WISER WISER Quantum Project

Quantum Galton Board

Monte Carlo Simulation via Quantum Circuits

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August 10, 2025

WISER 2025 Quantum Program

Project Goals:

- Implement Universal Statistical Simulator
- Generate quantum circuits for Monte Carlo problems
- Explore quantum advantage in statistical sampling

Completed Tasks:

- Task 1: Literature Review & Theory
- Task 2: General Algorithm Implementation
- Task 3: Alternative Distributions

Key Innovation

Quantum Superposition

Explore all possible paths simultaneously through quantum parallelism

Applications

High-Dimensional Problems

- Particle transport
- Quantum systems
- PDE solutions

Quantum vs Classical Galton Board

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Classical Approach:

- Ball follows single path
- Sequential peg interactions
- Probability: $P(k) = \frac{1}{2^n}\binom{n}{k}$
- Gaussian distribution emerges

Quantum Approach:

- Superposition explores all paths
- Parallel computation of outcomes
- Measurement collapses to sample

Circuit Architecture

Single Peg Design:

1. Initialize ball: X gate $\rightarrow |0100\rangle$
2. Superposition: $H \rightarrow \frac{1}{\sqrt{2}}(|0100\rangle + |0101\rangle)$
3. Routing: CSWAP + CNOT operations
4. Result: $\frac{1}{\sqrt{2}}(|1001\rangle + |0011\rangle)$

Resource Requirements:

- **Qubits:** $2n + 2$
- **Gates:** $O(n^2)$ complexity
- **Depth:** Linear in n

Design Philosophy: Clean, scalable architecture with reusable components

Core Functions:

```python
def peg(qc: QuantumCircuit, qubit: int):
    """Single peg with equal left/right probability"""
    qc.cswap(0, qubit, qubit-1)   # Left deflection
    qc.cx(qubit, 0)               # Update control
    qc.cswap(0, qubit, qubit+1)   # Right deflection

def level(qc: QuantumCircuit, l: int, n: int):
    """Row of pegs with control management"""
    peg(qc, n+1-l)                # First peg
    for i in range(n+3-l, n+l+2, 2):
        qc.cx(i-1, 0)             # Reset superposition
        peg(qc, i)                # Additional pegs
```

Algorithm Flow

1. Initialization
Ball placement in quantum register

2. Level Construction
Systematic peg placement per level

3. Control Management
Reset between levels for independence

4. Measurement
Final state collapse to distribution

Task 2: Verification Results

Testing Strategy:

- Multiple configurations: 2, 4, 6, 8 levels
- Qiskit AerSimulator with 1024 shots
- Statistical comparison with theory
- Visual distribution analysis

Verification Metrics:

- Close-to-perfect Gaussian emergence
- Left-right symmetry confirmed
- Peak centered at $n/2$ position

Key Achievements:

General algorithm works for any n
Circuit compilation successful
Theoretical predictions matched
Clean modular code architecture
Comprehensive documentation

Performance

Scalability Demonstrated:

Successfully tested up to 8-level boards with excellent statistical agreement

Design Strategy:

- Add one peg per row at right
- Add systematic SWAP operations at left
- Create cumulative bias effect
- Maintain quantum superposition

```
def expo_qgb(n: int) -> QuantumCircuit:
    qc = QuantumCircuit(2*n+2, n+1)
    qc.x(n+1)  # Initialize ball
    for l in range(n):
        qc.reset(0)
        qc.h(0)  # Standard superposition
        peg(qc, n+1-l)  # First peg of level
        # Key modification: SWAP operations
        for i in range(n+3-l, n+l+2, 2):
            qc.swap(i, i+1)  # Shift probability
```

Mechanism

SWAP Effect:

Creates systematic bias toward one edge of the distribution

Result:

Exponential decay pattern emerges from quantum interference

Key Features

- Asymmetric probability distribution
- Quantum coherence maintained
- Exponential-like profile achieved

Quantum Walk Theory:

- Different from classical random walk
- Quantum interference creates non-classical behavior
- Coherent evolution across all levels
- Entanglement between position and control

```
def hadamard_walk_qgb(n: int) -> QuantumCircuit:
    qc = QuantumCircuit(2*n+2, n+1)
    qc.x(n+1)  # Initialize ball
    for l in range(n):
        qc.h(0)  # No reset - key difference!
        level(qc, l, n)  # Standard level
        # Additional entanglement
        qc.cx(n+l+2, 0)  # Position-dependent control
```

Innovation

No Control Reset:

Maintains quantum coherence between levels for interference effects

Entanglement:

Position-dependent operations create complex quantum correlations

Expected Results

- Quantum interference patterns
- Non-classical distribution shape
- Coherent quantum evolution
- Potentially faster spreading

Distribution Comparison & Results

Standard QGB

Gaussian Profile

- Symmetric bell curve
- Peak at center bins
- Variance $\propto \sqrt{n}$
- Classical binomial limit

Success: Theory matched
Shots: 1024 samples
Levels: Up to 8 tested

Exponential QGB

Decay Pattern

- Asymmetric distribution
- Higher edge probability
- Exponential-like profile
- SWAP-induced bias

Success: Non-Gaussian
Method: SWAP operations
Effect: Clear asymmetry

Hadamard Walk

Quantum Walk

- Interference effects
- Coherent evolution
- Non-classical spread
- Entangled dynamics

Success: Quantum behavior
Method: No control reset
Effect: Interference patterns

Technical Achievement: All three distributions successfully implemented with distinct quantum signatures and clear differentiation from classical expectations

Technical Excellence

Code Quality:

- Comprehensive docstrings
- Type hints throughout
- Modular, reusable design
- Systematic testing approach
- Clear variable naming

Development Stack:

- Qiskit quantum framework
- AerSimulator for noiseless simulation
- Matplotlib for visualization
- Python best practices

Performance Analysis

Resource Scaling:

- $2n + 2$ qubits required
- $O(n^2)$ gate complexity
- Linear circuit depth
- Polynomial classical simulation time

Validation Methods

Verification Strategy:

- Multiple level configurations
- Statistical convergence analysis
- Visual distribution inspection
- Theoretical comparison

Project Impact & Achievements

Completed Deliverables:

- **Task 1:** Comprehensive 2-page review
- **Task 2:** General n-level algorithm
- **Task 3:** Exponential & Hadamard walk distributions

Key Insights:

- Quantum superposition enables parallel path exploration
- Circuit complexity manageable for moderate sizes
- Clear quantum advantage in statistical sampling

Future Directions

Tasks 4 & 5 Extensions:

- Noise model implementation
- Error mitigation strategies
- Quantitative accuracy metrics
- Statistical uncertainty analysis

Research Impact

Broader Applications:

- Monte Carlo method acceleration
- High-dimensional sampling problems
- Quantum algorithm development
- Statistical simulation frameworks

Thank You

We hope you liked our project.

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Available for technical discussions and follow-up questions