mapsto & (101101). \end{array}$$

\begin{array}{rclccrcl} (000) & \mapsto & (000000) & & & (100) & \mapsto & (111100) \\ (001) & \mapsto & (001111) & & & (101) & \mapsto & (110011) \\ (010) & \mapsto & (011110) & & & (110) & \mapsto & (100010) \\ (011) & \mapsto & (010001) & & & (111) & \mapsto & (101101). \end{array}

5554. ⠷011011⠾

(011011)

(011011)

5555. ⠷⠁ ⠰⠴ ⠠⠁ ⠂ ⠤ ⠠⠁ ⠰⠝⠤⠂ ⠐⠾

$(a_0, a_1, \ldots, a_{n-1})$

(a_0, a_1, \ldots, a_{n - 1})

5556. ⠋⠷⠭⠾ ⠶ ⠁⠰⠴⠬⠁⠂⠭⠬⠄⠄⠄⠬⠁⠰⠝⠤⠂⠐⠭⠘⠝⠤⠂

$f(x) = a_0 + a_1 x + \cdots + a_{n-1} x^{n-1}$

f(x) = a_0 + a_1 x + \cdots + a_{n-1} x^{n - 1}

5557. ⠷10011⠾

(10011)

(10011)

5558. ⠂⠬⠴⠭⠬⠴⠭⠘⠆ ⠐⠬⠂⠭⠘⠒ ⠐⠬⠂⠭⠘⠲ ⠐ ⠶ ⠂⠬⠭⠘⠒ ⠐⠬⠭⠘⠲

$1 + 0x + 0x^2 + 1x^3 + 1x^4 = 1 + x^3 + x^4$

1 + 0 x + 0 x^2 + 1 x^3 + 1 x^4 = 1 + x^3 + x^4

5559. ⠋⠷⠭⠾ ⠈⠑ ⠨⠰⠵⠆ ⠈⠷⠭⠈⠾

$f(x) \in \mathbb{Z}_2[x]$

f(x) \in {\mathbb Z}_2[x]

5560. ⠙⠑⠛⠋⠷⠭⠾ ⠐⠅ ⠝

$\deg f(x) < n$

\deg f(x) \lt n

5561. ⠭⠬⠭⠘⠆⠐⠬⠭⠘⠲

$x + x^2 + x^4$

x + x^2 + x^4

5562. ⠷01101⠾

(01101)

(01101)

5563. ⠝⠤⠅

$n - k$

n - k

5564. ⠷⠁⠴⠠⠄⠄⠄⠠⠁⠰⠅⠤⠂⠐⠾

$(a_0, \ldots, a_{k-1})$

(a_0, \ldots, a_{k - 1})

5565. ⠋⠷⠭⠾ ⠨⠅ ⠁⠴⠬⠁⠂⠭⠬⠄⠄⠄⠬⠁⠰⠅⠤⠂⠐⠭⠘⠅⠤⠂

$f(x) = a_0 + a_1 x + \cdots + a_{k-1} x^{k-1}$

f(x) = a_0 + a_1 x + \cdots + a_{k - 1} x^{k - 1}

5566. ⠛⠷⠭⠾ ⠨⠅ ⠂⠬⠭⠘⠒

$g(x) = 1 + x^3$

g(x)= 1 + x^3

5567. ⠷⠁⠴⠠⠁⠂⠠⠁⠆⠐⠾

(a_0, a_1, a_2)

(a_0, a_1, a_2)

5568. ⠋⠷⠭⠾ ⠨⠅ ⠁⠴⠬⠁⠂⠭⠬⠁⠆⠭⠘⠆

$f(x) = a_0 + a_1 x + a_2 x^2$

f(x) = a_0 + a_1 x + a_2 x^2

5569. ⠂⠬⠭⠘⠒

$1 + x^3$

1 + x^3

5570. ⠨⠋⠸⠒⠈⠰⠵⠰⠆⠘⠒⠐ ⠫⠕ ⠈⠰⠵⠰⠆⠘⠖

$\phi : \mathbb{Z}_2^3 \rightarrow \mathbb{Z}_2^6$

\phi : {\mathbb Z}_2^3 \rightarrow {\mathbb Z}_2^6

5571. ⠨⠋⠸⠒⠋⠷⠭⠾ ⠫⠳⠒⠒⠕ ⠛⠷⠭⠾⠋⠷⠭⠾

$\phi : f(x) \mapsto g(x) f(x)$

\phi : f(x) \mapsto g(x) f(x)

5572. ⠨⠋⠷⠁⠴⠠⠁⠂⠠⠁⠆⠐⠾ ⠨⠅ ⠷000000⠾

$\phi(a_0, a_1, a_2) = (000000)$

\phi (a_0, a_1, a_2) = (000000)

5573.

$$a_0 + a_1 x + a_2 x^2$$

```
a_0 + a_1 x + a_2 x^2
```

5574.

$$\ker \phi = \{(000)\}$$

```
\ker \phi = \{ (000) \}
```

5575.

$$H = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

```
H = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 &
0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}
```

5576.

$$x^n - 1 = (x - 1)(x^{n-1} + \cdots + x + 1)$$

```
x^n - 1 = (x - 1)( x^{n - 1} + \cdots + x + 1)
```

5577.

$$R_n = \mathbb{Z}_2[x]/\langle x^n - 1 \rangle$$

```
R_n = {\mathbb Z}_2[x]/ \langle x^n - 1 \rangle
```

5578.

$$f(t) = a_0 + a_1 t + \cdots + a_{n-1} t^{n-1}$$

```
f(t) = a_0 + a_1 t + \cdots + a_{n - 1} t^{n - 1}
```

5579.

$$t^n = 1$$

```
t^n = 1
```

5580.

$$\mathbb{Z}[x]/\langle x^n - 1 \rangle$$

```
{\mathbb Z}[x] / \langle x^n - 1 \rangle
```

5581.

$$t f(t) = a_{n-1} + a_0 t + \cdots + a_{n-2} t^{n-1}$$

```
tf(t) = a_{n - 1} + a_0 t + \cdots + a_{n - 2} t^{n - 1}
```

5582.

$$f(t)$$

```
f(t)
```

5583.

$$R_n = \mathbb{Z}[x]/\langle x^n - 1 \rangle$$

```
R_n = {\mathbb Z}[x] / \langle x^n - 1 \rangle
```

5584.

$$t f(t)$$

```
t f(t)
```

5585. ⠞⠘⠅ ⠐⠋⠷⠞⠾

$t^k f(t)$

t^k f(t)

5586. ⠋⠷⠞⠾⠠⠞⠋⠷⠞⠾⠠⠞⠘⠆⠐⠋⠷⠞⠾⠠⠄⠄ ⠠⠞⠘⠝⠤⠂ ⠐⠋⠷⠞⠾

$f(t), tf(t), t^2f(t), \ldots, t^{n-1}f(t)$

f(t), tf(t), t^2f(t), \ldots, t^{n-1}f(t)

5587. ⠏⠷⠞⠾

$p(t)$

p(t)

5588. ⠏⠷⠞⠾⠋⠷⠞⠾

$p(t)f(t)$

p(t)f(t)

5589. ⠈ ⠠⠵⠆ ⠈⠷⠭ ⠈⠾⠸⠌ ⠨⠣⠭⠘⠝⠐⠬⠂⠨⠜

$\mathbb{Z}_2[x]/\langle x^n + 1\rangle$

{\mathbb Z}_2[x]/\langle x^n + 1\rangle

5590. ⠷⠁⠂ ⠠⠄⠄ ⠠⠁ ⠰⠝⠤⠂ ⠠⠁⠴⠾

$(a_1, \ldots, a_{n-1}, a_0)$

(a_1, \ldots, a_{n-1}, a_0)

5591. ⠨⠋⠸⠒ ⠈ ⠠⠵⠆ ⠈⠷⠭ ⠈⠾ ⠫⠕ ⠠⠗⠰⠝

$\phi : \mathbb{Z}_2[x] \rightarrow R_n$

\phi : {\mathbb Z}_2[x] \rightarrow R_n

5592. ⠨⠋⠈⠷⠋⠷⠭⠾⠈⠾ ⠨⠅ ⠋⠷⠞⠾

$\phi[f(x)] = f(t)$

\phi[f(x)] = f(t)

5593. ⠭⠘⠝⠐⠤⠂

$x^n - 1$

x^n - 1

5594. ⠨⠋⠷⠠⠊⠾

$\phi(I)$

\phi(I)

5595. ⠨⠣⠭⠘⠝⠐⠤⠂⠨⠜

$\langle x^n - 1\rangle$

\langle x^n - 1 \rangle

5596. ⠠⠊ ⠨⠅ ⠨⠣⠛⠷⠭⠾⠨⠜

$I = \langle g(x)\rangle$

I = \langle g(x) \rangle

5597. ⠠⠉ ⠨⠅ ⠠⡗⠿⠆ ⠭⠑⠢⠄ ⠛⠷⠞⠾⠠⡗⠿⠆ ⠭⠑⠠⠄ ⠨⠅ StartSet⠋⠷⠞⠾⠛⠷⠞⠾⠒⠋⠷⠞⠾ ⠈⠑ ⠠⠗⠰⠝ ⠐and⠛⠷⠭⠾

$C = \langle g(t) \rangle = \{ f(t)g(t) : f(t) \in R_n \text{ and } g(x) \mid (x^n - 1) \text{ in } {\mathbb Z}_2[x] \}$

```
C = \langle g(t) \rangle = \{ f(t)g(t) : f(t) \in R_n \text{ and
} g(x) \mid (x^n - 1) \text{ in } {\mathbb Z}_2[x] \}
```

5598. ⠭⠘⠶⠐⠤⠂

$x^7 - 1$

```
x^7 - 1
```

5599. ⠭⠘⠶⠐⠤⠂ ⠨⠅ ⠷⠂⠬⠭⠾⠷⠂⠬⠭⠬⠭⠘⠒⠐⠾⠷⠂⠬⠭⠘⠆⠐⠬⠭⠘⠒⠐⠾

$x^7 - 1 = (1 + x)(1 + x + x^3)(1+ x^2 + x^3)$

```
x^7 - 1 = (1 + x)(1 + x + x^3)(1+ x^2 + x^3)
```

5600. ⠛⠷⠞⠾ ⠨⠅ ⠷⠂⠬⠞⠬⠞⠘⠒⠐⠾

$g(t) = (1 + t + t^3)$

```
g(t) = (1 + t + t^3)
```

5601. ⠠⠗⠶

R_7

```
R_7
```

5602. ⠷⠶⠠⠲⠾

$(7, 4)$

```
(7, 4)
```

5603. ⠛⠷⠞⠾

$g(t)$

```
g(t)
```

5604. ⠞⠘⠆

t^2

```
t^2
```

5605. ⠞⠘⠒

t^3

```
t^3
```

5606. ⠠⠛ ⠨⠅ ⠠⠷⠼⠂ ⠼⠴ ⠼⠴ ⠼⠴⠠⠾ ⠠⠷⠼⠂ ⠼⠂ ⠼⠴ ⠼⠴⠠⠾ ⠠⠷⠼⠴ ⠼⠂ ⠼⠂ ⠼⠴⠠⠾
⠠⠷⠼⠂ ⠼⠴ ⠼⠂ ⠼⠂⠠⠾ ⠠⠷⠼⠴ ⠼⠂ ⠼⠴ ⠼⠂⠠⠾ ⠠⠷⠼⠴ ⠼⠴ ⠼⠂ ⠼⠴⠠⠾
⠠⠷⠼⠴ ⠼⠴ ⠼⠴ ⠼⠂⠠⠾

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
G = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1
& 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0
& 0 & 1 \end{pmatrix}
```

5607.

t^k

`t^k`

5608.

$x^n - 1 = g(x)h(x)$

`x^n - 1 = g(x) h(x)`

5609.

$g(x) = g_0 + g_1 x + \cdots + g_{n-k} x^{n-k}$

`g(x) = g_0 + g_1 x + \cdots + g_{n-k} x^{n-k}`

5610.

$h(x) = h_0 + h_1 x + \cdots + h_k x^k$

`h(x) = h_0 + h_1 x + \cdots + h_k x^k`

5611.

$$G = \begin{pmatrix} g_0 & 0 & \cdots & 0 \\ g_1 & g_0 & \cdots & 0 \\ \vdots & \vdots &\ddots & \vdots \\ g_{n-k} & g_{n-k-1} & \cdots & g_0 \\ 0 & g_{n-k} & \cdots & g_{1} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & g_{n-k} \end{pmatrix}$$

```
G = \begin{pmatrix} g_0 & 0 & \cdots & 0 \\ g_1 & g_0 & \cdots &
0 \\ \vdots & \vdots &\ddots & \vdots \\ g_{n-k} & g_{n-k-1} & \cdots
& g_0 \\ 0 & g_{n-k} & \cdots & g_{1} \\ \vdots & \vdots & \ddots
& \vdots \\ 0 & 0 & \cdots & g_{n-k} \end{pmatrix}
```

5612.

$(n - k) \times n$

`(n - k) \times n`

5613.

$$H = \begin{pmatrix} 0 & \cdots & 0 & 0 & h_k & \cdots & h_0 \\ 0 & \cdots & 0 & h_k & \cdots & h_0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ h_k & \cdots & h_0 & 0 & 0 & \cdots & 0 \end{pmatrix}$$

```
H = \begin{pmatrix} 0 & \cdots & 0 & 0 & h_k & \cdots & h_0 \\ 0
& \cdots & 0 & h_k & \cdots & h_0 & 0 \\ \cdots & \cdots & \cdots
& \cdots & \cdots & \cdots & \cdots \\ h_k & \cdots & h_0 & 0 &
0 & \cdots & 0 \end{pmatrix}
```

5614.

$C = \langle g(t) \rangle$

`C = \langle g(t) \rangle`

5615. ⠠⠓⠠⠛ ⠶ ⠼⠴

$HG = 0$

HG = 0

5616. ⠭⠘⠶⠐⠤⠂ ⠶ ⠛⠷⠭⠾⠓⠷⠭⠾ ⠶ ⠷⠂⠬⠭⠬⠭⠘⠒⠐⠾⠷⠂⠬⠭⠬⠭⠘⠆⠐⠬⠭⠘⠲⠐⠾

$x^7 - 1 = g(x)h(x) = (1 + x + x^3)(1 + x + x^2 + x^4)$

x^7 - 1 = g(x) h(x) = (1 + x + x^3)(1 + x + x^2 + x^4)

5617. ⠠⠓ ⠶ ⠠⠷⠼⠴ ⠼⠴ ⠼⠂ ⠼⠴ ⠼⠂ ⠼⠂ ⠼⠂ ⠠⠾ ⠠⠷⠼⠴ ⠼⠂ ⠼⠴ ⠼⠂ ⠼⠂ ⠼⠂ ⠼⠴⠠⠾
⠠⠷⠼⠂ ⠼⠴ ⠼⠂ ⠼⠂ ⠼⠂ ⠼⠴ ⠼⠴⠠⠾

$$H = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

H = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 &
1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}

5618. ⠠⠷⠼⠂ ⠼⠂ ⠄⠄⠄ ⠼⠂ ⠠⠾ ⠠⠷⠉⠁ ⠉⠆ ⠄⠄⠄ ⠉⠵⠠⠾ ⠠⠷⠉⠁⠑ ⠉⠆⠑ ⠄⠄⠄ ⠉⠵⠑ .
⠠⠷⠑⠄⠄⠄ ⠑⠄⠄⠄ ⠆⠄⠄⠄ ⠑⠄⠄⠄ ⠠⠾ ⠠⠷⠉⠁⠙⠤⠁ ⠉⠆⠙⠤⠁ ⠄⠄⠄ ⠉⠵⠙⠤⠁ ⠠⠾

$$\begin{pmatrix} 1 & 1 & \cdots & 1 \\ \alpha_1 & \alpha_2 & \cdots & \alpha_n \\ \alpha_1^2 & \alpha_2^2 & \cdots & \alpha_n^2 \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_1^{n-1} & \alpha_2^{n-1} & \cdots & \alpha_n^{n-1} \end{pmatrix}$$

\begin{pmatrix} 1 & 1 & \cdots & 1 \\ \alpha_1 & \alpha_2 & \cdots
& \alpha_n \\ \alpha_1^2 & \alpha_2^2 & \cdots & \alpha_n^2 \\ \vdots
& \vdots & \ddots & \vdots \\ \alpha_1^{n-1} & \alpha_2^{n-1} &
\cdots & \alpha_n^{n-1} \end{pmatrix}

5619. ⠙⠑⠞⠠⠷⠼⠂ ⠼⠂ ⠄⠄⠄ ⠼⠂ ⠠⠾ ⠠⠷⠉⠁ ⠉⠆ ⠄⠄⠄ ⠉⠵⠂⠠⠾ ⠠⠷⠉⠁⠑⠂ ⠉⠆⠑⠂ ⠄⠄⠄ ⠉
⠠⠷⠑⠄⠄⠄ ⠑⠄⠄⠄ ⠆⠄⠄⠄ ⠑⠄⠄⠄ ⠠⠾ ⠠⠷⠉⠁⠙⠤⠁⠂ ⠉⠆⠙⠤⠁⠂ ⠄⠄⠄ ⠉⠵⠙⠤⠁⠂⠠⠾ ⠶

$$\det \begin{pmatrix} 1 & 1 & \cdots & 1 \\ \alpha_1 & \alpha_2 & \cdots & \alpha_n \\ \alpha_1^2 & \alpha_2^2 & \cdots & \alpha_n^2 \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_1^{n-1} & \alpha_2^{n-1} & \cdots & \alpha_n^{n-1} \end{pmatrix} = \prod_{1 \leq j < i \leq n} (\alpha_i - \alpha_j)$$

\det \begin{pmatrix} 1 & 1 & \cdots & 1 \\ \alpha_1 & \alpha_2 &
\cdots & \alpha_n \\ \alpha_1^2 & \alpha_2^2 & \cdots & \alpha_n^2
\\ \vdots & \vdots & \ddots & \vdots \\ \alpha_1^{n-1} & \alpha_2^{n-1}
& \cdots & \alpha_n^{n-1} \end{pmatrix} = \prod_{1 \leq j \lt i
\leq n} (\alpha_i - \alpha_j)

5620. ⠉⠆⠤⠉⠂

$\alpha_2 - \alpha_1$

\alpha_2 - \alpha_1

5621. ⠏⠷⠭⠾ ⠶ ⠙⠑⠞⠠⠷⠼⠂ ⠼⠂ ⠄⠄⠄ ⠼⠂ ⠼⠂ ⠠⠾ ⠠⠷⠉⠁ ⠉⠆ ⠄⠄⠄ ⠉⠵⠤⠁⠂ ⠭⠠⠾
⠠⠷⠉⠁⠑⠂ ⠉⠆⠑⠂ ⠄⠄⠄ ⠉⠵⠤⠑⠁⠂ ⠭⠑⠂⠠⠾ ⠠⠷⠑⠄⠄⠄ ⠑⠄⠄⠄ ⠆⠄⠄⠄ ⠑⠄⠄⠄ ⠑⠄⠄⠄
⠠⠷⠉⠁⠙⠤⠁⠂ ⠉⠆⠙⠤⠁⠂ ⠄⠄⠄ ⠉⠵⠤⠙⠤⠁⠂ ⠭⠙⠤⠁⠠⠾

$$p(x) = \det \begin{pmatrix} 1 & 1 & \cdots & 1 & 1 \\ \alpha_1 & \alpha_2 & \cdots & \alpha_{n-1} & x \\ \alpha_1^2 & \alpha_2^2 & \cdots & \alpha_{n-1}^2 & x^2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \alpha_1^{n-1} & \alpha_2^{n-1} & \cdots & \alpha_{n-1}^{n-1} & x^{n-1} \end{pmatrix}$$

p(x) = \det \begin{pmatrix} 1 & 1 & \cdots & 1 & 1 \\ \alpha_1 &
\alpha_2 & \cdots & \alpha_{n-1} & x \\ \alpha_1^2 & \alpha_2^2
& \cdots & \alpha_{n-1}^2 & x^2 \\ \vdots & \vdots & \ddots & \vdots
& \vdots \\ \alpha_1^{n-1} & \alpha_2^{n-1} & \cdots & \alpha_{n-1}^{n-1}
& x^{n-1} \end{pmatrix}

5622. ⠏⠂ ⠤ ⠠⠏ ⠰⠏⠤⠂ ⠂

$\alpha_1, \ldots, \alpha_{n-1}$

\alpha_1, \ldots, \alpha_{n-1}

5623. ⠏⠷⠭⠾ ⠶ ⠷⠭⠤⠏⠂⠽⠷⠭⠤⠏⠆⠾⠄⠄⠄⠷⠭⠤⠏ ⠰⠏⠤⠂ ⠐⠽⠏

$p(x) = (x - \alpha_1)(x - \alpha_2) \cdots (x - \alpha_{n-1})\beta$

p(x) = (x - \alpha_1)(x - \alpha_2) \cdots (x - \alpha_{n-1}) \beta

5624. ⠏ ⠶ ⠷⠤⠂⠾⠘⠝⠬⠝⠐⠙⠑⠞⠠⠷⠼ ⠼⠂ ⠄⠄⠄ ⠼⠂ ⠠⠽ ⠠⠷⠏⠂ ⠏⠆ ⠄⠄⠄ ⠏ ⠰⠏⠤⠂ ⠂⠠⠽
⠠⠷⠏⠂⠑⠆⠂ ⠏⠆⠑⠆⠂ ⠄⠄⠄ ⠏ ⠰⠏⠤⠂⠑⠆⠂⠠⠽⠠⠷⠳⠄⠄ ⠳⠄⠄⠄ ⠱⠄⠄⠄ ⠳⠄⠄⠄⠠⠽
⠠⠷⠏⠂⠘⠝⠤⠆⠂ ⠏⠆⠘⠝⠤⠆⠂ ⠄⠄⠄ ⠏ ⠰⠏⠤⠂⠘⠝⠤⠆⠠⠽

$$\beta = (-1)^{n+n} \det \begin{pmatrix} 1 & 1 & \cdots & 1 \\ \alpha_1 & \alpha_2 & \cdots & \alpha_{n-1} \\ \alpha_1^2 & \alpha_2^2 & \cdots & \alpha_{n-1}^2 \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_1^{n-2} & \alpha_2^{n-2} & \cdots & \alpha_{n-1}^{n-2} \end{pmatrix}$$

\beta = (-1)^{n + n} \det \begin{pmatrix} 1 & 1 & \cdots & 1 \\
\alpha_1 & \alpha_2 & \cdots & \alpha_{n-1} \\ \alpha_1^2 & \alpha_2^2
& \cdots & \alpha_{n-1}^2 \\ \vdots & \vdots & \ddots & \vdots \\
\alpha_1^{n-2} & \alpha_2^{n-2} & \cdots & \alpha_{n-1}^{n-2} \end{pmatrix}

5625. ⠏ ⠶ ⠷⠤⠂⠾⠘⠝⠬⠝⠐⠨⠏⠰⠂⠀⠨⠅⠀⠚⠀⠐⠅⠀⠊⠀⠨⠅⠀⠝⠤⠂⠐⠷⠏⠰⠊⠤⠏⠰⠚⠾

$\beta = (-1)^{n+n} \prod_{1 \leq j < i \leq n-1} (\alpha_i - \alpha_j)$

\beta = (-1)^{n+n} \prod_{1 \leq j \lt i \leq n - 1} (\alpha_i -
\alpha_j)

5626. ⠭ ⠶ ⠏ ⠰⠝

$x = \alpha_n$

x = \alpha_n

5627. ⠺

ω

\omega

5628. ⠎⠬⠂

$s + 1$

s + 1

5629. ⠛⠷⠺⠘⠗⠐⠾ ⠶ ⠛⠷⠺⠘⠗⠬⠂⠐⠾ ⠶ ⠄⠄⠄ ⠶ ⠛⠷⠺⠘⠗⠬⠎⠤⠂⠐⠾ ⠶ ⠴

$g(\omega^r) = g(\omega^{r+1}) = \cdots = g(\omega^{r+s-1}) = 0$

g(\omega^r) = g(\omega^{r + 1}) = \cdots = g(\omega^{r + s - 1})
= 0

5630. ⠋⠷⠭⠾ ⠨⠅ ⠁⠰⠊⠴⠐⠭⠘⠊⠴⠐⠬⠁⠰⠊⠂⠐⠭⠘⠊⠂⠐⠬⠄⠄⠄⠬⠁⠰⠊⠰⠰⠎⠤⠂⠐⠭⠘⠊⠘⠰⠎⠤⠂

$$f(x) = a_{i_0} x^{i_0} + a_{i_1} x^{i_1} + \cdots + a_{i_{s-1}} x^{i_{s-1}}$$

f(x) = a_{i_0} x^{i_0} + a_{i_1} x^{i_1} + \cdots + a_{i_{s - 1}}
x^{i_{s - 1}}

5631. ⠋⠷⠨⠕⠘⠗⠐⠾ ⠨⠅ ⠋⠷⠨⠕⠘⠗⠬⠂⠐⠾ ⠨⠅ ⠄⠄⠄ ⠨⠅ ⠋⠷⠨⠕⠘⠗⠬⠎⠤⠂⠐⠾ ⠨⠅ ⠴

$$f(\omega^r) = f(\omega^{r+1}) = \cdots = f(\omega^{r+s-1}) = 0$$

f(\omega^r) = f(\omega^{r + 1}) = \cdots = f(\omega^{r + s - 1})
= 0

5632. ⠷⠁⠰⠊⠴⠐⠠⠁⠰⠊⠂⠐⠠⠄⠠⠁⠰⠊⠰⠰⠎⠤⠂⠰⠐⠾

$$(a_{i_0}, a_{i_1}, \ldots, a_{i_{s-1}})$$

(a_{i_0}, a_{i_1}, \ldots, a_{i_{s - 1}})

5633. ⠠⠷⠷⠨⠕⠘⠊⠴⠐⠾⠘⠗ ⠷⠨⠕⠘⠊⠂⠐⠾⠘⠗ ⠄⠄⠄ ⠷⠨⠕⠘⠊⠘⠰⠎⠤⠂⠘⠐⠾⠘⠗⠠⠾
⠠⠷⠷⠨⠕⠘⠊⠴⠐⠾⠘⠗⠬⠂ ⠷⠨⠕⠘⠊⠂⠐⠾⠘⠗⠬⠂ ⠄⠄⠄ ⠷⠨⠕⠘⠊⠘⠰⠎⠤⠂⠘⠐⠾⠘⠗⠬⠂⠠⠾
⠠⠷⠣⠄⠄⠄ ⠣⠄⠄⠄ ⠰⠄⠄⠄ ⠣⠄⠄⠄⠠⠾ ⠠⠷⠷⠨⠕⠘⠊⠴⠐⠾⠘⠗⠬⠎⠤⠂ ⠷⠨⠕⠘⠊⠂⠐⠾⠘⠗⠬⠎⠤⠂ ⠄⠄⠄ ⠷⠨⠕

$$\begin{pmatrix} (\omega^{i_0})^r & (\omega^{i_1})^r & \cdots & (\omega^{i_{s-1}})^r \\ (\omega^{i_0})^{r+1} & (\omega^{i_1})^{r+1} & \cdots & (\omega^{i_{s-1}})^{r+1} \\ \vdots & \vdots & \ddots & \vdots \\ (\omega^{i_0})^{r+s-1} & (\omega^{i_1})^{r+s-1} & \cdots & (\omega^{i_{s-1}})^{r+s-1} \end{pmatrix}$$

\begin{pmatrix} (\omega^{i_0})^r & (\omega^{i_1})^r & \cdots & (\omega^{i_{s-1}})^r
\\ (\omega^{i_0})^{r+1} & (\omega^{i_1})^{r+1} & \cdots & (\omega^{i_{s-1}})^{r+1}
\\ \vdots & \vdots & \ddots & \vdots \\ (\omega^{i_0})^{r+s-1} &
(\omega^{i_1})^{r+s-1} & \cdots & (\omega^{i_{s-1}})^{r+s-1} \end{pmatrix}

5634. ⠁⠰⠊⠴⠐ ⠨⠅ ⠁⠰⠊⠂⠐ ⠨⠅ ⠄⠄⠄ ⠨⠅ ⠁⠰⠊⠰⠰⠎⠤⠂⠐ ⠨⠅ ⠴

$$a_{i_0} = a_{i_1} = \cdots = a_{i_{s-1}} = 0$$

a_{i_0} = a_{i_1} = \cdots = a_{i_{s - 1}} = 0

5635. ⠼231

$$231$$

231

5636. 231⠬24 ⠨⠅ 255 ⠨⠅ ⠆⠘⠦⠐⠤⠂

$$231 + 24 = 255 = 2^8 - 1$$

231 + 24 = 255 = 2^8-1

5637. ⠷255⠠231⠾

$$(255, 231)$$

(255, 231)

5638. ⠙ ⠨⠅ ⠆⠗⠬⠂

$$d = 2r + 1$$

d = 2r + 1

5639. ⠍⠰⠊⠐⠷⠭⠾

$$m_i(x)$$

m_i(x)

5640. ⠨⠺⠘⠊

ω^i

```
\omega^i
```

5641. ⠛⠷⠭⠾ ⠨⠅ ⠇⠉⠍⠈⠷⠍⠂⠷⠭⠾⠠⠍⠆⠷⠭⠾⠠ ⠄⠄⠄⠠ ⠍⠰⠆⠗⠷⠭⠾⠈⠾

$g(x) = \mathrm{lcm}[m_1(x), m_2(x), \ldots, m_{2r}(x)]$

```
g(x) = \lcm[ m_1(x), m_{2}(x), \ldots, m_{2r}(x)]
```

5642. ⠈⠣⠛⠷⠞⠾⠈⠜

$\langle g(t) \rangle$

```
\langle g(t) \rangle
```

5643. ⠋⠷⠨⠺⠘⠊⠐⠾ ⠨⠅ ⠴

$f(\omega^i) = 0$

```
f( \omega^i) = 0
```

5644. ⠂ ⠐⠅⠱ ⠊ ⠐⠅ ⠙

$1 \leq i < d$

```
1 \leq i \lt d
```

5645. ⠠⠓ ⠨⠅ ⠠⠷⠷⠂ ⠨⠺ ⠨⠺⠘⠆⠂ ⠄⠄⠄ ⠨⠺⠘⠝⠤⠂⠐⠠⠾ ⠠⠷⠷⠂ ⠨⠺⠘⠆⠂ ⠨⠺⠘⠲⠂ ⠄⠄⠄ ⠨⠺⠘⠷⠝⠤⠂
⠠⠷⠷⠂ ⠨⠺⠘⠒⠂ ⠨⠺⠘⠖⠂ ⠄⠄⠄ ⠨⠺⠘⠷⠝⠤⠂⠾⠷⠒⠾⠐⠠⠾ ⠠⠷⠳⠄⠄⠄ ⠳⠄⠄⠄ ⠳⠄⠄⠄ ⠒⠄⠄⠄ ⠳⠄⠄⠄
⠠⠷⠷⠂ ⠨⠺⠘⠆⠗⠂ ⠨⠺⠘⠲⠗⠂ ⠄⠄⠄ ⠨⠺⠘⠷⠝⠤⠂⠾⠷⠆⠗⠾⠠⠾

$$H = \begin{pmatrix} 1 & \omega & \omega^2 & \cdots & \omega^{n-1}\\ 1 & \omega^2 & \omega^{4} & \cdots & \omega^{(n-1)(2)} \\ 1 & \omega^3 & \omega^{6} & \cdots & \omega^{(n-1)(3)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \omega^{2r} & \omega^{4r} & \cdots & \omega^{(n-1)(2r)} \end{pmatrix}$$

```
H = \begin{pmatrix} 1 & \omega & \omega^2 & \cdots & \omega^{n-1}\\
1 & \omega^2 & \omega^{4} & \cdots & \omega^{(n-1)(2)} \\ 1 & \omega^3
& \omega^{6} & \cdots & \omega^{(n-1)(3)} \\ \vdots & \vdots & \vdots
& \ddots & \vdots \\ 1 & \omega^{2r} & \omega^{4r} & \cdots & \omega^{(n-1)(2r)}
\end{pmatrix}
```

5646. ⠛⠷⠭⠾⠳⠋⠷⠭⠾

$g(x) \mid f(x)$

```
g(x) \mid f(x)
```

5647. ⠊ ⠨⠅ ⠂⠠ ⠄⠄⠄⠠ ⠆⠗

$i = 1, \ldots, 2r$

```
i = 1, \ldots, 2r
```

5648. ⠛⠷⠨⠺⠘⠊⠐⠾ ⠨⠅ ⠴

$g(\omega^i) = 0$

```
g( \omega^i ) = 0
```

5649. ⠂ ⠐⠅⠱ ⠊ ⠐⠅⠱ ⠙

$1 \leq i \leq d$

```
1 \leq i \leq d
```

5650. ⠋⠷⠞⠾ ⠨⠅ ⠁⠴⠬⠁⠂⠞⠬⠄⠄⠄⠬⠁⠰⠝⠤⠂⠐⠧⠞⠘⠝⠤⠂

$f(t) = a_0 + a_1 t + \cdots + a_{n-1} v t^{n-1}$

f(t) = a_0 + a_1 t + \cdots + a_{n - 1}v t^{n - 1}

5651. ⠸⠭ ⠨⠅ ⠷⠁⠴⠁⠂⠄⠄⠄⠁⠰⠝⠤⠂⠐⠾⠘⠞

$\mathbf{x} = (a_0 a_1 \cdots a_{n-1})^{\mathsf{t}}$

{\mathbf x} = (a_0 a_1 \cdots a_{n - 1})^\transpose

5652. ⠠⠓⠸⠭ ⠨⠅ ⠁⠴⠬⠁⠂⠨⠺⠬⠄⠄⠄⠬⠁⠰⠝⠤⠂⠐⠨⠺⠘⠝⠤⠂ ⠁⠴⠬⠁⠂⠨⠺⠘⠆⠐⠬⠄⠄⠄⠬⠁⠰⠝⠤⠂⠐⠷⠨⠺⠘⠆⠐⠾⠘⠝⠤⠂
⠄⠄⠄ ⠁⠴⠬⠁⠂⠨⠺⠘⠆⠗⠐⠬⠄⠄⠄⠬⠁⠰⠝⠤⠂⠐⠷⠨⠺⠘⠆⠗⠐⠾⠘⠝⠤⠂ ⠨⠅ ⠋⠷⠨⠺⠾
⠋⠷⠨⠺⠘⠆⠐⠾⠄⠄⠄⠋⠷⠨⠺⠘⠆⠗⠐⠾ ⠨⠅ ⠼⠴

$$H\mathbf{x} = \begin{pmatrix} a_0 + a_1\omega + \cdots + a_{n-1}\omega^{n-1} \\ a_0 + a_1\omega^2 + \cdots + a_{n-1}(\omega^2)^{n-1} \\ \vdots \\ a_0 + a_1\omega^{2r} + \cdots + a_{n-1}(\omega^{2r})^{n-1} \end{pmatrix} = \begin{pmatrix} f(\omega) \\ f(\omega^2) \\ \vdots \\ f(\omega^{2r}) \end{pmatrix} = 0$$

H {\mathbf x} = \begin{pmatrix} a_0 + a_1 \omega + \cdots + a_{n-1}
\omega^{n-1} \\ a_0 + a_1 \omega^2 + \cdots + a_{n-1} (\omega^2)^{n-1}
\\ \vdots \\ a_0 + a_1 \omega^{2r} + \cdots + a_{n-1} (\omega^{2r})^{n-1}
\end{pmatrix} = \begin{pmatrix} f(\omega) \\ f(\omega^2) \\ \vdots
\\ f(\omega^{2r}) \end{pmatrix} = 0

5653. ⠋⠷⠨⠺⠘⠊⠐⠾ ⠨⠅ ⠼⠴

$f(\omega^i) = 0$

f(\omega^i) = 0

5654. ⠛⠷⠞⠾ ⠨⠅ ⠇⠉⠍⠈⠷⠍⠂⠷⠞⠾⠠⠄⠄⠄⠠⠍⠰⠆⠗⠐⠷⠞⠾⠈⠾

$g(t) = \mathrm{lcm}[m_1(t), \ldots, m_{2r}(t)]$

g(t) = \lcm[m_1(t),\ldots, m_{2r}(t)]

5655. ⠭⠘15⠐⠤⠂ ⠈⠑ ⠈⠰⠵⠆⠈⠷⠭⠈⠾

$x^{15} - 1 \in \mathbb{Z}_2[x]$

x^{15} - 1 \in {\mathbb Z}_2[x]

5656. ⠭⠘15⠐⠤⠂ ⠨⠅ ⠷⠭⠬⠂⠾⠷⠭⠘⠆⠐⠬⠭⠬⠂⠾⠷⠭⠘⠲⠐⠬⠭⠬⠂⠾⠷⠭⠘⠲⠐⠬⠭⠘⠒⠐⠬⠂⠾⠷⠭⠘⠲⠐⠬⠭⠘⠒⠐⠬⠭⠘⠆⠐⠬⠭⠬⠂⠾

$x^{15} - 1 = (x+1)(x^2+x+1)(x^4+x+1)(x^4+x^3+1)(x^4+x^3+x^2+x+1)$

x^{15} - 1 = (x + 1)(x^2 + x + 1)(x^4 + x + 1)(x^4 + x^3 + 1)(x^4
+ x^3 + x^2 + x + 1)

5657. ⠂⠬⠭⠬⠭⠘⠲

$1 + x + x^4$

1 + x + x^4

5658. StartSet⠁⠴⠬⠁⠂⠨⠺⠬⠁⠆⠨⠺⠘⠆⠐⠬⠁⠒⠨⠺⠘⠒ ⠸⠒ ⠁⠊ ⠈⠑ ⠈⠰⠵⠆ and⠂⠬⠨⠺⠬⠨⠺⠘⠲ ⠨⠅ ⠼⠴EndSet

$\{a_0 + a_1\omega + a_2\omega^2 + a_3\omega^3 : a_i \in \mathbb{Z}_2 \text{ and } 1 + \omega + \omega^4 = 0\}$

\{ a_0 + a_1 \omega + a_2 \omega^2 + a_3 \omega^3 : a_i \in {\mathbb
Z}_2 \text{ and } 1 + \omega + \omega^4 = 0 \}

5659. ⠍⠂⠷⠭⠾ ⠨⠅ ⠂⠬⠭⠬⠭⠘⠲

$m_1(x) = 1 + x + x^4$

m_1(x) = 1 + x + x^4

5660. ⠨⠕⠘⠆

ω^2

\omega^2

5661. ⠨⠕⠘⠲

ω^4

\omega^4

5662. ⠍⠂⠷⠭⠾

$m_1(x)$

m_1(x)

5663. ⠨⠕⠘⠒

ω^3

\omega^3

5664. ⠍⠆⠷⠭⠾ ⠨⠅ ⠂⠬⠭⠬⠭⠘⠆⠐⠬⠭⠘⠒⠐⠬⠭⠘⠲

$m_2(x) = 1 + x + x^2 + x^3 + x^4$

m_2(x) = 1 + x + x^2 + x^3 + x^4

5665. ⠛⠷⠭⠾ ⠨⠅ ⠍⠂⠷⠭⠾⠍⠆⠷⠭⠾ ⠨⠅ ⠂⠬⠭⠘⠲⠐⠬⠭⠘⠖⠐⠬⠭⠘⠶⠐⠬⠭⠘⠦

$g(x) = m_1(x)m_2(x) = 1 + x^4 + x^6 + x^7 + x^8$

g(x) = m_1(x) m_2(x) = 1 + x^4 + x^6 + x^7 + x^8

5666. ⠍⠆⠷⠭⠾

$m_2(x)$

m_2(x)

5667. ⠭⠘15⠐⠤⠂

$x^{15} - 1$

x^{15} - 1

5668. ⠷15⠠⠶⠾

$(15, 7)$

(15, 7)

5669. ⠭⠘15⠐⠤⠂ ⠨⠅ ⠛⠷⠭⠾⠓⠷⠭⠾

$x^{15} - 1 = g(x)h(x)$

x^{15} -1 = g(x)h(x)

5670. ⠓⠷⠭⠾ ⠨⠅ ⠂⠬⠭⠘⠲⠐⠬⠭⠘⠖⠐⠬⠭⠘⠶

$h(x) = 1 + x^4 + x^6 + x^7$

h(x) = 1 + x^4 + x^6 + x^7

5671. $($

\left(

5672. ⠈⠷⠠⠛⠠⠋⠷⠒⠘⠖⠐⠾⠱⠠⠛⠠⠋⠷⠒⠘⠒⠐⠾⠈⠾

$[\mathrm{GF}(3^6) : \mathrm{GF}(3^3)]$

[\gf(3^6) : \gf(3^3)]

5673. ⠈⠷ ⠠⠛ ⠠⠋⠷128⠾⠱ ⠠⠛ ⠠⠋⠷16⠾ ⠈⠾

$[\mathrm{GF}(128) : \mathrm{GF}(16)]$

`[\gf(128): \gf(16)]`

5674. ⠈⠷ ⠠⠛ ⠠⠋⠷625⠾⠱ ⠠⠛ ⠠⠋⠷25⠾ ⠈⠾

$[\mathrm{GF}(625) : \mathrm{GF}(25)]$

`[\gf(625) : \gf(25) ]`

5675. ⠈⠷ ⠠⠛ ⠠⠋⠷⠏ ⠘12 ⠐⠾⠱ ⠠⠛ ⠠⠋⠷⠏ ⠘⠆ ⠐⠾ ⠈⠾

$[\mathrm{GF}(p^{12}) : \mathrm{GF}(p^2)]$

`[\gf(p^{12}): \gf(p^2)]`

5676. ⠈⠷ ⠠⠛ ⠠⠋⠷⠏ ⠘⠍ ⠐⠾⠱ ⠠⠛ ⠠⠋⠷⠏ ⠘⠝ ⠐⠾ ⠈⠾

$[\mathrm{GF}(p^m) : \mathrm{GF}(p^n)]$

`[\gf(p^m): \gf(p^n)]`

5677. ⠠⠛ ⠠⠋⠷⠏ ⠘30 ⠐⠾

$\mathrm{GF}(p^{30})$

`\gf(p^{30})`

5678. ⠭ ⠘⠒ ⠐⠬⠭ ⠘⠆ ⠐⠬⠂

$x^3 + x^2 + 1$

`x^3 + x^2 + 1`

5679. ⠈ ⠠⠨⠤ ⠈⠷⠭ ⠈⠾⠸⠌ ⠳⠪⠱ ⠿⠈⠢⠄ ⠏⠷⠭⠾⠠ ⠳⠪⠱ ⠿⠈⠂⠄

$\mathbb{Z}_3[x]/\langle p(x) \rangle$

`{\mathbb Z}_3[x]/ \langle p(x) \rangle`

5680. ⠭ ⠘⠢ ⠐⠤⠂

$x^5 - 1$

`x^5- 1`

5681. ⠭ ⠘⠖ ⠐⠬⠭ ⠘⠢ ⠐⠬⠭ ⠘⠲ ⠐⠬⠭ ⠘⠒ ⠐⠬⠭ ⠘⠆ ⠐⠬⠭⠬⠂

$x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$

`x^6 + x^5 + x^4 + x^3 + x^2 + x + 1`

5682. ⠭ ⠘⠔ ⠐⠤⠂

$x^9 - 1$

`x^9 - 1`

5683. ⠭ ⠘⠲ ⠐⠬⠭ ⠘⠒ ⠐⠬⠭ ⠘⠆ ⠐⠬⠭⠬⠂

$x^4 + x^3 + x^2 + x + 1$

`x^4 +x^3 + x^2 + x + 1`

5684. ⠭ ⠘⠢ ⠐⠤⠂ ⠶ ⠷⠭⠬⠂⠾⠷⠭ ⠘⠲ ⠐⠬⠭ ⠘⠒ ⠐⠬⠭ ⠘⠆ ⠐⠬⠭⠬⠂⠾

$x^5 - 1 = (x+1)(x^4 + x^3 + x^2 + x + 1)$

`x^5 -1 = (x+1)(x^4+x^3 + x^2 + x+ 1)`

5685. ⠭ ⠘⠔ ⠐⠤⠂ ⠶ ⠷⠭⠬⠂⠾⠷⠭ ⠘⠆ ⠐⠬⠭⠬⠂⠾⠷⠭ ⠘⠖ ⠐⠬⠭ ⠘⠒ ⠐⠬⠂⠾

$x^9 - 1 = (x+1)(x^2 + x + 1)(x^6 + x^3 + 1)$

`x^9 -1 = (x+1)( x^2 + x+ 1)(x^6+x^3+1)`

5686.

$\mathbb{Z}_2[x]/\langle x^3 + x + 1 \rangle \cong \mathbb{Z}_2[x]/\langle x^3 + x^2 + 1 \rangle$

```
{\mathbb Z}_2[x] / \langle x^3 + x + 1 \rangle \cong {\mathbb Z}_2[x]
/ \langle x^3 + x^2 + 1 \rangle
```

5687. 10

$n = 6, 7, 8, 10$

```
n = 6, 7, 8, 10
```

5688.

$\langle t + 1 \rangle$

```
\langle t + 1 \rangle
```

5689.

$x^7 - 1 = (x + 1)(x^3 + x + 1)(x^3 + x^2 + 1)$

```
x^7 - 1 = (x + 1)( x^3 + x + 1)(x^3 + x^2 + 1)
```

5690.

$(a - b)^{p^n} = a^{p^n} - b^{p^n}$

```
(a - b)^{p^n} = a^{p^n} - b^{p^n}
```

5691.

$F \subset E \subset K$

```
F \subset E \subset K
```

5692.

$p(x) \in E[x]$

```
p(x) \in E[x]
```

5693.

q^n

```
q^n
```

5694.

$\beta = a_0 + a_1 \alpha + \cdots + a_{n-1} \alpha^{n-1}$

```
\beta = a_0 + a_1 \alpha + \cdots + a_{n - 1} \alpha^{n - 1}
```

5695.

$(a_0, a_1, \ldots, a_{n-1})$

```
(a_0, a_1, \ldots, a_{n - 1})
```

5696.

$\Phi : \mathrm{GF}(p^n) \rightarrow \mathrm{GF}(p^n)$

```
\Phi : \gf(p^n) \rightarrow \gf(p^n)
```

5697.

$\Phi : \alpha \mapsto \alpha^p$

```
\Phi : \alpha \mapsto \alpha^p
```

5698. ⠁⠈⠑⠠⠛⠠⠋⠷⠏⠘⠝⠐⠾

$a \in \mathrm{GF}(p^n)$

a \in \gf(p^n)

5699. ⠳⠠⠑⠳ ⠨⠅ ⠏⠘⠗

$|E| = p^r$

|E| = p^r

5700. ⠳⠠⠋⠳ ⠨⠅ ⠏⠘⠎

$|F| = p^s$

|F| = p^s

5701. ⠠⠑⠨⠩⠠⠋

$E \cap F$

E \cap F

5702. ⠷⠏⠤⠂⠾⠯⠸⠇⠤⠂⠷⠍⠕⠙⠏⠾

$(p-1)! \equiv -1 \pmod{p}$

(p-1)! \equiv -1 \pmod{p}

5703. ⠭⠘⠏⠤⠂⠐⠤⠂

$x^{p-1} - 1$

x^{p-1} - 1

5704. ⠄⡷⠭⠆⠶⠑⠦⠄⠋⠷⠞⠾⠄⡷⠭⠆⠶⠑⠔⠄

$\langle f(t) \rangle$

\langle f(t) \rangle

5705. ⠄⡷⠭⠆⠶⠑⠦⠄⠛⠷⠞⠾⠄⡷⠭⠆⠶⠑⠔⠄⠸⠐⠅⠄⡷⠭⠆⠶⠑⠦⠄⠋⠷⠞⠾⠄⡷⠭⠆⠶⠑⠔⠄

$\langle g(t) \rangle \subset \langle f(t) \rangle$

\langle g(t) \rangle \subset \langle f(t) \rangle

5706. ⠛⠷⠭⠾ ⠨⠅ ⠛⠴⠬⠛⠂⠭⠬⠄⠄⠄⠬⠛⠰⠝⠤⠅⠐⠭⠘⠝⠤⠅

$g(x) = g_0 + g_1 x + \cdots + g_{n-k} x^{n-k}$

g(x) = g_0 + g_1 x + \cdots + g_{n - k} x^{n - k}

5707. ⠷⠝⠤⠅⠾⠡⠝

$(n-k) \times n$

(n-k) \times n

5708. ⠉⠷⠞⠾ ⠨⠅ ⠉⠴⠬⠉⠂⠞⠬⠄⠄⠄⠬⠉⠰⠝⠤⠂⠐⠞⠘⠝⠤⠂

$c(t) = c_0 + c_1 t + \cdots + c_{n-1} t^{n-1}$

c(t) = c_0 + c_1 t + \cdots + c_{n-1} t^{n-1}

5709. ⠺⠷⠞⠾ ⠨⠅ ⠺⠴⠬⠺⠂⠞⠬⠄⠄⠄⠺⠰⠝⠤⠂⠐⠞⠘⠝⠤⠂

$w(t) = w_0 + w_1 t + \cdots w_{n-1} t^{n-1}$

w(t) = w_0 + w_1 t + \cdots w_{n-1} t^{n-1}

5710. ⠺⠷⠞⠾ ⠨⠅ ⠉⠷⠞⠾⠬⠑⠷⠞⠾

$w(t) = c(t) + e(t)$

w(t) = c(t) + e(t)

5711. ⠑⠷⠞⠾ ⠨⠅ ⠞⠘⠁⠂⠐⠬⠞⠘⠁⠆⠐⠬⠄⠄⠄⠬⠞⠘⠁⠰⠅

$e(t) = t^{a_1} + t^{a_2} + \cdots + t^{a_k}$

```
e(t) = t^{a_1} + t^{a_2} + \cdots + t^{a_k}
```

5712. ⠉⠷⠞⠾

$c(t)$

```
c(t)
```

5713. ⠺⠷⠞⠾

$w(t)$

```
w(t)
```

5714. ⠺⠷⠨⠺⠘⠊⠐⠾ ⠨⠅ ⠎⠰⠊

$w(\omega^i) = s_i$

```
w( \omega^i ) = s_i
```

5715. ⠎⠂⠠ ⠄⠄⠄⠠ ⠎⠰⠆⠗⠐

$s_1, \ldots, s_{2r}$

```
s_1, \ldots, s_{2r}
```

5716. ⠎⠰⠊⠐ ⠨⠅ ⠴

$s_i = 0$

```
s_i = 0
```

5717. ⠎⠰⠊⠐ ⠨⠅ ⠺⠷⠨⠺⠘⠊⠐⠾ ⠨⠅ ⠑⠷⠨⠺⠘⠊⠐⠾ ⠨⠅ ⠨⠺⠘⠊⠁⠰⠂⠐⠬⠨⠺⠘⠊⠁⠰⠆⠐⠬⠄⠄⠄⠬⠨⠺⠘⠊⠁⠰⠅

$s_i = w(\omega^i) = e(\omega^i) = \omega^{ia_1} + \omega^{ia_2} + \cdots + \omega^{ia_k}$

```
s_i = w( \omega^i) = e( \omega^i) = \omega^{i a_1} + \omega^{i a_2}
+ \cdots + \omega^{i a_k}
```

5718. ⠎⠷⠭⠾ ⠨⠅ ⠷⠭⠬⠨⠺⠘⠁⠂⠐⠾⠷⠭⠬⠨⠺⠘⠁⠆⠐⠾⠄⠄⠄⠷⠭⠬⠨⠺⠘⠁⠰⠅⠐⠾

$s(x) = (x + \omega^{a_1})(x + \omega^{a_2}) \cdots (x + \omega^{a_k})$

```
s(x) = (x + \omega^{a_1})(x + \omega^{a_2}) \cdots (x + \omega^{a_k})
```

5719. ⠷15 ⠠⠶⠾

$(15,7)$

```
(15,7)
```

5720. ⠎⠷⠭⠾ ⠨⠅ ⠷⠭⠬⠨⠺⠘⠁⠂⠐⠾⠷⠭⠬⠨⠺⠘⠁⠆⠐⠾

$s(x) = (x + \omega^{a_1})(x + \omega^{a_2})$

```
s(x) = (x + \omega^{a_1})(x + \omega^{a_2})
```

5721. ⠎⠷⠭⠾ ⠨⠅ ⠭⠘⠆⠐⠬⠎⠂⠭⠬⠷⠎⠂⠘⠆⠐⠬⠹⠎⠒⠌⠎⠂⠼⠾

$s(x) = x^2 + s_1 x + \left(s_1^2 + \frac{s_3}{s_1} \right)$

```
s(x) = x^2 + s_1 x + \left( s_1^2 + \frac{s_3}{s_1} \right)
```

5722. ⠺⠷⠞⠾ ⠨⠅ ⠂⠬⠞⠘⠆⠐⠬⠞⠘⠲⠐⠬⠞⠘⠢⠐⠬⠞⠘⠶⠐⠬⠞⠘12⠐⠬⠞⠘13

$w(t) = 1 + t^2 + t^4 + t^5 + t^7 + t^{12} + t^{13}$

```
w(t) = 1 + t^2 +t^4 + t^5 + t^7 + t^{12} + t^{13}
```

5723. ⠢⠘⠆

5^2

`5^2`

5724. ⠏⠷⠭⠾ ⠨⠅ ⠭⠘25⠐⠤⠭

$p(x)=x^{25}-x$

`p(x)=x^{25}-x`

5725. ⠆⠘⠶

2^7

`2^7`

5726. ⠒⠘⠖

3^6

`3^6`

5727. ⠒⠘⠆

3^2

`3^2`

5728. ⠼⠆⠳⠼⠖

$2|6$

`2|6`

5729. ⠈⠠⠟⠷⠜⠒⠻⠠⠜⠶⠻⠾

${\mathbb Q}(\sqrt{3},\sqrt{7})$

`{\mathbb Q}(\sqrt{3},\sqrt{7})`

5730. ⠭⠘⠆⠐⠤⠒

x^2-3

`x^2-3`

5731. ⠏ ⠨⠅ ⠆⠠⠼⠒⠠⠼⠢⠠⠼⠶

$p=2,3,5,7$

`p=2,3,5,7`

5732. ⠒ ⠅⠱ ⠝ ⠅⠱ 10

$3\leq n\leq 10$

`3\leq n\leq 10`

5733. ⠠⠅ ⠨⠅ ⠨⠷⠃⠈⠑⠠⠑⠳⠨⠞⠷⠃⠾ ⠨⠅ ⠃⠨⠾

$K=\{b\in E\mid\tau(b)=b\}$

`K=\{b\in E\mid\tau(b)=b\}`

5734. ⠠⠑ ⠨⠅ ⠠⠛⠠⠋⠷⠒⠘⠖⠐⠾

$E=GF(3^6)$

`E=GF(3^6)`

5735. ⠁ ⠈⠑ ⠠⠋

$a\in F$

`a\in F`

5736. ⠠⠎⠑⠞⠷⠾

$Set()$

```
Set()
```

5737. ⠭⠘⠢⠐⠤⠂ ⠨⠅ ⠴

$x^5 - 1 = 0$

```
x^5 - 1 = 0
```

5738. ⠭⠘⠖⠐⠤⠭⠘⠒⠐⠤⠖ ⠨⠅ ⠴

$x^6 - x^3 - 6 = 0$

```
x^6 - x^3 - 6 = 0
```

5739. ⠁⠭⠘⠢⠐⠬⠃⠭⠘⠲⠐⠬⠉⠭⠘⠒⠐⠬⠙⠭⠘⠆⠐⠬⠑⠭⠬⠋ ⠨⠅ ⠴

$ax^5 + bx^4 + cx^3 + dx^2 + ex + f = 0$

```
a x^5 + b x^4 +c x^3 + d x^2 + e x + f = 0
```

5740. ⠨⠎⠘⠤⠂

σ^{-1}

```
\sigma^{-1}
```

5741. ⠨⠎⠱ ⠠⠑ ⠫⠕ ⠠⠑

$\sigma : E \rightarrow E$

```
\sigma : E \rightarrow E
```

5742. ⠨⠎⠷⠨⠁⠾ ⠨⠅ ⠨⠁

$\sigma(\alpha) = \alpha$

```
\sigma( \alpha ) = \alpha
```

5743. ⠨⠞⠷⠨⠁⠾ ⠨⠅ ⠨⠁

$\tau(\alpha) = \alpha$

```
\tau( \alpha ) = \alpha
```

5744. ⠨⠎⠨⠞⠷⠨⠁⠾ ⠨⠅ ⠨⠎⠷⠨⠁⠾ ⠨⠅ ⠨⠁

$\sigma\tau(\alpha) = \sigma(\alpha) = \alpha$

```
\sigma \tau( \alpha ) = \sigma( \alpha) = \alpha
```

5745. ⠨⠎⠘⠤⠂⠐⠷⠨⠁⠾ ⠨⠅ ⠨⠁

$\sigma^{-1}(\alpha) = \alpha$

```
\sigma^{-1}( \alpha ) = \alpha
```

5746. ⠠⠁⠥⠞⠷⠠⠑⠾

$\mathrm{Aut}(E)$

```
\aut(E)
```

5747. ⠠⠛⠷⠠⠑⠌⠠⠋⠾ ⠨⠅ StartSet ⠨⠎ ⠈⠑ ⠠⠁⠥⠞⠷⠠⠑⠾⠱⠨⠎⠷⠨⠁⠾ ⠨⠅ ⠨⠁ forall ⠨⠁ ⠈⠑ ⠠⠋EndSet

$G(E/F) = \{\sigma \in \mathrm{Aut}(E) : \sigma(\alpha) = \alpha \text{ for all } \alpha \in F\}$

```
G(E/F) = \{ \sigma \in \aut(E) : \sigma(\alpha) = \alpha \text{
for all } \alpha \in F \}
```

5748. ⠠⠛⠷⠠⠑⠌⠠⠋⠾

$G(E/F)$

G(E/F)

5749. ⠨⠎⠐⠂⠁ ⠬⠃⠊ ⠫⠳⠒⠒⠕ ⠁⠤⠃⠊

$\sigma : a + bi \mapsto a - bi$

\sigma : a + bi \mapsto a - bi

5750. ⠨⠎⠷⠁⠾ ⠨⠅ ⠨⠎⠷⠁⠬⠴⠊⠾ ⠨⠅ ⠁⠤⠴⠊ ⠨⠅ ⠁

$\sigma(a) = \sigma(a + 0i) = a - 0i = a$

\sigma(a) = \sigma(a + 0i) = a - 0i = a

5751. ⠠⠛⠷⠈⠠⠉⠌⠈⠠⠗⠾

$G(\mathbb{C}/\mathbb{R})$

G({\mathbb C} / {\mathbb R})

5752. ⠈⠠⠟ ⠸⠐⠅ ⠈⠠⠟⠷⠜⠢⠻⠾ ⠸⠐⠅ ⠈⠠⠟⠷⠜⠒⠻⠠ ⠜⠢⠻⠾

$\mathbb{Q} \subset \mathbb{Q}(\sqrt{5}\,) \subset \mathbb{Q}(\sqrt{3}, \sqrt{5}\,)$

{\mathbb Q} \subset {\mathbb Q}(\sqrt{5}\,) \subset {\mathbb Q}(
\sqrt{3}, \sqrt{5}\,)

5753. ⠁⠠ ⠃ ⠈⠑ ⠈⠠⠟⠷⠜⠢⠻⠾

$a, b \in \mathbb{Q}(\sqrt{5}\,)$

a, b \in {\mathbb Q}(\sqrt{5}\,)

5754. ⠨⠎⠷⠁⠬⠃⠜⠒⠻⠾ ⠨⠅ ⠁⠤⠃⠜⠒⠻

$\sigma(a + b\sqrt{3}\,) = a - b\sqrt{3}$

\sigma(a + b \sqrt{3}\,) = a - b \sqrt{3}

5755. ⠈⠠⠟⠷⠜⠒⠻⠠ ⠜⠢⠻⠾

$\mathbb{Q}(\sqrt{3}, \sqrt{5}\,)$

{\mathbb Q}(\sqrt{3}, \sqrt{5}\,)

5756. ⠈⠠⠟⠷⠜⠢⠻⠾

$\mathbb{Q}(\sqrt{5}\,)$

{\mathbb Q}(\sqrt{5}\,)

5757. ⠨⠞⠷⠁⠬⠃⠜⠢⠻⠾ ⠨⠅ ⠁⠤⠃⠜⠢⠻

$\tau(a + b\sqrt{5}\,) = a - b\sqrt{5}$

\tau(a + b \sqrt{5}\,) = a - b \sqrt{5}

5758. ⠨⠍ ⠨⠅ ⠨⠎⠨⠞

$\mu = \sigma\tau$

\mu = \sigma \tau

5759. StartSet⠊⠙⠠ ⠨⠎⠠ ⠨⠞⠠ ⠨⠍EndSet

$\{\mathrm{id}, \sigma, \tau, \mu\}$

\{ \identity, \sigma, \tau, \mu \}

5760. StartLayout1stRow ⠊⠙ ⠨⠎ ⠨⠞ ⠨⠍2ndRow⠊⠙ ⠊⠙ ⠨⠎ ⠨⠞ ⠨⠍3rdRow⠨⠎ ⠨⠎ ⠊⠙ ⠨⠍

$$\begin{array}{c|cccc} & \mathrm{id} & \sigma & \tau & \mu \\ \hline \mathrm{id} & \mathrm{id} & \sigma & \tau & \mu \\ \sigma & \sigma & \mathrm{id} & \mu & \tau \\ \tau & \tau & \mu & \mathrm{id} & \sigma \\ \mu & \mu & \tau & \sigma & \mathrm{id} \end{array}$$

```
\begin{array}{c|cccc} & \identity & \sigma & \tau & \mu \\ \hline
\identity & \identity & \sigma & \tau & \mu \\ \sigma & \sigma &
\identity & \mu & \tau \\ \tau & \tau & \mu & \identity & \sigma
\\ \mu & \mu & \tau & \sigma & \identity \end{array}
```

5761. ⠈ ⠠⠟⠷⠜⠒⠻ ⠠⠜⠢⠻⠾

$\mathbb{Q}(\sqrt{3}, \sqrt{5}\,)$

```
{\mathbb Q}( \sqrt{3}, \sqrt{5}\, )
```

5762. StartSet⠂ ⠠⠜⠒⠻ ⠠⠜⠢⠻ ⠠⠜15⠻EndSet

$\{ 1, \sqrt{3}, \sqrt{5}, \sqrt{15}\, \}$

```
\{ 1, \sqrt{3}, \sqrt{5}, \sqrt{15}\, \}
```

5763. ⠳⠠⠛⠷⠈ ⠠⠟⠷⠜⠒⠻ ⠠⠜⠢⠻⠾⠌⠈ ⠠⠟⠾⠳ ⠨⠅ ⠈⠷⠈ ⠠⠟⠷⠜⠒⠻ ⠠⠜⠢⠻⠾⠱⠈ ⠠⠟⠾⠈⠾ ⠨⠅ ⠲

$|G({\mathbb Q}(\sqrt{3}, \sqrt{5}\,) /{\mathbb Q})| = [{\mathbb Q}(\sqrt{3}, \sqrt{5}\,):{\mathbb Q})] = 4$

```
|G( {\mathbb Q}( \sqrt{3}, \sqrt{5}\, ) /{\mathbb Q})| = [{\mathbb
Q}(\sqrt{3}, \sqrt{5}\, ):{\mathbb Q})] = 4
```

5764. ⠋⠷⠭⠾ ⠨⠅ ⠁⠴⠬⠁⠂⠭⠬⠁⠆⠭⠘⠆⠐⠬⠄⠄⠄⠬⠁⠰⠝⠭⠘⠝

$f(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$

```
f(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n
```

5765. ⠨⠎ ⠈⠑ ⠠⠛⠷⠠⠑⠌⠠⠋⠾

$\sigma \in G(E/F)$

```
\sigma \in G(E/F)
```

5766. ⠨⠎⠷⠨⠁⠾

$\sigma(\alpha)$

```
\sigma( \alpha )
```

5767. ⠤⠜⠆⠻

$-\sqrt{2}$

```
-\sqrt{2}
```

5768. ⠨⠎⠱⠠⠋⠷⠨⠁⠾ ⠫⠕ ⠠⠋⠷⠨⠃⠾

$\sigma : F(\alpha) \rightarrow F(\beta)$

```
\sigma : F( \alpha ) \rightarrow F( \beta )
```

5769. ⠳⠠⠛⠷⠠⠑⠌⠠⠋⠾⠳ ⠨⠅ ⠈⠷⠠⠑⠱⠠⠋⠈⠾

$|G(E/F)| = [E:F]$

```
|G(E/F)| = [E:F]
```

5770. ⠠⠋ ⠸⠐⠅ ⠠⠋⠷⠨⠁⠾ ⠸⠐⠅ ⠠⠑

$F \subset F(\alpha) \subset E$

```
F \subset F( \alpha ) \subset E
```

5771. ⠈⠷⠠⠑⠱⠠⠋⠷⠨⠁⠾⠈⠾ ⠨⠅ ⠝⠌⠗and⠈⠷⠠⠋⠷⠨⠁⠾⠱⠠⠋⠈⠾ ⠨⠅ ⠗

$$[E:F(\alpha)] = n/r \quad \text{and} \quad [F(\alpha):F] = r$$

`[E: F(\alpha)] = n/r \quad \text{and} \quad [F(\alpha): F] = r`

5772. ⠠⠋⠸⠪⠠⠋⠷⠨⠃⠾⠸⠪⠠⠑

$$F \subset F(\beta) \subset E$$

`F \subset F( \beta ) \subset E`

5773. ⠨⠎⠱⠠⠋⠷⠨⠁⠾ ⠫⠕ ⠠⠋⠷⠨⠃⠾

$$\sigma : F(\alpha) \rightarrow F(\beta)$$

`\sigma: F( \alpha ) \rightarrow F( \beta )`

5774. ⠈⠷⠠⠑⠱⠠⠋⠷⠨⠁⠾⠈⠾ ⠨⠅ ⠝⠌⠗ ⠐⠅ ⠝

$$[E:F(\alpha)] = n/r < n$$

`[E: F(\alpha)] = n/r \lt n`

5775. ⠳⠠⠛⠷⠠⠑⠌⠠⠋⠷⠨⠁⠾⠾⠳ ⠨⠅ ⠈⠷⠠⠑⠱⠠⠋⠷⠨⠁⠾⠈⠾

$$|G(E/F(\alpha))| = [E:F(\alpha)]$$

`|G(E/F(\alpha))| = [E:F(\alpha)]`

5776. ⠈⠷⠠⠑⠱⠠⠋⠈⠾ ⠨⠅ ⠈⠷⠠⠑⠱⠠⠋⠷⠨⠁⠾⠈⠾⠈⠷⠠⠋⠷⠨⠁⠾⠱⠠⠋⠈⠾ ⠨⠅ ⠝

$$[E:F] = [E:F(\alpha)][F(\alpha):F] = n$$

`[E:F] = [E:F(\alpha)] [F( \alpha):F] = n`

5777. ⠈⠷⠠⠑⠱⠠⠋⠈⠾ ⠨⠅ ⠅

$$[E:F] = k$$

`[E:F]=k`

5778. ⠝⠅ ⠨⠅ ⠍

$$nk = m$$

`nk = m`

5779. ⠳⠠⠛⠷⠠⠑⠌⠠⠋⠾⠳ ⠨⠅ ⠅

$$|G(E/F)| = k$$

`|G(E/F)| = k`

5780. ⠨⠎⠷⠨⠁⠾ ⠨⠅ ⠨⠁⠘⠏⠘⠘⠝

$$\sigma(\alpha) = \alpha^{p^n}$$

`\sigma(\alpha) = \alpha^{p^n}`

5781. ⠨⠎⠷⠨⠁⠬⠨⠃⠾ ⠨⠅ ⠷⠨⠁⠬⠨⠃⠾⠘⠏⠘⠘⠝⠐ ⠨⠅ ⠨⠁⠘⠏⠘⠘⠝⠐⠬⠨⠃⠘⠏⠘⠘⠝⠐ ⠨⠅ ⠨⠎⠷⠨⠁⠾⠬⠨⠎⠷

$$\sigma(\alpha+\beta) = (\alpha+\beta)^{p^n} = \alpha^{p^n} + \beta^{p^n} = \sigma(\alpha) + \sigma(\beta)$$

`\sigma(\alpha + \beta) = (\alpha + \beta)^{p^n} = \alpha^{p^n} +`
`\beta^{p^n} = \sigma(\alpha) + \sigma(\beta)`

5782. ⠨⠎⠷⠨⠁⠨⠃⠾ ⠨⠅ ⠨⠎⠷⠨⠁⠾⠨⠎⠷⠨⠃⠾

$$\sigma(\alpha\beta) = \sigma(\alpha)\sigma(\beta)$$

`\sigma(\alpha \beta) = \sigma( \alpha ) \sigma( \beta )`

5783. ⠨⠎⠘⠅⠐⠷⠨⠁⠾ ⠨⠅ ⠨⠁⠘⠏⠘⠘⠝⠅⠐ ⠨⠅ ⠨⠁⠘⠏⠘⠘⠍⠐ ⠨⠅ ⠨⠁

$\sigma^k(\alpha) = \alpha^{p^{nk}} = \alpha^{p^m} = \alpha$

\sigma^k(\alpha) = \alpha^{p^{nk}} = \alpha^{p^m} = \alpha

5784. ⠠⠛⠷⠠⠑⠸⠌⠠⠋⠾

$G(E/F)$

G(E/F)

5785. ⠨⠎⠘⠗

σ^r

\sigma^r

5786. ⠂ ⠐⠅⠱ ⠗ ⠐⠅ ⠅

$1 \leq r < k$

1 \leq r \lt k

5787. ⠭⠘⠏⠘⠘⠝⠗⠐⠤⠭

$x^{p^{nr}} - x$

x^{p^{nr}} - x

5788. ⠠⠓ ⠨⠅ StartSet⠊⠙⠠⠨⠎⠠⠨⠞⠠⠨⠍EndSet

$H = \{\mathrm{id}, \sigma, \tau, \mu\}$

H = \{ \identity, \sigma, \tau, \mu \}

5789. ⠠⠛⠷⠈⠠⠟⠷⠜⠒⠻⠠⠠⠜⠢⠻⠾⠸⠌⠈⠠⠟⠾

$G(\mathbb{Q}(\sqrt{3}, \sqrt{5}\,)/\mathbb{Q})$

G({\mathbb Q}(\sqrt{3}, \sqrt{5}\,)/{\mathbb Q})

5790. ⠳⠠⠓⠳ ⠨⠅ ⠈⠷⠈⠠⠟⠷⠜⠒⠻⠠⠠⠜⠢⠻⠾⠳⠈⠠⠟⠈⠾ ⠨⠅ ⠳⠠⠛⠷⠈⠠⠟⠷⠜⠒⠻⠠⠠⠜⠢⠻⠾⠸⠌⠈⠠⠟⠾⠳ ⠨⠅ ⠲

$|H| = [\mathbb{Q}(\sqrt{3}, \sqrt{5}\,) : \mathbb{Q}] = |G(\mathbb{Q}(\sqrt{3}, \sqrt{5}\,)/\mathbb{Q})| = 4$

|H| = [{\mathbb Q}(\sqrt{3}, \sqrt{5}\,):{\mathbb Q}] = |G({\mathbb
Q}(\sqrt{3}, \sqrt{5}\,)/{\mathbb Q})| = 4

5791. ⠋⠷⠭⠾ ⠨⠅ ⠭⠘⠲⠐⠬⠭⠘⠒⠐⠬⠭⠘⠆⠐⠬⠭⠬⠂

$f(x) = x^4 + x^3 + x^2 + x + 1$

f(x) = x^4 + x^3 + x^2 + x + 1

5792. ⠷⠭⠤⠂⠾⠋⠷⠭⠾ ⠨⠅ ⠭⠘⠢⠐⠤⠂

$(x - 1)f(x) = x^5 - 1$

(x -1)f(x) = x^5 - 1

5793. ⠊ ⠨⠅ ⠂⠠ ⠄⠄ ⠠⠲

$i = 1, \ldots, 4$

i = 1, \ldots, 4

5794. ⠨⠺ ⠨⠅ ⠉⠕⠎⠷⠆⠨⠏⠸⠌⠢⠾⠬⠊⠎⠊⠝⠷⠆⠨⠏⠸⠌⠢⠾

$\omega = \cos(2\pi/5) + i\sin(2\pi/5)$

\omega = \cos(2 \pi / 5) + i \sin(2 \pi / 5)

5795. ⠈⠠⠟⠷⠨⠺⠾

$\mathbb{Q}(\omega)$

{\mathbb Q}(\omega)

5796. ⠈⠠⠟⠷⠨⠺⠾

$\mathbb{Q}(\omega)$

{\mathbb Q}(\omega)

5797. ⠨⠎⠰⠊⠷⠨⠺⠾ ⠨⠅ ⠨⠺⠘⠊

$\sigma_i(\omega) = \omega^i$

\sigma_i(\omega) = \omega^i

5798. ⠠⠛⠷⠈⠠⠟⠷⠨⠺⠾⠸⠌⠈⠠⠟⠾

$G(\mathbb{Q}(\omega)/\mathbb{Q})$

G({\mathbb Q}(\omega) / {\mathbb Q})

5799. ⠈⠷⠈⠠⠟⠷⠨⠺⠾⠸⠒⠈⠠⠟⠈⠾ ⠨⠅ ⠳⠠⠛⠷⠈⠠⠟⠷⠨⠺⠾⠸⠌⠈⠠⠟⠾⠳ ⠨⠅ ⠼⠙

$[\mathbb{Q}(\omega) : \mathbb{Q}] = |G(\mathbb{Q}(\omega)/\mathbb{Q})| = 4$

[{\mathbb Q}(\omega) : {\mathbb Q}] = | G({\mathbb Q}(\omega)
/ {\mathbb Q})| = 4

5800. ⠠⠛⠷⠈⠠⠟⠷⠨⠺⠾⠸⠌⠈⠠⠟⠾ ⠐⠨⠅ ⠈⠠⠵⠙

$G(\mathbb{Q}(\omega)/\mathbb{Q}) \cong \mathbb{Z}_4$

G({\mathbb Q}(\omega) / {\mathbb Q})\cong {\mathbb Z}_4

5801. ⠋⠷⠭⠾ ⠨⠅ ⠷⠭⠤⠨⠁⠂⠾⠘⠝⠰⠂⠐⠷⠭⠤⠨⠁⠆⠾⠘⠝⠰⠆⠐⠄⠄⠄⠷⠭⠤⠨⠁⠰⠗⠾⠘⠝⠰⠰⠗⠐ ⠨⠅ ⠨⠠⠏⠰⠊⠨⠅⠂

$f(x) = (x - \alpha_1)^{n_1}(x - \alpha_2)^{n_2} \cdots (x - \alpha_r)^{n_r} = \prod_{i=1}^{r}(x - \alpha_i)^{n_i}$

f(x) = (x - \alpha_1)^{n_1} (x - \alpha_2)^{n_2} \cdots (x - \alpha_r)^{n_r}
= \prod_{i = 1}^{r} (x - \alpha_i)^{n_i}

5802. ⠛⠉⠙⠷⠋⠷⠭⠾⠠⠋⠄⠷⠭⠾⠾ ⠨⠅ ⠼⠂

$\gcd(f(x), f'(x)) = 1$

\gcd(f(x), f'(x)) = 1

5803. ⠋⠷⠭⠾ ⠌⠨⠅ ⠛⠷⠭⠘⠏⠐⠾

$f(x) \neq g(x^p)$

f(x) \neq g(x^p)

5804. ⠙⠑⠛⠀⠋⠄⠷⠭⠾ ⠐⠅ ⠙⠑⠛⠀⠋⠷⠭⠾

$\deg f'(x) < \deg f(x)$

\deg f'(x) \lt \deg f(x)

5805. ⠛⠉⠙⠷⠋⠷⠭⠾⠠⠋⠄⠷⠭⠾⠾ ⠌⠨⠅ ⠼⠂

$\gcd(f(x), f'(x)) \neq 1$

\gcd(f(x), f'(x)) \neq 1

5806. ⠋⠷⠭⠾ ⠨⠅ ⠁⠴⠬⠁⠂⠭⠘⠏⠐⠬⠁⠆⠭⠘⠆⠏⠐⠬⠄⠄⠄⠬⠁⠰⠝⠭⠘⠝⠏

$f(x) = a_0 + a_1 x^p + a_2 x^{2p} + \cdots + a_n x^{np}$

f(x) = a_0 + a_1 x^p + a_2 x^{2p} + \cdots + a_n x^{np}

5807. ⠈⠠⠟⠷⠣⠒⠜⠢⠻⠠⠜⠢⠻⠊⠾⠀⠨⠅⠀⠈⠠⠟⠷⠣⠖⠜⠢⠻⠊⠾

$\mathbb{Q}(\sqrt[3]{5}, \sqrt{5}\, i) = \mathbb{Q}(\sqrt[6]{5}\, i)$

```
{\mathbb Q}( \sqrt[3]{5}, \sqrt{5}\, i ) = {\mathbb Q}( \sqrt[6]{5}\,
i )
```

5808. ⠠⠑⠀⠨⠅⠀⠠⠋⠷⠨⠁⠾

$E = F(\alpha)$

```
E=F( \alpha )
```

5809. ⠠⠋⠷⠨⠁⠠⠨⠃⠾

$F(\alpha, \beta)$

```
F(\alpha, \beta)
```

5810. ⠨⠁⠀⠨⠅⠀⠨⠁⠂⠠⠀⠄⠄⠄⠠⠀⠨⠁⠰⠝

$\alpha = \alpha_1, \ldots, \alpha_n$

```
\alpha = \alpha_1, \ldots, \alpha_n
```

5811. ⠨⠃⠀⠨⠅⠀⠨⠃⠂⠠⠀⠄⠄⠄⠠⠀⠨⠃⠰⠍

$\beta = \beta_1, \ldots, \beta_m$

```
\beta = \beta_1, \ldots, \beta_m
```

5812. ⠁⠀⠌⠨⠅⠀⠹⠨⠁⠰⠊⠐⠤⠨⠁⠌⠨⠃⠤⠨⠃⠰⠚⠐⠼

$a \neq \frac{\alpha_i - \alpha}{\beta - \beta_j}$

```
a \neq \frac{\alpha_i - \alpha}{\beta - \beta_j}
```

5813. ⠚⠀⠌⠨⠅⠀⠂

$j \neq 1$

```
j \neq 1
```

5814. ⠁⠷⠨⠃⠤⠨⠃⠰⠚⠐⠾⠀⠌⠨⠅⠀⠨⠁⠰⠊⠐⠤⠨⠁

$a(\beta - \beta_j) \neq \alpha_i - \alpha$

```
a( \beta - \beta_j ) \neq \alpha_i - \alpha
```

5815. ⠨⠛⠀⠨⠅⠀⠨⠁⠬⠁⠨⠃

$\gamma = \alpha + a\beta$

```
\gamma = \alpha + a \beta
```

5816. ⠨⠛⠀⠨⠅⠀⠨⠁⠬⠁⠨⠃⠀⠌⠨⠅⠀⠨⠁⠰⠊⠐⠬⠁⠨⠃⠰⠚⠐⠆

$\gamma = \alpha + a\beta \neq \alpha_i + a\beta_j;$

```
\gamma = \alpha + a \beta \neq \alpha_i + a \beta_j;
```

5817. ⠨⠛⠤⠁⠨⠃⠰⠚⠀⠌⠨⠅⠀⠨⠁⠰⠊

$\gamma - a\beta_j \neq \alpha_i$

```
\gamma - a \beta_j \neq \alpha_i
```

5818. ⠊⠠⠚

i, j

```
i, j
```

5819. ⠓⠷⠭⠾ ⠈⠑ ⠠⠋⠷⠨⠛⠾⠈⠷⠭⠈⠾

$h(x) \in F(\gamma)[x]$

h(x) \in F(\gamma)[x]

5820. ⠓⠷⠭⠾ ⠨⠅ ⠋⠷⠨⠛⠤⠁⠭⠾

$h(x) = f(\gamma - ax)$

h(x) = f(\gamma - ax)

5821. ⠓⠷⠨⠃⠾ ⠨⠅ ⠋⠷⠨⠁⠾ ⠨⠅ ⠴

$h(\beta) = f(\alpha) = 0$

h(\beta) = f(\alpha) = 0

5822. ⠓⠷⠨⠃⠰⠚⠐⠾ ⠌⠨⠅ ⠴

$h(\beta_j) \neq 0$

h(\beta_j) \neq 0

5823. ⠠⠋⠷⠨⠛⠾⠈⠷⠭⠈⠾

$F(\gamma)[x]$

F(\gamma)[x]

5824. ⠠⠋⠷⠨⠛⠾

$F(\gamma)$

F(\gamma)

5825. ⠨⠃ ⠈⠑ ⠠⠋⠷⠨⠛⠾

$\beta \in F(\gamma)$

\beta \in F(\gamma)

5826. ⠨⠁ ⠨⠅ ⠨⠛⠤⠁⠨⠃

$\alpha = \gamma - a\beta$

\alpha = \gamma - a \beta

5827. ⠠⠋⠷⠨⠁⠠⠨⠃⠾ ⠨⠅ ⠠⠋⠷⠨⠛⠾

$F(\alpha, \beta) = F(\gamma)$

F(\alpha, \beta) = F(\gamma)

5828. StartSet ⠨⠎⠰⠊ ⠐⠒⠊ ⠈⠑ ⠠⠊EndSet

$\{\sigma_i : i \in I\}$

\{\sigma_i : i \in I \}

5829. ⠠⠋⠘⠸⠪⠨⠎⠘⠰⠊⠘⠸⠻⠐ ⠨⠅ StartSet⠁ ⠈⠑ ⠠⠋⠸⠒ ⠨⠎⠰⠊⠐⠷⠁⠾ ⠨⠅ ⠁ forall ⠨⠎⠰⠊ ⠐EndSet

$F_{\{\sigma_i\}} = \{a \in F : \sigma_i(a) = a \text{ for all } \sigma_i\}$

F_{\{\sigma_i \}} = \{ a \in F : \sigma_i(a) = a \text{ for all
} \sigma_i \}

5830. ⠨⠎⠰⠊⠐⠷⠁⠾ ⠨⠅ ⠁

$\sigma_i(a) = a$

\sigma_i(a) = a

5831. ⠨⠎⠰⠊⠐⠷⠃⠾ ⠨⠅ ⠃

$\sigma_i(b) = b$

\sigma_i(b)=b

5832. ⠨⠎⠰⠊⠐⠷⠁⠬⠤⠃⠾ ⠨⠅ ⠨⠎⠰⠊⠐⠷⠁⠾⠬⠤⠨⠎⠰⠊⠐⠷⠃⠾ ⠨⠅ ⠁⠬⠤⠃

$\sigma_i(a \pm b) = \sigma_i(a) \pm \sigma_i(b) = a \pm b$

\sigma_i(a \pm b) = \sigma_i(a) \pm \sigma_i(b) = a \pm b

5833. ⠨⠎⠰⠊⠐⠷⠁⠃⠾ ⠨⠅ ⠨⠎⠰⠊⠐⠷⠁⠾⠨⠎⠰⠊⠐⠷⠃⠾ ⠨⠅ ⠁⠃

$\sigma_i(ab) = \sigma_i(a)\sigma_i(b) = ab$

\sigma_i(a b) = \sigma_i(a) \sigma_i(b) = a b

5834. ⠨⠎⠰⠊⠐⠷⠁⠘⠤⠂⠐⠾ ⠨⠅ ⠈⠷⠨⠎⠰⠊⠐⠷⠁⠾⠈⠾⠘⠤⠂⠐ ⠨⠅ ⠁⠘⠤⠂

$\sigma_i(a^{-1}) = [\sigma_i(a)]^{-1} = a^{-1}$

\sigma_i(a^{-1}) = [\sigma_i(a)]^{-1} = a^{-1}

5835. ⠨⠎⠰⠊⠐⠷⠴⠾ ⠨⠅ ⠴

$\sigma_i(0) = 0$

\sigma_i(0) = 0

5836. ⠨⠎⠰⠊⠐⠷⠂⠾ ⠨⠅ ⠂

$\sigma_i(1) = 1$

\sigma_i(1)=1

5837. ⠠⠁⠥⠞⠷⠠⠋⠾

$\mathrm{Aut}(F)$

\aut(F)

5838. ⠠⠋⠰⠠⠛⠐ ⠨⠅ StartSet ⠨⠁ ⠈⠑ ⠠⠋⠒ ⠨⠎⠷⠨⠁⠾ ⠨⠅ ⠨⠁ forall ⠨⠎ ⠈⠑ ⠠⠛EndSet

$F_G = \{\alpha \in F : \sigma(\alpha) = \alpha$ for all $\sigma \in G\}$

F_G = \{ \alpha \in F : \sigma(\alpha) = \alpha \text{ for all
} \sigma \in G \}

5839. ⠠⠋⠰⠸⠷⠨⠎⠰⠰⠊⠰⠸⠾

$F_{\{\sigma_i\}}$

F_{ \{\sigma_i \} }

5840. ⠸⠷⠨⠎⠰⠊⠐⠸⠾

$\{\sigma_i\}$

\{ \sigma_i \}

5841. ⠠⠋⠰⠠⠛

F_G

F_G

5842. ⠨⠎⠸⠒ ⠈⠠⠟⠷⠜⠒⠻⠠⠠⠜⠢⠻⠾ ⠫⠕ ⠈⠠⠟⠷⠜⠒⠻⠠⠠⠜⠢⠻⠾

$\sigma : \mathbb{Q}(\sqrt{3}, \sqrt{5}\,) \rightarrow \mathbb{Q}(\sqrt{3}, \sqrt{5}\,)$

\sigma : {\mathbb Q}(\sqrt{3}, \sqrt{5}\,) \rightarrow {\mathbb
Q}(\sqrt{3}, \sqrt{5}\,)

5843. ⠤⠜⠒⠻

$-\sqrt{3}$

-\sqrt{3}

5844. ⠠⠑⠰⠠⠛⠷⠠⠑⠌⠠⠋⠾⠐ ⠨⠅ ⠠⠋

$E_{G(E/F)} = F$

E_{G(E/F)} = F

5845. ⠠⠛ ⠨⠅ ⠠⠛⠷⠠⠑⠌⠠⠋⠾

$G = G(E/F)$

G = G(E/F)

5846. ⠠⠋ ⠸⠐⠅ ⠠⠑⠰⠠⠛⠐ ⠸⠐⠅ ⠠⠑

$F \subset E_G \subset E$

F \subset E_G \subset E

5847. ⠠⠑⠰⠠⠛

E_G

E_G

5848. ⠠⠛⠷⠠⠑⠌⠠⠋⠾ ⠨⠅ ⠠⠛⠷⠠⠑⠌⠠⠑⠰⠠⠛⠐⠾

$G(E/F) = G(E/E_G)$

G(E/F) = G(E/E_G)

5849. ⠳⠠⠛⠳ ⠨⠅ ⠈⠷⠠⠑⠱⠠⠑⠰⠠⠛⠐⠈⠾ ⠨⠅ ⠈⠷⠠⠑⠱⠠⠋⠈⠾

$|G| = [E : E_G] = [E : F]$

|G| = [E: E_G] =[E:F]

5850. ⠈⠷⠠⠑⠰⠠⠛⠐⠱⠠⠋⠈⠾ ⠨⠅ ⠂

$[E_G : F] = 1$

[E_G : F] =1

5851. ⠠⠑⠰⠠⠛⠐ ⠨⠅ ⠠⠋

$E_G = F$

E_G = F

5852. ⠠⠋ ⠨⠅ ⠠⠑⠰⠠⠛

$F = E_G$

F = E_G

5853. ⠈⠷⠠⠑⠱⠠⠋⠈⠾ ⠅⠱ ⠳⠠⠛⠳

$[E : F] \leq |G|$

[E:F] \leq |G|

5854. ⠍⠐ ⠤ ⠄⠍ ⠆⠝⠢⠂ ⠐

$\alpha_1, \ldots, \alpha_{n+1}$

\alpha_1, \ldots, \alpha_{n + 1}

5855. ⠁⠂⠨⠁⠂⠬⠁⠆⠨⠁⠆⠬⠄⠄⠄⠬⠁⠰⠝⠬⠂⠐⠨⠁⠰⠝⠬⠂⠐ ⠨⠅ ⠴

$a_1\alpha_1 + a_2\alpha_2 + \cdots + a_{n+1}\alpha_{n+1} = 0$

```
a_1 \alpha_1 + a_2 \alpha_2 + \cdots + a_{n + 1} \alpha_{n + 1}
= 0
```

5856. ⠨⠎⠂ ⠨⠅ ⠊⠙⠠⠨⠎⠆⠠⠄⠠⠨⠎⠰⠝⠐

$\sigma_1 = \mathrm{id}, \sigma_2, \ldots, \sigma_n$

```
\sigma_1 = \identity, \sigma_2, \ldots, \sigma_n
```

5857. ⠭⠰⠊⠐ ⠨⠅ ⠁⠰⠊

$x_i = a_i$

```
x_i = a_i
```

5858. ⠊ ⠨⠅ ⠂⠠⠼⠆⠠⠄⠠⠝⠬⠂

$i = 1, 2, \ldots, n + 1$

```
i = 1, 2, \ldots, n + 1
```

5859. ⠨⠎⠂

σ_1

```
\sigma_1
```

5860. ⠁⠂ ⠨⠅ ⠂

$a_1 = 1$

```
a_1 = 1
```

5861. ⠨⠁⠆⠠⠄⠠⠨⠁⠰⠝⠬⠂⠐

$\alpha_2, \ldots, \alpha_{n+1}$

```
\alpha_2, \ldots, \alpha_{n + 1}
```

5862. ⠨⠎⠰⠊⠐⠷⠁⠆⠾ ⠌⠨⠅ ⠁⠆

$\sigma_i(a_2) \neq a_2$

```
\sigma_i( a_2 ) \neq a_2
```

5863. ⠭⠂ ⠨⠅ ⠨⠎⠰⠊⠐⠷⠁⠂⠾ ⠨⠅ ⠂

$x_1 = \sigma_i(a_1) = 1$

```
x_1 = \sigma_i(a_1) = 1
```

5864. ⠭⠆ ⠨⠅ ⠨⠎⠰⠊⠐⠷⠁⠆⠾

$x_2 = \sigma_i(a_2)$

```
x_2 = \sigma_i(a_2)
```

5865. ⠭⠰⠝⠬⠂⠐ ⠨⠅ ⠨⠎⠰⠊⠐⠷⠁⠰⠝⠬⠂⠐⠾

$x_{n+1} = \sigma_i(a_{n+1})$

```
x_{n + 1} = \sigma_i(a_{n+1} )
```

5866. ⠁⠂⠠⠄⠠⠁⠰⠝⠬⠂⠐ ⠈⠑ ⠠⠋

$a_1, \ldots, a_{n+1} \in F$

```
a_1, \ldots, a_{n + 1} \in F
```

5867. ⠠⠑ ⠶ ⠠⠋⠷⠨⠁⠾

$E = F(\alpha)$

E = F(\alpha)

5868. ⠳⠠⠛⠷⠠⠑⠌⠠⠋⠾⠳ ⠶ ⠈⠷⠠⠑⠒⠠⠋⠈⠾

$|G(E/F)| = [E : F]$

| G(E/F)| = [E:F]

5869. ⠨⠁⠂ ⠶ ⠨⠁ ⠠⠨⠁⠆ ⠄⠄ ⠠⠨⠁ ⠰⠝⠂

$\alpha_1 = \alpha, \alpha_2, \ldots, \alpha_n$

\alpha_1 = \alpha, \alpha_2, \ldots, \alpha_n

5870. ⠛⠷⠭⠾ ⠶ ⠐⠠⠏⠭⠆⠰⠋⠠⠝⠊ ⠶ ⠐⠣⠏⠻⠷⠭⠤⠨⠁ ⠰⠊⠐⠾

$g(x) = \prod_{i=1}^{n}(x - \alpha_i)$

g(x) = \prod_{i = 1}^{n} (x -\alpha_i)

5871. ⠙⠑⠛⠛⠷⠭⠾ ⠨⠣ ⠙⠑⠛⠋⠷⠭⠾

$\deg g(x) \leq \deg f(x)$

\deg g(x) \leq \deg f(x)

5872. ⠋⠷⠭⠾ ⠶ ⠛⠷⠭⠾

$f(x) = g(x)$

f(x) = g(x)

5873. ⠠⠋ ⠶ ⠠⠅ ⠆⠠⠛

$F = K_G$

F = K_G

5874. ⠠⠛ ⠶ ⠠⠛⠷⠠⠅⠌⠠⠋⠾

$G = G(K/F)$

G = G(K/F)

5875. ⠠⠛⠷⠠⠅⠌⠠⠋⠾

$G(K/F)$

G(K/F)

5876. ⠈⠷⠠⠅⠒⠠⠋⠈⠾ ⠨⠣ ⠳⠠⠛⠳ ⠨⠣ ⠳⠠⠛⠷⠠⠅⠌⠠⠋⠾⠳ ⠶ ⠈⠷⠠⠅⠒⠠⠋⠈⠾

$[K : F] \leq |G| \leq |G(K/F)| = [K : F]$

[K : F] \leq |G| \leq |G(K/F)| = [K:F]

5877. ⠠⠛⠷⠈⠠⠟⠷⠜⠤⠩⠠⠜⠱⠻⠂⠈⠠⠟⠾

$G(\mathbb{Q}(\sqrt{3}, \sqrt{5}\,)/\mathbb{Q})$

G({\mathbb Q}(\sqrt{3}, \sqrt{5}\,) /{\mathbb Q})

5878. ⠠⠛⠷⠈⠠⠟⠷⠜⠤⠩⠠⠜⠱⠻⠂⠈⠠⠟⠾

$G(\mathbb{Q}(\sqrt{3}, \sqrt{5}\,)/\mathbb{Q})$

G({\mathbb Q(\sqrt{3}, \sqrt{5}\,) / {\mathbb Q})}

5879. ⠠⠅ ⠫⠳⠒⠒⠕ ⠠⠛⠷⠠⠑⠌⠠⠅⠾

$K \mapsto G(E/K)$

K \mapsto G(E/K)

5880. ⠠⠋ ⠸⠐⠅ ⠠⠅ ⠸⠐⠅ ⠠⠑

$F \subset K \subset E$

F \subset K \subset E

5881. ⠈⠷⠠⠑⠐⠒⠠⠅⠈⠾ ⠨⠅ ⠳⠠⠛⠷⠠⠑⠌⠠⠅⠾⠳and ⠈⠷⠠⠅⠐⠒⠠⠋⠈⠾ ⠨⠅ ⠈⠷⠠⠛⠷⠠⠑⠌⠠⠋⠾⠐⠒⠠⠛⠷⠠⠑⠌⠠⠅⠾⠈

$[E:K] = |G(E/K)|$ and $[K:F] = [G(E/F):G(E/K)]$

[E:K] = |G(E/K)| \text{ and } [K:F] = [G(E/F):G(E/K)]

5882. ⠠⠋ ⠸⠐⠅ ⠠⠅ ⠸⠐⠅ ⠠⠇ ⠸⠐⠅ ⠠⠑

$F \subset K \subset L \subset E$

F \subset K \subset L \subset E

5883. ⠨⠷⠊⠙⠨⠾ ⠸⠐⠅ ⠠⠛⠷⠠⠑⠌⠠⠇⠾ ⠸⠐⠅ ⠠⠛⠷⠠⠑⠌⠠⠅⠾ ⠸⠐⠅ ⠠⠛⠷⠠⠑⠌⠠⠋⠾

$\{\mathrm{id}\} \subset G(E/L) \subset G(E/K) \subset G(E/F)$

\{ \identity \} \subset G(E/L) \subset G(E/K) \subset G(E/F)

5884. ⠠⠛⠷⠠⠑⠌⠠⠅⠾

$G(E/K)$

G(E/K)

5885. ⠠⠛⠷⠠⠅⠌⠠⠋⠾ ⠈⠱⠈⠱ ⠠⠛⠷⠠⠑⠌⠠⠋⠾⠌⠠⠛⠷⠠⠑⠌⠠⠅⠾

$G(K/F) \cong G(E/F)/G(E/K)$

G(K/F) \cong G(E/F) / G(E/K)

5886. ⠠⠛⠷⠠⠑⠌⠠⠅⠾ ⠨⠅ ⠠⠛⠷⠠⠑⠌⠠⠇⠾ ⠨⠅ ⠠⠛

$G(E/K) = G(E/L) = G$

G(E/K) = G(E/L) = G

5887. ⠠⠅ ⠨⠅ ⠠⠇

$K = L$

K=L

5888. ⠠⠛⠷⠠⠑⠌⠠⠅⠾ ⠨⠅ ⠠⠛

$G(E/K) = G$

G(E/K) = G

5889. ⠳⠠⠛⠷⠠⠑⠌⠠⠅⠾⠳ ⠨⠅ ⠈⠷⠠⠑⠐⠒⠠⠅⠈⠾

$|G(E/K)| = [E:K]$

|G(E/K)| = [E:K]

5890. ⠳⠠⠛⠷⠠⠑⠌⠠⠋⠾⠳ ⠨⠅ ⠈⠷⠠⠛⠷⠠⠑⠌⠠⠋⠾⠐⠒⠠⠛⠷⠠⠑⠌⠠⠅⠾⠈⠾⠡⠳⠠⠛⠷⠠⠑⠌⠠⠅⠾⠳ ⠨⠅ ⠈⠷⠠⠑⠐⠒⠠⠋⠈

$|G(E/F)| = [G(E/F):G(E/K)] \cdot |G(E/K)| = [E:F] = [E:K][K:F]$

|G(E/F)| = [G(E/F):G(E/K)] \cdot |G(E/K)| = [E:F] = [E:K][K:F]

5891. ⠈⠷⠠⠅⠐⠒⠠⠋⠈⠾ ⠨⠅ ⠈⠷⠠⠛⠷⠠⠑⠌⠠⠋⠾⠐⠒⠠⠛⠷⠠⠑⠌⠠⠅⠾⠈⠾

$[K:F] = [G(E/F):G(E/K)]$

[K:F] = [G(E/F):G(E/K)]

5892. ⠨⠎⠘⠤⠂⠐⠨⠞⠨⠎

$\sigma^{-1}\tau\sigma$

\sigma^{-1} \tau \sigma

5893. ⠨⠎⠘⠤⠂⠐⠨⠞⠨⠎⠷⠨⠁⠾ ⠨⠅ ⠨⠁

$\sigma^{-1}\tau\sigma(\alpha) = \alpha$

`\sigma^{-1} \tau \sigma( \alpha) = \alpha`

5894. ⠨⠞⠷⠨⠎⠷⠨⠁⠾⠾ ⠨⠅ ⠨⠎⠷⠨⠁⠾

$\tau(\sigma(\alpha)) = \sigma(\alpha)$

`\tau( \sigma( \alpha )) = \sigma( \alpha )`

5895. ⠠⠋ ⠨⠅ ⠠⠅⠰⠠⠛⠷⠠⠅⠌⠠⠋⠾

$F = K_{G(K/F)}$

`F = K_{G(K/F)}`

5896. ⠨⠞⠈⠑⠠⠛⠷⠠⠑⠌⠠⠅⠾

$\tau \in G(E/K)$

`\tau \in G(E/K)`

5897. ⠨⠞overbar⠈⠑⠠⠛⠷⠠⠑⠌⠠⠅⠾

$\overline{\tau} \in G(E/K)$

`\overline{\tau} \in G(E/K)`

5898. ⠨⠞⠨⠎ ⠨⠅ ⠨⠎⠨⠞overbar

$\tau\sigma = \sigma\overline{\tau}$

`\tau \sigma = \sigma \overline{\tau}`

5899. ⠨⠞⠷⠨⠎⠷⠨⠁⠾⠾ ⠨⠅ ⠨⠎⠷ModifyingAbove⠨⠞With ⠷⠨⠁⠾⠾ ⠨⠅ ⠨⠎⠷⠨⠁⠾⠸⠆

$\tau(\sigma(\alpha)) = \sigma(\overline{\tau}(\alpha)) = \sigma(\alpha);$

`\tau( \sigma( \alpha ) ) = \sigma( \overline{\tau}( \alpha ) ) = \sigma( \alpha );`

5900. ⠨⠎overbar

$\overline{\sigma}$

`\overline{\sigma}`

5901. ⠨⠎⠷⠨⠁⠾⠈⠑⠠⠅

$\sigma(\alpha) \in K$

`\sigma( \alpha ) \in K`

5902. ⠨⠎overbar⠈⠑⠠⠛⠷⠠⠅⠌⠠⠋⠾

$\overline{\sigma} \in G(K/F)$

`\overline{\sigma} \in G(K/F)`

5903. ModifyingAbove⠨⠎With ⠷⠨⠃⠾ ⠨⠅ ⠨⠃

$\overline{\sigma}(\beta) = \beta$

`\overline{\sigma}(\beta) = \beta`

5904. ⠠⠛⠷⠠⠅⠌⠠⠋⠾ ⠈⠱⠈⠱ ⠠⠛⠷⠠⠑⠌⠠⠋⠾⠌⠠⠛⠷⠠⠑⠌⠠⠅⠾

$G(K/F) \cong G(E/F)/G(E/K)$

`G(K/F) \cong G(E/F) / G(E/K)`

5905. ⠨⠎⠰⠠⠅

σ_K

\sigma_K

5906. ⠨⠎⠰⠠⠅⠀⠐⠈⠑⠀⠠⠛⠷⠠⠅⠌⠠⠋⠾

$\sigma_K \in G(K/F)$

\sigma_K \in G(K/F)

5907. ⠨⠋⠸⠒⠠⠛⠷⠠⠑⠌⠠⠋⠾⠀⠫⠕⠀⠠⠛⠷⠠⠅⠌⠠⠋⠾

$\phi:G(E/F) \rightarrow G(K/F)$

\phi:G(E/F) \rightarrow G(K/F)

5908. ⠨⠎⠀⠫⠳⠒⠒⠕⠀⠨⠎⠰⠠⠅

$\sigma \mapsto \sigma_K$

\sigma \mapsto \sigma_K

5909. ⠨⠋⠷⠨⠎⠨⠞⠾⠀⠨⠅⠀⠷⠨⠎⠨⠞⠾⠰⠠⠅⠐⠀⠨⠅⠀⠨⠎⠰⠠⠅⠐⠨⠞⠰⠠⠅⠐⠀⠨⠅⠀⠨⠋⠷⠨⠎⠾⠨⠋⠷⠨⠞⠾

$\phi(\sigma \tau) = (\sigma \tau)_K = \sigma_K \tau_K = \phi(\sigma) \phi(\tau)$

\phi(\sigma \tau) = (\sigma \tau)_K = \sigma_K \tau_K = \phi(
\sigma) \phi(\tau)

5910. ⠳⠠⠛⠷⠠⠑⠌⠠⠋⠾⠳⠌⠳⠠⠛⠷⠠⠑⠌⠠⠅⠾⠳⠀⠨⠅⠀⠈⠷⠠⠅⠸⠒⠠⠋⠈⠾⠀⠨⠅⠀⠳⠠⠛⠷⠠⠅⠌⠠⠋⠾⠳

$|G(E/F)|/|G(E/K)| = [K:F] = |G(K/F)|$

|G(E/F)| / |G(E/K)| = [K:F] = |G(K/F)|

5911. ⠋⠷⠭⠾⠀⠨⠅⠀⠭⠘⠲⠐⠤⠆

$f(x) = x^4 - 2$

f(x) = x^4 - 2

5912. ⠭⠘⠲⠐⠤⠆

$x^4 - 2$

x^4-2

5913. ⠈⠠⠟⠷⠣⠲⠠⠜⠆⠻⠠⠊⠾

$\mathbb{Q}(\sqrt[4]{2}, i)$

{\mathbb Q}(\sqrt[4]{2}, i)

5914. ⠷⠭⠘⠆⠐⠬⠜⠆⠻⠾⠷⠭⠘⠆⠐⠤⠜⠆⠻⠾

$(x^2 + \sqrt{2}\,)(x^2 - \sqrt{2}\,)$

(x^2 + \sqrt{2}\,)(x^2 - \sqrt{2}\,)

5915. ⠬⠤⠣⠲⠜⠆⠻

$\pm \sqrt[4]{2}$

\pm \sqrt[4]{2}

5916. ⠬⠤⠣⠲⠜⠆⠻⠊

$\pm \sqrt[4]{2}\, i$

\pm \sqrt[4]{2}\, i

5917. ⠣⠲⠜⠆⠻

$\sqrt[4]{2}$

`\sqrt[4]{2}`

5918. ⠈⠠⠟⠷⠣⠲⠜⠆⠻⠾

${\mathbb Q}(\sqrt[4]{2}\,)$

`{\mathbb Q}(\sqrt[4]{2}\, )`

5919. ⠈⠠⠟⠷⠣⠲⠜⠆⠻⠾⠷⠊⠾ ⠨⠅ ⠈⠠⠟⠷⠣⠲⠜⠆⠻⠠⠊⠾

${\mathbb Q}(\sqrt[4]{2}\,)(i) = {\mathbb Q}(\sqrt[4]{2}, i)$

`{\mathbb Q}(\sqrt[4]{2}\, )(i) = {\mathbb Q}( \sqrt[4]{2}, i )`

5920. ⠈⠷⠈⠠⠟⠷⠣⠲⠜⠆⠻⠾⠱⠈⠠⠟⠈⠾ ⠨⠅ ⠲

$[{\mathbb Q}(\sqrt[4]{2}\,) : {\mathbb Q}] = 4$

`[ {\mathbb Q}( \sqrt[4]{2}\, ) : {\mathbb Q}] = 4`

5921. ⠈⠠⠟⠷⠣⠲⠜⠆⠻⠾

${\mathbb Q}(\sqrt[4]{2}\,)$

`{\mathbb Q}( \sqrt[4]{2}\, )`

5922. ⠈⠷⠈⠠⠟⠷⠣⠲⠜⠆⠻⠠⠊⠾⠱⠈⠠⠟⠷⠣⠲⠜⠆⠻⠾⠈⠾ ⠨⠅ ⠆

$[{\mathbb Q}(\sqrt[4]{2}, i): {\mathbb Q}(\sqrt[4]{2}\,)] = 2$

`[ {\mathbb Q}( \sqrt[4]{2}, i ): {\mathbb Q}(\sqrt[4]{2}\, )] =`
`2`

5923. ⠈⠷⠈⠠⠟⠷⠣⠲⠜⠆⠻⠠⠊⠾⠱⠈⠠⠟⠈⠾ ⠨⠅ ⠦

$[{\mathbb Q}(\sqrt[4]{2}, i):{\mathbb Q}] = 8$

`[ {\mathbb Q}( \sqrt[4]{2}, i ):{\mathbb Q}] = 8`

5924. StartSet⠂ ⠠⠣⠲⠜⠆⠻⠠⠷⠣⠲⠜⠆⠻⠾⠘⠆⠐⠠⠷⠣⠲⠜⠆⠻⠾⠘⠒⠐⠠⠊⠠⠊⠣⠲⠜⠆⠻⠠⠊⠷⠣⠲⠜⠆⠻⠾⠘⠆⠐⠠⠊⠷⠣⠲⠜

$\{ 1, \sqrt[4]{2}, (\sqrt[4]{2}\,)^2, (\sqrt[4]{2}\,)^3, i, i \sqrt[4]{2}, i (\sqrt[4]{2}\,)^2, i(\sqrt[4]{2}\,)^3 \}$

`\{ 1, \sqrt[4]{2}, (\sqrt[4]{2}\, )^2, (\sqrt[4]{2}\, )^3, i, i`
`\sqrt[4]{2}, i (\sqrt[4]{2}\, )^2, i(\sqrt[4]{2}\, )^3 \}`

5925. ⠈⠠⠟⠷⠣⠲⠜⠆⠻⠠⠊⠾

${\mathbb Q}(\sqrt[4]{2}, i)$

`{\mathbb Q}( \sqrt[4]{2}, i)`

5926. ⠨⠎⠷⠣⠲⠜⠆⠻⠾ ⠨⠅ ⠊⠣⠲⠜⠆⠻

$\sigma(\sqrt[4]{2}\,) = i \sqrt[4]{2}$

`\sigma( \sqrt[4]{2}\, ) = i \sqrt[4]{2}`

5927. ⠨⠎⠷⠊⠾ ⠨⠅ ⠊

$\sigma(i) = i$

`\sigma( i ) = i`

5928. ⠨⠞⠷⠊⠾ ⠨⠅ ⠤⠊

$\tau(i) = -i$

`\tau(i ) = -i`

5929. StartSet⠈⠙ ⠠⠨⠎ ⠠⠨⠎⠘⠆ ⠐⠠⠨⠎⠘⠒⠐ ⠠⠨⠞ ⠠⠨⠎⠨⠞ ⠠⠨⠎⠘⠆ ⠐⠨⠞ ⠠⠨⠎⠘⠒⠐⠨⠞EndSet

$\{\mathrm{id}, \sigma, \sigma^2, \sigma^3, \tau, \sigma\tau, \sigma^2\tau, \sigma^3\tau\}$

\{ \identity, \sigma, \sigma^2, \sigma^3, \tau, \sigma \tau, \sigma^2
\tau, \sigma^3 \tau \}

5930. ⠨⠞⠘⠆⠐ ⠨⠅ ⠊⠙

$\tau^2 = \mathrm{id}$

\tau^2 = \identity

5931. ⠨⠎⠘⠲⠐ ⠨⠅ ⠊⠙

$\sigma^4 = \mathrm{id}$

\sigma^4 = \identity

5932. ⠨⠞⠨⠎⠨⠞ ⠨⠅ ⠨⠎⠘⠤⠂

$\tau\sigma\tau = \sigma^{-1}$

\tau \sigma \tau = \sigma^{-1}

5933. ⠁⠭⠘⠆⠐⠬⠃⠭⠬⠉ ⠨⠅ ⠴

$ax^2 + bx + c = 0$

a x^2 + b x +c = 0

5934. ⠠⠋ ⠨⠅ ⠠⠋⠴⠸⠐⠅ ⠠⠋⠂ ⠸⠐⠅ ⠠⠋⠆ ⠸⠐⠅ ⠄⠄⠄ ⠸⠐⠅ ⠠⠋⠰⠗⠐ ⠨⠅ ⠠⠑

$F = F_0 \subset F_1 \subset F_2 \subset \cdots \subset F_r = E$

F = F_0 \subset F_1 \subset F_2 \subset \cdots \subset F_r = E

5935. ⠊ ⠨⠅ ⠂ ⠠⠆⠠ ⠄⠄ ⠠⠗

$i = 1, 2, \ldots, r$

i = 1, 2, \ldots, r

5936. ⠠⠋⠰⠊⠐ ⠨⠅ ⠠⠋⠰⠊⠤⠂⠐ ⠷⠨⠁ ⠰⠊⠐⠾

$F_i = F_{i-1}(\alpha_i)$

F_i = F_{i - 1}(\alpha_i)

5937. ⠨⠁ ⠰⠊⠘⠝⠘⠰⠊⠐⠈⠑⠠⠋⠰⠊⠤⠂

$\alpha_i^{n_i} \in F_{i-1}$

\alpha_i^{n_i} \in F_{i-1}

5938. ⠭⠘⠝⠐⠤⠁

$x^n - a$

x^n - a

5939. ⠭⠘⠝⠐⠤⠂

$x^n - 1$

x^n -1

5940. ⠂⠐ ⠠⠨⠺ ⠠⠨⠺⠘⠆ ⠐⠠⠄⠄ ⠠⠨⠺⠘⠝⠤⠂

$1, \omega, \omega^2, \ldots, \omega^{n-1}$

1, \omega, \omega^2, \ldots, \omega^{n - 1}

5941. ⠨⠺ ⠨⠅ ⠉⠕⠎ ⠷⠹⠆⠨⠏⠌⠝⠼⠾ ⠬⠊⠎⠊⠝ ⠷⠹⠆⠨⠏⠌⠝⠼⠾

$\omega = \cos\left(\frac{2\pi}{n} \right) + i \sin\left(\frac{2\pi}{n} \right)$

\omega = \cos\left(\frac{2 \pi}{n} \right) + i \sin\left(\frac{2
\pi}{n} \right)

5942. ⠠⠓⠰⠊⠬⠂ ⠸⠌⠠⠓⠰⠊

H_{i+1}/H_i

H_{i+1} /H_i

5943. ⠸⠣⠊⠙⠸⠜ ⠸⠐⠅ ⠠⠁⠆ ⠸⠐⠅ ⠠⠎⠆

$\{\mathrm{id}\} \subset A_3 \subset S_3$

\{ \identity \} \subset A_3 \subset S_3

5944. ⠣⠝⠜⠁⠻ ⠠⠨⠺⠣⠝⠜⠁⠻ ⠠⠄ ⠄⠄⠄ ⠠⠨⠺⠘⠝⠤⠂⠐⠣⠝⠜⠁⠻

$\sqrt[n]{a}, \omega \sqrt[n]{a}, \ldots, \omega^{n-1} \sqrt[n]{a}$

\sqrt[n]{a}, \omega \sqrt[n]{a}, \ldots, \omega^{n-1} \sqrt[n]{a}

5945. ⠨⠵

ζ

\zeta

5946. ⠨⠵⠠ ⠨⠺⠨⠵⠠ ⠄⠄⠄ ⠠⠨⠺⠘⠝⠤⠂⠐⠨⠵

$\zeta, \omega\zeta, \ldots, \omega^{n-1}\zeta$

\zeta, \omega \zeta, \ldots, \omega^{n - 1} \zeta

5947. ⠠⠑ ⠨⠅ ⠠⠋⠷⠨⠵⠾

$E = F(\zeta)$

E = F(\zeta)

5948. ⠨⠎⠷⠨⠵⠾ ⠨⠅ ⠨⠺⠘⠊⠐⠨⠵

$\sigma(\zeta) = \omega^i \zeta$

\sigma(\zeta) = \omega^i \zeta

5949. ⠨⠞⠷⠨⠵⠾ ⠨⠅ ⠨⠺⠘⠚⠐⠨⠵

$\tau(\zeta) = \omega^j \zeta$

\tau(\zeta) = \omega^j \zeta

5950. ⠨⠎⠨⠞⠷⠨⠵⠾ ⠨⠅ ⠨⠎⠷⠨⠺⠘⠚⠐⠨⠵⠾ ⠨⠅ ⠨⠺⠘⠚⠐⠨⠎⠷⠨⠵⠾ ⠨⠅ ⠨⠺⠘⠊⠬⠚⠐⠨⠵ ⠨⠅ ⠨⠺⠘⠊⠐⠨⠞⠷⠨⠵⠾

$\sigma\tau(\zeta) = \sigma(\omega^j\zeta) = \omega^j\sigma(\zeta) = \omega^{i+j}\zeta = \omega^i\tau(\zeta) = \tau(\omega^i\zeta) = \tau\sigma(\zeta)$

\sigma \tau(\zeta) = \sigma(\omega^j \zeta) = \omega^j \sigma(
\zeta) = \omega^{i+j} \zeta = \omega^i \tau(\zeta) = \tau(\omega^i
\zeta) = \tau \sigma(\zeta)

5951. ⠨⠺⠨⠁

$\omega\alpha$

\omega \alpha

5952. ⠨⠺ ⠨⠅ ⠷⠨⠺⠨⠁⠾⠸⠌⠨⠁

$\omega = (\omega\alpha)/\alpha$

\omega = (\omega \alpha)/ \alpha

5953. ⠠⠅⠀⠨⠅⠀⠠⠋⠷⠨⠺⠾

$K = F(\omega)$

K = F(\omega)

5954. ⠠⠛⠷⠠⠋⠷⠨⠺⠾⠸⠌⠠⠋⠾

$G(F(\omega)/F)$

G(F(\omega)/ F)

5955. ⠨⠎⠷⠨⠺⠾

$\sigma(\omega)$

\sigma(\omega)

5956. ⠨⠎⠷⠨⠺⠾⠀⠨⠅⠀⠨⠺⠘⠊

$\sigma(\omega) = \omega^i$

\sigma(\omega) = \omega^i

5957. ⠨⠞⠷⠨⠺⠾⠀⠨⠅⠀⠨⠺⠘⠚

$\tau(\omega) = \omega^j$

\tau(\omega) = \omega^j

5958. ⠠⠛⠷⠠⠋⠷⠨⠺⠾⠸⠌⠠⠋⠾

$G(F(\omega)/F)$

G(F(\omega)/F)

5959. ⠨⠎⠨⠞⠷⠨⠺⠾⠀⠨⠅⠀⠨⠎⠷⠨⠺⠘⠚⠐⠾⠀⠨⠅⠀⠈⠷⠨⠎⠷⠨⠺⠾⠈⠾⠘⠚⠐⠀⠨⠅⠀⠨⠺⠘⠊⠚⠐⠀⠨⠅⠀⠈⠷⠨⠞⠷⠨⠺⠾⠈⠾

$\sigma\tau(\omega) = \sigma(\omega^j) = [\sigma(\omega)]^j = \omega^{ij} = [\tau(\omega)]^i = \tau(\omega^i) = \tau\sigma(\omega)$

\sigma \tau(\omega) = \sigma(\omega^j) = [\sigma(\omega)]^j
= \omega^{ij} = [\tau(\omega)]^i = \tau(\omega^i) = \tau \sigma(
\omega)

5960. ⠠⠛⠷⠠⠋⠷⠨⠺⠾⠸⠌⠠⠋⠾

$G(F(\omega)/F)$

G(F(\omega) / F)

5961. ⠨⠷⠊⠙⠨⠾⠀⠸⠐⠅⠀⠠⠛⠷⠠⠑⠸⠌⠠⠋⠷⠨⠺⠾⠾⠀⠸⠐⠅⠀⠠⠛⠷⠠⠑⠸⠌⠠⠋⠾

$\{\mathrm{id}\} \subset G(E/F(\omega)) \subset G(E/F)$

\{ \identity \} \subset G(E/ F(\omega)) \subset G(E/F)

5962. ⠠⠛⠷⠠⠑⠸⠌⠠⠋⠷⠨⠺⠾⠾

$G(E/F(\omega))$

G(E/F(\omega))

5963. ⠠⠛⠷⠠⠑⠸⠌⠠⠋⠾⠸⠌⠠⠛⠷⠠⠑⠸⠌⠠⠋⠷⠨⠺⠾⠾⠀⠈⠱⠈⠱⠀⠠⠛⠷⠠⠋⠷⠨⠺⠾⠸⠌⠠⠋⠾

$G(E/F)/G(E/F(\omega)) \cong G(F(\omega)/F)$

G(E/F) /G(E/F(\omega)) \cong G(F(\omega)/F)

5964. ⠠⠋⠀⠨⠅⠀⠠⠅⠴⠀⠸⠐⠅⠀⠠⠅⠂⠀⠸⠐⠅⠀⠠⠅⠆⠀⠸⠐⠅⠀⠄⠄⠄⠀⠸⠐⠅⠀⠠⠅⠰⠗⠐⠀⠨⠅⠀⠠⠅

$F = K_0 \subset K_1 \subset K_2 \subset \cdots \subset K_r = K$

F = K_0 \subset K_1 \subset K_2 \subset \cdots \subset K_r = K

5965. ⠠⠅⠰⠊

K_i

`K_i`

5966. ⠠⠅⠰⠊⠤⠂

K_{i-1}

`K_{i-1}`

5967. ⠠⠅ ⠰⠂⠱ ⠠⠑

$K \supseteq E$

`K \supseteq E`

5968. ⠠⠅⠂

K_1

`K_1`

5969. ⠭⠘⠝⠂⠐⠤⠨⠁⠂⠘⠝⠂

$x^{n_1} - \alpha_1^{n_1}$

`x^{n_1} - \alpha_1^{n_1}`

5970. ⠨⠁⠂⠠ ⠨⠁⠂⠨⠺⠠ ⠨⠁⠂⠨⠺⠘⠆⠐⠠ ⠄⠄⠄⠠ ⠨⠁⠂⠨⠺⠘⠝⠂⠤⠂

$\alpha_1, \alpha_1 \omega, \alpha_1 \omega^2, \ldots, \alpha_1 \omega^{n_1 - 1}$

`\alpha_1, \alpha_1 \omega, \alpha_1 \omega^2, \ldots, \alpha_1 \omega^{n_1 - 1}`

5971. ⠠⠅⠂ ⠨⠅ ⠠⠋⠷⠨⠁⠰⠮⠐⠾

$K_1 = F(\alpha_!)$

`K_1 = F(\alpha_!)`

5972. ⠨⠃⠠ ⠨⠺⠨⠃⠠ ⠄⠄⠄⠠ ⠨⠺⠘⠝⠂⠤⠂

$\beta, \omega \beta, \ldots, \omega^{n_1-1}$

`\beta, \omega \beta, \ldots, \omega^{n_1-1}`

5973. ⠠⠅⠂ ⠨⠅ ⠠⠋⠷⠨⠺⠨⠃⠾

$K_1 = F(\omega \beta)$

`K_1 = F(\omega \beta)`

5974. ⠠⠅⠰⠊⠐ ⠰⠂⠱ ⠠⠋⠰⠊

$K_i \supseteq F_i$

`K_i \supseteq F_i`

5975. ⠠⠋ ⠨⠅ ⠠⠋⠴ ⠸⠐⠅ ⠠⠋⠂ ⠸⠐⠅ ⠄⠄⠄ ⠸⠐⠅ ⠠⠋⠰⠝⠐ ⠨⠅ ⠠⠑

$F = F_0 \subset F_1 \subset \cdots \subset F_n = E$

`F = F_0 \subset F_1 \subset \cdots \subset F_n = E`

5976. ⠠⠋⠰⠊⠤⠂

F_{i-1}

`F_{i - 1}`

5977. ⠠⠛⠷⠠⠑⠌⠠⠋⠰⠊⠐⠾

$G(E/F_i)$

G(E/F_i)

5978. ⠠⠛⠷⠠⠑⠌⠠⠋⠰⠊⠤⠂⠐⠾

$G(E/F_{i-1})$

G(E/F_{i - 1})

5979. ⠨⠷⠊⠙⠨⠾ ⠸⠐⠅ ⠠⠛⠷⠠⠑⠌⠠⠋⠰⠝⠤⠂⠐⠾ ⠸⠐⠅ ⠄⠄⠄ ⠸⠐⠅ ⠠⠛⠷⠠⠑⠌⠠⠋⠂⠾ ⠸⠐⠅ ⠠⠛⠷⠠⠑⠌⠠⠋⠾

$\{\mathrm{id}\} \subset G(E/F_{n-1}) \subset \cdots \subset G(E/F_1) \subset G(E/F)$

\{ \identity \} \subset G(E/F_{n - 1}) \subset \cdots \subset G(E/F_1)
\subset G(E/F)

5980. ⠠⠛⠷⠠⠑⠌⠠⠋⠰⠊⠤⠂⠐⠾⠌⠠⠛⠷⠠⠑⠌⠠⠋⠰⠊⠐⠾ ⠈⠱⠈⠱ ⠠⠛⠷⠠⠋⠰⠊⠐⠌⠠⠋⠰⠊⠤⠂⠐⠾

$G(E/F_{i-1})/G(E/F_i) \cong G(F_i/F_{i-1})$

G(E/F_{i - 1})/G(E/F_i) \cong G(F_i/F_{i - 1})

5981. ⠠⠛⠷⠠⠋⠰⠊⠐⠌⠠⠋⠰⠊⠤⠂⠐⠾

$G(F_i/F_{i-1})$

G(F_i/F_{i - 1})

5982. ⠠⠎⠰⠏

S_p

S_p

5983. ⠨⠎ ⠨⠅ ⠷⠂⠆⠾

$\sigma = (12)$

\sigma = (1 2)

5984. ⠨⠞⠘⠝

τ^n

\tau^n

5985. ⠂ ⠅⠱ ⠝ ⠐⠅ ⠏

$1 \leq n < p$

1 \leq n \lt p

5986. ⠨⠍ ⠨⠅ ⠨⠞⠘⠝⠐ ⠨⠅ ⠷⠂⠆⠊⠒⠄⠊⠰⠏⠐⠾

$\mu = \tau^n = (12i_3 \ldots i_p)$

\mu = \tau^n = (1 2 i_3 \ldots i_p)

5987. ⠷⠂⠆⠾⠷12⠊⠒⠄⠊⠰⠏⠐⠾ ⠨⠅ ⠷⠆⠊⠒⠄⠊⠰⠏⠐⠾

$(12)(12i_3 \ldots i_p) = (2i_3 \ldots i_p)$

(1 2)(12 i_3\ldots i_p) = (2 i_3\ldots i_p)

5988. ⠷⠆⠊⠒⠄⠊⠰⠏⠐⠾⠘⠅ ⠐⠷12⠾⠷⠆⠊⠒⠄⠊⠰⠏⠐⠾⠘⠤⠅⠐ ⠨⠅ ⠷⠂⠊⠰⠅⠐⠾

$(2i_3 \ldots i_p)^k(12)(2i_3 \ldots i_p)^{-k} = (1i_k)$

(2i_3 \ldots i_p)^k(12)(2i_3 \ldots i_p)^{-k} = (1i_k)

5989. ⠷⠂ ⠝⠾

$(1n)$

`(1n)`

5990. ⠷⠂ ⠚⠾⠷⠂ ⠊⠾⠷⠂ ⠚⠾ ⠶ ⠷⠊⠚⠾

$(1j)(1i)(1j) = (ij)$

`(1j)(1 i)(1 j) = (i j)`

5991. ⠋⠷⠭⠾ ⠶ ⠭⠘⠢⠐⠤⠖⠭⠘⠒⠐⠤27⠭⠤⠒

$f(x) = x^5 - 6x^3 - 27x - 3$

`f(x) = x^5 - 6 x^3 - 27 x - 3`

5992. ⠋⠷⠭⠾ ⠶ ⠭⠘⠢⠐⠤⠖⠭⠘⠒⠐⠤27⠭⠤⠒ ⠈⠑ ⠈⠠⠟⠈⠷⠭⠈⠾

$f(x) = x^5 - 6x^3 - 27x - 3 \in \mathbb{Q}[x]$

`f(x) = x^5 - 6 x^3 - 27 x - 3 \in {\mathbb Q}[x]`

5993. ⠋⠄⠷⠭⠾ ⠶ ⠢⠭⠘⠲⠐⠤18⠭⠘⠆⠐⠤27

$f'(x) = 5x^4 - 18x^2 - 27$

`f'(x) = 5 x^4 - 18 x^2 - 27`

5994. ⠋⠄⠷⠭⠾ ⠶ ⠴

$f'(x) = 0$

`f'(x) = 0`

5995. ⠭ ⠶ ⠬⠤⠜⠹⠖⠜⠖⠻⠬⠔⠌⠢⠼⠻

$x = \pm\sqrt{\frac{6\sqrt{6}+9}{5}}$

`x = \pm \sqrt{ \frac{6 \sqrt{6} + 9 }{5} }`

5996. ⠤⠆

-2

`-2`

5997. ⠠⠛⠷⠠⠅⠌⠈⠠⠟⠾

$G(K/\mathbb{Q})$

`G(K/{\mathbb Q})`

5998. ⠨⠎ ⠈⠑ ⠠⠛⠷⠠⠅⠌⠈⠠⠟⠾

$\sigma \in G(K/\mathbb{Q})$

`\sigma \in G(K/{\mathbb Q})`

5999. ⠨⠎⠷⠁⠾ ⠶ ⠃

$\sigma(a) = b$

`\sigma(a) = b`

6000. ⠁ ⠬⠃⠊ ⠫⠳⠒⠒⠕ ⠁ ⠤⠃⠊

$a + bi \mapsto a - bi$

`a + bi \mapsto a - bi`

6001. ⠠⠛⠷⠠⠅⠌⠈⠠⠟⠾

$G(K/\mathbb{Q})$

`G(K/{\mathbb Q} )`

6002. ⠈⠷⠈⠠⠟⠷⠰⠁⠾⠱⠈⠠⠟⠈⠾ ⠰⠆ ⠢

$[\mathbb Q(\alpha) : \mathbb Q] = 5$

`[\mathbb Q(\alpha) : \mathbb Q] = 5`

6003. ⠈⠠⠟⠷⠰⠁⠾

$\mathbb Q(\alpha)$

`\mathbb Q(\alpha)`

6004. ⠈⠷⠠⠅⠱⠈⠠⠟⠈⠾

$[K : \mathbb Q]$

`[K : \mathbb Q]`

6005. ⠈⠷⠠⠅⠱⠈⠠⠟⠈⠾ ⠰⠆ ⠳⠠⠛⠷⠠⠅⠌⠈⠠⠟⠾⠳

$[K : \mathbb Q] = |G(K/{\mathbb Q})|$

`[K : \mathbb Q] = |G(K/{\mathbb Q})|`

6006. ⠠⠛⠷⠠⠅⠌⠈⠠⠟⠾⠸⠪ ⠠⠎⠢

$G(K/{\mathbb Q}) \subset S_5$

`G(K/{\mathbb Q}) \subset S_5`

6007. ⠈⠠⠉⠈⠷⠭⠈⠾

${\mathbb C}[x]$

`{\mathbb C}[x]`

6008. ⠠⠑ ⠰⠆ ⠈⠠⠉⠷⠰⠁⠾

$E = {\mathbb C}(\alpha)$

`E = {\mathbb C}( \alpha )`

6009. ⠋⠷⠭⠾⠷⠭⠘⠆⠐⠬⠂⠾

$f(x)(x^2 + 1)$

`f(x)(x^2 + 1)`

6010. ⠠⠛⠷⠠⠇⠌⠈⠠⠗⠾

$G(L/{\mathbb R})$

`G(L/{\mathbb R})`

6011. ⠠⠇⠸⠕ ⠠⠅⠸⠕ ⠈⠠⠗

$L \supset K \supset {\mathbb R}$

`L \supset K \supset {\mathbb R}`

6012. ⠳⠠⠛⠷⠠⠇⠌⠠⠅⠾⠳ ⠰⠆ ⠈⠷⠠⠇⠱⠠⠅⠈⠾

$|G(L / K)| =[L:K]$

`|G( L / K )| =[L:K]`

6013. ⠈⠷⠠⠇⠱⠈⠠⠗⠈⠾ ⠰⠆ ⠈⠷⠠⠇⠱⠠⠅⠈⠾⠈⠷⠠⠅⠱⠈⠠⠗⠈⠾

$[L : {\mathbb R}] = [L:K][K:{\mathbb R}]$

`[L : {\mathbb R}] = [L:K][K:{\mathbb R}]`

6014. ⠈⠷⠠⠅⠱⠈⠠⠗⠈⠾

$[K:{\mathbb R}]$

`[K:{\mathbb R}]`

6015. ⠠⠅ ⠨⠅ ⠈⠠⠗⠷⠨⠃⠾

$K = \mathbb{R}(\beta)$

`K = {\mathbb R}(\beta)`

6016. ⠠⠅ ⠨⠅ ⠈⠠⠗

$K = \mathbb{R}$

`K = {\mathbb R}`

6017. ⠠⠛⠷⠠⠇⠌⠈⠠⠉⠾

$G(L/\mathbb{C})$

`G(L / {\mathbb C})`

6018. ⠠⠇ ⠌⠨⠅ ⠈⠠⠉

$L \neq \mathbb{C}$

`L \neq {\mathbb C}`

6019. ⠳⠠⠛⠷⠠⠇⠌⠈⠠⠉⠾⠳ ⠨⠂⠱ ⠆

$|G(L/\mathbb{C})| \geq 2$

`|G(L / {\mathbb C})| \geq 2`

6020. ⠠⠛⠷⠠⠇⠌⠈⠠⠉⠾

$G(L/\mathbb{C})$

`G(L/{\mathbb C})`

6021. ⠈⠷⠠⠑⠱⠈⠠⠉⠈⠾ ⠨⠅ ⠆

$[E:\mathbb{C}] = 2$

`[E:{\mathbb C}] = 2`

6022. ⠨⠛⠈⠑⠠⠑

$\gamma \in E$

`\gamma \in E`

6023. ⠭⠘⠆⠐⠬⠃⠭⠬⠉

$x^2 + bx + c$

`x^2 + b x + c`

6024. ⠷⠤⠃⠬⠤⠜⠃⠘⠆⠐⠤⠲⠉⠻⠾⠌⠆

$(-b \pm \sqrt{b^2 - 4c}\,)/2$

`( - b \pm \sqrt{b^2 - 4c}\, ) / 2`

6025. ⠃⠘⠆⠐⠤⠲⠉

$b^2 - 4c$

`b^2 - 4 c`

6026. ⠠⠇ ⠨⠅ ⠈⠠⠉

$L = \mathbb{C}$

`L = {\mathbb C}`

6027. ⠠⠛⠷⠈⠠⠟⠷⠜30⠻⠾⠌⠈⠠⠟⠾

$G(\mathbb{Q}(\sqrt{30}\,)/\mathbb{Q})$

`G({\mathbb Q}(\sqrt{30}\, ) / {\mathbb Q})`

6028. ⠠⠛⠷⠈⠠⠟⠷⠣⠲⠜⠢⠻⠾⠌⠈⠠⠟⠾

$G(\mathbb{Q}(\sqrt[4]{5}\,)/\mathbb{Q})$

`G({\mathbb Q}(\sqrt[4]{5}\, ) / {\mathbb Q})`

6029. ⠠⠛⠷⠈⠠⠟⠷⠜⠆⠻⠠⠜⠒⠻⠠⠜⠢⠻⠾⠌⠈⠠⠟⠾

$G(\mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5}\,)/\mathbb{Q})$

`G( {\mathbb Q}(\sqrt{2}, \sqrt{3}, \sqrt{5}\, )/ {\mathbb Q} )`

6030. ⠠⠛⠷⠈⠠⠟⠷⠜⠆⠻⠠⠣⠒⠜⠆⠻⠠⠊⠾⠌⠈⠠⠟⠾

$G(\mathbb{Q}(\sqrt{2}, \sqrt[3]{2}, i)/\mathbb{Q})$

`G({\mathbb Q}(\sqrt{2}, \sqrt[3]{2}, i) / {\mathbb Q})`

6031. ⠠⠛⠷⠈⠠⠟⠷⠜⠖⠻⠠⠊⠾⠌⠈⠠⠟⠾

$G(\mathbb{Q}(\sqrt{6}, i)/\mathbb{Q})$

`G({\mathbb Q}(\sqrt{6}, i) / {\mathbb Q})`

6032. ⠈⠠⠵⠆⠈⠡⠈⠠⠵⠆⠈⠡⠈⠠⠵⠆

$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$

`{\mathbb Z}_2 \times {\mathbb Z}_2 \times {\mathbb Z}_2`

6033. ⠭⠘⠒⠐⠬⠆⠭⠘⠆⠤⠭⠤⠆

$x^3 + 2x^2 - x - 2$

`x^3 + 2 x^2 - x - 2`

6034. ⠭⠘⠲⠐⠬⠆⠭⠘⠆⠬⠂

$x^4 + 2x^2 + 1$

`x^4 + 2 x^2 + 1`

6035. ⠭⠘⠲⠐⠬⠭⠘⠆⠐⠬⠂

$x^4 + x^2 + 1$

`x^4 + x^2 + 1`

6036. ⠭⠘⠒⠐⠬⠭⠘⠆⠐⠬⠂

$x^3 + x^2 + 1$

`x^3 +x^2 + 1`

6037. ⠭⠘⠒⠐⠬⠆⠭⠘⠆⠐⠤⠭⠤⠆ ⠶ ⠷⠭⠤⠂⠾⠷⠭⠬⠂⠾⠷⠭⠬⠆⠾

$x^3 + 2x^2 - x - 2 = (x - 1)(x + 1)(x + 2)$

`x^3 + 2 x^2 - x - 2 = (x - 1)(x + 1)(x + 2)`

6038. ⠈⠠⠵⠒

$\mathbb{Z}_3$

`\mathbb Z_3`

6039. ⠭⠘⠲⠐⠬⠭⠘⠆⠐⠬⠂ ⠶ ⠷⠭⠬⠂⠾⠘⠆⠐⠷⠭⠬⠆⠾⠘⠆

$x^4 + x^2 + 1 = (x + 1)^2 (x + 2)^2$

`x^4 + x^2 + 1 = (x + 1)^2 (x + 2)^2`

6040. ⠠⠛⠠⠋⠷729⠾

$\mathrm{GF}(729)$

`\gf(729)`

6041. ⠠⠛⠠⠋⠷⠔⠾

$\mathrm{GF}(9)$

`\gf(9)`

6042. ⠈⠷⠠⠛⠠⠋⠷729⠾⠱⠠⠛⠠⠋⠷⠔⠾⠈⠾⠀⠨⠅⠀⠈⠷⠠⠛⠠⠋⠷729⠾⠱⠠⠛⠠⠋⠷⠒⠾⠈⠾⠌⠈⠷⠠⠛⠠⠋⠷⠔⠾⠱⠠⠛⠠⠋⠷

$[\mathrm{GF}(729) : \mathrm{GF}(9)] = [\mathrm{GF}(729) : \mathrm{GF}(3)]/[\mathrm{GF}(9) : \mathrm{GF}(3)] = 6/2 = 3$

`[\gf(729): \gf(9)] = [\gf(729): \gf(3)] /[\gf(9): \gf(3)] = 6/2`
`= 3`

6043. ⠠⠛⠷⠠⠛⠠⠋⠷729⠾⠌⠠⠛⠠⠋⠷⠔⠾⠾⠀⠈⠱⠈⠱⠀⠈⠠⠵⠒

$G(\mathrm{GF}(729)/\,\mathrm{GF}(9)) \cong \mathbb{Z}_3$

`G(\gf(729)/ \gf(9)) \cong {\mathbb Z}_3`

6044. ⠠⠛⠷⠠⠛⠠⠋⠷729⠾⠌⠠⠛⠠⠋⠷⠔⠾⠾

$G(\mathrm{GF}(729)/\,\mathrm{GF}(9))$

`G(\gf(729)/ \gf(9))`

6045. ⠨⠎⠰⠒⠰⠘⠖⠐⠷⠨⠁⠾⠀⠨⠅⠀⠨⠁⠘⠒⠘⠘⠖⠐⠀⠨⠅⠀⠨⠁⠘729

$\sigma_{3^6}(\alpha) = \alpha^{3^6} = \alpha^{729}$

`\sigma_{3^6}( \alpha) = \alpha^{3^6} = \alpha^{729}`

6046. ⠨⠁⠀⠈⠑⠀⠠⠛⠠⠋⠷729⠾

$\alpha \in \mathrm{GF}(729)$

`\alpha \in \gf(729)`

6047. ⠭⠘⠢⠀⠐⠤12⠭⠘⠆⠀⠬⠆

$x^5 - 12x^2 + 2$

`x^5 - 12 x^2 + 2`

6048. ⠭⠘⠢⠀⠐⠤⠲⠭⠘⠲⠬⠆⠀⠭⠬⠆

$x^5 - 4x^4 + 2x + 2$

`x^5 - 4 x^4 + 2 x + 2`

6049. ⠭⠘⠒⠀⠐⠤⠢

$x^3 - 5$

`x^3 - 5`

6050. ⠭⠘⠲⠀⠐⠤⠭⠘⠆⠀⠐⠤⠖

$x^4 - x^2 - 6$

`x^4 - x^2 - 6`

6051. ⠭⠘⠢⠀⠐⠬⠂

$x^5 + 1$

`x^5 + 1`

6052. ⠷⠭⠘⠆⠀⠐⠤⠆⠾⠷⠭⠘⠆⠀⠐⠬⠆⠀⠾

$(x^2 - 2)(x^2 + 2)$

`(x^2 - 2)(x^2 + 2)`

6053. ⠭ ⠘⠦ ⠐⠤⠂

$x^8 - 1$

```
x^8 - 1
```

6054. ⠭ ⠘⠦ ⠐⠬⠂

$x^8 + 1$

```
x^8 + 1
```

6055. ⠭ ⠘⠲ ⠐⠤⠒⠭ ⠘⠆ ⠤10

$x^4 - 3x^2 - 10$

```
x^4 - 3 x^2 -10
```

6056. ⠭ ⠘⠲ ⠐⠤⠂

$x^4 - 1$

```
x^4 - 1
```

6057. ⠭ ⠘⠲ ⠐⠤⠦⠭ ⠘⠆ ⠬15

$x^4 - 8x^2 + 15$

```
x^4 - 8 x^2 + 15
```

6058. ⠭ ⠘⠲ ⠐⠤⠆ ⠭ ⠘⠆ ⠤15

$x^4 - 2x^2 - 15$

```
x^4 - 2 x^2 - 15
```

6059. ⠭ ⠘⠒ ⠐⠤⠆

$x^3 - 2$

```
x^3 - 2
```

6060. ⠈⠷ ⠠⠑⠱ ⠠⠋ ⠈⠾

$[E : F]$

```
[E:F]
```

6061. ⠋⠷⠭⠾ ⠈⠑ ⠈ ⠠⠟ ⠈⠷⠭ ⠈⠾

$f(x) \in \mathbb{Q}[x]$

```
f(x) \in{\mathbb Q}[x]
```

6062. ⠏ ⠕⠱ ⠢

$p \geq 5$

```
p \geq 5
```

6063. ⠈ ⠠⠵ ⠰⠏ ⠐⠷⠞⠾

$\mathbb{Z}_p(t)$

```
{\mathbb Z}_p(t)
```

6064. ⠋⠷⠭⠾ ⠶ ⠭ ⠘⠏ ⠐⠤⠞

$f(x) = x^p - t$

```
f(x) = x^p - t
```

6065. ⠈ ⠠⠵ ⠰⠏ ⠐⠷⠞⠾ ⠈⠷⠭ ⠈⠾

$\mathbb{Z}_p(t)[x]$

```
{\mathbb Z}_p(t)[x]
```

6066. ⠨⠎⠷⠠⠅⠾ ⠨⠅ ⠠⠇

$\sigma(K) = L$

```
\sigma(K) = L
```

6067. ⠠⠛⠷⠠⠑⠌⠠⠇⠾

$G(E/L)$

```
G(E/L)
```

6068. ⠨⠎ ⠈⠑ ⠠⠁⠥⠞⠷⠈⠠⠗⠾

$\sigma \in \mathrm{Aut}(\mathbb{R})$

```
\sigma \in \aut( {\mathbb R} )
```

6069. ⠨⠎⠷⠁⠾ ⠨⠂ ⠴

$\sigma(a) > 0$

```
\sigma( a) > 0
```

6070. ⠭⠘⠒⠐⠬⠭⠘⠆⠐⠬⠂ ⠈⠑ ⠈⠠⠵⠆⠈⠷⠭⠈⠾

$x^3 + x^2 + 1 \in \mathbb{Z}_2[x]$

```
x^3 + x^2 + 1 \in {\mathbb Z}_2[x]
```

6071. ⠉⠓⠁⠗⠷⠠⠋⠾ ⠌⠨⠅ ⠆

$\mathrm{char}(F) \neq 2$

```
\chr(F) \neq 2
```

6072. ⠋⠷⠭⠾ ⠨⠅ ⠁⠭⠘⠆⠐⠬⠃⠭⠐⠬⠉

$f(x) = ax^2 + bx + c$

```
f(x) = a x^2 + b x + c
```

6073. ⠨⠁ ⠨⠅ ⠃⠘⠆⠐⠤⠲⠁⠉

$\alpha = b^2 - 4ac$

```
\alpha = b^2 - 4ac
```

6074. ⠨⠠⠋⠰⠏⠐⠷⠭⠾ ⠨⠅ ⠹⠭⠘⠏⠐⠤⠂⠌⠭⠤⠂⠼ ⠨⠅ ⠭⠘⠏⠤⠂⠐⠬⠭⠘⠏⠤⠆⠐⠬⠄⠄⠄⠬⠭⠬⠂

$\Phi_p(x) = \frac{x^p - 1}{x - 1} = x^{p-1} + x^{p-2} + \cdots + x + 1$

```
\Phi_p(x) = \frac{x^p - 1}{x - 1} = x^{p - 1} + x^{p - 2} + \cdots
+ x + 1
```

6075. ⠨⠕⠠ ⠨⠕⠘⠆⠐⠠ ⠄⠄⠠ ⠨⠕⠘⠏⠤⠂

$\omega, \omega^2, \ldots, \omega^{p-1}$

```
\omega, \omega^2, \ldots, \omega^{p-1}
```

6076. ⠠⠛⠷⠈⠠⠟⠷⠨⠕⠾⠌⠈⠠⠟⠾

$G(\mathbb{Q}(\omega)/\mathbb{Q})$

```
G( {\mathbb Q}( \omega ) / {\mathbb Q} )
```

6077. ⠨⠕⠠ ⠨⠕⠘⠆⠐⠠ ⠄⠄⠠ ⠨⠕⠘⠏⠤⠂

$\omega, \omega^2, \ldots, \omega^{p-1}$

```
\omega, \omega^2, \ldots, \omega^{p - 1}
```

6078. ⠹⠀⠌⠭⠀⠀⠂

$\omega \neq 1$

\omega \neq 1

6079. ⠅⠠⠏⠀⠵⠏

Φ_p

\Phi_p

6080. ⠅⠠⠏⠀⠵⠏⠀⠐⠷⠀⠹⠐⠀⠑⠁⠀⠐⠾

$\Phi_p(\omega^i)$

\Phi_p(\omega^i)

6081. ⠏⠃⠀⠵⠁⠀⠐⠢⠁⠠⠯⠷⠀⠹⠐⠾⠀⠀⠫⠕⠀⠀⠈⠠⠯⠷⠀⠹⠐⠀⠑⠁⠀⠐⠾

$\phi_i : \mathbb{Q}(\omega) \rightarrow \mathbb{Q}(\omega^i)$

\phi_i: {\mathbb Q}(\omega) \rightarrow {\mathbb Q}(\omega^i)

6082. ⠏⠃⠀⠵⠁⠀⠐⠷⠁⠀⠰⠤⠣⠀⠹⠐⠤⠄⠄⠄⠤⠃⠀⠵⠏⠤⠆⠀⠐⠹⠊⠏⠤⠆⠀⠐⠾⠀⠀⠶⠀⠀⠈⠰⠤⠣⠀⠹⠑⠁⠐⠤⠄⠄⠄⠤⠉⠀⠵⠏⠤⠆⠀⠐⠷⠀⠹⠑

$\phi_i(a_0 + a_1\omega + \cdots + a_{p-2}\omega^{p-2}) = a_0 + a_1\omega^i + \cdots + c_{p-2}(\omega^i)^{p-2}$

\phi_i(a_0 + a_1 \omega + \cdots + a_{p - 2} \omega^{p - 2}) = a_0
+ a_1 \omega^i + \cdots + c_{p - 2} (\omega^i)^{p - 2}

6083. ⠁⠀⠵⠁⠐⠀⠑⠢⠀⠈⠠⠯

$a_i \in \mathbb{Q}$

a_i \in {\mathbb Q}

6084. ⠏⠉⠆

ϕ_2

\phi_2

6085. ⠠⠛⠷⠀⠈⠠⠯⠷⠀⠹⠐⠾⠌⠈⠀⠈⠠⠯⠾

$G(\mathbb{Q}(\omega)/\mathbb{Q})$

G({\mathbb Q}(\omega)/{\mathbb Q})

6086. StartSet ⠹⠐⠀⠠⠹⠐⠀⠑⠆⠀⠐⠀⠤⠤⠀⠠⠹⠐⠀⠑⠏⠤⠤⠐⠀⠐EndSet

$\{\omega, \omega^2, \ldots, \omega^{p-1}\}$

\{ \omega, \omega^2, \ldots, \omega^{p - 1} \}

6087. ⠈⠠⠯⠷⠀⠹⠐⠾

$\mathbb{Q}(\omega)$

{\mathbb Q}(\omega)

6088. ⠠⠛⠷⠀⠈⠠⠯⠷⠀⠹⠐⠾⠌⠈⠀⠈⠠⠯⠾

$G(\mathbb{Q}(\omega)/\mathbb{Q})$

G({\mathbb Q}(\omega) / {\mathbb Q})

6089. ⠅⠠⠙⠀⠀⠶⠀⠀⠐⠄⠀⠳⠭⠆⠀⠆⠀⠰⠏⠄⠀⠫⠐⠁⠀⠀⠐⠆⠀⠀⠘⠻⠷⠀⠉⠀⠵⠁⠀⠐⠤⠀⠉⠀⠵⠚⠀⠐⠾

$\Delta = \prod_{i<j}(\alpha_i - \alpha_j)$

\Delta = \prod_{i \lt j} (\alpha_i - \alpha_j)

6090. ⠨⠠⠙⠘⠆

Δ^2

```
\Delta^2
```

6091. ⠋⠷⠭⠾ ⠨⠅ ⠭⠘⠆ ⠐⠬⠃ ⠭⠬⠉

$f(x) = x^2 + bx + c$

```
f(x) = x^2 + b x + c
```

6092. ⠨⠠⠙⠘⠆⠐ ⠨⠅ ⠃⠘⠆ ⠐⠤⠲⠉

$\Delta^2 = b^2 - 4c$

```
\Delta^2 = b^2 - 4c
```

6093. ⠋⠷⠭⠾ ⠨⠅ ⠭⠘⠒⠐⠬⠏⠭⠬⠟

$f(x) = x^3 + px + q$

```
f(x) = x^3 + p x + q
```

6094. ⠨⠠⠙⠘⠆⠐ ⠨⠅ ⠤⠲⠏⠘⠒⠐⠤27⠟⠘⠆

$\Delta^2 = -4p^3 - 27q^2$

```
\Delta^2 = - 4p^3 - 27q^2
```

6095. ⠎⠷⠨⠠⠙⠾ ⠨⠅ ⠤⠨⠠⠙

$\sigma(\Delta) = -\Delta$

```
\sigma( \Delta ) = -\Delta
```

6096. ⠎⠷⠨⠠⠙⠾ ⠨⠅ ⠨⠠⠙

$\sigma(\Delta) = \Delta$

```
\sigma( \Delta ) = \Delta
```

6097. ⠨⠠⠙ ⠈⠑ ⠠⠋

$\Delta \in F$

```
\Delta \in F
```

6098. ⠭ ⠘⠒ ⠐⠬⠆ ⠭⠤⠲

$x^3 + 2x - 4$

```
x^3 + 2 x - 4
```

6099. ⠭ ⠘⠒ ⠐⠬⠭⠤⠒

$x^3 + x - 3$

```
x^3 + x -3
```

6100. ⠏⠷⠭⠾ ⠨⠅ ⠭ ⠘⠲ ⠐⠤⠆

$p(x) = x^4 - 2$

```
p(x)=x^4-2
```

6101. ⠆ ⠘one-fourth ⠐ ⠨⠅ ⠣⠲⠜⠆ ⠻

$2^{\frac{1}{4}} = \sqrt[4]{2}$

```
2^{\frac{1}{4}}=\sqrt[4]{2}
```

6102. ⠆ ⠘one-fourth ⠐⠊ ⠨⠅ ⠣⠲⠜⠆ ⠻⠊

$2^{\frac{1}{4}}i = \sqrt[4]{2}i$

```
2^{\frac{1}{4}}i=\sqrt[4]{2}i
```

6103. ⠨⠞⠷⠉⠾

$\tau(c)$

\tau(c)

6104. ⠨⠞⠷⠉⠘⠅⠐⠾ ⠨⠅ ⠷⠨⠞⠷⠉⠾⠾⠘⠅

$\tau(c^k)=(\tau(c))^k$

\tau(c^k)=(\tau(c))^k

6105. ⠣⠲⠜⠆⠻⠊

$\sqrt[4]{2}i$

\sqrt[4]{2}i

6106. ⠈⠠⠟⠷⠣⠲⠜⠆⠻⠊⠾

${\mathbb Q}(\sqrt[4]{2}i)$

{\mathbb Q}(\sqrt[4]{2}i)

6107. ⠣⠲⠜⠆⠻⠊⠤⠣⠲⠜⠆⠻ ⠨⠅ ⠷⠂⠤⠊⠾⠣⠲⠜⠆⠻

$\sqrt[4]{2}i - \sqrt[4]{2} = (1-i)\sqrt[4]{2}$

\sqrt[4]{2}i - \sqrt[4]{2} = (1-i)\sqrt[4]{2}

6108. ⠈⠠⠟⠷⠣⠲⠜⠆⠻⠊⠤⠣⠲⠜⠆⠻⠾ ⠨⠅ ⠈⠠⠟⠷⠷⠂⠤⠊⠾⠣⠲⠜⠆⠻⠾

${\mathbb Q}(\sqrt[4]{2}i - \sqrt[4]{2}) = {\mathbb Q}((1-i)\sqrt[4]{2})$

{\mathbb Q}(\sqrt[4]{2}i - \sqrt[4]{2}) = {\mathbb Q}((1-i)\sqrt[4]{2})

6109. ⠭⠘⠲⠐⠬⠦

x^4+8

x^4+8

6110. ⠷⠂⠤⠊⠾⠣⠲⠜⠆⠻

$(1-i)\sqrt[4]{2}$

(1-i)\sqrt[4]{2}

6111. ⠭⠘⠲⠐⠬⠦

$x^4 + 8$

x^4 + 8

6112. ⠈⠠⠟⠷⠜⠆⠻⠾

${\mathbb Q}(\sqrt{2})$

{\mathbb Q}(\sqrt{2})

6113. ⠤14

-14

-14

6114. ⠈⠠⠟⠷⠣⠲⠜⠆⠻⠾

${\mathbb Q}(\sqrt[4]{2})$

{\mathbb Q}(\sqrt[4]{2})

6115. ⠠⠓ ⠨⠅ ⠄⡳⠭⠆⠶⠑⠦⠄⠷⠂⠠⠲⠾⠄⡳⠭⠆⠶⠑⠔⠄

$H=\langle(1,4)\rangle$

H=\langle(1,4)\rangle

6116. ⠏⠷⠭⠾⠀⠨⠅⠀⠭⠘⠒⠐⠤⠖⠭⠘⠆⠐⠬12⠭⠤10

$p(x) = x^3 - 6x^2 + 12x - 10$

p(x)=x^3-6x^2+12x-10

6117. ⠭⠘⠢⠐⠤⠭⠤⠂

$x^5 - x - 1$

x^5-x-1

6118. ⠏⠷⠭⠾⠀⠨⠅⠀⠭⠘⠲⠐⠬⠭⠬⠂

$p(x) = x^4 + x + 1$

p(x)=x^4+x+1

6119. ⠭⠘⠖⠐⠬⠭⠘⠆⠐⠬⠆⠭⠬⠂

$x^6 + x^2 + 2x + 1$

x^6+x^2+2x+1

6120. ⠏⠷⠭⠾⠀⠨⠅⠀⠭⠘⠶⠐⠤⠶⠭⠬⠒

$p(x) = x^7 - 7x + 3$

p(x)=x^7-7x+3

6121. ⠽⠘⠆⠐⠀⠨⠅⠀⠭⠷81⠭⠘⠢⠐⠬396⠭⠘⠲⠐⠬738⠭⠘⠒⠐⠬660⠭⠘⠆⠐⠬269⠭⠬48⠾

$y^2 = x(81x^5 + 396x^4 + 738x^3 + 660x^2 + 269x + 48)$

y^2 = x(81x^5 + 396x^4 + 738x^3 + 660x^2 + 269x + 48)

6122. ⠠⠏⠠⠎⠠⠇⠷⠆⠠⠶⠾

$PSL(2,7)$

PSL(2,7)

6123. ⠠⠎⠠⠇⠷⠆⠠⠶⠾

$SL(2,7)$

SL(2,7)

6124. StartSet ⠠⠊⠆⠠⠤⠠⠊⠆ EndSet

$\{I_2, -I_2\}$

\{I_2,-I_2\}

6125. ⠼168

168

168